Temă ALGAD

5. $S = \{(x,y,z) \mid 2x - y - z = 0,\ 4x + z = 0\} \subset \mathbb{R}^3$

Verificare subspațiu vectorial:

Fie $x = (x_1, x_2, x_3) \in S$, $2x_1 - x_2 - x_3 = 0$, $4x_1 + x_3 = 0$.

$y = (y_1, y_2, y_3) \in S$, $2y_1 - y_2 - y_3 = 0$, $4y_1 + y_3 = 0$.

? $x + y = (x_1 + y_1, x_2 + y_2, x_3 + y_3) \in S$

$$2(x_1+y_1) - (x_2+y_2) - (x_3+y_3) = 0.$$
$$2x_1 + 2y_1 - x_2 - y_2 - x_3 - y_3 = 0.$$
$$\underbrace{2x_1 - x_2 - x_3}_{0} + \underbrace{2y_1 - y_2 - y_3}_{0} = 0 \quad (A) \quad (1)$$
$$4(x_1+y_1) + (x_3+y_3) = 0.$$
$$4x_1 + 4y_1 + x_3 + y_3 = 0.$$
$$\underbrace{4x_1 + x_3}_{0} + \underbrace{4y + y_3}_{0} = 0. \quad (A) \quad (2)$$

$(1), (2) \Rightarrow x + y \in S$ (5)

? $\alpha \cdot x \in S$

$$x = (x_1, x_2, x_3)$$
$$\alpha x = (\alpha x_1, \alpha x_2, \alpha x_3)$$
$$2\alpha x_1 - \alpha x_2 - \alpha x_3 = 0$$
$$\alpha(\underbrace{2x_1 - x_2 - x_3}_{0}) = 0 \quad (A) \quad (3)$$
$$4\alpha x_1 + \alpha x_3 = 0$$
$$\alpha(\underbrace{4x_1 + x_3}_{A}) = 0 \quad (4)$$

$(3), (4) \Rightarrow \alpha \cdot x \in S$ (6)

$(5), (6) \Rightarrow S$ subspațiu vectorial

Bază:

$v = (x, y, z)$

$$\begin{cases} 2x - y - z = 0 \\ 4x + z = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ 4 & 0 & 1 & 0 \end{array}\right)$$

$\Delta_2 = \begin{vmatrix} -1 & -1 \\ 0 & 1 \end{vmatrix} = -1 \neq 0 \Rightarrow$ sist. compatibil simplu nedet.

y, z - necunoscute principale

x - necunoscută secundară

Notăm $x = \alpha$.

$$\begin{cases} -y - z = -2\alpha \\ z = -4\alpha \end{cases}$$

$-y + 4\alpha = -2\alpha$

$y = 6\alpha$

$v = (\alpha, 6\alpha, -4\alpha) = \alpha(1, 6, -4)$

$v = \alpha \cdot e_1'$, $e_1' = (1, 6, -4)$

$e_1' \to$ sistem de generatori

Studiem liniar independența:

$A = \begin{pmatrix} 1 \\ 6 \\ -4 \end{pmatrix}$ $\det A = 1 =$ nr. vectori Li $\Rightarrow \{e_1'\}$ liniar independ.

$B' = \{e_1'\}$

$\dim S = \operatorname{card} B' = 1.$

6. $U = \{(x_1, x_2, x_3) | x_1 + x_2 - x_3 = 0\}$
$V = \{(x_1, x_2, x_3) | x_1 - 2x_3 = 0\}$

$v \in U \Rightarrow v = (x_1, x_2, x_3)$

$x_1 + x_2 - x_3 = 0.$

$(\underline{1} \;\; 1 \;\; -1 \;\vdots\; 0)$

$\Delta = 1 \Rightarrow$ sist. compatibil dublu nedet.

x_1 - nec. principala

$x_2 = \alpha$

$x_3 = \beta$

$\Rightarrow x_1 = -\alpha + \beta$

$v = (-\alpha + \beta, \alpha, \beta) = (-\alpha, \alpha, 0) + (\beta, 0, \beta)$
$= \alpha(-1, 1, 0) + \beta(1, 0, 1)$

$v = \alpha \cdot e_1' + \beta \cdot e_2'$ $\quad e_1' = (-1, 1, 0)$
$e_2' = (1, 0, 1)$

Studiem liniar independenta:

$A = \begin{pmatrix} -1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$ rang $A = 2 \Rightarrow e_1', e_2'$ liniar independenți

$B' = \{e_1', e_2'\}$ bază în U.
$\dim U = 2$.

