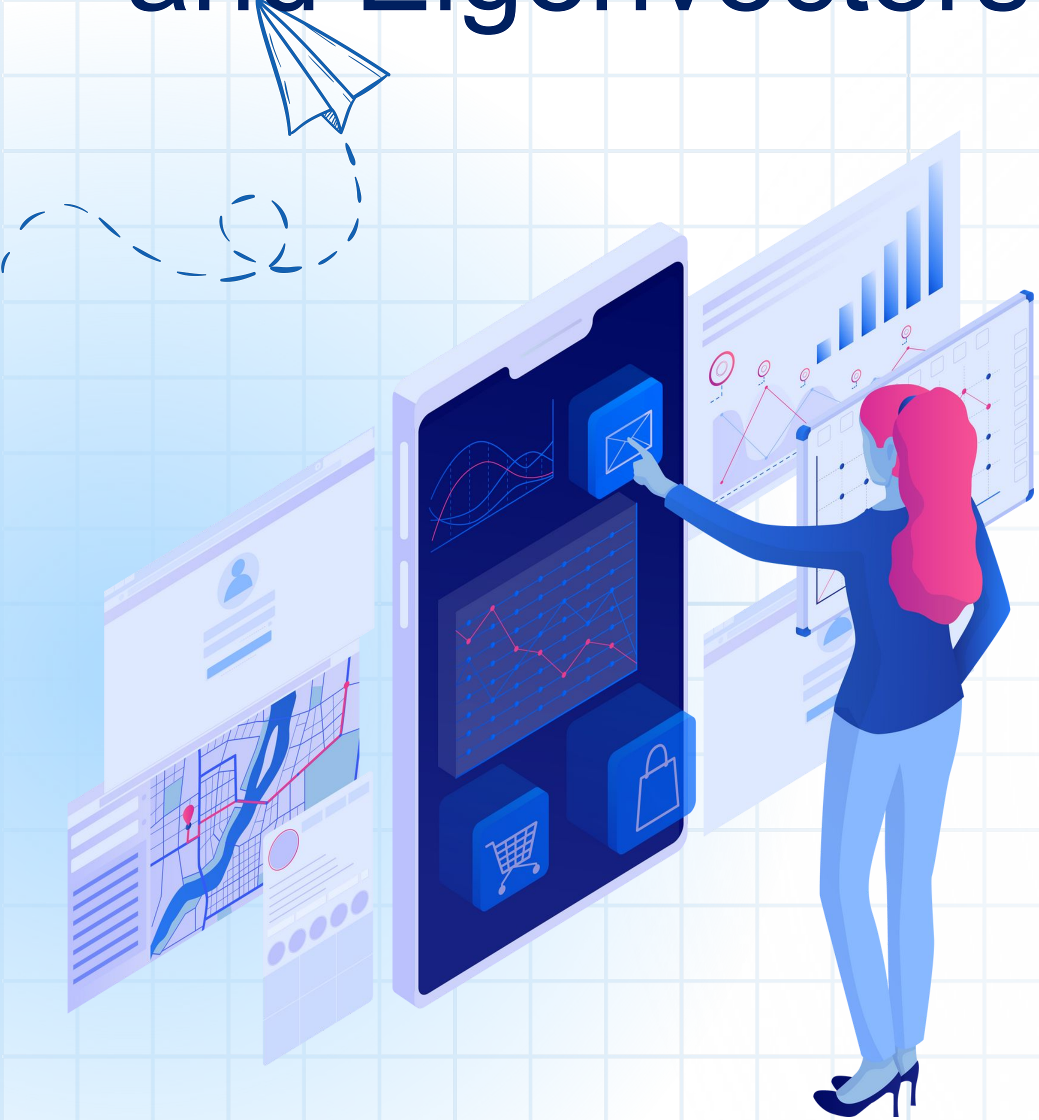
BS./BSc.

in Applied AI and Data Science

Linear algebra and numerical analysis

Module 3.2 Computation of Eigenvalues and Eigenvectors



1. **Eigenvalues of a Square Matrix**
2. **Eigenvectors and applications**
3. **Eigenvalues, Eigenvectors and Matrix Diagonalization**
4. **Vector space and linear independence**

Eigenvalues and Eigenvectors in real world

Eigenvalues and Eigenvectors are indispensable to data science

Image Compression: Eigenvalues and eigenvectors are used in image compression techniques to represent images in a more compact form.

