BS./BSc.

in Applied AI and Data Science

Linear algebra and numerical analysis

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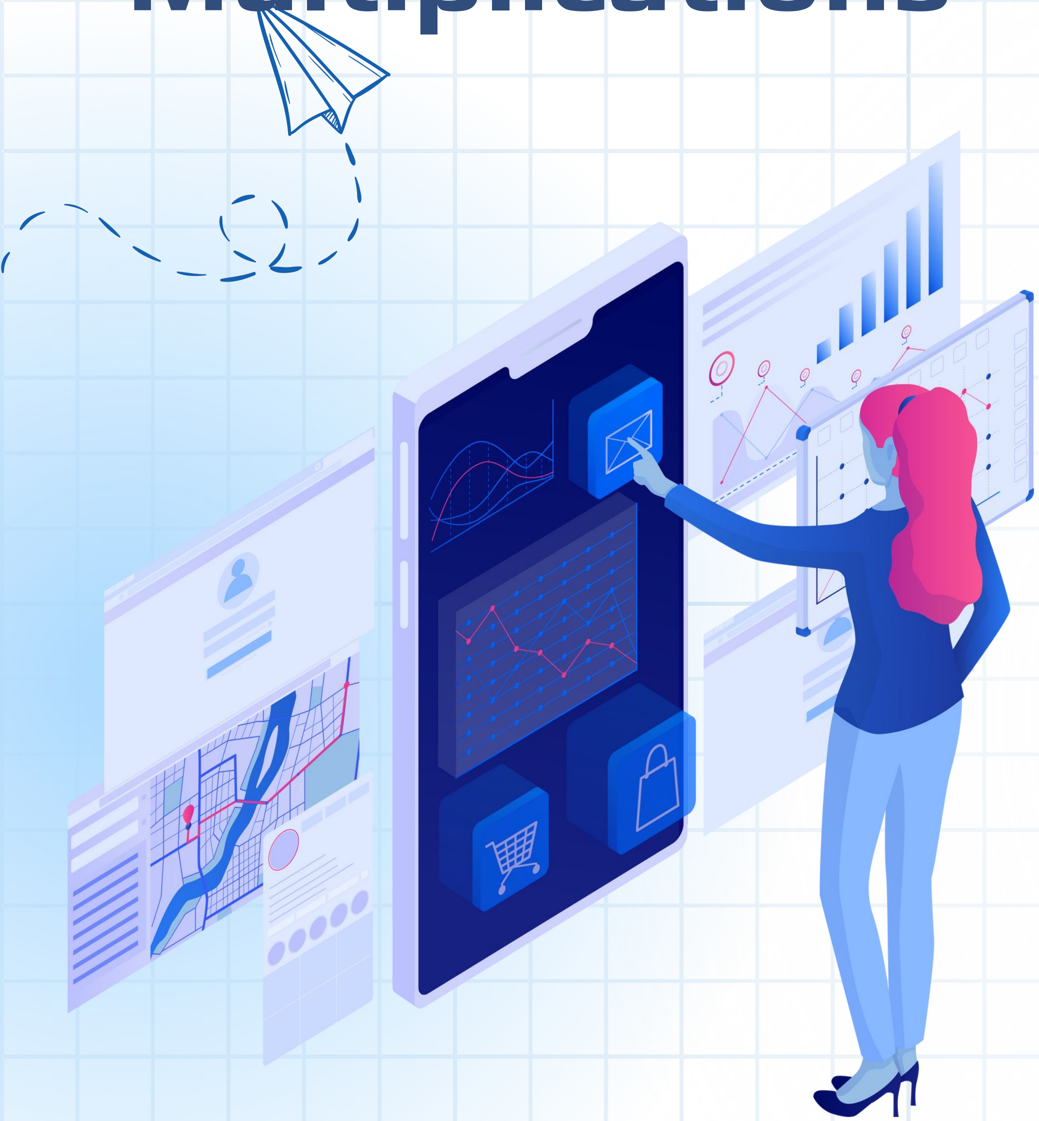
Module 2.1: Matrix Transpose and Multiplications



1. Matrix Algebra - Addition and Subtraction
2. Matrix Algebra – Shifting of Matrices & Scalar multiplication
3. Matrix Algebra - Matrix Multiplication
4. Transpose of a Matrix

Learning Objectives

Basics operations with Matrices

Understand the basics of Matrix Multiplication

Everyday Examples

Recognize application of Matrix Multiplications in everyday life

codes

Python code and syntax to solve Matrix Multiplication

Hands on

Gain hands-on experience in solving simple Matrix Multiplication problems

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Matrix operations in daily life

“Matrix Algebra Is Everywhere! 🌍” Please see

Examples below:

- **Image processing:** All images can be represented in matrix format
- **Computer Graphics:** In computer graphics every element is represented by a matrix.
- **Digital world:** Audio, video and image compression, including MP3, JPEG and MPEG video.

