AIL1020 Foundations of Statistics & Probability

Module 03

Introduction to Probability

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Learning Outcomes

By the end of this video, the students will be able to

define probability, identify sample spaces and events, and calculate probabilities in real-world scenarios

distinguish between permutations and combinations and solve problems involving arrangements and selections.

Probability

Probability measures how likely an event is to occur,
ranging from **0 (impossible)** to **1 (certain).**

***P*(Event)** = (Number of favorable outcomes) / (Total number of outcomes)

Example: Tossing a fair coin

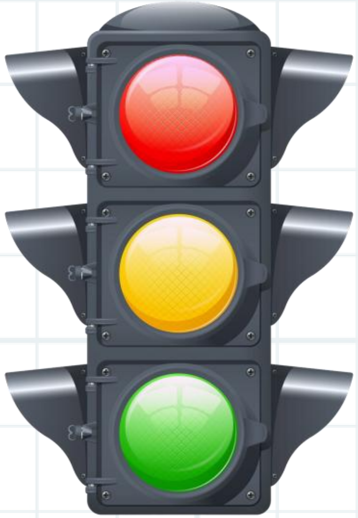
Sample Space: {Heads, Tails}

***P*(Head) = 1/2**

Probability is at the core of decision-making under uncertainty: from choosing the right treatment in healthcare to predicting rain before a trek.

Probability

Imagine reaching a traffic light with an equal chance of showing **Red, Yellow, or Green.**



***P*(Event)** = (Number of favorable outcomes) / (Total number of outcomes)

Sample Space: {Red, Green, Yellow}

***P*(Red) = *P*(Yellow) = *P(Green*) = 1/3**

***P*(Not Red) = 1 – *P*(Red)**

Simple models like this help optimize travel routes using probability-based simulations.

Probability

Sample Space and Events

A **sample space** (S) is the set of all possible outcomes.

An **event** (E) is a subset of S.

Example: Rolling a die

- $S = \{1, 2, 3, 4, 5, 6\}$
- E = Even numbers = $\{2, 4, 6\}$
- $P(E) = 3/6 = 1/2$

Types of Events

Independent Events: Events that do not affect each other.
Example: Tossing two coins.

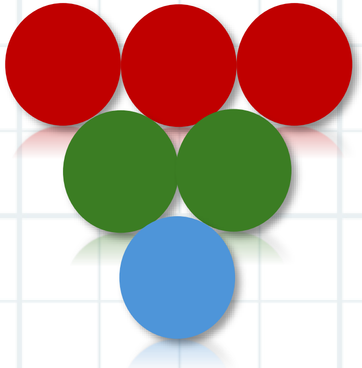
Dependent Events: The outcome of one affects the other.
Example: Drawing two cards without replacement.

Mutually Exclusive Events: Cannot happen at the same time.
Example: Drawing a red or black card from a single deck.

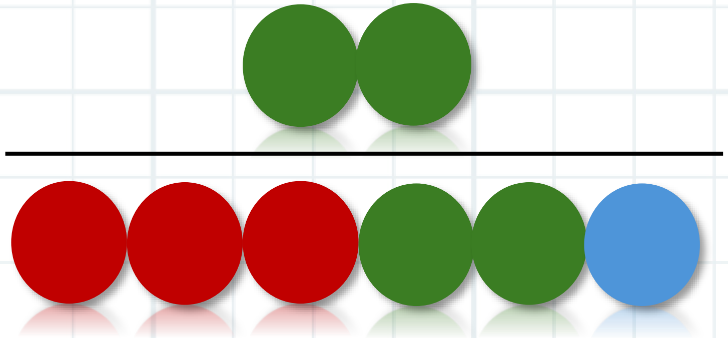
Exhaustive Events: Cover the entire sample space.
Example: Rolling any number from 1–6 on a die.

Quick Recap of Probability-based Problems

A bag contains 3 red, 2 green, and 1 blue marble.



$P(\text{Green})$ =



$P(\text{Green}) = 2/6 = \mathbf{1/3}$

Product quality control uses probability to predict defect rates across mixed batches.

The Principle of Counting

If event A has *m* outcomes and B has *n* outcomes: **Total outcomes = *m* × *n***

Example: 3 shirts × 2 pants = 6 outfits

Permutations (Order matters!)

Permutations refer to the number of ways to arrange a set of items **when the order of arrangement is important**.

$$P(n, r) = \frac{n!}{(n-r)!}$$

n is the total number of items

r is the number of items to arrange

***Example*:** There are 5 students in a class, and we want to assign first, second, and third positions for a competition.

Since positions (1st, 2nd, 3rd) matter, this is a permutation.

$$P(5,3) = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{120}{2} = 60$$

There are **60** different ways to assign the three ranks.