Google's PageRank Algorithm: The **PageRank** algorithm, which determines the importance of web pages, relies heavily on eigenvalues and eigenvectors.

The **algorithm** represents the web as a **graph**, and each page's **PageRank** is determined by the eigenvector corresponding to the largest eigenvalue of the link matrix.

Learning through examples

Determine if the given vectors v and u are eigenvectors of A? If yes, find the eigenvalue of A associated to the eigenvector.

$$\mathbf{A} = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}.$$

$$\mathbf{A}\mathbf{v} = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 2 \end{bmatrix}$$

$$= 2 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

$$= 2\mathbf{v}$$

Estimation of Eigenvalues

Hence, in the previous example, Av = 2v and thus v is an eigenvector of A with corresponding eigenvalue λ = 2.

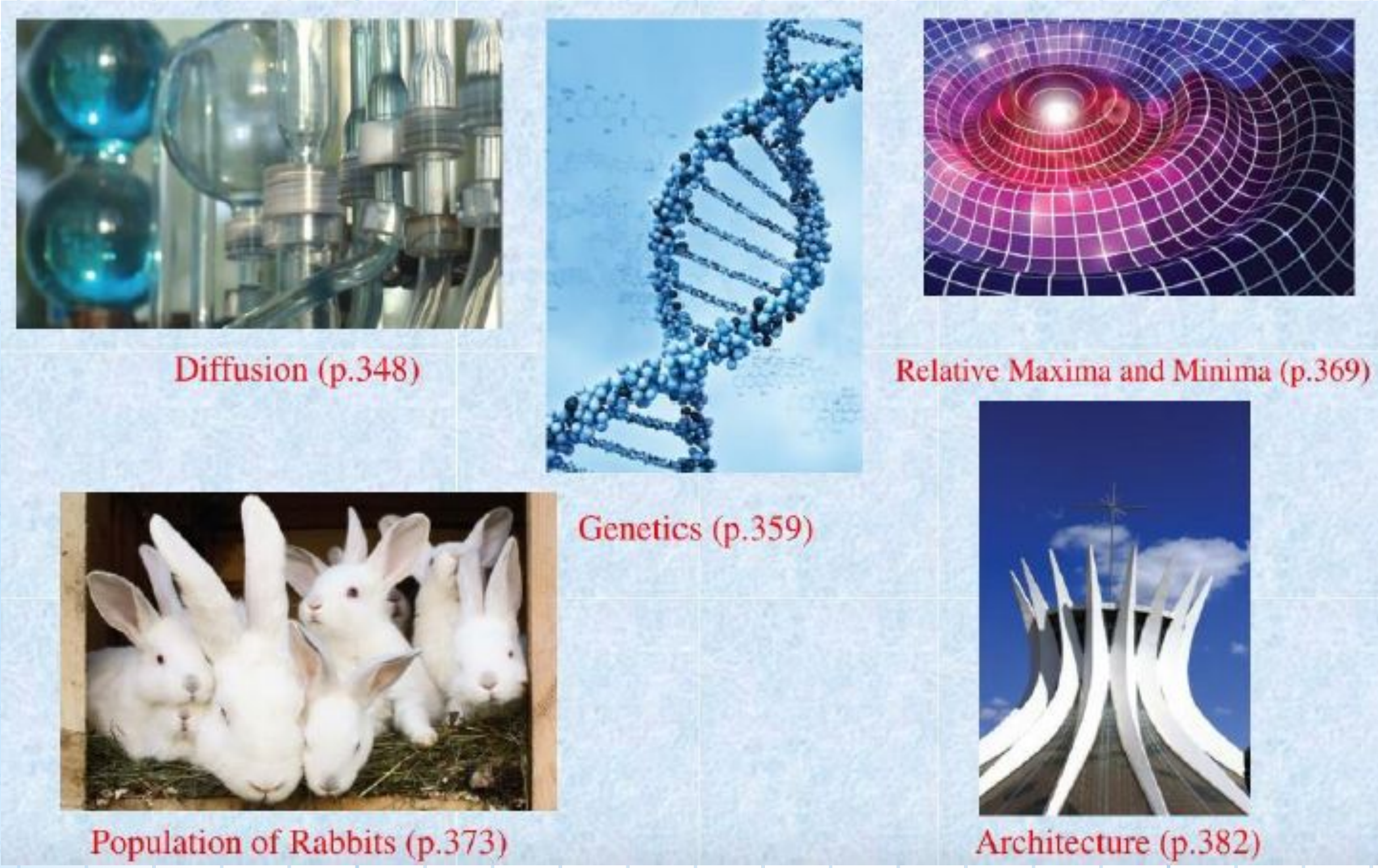
$$\mathbf{Au} = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 4 \end{bmatrix}$$

