BS./BSc.

in Applied AI and Data Science

Linear algebra and numerical analysis

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Let's dive into and learn:

1. Understanding systems of linear equation
2. Exploring real-world examples
3. Learning how vector algebra and geometry is linked with linear equations
4. Learning matrix to solve linear equations and real-world applications

Learning Objectives

Basics of linear equations

Understand the basics of linear system of equations

Everyday Examples

Recognize application of linear equations in everyday life

codes

Understand what is python code and syntax to solve linear system of equations

Hands on

Gain hands-on experience on solving simple linear equations

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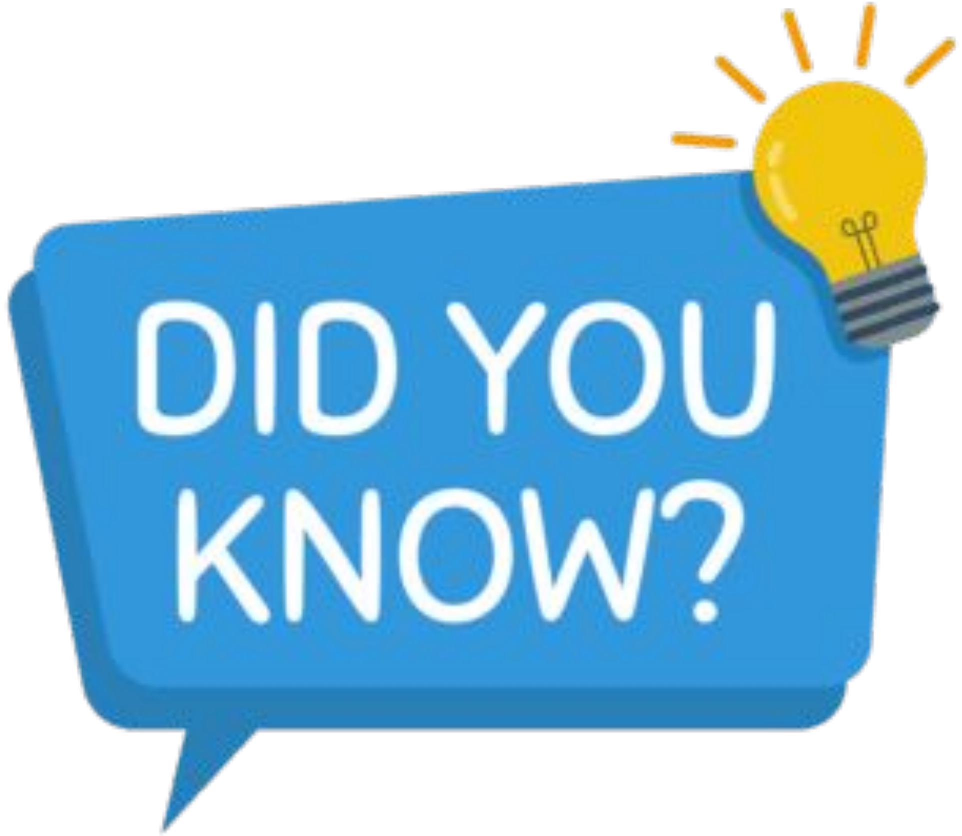
Welcome to Real-world examples of linear equations

What is a linear system of Equations?



Did you know, that your daily schedule and calendar is also a linear system of equations with many variables?

Linear systems in daily life

"Linear Equations Are Everywhere! 🌍" Please see Examples below:

- Spreading of viruses: They can be used to model the spreading of viruses in a network.
- Sports: They are used to solve geometrical problems such as lines, parabolas, etc.
- Travel: They are used to calculate the speed, distance, and time of a moving object

Basic Components of a Linear Equation

A linear equation has the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

$a_1, \ldots, a_n$ and b are the **coefficients**, $x_1, \ldots, x_n$ are the **variables** or **unknowns**, and n is the **dimension**, or number of variables.

For example,

- $2x_1 + 4x_2 = 4$ is a line in two dimensions
- $3x_1 + 2x_2 + x_3 = 6$ is a plane in three dimensions

Systems of linear equation

When we have more than one linear equation, we have a **linear system** of equations. For example, a linear system with two equations is

$$\begin{array}{lllll} x_1 & + 1.5x_2 & + \pi x_3 & = & 4 \\ 5x_1 & & + 7x_3 & = & 5 \end{array}$$

Definition: Solution to a Linear System

The set of all possible values of $x_1, x_2, \ldots x_n$ that satisfy all equations is the **solution** to the system.

