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Emergent dynamics of a macroeconomic agent based model with capital and credit

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A B S T R A C T

In this paper we present and discuss a Macroeconomic Agent-Based Model with Capital and Credit (CC-MABM) which builds upon the framework put forward by Delli Gatti et al. (2011). The novelty of this model with respect to the previous framework consists in the introduction of a stylized supply chain where upstream firms – i.e. producers of capital goods (K-firms) – supply a *durable and sticky input* (capital) to the downstream firms, who produce consumption goods (C-firms) to be sold to households. Both C-firms and K-firms resort to bank loans to satisfy their financing needs. There are two-way feedbacks between firms and markets which yield interesting emerging properties at the macro level. We show that the interaction of upstream and downstream firms and the evolution of their financial conditions – in a nutshell: *Capital and Credit* – are essential ingredients of a "crisis" i.e. a sizable slump followed by a long recovery.

1. Introduction

In macroeconomic model building, very often researchers start from simple models where capital goods are absent. This is the case, for instance, of the simplest New Keynesian DSGE model presented in Clarida et al. (1999) (hereafter CGG) where firms produce only consumption goods using only labor as an input. This is also the case, in the agent based literature, of the Macro Agent Based Model (MABM) developed in Delli Gatti et al. (2011).[1]

Toy models in which the only final use of goods is consumption and the only input is labor are easier to interpret and sometimes sufficient to answer deep research questions but they are surely inadequate when one wants to replicate empirical business cycle facts. As it is well known, in fact, changes in capital and in inventories play a major role in shaping the dynamic pattern of GDP. The first reason why we want to move up the ladder of complexity in macroeconomic model building, therefore, is simply *realism*, i.e. the need to reproduce macroeconomic reality as closely as possible.

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[1] In CGG, macroeconomic equilibrium yields the equality of consumption expenditure and aggregate output (so that involuntary inventories and saving are absent). In Delli Gatti et al. (2011) the macroeconomic equilibrium condition is not imposed ex ante (i.e. there is no *top down coordinating device*, as often assumed in agent based models) so that aggregate output may be absorbed by consumption expenditure or end up in involuntary inventories. Hence, by construction, aggregate saving is equal to inventories.

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A second and no less important reason is that the financing decisions of firms enter into the picture in a significant way only when investment is considered. The balance sheet of the firm is, in this case, properly defined: capital and liquidity show up in the assets' side, external finance (debt) on the liabilities side, equity or net worth being defined by the difference between the two. Decisions concerning investment and decisions concerning *financial structure* of the investing firm, in fact, are deeply interrelated. Capital should be incorporated into a macroeconomic model if we want to take into account financial factors in an appropriate conceptual setting.[2]

The NK literature has rapidly gone beyond the simple CGG framework. Starting from the pioneering work of Bernanke et al. (1999) (hereafter BGG), a large literature has developed which takes investment and financing decisions into account in the presence of *financial frictions*. The architecture of these models is rather sophisticated (even if they retain the representative agent assumption), in some cases so complicated that the overall picture becomes blurred and the interpretation of results difficult.

In the agent based literature, too, macroeconomic models have incorporated capital and investment. For instance, in the *EURACE* framework (Cincotti et al., 2010; Dawid et al., 2012) firms need heterogeneous capital goods and heterogeneous labor services to produce consumption goods. The use of high quality capital goods requires the employment of skilled workers, hence qualities of capital goods and skills of workers are complements in production. In the *Keynes meeting Schumpeter* (K&S) framework, Dosi et al. (2010) assume that firms use machine tools of different vintages and with different productivities to produce consumption goods. Also in the computational literature, therefore, the architecture of macroeconomic models is sometimes so complicated that the resulting emerging properties become difficult to explore.

In this paper we present a MABM *with Capital and Credit* (CC-MABM) which builds upon the MABM developed in Delli Gatti et al. (2011). In the CC-MABM there are four categories of agents: households, firms producing consumption goods (C-firms), firms producing capital goods (K-firms) and banks. The corporate sector describes a stylized supply chain: the *upstream sector*, consisting of K-firms that supply a *durable and sticky input* (capital) to the *downstream sector* consisting of C-firms. Both C-firms and K-firms resort to bank loans to satisfy their financing needs.

The CC-MABM may be considered as a simpler and shorter route (with respect to EURACE or K&S) to introduce capital and investment in a MABM. In our model capital goods (and labor) are not differentiated: the productivity of labor and of capital is uniform across firms and workers. This relatively simple architecture, however, generates two-way feedbacks between markets and sectors which yield interesting emerging properties at the macro level.

The time series of GDP computed on artificial data fluctuates around a "long run mean" for an extended time window which we characterize as *normal times*. Occasionally, however, GDP falls dramatically bottoming out only after many periods of contraction. At the trough, GDP and employment are at least 15% lower than the pre-recession level. The recovery, then, is slow and painful; it takes a long time for GDP and employment to go back to normal. A dramatic slump followed by a long recovery is a *crisis* in our terminology.

Where does a crisis come from? We show that the interaction of upstream and downstream firms and the evolution of their financial conditions – in a nutshell: *Capital and Credit* – are essential ingredients of a "crisis". If they were absent the volatility of GDP would be limited and no sizable slump would occur.

The paper is organized as follows. Section 2 briefly surveys the literature. In Section 3 we sketch the basic features of the model. In Sections 4, 5, 6 and 8 we present the assumptions concerning the behavior of households, C-firms, K-firms and banks respectively. In Section 7 we define the financing gap and the demand for loans. Section 9 is devoted to a discussion of the accounting framework and the interrelated balance sheets of the main categories of agents. Section 10 is devoted to a discussion of the results of the simulations. Section 11 concludes.

2. Related literature

The need to move up in the ladder of increasing complexity by incorporating capital and credit in macroeconomic models is evident both in NK and in AB literatures.[3]

In the NK-DSGE literature, the canonical model popularized by CGG in which there are only consumption goods, the only input is labor and financing decisions are absent has been superseded by more sophisticated models *with capital* and *financial frictions* starting from BGG. A large literature has developed from this pioneering paper. A couple of recent and remarkable additions to this literature are Christiano et al. (2010) (hereafter CMR), Gertler and Kiyotaki (2010) (hereafter GK).

In BGG (i) "entrepreneurs" (or wholesale goods producers) use capital and labor to produce and sell a homogeneous good to "retailers", (ii) retailers differentiate the good and sell final goods to households (in the form of consumption goods) or to capital producing firms (K-firms) (in the form of investment goods), (iii) K-firms use investment goods and undepreciated capital (sold by entrepreneurs to K-firms) to produce and sell new capital to the entrepreneurs.

[2] In Delli Gatti et al. (2011), in the absence of investment, the firm's financing decision consists in funding the wage bill (a setting which recalls the classical wages-fund theory). The firm seeks external finance (bank loans) to complement internal funds (net worth) in order to anticipate wages to employees. In this simplified setting, by construction, only liquidity shows up in the assets side of the firm's balance sheet, which is then used to pay wages.

[3] The remark does not apply to Real Business Cycle models in which capital is playing a key role by construction. Financing decisions, however, are generally overlooked in relatively simple RBC models.

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In CMR the architecture is even more complex: (i) intermediate goods firms use capital and labor to produce and sell differentiated goods to final goods producers, (ii) final goods producers in turn sell final goods to households (consumption goods) and to K-firms (investment goods), (iii) K-firms use investment goods and undepreciated capital to produce and sell new capital to the entrepreneurs, (iv) finally entrepreneurs rent capital to intermediate goods producers. As far as financing decisions are concerned, both in BGG and in CMR entrepreneurs ask for loans to financial intermediaries in order to finance the difference between the cost of investment and internal resources (net worth).

In GK, this part of the architecture is streamlined: (i) final goods producers use capital and labor to produce and sell undifferentiated final goods to households (consumption goods) and to K-firms (investment goods), (ii) K-firms use investment goods and undepreciated capital to produce and sell new capital to the final goods producers. As far as financing is concerned, it is the final goods producers who ask for loans to financial intermediaries.

There are different ways of modeling capital also in the AB literature. For instance, in the *EURACE* framework (Cincotti et al., 2010, 2012; Deissenberg et al., 2008; Dawid and Neugart, 2010; Dawid et al., 2012) final goods producers use capital goods (supplied by K-firms) characterized by different qualities/productivities and hire workers characterized by different skills to produce and sell consumption goods to households. Moreover, the technological frontier in the production of capital goods is pushed up systematically over time by a stochastic (innovation) process.

Along similar lines, in the K&S framework, Dosi et al. (2014, 2013, 2010) and Napoletano et al. (2012) assume that consumption goods producers use labor and machine tools (supplied by K-firms) to produce and sell consumption goods to households. Machine tools are characterized by different productivities. K-producers, in fact, spend resources in R&D to discover new products and increase the productivity of the machines they produce. In both the EURACE and the K&S frameworks, embodied technical progress plays a key role in driving fluctuations and (long run) growth in the model.

In the present paper we introduce capital and investment in the MABM developed by Delli Gatti et al. (2011) in a more straightforward (but less realistic) way. First, capital goods (and labor) are not differentiated. Second, firms producing capital goods do not engage in R&D and therefore technical progress does not trickle down in the macro economy through machine tool purchases on the part of final goods producers. The reason for this simpler approach is in itself simple: we want to explore only the role of capital – characterized as a *durable and sticky* input – in shaping GDP fluctuations at business cycle frequency. We are not interested (for the moment) in modeling the changes in GDP at low frequencies. Thanks to this different research question, we end up with a much leaner architecture: in a stylized supply chain K-firms play the role of *upstream suppliers* of a durable and sticky input (capital) to C-firms, who play the role of *downstream customers* of K-firms and suppliers of consumption goods to households. Both C-firms and K-firms resort to bank loans to fill their financing gap. This modeling approach has remarkable repercussions on the properties of the business cycle as we will see in the section on simulations.

3. The model

We consider an economy populated by households, firms and banks. The household sector consists of workers and "capitalists". Workers supply labor and buy consumption goods. Capitalists are the owners of firms (for simplicity we assume that there is one capitalist per firm). They get dividends and buy consumption goods (therefore they behave as *rentiers*). Both workers and capitalists save and accumulate financial wealth in the form of deposits at banks.

The corporate sector consists of producers of consumption goods (C-firms) and producers of capital goods (K-firms). C-firms demand labor and capital goods in order to produce and sell consumption goods to households. For simplicity we assume that K-firms do not need durable inputs: they demand only labor in order to produce and sell capital goods to C-firms. Technology, however, is linear in both cases: in the C-sector, capital and labor are perfect complements (Leontief production function); in the K-sector the productivity of labor is given and constant. Hence total cost is linear in production and marginal cost is given and constant. If internal financial resources are not sufficient to finance costs firms resort to bank loans to fill the financing gap.

Banks receive deposits from households and extend loans to firms. For the sake of simplicity, we reduce the cardinality of the set of banks to one.[4] Households' deposits are not remunerated. As to loans, the bank has to decide both the price (interest rate) and the quantity of credit to be extended to each borrowing firm. The bank will provide funds on demand (up to a maximum amount) to each firm at an interest rate which is different from one borrower to another depending on the firm's financial fragility.

There are markets for consumption goods, capital goods, labor, credit and deposits. Fig. 1 presents a visualization of the macro-economy.

The markets for goods and labor are characterized by a *search and matching* mechanism. Since we have assumed that there is only one bank, the credit and deposits markets are the only markets which are not modeled by means of a search and matching mechanism.

We assume that on the markets for goods and labor neither sellers nor buyers have complete information about business conditions, e.g. posted prices and available quantities. They incur *transaction costs* to explore the market and therefore engage in *local interaction* with a finite set of potential trading partners. Each consumer (worker, capitalist) visits a random

[4] This simplifying assumption can be easily relaxed. In Delli Gatti et al. (2011), for instance, there is a finite set of banks.