There is no scalar λ such that

$$\begin{bmatrix} 0 \\ 6 \\ 4 \end{bmatrix} = \lambda \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}.$$

Therefore, u is not an eigenvector of A.

Applications where you need to compute Eigenvalues



Source: Elementary Linear Algebra

Stepping through Eigenvalues

$$A = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \quad x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Ax_1 = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \underset{\text{Eigenvalue}}{2}\underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\text{Eigenvector}} = 2x_1$$

$$Ax_2 = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \underset{\text{Eigenvalue}}{-1}\underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\text{Eigenvector}} = (-1)x_2$$

Geometrical interpretation of Eigenvalues

- Eigenvalue and eigenvector:

A : an $n \times n$ matrix

λ : a scalar

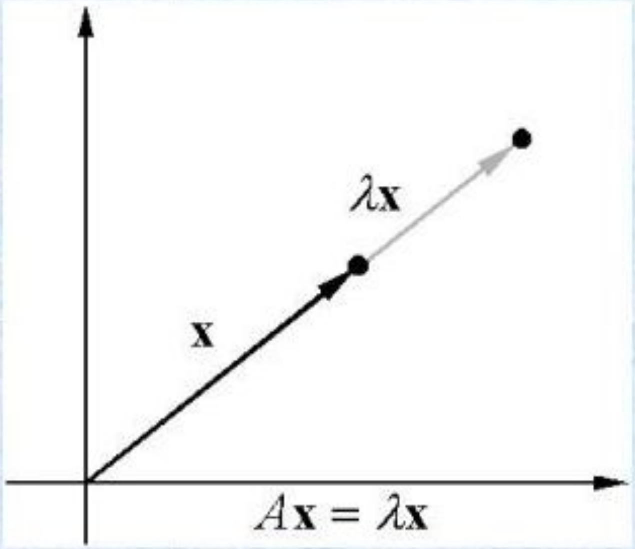
$\boldsymbol{x}$: a nonzero vector in R^n

Eigenvalue

$$Ax = \lambda x$$

Eigenvector

- Geometrical Interpretation



How to find eigenvectors/eigenvalues of a matrix A?

Let A be an $n \times n$ matrix with eigenvalue λ and corresponding eigenvector $\mathbf{x}$. Thus $A\mathbf{x} = \lambda\mathbf{x}$. This equation may be written

$$A\mathbf{x} - \lambda\mathbf{x} = \mathbf{0}$$

given

$$(A - \lambda I_n)\mathbf{x} = \mathbf{0}$$

Solving the equation $|A - \lambda I_n| = 0$ for λ leads to all the eigenvalues of A.

On expending the determinant $|A - \lambda I_n|$, we get a polynomial in λ.

This polynomial is called the **characteristic polynomial** of A.

The equation $|A - \lambda I_n| = 0$ is called the **characteristic equation** of A.

Determining eigenvalues of matrix A

$$\mathbf{A} = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

$$\mathbf{Ax} = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{aligned} -5x_1 + 2x_2 &= \lambda x_1 \\ 2x_1 - 2x_2 &= \lambda x_2. \end{aligned}$$

$$\begin{aligned} (-5-\lambda)x_1 + \quad 2x_2 \quad &= 0 \\ 2x_1 \quad + (-2-\lambda)x_2 &= 0. \end{aligned}$$

Characteristic determinant

By Cramer's theorem, the linear system in the previous slide has a nontrivial solution if and only if its coefficient determinant is zero, that is,

$$D(\lambda) = \det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} -5-\lambda & 2 \\ 2 & -2-\lambda \end{vmatrix} = (-5-\lambda)(-2-\lambda) - 4 = \lambda^2 + 7\lambda + 6 = 0.$$

We call $D(\lambda)$ the **characteristic determinant** or, if expanded, the **characteristic polynomial**, and $D(\lambda) = 0$ the **characteristic equation** of $\mathbf{A}$. The solutions of this quadratic equation are $\lambda_1 = -1$ and $\lambda_2 = -6$. These are the eigenvalues of $\mathbf{A}$.