A system can have a unique solution, no solution, or an infinite number of solutions.

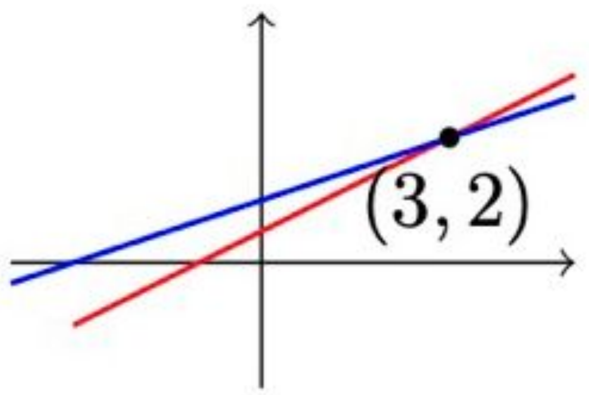
Linear Equations in 2D and geo

Consider the following systems. How are they different from each other?

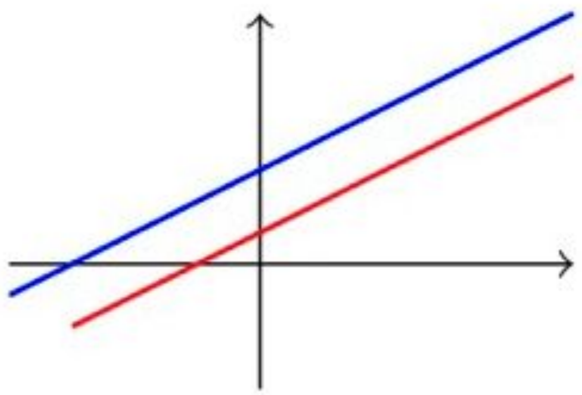
$$\begin{aligned} x_1 - 2x_2 &= -1 \\ -x_1 + 3x_2 &= 3 \end{aligned}$$

$$\begin{aligned} x_1 - 2x_2 &= -1 \\ -x_1 + 2x_2 &= 3 \end{aligned}$$

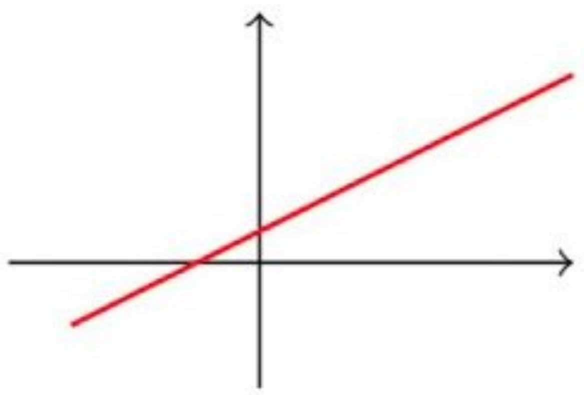
$$\begin{aligned} x_1 - 2x_2 &= -1 \\ -x_1 + 2x_2 &= 1 \end{aligned}$$



non-parallel lines



parallel lines



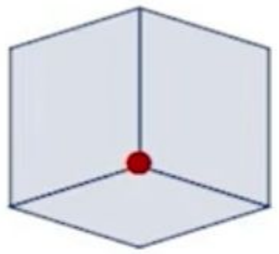
identical lines

Extending Linear Equations in three dimensions

Visualizing in 3D using linear Equations

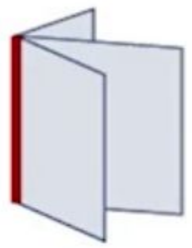
An equation $a_1x_1 + a_2x_2 + a_3x_3 = b$ defines a plane in $\mathbb{R}^3$. The **solution** to a system of **three equations** is the set of points were all planes intersect.

planes intersect at a point



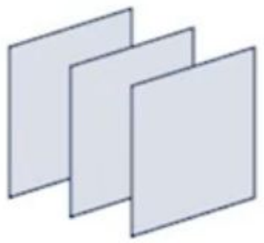
unique solution

planes intersect on a line



infinite number of solutions

parallel planes



no solution

Linear weighted combination and connection with vectors

- *A linear weighted combination is a way of mixing information from multiple variables, with some variables contributing more than others.*

