AIL1020 Foundations of Statistics & Probability

Module 03

Conditional Probability

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Learning Outcomes

By the end of this video, you will be able to

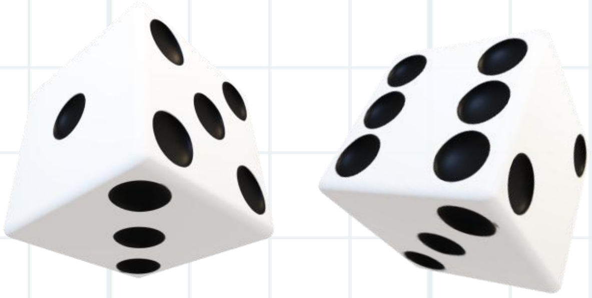
Define and explain conditional probability using everyday examples and Venn diagrams.

Apply the formula for conditional probability to solve problems.

Conditional Probability

The probability of an event *A* **given** that another event *B* has occurred: $P(A|B)$

Example



A B

Conditional Probability

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Example

Suppose we observe that one of the die lands on side 3.

Given this information, **what is the probability that the sum of the two dice equals 8?**

There can be at most 6 possible outcomes of our experiment:

(1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3)

A B

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A B

Probability that the sum of the two dice equals 8, given there is 3 on one of the die

$$\boldsymbol{P(A|B) = \frac{1}{6}}$$

Conditional Probability

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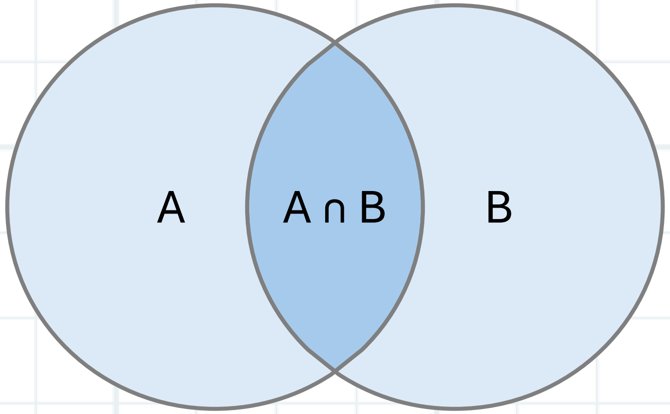

The overlap $A \cap B$ shows where both *A* and *B* occur.

When we condition on *B*

Conditional Probability

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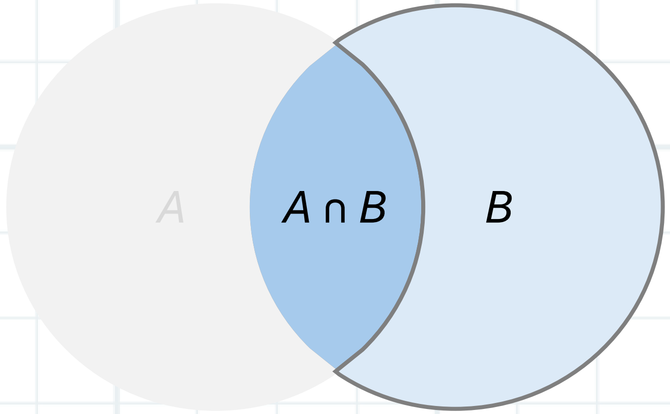


The overlap *A* ∩ *B* shows where both *A* and *B* occur.

When we condition on *B*

Within B, what fraction lies in A?

$$P(A|B) = \frac{\text{Area of } A \cap B}{\text{Area of } B}$$

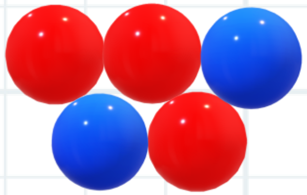
Conditional Probability $$P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ when } P(B) > 0$$

Conditional Probability

Example 1

A bag has 3 red and 2 blue balls. One ball is drawn, not replaced, and another ball is drawn.

What is the probability the second ball is blue given the first was red?



Conditional Probability

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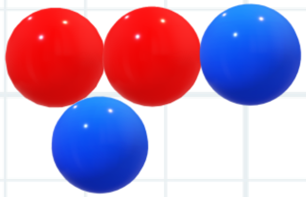
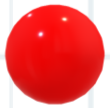


P(Second green | First red) **= 2/4**

= 1/2



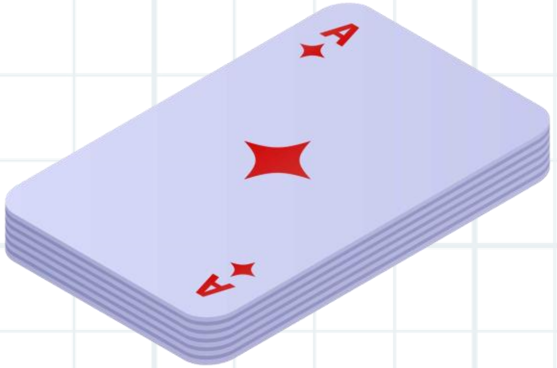
Given the first was red

Probability that second is **blue**?

Conditional Probability

Example 2

In a card deck, what's the probability the second card is a king **given the first card was a king**?



Total cards: 52

First king drawn → remaining kings = 3, total = 51

$P(\text{King}_2 \mid \text{King}_1) = 3/51$

Conditional Probability

***Example** 3*

A bin contains 5 defective (that immediately fail when put in use), 10 partially defective (that fail after a couple of hours of use), and 25 acceptable transistors. **A transistor is chosen at random from the bin and put into use. If it does not immediately fail, what is the probability it is acceptable?**

05	10	25
Defective (immediate fails)	Partially defective (fails after a few hours)	Acceptable

Total = 40

P(acceptable | not defective)

$$= \frac{P(\text{acceptable \& not defective})}{P(\text{not defective})}$$

$$= \frac{P(\text{acceptable})}{P(\text{not defective})} = \frac{25/40}{35/40} = \mathbf{5/7}$$

Alternatively – using reduced sample space

***P*(acceptable)** given containing 25 acceptable and 10 partially defective transistors = 25/35 = 5/7

Conditional Probability

***Example* 4**

The organization that Jones works for is running a father–son dinner for those employees having at least one son. Each of these employees is invited to attend along with his youngest son.

If Jones is known to have two children, what is the conditional probability that they are both boys given that he is invited to the dinner?

Assume that the sample space S is given by $S = \{(b, b), (b, g), (g, b), (g, g)\}$ and all outcomes are equally likely [(b, g) means, for instance, that the younger child is a boy and the older child is a girl].

Since Jones was invited, we know that he has at least one son.

B: Event that both children are boys

A: At least one of them is a boy

$$P(B|A) = \frac{P(BA)}{P(A)} = \frac{P(\{(b,b)\})}{P(\{(b,b),(b,g),(g,b)\})} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Summary

Conditional probability and some examples

In the next video

Bayes' Theorem

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Thank you.