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AIL1020

Foundations of Statistics & Probability

Instructor

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Module 02

Measures of Central Tendency Contd.

Normal and paired datasets

Recap

Descriptive Statistics

Chebychev’s Inequality

In this video,

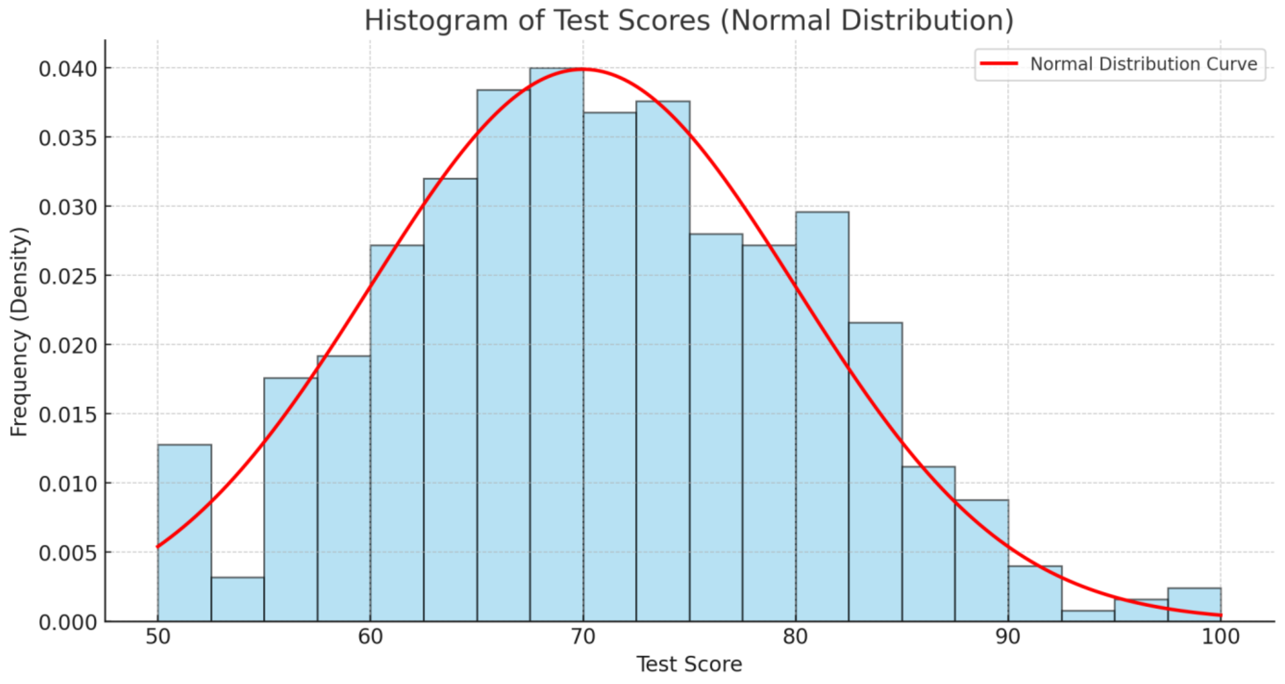
Normal Distribution of Dataset

Paired dataset

Normal Distribution (dataset)

The **normal distribution** describes how data points in a dataset tend to cluster around a central value, with most data points falling close to the mean and fewer occurring as you move further away.

This results in the characteristic **bell-shaped curve** when the data is plotted as a histogram.