- *This fundamental operation is also sometimes called linear mixture or weighted combination (the linear part is assumed). Sometimes, the term coefficient is used instead of weight.*

$$\mathbf{w} = \lambda_1 \mathbf{v}_1 + \lambda_2 \mathbf{v}_2 + \ldots + \lambda_n \mathbf{v}_n$$

$$\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = -3, \quad \mathbf{v_1} = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix},\ \mathbf{v_2} = \begin{bmatrix} -4 \\ 0 \\ -4 \end{bmatrix},\ \mathbf{v_3} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

$$\mathbf{w} = \lambda_1 \mathbf{v_1} + \lambda_2 \mathbf{v_2} + \lambda_3 \mathbf{v_3} = \begin{bmatrix} -7 \\ -4 \\ -13 \end{bmatrix}$$

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Coding linear mixing of vectors

"How do add a car moving with three different velocities in different parts of the city🚗"

Complete Workflow:

```
l1 = 1
l2 = 2
l3 = -3
v1 = np.array([4,5,1])
v2 = np.array([-4,0,-4])
v3 = np.array([1,3,2])
l1*v1 + l2*v2 + l3*v3
```

To do Activity: **Exercise 1**

1. In each part, determine whether the equation is linear in x_1 x_2, and x_3.

a. $x_1 + 5x_2 - \sqrt{2}x_3 = 1$

b. $x_1 + 3x_2 + x_1x_3 = 2$

c. $x_1 = -7x_2 + 3x_3$

d. $x_1^{-2} + x_2 + 8x_3 = 5$

e. $x_1^{3/5} - 2x_2 + x_3 = 4$

f. $\pi x_1 - \sqrt{2}x_2 = 7^{1/3}$

Mastering matrices and matrix algebra

- A matrix is a vector taken to the next level.
- Matrices are highly versatile mathematical objects.
- They can store sets of linear equations, geometrical transformations, and the positions of particles over time, financial records, and myriad other things.
- In data science, matrices are sometimes called data tables, in which rows correspond to observations (e.g., customers) and columns correspond to features (e.g., purchases).

A brief history of Matrix

"History of Matrix"

- The history of matrices goes back to **ancient times** , but the term **“Matrix”** was not applied to the concept till **1850**
- . Matrix is the **Latin** word for womb and it retains that sense in **English**
- It can also mean more generally any place where **something is formed or produced**

Hands-on Matrix representation

"Let's Use Matrix to Solve a Problem!"

- I show you here three matrices, A, B, C. Matrices are generally named so you can distinguish one matrix from another in a discussion or text.

$$A = \begin{bmatrix} 2 & 3 \\ 6 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 2 & 0 & 0 & 3 \\ -4 & 8 & 3 & 2 & 0 & 5 \\ 2 & 0 & 1 & 0 & 20 & 4 \\ -3 & 6 & 8 & 5 & 9 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

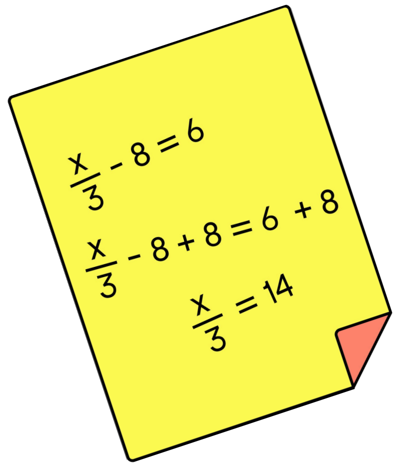
Basic Matrix operations

Matrix Operation:

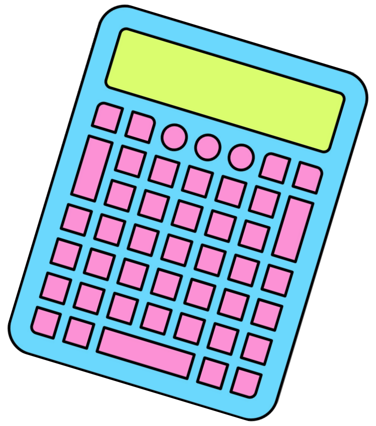
For example, if you want to construct a matrix with three rows and four columns in which each element is the sum of the digits in its index, you write: A is a 3 by 4 matrix where $a_{ij} = i + j$.