$u \in V \Rightarrow u = (x_1, x_2, x_3)$

$x_1 - 2x_3 = 0.$

$x_1 = 2x_3$

$u = (2x_3, x_2, x_3) = x_3(2, 0, 1) + x_2(0, 1, 0)$

$v = x_3 \cdot e_3' + x_2 \cdot e_4'$ $\quad e_3' = (2, 0, 1)$
$e_4' = (0, 1, 0)$

Studiem li:

$A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$

rang $A = 2$.
$\Rightarrow e_3', e_4'$ Li
$B = \{e_3', e_4'\}$

$U+V$:

$U = L\{e_1', e_2'\}$

$V = L\{e_3', e_4'\}$

$U+V = \{\alpha_1 v_1 + \alpha_2 v_2 \mid \alpha_1, \alpha_2 \in \mathbb{R}, v_1 \in U, v_2 \in V\}$

$U+V = L\{e_1', e_2', e_3', e_4'\}$

Studiem Li:

$A = \begin{pmatrix} -1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \qquad \Delta_3 = \begin{vmatrix} -1 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 1 \neq 0$

rang $A = 3 \Rightarrow \{e_1', e_2', e_3'\}$ Li

$B = \{e_1', e_2', e_3'\}$ bază în $U+V$

$\dim(U+V) = 3$

$U \cap V$:

$U = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 + x_2 - x_3 = 0\}$

$V = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 - 2x_3 = 0\}$

$U \cap V = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 + x_2 - x_3 = 0, x_1 - 2x_3 = 0\}$

$\begin{cases} x_1 + x_2 - x_3 = 0 \\ x_1 - 2x_3 = 0 \end{cases}$

$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & 0 & -2 & 0 \end{array}\right)$

x_1, x_2 - necunoscute principale

$x_3 = \alpha$

$\Rightarrow x_1 = 2\alpha$

$x_2 = -\alpha$

$v \in U \cap V \Rightarrow v = (2\alpha, -\alpha, \alpha) = \alpha(2, -1, 1) = \alpha \cdot e_1'$

e_1' - liniar independent $\Rightarrow B = \{e_1'\}$ bază în $U \cap V$

Teorema Grassman:

$$\dim(U+V) + \dim(U\cap V) = \dim U + \dim V$$

$$3+1 = 2+2 \Leftrightarrow 4=4 \ (A).$$

8. $B' = \{e_1' = (1,1,0),\ e_2' = (1,0,1),\ e_3' = (1,0,-1)\}$

$B'' = \{e_1'' = (1,0,0),\ e_2'' = (1,1,0),\ e_3' = (1,1,1)\}$

Pentru B': card $B' = 3$ (1)

Studiem liniar independența:

$$A' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

$\det A' = 1+1 = 2 \neq 0$

rang $A' = 3 \Rightarrow B'$ sistem liniar independent (2)

(1), (2) $\Rightarrow B'$ bază a lui $\mathbb{R}^3$

Pentru B'' card $B'' = 3$ (3)

Studiem liniar independența:

$$A'' = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$\det A'' = 1 \neq 0$

rang $A'' = 3 \Rightarrow B''$ sistem liniar independent (4)