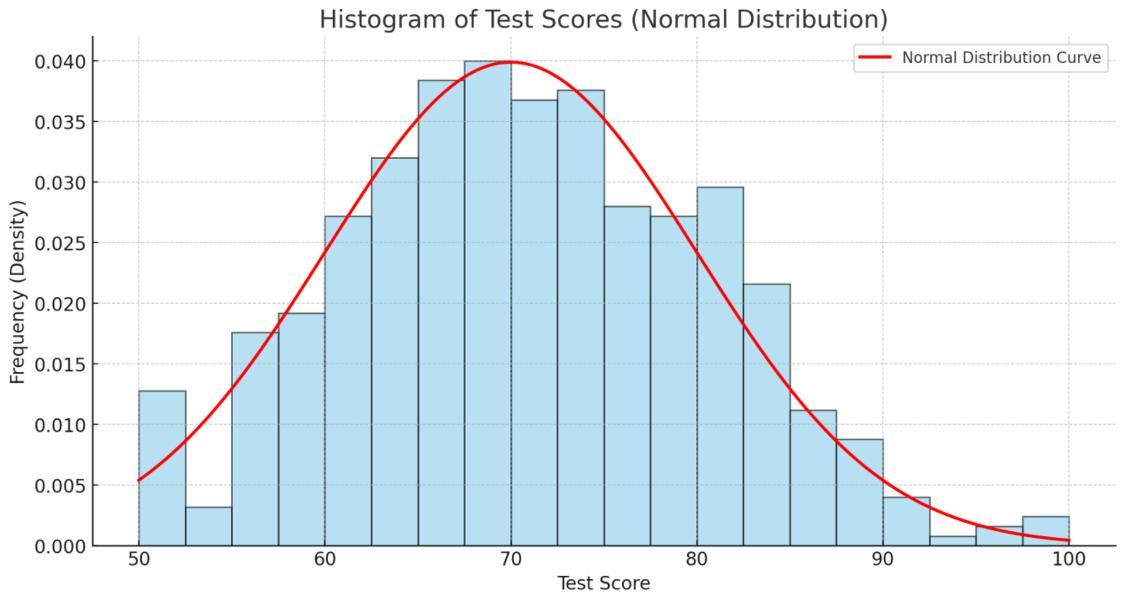


Key Properties of a Normally distributed Dataset

Symmetry: The dataset is evenly distributed around the mean.

Mean, Median, and Mode coincide.

Most data points fall within a predictable range.



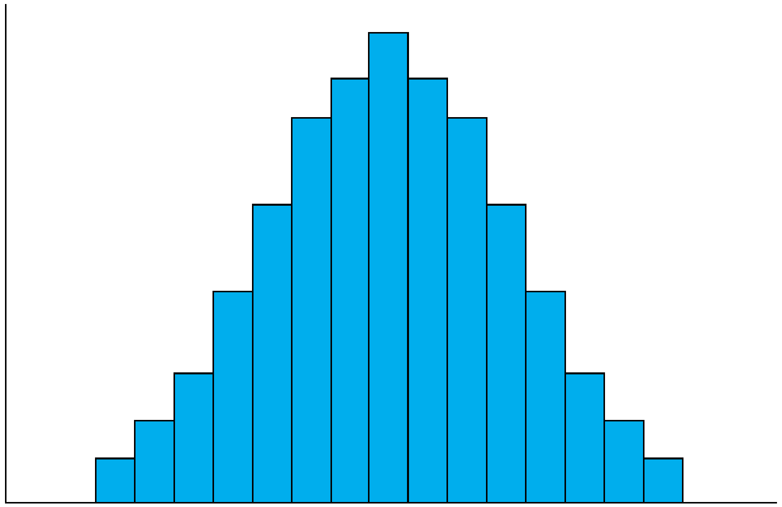
Visualization

A histogram of a dataset that follows a normal distribution shows a peak at the mean and gradually tapers off on both sides:

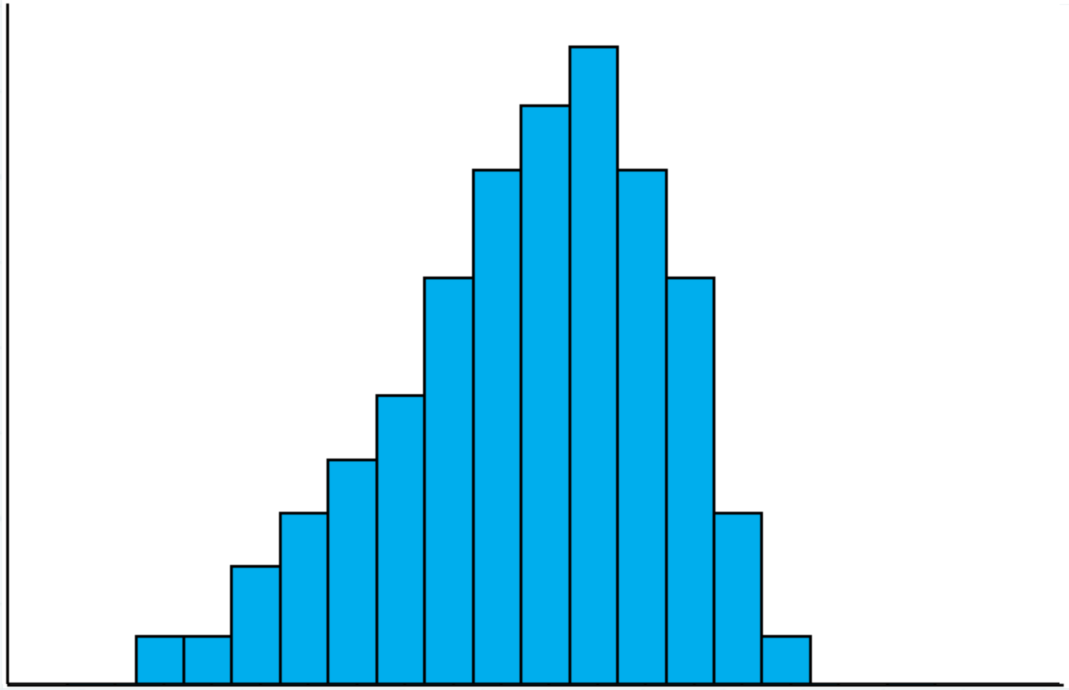
68% of the data falls within one standard deviation (σ) of the mean.

95% falls within two standard deviations.

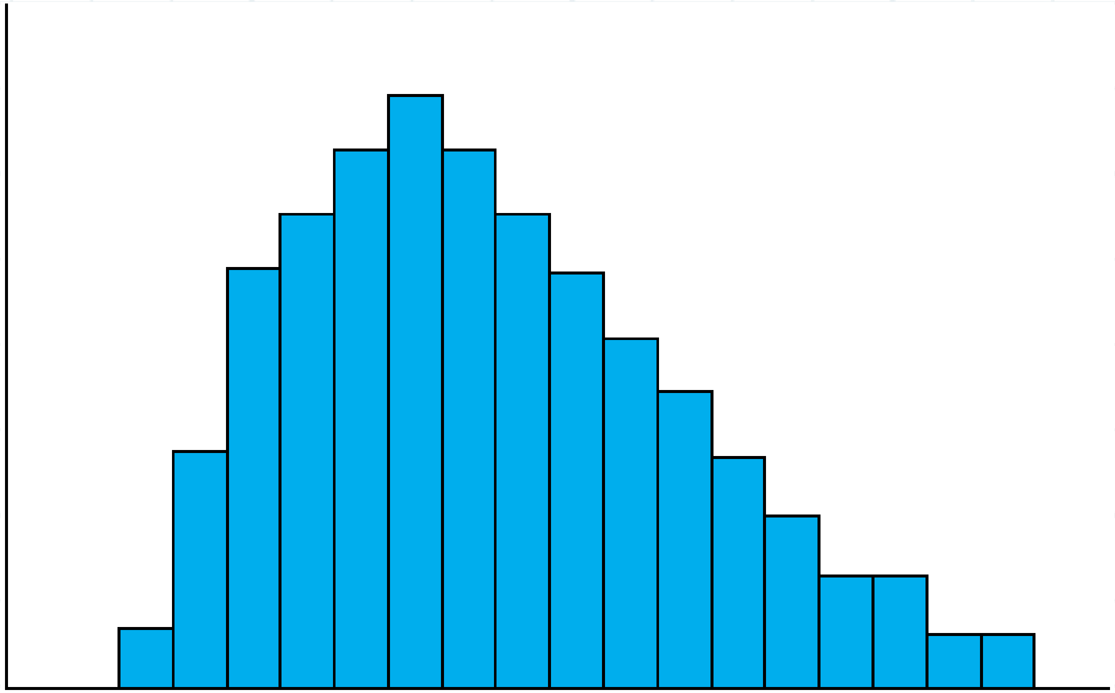
99.7% falls within three standard deviations.