Eigenspace

- Thm 7.1: (The eigenspace of A corresponding to λ)

  If A is an $n\times n$ matrix with an eigenvalue λ, then the set of all eigenvectors of λ together with the zero vector is a subspace of R^n. This subspace is called **the eigenspace of λ** .

Pf:

x_1 and x_2 are eigenvectors corresponding to λ

$(i.e.\ \ Ax_1 = \lambda x_1,\ \ Ax_2 = \lambda x_2)$

(1) $A(x_1 + x_2) = Ax_1 + Ax_2 = \lambda x_1 + \lambda x_2 = \lambda(x_1 + x_2)$

$(i.e.\ \ x_1 + x_2$ is an eigenvector corresponding to $\lambda)$

(2) $A(cx_1) = c(Ax_1) = c(\lambda x_1) = \lambda(cx_1)$

$(i.e.\ \ cx_1$ is an eigenvector corresponding to $\lambda)$

Eigenspace and eigenvalues

Find the eigenvalues and corresponding eigenspaces of

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

If $\quad \mathrm{v} = (x, y)$

$$A\mathrm{v} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$$

For a vector on the x-axis

Eigenvalue $\lambda_1 = -1$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ 0 \end{bmatrix} = \begin{bmatrix} -x \\ 0 \end{bmatrix} = -1\begin{bmatrix} x \\ 0 \end{bmatrix}$$

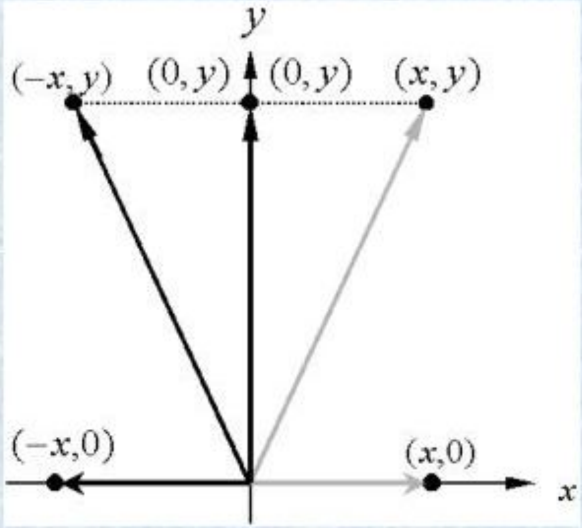
Eigenspace and eigenvalues

For a vector on the y-axis

Eigenvalue $\lambda_2 = 1$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ y \end{bmatrix} = 1\begin{bmatrix} 0 \\ y \end{bmatrix}$$

Geometrically, multiplying a vector (x, y) in R^2 by the matrix A corresponds to a reflection in the y-axis.



The eigenspace corresponding to $\lambda_1 = -1$ is the x-axis.

The eigenspace corresponding to $\lambda_2 = 1$ is the y-axis.

Eigenspace and eigenvalues

- Thm 7.2: (Finding eigenvalues and eigenvectors of a matrix $A \in M_{n\times n}$)

  Let A is an $n\times n$ matrix.

  (1) An eigenvalue of A is a scalar λ such that $\det(\lambda I - A) = 0$.

  (2) The eigenvectors of A corresponding to λ are <u>the nonzero solutions</u> of $(\lambda I - A)x = 0$.

- Note:

  $Ax = \lambda x \;\Rightarrow\; (\lambda I - A)x = 0$ (homogeneous system)

  If $(\lambda I - A)x = 0$ has nonzero solutions iff $\det(\lambda I - A) = 0$.

Question 1

Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$$

Solution

Let us first derive the characteristic polynomial of *A*.

We get

$$A-\lambda I_2=\begin{bmatrix}-4 & -6\\ 3 & 5\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}-4-\lambda & -6\\ 3 & 5-\lambda\end{bmatrix}$$

$$|A-\lambda I_2|=(-4-\lambda)(5-\lambda)+18=\lambda^2-\lambda-2$$

We now solve the characteristic equation of *A*.

The eigenvalues of *A* are 2 and –1.