(3), (4) $\Rightarrow B''$ bază a lui $\mathbb{R}^3$

Matricea de trecere de la baza B' la baza $B'' = C$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right) \underset{L_2 \to L_2 - L_1}{\sim} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right) \underset{L_2 \to (-1)L_2}{\sim}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right) \underset{L_3 \to L_3 - L_2}{\overset{L_1 \to L_1 - L_2}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right)$$

$$\underset{L_3 \to -\frac{1}{2}L_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 & 0 & -1/2 \end{array}\right) \underset{L_2 \to L_2 - L_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 & 0 & -1/2 \end{array}\right)$$

$$\underbrace{}_{I_3} \qquad \underbrace{}_{C}$$

Matricea de trecere de la B' la B''

$$C = \begin{pmatrix} 0 & 1 & 1 \\ 1/2 & 0 & 1/2 \\ 1/2 & 0 & -1/2 \end{pmatrix}$$

Coordonatele lui $v = (2,-1,1)$ în B':

$$\alpha e_1' + \beta e_2' + \gamma e_3' = v$$

$$\alpha(1,1,0) + \beta(1,0,1) + \gamma(1,0,-1) = (2,-1,1)$$

$$\begin{cases} \alpha + \beta + \gamma = 2. \\ \alpha = -1 \\ \beta - \gamma = 1. \end{cases}$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$\det A = 2.$

$\alpha = -1$

$$\Delta_\beta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = 1 + 1 + 2 = 4 \qquad \beta = 2$$

$$\Delta_\gamma = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 2 + 1 - 1 = 2 \qquad \gamma = 1.$$

$v = (-1, 2, 1)_{B'}$

Coordonatele lui $v=(2,-1,1)$ în B''

$\alpha e_1'' + \beta e_2'' + \gamma e_3'' = v$.

$\alpha(1,0,0) + \beta(1,1,0) + \gamma(1,1,1) = (2,-1,1)$

$$\begin{cases} \alpha+\beta+\gamma = 2 \Rightarrow \alpha = 3 \\ \beta+\gamma = -1 \Rightarrow \beta = -2 \\ \gamma = 1 \end{cases}$$

$v = (3,-2,1)_{B''}$

9. $\langle f,g\rangle = \int_0^1 x\cdot f(x)\cdot g(x)\,dx$, $f,g \in C^0[0;1]$

$B = \{v_1 = 1,\ v_2 = e^x,\ v_3 = e^{-x}\}$

$\langle\cdot,\cdot\rangle$ - produs scalar dacă:

(1) $\langle f,g\rangle = \langle g,f\rangle$, $\forall f,g \in C^0[0;1]$

$\int_0^1 x\cdot f(x)\cdot g(x)\,dx = \int_0^1 x\cdot g(x)\cdot f(x)\,dx$ (A)

(2) $\langle \alpha f,g\rangle = \alpha\langle f,g\rangle$

$\int_0^1 \alpha\cdot x\cdot f(x)\cdot g(x)\,dx = \alpha\int_0^1 x\cdot f(x)\cdot g(x)\,dx$ (A)

(3) $\langle f+f',g\rangle = \langle f,g\rangle + \langle f',g\rangle$

$\int_0^1 x\cdot (f(x)+f'(x))\cdot g(x)\,dx =$

$= \int_0^1 x\cdot f(x)\cdot g(x)\,dx + \int_0^1 x\cdot f'(x)\cdot g(x)\,dx$ (A)

(4) $\langle f,f\rangle \geq 0$, $\forall f \in C^0[0;1]$

Egalitatea are loc $\Leftrightarrow f=0$.

$\langle f,f\rangle \geq 0 \Leftrightarrow \int_0^1 x\cdot f(x)\cdot f(x)\,dx \geq 0$ (A) $x\in[0;1]$

$\left.\begin{array}{l}\langle f,f\rangle = 0 \Leftrightarrow \int_0^1 x\cdot f(x)\cdot f(x) = 0 \\ f\text{-continuă}\end{array}\right| \Rightarrow f(x)=0,\ \forall x\in[0;1]$

Ortonormare:

$f_1 = v_1 = 1.$

$f_2 = v_2 - \frac{\langle v_2, f_1 \rangle}{\|f_1\|^2} \cdot f_1$

$\langle v_2, f_1 \rangle = \int_0^1 x \cdot v_2(x) \cdot f_1(x)\, dx$

$= \int_0^1 x \cdot e^x\, dx$

$f = x \Rightarrow f' = 1.$

$g' = e^x \Rightarrow g = e^x$

$\langle v_2, f_1 \rangle = x \cdot e^x \Big|_0^1 - \int_0^1 e^x\, dx$

$= 1 \cdot e - e^x \Big|_0^1 = e - e + 1 = 1.$

$\|f_1\|^2 = \langle f_1, f_1 \rangle = \int_0^1 x \cdot f_1(x) \cdot f_1(x)\, dx$

$= \int_0^1 x\, dx = \frac{x^2}{2} \Big|_0^1 = \frac{1}{2}.$

$f_2 = e^x - \frac{1}{1/2} \cdot 1 = e^x - 2.$

$f_3 = v_3 - \frac{\langle v_3, f_1 \rangle}{\|f_1\|^2} \cdot f_1 - \frac{\langle v_3, f_2 \rangle}{\|f_2\|^2} \cdot f_2.$

$\langle v_3, f_1 \rangle = \int_0^1 x \cdot v_3(x) \cdot f_1(x)\, dx$

$= \int_0^1 x \cdot e^{-x}\, dx$

$f = x \Rightarrow f' = 1$

$g' = e^{-x} \Rightarrow g = -e^{-x}$

$\langle v_3, f_1 \rangle = -x \cdot e^{-x} \Big|_0^1 + \int_0^1 e^{-x}\, dx$

$= -e^{-1} - e^{-x} \Big|_0^1$

$= -\frac{1}{e} - \frac{1}{e} + 1 = -\frac{2}{e} + 1$

$$\langle v_3, f_2 \rangle = \int_0^1 x \cdot v_3(x) \cdot f_2(x)\,dx$$

$$= \int_0^1 x \cdot e^{-x} \cdot (e^x - 2)\,dx$$

$$= \int_0^1 x \cdot e^{-x} \cdot e^x\,dx - 2\int_0^1 x \cdot e^{-x}\,dx$$

$$= \int_0^1 x\,dx - 2\int_0^1 x \cdot e^{-x}\,dx$$

$$= \frac{1}{2} - 2 \cdot \left(-\frac{2}{e} + 1\right)$$

$$= \frac{1}{2} + \frac{4}{e} - 2 = \frac{4}{e} - \frac{3}{2}$$

$$\|f_2\|^2 = \langle f_2, f_2 \rangle = \int_0^1 x \cdot (e^x - 2)^2\,dx$$

$$= \int_0^1 x \cdot (e^{2x} - 4e^x + 4)\,dx$$

$$= \underbrace{\int_0^1 x \cdot e^{2x}\,dx}_{I_1} - \underbrace{4\int_0^1 x \cdot e^x}_{I_2} + \underbrace{4\int_0^1 x\,dx}_{I_3}$$

$$I_1 = \int_0^1 x \cdot e^{2x}\,dx$$

$$f = x \Rightarrow f' = 1$$

$$g' = e^{2x} \Rightarrow g = \frac{1}{2}e^{2x}$$

$$I_1 = \frac{1}{2} \cdot x \cdot e^{2x}\Big|_0^1 - \frac{1}{2}\int_0^1 e^{2x}\,dx$$

$$= \frac{1}{2} \cdot e^2 - \frac{1}{4}e^{2x}\Big|_0^1$$

$$= \frac{1}{2}e^2 - \frac{1}{4} \cdot e^2 + \frac{1}{4} = \frac{e^2+1}{4}$$

$$I_2 = 4$$

$$I_3 = 4 \cdot \frac{x^2}{2}\Big|_0^1 = 2$$

$$\|f_2\|^2 = \frac{e^2+1}{4} - 4 + 2 = \frac{e^2+1}{4} - 2 = \frac{e^2-7}{4}$$

$$f_3 = e^{-x} - \frac{-\frac{2}{e}+1}{\frac{1}{2}} - \frac{\frac{4}{e}-\frac{3}{2}}{\frac{e^2-7}{4}}\cdot(e^x-2)$$

