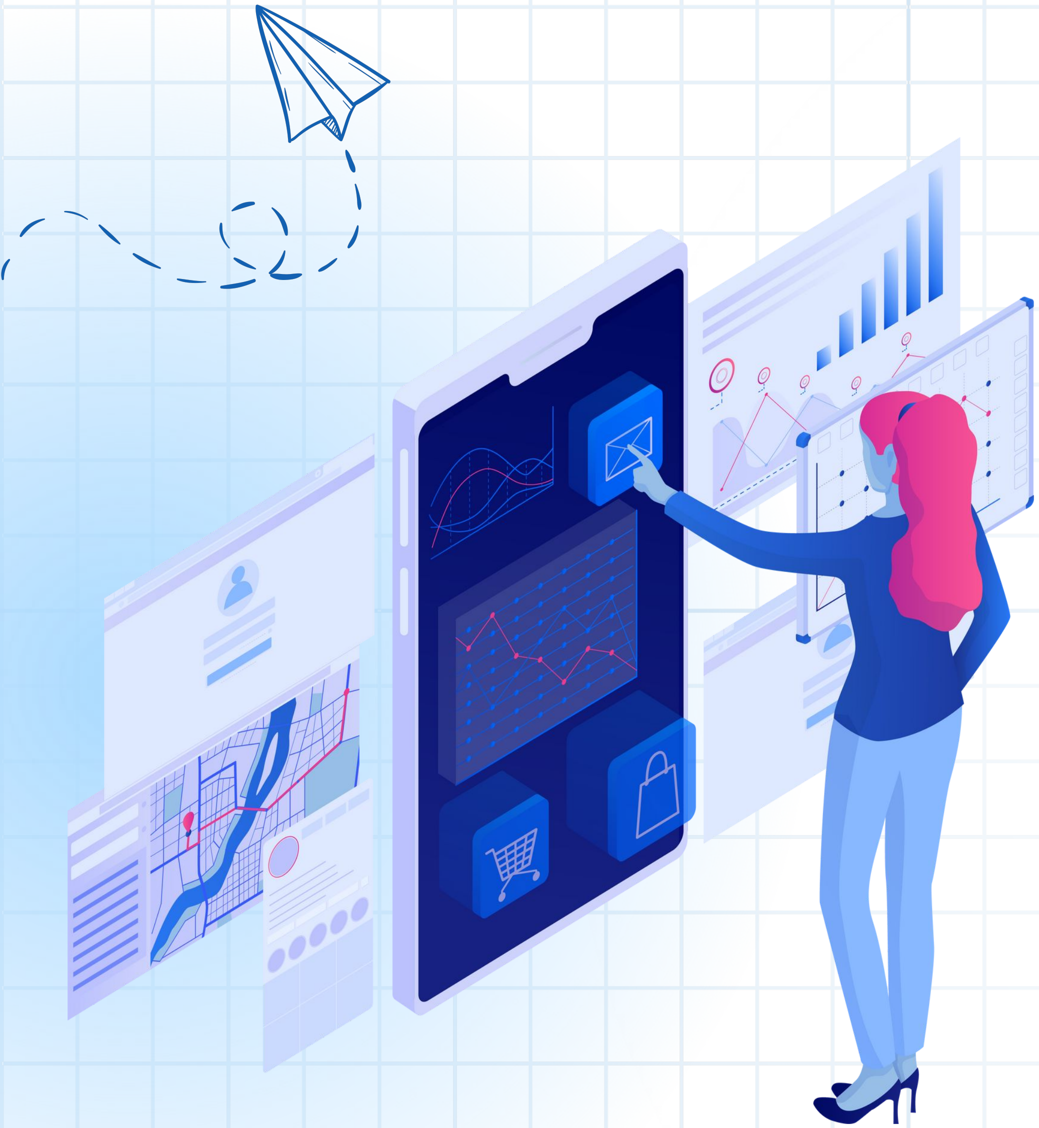
BS./BSc.