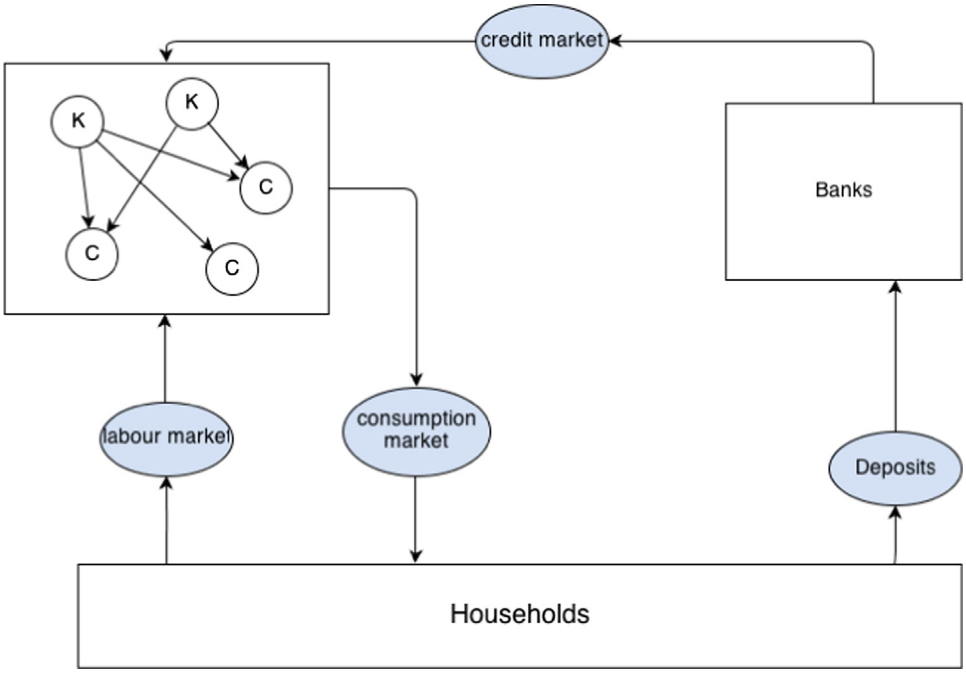

Fig. 1. The macroeconomy: agents and markets.

sample of C-firms, ranks them in ascending order of price and demands C-goods starting from the firm who has posted the lowest price. This search and matching mechanism implies that firms charging lower prices will receive larger orders and will be more likely to find buyers. A similar mechanism is at work also in the market for capital goods. Each C-firm selects at random a set of K-producers, ranks them in ascending order of price and demands capital goods starting from the firm who has posted the lowest price.

Both C-firms and K-firms have market power on their local market. In the present framework, this is due to transaction costs which force buyers to visit only a limited number of sellers so that goods, in the buyers' eyes, are imperfect substitutes even if they are not differentiated.[5]

Thanks to market power, each firm behaves as a monopolist on her own local market. If the firm knew the demand curve, she would choose an optimal mark up (i.e. ratio of the individual price to marginal cost) and an optimal scale of activity, profits would be maximized and demand would be always equal to supply. We assume, however, that *the firm does not know the demand curve* so that she cannot choose her mark up optimally. Since technology is linear, however, once the firm has chosen the price (and therefore the mark up) she will try to maximize output, the only constraint being the demand for her good.[6] She may end up, however, with an inventory of unsold goods – if she has overestimated demand – or a queue of unsatisfied consumers (if demand has been underestimated). Both situations are costly for the firm, so that we can assume that the firm is always adapting to market conditions in order to minimize the distance between production and demand. Moreover, due to implicit Bertrand competition, she will also try to minimize the distance between her own price and the average price charged by competitors.

On the labor market firms post-vacancies and unemployed workers visit a random sample of firms in search of a job. For simplicity, in this paper we assume that the wage posted is constant and uniform across firms.[7] As a consequence unemployed workers do not rank firms and stop searching whenever a matching opportunity emerges, i.e. when a firm they visit has posted a vacancy.

Notice that in the markets considered so far (labor and goods) in each period the searching agent selects at random *a new set* of potential trading partners. Consumer C_1, for instance, may visit firms F_1 and F_4 today and firms F_2 and F_5 tomorrow. Therefore there is no "brand loyalty" which links a specific consumer to a specific firm in a sort of implicit partnership. The network of transactions (if any) generated by this mechanism is reshuffled every period. Due to search and matching and local interaction, on the market for C-goods and K-goods unsold goods at some firms may coexist with queues of unsatisfied

[5] Of course we could assume that market power is due to product differentiation – as in the standard Dixit and Stiglitz (1977) model of monopolistic competition.

[6] In so doing, the firm behaves as a myopic optimizer (she is indeed doing her best to maximize profits period by period).

[7] This is of course a simplifying assumption. In Delli Gatti et al. (2011), for instance, each firm offers a wage consisting of (i) a uniform minimum wage which is indexed to inflation and may capture the effect of nationwide collective bargaining and (ii) a firm-specific component which is increasing with the number of vacancies posted.

customers at some other firms. Moreover, some households may end up spending less than planned and accumulating involuntary saving. On the labor market redundant labor force at some firms and unemployment may coexist with unfilled vacancies at other firms.

4. Households

The household sector consists of workers and capitalists.

4.1. Workers

Workers are suppliers on the labor market and buyers on the C-market.

The h th worker $(h=1,2,\ldots,H)$ supplies inelastically one unit of labor.

If unemployed the worker looks for a job on the labor market by visiting Z_e firms chosen at random and applying to those with open vacancies. Since the posted wage is given and uniform across firms, the unemployed worker will accept a job from the first firm with open vacancies he has the chance to visit. If he does not find a vacancy to fill, he remains unemployed and will fund his consumption (see below) dis-saving, i.e. consuming out of accumulated wealth.

If he succeeds in finding a job, he will receive the wage w until he is fired. For simplicity we assume that there are neither hiring nor firing costs.[8] If a firm wants to scale down activity in a certain period, she can fire workers at no costs. Fired workers become unemployed and start searching for a job in the same period.

4.2. Capitalists

For simplicity, we assume that each firm (both in the C-sector and in the K-sector) is owned by a capitalist. There are therefore as many capitalists as there are firms, say F.[9] Each capitalist receives income in the form of dividends if the firm he owns is making profits after interest payments. Dividends accruing to the fth capitalist are defined as $\tau\pi_{f,t-1}$ where $\tau\in(0,1)$ is the dividend payout ratio and $\pi_{f,t}$ are profits. If the firm makes losses that deplete her net worth, she will go bankrupt and be replaced by a new firm. In this case we assume that the owner of the bankrupt firm employs his personal wealth to provide equity to the entrant firm. In other words, the capitalist is de facto re-capitalizing the defaulting firm to make her survive. Each firm is therefore a dynasty and the number of firms is constant.[10]

4.3. Consumers

In the C-market, workers and capitalists together behave as F+H consumers. In each period the cth consumer receives income $Y_{c,t}$ where

$$Y_{c,t}=\begin{cases} w & \text{if the consumer is a worker with an active labor contract,}\\ \tau\pi_{f,t-1} & \text{if the consumer is a capitalist receiving dividends.}\end{cases} \tag{4.1}$$

In a bounded rationality setting, consumers' behavior follows a rule of thumb (heuristic) which is based on the following ingredients.

First, the consumer estimates *human wealth* $\overline{Y}_{c,t}$ – which is a proxy for future expected income – using an adaptive mechanism: $\overline{Y}_{c,t}=\xi\overline{Y}_{c,t-1}+(1-\xi)Y_{c,t}$ where $\xi\in(0,1)$ is a memory parameter. By iterating, it is easy to see that human wealth is a weighted average of current and past incomes with exponentially decaying weights.

Second, the consumer determines the *budget allocated to consumption*: $C_{c,t}=\overline{Y}_{c,t}+\chi D_{c,t}$ where $D_{c,t}$ is the consumer's non-human (i.e. financial) wealth (deposited at the bank) and $\chi\in(0,1)$ is the fraction of financial wealth used in consumption.[11] If the consumer does not receive income – for instance because a worker becomes unemployed or a capitalist does not

[8] This simplifying assumption can be easily relaxed. The presence of hiring and firing costs would reduce labor mobility and lead to labor hoarding. We leave the exploration of this issue to future research.

[9] $F=F_c+F_k$ where F_c is the number of C-firms and F_k is the number of K-firms.

[10] By splitting the household sector into workers and capitalists, we have simplified the problem of firms' ownership, dividend distribution and recapitalization of defaulting firms to the greatest possible extent. As an alternative, we could have maintained that households supply labor and own firms at the same time. This alternative assumption, however, would have posed the problem of attributing ownership rights, dividends and recapitalization commitments to heterogeneous households.

[11] This simple rule of thumb is reminiscent of the permanent income/life cycle hypothesis. As it is well known, inter-temporal optimization and rational expectations yield the following results: (i) current consumption is equal to *permanent income*, (ii) permanent income is proportional to the sum of financial wealth and human wealth where the latter is the present value of future expected incomes, (iii) the coefficient of proportionality (of permanent income to wealth) is a function of the real interest rate and the rate of time preference (under some restrictions it coincides with the real interest rate). The interpretation is straightforward: consumption is equal to the annuity value of total (financial and human) wealth, i.e. permanent income. A rough approximation of this coefficient, determined in an inter-temporally optimal way, is something around 5%. Carrol (2009, 1997, 1992) has shown that in the presence of uncertainty about future labor incomes and precautionary savings, consumption is a concave function of *cash-on-hand* i.e. the sum of human capital and beginning of period wealth (Deaton, 1991). In Carroll's framework, for low values of wealth, the propensity to consume (out of wealth) is quite high due to the precautionary saving motive: a reduction of wealth would in fact lead to a steep reduction of consumption to rebuild wealth.

receive dividends – he will de-cumulate his financial wealth to form a consumption budget. Savings, i.e. the difference between current income and consumption expenditure are used to accumulate financial wealth, i.e. $D_{c,t} = D_{c,t-1} + Y_{c,t} - C_{c,t}$.

Third, once they have determined their consumption budget, consumers visit C-firms in order to purchase goods.

The hth consumer visits Z_c randomly selected firms, ranks them in ascending order of posted price and demands consumption goods starting from the firm charging the lowest price. If he does not exhaust the consumption budget at the first firm, he will move up to the second firm in the ranking and so on. If he exhausts the purchasing opportunities (i.e. the visits to the Z_c firms) without exhausting the consumption budget, he will save involuntarily. Hence, firms posting lower prices are more likely to find customers and will receive larger orders on average. This implies that there is an implicit *negative elasticity* of the demand for the good produced by the ith C-firm to the relative price $P_{i,t}/P_t$ where $P_{i,t}$ is the price charged by the ith firm and P_t is the average price. Assuming that the individual firm is "negligible", the average price is approximately equal to the price charged by the "competitors". Notice however that both the price elasticity and the absolute level of demand at each price for each C-good are not constant. Firm F_1, for instance, may be visited by consumers C_1 and C_3 today and by consumers C_2 and C_5 tomorrow. The search and matching mechanism leads to the coexistence of queues of unsatisfied consumers (involuntary savers) at some firms and involuntary inventories of unsold goods at some other firms.

5. C-firms

The ith firm in the C-sector $(i = 1, 2, \ldots, F_c)$ produces a C-good using labor and capital. Imperfect information and transaction costs force the firm to explore a limited portion of the price-quantity territory around the current position – the status quo – in order to adapt to the market environment.

At the beginning of time t the ith firm sets the status quo, i.e. the pair $(P_{i,t}, Y_{i,t})$ where $P_{i,t}$ represents the firm's selling price and $Y_{i,t}$ is the firm's *current production*. At the end of period t the firm learns also the average price P_t.[12] Once production has been carried out and search and matching has taken place the ith firm can observe the amount of consumption goods actually sold $Q_{i,t} = \min(Y_{i,t}, Y^d_{i,t})$ where $Y^d_{i,t}$ is *actual demand*.

Since sales occur only after the firm has carried out production, actual demand can differ from current production. If $Q_{i,t} = Y^d_{i,t}$, the difference between current production and actual demand $\Delta_{i,t} = Y_{i,t} - Y^d_{i,t}$ is positive and represents involuntary inventories. This is a signal of excess supply (sales are equal to actual demand which is smaller than production), i.e. of a positive *forecasting error* (demand has been overestimated).