Histogram of Dataset “Skewed”

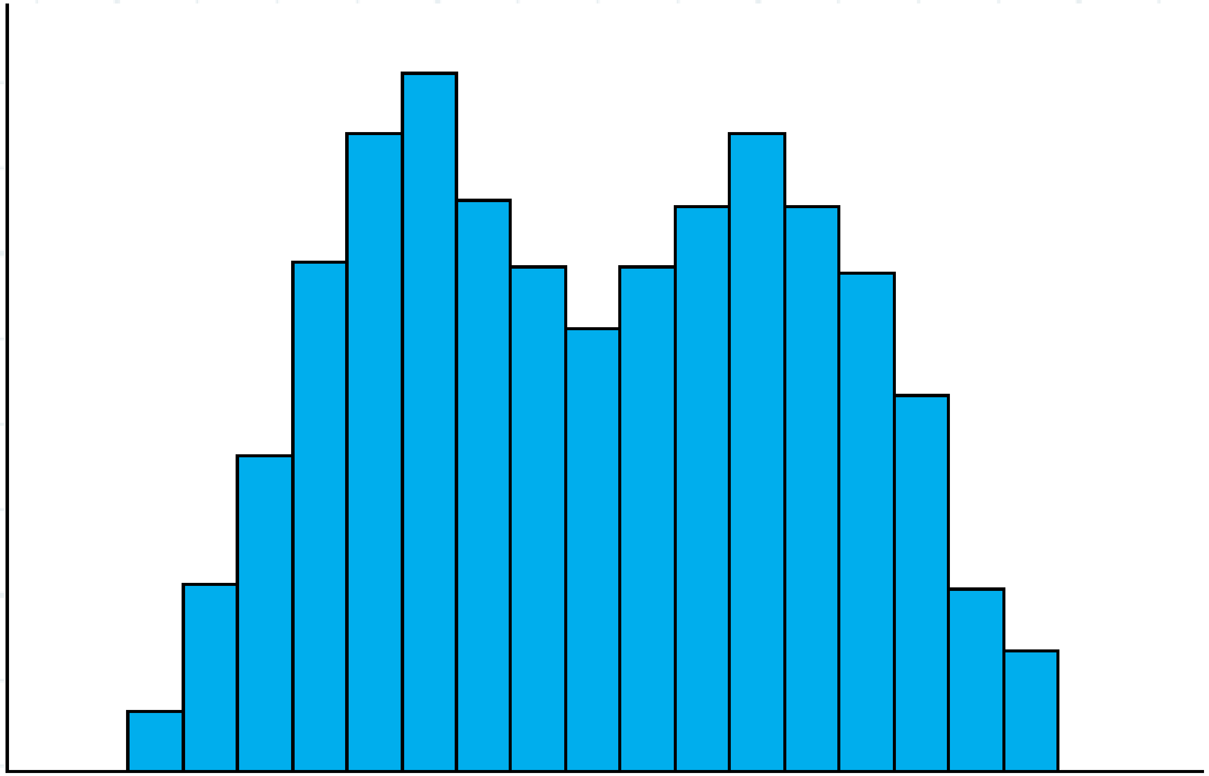


Histogram of dataset skewed to the left



Histogram of dataset skewed to the right

Bimodal Histogram



A histogram with two local peaks: *Bimodal* Dataset

Normal Distribution in Real-world Scenarios

Natural Phenomenon

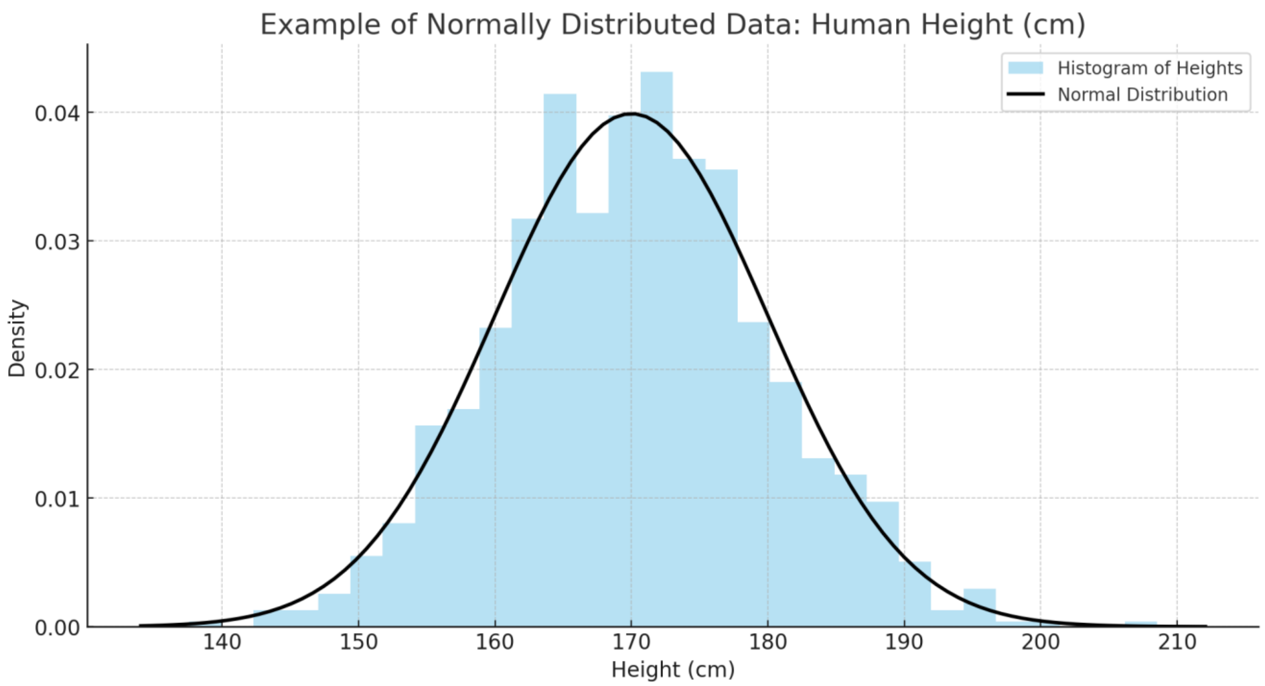
Measurement errors

Social and economic phenomenon

Biological processes