in Applied AI and Data Science

Linear algebra and numerical analysis

Module 02: Matrix Algebra



1. Matrix Algebra - Addition and Subtraction
2. Matrix Algebra - Scalar Multiplication
3. Matrix Algebra - Matrix Multiplication
4. Transpose of a Matrix

Learning Objectives

Basics operations with Matrices

Understand the basics of Matrix Algebra

Everyday Examples

Recognize application of Matrix Algebra in everyday life

codes

Python code and syntax to solve Matrix Algebra

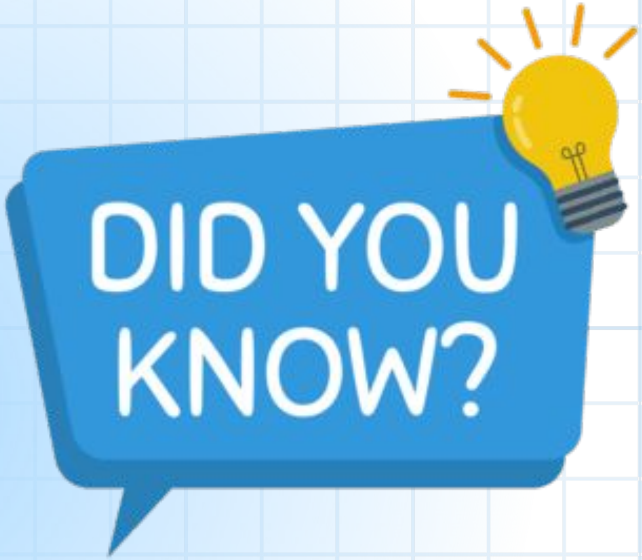
Hands on

Gain hands-on experience in solving simple Matrix Algebra problems

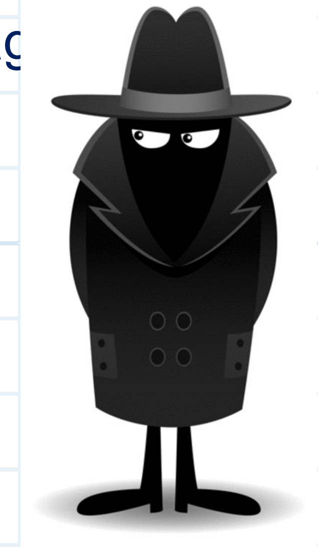
Powered by Futurense

Welcome to Real-world examples of Matrix operations

How can we use matrices to encode and decode messages ?



cryptogram is a message written according to a secret code. (The Greek word *kryptos* means "hidden.") Matrix multiplication can be used to encode and decode messag



Matrix operations in daily life

“Matrix Algebra Is Everywhere! 🌍” Please see

Examples below:

- **Image processing:** All images can be represented in matrix format
- **Computer Graphics:** In computer graphics every element is represented by a matrix.
- **Digital world:** Audio, video and image compression, including MP3, JPEG and MPEG video.

Recap: Basic Matrix notation

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.
Math	2	3	2	4	1	4	2
History	0	3	1	4	3	2	2
Language	4	1	3	1	0	0	2

If we suppress the headings, then we are left with the following rectangular array of numbers with three rows and seven columns, called a "matrix":

$$\begin{bmatrix} 2 & 3 & 2 & 4 & 1 & 4 & 2 \\ 0 & 3 & 1 & 4 & 3 & 2 & 2 \\ 4 & 1 & 3 & 1 & 0 & 0 & 2 \end{bmatrix}$$

Key Matrix Properties

- If $\mathbf{A} = \mathbf{B}$, then $\mathbf{B} = \mathbf{A}$ for all $\mathbf{A}$ and $\mathbf{B}$
- If $\mathbf{A} = \mathbf{B}$, and $\mathbf{B} = \mathbf{C}$, then $\mathbf{A} = \mathbf{C}$ for all $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