Combination (Order doesn't matter)

Combinations count the number of ways to choose items **without considering order.**

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

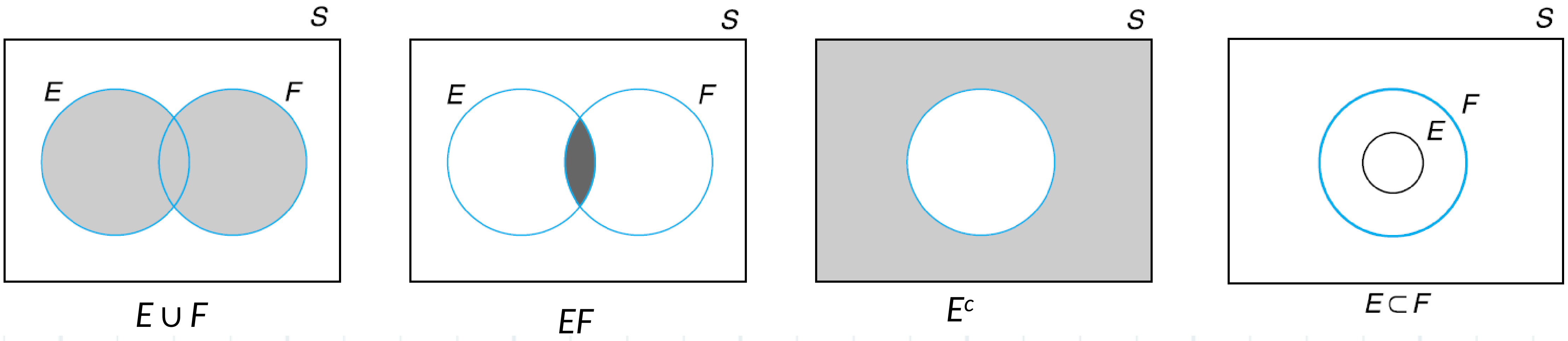
n is the total number of items

r is the number of items to arrange

***Example*:** There are 5 students in a class, and we want to select any three to form a team.

$$C(5,3) = \frac{5!}{3!2!} = \frac{120}{6\times 2} = 10$$

Venn Diagram Representation



Commutative law	$E \cup F = F \cup E$	$EF = FE$
Associative law	$(E \cup F) \cup G = E \cup (F \cup G)$	$(EF)G = E(FG)$
Distributive law	$(E \cup F)G = EG \cup FG$	$EF \cup G = (E \cup G)(F \cup G)$

Ross, Sheldon M. Introduction to Probability and Statistics for Engineers and Scientists. 6th ed., Academic Press, 2021.

Axioms of Probability

Axiom 1.

$$0 \leq P(E) \leq 1$$

Axiom 2.

$$P(S) = 1$$

Axiom 3. For any sequence of mutually exclusive events $E_1, E_2, \ldots$ (that is, events for which $E_i E_j = \emptyset$ when $i \neq j$),

$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i), \qquad n = 1, 2, \ldots, \infty$$

We call $P(E)$ the probability of the event E.

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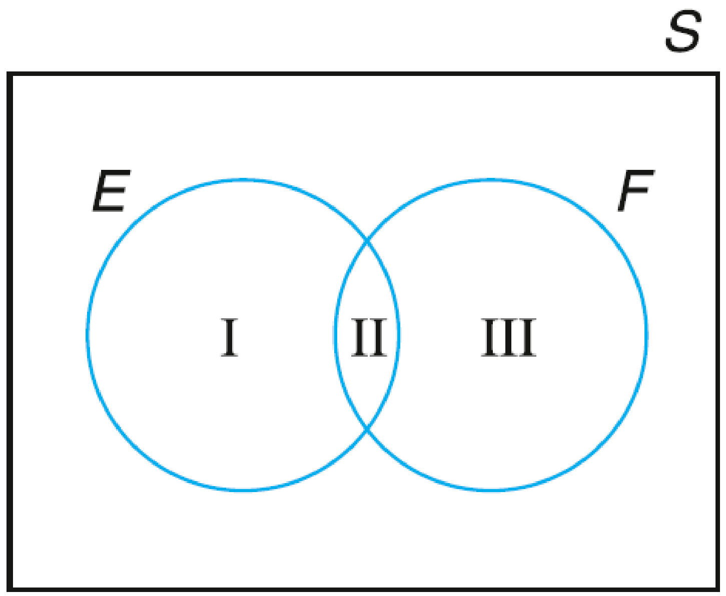
Axioms of Probability

$$P(E^c) = 1 - P(E)$$

$$P(E \cup F) = P(E) + P(F) - P(EF)$$

The *odds* of an event A is defined by

$$\frac{P(A)}{P(A^c)} = \frac{P(A)}{1 - P(A)}$$



The odds of an event *A* tells how much more likely it is that *A* occurs than that it does not occur.

Ross, Sheldon M. Introduction to Probability and Statistics for Engineers and Scientists. 6th ed., Academic Press, 2021.

Sample spaces having equally likely outcomes

For a large number of experiments, it is natural to assume that each point in the sample space is equally likely to occur. That is, for many experiments whose sample space S is a finite set, say $S = \{1, 2, \ldots, N\}$, it is often natural to assume that

$$P(\{1\}) = P(\{2\}) = \cdots = P(\{N\}) = p \quad \text{(say)}$$

Now it follows from Axioms 2 and 3 that

$$1 = P(S) = P(\{1\}) + \cdots + P(\{N\}) = Np$$

which shows that

$$P(\{i\}) = p = 1/N$$

From this it follows from Axiom 3 that for any event E,

$$P(E) = \frac{\text{Number of points in } E}{N}$$

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Summary

Recap of basics of probability

Recap of permutation and combination

In the next video

Conditional Probability and Bayes’ Theorem

Thank you.

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