For simplicity we assume that C-goods are *non-storable*, hence the firm cannot carry inventories over from today to tomorrow and satisfy future demand. If the firm ends up with a positive inventory she gets rid of the unsold goods at zero costs.[13]

If $Q_{i,t} = Y_{i,t}$, the difference between current production and actual demand $\Delta_{i,t}$ is negative and its absolute value measures the demand of unsatisfied consumers. This is a signal of excess demand (sales are equal to production which is smaller than actual demand), i.e. a negative forecasting error (demand has been underestimated). No inventory accumulation and no queues of unsatisfied customers signal "equilibrium" (hence no forecasting error).

5.1. Price and quantity decision

The firm receives two signals: the price charged by competitors P_t and the forecasting error $\Delta_{i,t}$. These signals capture – albeit imprecisely – the distance between the firm's actual position (i.e. the status quo) and the "benchmark" i.e. a situation in which all firms charge the same price and demand equals supply, i.e. $P_{i,t} = P_t$ and $Y_{i,t} = Y^d_{i,t}$.[14] On the basis of these signals the firm decides whether to change the price or the quantity.

As to quantity, in t the firm sets *desired production* $Y^*_{i,t+1}$ for $t+1$ at the level of *expected future demand* i.e. $Y^*_{i,t+1} = Y^e_{i,t+1}$. Expectations on future demand, in turn, are based on the forecasting error $\Delta_{i,t}$. We assume that the firm decides to update the desired scale of activity as follows:

$$Y^*_{i,t+1} = Y^e_{i,t+1} = \begin{cases} Y_{i,t} + \rho(-\Delta_{i,t}) & \text{if } \Delta_{i,t} \le 0 \text{ and } P_{i,t} \ge P_t \\ Y_{i,t} - \rho\Delta_{i,t} & \text{if } \Delta_{i,t} > 0 \text{ and } P_{i,t} < P_t \end{cases} \tag{5.1}$$

where ρ is a positive parameter, $\rho \in (0,1)$. If $\Delta_{i,t} < 0$, demand is higher than production in t and the firm revises expected demand and desired production up.[15] On the contrary, if $\Delta_{i,t} > 0$, demand is smaller than production and the firm revises

[12] We assume that information concerning the average price is publicly available.

[13] This assumption simplifies the complexity of the model at the cost of ignoring the role of the inventory cycle in business cycle fluctuations. We leave the exploration of this issue to future research.

[14] The benchmark coincides with the outcome of Bertrand competition in a Dixit–Stiglitz framework in which the technology and the elasticity of demand are uniform across firms. In such a setting, the Bertrand competition leads to a symmetric Nash equilibrium in which all firms charge the same price, i.e. $P_{i,t} = P_t$ and each firm produces the same quantity, i.e. a fraction $1/F$ of total outputs.

[15] This quantity adjustment takes place if the individual price is higher than the average price. If $\Delta_{i,t} < 0$ and the individual price is lower than the average price, on the contrary, the firm will increase the individual price, keeping the quantity at the original level $Y_{i,t}$. The increase of the individual price will mitigate excess demand for the good in question.

expected demand and desired production down.[16] Recalling that $\Delta_{i,t}$ is the forecasting error, it is easy to conclude that firms compute *expected future demand* following a simple *adaptive rule*: $Y^e_{i,t+1} = Y^e_{i,t} + \rho(Y^d_{i,t} - Y^e_{i,t})$. By iterating, it is easy to see that expected future demand is a weighted average of current and past quantities with exponentially decaying weights.

As to the price, we assume the following updating rule:

$$P_{i,t+1} = \begin{cases} P_{i,t}(1+\eta_{i,t+1}) & \text{if } \Delta_{i,t} \le 0 \text{ and } P_{i,t} < P_t \\ P_{i,t}(1-\eta_{i,t+1}) & \text{if } \Delta_{i,t} > 0 \text{ and } P_{i,t} \ge P_t \end{cases} \tag{5.2}$$

where $\eta_{i,t+1}$ is a positive parameter drawn from a time invariant uniform distribution with support $(0, 0.1)$.

The firm tries to catch up with the competitors, raising the individual price when it is below the average price[17] and slashing it when it is above.[18] The magnitude of the price change is stochastic. This is the source of the idiosyncratic shocks in the model.

The firm's position in the price-quantity space is represented in Fig. 2, where point A represents the benchmark. The firm's position in the price-quantity space at the end of period t, given the available information, is described by the status quo. We can partition the plane into four regions (a, b, c, d). The firm will take a price or a quantity decision depending on the region of the plane describing her situation. In particular the firm will change the price if she is in region a or in region b in Fig. 2; she will change the quantity if she is in region c or d. In each of the four scenarios the firm changes either the price or the quantity but not both at the same time, i.e. adjustment is partial and asymmetric.[19] Finally we assume that the firm cannot set the price below a certain threshold (AC in Fig. 2) which covers average cost. Moreover, the firm will not choose a quantity below the level that guarantees the employment of at least one worker.

5.2. Production

The ith firm produces output $Y_{i,t}$ by means of capital $K_{i,t}$ and labor $N_{i,t}$. For simplicity we assume a Leontief technology. *In a condition of full capacity utilization* output will be

$$\hat{Y}_{i,t} = \min(\alpha N_{i,t}, \kappa K_{i,t}) \tag{5.3}$$

where α and κ are the productivities of labor and capital respectively, both exogenous, constant and uniform across firms. We assume that labor is always abundant, so that $\hat{Y}_{i,t} = \kappa K_{i,t}$. Hence labor requirements at full capacity utilization (i.e. the technically efficient level of employment) is $\hat{N}_{i,t} = (\kappa/\alpha)K_{i,t}$, where κ/α is the reciprocal of the (given and constant) capital/labor ratio.

When capacity is not fully utilized, only a fraction $\omega_{i,t} \in (0,1)$ of the capital stock will be used in production. $\omega_{i,t}$ represents the *rate of capacity utilization*. Hence actual production will be

$$Y_{i,t} = \omega_{i,t}\hat{Y}_{i,t} = \omega_{i,t}\kappa K_{i,t} \tag{5.4}$$

and labor requirement will be $N_{i,t} = \omega_{i,t}\hat{N}_{i,t} = (\kappa/\alpha)\omega_{i,t}K_{i,t}$.

Physical capital depreciates at the rate $\delta \in (0,1)$. However, only capital which is actually used in production depreciates. Therefore the law of motion of capital installed at the ith firm is

$$K_{i,t+1} = (1 - \delta\omega_{i,t})K_{i,t} + I_{i,t} \tag{5.5}$$

where $\delta\omega_{i,t+1}$ is the actual depreciation rate and $I_{i,t}$ is the investment i.e. the purchase of new capital goods.

5.3. Investment

At the beginning of time t the ith firm has to decide investment $I_{i,t}$. Investment allows the firm to adjust the capital stock. Capital adjustment, however, is not immediate (i) costly and (ii) time consuming.

First, we assume that there are *adjustment costs* of the capital stock such that only a fraction $\gamma \in (0,1)$ of C-firms is capable of purchasing new capital goods in each period. This is tantamount to assuming that the probability of investing is γ per period or that the firm can invest only in one period out of $1/\gamma$.[20] Second, capital is fixed in the "short run": new capital goods bought at the beginning of time t will be part of the capital stock only at the end of the same period and therefore will be available in period $t+1$. These difficulties in adjustment make capital a durable and *sticky* input.

[16] This quantity adjustment takes place if the individual price is lower than the average price. If $\Delta_{i,t} > 0$ and the individual price is higher than the average price, on the contrary, the firm will slash the individual price, keeping the quantity constant. The reduction of the individual price will boost demand.

[17] This price adjustment takes place if $\Delta_{i,t} < 0$ so that the increase of the individual price will mitigate excess demand.

[18] This price adjustment takes place if $\Delta_{i,t} > 0$ so that the reduction of the individual price will boost demand.

[19] Empirical surveys of managers' pricing and quantity decisions point in this direction, see e.g. Kawasaki et al. (1982) and Bhaskar et al. (1993).

[20] It is widely recognized that adjustment costs are a source of stickiness of the capital stock. In the present setting these adjustment costs play the same role as menu costs in Calvo pricing.

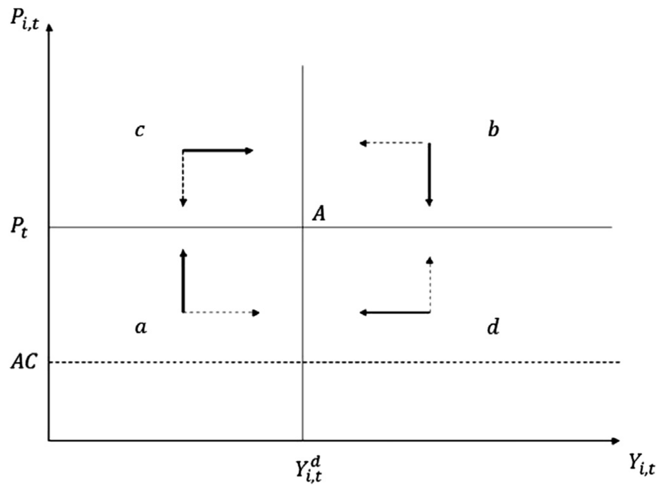

Fig. 2. The firm changes either the price or the quantity depending on her position in the price-quantity space.

Investment and price/quantity decisions are taken simultaneously so that, at the moment of investing in t, the firm is uncertain about the capital she needs to carry out her future production plans. The firm's information set, in fact, is incomplete: she has not observed yet whether her production decision in t has matched demand or not.

Due to uncertainty, the firm abstracts from the short run fluctuations of capital and grounds her investment decision on the stock of capital she has used *on average* in her lifetime, evaluated according to the following adaptive rule: $\overline{K}_{i,t-1} = \nu \overline{K}_{i,t-2} + (1-\nu)\omega_{i,t-1}K_{i,t-1}$ where $\nu \in (0,1)$. By iterating, it is easy to see that $\overline{K}_{i,t-1}$ is a weighted average of past utilized capital with exponentially decaying weights.

In the absence of investment, capital would decrease due to the use in production. The firm, therefore, has to form expectations on the investment needed to replace worn out capital. In so doing, the firm has to take into account the fact that investment occurs only one period out of $1/\gamma$. Therefore investment planned to replace depreciated capital is

$$I^r_{i,t} = \frac{\delta}{\gamma}\overline{K}_{i,t-1} \tag{5.6}$$

Moreover, the firm has to decide the net addition to capital. In order to do so, the firm uses a *desired long term rate of capital utilization* $\overline{\omega}$. Therefore the capital desired at the investing stage t for $t+1$ is $\overline{K}_{i,t-1}/\overline{\omega}$.

Hence total investment will be

$$I_{i,t} = \left(\frac{1}{\overline{\omega}} + \frac{\delta}{\gamma}\right)\overline{K}_{i,t-1} - K_{i,t} \tag{5.7}$$

Plugging (5.7) in (5.5) and rearranging one gets

$$K_{i,t+1} = \left(\frac{1}{\overline{\omega}} + \frac{\delta}{\gamma}\right)\overline{K}_{i,t-1} - \delta\omega_{i,t}K_{i,t} \tag{5.8}$$

Notice that durability and stickiness imply that the capital stock available for production at the beginning of period $t+1$ keeps memory of the entire history of the capital stock in the past.

5.4. Labor requirement and vacancies

Let us assume that, on the basis of expected demand, the firm decides to produce $Y^*_{i,t+1}$ in $t+1$. Hence desired capital is $K^*_{i,t+1} = Y^*_{i,t+1}/\kappa$. Given the availability of capital at the same date, $K_{i,t+1}$, there are two scenarios.

If the capital stock is large – i.e. $K_{i,t+1} > K^*_{i,t+1}$ – the firm could reach the desired scale of activity by setting capacity utilization at the required level: $\omega^*_{i,t+1} = K^*_{i,t+1}/K_{i,t+1}$. Then desired employment would be $N^*_{i,t+1} = K^*_{i,t+1}\kappa/\alpha = K_{i,t+1}\omega^*_{i,t+1}\kappa/\alpha$.