If $\mathbf{A} = \mathbf{B}$ then $a_{ij} = b_{ij}$

Matrix addition and subtraction

The sum or difference of two matrices, **A** and **B** of the same size yields a matrix **C** of the same size

$$c_{ij} = a_{ij} + b_{ij}$$

Matrices of different sizes cannot be added or subtracted

Real world application of Matrix addition and subtraction

For example, if matrix J represents the number of life insurance policies, car insurance policies, and homeowner's insurance policies sold by eight different agents in January, and matrix F represents the number of polices sold by those same agents in February, then J + F represents the total number of polices for each agent.

$$J + F = \begin{array}{c} \begin{array}{ccc} LI & CI & HI \end{array} \\ \begin{bmatrix} 12 & 23 & 14 \\ 20 & 29 & 38 \\ 3 & 6 & 10 \\ 15 & 12 & 2 \\ 90 & 5 & 16 \\ 40 & 40 & 40 \\ 0 & 0 & 83 \\ 16 & 26 & 39 \end{bmatrix} \end{array} + \begin{array}{c} \begin{array}{ccc} LI & CI & HI \end{array} \\ \begin{bmatrix} 13 & 33 & 22 \\ 10 & 19 & 8 \\ 30 & 0 & 20 \\ 0 & 0 & 0 \\ 60 & 15 & 26 \\ 30 & 40 & 50 \\ 0 & 0 & 69 \\ 12 & 48 & 11 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} LI & CI & HI \end{array} \\ \begin{bmatrix} 25 & 56 & 36 \\ 30 & 48 & 46 \\ 33 & 6 & 30 \\ 15 & 12 & 2 \\ 150 & 20 & 42 \\ 70 & 80 & 90 \\ 0 & 0 & 152 \\ 28 & 74 & 50 \end{bmatrix} \end{array}$$

More example of matrix operations

$$\begin{bmatrix} 1 & 2 & 5 \\ 9 & 8 & 7 \end{bmatrix} + \begin{bmatrix} -1 & 0 & -5 \\ 3 & a & \pi \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 12 & 8+a & 7+\pi \end{bmatrix}$$

Needless to say, matrix subtraction works exactly the same way, except with a minus sign instead of a plus sign.

Matrix addition is commutative

Like vector addition, matrix addition is commutative, meaning that

$$\mathbf{C} = \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

Example : For matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$$A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$\Rightarrow A + B = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

$$B + A = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow B + A = \begin{bmatrix} (5+1) & (6+2) \\ (7+3) & (8+4) \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

Question

"How to add matrices using Python"

Complete Workflow:

import numpy **as** np

 # create random matrices

- A **=** np**.**random**.**randn(5,4)
- B **=** np**.**random**.**randn(5,3)
- C **=** np**.**random**.**randn(5,4)

 # try to add them

- A**+**B
- A**+**C

Answer

"How to add matrices using Python"

Complete Workflow:

import numpy **as** np

 # create random matrices

- A = np.random.randn(5,4)
- B = np.random.randn(5,3)
- C = np.random.randn(5,4)

 # try to add them

- A+B
- A+C

```
In [3]: A= np.random.randn(5,4)
Out[3]: A=
array([[-0.78744152, -1.46661041,  2.34016363,  1.56843125],
       [-0.62755523, -0.02645865,  1.35160825,  0.59611772],
       [-0.05722104, -0.14407712,  0.30686938, -0.23179142],
       [-0.25340969, -0.29993093,  2.24714325,  0.57879988],
       [ 1.21139259,  0.91730765, -0.55045054, -0.03369767]])
```

Try yourself to get matrix B and then try to add A+B.

Recap

- Converting data in terms of Matrix representations allows us to solve numerous real-world problems.
- Matrix addition and subtractions are fundamental operations dealing with matrix algebra.
- Scalar multiplication and shifting of matrices are cornerstones of numerically solving matrix equations.
- Python coding of matrix operations using NumPy arrays provides a convenient way to approach solutions.

Coming up next................

Matrix Algebra: Scalar Matrix Multiplication, Shifting of Matrices and Matrix Multiplication