"Shifting" a Matrix

There is a linear-algebra way to add a scalar to a square matrix, and that is called shifting a matrix. It works by adding a constant value to the diagonal, which is implemented by adding a scalar multiplied identity matrix:

$$\mathbf{A} + \lambda\mathbf{I}$$

$$\begin{bmatrix} 4 & 5 & 1 \\ 0 & 1 & 11 \\ 4 & 9 & 7 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 5 & 1 \\ 0 & 7 & 11 \\ 4 & 9 & 13 \end{bmatrix}$$

Shifting operation in Python

```
A = np.array([ [4,5,1],[0,1,11],[4,9,7] ])
s = 6
A + s # NOT shifting!
A + s*np.eye(len(A)) # shifting
```

- Only the diagonal elements change; the rest of the matrix is unadulterated by shifting.
- In practice, one shifts a relatively small amount to preserve as much information as possible in the matrix while benefiting from the effects of shifting, including increasing the numerical stability of the matrix
- This will be covered in more detail in later modules dealing with numerical Linear Algebra

Matrix Scalar Multiplication

Matrices can be multiplied by a scalar (constant or single element)

Let k be a scalar quantity; then **kA = Ak**

Ex. If k=4 and $A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \\ 2 & -3 \\ 4 & 1 \end{bmatrix}$

$$4 \times \begin{bmatrix} 3 & -1 \\ 2 & 1 \\ 2 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & 1 \\ 2 & -3 \\ 4 & 1 \end{bmatrix} \times 4 = \begin{bmatrix} 12 & -4 \\ 8 & 4 \\ 8 & -12 \\ 16 & 4 \end{bmatrix}$$

General Properties of Scalar Multiplication

Properties:

- $k(\mathbf{A} + \mathbf{B}) = k\mathbf{A} + k\mathbf{B}$
- $(k + g)\mathbf{A} = k\mathbf{A} + g\mathbf{A}$
- $k(\mathbf{AB}) = (k\mathbf{A})\mathbf{B} = \mathbf{A}(k)\mathbf{B}$
- $k(g\mathbf{A}) = (kg)\mathbf{A}$

Question

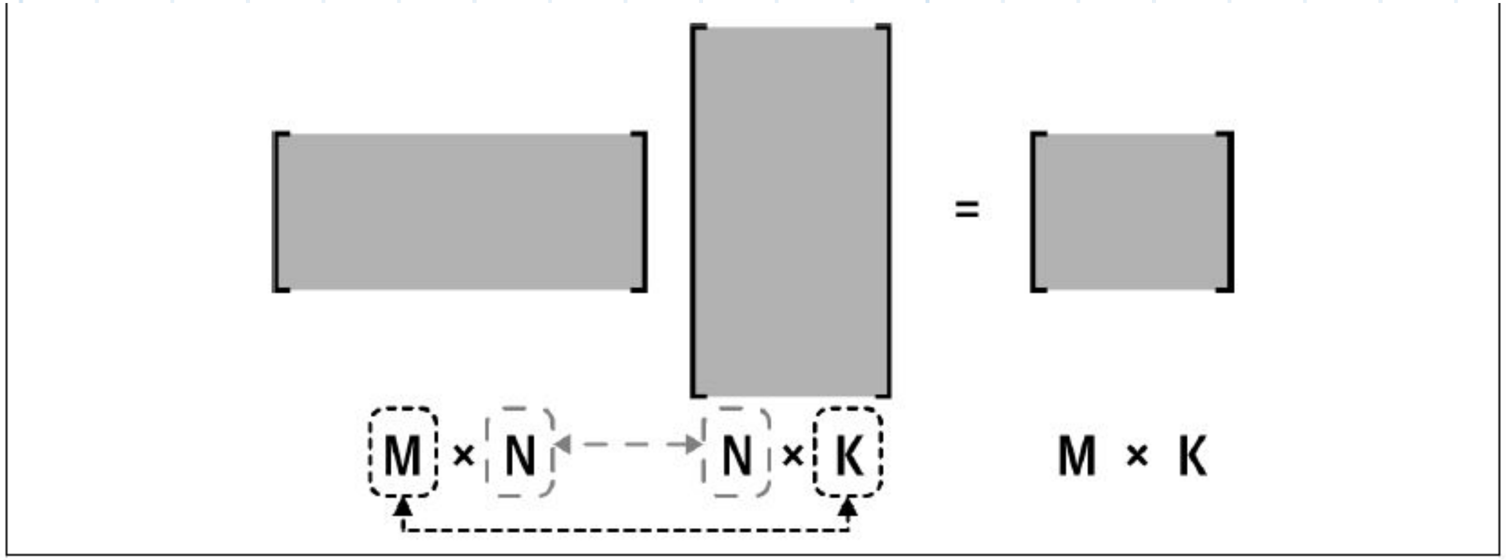
Shift the following matrices according to the specified

$$\text{a) } \begin{bmatrix} .4 & 3 & 0 \\ 2 & 1 & 0 \\ .1 & .4 & 0 \end{bmatrix}, \ \lambda = .1$$

$$\text{b) } \begin{bmatrix} 43 & 42 & 42 \\ 234 & 746 & 12 \\ 0 & 33 & 1001 \end{bmatrix}, \ \lambda = -1$$

Matrix Multiplication basics

- Matrix multiplication involves two different operations: multiplication and addition.