If, on the contrary, the capital stock is insufficient – i.e. $K_{i,t+1} < K^*_{i,t+1}$ – the firm will use machinery and equipment at full capacity ($\omega_{i,t+1} = 1$) but the desired scale of activity will not be reached. Desired employment in this case will be $N^*_{i,t+1} = K_{i,t+1}\kappa/\alpha$.

Desired employment therefore is determined as follows:

$$N^*_{i,t+1} = \min\left(K^*_{i,t+1}\frac{\kappa}{\alpha}, K_{i,t+1}\frac{\kappa}{\alpha}\right) \tag{5.9}$$

Given desired employment and actual employment, i.e. $N_{i,t}$, at the beginning of period $t+1$ the firm decides vacancies to post as follows:

$$v_{i,t+1} = \max(N^*_{i,t+1} - N_{i,t}, 0) \tag{5.10}$$

A vacancy is filled only if the firm is visited by an unemployed worker. Since the nominal wage is given and uniform, the probability of filling the vacancy depends only on the number of unemployed workers visiting the firm. Thus actual employment $N_{i,t}$ may be smaller than desired employment $N^*_{i,t+1}$, the difference between the two being unfilled vacancies. On the contrary, if $N^*_{i,t+1} < N_{i,t}$, i.e. the number of actual workers exceeds the level of desired employment, the firm selects at random $n_{i,t+1} = N_{i,t} - N^*_{i,t+1}$ workers and fires them.

5.4.1. A numerical example

Consider a simplified time frame in which the past consists of two periods only ($t-2$ and $t-1$). Consider a firm with capital equal to 95 in $t-2$ and 105 in $t-1$.[21] Giving equal weights to all the periods, i.e. setting $\nu = 0.5$, average capital in $t-1$ is $\overline{K}_{i,t-1} = 100$. Suppose also that $\gamma = 0.25$ so that the firm can invest in one period out of four.[22]

Suppose investment in $t-1$ was 10.1. Hence capital in t will be equal to $K_{i,t} = (1-0.02)105 + 10.1 = 113$. Assuming that the desired capacity utilization rate is $\overline{\omega} = 0.85$, desired capital for $t+1$ is 100/0.85=117.65 so that the desired net addition to capital is 117.65 − 113=4.65. Suppose that capital depreciates by $\delta = 2\%$ per period. In order to keep the average capital constant the firm has to plan $I^r_{i,t} = 8$ of capital replacement. Therefore total planned investment in t will be 4.65+8=12.65 and capital available in $t+1$ will be $K_{i,t+1} = (1-0.02)113 + 12.65 = 123.4$.[23]

Assuming that the productivity of capital is $\kappa = 1/3$, production in t will be 113/3=37.67. Assuming that the productivity of labor is $\alpha = 1/2$, desired employment will be $37.67 \times 2 \approx 75$. For simplicity we assume that this is also current employment in t. Consider now three scenarios.

- Suppose that on the basis of the signals received, the firm sets desired production in $t+1$ at 30, i.e. she wants to scale down activity. The stock of capital needed to produce this amount is $30 \times 3 = 90$. The firm therefore uses only a fraction 90/123.4=0.73 of the available capital stock. The desired level of employment will be $30 \times 2 = 60$. Since the workforce is 75, the firm will fire 15 workers.
- Suppose now that the firm wants to increase production slightly: desired output is 40 in $t+1$. The stock of capital needed to produce this amount is $40 \times 3 = 120$. Since capital available in $t+1$ is 123.4, there is enough capital to reach the desired level of production. The firm sets the desired rate of capacity utilization at 120/123.4=0.97. The desired level of employment is $40 \times 2 = 80$. The firm will post 5 vacancies.
- Suppose now that the desired increase in production is sizable: desired production in $t+1$ is 50. The stock of capital needed to produce this amount is $50 \times 3 = 150$. There is no enough capital to reach this level of production. The firm tries to get as close as possible to the desired scale of activity using all the capital available at full capacity, i.e. 123.4. Total output will be 123.4/3=41.13. The desired level of employment is $41.13 \times 2 \approx 82$. The workforce being 75, the firm will post 7 vacancies.

Summing up, the firm enjoys full flexibility in adjusting production when production must be scaled down (first scenario) and when output should be increased but "not too much" (second scenario). When the desired increase in output is sizable and/or past investment has been relatively low, the firm hits a capacity constraint (third scenario).

Actual production may be smaller than desired if the firm does not succeed in achieving the "appropriate" level of capital (as in the third scenario) but also in the second scenario if the firm does not succeed in achieving the "appropriate" level employment (for instance because of unfilled vacancies) and/or she does not get enough credit.

6. K-firms

In the market for K-goods (K-market for short), C-firms are buyers of capital goods sold by K-firms. We assume imperfect information and the existence of transaction costs also in the K-market so that price and quantity are decided by K-firms following adaptive rules similar to those described above for C-firms (see Section 5.1).

There is a crucial difference, however. Since capital is durable, we assume that K-goods are storable, hence the K-firm can carry inventories over from today to tomorrow and satisfy future demand. The level of inventories available in period t will be denoted by $Y^k_{j,t}$.

At the beginning of time t the jth firm in the K-sector ($j = 1, 2, \ldots, F_k$) sets the status quo ($P_{j,t}, Y_{j,t}$) where $P_{j,t}$ and $Y_{j,t}$ are the price and the quantity of K-goods produced by the jth firm.

Once production has been carried out and search and matching has taken place the firm can observe the amount of K-goods actually sold $Q_{j,t} = \min(Y^s_{j,t}, Y^d_{j,t})$ where $Y^d_{j,t}$ is the actual demand, and $Y^s_{j,t} = Y_{j,t} + Y^k_{j,t}$ is the total supply of goods by the jth firm, i.e. the sum of total production in t and inventories available in the same period. The change in inventories in

[21] In this section, for simplicity we will assume that capital is fully utilized unless explicitly stated otherwise.

[22] In this numerical example we give parameters the same numerical values used in the simulations, see Table 2.

[23] This number can be retrieved also from Eq. (5.8).

period t is equal to the difference between the level of inventories at the end of period t, $Y^s_{j,t}-Y^d_{j,t}$, and the level of inventories available at the beginning of period t, $Y^k_{j,t}$:

$$\Delta_{j,t}=Y^s_{j,t}-Y^d_{j,t}-Y^k_{j,t} \tag{6.1}$$

which, given the definition of total supply, can be written as

$$\Delta_{j,t}=Y_{j,t}-Y^d_{j,t} \tag{6.2}$$

The change in inventories measures the forecasting error of the firm. The law of motion of inventories is

$$Y^k_{j,t+1}=(Y^k_{j,t}+\Delta_{j,t})(1-\delta^k) \tag{6.3}$$

where δ^k is the depreciation rate of inventories.[24] At the beginning of each period, the inventories resulting from last period activity depreciate at a constant depreciation rate δ^k. The law of motion described in Eq. (6.3) is a theoretical law of motion. This is because the actual decumulation of inventories has a physical limit in the stock of inventories. Therefore, $\Delta_{j,t}$ translates into involuntary inventory accumulation if positive, i.e. if the firm has produced "too much" (with respect to actual demand). Since $Y^k_{j,t+1}$ must be greater than or equal to zero, if $\Delta_{j,t}$ happens to be negative (i.e. if the firm has produced "too little"), it translates into a reduction in inventories up to a limit. In words: if demand is greater than output, the firm can satisfy excess demand by shedding inventories up to a limit represented by the entire stock of inventories carried over from the past. If excess demand is greater than this threshold, there will be a queue of unsatisfied C-firms.

The actual change in inventories will be denoted by

$$\Delta^k_{j,t+1}=Y^k_{j,t+1}-Y^k_{j,t}=\Delta_{j,t}(1-\delta^k)-\delta^k Y^k_{j,t} \tag{6.4}$$

6.1. K-price and quantity decision

The firm receives two signals: the price charged by competitors and the change of inventories/queues. On the basis of these two signals the firm changes the price or the quantity. The firm adopts the following adaptive rule in order to change the price:

$$P_{j,t+1}=\begin{cases} P_{j,t}(1+\eta_{j,t+1}) & \text{if } \Delta_{j,t}\le 0 \text{ and } P_{j,t}<P_{k,t} \\ P_{j,t}(1-\eta_{j,t+1}) & \text{if } \Delta_{j,t}>0 \text{ and } P_{j,t}>P_{k,t} \end{cases} \tag{6.5}$$

where $\eta_{j,t+1}\in(0,0.1)$

As to quantity adjustment, in the case of K-firms, desired production is not equal by construction to expected demand (as was the case for C-firms) because K-firms can satisfy at least part of expected demand using inventories carried over from the past. In symbols: $Y^*_{j,t+1}+Y^k_{j,t+1}=Y^e_{j,t+1}$. Hence the adaptive rule adopted to adjust quantity is

$$Y^*_{j,t+1}=\begin{cases} Y_{j,t}+\rho(-\Delta_{j,t})-Y^k_{j,t+1} & \text{if } \Delta_{j,t}\le 0 \text{ and } P_{j,t}>P_{k,t} \\ Y_{j,t}-\rho\Delta_{j,t}-Y^k_{j,t+1} & \text{if } \Delta_{j,t}>0 \text{ and } P_{j,t}<P_{k,t} \end{cases} \tag{6.6}$$

where ρ is a positive parameter, $\rho\in(0,1)$.

If $\Delta_{j,t}<0$ (and the individual price is higher than the average price), the firm revises expected demand up. The increase in expected demand does not translate into an increase of production of the same size because the K-firm can satisfy at least part of the increase in demand using inventories carried over from the past.

If $\Delta_{i,t}>0$ (and the individual price is lower than the average price), the firm revises expected demand down. The reduction in expected demand translates into a reduction of production greater than the reduction in demand because the K-firm satisfies part of demand getting rid of inventories carried over from the past.

The K-market is modeled as a search and matching mechanism: each C-firm randomly selects Z_k K-firms, sorts them by selling price and demand capital goods starting from the firm charging the lowest price. K-firms with lower prices, therefore, will receive on average higher demand for capital goods. The search and matching mechanism implies coexistence of queues of unsatisfied C-firms at some K-firms and involuntary inventories of K-goods at some other firms.

6.2. K-production, labor requirement and vacancies

The jth firm produces output $Y_{j,t}$ by means of a linear technology:

$$Y_{j,t}=\alpha N_{j,t} \tag{6.7}$$

[24] We assume that inventories depreciate faster than capital installed: $\delta^k>\delta$.

where α is the productivity of labor, exogenous, constant and uniform across firms.[25] Desired employment will be $N^*_{i,t+1} = Y^*_{i,t+1}/\alpha$.

Given the desired employment and the amount of labor already employed (i.e. $N_{j,t}$) at the beginning of period $t+1$, i.e. before the labor market opens, the firm decides vacancies to post according to:

$$v_{j,t+1} = \max(N^*_{j,t+1} - N_{j,t}, 0) \tag{6.8}$$

These vacancies add to those posted by C-firms. On the labor market C-firms and K-firms are competing for labor services: labor is a homogeneous input. Unemployed workers search for a job visiting firms irrespective of the type of goods they produce. In other words, the unemployed worker is indifferent between finding a job at a C-firm or a K-firm.

7. The financing gap

Each firm has a funding problem, irrespective of the type of goods she produces. If liquid resources, $D_{f,t-1}, f = 1, 2 \ldots F$, are in short supply with respect to expenditure $X_{f,t}$ there is a positive *financing gap* equal to $F_{f,t} = X_{f,t} - D_{f,t-1}$ in which the firm tries to fill by asking a loan to a bank. If $F_{f,t} < 0$ the firm can finance expenditure with internal financial resources and therefore she will not ask for a loan.

Expenditure for a C-firm is the sum of the wage bill $wN_{i,t}$ and investment expenditure $P_{k,t-1}I_{i,t}$ where $P_{k,t-1}$ represents the price index of capital goods.[26] In symbols: $X_{i,t} = wN_{i,t} + P_{k,t-1}I_{i,t}$. Expenditure for a K-firm boils down to the wage bill $X_{j,t} = wN_{j,t}$ as she does not use capital to carry on production.