The corresponding eigenvectors are found by using these values of λ in the equation$(A-\lambda I_2)\mathbf{x}=\mathbf{0}$. There are many eigenvectors corresponding to each eigenvalue.

Question 2

Find the characteristic polynomial of matrix A. What are the eigenvalues of A?

$$A = \begin{bmatrix} -2 & 4 \\ -6 & 8 \end{bmatrix}.$$

Solution

$$\mathbf{A} - \lambda\mathbf{I} = \begin{bmatrix} -2 & 4 \\ -6 & 8 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} -2-\lambda & 4 \\ -6 & 8-\lambda \end{bmatrix}.$$

Therefore,

$$\begin{aligned} p(\lambda) &= \det(\mathbf{A} - \lambda\mathbf{I}) \\ &= \begin{vmatrix} -2-\lambda & 4 \\ -6 & 8-\lambda \end{vmatrix} \\ &= (-2-\lambda)(8-\lambda) + 24 \\ &= \lambda^2 - 6\lambda + 8 \\ &= (\lambda - 4)(\lambda - 2) \end{aligned}$$

The roots of p(λ) are λ1 = 4 and λ2 = 2. Therefore, the eigenvalues of A are λ1 = 4 and λ2 = 2.

Powered by Futurense

Recap

- ❖ Eigenvalues and eigenvectors are fundamental concepts in linear algebra with widespread applications across various scientific and engineering fields.
- ❖ **Eigenvectors:** These are special vectors that, when a linear transformation (represented by a matrix) is applied to them, only change in scale, not direction.
- ❖ In simpler terms, they are vectors that remain on the same line after a transformation.
- ❖ **Eigenvalues:**

These are the scalar factors by which the eigenvectors are scaled during the linear transformation. They indicate how much the eigenvectors are stretched or shrunk.

Key Applications:

•**Principal Component Analysis (PCA):**

- Used in data analysis and machine learning for dimensionality reduction.
- Eigenvectors identify the directions of maximum variance in data, and eigenvalues indicate the magnitude of that variance.

Coming up next................

- ❖ Python coding of eigenvalues and eigenvectors
- ❖ Concepts of matrix diagonalization

Module 3.3: Recap eigenvalues and eigenvectors

Find the eigenvalues of

$$\mathbf{A} = \begin{bmatrix} -4 & -6 & -7 \\ 3 & 5 & 3 \\ 0 & 0 & 3 \end{bmatrix}.$$

Then

$$\det(\mathbf{A} - \lambda\mathbf{I}) = (-4-\lambda)\begin{vmatrix} 5-\lambda & 3 \\ -\lambda & 3-\lambda \end{vmatrix} - 3\begin{vmatrix} -6 & -7 \\ -\lambda & 3-\lambda \end{vmatrix}$$
$$= (-4-\lambda)[(3-\lambda)(5-\lambda)+3\lambda] - 3[-6(3-\lambda)-7\lambda]$$
$$= \lambda^3 - 4\lambda^2 + \lambda + 6$$

Factor the characteristic polynomial:

$$p(\lambda) = \lambda^3 - 4\lambda^2 + \lambda + 6 = (\lambda-2)(\lambda-3)(\lambda+1)$$

Therefore, the eigenvalues of $\mathbf{A}$ are

$$\lambda_1 = 2, \quad \lambda_2 = 3, \quad \lambda_3 = -1.$$

Python implementation of eigenvalues and eigenvectors

To eigendecompose a square matrix, you first find the eigenvalues, and then use each eigenvalue to find its corresponding eigenvector. The eigenvalues are like keys that you insert into the matrix to unlock the mystical eigenvector. Finding the eigenvalues of a matrix is super easy in Python:

```
matrix = np.array([
            [1,2],
            [3,4]
            ])

# get the eigenvalues
evals = np.linalg.eig(matrix)[0]
```

The two eigenvalues (rounded to the nearest hundredth) are −0.37 and 5.37

But the important question isn't which function returns the eigenvalues; instead, the important question is how are the eigenvalues of a matrix identified?