I explain the × sign in the next section. And here's matrix A:

$$A = \begin{bmatrix} 1+1 & 1+2 & 1+3 & 1+4 \\ 2+1 & 2+2 & 2+3 & 2+4 \\ 3+1 & 3+2 & 3+3 & 3+4 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$$



Exercise 2



Sales figures for three products I, II, III in a store on Monday (Mon), Tuesday (Tues), may for each week be arranged in a matrix

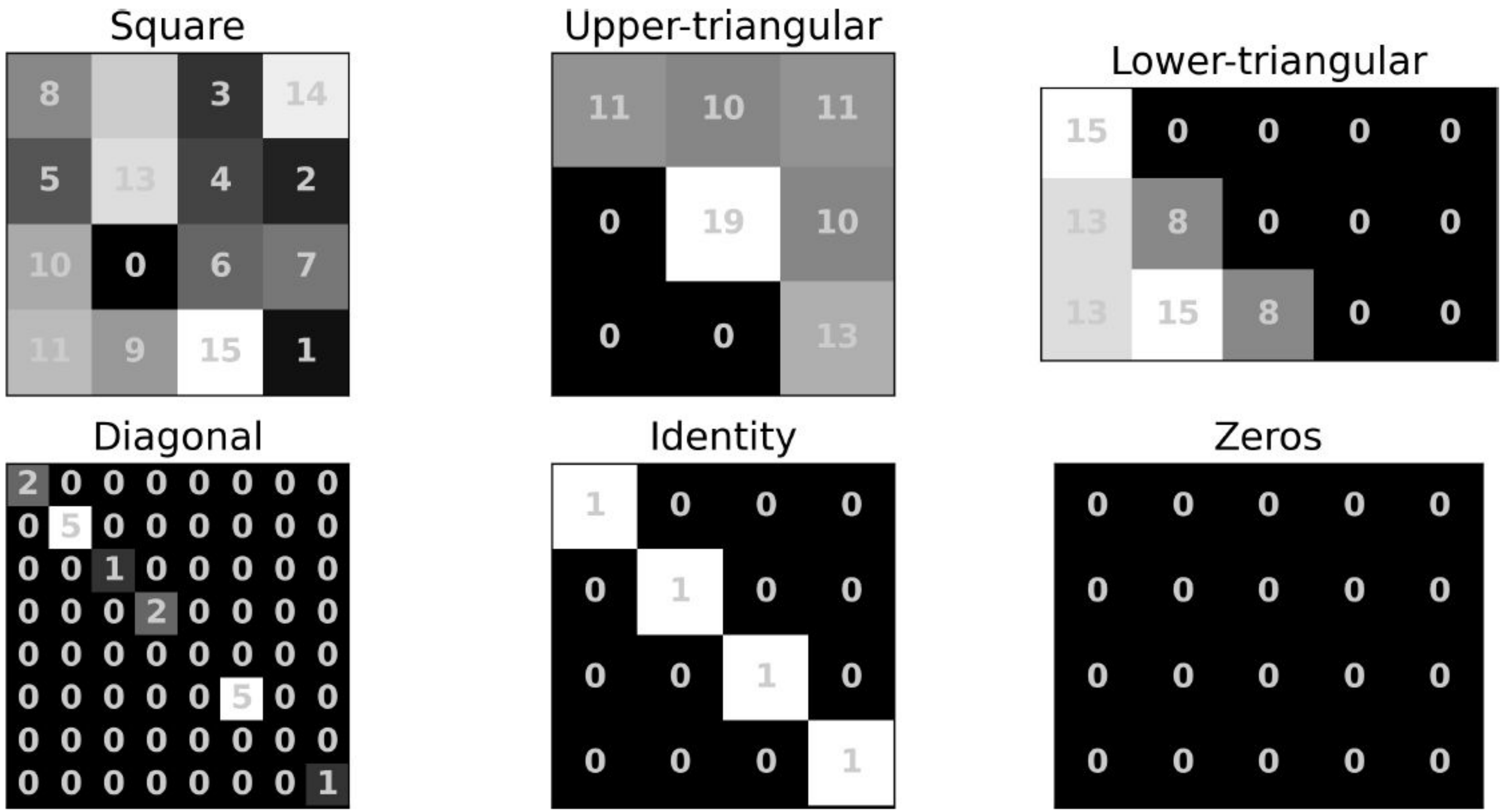
$$\begin{array}{cc} & \begin{array}{ccccccc} \text{Mon} & \text{Tues} & \text{Wed} & \text{Thur} & \text{Fri} & \text{Sat} & \text{Sun} \end{array} \\ \mathbf{A} = & \begin{bmatrix} 40 & 33 & 81 & 0 & 21 & 47 & 33 \\ 0 & 12 & 78 & 50 & 50 & 96 & 90 \\ 10 & 0 & 0 & 27 & 43 & 78 & 56 \end{bmatrix} \cdot \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \end{array} \end{array}$$