In other words, the ith C-firm asks for a bank loan only if there is a financing gap defined as follows:

$$F_{i,t} = \max(wN_{i,t} + P_{k,t-1}I_{i,t} - D_{i,t-1}, 0) \tag{7.1}$$

The jth K-firm's financing gap is defined as follows:

$$F_{j,t} = \max(wN_{j,t} - D_{j,t-1}, 0) \tag{7.2}$$

8. The bank

For the sake of simplicity we assume that there is only one commercial bank. Households, capitalists and firms hold deposits at the bank to manage their liquidity. The bank accepts deposits in unlimited amount at zero interest rate. On the other hand, firms demand credit according to their production plans and the resulting financing gaps (as shown above). The bank has to decide both the price (interest rate) and the quantity of loans to be supplied to each firm on the basis of her assessment of the firm's financial fragility proxied by *leverage*.

We assume that the bank has full access to past individual data on the financial fragility (measured by leverage) and the state (surviving or bankrupt) of each firm she has extended a loan to over a certain time span $\hat{T}$. For each firm $f = 1, 2 \ldots . F$ in each period the bank can compute the following *leverage ratio*:

$$\lambda_{f,t} = \frac{L_{f,t-1} + F_{f,t}}{E_{f,t-1} + L_{f,t-1} + F_{f,t}} \tag{8.1}$$

where $L_{f,t-1}$ and $E_{f,t-1}$ are the end-of-period values of debt and equity respectively (by construction, this sum coincides with the total nominal value of assets in $t-1$) and $F_{f,t}$ represents the current financing gap (i.e. the demand for a new loan). Notice that $0 \le \lambda_{f,t} < 1$. If $\lambda_{f,t} = 0$ the firm is self-financed (maximum financial robustness): total assets will be equal to net worth (in $t-1$). On the contrary when $\lambda_{f,t} \to 1$, equity tends to zero (maximum financial fragility): total assets tend to total loans.[27]

Using these data, the bank will populate two datasets for C-firms and K-firms respectively. Using the dataset for C-firms the bank will estimate a logistic regression between the individual bankruptcy probability ϕ_c and the individual leverage λ_c for C-firms: $\phi_c = f^c(\lambda_c)$. Following the same procedure and using the dataset for K-firms, the bank will also estimate the logistic relationship between the bankruptcy probability ϕ_k and the individual leverage λ_k for K-firms: $\phi_k = f^k(\lambda_k)$.[28] The estimated default probability turns out to be an increasing convex function of leverage both for C-firms and K-firms. In both cases, for values of λ tending to 1 the estimated probability of bankruptcy tends to a level well below 1 (around 0.1 in the

[25] We assume that the productivity of labor is the same, i.e. α, in both sectors.

[26] Since the C-firm in t does not have sufficient information about the prices charged by K-firms (a decision also taken in t), in order to evaluate the financing gap she uses the average of the prices of K-goods charged in the previous period.

[27] The definition of leverage we adopt here for convenience is a monotonic increasing transformation of the usual definition according to which leverage is the ratio of assets to equity: $\hat{\lambda}_{f,t} = (L_{f,t-1} + E_{f,t-1} + F_{f,t})/E_{f,t-1}$. Our measure of leverage has the advantage of ranging between zero and one.

[28] We assume that, in the estimation of the default probability, the bank will make use only of the last $\hat{T}$ periods. Each period the logistic relationship will be re-estimated discarding the oldest observation and incorporating the newest one and therefore maintaining the length of the time window and the size of the dataset unaltered. The reason for building up two different datasets is that the structural features of C-firms and K-firms are indeed quite different. Since a minimum number of data points, i.e. bankruptcies, are necessary to obtain a good fit and since the number of C-firms is 4 times the number of K-firms in the simulations, also the time span $\hat{T}$ used in the estimation will be different for the two sectors.

C-sector and around 0.5 in the K-sector). The grim reality is that when λ tends to 1 the firm is indeed on the verge of bankruptcy (because equity is going to be depleted soon) but this risk is statistically underestimated by the logistic regression the bank uses to assess the riskiness of the firm.

8.1. Interest rates

We assume that the loan repayment schedule is the same for all the firms and is defined as follows. Suppose the firm receives a loan of 1 dollar in period zero. In each subsequent period, denoted by t, the firm will pay back a share $\theta(1-\theta)^t$ of the loan.[29]

Let us define a *risk free* firm as a firm that survives (does not go bankrupt) for an infinite number of periods. The total payment due by the risk free firm is

$$R=\sum_{t=0}^{\infty}(\theta+r)(1-\theta)^t \tag{8.2}$$

where r is the *risk free* rate, which we take as exogenous. The risk free rate is the natural candidate to play the role of the policy rate (the instrument of monetary policy) in our framework.

Expanding the geometric series in (8.2), we get

$$R=\left(1+\frac{r}{\theta}\right) \tag{8.3}$$

where R is the gross rate of return on lending to a *risk free* firm while r/θ is the net rate of return.

Consider now a generic *risky* firm $f=1,2,\dots F$. The bank will retrieve the probability of bankruptcy of the firm from the logistic regression: $\phi_{f,t}=f(\lambda_{f,t})$. The expected number of periods that the firm will survive (i.e. the time to default) is the reciprocal of the bankruptcy probability:

$$T_{f,t}=\frac{1}{\phi_{f,t}}=\frac{1}{f(\lambda_{f,t})} \tag{8.4}$$

By definition, the payment due by a firm with bankruptcy probability $\phi_{f,t}$ on a loan of 1 dollar – i.e. the gross rate of return on lending to the firm – is equal to:

$$R_{f,t}=\sum_{t=0}^{T_{f,t}}(\theta+r_{f,t})(1-\theta)^t \tag{8.5}$$

where $r_{f,t}$ is the interest rate the bank will charge the risky firm, which will be determined momentarily. Expanding the geometric series we get

$$R_{f,t}=(\theta+r_{f,t})\frac{1-(1-\theta)^{T_{f,t}+1}}{\theta} \tag{8.6}$$

From the relation above it is clear that the return on lending to a *risky* firm depends on the expected survival time $T_{f,t}$, which depends on the estimated bankruptcy probability and in the end on the firm's leverage.

For simplicity, we assume that the bank is risk neutral: she will lend to a risky firm inasmuch as the return on lending to that firm is not smaller than the return on investing in an alternative risk free asset which measures the opportunity cost of lending.[30] We assume that the opportunity cost of lending is a multiple of the return on lending to a risk free firm: $R\mu$ where $\mu>1$.

Once all the arbitrage opportunities have been exploited, the following equality will hold true for the fth firm: $R_{f,t}=R\mu$. Using (8.3) and (8.6) and denoting with $\Xi(\theta,T_{f,t})$ the expression $1-(1-\theta)^{T_{f,t}+1}/\theta$, from the no-arbitrage condition above we obtain the interest rate charged by the bank to the risky firm:

$$r_{f,t}=\mu\left[\frac{1+\dfrac{r}{\theta}}{\Xi(\theta,T_{f,t})}-\theta\right] \tag{8.7}$$

From (8.7) we infer that the interest rate charged by the bank to the fth firm is increasing with the risk free rate and decreasing with the time to default (and therefore increasing with the firm's leverage). An increase of the risk free rate raises the financial costs for the firm (cost channel in the transmission of monetary policy shock). If the firm is already quite fragile

[29] Ignoring interest payments, if the firm paid θ per period the loan of 1 dollar would be extinguished in $1/\theta$ periods, i.e. $1/\theta$ would be the maturity of the loan. In the repayment schedule we consider, the agent pays θ in the first period, $\theta(1-\theta)$ in the second period and so on. This means that the loan will be completely reimbursed only asymptotically on an infinite time horizon. The implicit maturity is $1/\theta$ but the borrower is entitled to extinguish the debt over an infinite horizon. By way of example, suppose $\theta=0.05$, i.e. the firm pays back 5% of the existing debt per period. The implicit maturity is 20 years. After 20 years, however, according to the repayment schedule here considered the firm has repaid approximately 2/3 of the loan.

[30] This is the definition of the *participation constraint* for the lender.

this interest rate policy will push the firm even closer to bankruptcy. In turn, if the firm goes bankrupt, the whole stock of credit is lost, hence the equity of the bank will decrease (see below).

Due to structural differences between the two sectors, we have assumed that the bank estimates two different logistic regressions to evaluate the probability of bankruptcy for C-firms and K-firms respectively. Taking this into account, we can specify the interest rate charged by the bank to a risky C-firm as

$$r_{i,t} = \mu \left[\frac{1+\dfrac{r}{\theta}}{\Xi(\theta, T_{i,t})} - \theta \right] \tag{8.8}$$

where $T_{i,t} = 1/\phi_{i,t} = 1/f^c(\lambda_{i,t})$.

Analogously, the interest rate charged by the bank to a risky K-firm is

$$r_{j,t} = \mu \left[\frac{1+\dfrac{r}{\theta}}{\Xi(\theta, T_{j,t})} - \theta \right] \tag{8.9}$$

where $T_{j,t} = 1/\phi_{j,t} = 1/f^k(\lambda_{i,t})$.

8.2. Supply of loans

We assume that the bank's risk manager aims at limiting the maximum loss deriving from commercial loans.[31] For each loan, we assume the maximum admissible loss is ζE_t^b where $0 < \zeta < 1$, and E_t^b represents the equity of the bank in period t. The expected loss from lending to the fth firm, on the other hand, is $\phi_f(\Delta L_{f,t} + L_{f,t-1})$ where ϕ_f is the bank's estimated probability of bankruptcy for firm f and $\Delta L_{f,t} = L_{f,t} - L_{f,t-1}$ is the new loan extended by the bank to the firm. Hence the following inequality must hold

$$\phi_f(\Delta L_{f,t} + L_{f,t-1}) \le \zeta E_t^b \tag{8.10}$$

Since $L_{f,t-1}$ is known, the only decision variable for the bank is the amount of loans she extends to the fth firm, i.e. $\Delta L_{f,t}$:

$$\Delta L_{f,t} \le \frac{\zeta E_t^b - \phi_f L_{f,t-1}}{\phi_f} \equiv \overline{F}_{f,t} \tag{8.11}$$

where $\overline{F}_{f,t}$ is the maximum size of the *new* loan the bank is willing to supply to firm f, given the estimated probability of default.[32]

If the loan demanded by the firm (i.e. the financing gap $F_{f,t}$) is smaller than the maximum loan the bank is willing to extend (i.e. $\overline{F}_{f,t}$), then the bank will accommodate the demand for credit: $\Delta L_{f,t} = F_{f,t}$. If, on the contrary, the financing gap ($F_{f,t}$) is greater than the maximum loan the bank is willing to extend, the bank will limit credit to the borrower: $\Delta L_{f,t} = \overline{F}_{f,t}$. In symbols:

$$\Delta L_{f,t} = \min(F_{f,t}, \overline{F}_{f,t}) \tag{8.12}$$

From the inequality above, given the definition of financing gap and (8.11) we can infer that an increase of the net worth of the bank and/or of the borrowing firm – so that the firm's leverage and the estimated probability of bankruptcy go down – relaxes the constraint represented by the maximum admissible loan supply and makes credit rationing less likely.

Although being very simple the bank's risk management is effective because it prevents the bank from taking too large a risk on the loan market.

For the sake of simplicity we assume that firms' owners own the bank as well, in particular each one holds the same number of shares. At the end of each period, if net profits are positive, the bank will distribute dividends. Retained profits, on the other hand, will be added to her equity.

9. Accounting

In this section we will describe the accounting framework of the model, focusing on the system of interrelated balance sheets. We will also show the changes in liquidity generated by the financial structure of the balance sheets. As far as households (workers and capitalists) are concerned, the situation is simple. The hth worker has (non-negative) wealth ($E_{h,t}$) which coincides with deposits ($D_{h,t}$), i.e. $E_{h,t} = D_{h,t}$, $h = 1, 2\ldots, H$. Hence, savings are deposited at the bank:

$$D_{h,t} = D_{h,t-1} + Y_{h,t} - C_{h,t} \tag{9.1}$$

The same assumption applies to capitalists.