Python implementation of eigenvalues and eigenvectors

$$
\begin{aligned}
\mathbf{Av} &= \lambda\mathbf{v} \\
\mathbf{Av} - \lambda\mathbf{v} &= \mathbf{0} \\
(\mathbf{A} - \lambda\mathbf{I})\mathbf{v} &= \mathbf{0}
\end{aligned}
$$

- The first equation is an exact repeat of the eigenvalue equation. In the second equation, we simply subtracted the right-hand side to set the equation to the zeros vector.
- The transition from the second to the third equation requires some explanation. The left-hand side of the second equation has two vector terms, both of which involve v. So, we factor out the vector. But that leaves us with subtracting a matrix and a scalar (A − λ), which is not a defined operation in linear algebra. So, instead, we shift the matrix by λ. That brings us to the third equation.
- What does that third equation mean? It means that the eigenvector is in the null space of the matrix shifted by its eigenvalue.

Recall Eigenvalue estimations

Believe it or not, that's the key to finding eigenvalues: shift the matrix by the unknown eigenvalue λ, set its determinant to zero, and solve for λ. Let's see how this looks for a 2×2 matrix:

$$\begin{aligned}
\left| \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| &= 0 \\
\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} &= 0 \\
(a-\lambda)(d-\lambda) - bc &= 0 \\
\lambda^2 - (a+d)\lambda + (ad-bc) &= 0
\end{aligned}$$

- *The matrix-vector multiplication acts like scalar-vector multiplication (the eigenvalue equation).*
- *We set the eigenvalue equation to the zeros vector, and factor out common terms.*
- *This reveals that the eigenvector is in the null space of the matrix shifted by the eigenvalue. We do not consider the zeros vector to be an eigenvector, which means the shifted matrix is singular.*
- *Therefore, we set the determinant of the shifted matrix to zero and solve for the unknown eigenvalue.*

The determinant of an eigenvalue-shifted matrix set to zero is called the characteristic polynomial of the matrix.

Python estimation of eigenvectors

As with eigenvalues, finding eigenvectors is super-duper easy in Python:

```
evals,evecs = np.linalg.eig(matrix)
print(evals), print(evecs)
```

```
[-0.37228132  5.37228132]

[[-0.82456484 -0.41597356]
 [ 0.56576746 -0.90937671]]
```

Key points of the concept of a Matrix diagonalization

Let A be a triangular matrix (either upper or lower). Then the eigenvalues of A are its diagonal entries.

Proof. We will prove the theorem for the case $n = 3$ and $\mathbf{A}$ is upper triangular; the general case is similar. Suppose then that $\mathbf{A}$ is a 3×3 upper triangular matrix:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

Then

$$\mathbf{A} - \lambda \mathbf{I} = \begin{bmatrix} a_{11} - \lambda & a_{12} & a_{13} \\ 0 & a_{22} - \lambda & a_{23} \\ 0 & 0 & a_{33} - \lambda \end{bmatrix}.$$

and thus the characteristic polynomial of $\mathbf{A}$ is

$$p(\lambda) = \det(\mathbf{A} - \lambda \mathbf{I}) = (a_{11} - \lambda)(a_{22} - \lambda)(a_{33} - \lambda)$$

and the roots of $p(\lambda)$ are

$$\lambda_1 = a_{11}, \; \lambda_2 = a_{22}, \; \lambda_3 = a_{33}.$$

In other words, the eigenvalues of $\mathbf{A}$ are simply the diagonal entries of $\mathbf{A}$. □

Powered by Futurense

Solutions

We now introduce a very special type of a triangular matrix, namely, a diagonal matrix.

A matrix D whose off-diagonal entries are all zero is called a diagonal matrix.

For example, here is 3×3 diagonal matrix

$$\mathrm{D} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -8 \end{bmatrix}.$$

Diagonal matrix continued…….

A diagonal matrix is clearly also a triangular matrix and therefore the eigenvalues of a diagonal matrix D are simply the diagonal entries of D. Moreover, the powers of a diagonal matrix are easy to compute. For example, if $\mathrm{D} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ then

$$\mathrm{D}^2 = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{bmatrix}$$

and similarly for any integer $k = 1, 2, 3, \ldots$, we have that

$$\mathrm{D}^k = \begin{bmatrix} \lambda_1^k & 0 \\ 0 & \lambda_2^k \end{bmatrix}.$$

Diagonalization

- Diagonalization problem:

  For a square matrix A, does there exist an invertible matrix P such that $P^{-1}AP$ is diagonal?

- Diagonalizable matrix:

  A square matrix A is called **diagonalizable** if there exists an invertible matrix P such that $P^{-1}AP$ is **a diagonal matrix**.