$$= e^{-x} - 2\left(-\frac{2}{e}+1\right) - 4\cdot\frac{\frac{4}{e}-\frac{3}{2}}{e^2-7}(e^x-2)$$

$$= e^{-x} + \frac{4}{e} - 2 - \frac{\cancel{4}^{\,2}(8-3e)}{\cancel{2}e(e^2-7)}\cdot(e^x-2)$$

$$= e^{-x} + \frac{4}{e} - 2 - \frac{16-6e}{e(e^2-7)}\cdot(e^x-2)$$

$$= \frac{e^{-x+1}+4-2e}{e} - \frac{(16-6e)\cdot(e^x-2)}{e(e^2-7)}$$

$$= \frac{(e^{-x+1}+4-2e)(e^2-7)-(16-6e)(e^x-2)}{e(e^2-7)}$$

$$= \frac{e^{-x+3} - 7\cdot e^{-x+1} + 4e^2 - 28 - 2e^3 - 14e - 16e^x + 32 + 6e^{x+1} - 12e}{e(e^2-7)}$$

10. $\{v_1 = (1,1,0),\ v_2 = (1,0,1),\ v_3 = (0,0,-1)\} \subset \mathbb{R}^3$

$f_1 = v_1 = (1,1,0)$

$$f_2 = v_2 - \frac{\langle v_2, f_1\rangle}{\|f_1\|^2}\cdot f_1$$

$\langle v_2, f_1\rangle = \langle (1,1,0),(1,0,1)\rangle = 1$

$\|f_1\|^2 = \langle f_1, f_1\rangle = 2$

$\|f_1\| = \sqrt{2}$

$$f_2 = (1,0,1) - \frac{1}{2}\cdot(1,1,0)$$

$= (1,0,1) - (1/2, 1/2, 0)$

$= (1/2, -1/2, 1)$

$$f_3 = v_3 - \frac{\langle v_3, f_1\rangle}{\|f_1\|^2}\cdot f_1 - \frac{\langle v_3, f_2\rangle}{\|f_2\|^2}\cdot f_2$$

$\langle v_3, f_1\rangle = \langle (0,0,-1), (1,1,0)\rangle = 0.$

$\langle v_3, f_2\rangle = \langle (0,0,-1), (1/2, -1/2, 1)\rangle = -1.$

$\|f_2\|^2 = \langle f_2, f_2\rangle = \frac{1}{4} + \frac{1}{4} + 1 = \frac{3}{2}$

$$f_3 = (0,0,-1) - \frac{1}{\frac{3}{2}}\cdot(1/2, -1/2, 1)$$

$$= (0,0,-1) - \frac{2}{3}(1/2, -1/2, 1)$$

$$= (0,0,-1) - (1/3, -1/3, 2/3)$$

$$= (-1/3, 1/3, -5/3)$$

$$e_1 = \frac{1}{\|f_1\|}\cdot f_1 = \frac{1}{\sqrt{2}}\cdot(1,1,0)$$

$$e_2 = \frac{1}{\|f_2\|}\cdot f_2 = \sqrt{\frac{2}{3}}\cdot(1/2, -1/2, 1)$$

$$e_3 = \frac{1}{\|f_3\|}\cdot f_3 = \frac{1}{\sqrt{3}}(-1/3, 1/3, -5/3)$$

$$\|f_3\|^2 = \langle f_3, f_3\rangle = \frac{1}{9} + \frac{1}{9} + \frac{25}{9} = \frac{27}{9} = 3$$

$B' = \{e_1, e_2, e_3\}$ bază ortonormată

11. $S = \{(x_1, x_2, x_3) \mid x_1 - x_2 - x_3 = 0,\ x_1 + 2x_2 - x_3 = 0\}$

$v = (x_1, x_2, x_3) \in S$

$$\begin{cases} x_1 - x_2 - x_3 = 0 \\ x_1 + 2x_2 - x_3 = 0 \end{cases}$$

$$\left(\begin{array}{ccc:c} 1 & -1 & -1 & 0 \\ \underline{1} & \underline{2} & -1 & 0 \end{array}\right)$$

x_1, x_2 - necunoscute principale

x_3 - necunoscută secundară

Notăm $x_3 = \alpha$

$$\begin{cases} x_1 - x_2 = \alpha \Rightarrow x_1 = \alpha + x_2 \\ x_1 + 2x_2 = \alpha \end{cases}$$