[31] An alternative modeling strategy for capital requirements is described in Teglio et al. (2012).

[32] The inequality (8.11) may be expressed also in terms of stocks: $L_{f,t} \le \zeta E_t^b / \phi_f \equiv \overline{L}_{f,t}$ where $\overline{L}_{f,t}$ is the maximum amount of loans (total outstanding debt of the firm) the bank is willing to supply to firm f.

Consider now the balance sheet of the generic C-firm. The balance sheet accounting identity is

$$v^k_{i,t}K_{i,t}+D_{i,t}=L_{i,t}+E_{i,t} \tag{9.2}$$

where $v^k_{i,t}$ is the book value of physical capital $K_{i,t}$, $D_{i,t}$ is the firm's liquidity (deposits at the bank), $L_{i,t}$ is the outstanding debt and $E_{i,t}$ is the net worth.

From this identity it is easy to infer that liquidity is updated as follows:

$$D_{i,t}=D_{i,t-1}+\Pi_{i,t}+\Delta L_{i,t}-\theta L_{i,t}-P_{k,t}I_{i,t} \tag{9.3}$$

where $\Pi_{i,t}$ are the firm's profits, $\theta L_{i,t}$ is the debt installment and $P_{k,t}I_{i,t}$ are expenditures in new capital goods evaluated at the current price of capital goods $P_{k,t}$. At the end of period t the profits of the ith C-firm will be

$$\Pi_{i,t}=P_{i,t}Q_{i,t}-(wN_{i,t}+v^k_{i,t}I^r_{i,t})-\hat{r}_{i,t}L_{i,t}-Div_{i,t} \tag{9.4}$$

where $Q_{i,t}=\min(Y_{i,t},Y^d_{i,t})$ is the amount of consumption goods actually sold,[33] $wN_{i,t}$ is the wage bill, $v^k_{i,t}I^r_{i,t}$ represents the cost of capital due to the replacement of worn out equipment, $I^r_{i,t}$ evaluated at the book value $v^k_{i,t}$, $\hat{r}_{i,t}L_{i,t}$ are interest payments on the stock of debt, $Div_{i,t}$ are dividends.

In order to understand the definition of interest payments, let us remind the reader that the bank extends loans to the same firm at different interest rates in different time periods – essentially because the firm's financial fragility may change over time, see (8.8). We define $\hat{r}_{i,t}$ as the average cost of outstanding debt, which is updated according to the following rule:

$$\hat{r}_{i,t}=\frac{\hat{r}_{i,t-1}L_{i,t-1}+r_{i,t}\Delta L_{i,t}}{L_{i,t-1}+\Delta L_{i,t}} \tag{9.5}$$

It is easy to see that the equation above can be written as follows:

$$\hat{r}_{i,t}=\hat{r}_{i,t-1}(1-x_{i,t})+r_{i,t}(x_{i,t}) \tag{9.6}$$

where $x_{i,t}=\Delta L_{it}/L_{it}$. By repeated substitution, it is easy to see that the current cost of debt turns out to be a weighted average of past interest rates with time-varying weights.

If equity turns negative, the firm goes bankrupt. Bankruptcy implies that debt and liquidity are absorbed by the bank and a new firm will enter the market (one to one replacement) with equity determined by the wealth of the owner of the bankrupt firm.[34]

Let us turn now to K-firms. The balance sheet identity of the generic K-firm is

$$D_{j,t}=E_{j,t}+L_{j,t} \tag{9.7}$$

where the meaning of the symbols is straightforward. From this identity it is easy to infer that liquidity is updated as follows:

$$D_{j,t}=D_{j,t-1}+\Pi_{j,t}+\Delta L_{j,t}-\theta L_{j,t} \tag{9.8}$$

At the end of period t the profits of the jth K-firm will be

$$\Pi_{j,t}=P_{j,t}Q_{j,t}-wN_{j,t}-\hat{r}_{j,t}L_{j,t}-Div_{j,t} \tag{9.9}$$

where $Q_{j,t}=\min(Y^s_{j,t},Y^d_{j,t})$ is the amount of capital goods actually sold.[35] Moreover

$$\hat{r}_{j,t}=\hat{r}_{j,t-1}(1-x_{j,t})+r_{j,t}(x_{j,t}) \tag{9.10}$$

where $x_{j,t}=\Delta L_{j,t}/L_{j,t}$. If the jth firm's equity turns negative, the firm goes bankrupt and will be replaced by a new firm with equity determined by the wealth of the owner of the bankrupt firm.

Let us turn now to the bank. The balance sheet accounting identity is

$$R^b_t+L_t=D_t+E^b_t \tag{9.11}$$

where R^b_t are the bank's reserves (bank's deposits at the central bank), L_t are total loans, D_t are total deposits and E^b_t is the bank's net worth.

At the end of period t the profits of the bank will be

$$\Pi^b_t=\sum_{s=1}^{F_s}r_{s,t}L_{s,t} \tag{9.12}$$

Since deposits are not remunerated, the bank does not incur costs. Therefore profits coincide with revenues, which in turn coincide with interest payments from solvent borrowers (the set of solvent borrowers consists of F_s elements).

[33] The product $P_{i,t}Q_{i,t}$ represents sale revenues. If the firm has ended up with positive inventories, i.e. $\Delta_{i,t}>0$, then $Q_{i,t}=Y_{i,t}-\Delta_{i,t}=Y^d_{i,t}$. If, on the other hand, there is a queue of unsatisfied consumers, i.e. $\Delta_{i,t}<0$, then $Q_{i,t}=Y_{i,t}$.

[34] Since there is no secondary market for capital goods we assume that the capital stock owned by the bankrupt firm is left to the firm replacing it.

[35] The same remarks on the relationship between sales, demand, supply and inventories that apply to the generic C-firm also apply to the generic K-firm, with the obvious caveat that capital goods are durable and therefore they will be stored and depreciate gradually.

Table 1
Interrelated balance sheets.

Balance sheet items	Households	C-firms	K-firms	Bank	Central bank	Total
Capital		K				K
Inventories		Δ^C	Δ^K			Δ
Deposits	D^H	D_C	D_K	$-D$		0
Reserves				R^b	$-R^b$	0
Loans		$-L_C$	$-L_K$	L		0
Gov. Bonds					B	B
Equity	$-E^H$	$-E^C$	$-E^K$	$-E^B$		$-(K+\Delta+B)$

K=physical capital evaluated at book value; Δ^C=inventories of C-goods; Δ^K=inventories of K-goods; D^H=households' deposits (workers and capitalists); D^C=deposits of C-firms; D^K=deposits of K-firms; D=total deposits ($D=D^H+D^C+D^K$); R^b=bank's reserves; L_C=loans to C-firms; L_C=loans to K-firms; L=total loans ($L=L^C+L^K$); B=Government bonds (assets of the central bank); E^H=households' wealth; E^C=equity of C-firms; E^K=equity of K-firms; E^B=equity of the bank.

Net worth will be updated as follows:

$$E_t^b = E_{t-1}^b + \Pi_t^b - \sum_{n=1}^{F_n} r_{n,t} L_{n,t} \tag{9.13}$$

The total value of interest payments due by insolvent borrowers (the set of insolvent borrowers consists of F_n elements) is *bad debt* and is recorded as a negative item in the determination of current bank's equity.

9.1. Balance sheets

Summing balance sheet items across agents in each group we get the aggregate balance sheets of households, C-firms, K-firms and the bank. In the background, the central bank provides liquidity to the bank (bank reserves). The aggregate balance sheets are interrelated as shown in Table 1.

In our economy, by construction there is no currency. Therefore base money (or High Powered Money, HPM) coincides with bank's liquidity (reserves) and money coincides with deposits. By assumption, in order to inject liquidity into the system, the central bank purchases external assets (e.g. government bonds).[36] The stock of HPM, R^b, therefore is set exogenously (it is equal to the value of external assets purchased by the central bank) and is constant over time. The actual amount of money D, on the contrary, is endogenously determined by the fluctuations of total credit and of the bank's equity base. In fact, from the balance sheet identity we get

$$R^b = D - L + E^B \tag{9.14}$$

In the equation above therefore, changes in the items on the RHS should sum up to zero.

In order to discuss the output of simulations (see next section) we will decompose deposits and loans so that the following partition of base money follows from the equation above:

$$R^b = D^H + M^C + M^K + E^B \tag{9.15}$$

where $M^C = D^C - L^C$ is liquidity in the hands of C-firms and $M^K = D^K - L^K$ is liquidity in the hands of K-firms. Notice moreover that $M^C = E^C - (K + \Delta^C)$ and $M^K = E^K - \Delta^K$.

10. Simulations

Table 2 reports the numerical values of the parameters used in the simulation and the initial conditions.

We have built a medium sized ABM with 3250 households (3000 workers and 250 capitalists) and 250 firms, of which 1/5 in the K-sector. Transaction costs are high in markets for goods: buyers visit a very small number of sellers ($Z_c = Z_k = 2$). They are slightly lower on the labor market: an unemployed worker, in fact, can visit Z_e=5 firms.

Overall there are 16 parameters, which have been generally calibrated using empirical regularities. In this section we briefly comment on some of them.

Adaptive behavior generally requires the computation of the weighted average of past data of a certain variable which will anchor the expectation of the future level of the variable in question (e.g. human wealth, expected demand, expected capital stock). The weight of the most recent data (*memory parameter*) is generally high. In our calibration it is indeed very high in the computation of human wealth ($\xi = 0.96$) and of expected demand ($\rho = 0.9$), much lower in the computation of average capital ($\nu = 0.5$).

The volatility of the individual prices is low (irrespective of the type of goods produced): the rate of change of the individual price η ranges between 0 and 10%.

[36] For simplicity, we do not model the economic determinants of the issuance of these assets.

Table 2
Parameters and initial conditions.

Parameter	Description	Value
T	Number of periods	3000
H	Number of workers	3000
F_c	Number of C-firms	200
F_k	Number of K-firms	50
Z_e	Number of firms visited by an unemployed worker	5
Z_c	Number of C-firms visited by a consumer	2
Z_k	Number of K-firms visited by a C-firm	2
ξ	Memory parameter (human wealth)	0.96
τ	Dividend payout ratio	0.2
χ	Fraction of wealth devoted to consumption	0.05
r	Risk free interest rate	0.01
ρ	Quantity adjustment parameter	0.9
η	Price adjustment parameter (random variable)	$U(0,0.1)$
μ	Bank's gross mark-up	1.2
α	Productivity of labor	0.5
κ	Productivity of capital	1/3
γ	Probability of investing	0.25
ζ	Bank's loss parameter	0.002
θ	Installment on Debt	0.05
δ	Depreciation of capital	0.02
ν	Memory parameter (investment)	0.5
$\overline{\omega}$	Desired capacity utilization rate	0.85
w	Wage	1
D_1^f	Initial liquidity of (all) the firms	10
K_1	Initial capital	10
Y_1^c	Initial production (C-firms)	5
Y_1^k	Initial production (K-firms)	3
E_1^b	Initial equity of the bank	3000
E_1^h	Initial households' personal assets	2

The productivity of capital κ is the reciprocal of the capital/output ratio, which is set at 3 according to a well known stylized fact. The productivity of labor α is set at 1/2. Therefore full employment GDP is equal to $3000 \times (1/2) = 1500$.

The long run desired rate of capital utilization is set at $\overline{\omega} = 85\%$ which is roughly in line with average capacity utilization in the USA before the Global Financial Crisis.

10.1. *Artificial vs observed time series*

We run simulations and generate artificial time series for different random seeds. Given a specific random seed, from aggregation of individual quantities we generate the time series of GDP which is characterized by irregular oscillations of modest amplitude along a long run average interrupted by recurrent, albeit infrequent, sizable slumps followed by long recoveries.

It is interesting to compare, at least from a qualitative point of view, the properties of the artificial and the observed time series. We will focus on four variables: GDP, consumption, investment and unemployment. The observed time series, retrieved from the FRED database, consist of quarterly data ranging from 1955 to 2013 for unemployment and from 1947 to 2013 for investment, consumption and GDP. The artificial time series are generated running the model 20 times for 3000 periods and discarding the first 1000 periods to get rid of transient behavior. The HP-filter has been applied both to artificial and observed time series in order to isolate the cyclical component. The results are shown in Tables 3 and 4.