  (P diagonalizes A)

- Notes:

  (1) If there exists an invertible matrix P such that $B = P^{-1}AP$, then two square matrices A and B are called **similar**.

  (2) The eigenvalue problem is related closely to the diagonalization problem.

Similar matrices have same

- Thm 7.4: (Similar matrices have the same eigenvalues)

  If A and B are similar $n \times n$ matrices, then they have the same eigenvalues.

Pf:

A and B are similar $\Rightarrow B = P^{-1}AP$

$$\begin{aligned}|\lambda I - B| &= |\lambda I - P^{-1}AP| = |P^{-1}\lambda IP - P^{-1}AP| = |P^{-1}(\lambda I - A)P| \\ &= |P^{-1}||\lambda I - A||P| = |P^{-1}||P||\lambda I - A| = |P^{-1}P||\lambda I - A| \\ &= |\lambda I - A|\end{aligned}$$

A and B have the same characteristic polynomial.

Thus A and B have the same eigenvalues.

Symmetric matrix and

Prove that a symmetric matrix is diagonalizable.

$$A = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$$

Pf: Characteristic equation:

$$|\lambda I - A| = \begin{vmatrix} \lambda - a & -c \\ -c & \lambda - b \end{vmatrix} = \lambda^2 - (a+b)\lambda + ab - c^2 = 0$$

As a quadratic in λ, this polynomial has a discriminant of

$$\begin{aligned}(a+b)^2 - 4(ab - c^2) &= a^2 + 2ab + b^2 - 4ab + 4c^2 \\ &= a^2 - 2ab + b^2 + 4c^2 \\ &= \underline{(a-b)^2 + 4c^2} \geq 0\end{aligned}$$

Symmetric matrix and diagonalization continued.........

(1) $(a-b)^2+4c^2=0$

$$\Rightarrow a=b,\ c=0$$

$$A=\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$$ is a matrix of diagonal.

(2) $(a-b)^2+4c^2>0$

The characteristic polynomial of A has two distinct real roots, which implies that A has two distinct real eigenvalues. Thus, A is diagonalizable.

Question

Let A be an $n \times n$ symmetric matrix. If λ_1 and λ_2 are distinct eigenvalues of A, then their corresponding eigenvectors x_1 and x_2 are orthogonal.

(Eigenvectors of a symmetric matrix)

Show that any two eigenvectors of $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ corresponding to distinct eigenvalues are orthogonal.

Solution

Sol: Characteristic function

$$|\lambda \mathrm{I} - A| = \begin{vmatrix} \lambda - 3 & -1 \\ -1 & \lambda - 3 \end{vmatrix} = \lambda^2 - 6\lambda + 8 = (\lambda - 2)(\lambda - 4) = 0$$

$$\Rightarrow \text{Eigenvalues}: \ \lambda_1 = 2, \lambda_2 = 4$$

$$(1)\ \lambda_1 = 2 \Rightarrow \lambda_1 \mathrm{I} - A = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow \mathrm{x}_1 = s\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \ s \neq 0$$

$$(2)\ \lambda_2 = 4 \Rightarrow \lambda_2 \mathrm{I} - A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \Rightarrow \mathrm{x}_2 = t\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ t \neq 0$$

$$\mathrm{x}_1 \cdot \mathrm{x}_2 = \begin{bmatrix} -s \\ s \end{bmatrix} \cdot \begin{bmatrix} t \\ t \end{bmatrix} = st - st = 0 \Rightarrow \mathrm{x}_1 \text{ and } \mathrm{x}_2 \text{ are orthogonal.}$$

Recap

Matrix diagonalization is a process in linear algebra that simplifies the study of square matrices. Here's a summary of its key aspects:

- **Transformation:**
  - It involves transforming a square matrix into a diagonal matrix. A diagonal matrix is one where all the entries outside the main diagonal are zero.
  - This transformation is achieved through a "similarity transformation."
- **Diagonalizable Matrices:**
  - Not all matrices can be diagonalized. A matrix is diagonalizable if it has a sufficient number of linearly independent eigenvectors.

Purpose and Significance:

- **Simplification:**
  - Diagonal matrices are much easier to work with than general matrices.
  - Operations like raising a matrix to a power become significantly simpler when the matrix is diagonalized.

Coming up next................

- Concepts of orthogonalization
- Concepts of vector spaces
- Concepts of linear independence

Thank you

Powered by
Futurense