$\alpha + x_2 + 2x_2 = \alpha.$

$3x_2 = 0 \Rightarrow x_2 = 0.$

$x_1 = \alpha.$

$v = (\alpha, 0, \alpha) = \alpha(1, 0, 1)$

$v = \alpha \cdot e_1'$, $e_1' = (1, 0, 1)$

$S = L\{e_1'\}$

$y \perp S \Rightarrow y \perp e_1' \Rightarrow \langle y, e_1' \rangle = 0.$

Fie $y = (y_1, y_2, y_3)$

$\langle y, e_1' \rangle = \langle (y_1, y_2, y_3), (1, 0, 1) \rangle = y_1 + y_3$

$y_1 + y_3 = 0. \Rightarrow y_3 = -y_1$

$y = (y_1, y_2, -y_1) = y_1(1, 0, -1) + y_2(0, 1, 0)$

$y = y_1 \cdot e_2' + y_2 \cdot e_2$, $e_2' = (1, 0, -1)$ $e_2 = (0, 1, 0)$

$S^\perp = L\{e_2', e_2\} = \{(x_1, x_2, x_3) \mid x_1 + x_3 = 0\}$

12. $S_1 = \{ v_1 = (1,2,3), v_2 = (2,-1,0) \}$ $\quad v = (1,1,1)$

$$A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{pmatrix} \quad D_2 = \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} = -6.$$

rang $A = 2 =$ nr. vectori Li $\Rightarrow \{v_1, v_2\}$ liniar independenți

$B = \{v_1, v_2\}$ bază în S_1.

$f_1 = v_1 = (1,2,3)$

$$f_2 = v_2 - \frac{\langle v_2, f_1 \rangle}{\|f_1\|^2} \cdot f_1.$$

$\langle v_1, f_1 \rangle = \langle (2,-1,0), (1,2,3) \rangle = 0$

$\|f_1\|^2 = \langle f_1, f_1 \rangle = 14 \Rightarrow \|f_1\| = \sqrt{14}.$

$f_2 = (2,-1,0)$

$\|f_2\|^2 = \langle f_2, f_2 \rangle = \langle (2,-1,0), (2,-1,0) \rangle = 5.$

$\|f_2\| = \sqrt{5}.$

$$e_1' = \frac{1}{\|f_1\|} \cdot f_1 = \frac{1}{\sqrt{14}} \cdot (1,2,3)$$

$$e_2' = \frac{1}{\|f_2\|} \cdot f_2 = \frac{1}{\sqrt{5}} \cdot (2,-1,0)$$

$B' = \{e_1', e_2'\}$ bază ortonormată

$w = pr_S v = \langle v, e_1' \rangle e_1' + \langle v, e_2' \rangle e_2'$

$$\langle v, e_1' \rangle = \langle (1,1,1), \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right) \rangle = \frac{6}{\sqrt{14}}$$

$$\langle v, e_2' \rangle = \langle (1,1,1), \left(\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}, 0\right) \rangle = \frac{1}{\sqrt{5}}$$

$$w = \frac{6}{\sqrt{14}} \cdot \frac{1}{\sqrt{14}} \cdot (1,2,3) + \frac{1}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} \cdot (2,-1,0)$$

$$= \frac{6}{14}(1,2,3) + \frac{1}{5}(2,-1,0)$$

$$= {}^{5/}\frac{3}{7}(1,2,3) + {}^{7\backslash}\frac{1}{5}(2,-1,0)$$

$$= \left(\frac{15}{35}, \frac{30}{35}, \frac{45}{35}\right) + \left(\frac{14}{35}, \frac{-7}{35}, 0\right)$$

$$= \left(\frac{29}{35}, \frac{23}{35}, \frac{45}{35}\right)$$