If the company has 10 stores, we can set up 10 such matrices, one for each store. Then, by adding corresponding entries of these matrices, we can get a matrix showing the total sales of each product on each day. Can you think of other data which can be stored in matrix form?

Special Matrices

- There is an infinite number of matrices because there is an infinite number of ways of organizing numbers into a matrix.
- But matrices can be described using a relatively small number of characteristics, which creates “families” or categories of matrices.
- These categories are important to know because they appear in certain operations or have certain useful properties.

Category of different types of



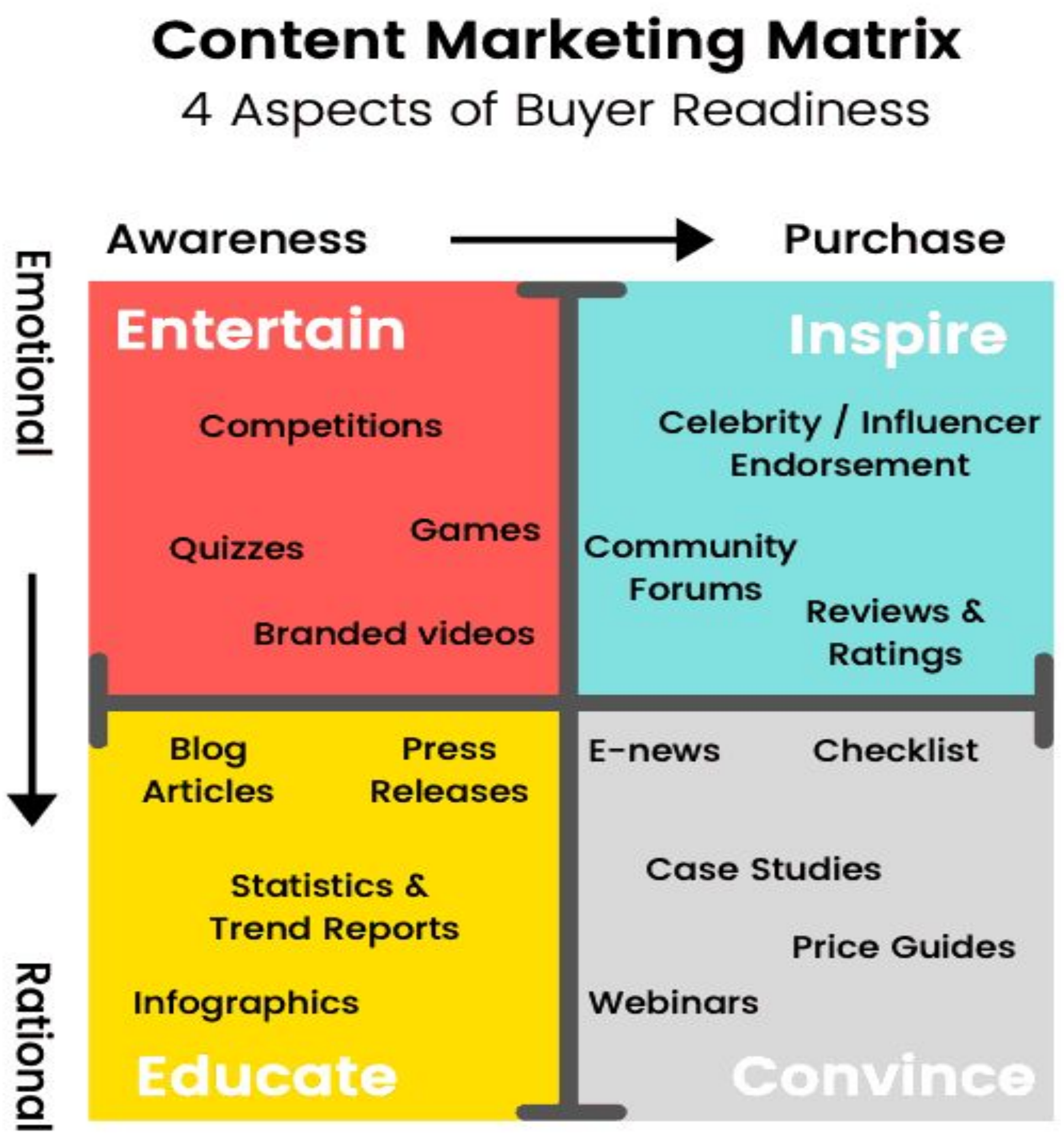
Applications of Matrix in real

"Are you on Instagram?"