The properties of the artificial time series are broadly comparable to those of the observed time series. Both in artificial and in real data the volatility of consumption is lower than the volatility of GDP, which in turn is lower than the volatility of investment and of unemployment.

The first lag autocorrelation of all the variables considered is positive and high both in artificial and in real data. This important feature is not grounded in the presence of an aggregate shock but emerges from the adaptive behavior of the agents and the self-organization of the macro-economy.[37]

Fig. 3 shows the autocorrelations in artificial and observed time series for the variables in question up to 20 lags. The autocorrelation functions look remarkably similar for artificial and real data, especially in the first lags.

In Fig. 4 we plot the autocorrelation of GDP and the correlation between GDP and investment, unemployment and consumption respectively at different lags. On the x-axis we report the length of the lag. On the y-axis we report the

[37] In Section 10.3 we will analyze the mechanisms that generate the positive autocorrelation of GDP.

Table 3
Standard deviation and first lag autocorrelation of the cyclical component of the observed time series. The data have been downloaded from FRED, the codes from the first row to the last are GDPC1, GPDIC96, PCECC9, LRUN64TTUSQ156N. All data are in real terms.

Observed Time series	Standard deviation	First lag autocorrelation
GDP	1.6613	0.8485
Investment	7.5422	0.7952
Consumption	1.2854	0.8176
Unemployment	13.6804	0.6454

Table 4
Standard deviation and first lag autocorrelation of the cyclical component of the simulated time series. Both standard deviation and first order autocorrelation are the averages over 20 runs. The model was run for 3000 periods but only the last 2000 periods were used to compute the statistics.

Simulated time series	Standard deviation	First lag autocorrelation
GDP	1.4369	0.6831
Investment	15.2645	0.5547
Consumption	1.1784	0.6778
Unemployment	17.3468	0.6530

correlation between the cyclical component of GDP at time t and the cyclical component of the aggregate variable at time $t-lag$. The maximum of the absolute value of the correlation indicates whether a given variable is mostly correlated with past, future or contemporaneous GDP. From the figure we infer that all the macroeconomic variables considered are coincident indicators of GDP. Moreover the off-peak behavior is similar between observed and artificial time series.

10.2. The emergence of a crisis

In Fig. 5 we report the time series of GDP computed on artificial data generated by one simulation. After a transient (not shown) GDP fluctuates irregularly around a "long run mean" for an extended time span which we characterize as *normal times*.[38] Normal times come to an abrupt end around period 1250 when GDP starts falling, bottoming out after 200 periods of contraction. At the trough, GDP is 30% lower than the pre-recession level and unemployment has jumped to 1/3 of the labor force: this is indeed a Great Depression! The recovery is slower. It takes 400 periods for GDP to go back to normal. For lack of a better term, we will label such an eventful macroeconomic episode – i.e. a dramatic slump followed by a long recovery – a *crisis*.[39]

This is a robust *emerging property* of the model, i.e. a recurring pattern across many simulations. Since the crisis does not originate from an exogenous aggregate shock – as usual in standard macro models – it is difficult to understand what is going on. Why the crisis? Which are the main determinants of the slump? Which are the forces that pull the economy out of the recession? In this section we will try to shed light on these mechanisms.

In order to do so, let us fix ideas on the initial conditions with the help of Table 2. According to Eq. (9.15), $R^b = D^H + M^C + M^K + E^B$ where $M^C = D^C - L^C$ is liquidity in the hands of C-firms and $M^K = D^K - L^K$ is liquidity in the hands of K-firms. Total liquidity R^B – i.e. base money created by the central bank and hold by the bank as reserves – is equal to 12,000. It will be constant throughout simulations.

In period 1 workers and capitalists have deposits equal to $2 \times 3000 = 6000$ and $2 \times 250 = 500$ respectively.[40] Therefore $D^H = 6500$. Since in the initial period there are no loans and each firm holds deposits equal to $D_1^f = 10$, liquidity in the hands of C-firms and K-firms are $D^C = M^C = 10 \times 200 = 2000$ and $D^K = M^K = 10 \times 50 = 500$ respectively. Hence total liquidity in the hands of firms is 2500. Total deposits are equal to $D = 9000$. The net worth of the bank is equal to the bank's net liquidity: $E^b = R^B - D = 3000$.

In period 1, therefore, more than half of total liquidity is in the hands of households, while 1/3 consists of the net liquidity of the bank. C-firms have liquidity for 1/6 of the total (approximately 17%) and K-firms for 1/24 (4%).

When the economy takes off, the bank starts extending loans to firms in both sectors. Firms use loans to expand production and use deposits to manage payments. The economy self-organizes towards a long run average GDP. Liquidity flows from one agent to another and from one macroeconomic sector (households, C-firms, K-firms and the bank) to the

[38] Full employment GDP is equal to 1500. The long run mean of GDP in normal times is about 1350. On average the unemployment rate in normal times is 10%.

[39] There is a crisis if at least one period among all simulated periods have more than 15% unemployment.

[40] The endowment of personal assets (net worth) in the hands of a household is denoted by E^h in Table 2. In the absence of housing wealth and of households' debt, personal assets – i.e. financial wealth – coincide with deposits: $E^h = D^h$. In the initial period, therefore $D^h = 2$.

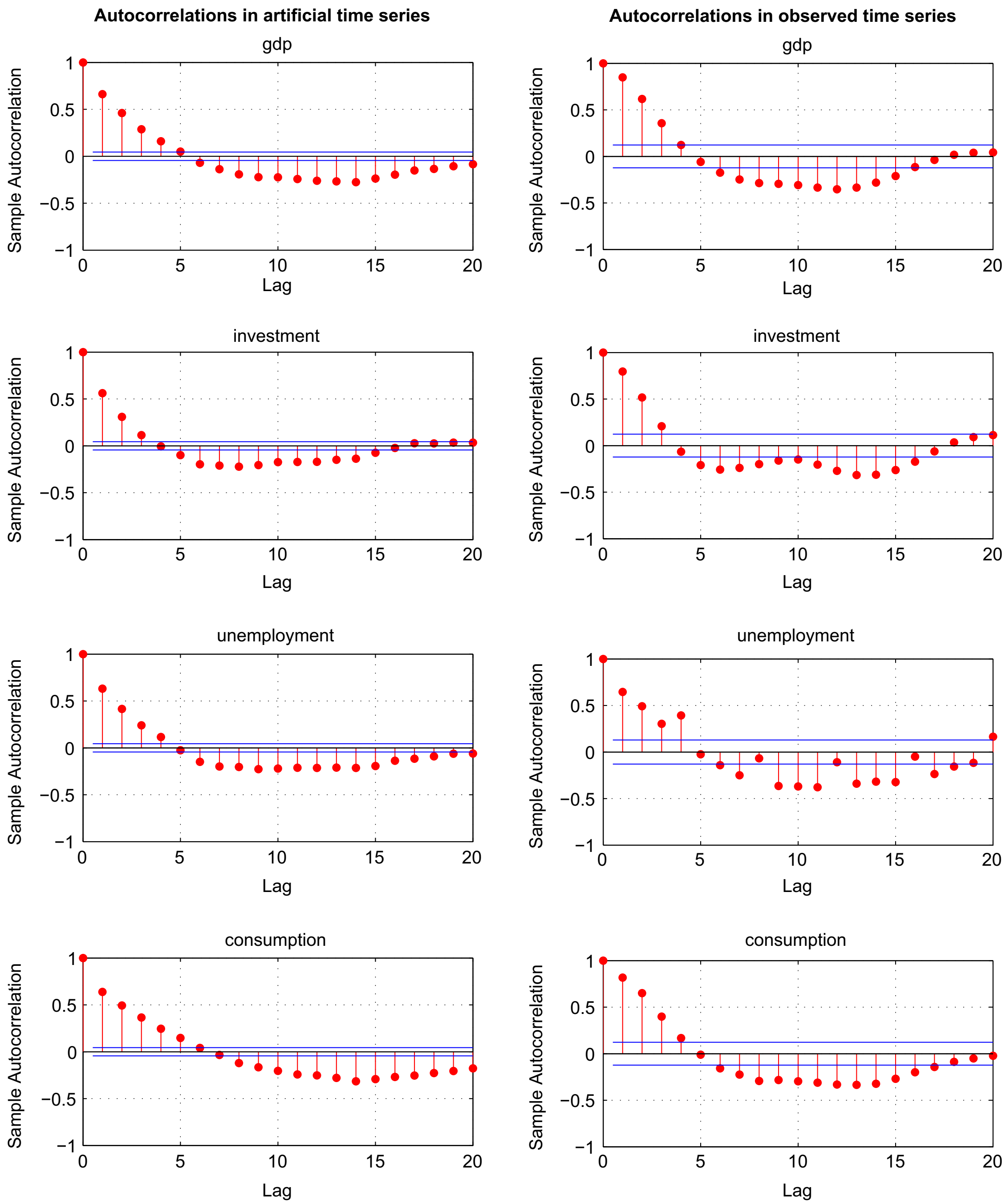

Fig. 3. GDP, investment, unemployment and consumption. *Left*: autocorrelations in artificial time series. *Right*: autocorrelations in observed time series.

other keeping total base money constant. The dynamic process of exchange of money can create an accumulation of liquid resources in one of the sectors and a dramatic drain in another sector. In the simulations, the volatility of GDP is low when changes in the composition of the money flow are relatively small. During a crisis, on the contrary, there are sizable swings in the composition of the money flows. To understand the behavior of the model, therefore, it is important to understand how base money flows among the sectors in the macro-economy.

Just before the crisis, towards the end of normal times i.e. around period 1250, liquidity in the hands of C-firms (M^C) represents approximately 30% of the total (see the right panel of Fig. 7). K-firms hold liquidity (M^K) approximately of the same size (see the right panel of Fig. 6).

In normal times, i.e. before period 1250, C-firms try to keep their activity at a fairly high level, investing in new capital goods at full speed. K-firms therefore experience excess demand for K-goods, which leads either to an increase