Industrial control processes

Financial data (in some cases)



Central Limit Theorem

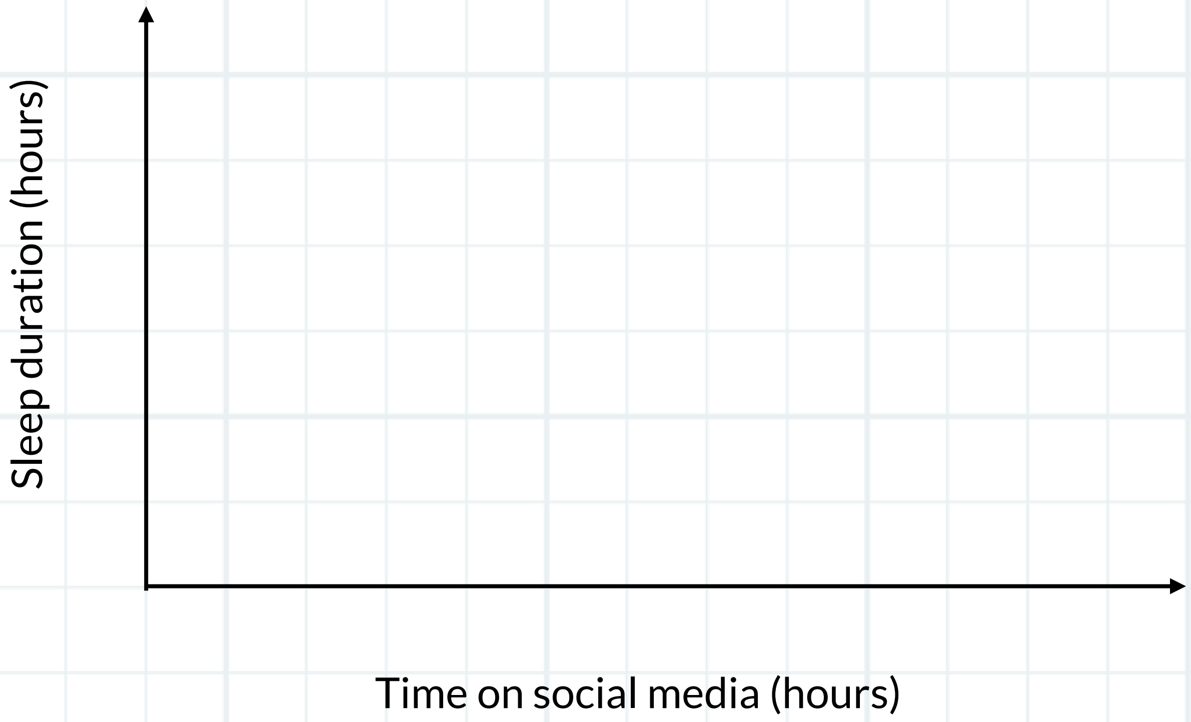
The Central Limit Theorem states that, given a sufficiently **large sample size**, the distribution of the sample mean (or sum) of a random variable will approach a **normal distribution (bell curve)**, **regardless of the original distribution** of the population.