Instagram mantra is a matrix operati

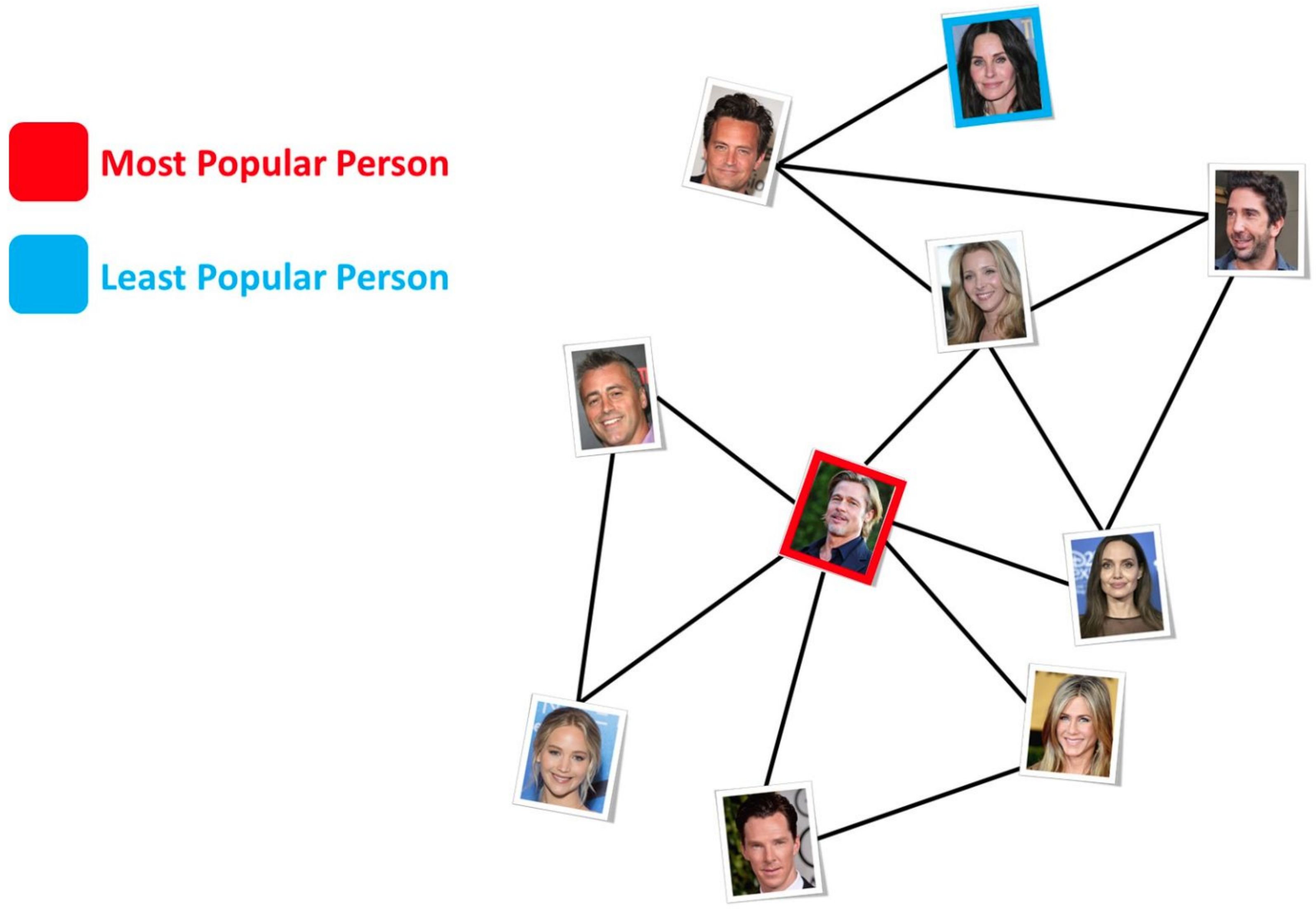
On images and filtering of images in

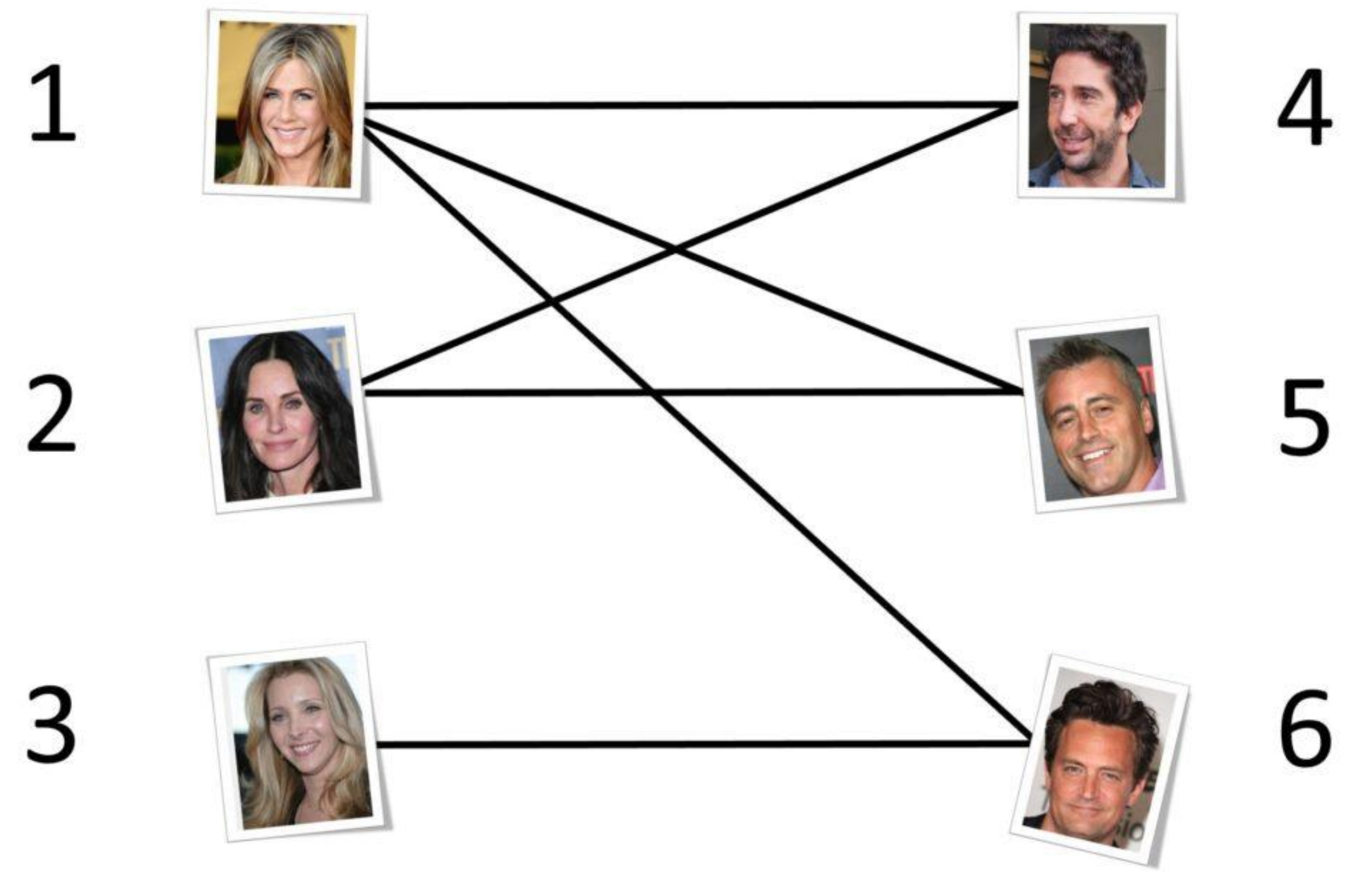
Understanding Matrix operations



Applications of Matrix in real

Matrices, Graphs and Recommender Systems

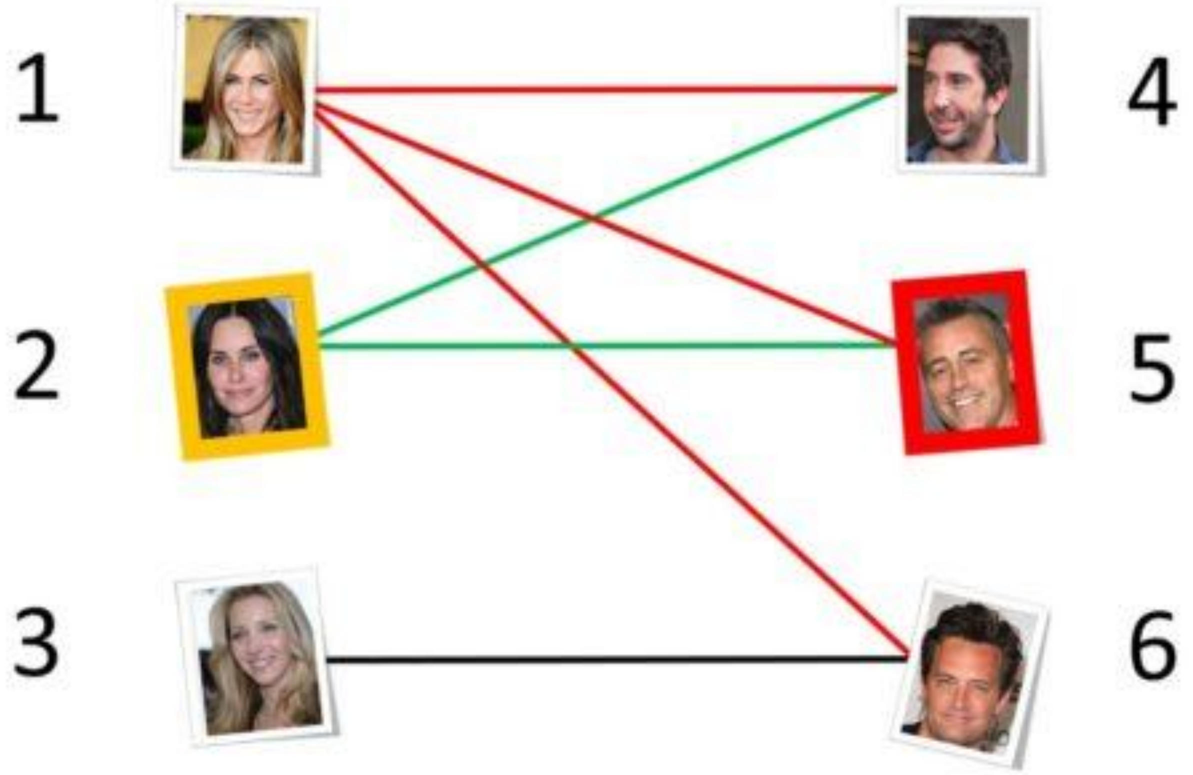




Applications of Matrix in real world

Matrices, Graphs and Recommender Systems

	1	2	3	4	5	6
1	0	0	0	1	1	1
2	0	0	0	1	1	0
3	0	0	0	0	0	1
4	1	1	0	0	0	0
5	1	1	0	0	0	0
6	1	0	1	0	0	0



Exercise 3

To do Activity:

Problem statement: Anil’s age is twice that of Sunil. Ten years ago, Anil's age was three times Sunil's age. Determine their current ages.

A. 20 Years
B. 30 Years
C. 10 Years
D. 35 Years

Recap

1. Linear equations enable us to solve real-world problems arising in data science.
2. Solving problems using vectors which are represented as linear equations.
3. How matrix formulation helps us in solving linear systems of equations.
4. Solution of real-world data science applications using matrix algebra.

Thank you