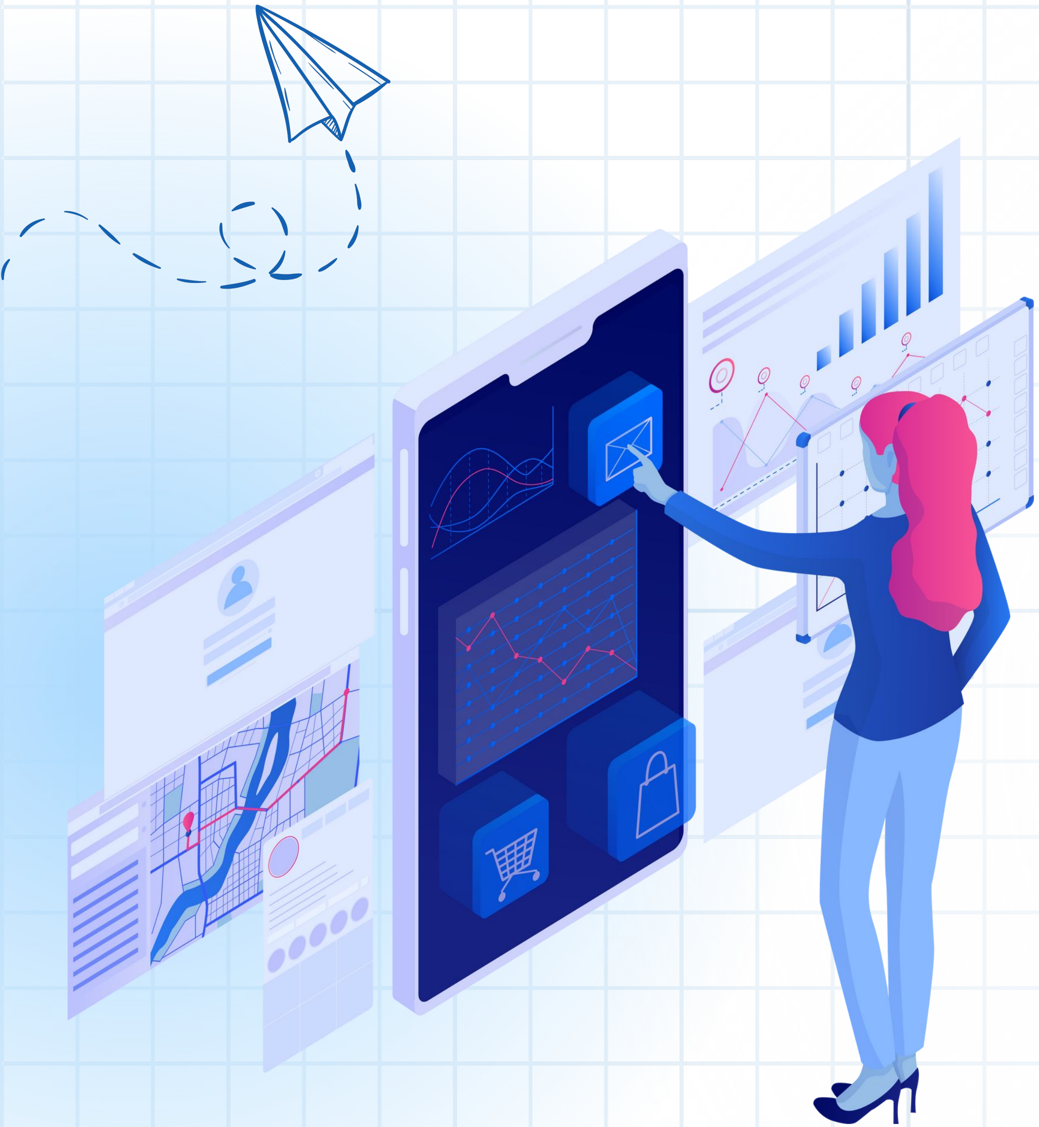
BS./BSc.

in Applied AI and Data Science

Linear algebra and numerical analysis

Module 2.1 Recap: Matrix Transpose



1. Matrix Algebra - Addition and Subtraction
2. Matrix Algebra – Shifting of Matrices & Scalar multiplication
3. Matrix Algebra - Matrix Multiplication
4. Transpose of a Matrix