- Matrix multiplication validity, visualized. Memorize this picture.

Key Matrix Multiplication Properties

MULTIPLICATION OF MATRICES

The product of two matrices is another matrix

Two matrices **A** and **B** must be **conformable** for multiplication to be possible i.e., the number of columns of **A** must equal the number of rows of **B**

Example.

$$\underset{(1\text{x}3)}{\mathbf{A}} \quad \text{x} \quad \underset{(3\text{x}1)}{\mathbf{B}} \quad = \quad \underset{(1\text{x}1)}{\mathbf{C}}$$

Restrictions on Matrix Multiplication

B x **A** = **Not possible!**

(2x1) (4x2)

A x **B** = **Not possible!**

(6x2) (6x3)

Example

A x **B** = **C**

(2x3) (3x2) (2x2)

First Example of Matrix Multiplication

Consider the matrices

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}$$

Matrix Multiplication step by step

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \boxed{26} & \square \end{bmatrix}$$

$$\boxed{(2 \cdot 4) + (6 \cdot 3) + (0 \cdot 5) = 26}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix} = \begin{bmatrix} \square & \square & \square & \boxed{13} \\ \square & \square & \square & \square \end{bmatrix}$$

$$\boxed{(1 \cdot 3) + (2 \cdot 1) + (4 \cdot 2) = 13}$$

Matrix Multiplication step by step

The computations for the remaining entries are

$$
\begin{aligned}
(1 \cdot 4) + (2 \cdot 0) + (4 \cdot 2) &= 12 \\
(1 \cdot 1) - (2 \cdot 1) + (4 \cdot 7) &= 27 \\
(1 \cdot 4) + (2 \cdot 3) + (4 \cdot 5) &= 30 \\
(2 \cdot 4) + (6 \cdot 0) + (0 \cdot 2) &= 8 \\
(2 \cdot 1) - (6 \cdot 1) + (0 \cdot 7) &= -4 \\
(2 \cdot 3) + (6 \cdot 1) + (0 \cdot 2) &= 12
\end{aligned}
$$

$$
AB = \begin{bmatrix} 12 & 27 & 30 & 13 \\ 8 & -4 & 26 & 12 \end{bmatrix}
$$

$$
\begin{array}{ccccc}
A & & B & & AB \\
m \times r & & r \times n & = & m \times n
\end{array}
$$

Inside

Outside

More Example of Matrix Multiplication

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 7 \end{bmatrix}\begin{bmatrix} 4 & 8 \\ 6 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} (1\times4)+(2\times6)+(3\times5) & (1\times8)+(2\times2)+(3\times3) \\ (4\times4)+(2\times6)+(7\times5) & (4\times8)+(2\times2)+(7\times3) \end{bmatrix}$$

$$= \begin{bmatrix} 31 & 21 \\ 63 & 57 \end{bmatrix}$$

Remember also:

IA = A

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 31 & 21 \\ 63 & 57 \end{bmatrix} = \begin{bmatrix} 31 & 21 \\ 63 & 57 \end{bmatrix}$$

Question

Matrix Algebra Summary

Assuming that matrices **A**, **B** and **C** are conformable for the operations indicated, the following are true:

1. **AI** = **IA** = **A**
2. **A**(**BC**) = (**AB**)**C** = **ABC** - (associative law)
3. **A**(**B**+**C**) = **AB** + **AC** - (first distributive law)
4. (**A**+**B**)**C** = **AC** + **BC** - (second distributive law)

Caution!

1. **AB** not generally equal to **BA**, **BA** may not be conformable
2. If **AB** = **0**, neither **A** nor **B** necessarily = **0**
3. If **AB** = **AC**, **B** not necessarily = **C**

Faster Matrix Multiplication Algorithm is invaluable for AI and Machine learning applications

nature

Article | Open access | Published: 05 October 2022

Discovering faster matrix multiplication algorithms with reinforcement learning

Alhussein Fawzi, Matej Balog, Aja Huang, Thomas Hubert, Bernardino Romera-Paredes, Mohammadamin Barekatain, Alexander Novikov, Francisco J. R. Ruiz, Julian Schrittwieser, Grzegorz Swirszcz, David Silver, Demis Hassabis & Pushmeet Kohli

Nature **610**, 47–53 (2022) | Cite this article

Recap

- Scalar multiplication and shifting of matrices are cornerstones of numerically solving matrix equations.
- Matrix Multiplication Algorithm that is invaluable for AI, Machine learning and data science applications
- Matrix Multiplication is crucial for understanding eigenvalue problems
- Python coding of matrix operations using NumPy arrays provides a convenient way to approach solutions.

Coming up next................

Matrix Algebra: Transpose, Inverse and Determinants of a Matrix

Thank you