This happens as the sample size *n* increases.

Paired Dataset

A **paired dataset** consists of two related variables collected for each individual or observation.

Paired datasets are best visualized using **Scatter plots**.



Example of Paired dataset

Friend	Time on Social Media (hours)	Sleep Duration (hours)
1	2	8
2	4	7
3	1	9
4	6	6
5	8	5
6	5	6.5
7	7	5.5
8	3	8
9	9	4.5
10	10	4

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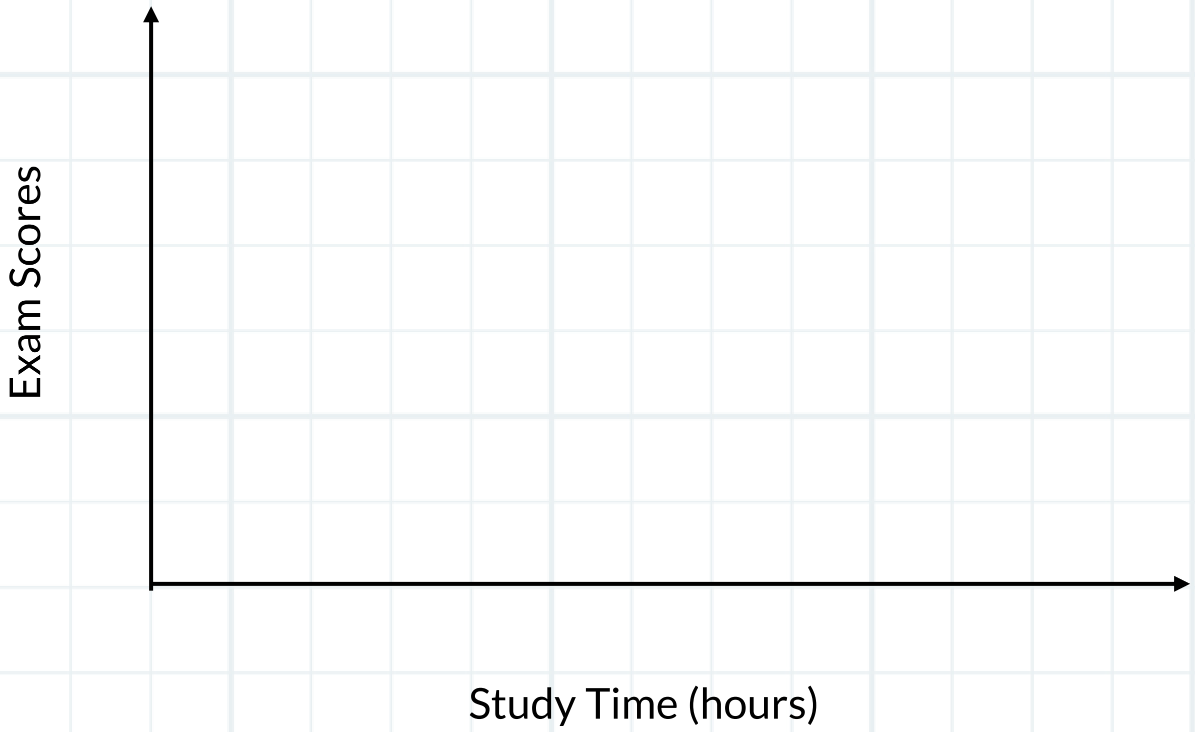


Another Example of Paired dataset

Friend	Study Time (hours)	Exam Scores
1	8.1	88.1
2	5.4	75.2
3	4.0	68.2
4	2.0	54.9
5	3.1	56.0
6	9.5	94.4
7	3.7	69.7
8	2.5	55.9
9	5.1	75.8
10	3.8	78.3

Scatter Plot

Scatterplots are a great visual tool to identify patterns.

The **shape** of the scatterplot represents the **direction and strength of the relationship** between the two variables.