Learning Objectives

Basics operations with Matrices

Understand the basics of Matrix Transpose

Everyday Examples

Recognize application of Matrix Transpose in everyday life

codes

Python code and syntax to solve Matrix Transpose

Hands on

Gain hands-on experience in solving simple Matrix Transpose problems

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Matrix transpose operations in daily life

“Matrix Algebra Is Everywhere! 🌍” Please see Examples below:

Data Analysis and Machine Learning

- **Scenario**:
  - Rows in a dataset represent **samples** (e.g., customers), and columns represent **features** (e.g., age, income, purchase history).
  - Some machine learning algorithms (e.g., Principal Component Analysis) require features to be rows and samples to be columns.
- **Solution**:
  - Transpose the matrix to switch rows (samples) and columns (features).

Transposing with matrix

- Transposing a matrix is like telling all the elements in the matrix to switch places.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 6 & 11 \\ 2 & 7 & 12 \\ 3 & 8 & 13 \\ 4 & 9 & 14 \\ 5 & 10 & 15 \end{bmatrix}$$

Matrix Transpose operation

To transpose:

Interchange rows and columns

The dimensions of $\mathbf{A}^T$ are the reverse of the dimensions of $\mathbf{A}$

$$A = {}_2A^3 = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 3 & 1 \end{bmatrix} \qquad 2 \times 3$$

$$A^T = {}_3A^{T\,2} = \begin{bmatrix} 2 & 5 \\ 4 & 3 \\ 7 & 1 \end{bmatrix} \qquad 3 \times 2$$

Matrix Transpose operation in Python

To transpose:You can easily take the transpose of a matrix using NumPy in Python. Here's how you can do it:

```
A = np.array([ [3,4,5],[1,2,3] ])
A_T1 = A.T # as method
A_T2 = np.transpose(A) # as function
```

```
print("Original Matrix:") print(matrix)
print("\nTranspose of the Matrix:")
print(transpose_matrix)
```

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Matrix Operations: LIVE EVIL (Order of Operations)

To transpose:

Matrix Operations: LIVE EVIL (Order of Operations)

$$(\mathbf{LIVE})^T = \mathbf{E}^T\mathbf{V}^T\mathbf{I}^T\mathbf{L}^T$$

$$\mathbf{A}^T = \begin{bmatrix} 0 & -4 & 9 & -8 \\ -1 & 6 & 5 & 8 \\ 8 & -10 & -2 & 4 \\ -7 & -9 & -3 & 7 \\ -4 & 6 & 5 & 7 \end{bmatrix}.$$

Question

Compute $(\mathbf{AB})^T$ and $\mathbf{B}^T\mathbf{A}^T$ if

$$\mathbf{A} = \begin{bmatrix} -2 & 1 & 0 \\ 3 & -1 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & 1 & 2 \\ -1 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

Solution

Solution. Compute $\mathbf{AB}$:

$$\mathbf{AB} = \begin{bmatrix} -2 & 1 & 0 \\ 3 & -1 & -3 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ -1 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 & -4 \\ -5 & 5 & 9 \end{bmatrix}$$

Next compute $\mathbf{B}^T\mathbf{A}^T$:

$$\mathbf{B}^T\mathbf{A}^T = \begin{bmatrix} -2 & -1 & 0 \\ 1 & -2 & 0 \\ 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 1 & -1 \\ 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -5 \\ -4 & 5 \\ -4 & 9 \end{bmatrix} = (\mathbf{AB})^T$$

Applying Matrix Algebra to a real-world problem: Matrix Power

Suppose that the 2004 state of land use in a city of 60 mi^2 of built-up area is

C: Commercially Used 25%   I: Industrially Used 20%   R: Residentially Used 55%.

Find the states in 2009, 2014, and 2019, assuming that the transition probabilities for 5-year intervals are given by the matrix $\mathbf{A}$ and remain practically the same over the time considered.

$$
\begin{array}{cc}
 & \begin{array}{ccc} \text{From C} & \text{From I} & \text{From R} \end{array} \\
\mathbf{A} = & \begin{bmatrix} 0.7 & 0.1 & 0 \\ 0.2 & 0.9 & 0.2 \\ 0.1 & 0 & 0.8 \end{bmatrix}
\end{array}
\begin{array}{l} \text{To C} \\ \text{To I} \\ \text{To R} \end{array}
$$

Solution

A is a stochastic matrix, that is, a square matrix with all entries nonnegative and all column sums equal to 1. Our example concerns a Markov process, that is, a process for which the probability of entering a certain state depends only on the last state occupied (and the matrix A), not on any earlier state.