Scatter Plot

***Slope* of the scatter plot**

A **steeper** slope (either positive or negative) usually indicates a **stronger correlation**.

A **flatter** slope (closer to zero) means a **weaker correlation**.

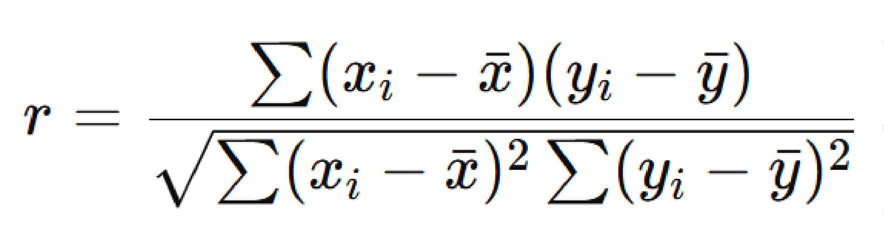
Descriptive Statistics Contd.

Correlation ≠ Causation

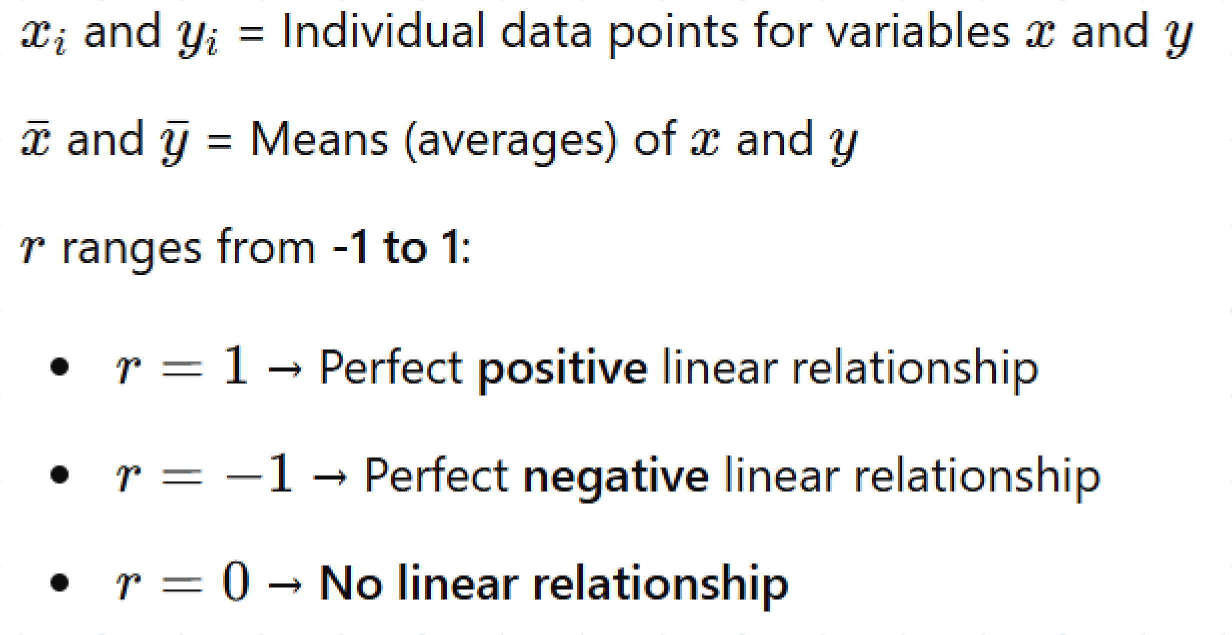
How can **correlation** be quantified?

Sample Correlation Coefficient (*r*)

The **sample correlation coefficient** measures the **strength and direction of a linear relationship** between two variables.



$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}}$$



x_i and y_i = Individual data points for variables x and y

$\bar{x}$ and $\bar{y}$ = Means (averages) of x and y

r ranges from **-1 to 1**:

- $r = 1$ → Perfect **positive** linear relationship
- $r = -1$ → Perfect **negative** linear relationship
- $r = 0$ → **No linear relationship**

Recap

Normal distribution

Paired dataset

- Scatterplots and correlation coefficients are great tools for visualizing and quantifying these patterns or relationships.

- Correlation helps identify relationships between variables, but you must always look for possible hidden factors before concluding causation.