Solution. From the matrix A and the 2004 state we can compute the 2009 state,

$$\begin{array}{c} C \\ I \\ R \end{array} \begin{bmatrix} 0.7 \cdot 25 + 0.1 \cdot 20 + 0 \cdot 55 \\ 0.2 \cdot 25 + 0.9 \cdot 20 + 0.2 \cdot 55 \\ 0.1 \cdot 25 + 0 \cdot 20 + 0.8 \cdot 55 \end{bmatrix} = \begin{bmatrix} 0.7 & 0.1 & 0 \\ 0.2 & 0.9 & 0.2 \\ 0.1 & 0 & 0.8 \end{bmatrix} \begin{bmatrix} 25 \\ 20 \\ 55 \end{bmatrix} = \begin{bmatrix} 19.5 \\ 34.0 \\ 46.5 \end{bmatrix}.$$

Solution continued.............

To explain: The 2009 figure for C equals 25% times the probability 0.7 that C goes into C, plus 20% times the probability 0.1 that I goes into C, plus 55% times the probability 0 that R goes into C. Together,

$$25 \cdot 0.7 + 20 \cdot 0.1 + 55 \cdot 0 = 19.5\,[\%]. \qquad \text{Also} \qquad 25 \cdot 0.2 + 20 \cdot 0.9 + 55 \cdot 0.2 = 34\,[\%].$$

Similarly, the new R is 46.5%. We see that the 2009 state vector is the column vector

$$\mathbf{y} = [19.5 \quad 34.0 \quad 46.5]^{\mathsf{T}} = \mathbf{Ax} = \mathbf{A}\,[25 \quad 20 \quad 55]^{\mathsf{T}}$$

where the column vector $\mathbf{x} = [25 \quad 20 \quad 55]^{\mathsf{T}}$ is the given 2004 state vector. Note that the sum of the entries of $\mathbf{y}$ is 100 [%]. Similarly, you may verify that for 2014 and 2019 we get the state vectors

$$\mathbf{z} = \mathbf{Ay} = \mathbf{A}(\mathbf{Ax}) = \mathbf{A}^2\mathbf{x} = [17.05 \quad 43.80 \quad 39.15]^{\mathsf{T}}$$

$$\mathbf{u} = \mathbf{Az} = \mathbf{A}^2\mathbf{y} = \mathbf{A}^3\mathbf{x} = [16.315 \quad 50.660 \quad 33.025]^{\mathsf{T}}.$$

Properties of Transposed Matrix

Properties of transposed matrices:

1. $(A+B)^T = A^T + B^T$
2. $(AB)^T = B^T A^T$
3. $(kA)^T = kA^T$
4. $(A^T)^T = A$

Properties of Transposed Matrix Example

$$(A * B)^T = B^T * A^T$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 2 \\ 4 & 5 & 6 \end{bmatrix}, A*B = \begin{bmatrix} 9 & 10 & 14 \\ 19 & 20 & 30 \\ 29 & 30 & 46 \end{bmatrix}, (A*B)^T = \begin{bmatrix} 9 & 19 & 29 \\ 10 & 20 & 30 \\ 14 & 30 & 46 \end{bmatrix},$$

$$A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}, B^T = \begin{bmatrix} 1 & 4 \\ 0 & 5 \\ 2 & 6 \end{bmatrix}, \text{and } B^T * A^T = \begin{bmatrix} 9 & 19 & 29 \\ 10 & 20 & 30 \\ 14 & 30 & 46 \end{bmatrix}$$

Question

Recap

- The transpose of a matrix is created by interchanging its rows and columns. Think of it as "flipping" the matrix over its main diagonal (the diagonal from the top-left to the bottom-right corner).
- Matrix transpose is a fundamental operation in linear algebra with applications in **Data manipulation,** reshaping data, especially in machine learning, **Geometry:** Representing reflections and rotations, **solving systems of equations, Computer graphics, and much more!**
- The order of the matrix changes when you take the transpose (unless it's a **square matrix**).

Coming up next................

Matrix Algebra: Symmetric and Square matrices, Inverse of a Matrix and Determinants

Thank you