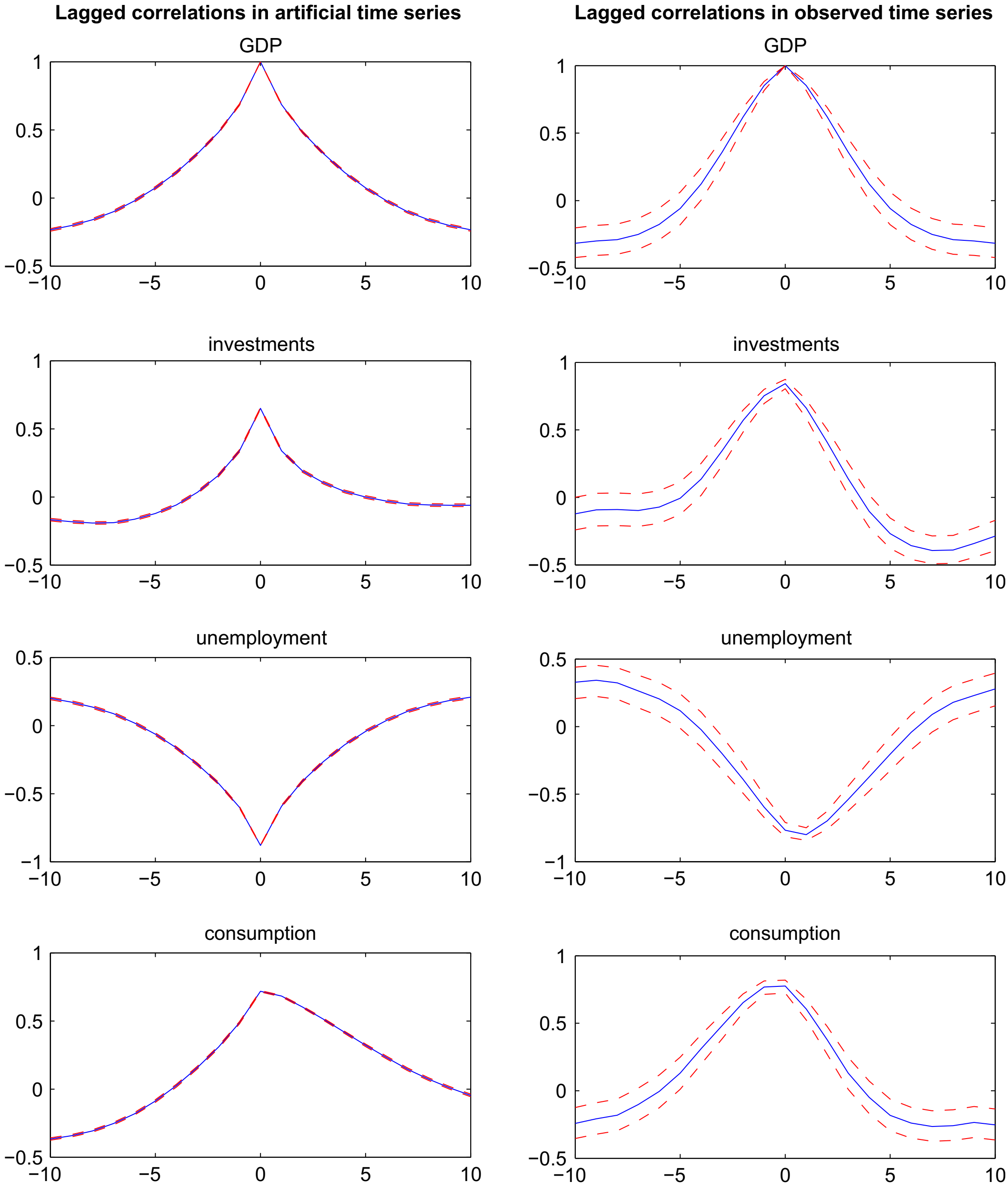

Fig. 4. The time series are treated with the hp-filter in order to isolate the cyclical component. The figures show the correlations between the cyclical component of GDP at time t and the cyclical component of GDP, investment, unemployment and consumption at $t-lag$ respectively. *Left*: artificial time series. *Right*: observed time series.

of desired production – if the individual price is higher than the average price of K-goods – or of the price – if the opposite holds true (see Eqs. (6.5) and (6.6)). Sooner or later the attempt to increase production of K-goods will be hindered by the tightness of the labor market, because the economy is geared to full employment. Hence excess demand for K-goods will ultimately push the prices of K-goods up. A tight labor market, coupled with booming investment leads to a steep increase of the average price of K-goods which peaks at 1300 (see the left panel of Fig. 6).

The steep increase in the price of capital goods moves liquidity out of the C-sector and into the K-sector. C-firms in fact increase the demand for credit and withdraw deposits to face the increasing bill due to the purchase of K-goods. As a

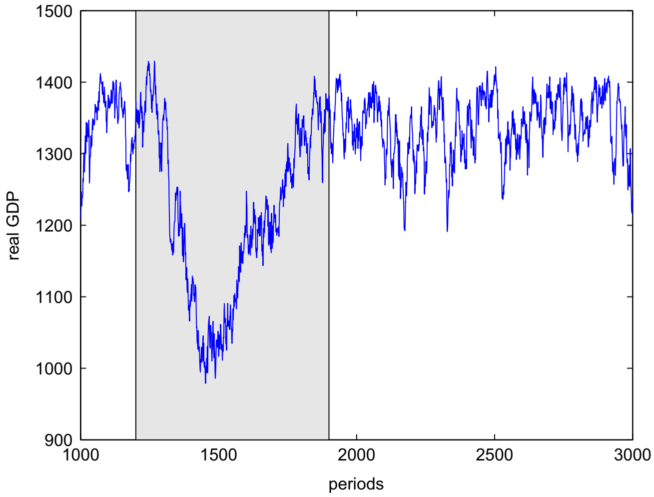

Fig. 5. Real GDP in a sample simulation. The shaded area highlights a crisis period.

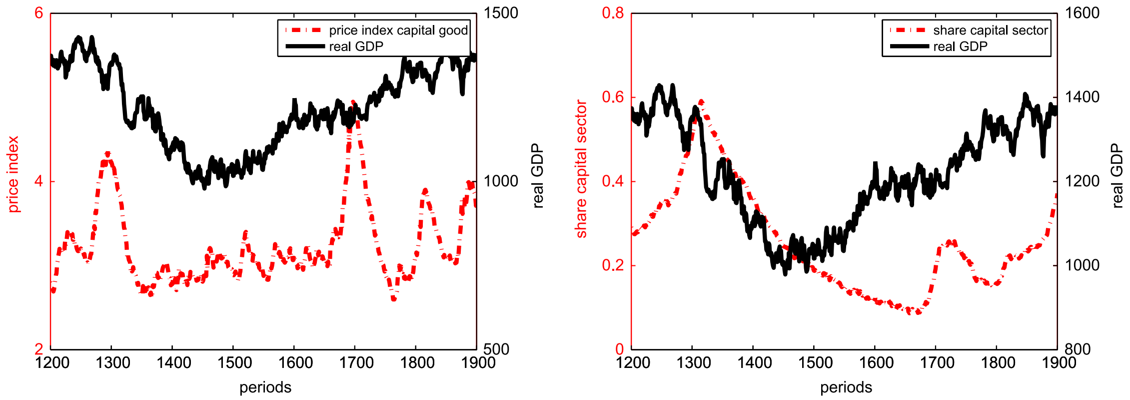

Fig. 6. *Left*: Capital good price. *Right*: Share of total money in the capital sector, net of loans.

consequence, the fraction of liquidity held by C-firms falls from 30% in period 1200 to -5% in period 1300.[41] Symmetrically, in the same period the fraction of liquidity accruing to K-firms jumps from 20% to 60% (see the right panel of Fig. 6).

The steep increase of the debt of C-firms from 4000 in period 1200 to 7000 in period 1300 is clearly shown in the left panel of Fig. 7, simultaneously the average estimated risk increases as shown in the left panel of Fig. 8. With the increase of debt and riskiness, C-firms run the risk of being refused credit. In fact the riskiness and the outstanding loans of borrowers determine the maximum amount of new loans the bank is willing to extend (see Eq. (8.11)). The amount of denied loans (credit rationing) increases sharply during the recession (see the right panel in Fig. 8). Credit rationing of C-firms plays the role of a dampening factor, reducing investment and C-good production. The consumption sector is the largest in the economic system so that the contraction of production in this sector reverberates throughout the economy, so that also the production of C-goods goes down. Hence the downturn.

During the recession, the price of K-goods goes down (until 1350), the debt of C-firms goes down (until 1500), liquidity in the hands of C-firms goes up (until 1700) and liquidity in the hands of K-firms goes down (until 1700). C-firms reduce capital and debt and increase deposits so that their liquidity goes up. Simultaneously, K-firms reduce production and prices. The slack of the demand for labor increases unemployment.

It is interesting to note that around period 1700 there is a second spike in capital good prices and a second spike in the share of money owned by the capital sector (see Fig. 6). At the same time, there is a drop in the share of liquidity held by C-firms (the right panel of Fig. 7). These phenomena do not trigger a new crisis but they are associated with a pause in the pace at which activity increases during the expansion.

[41] At the end of Section 9.1, we have derived the following identity: $M^C = D^C - L^C = E^C - (K + \Delta^C)$. Hence liquidity M^C becomes negative when the outstanding debt towards the bank becomes greater than deposits, which also implies that net worth becomes smaller than the market value of the capital stock and the change in inventories.

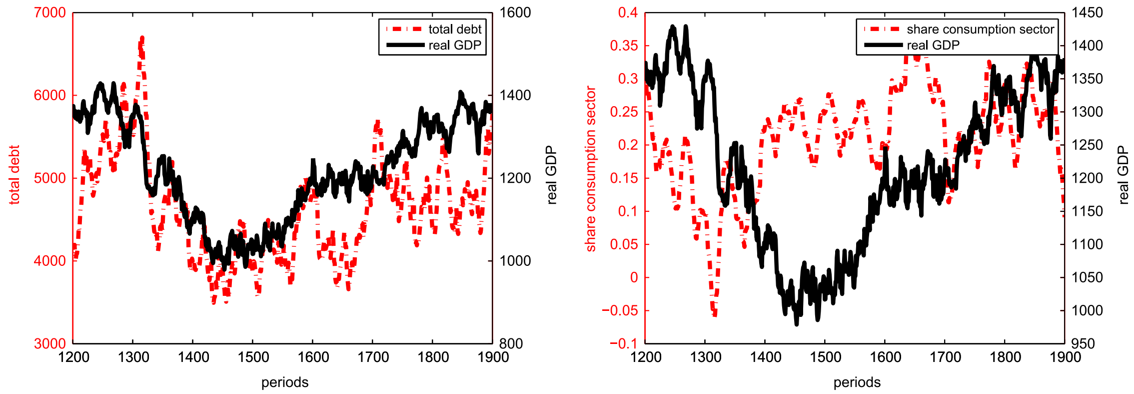

Fig. 7. *Left*: Total debt in the consumption sector. *Right*: Share of total money in the consumption sector, net of loans.

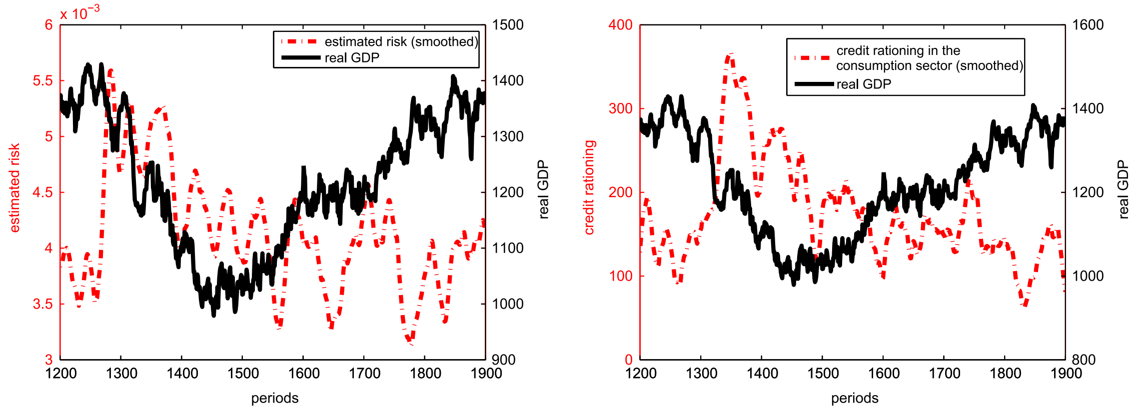

Fig. 8. *Left*: mean estimated risk of C-firms. *Right*: credit rationing in the consumption sector (smoothed).

Where does the recovery come from? GDP bounces back at the same time as the debt of C firms around period 1500. Around this period C-firms start getting into debt again. Banks relax the terms for extending loans and credit rationing goes down (from 1350). The share of money in the consumption sector keeps going up during the recovery (deposits growing faster than debt) and the share of money in the K-sector keeps going down.

Credit plays a crucial role, alongside capital, in the emergence and unfolding of a crisis in our artificial economy. In the left panel of Fig. 9 we report the correlation between unemployment in period t and total debt (loans extended to C-firms and K-firms) at different lags computed on artificial data.[42] The correlation reaches a global maximum ($+0.4$) at $t-5$ and a global minimum just after $t+5$ meaning that a high debt in the recent past implies high unemployment today but high unemployment today implies low debt in the future.

This is true also of the real economy. In the right panel of Fig. 9 we report the correlation between unemployment in period t and total (commercial and industrial) loans in the USA retrieved from the FRED database at different lags computed on real data.[43] The correlation reaches a global maximum ($+0.6$) at $t-10$ and a global minimum around $t+4$.[44] The empirical correlation structure is therefore qualitatively similar to the artificial correlation structure. A recession today is correlated with the build up of debt in the past, essentially because the high debt of firms creates an incentive for banks to limit credit and therefore leads to credit rationing. A low level of aggregate economic activity today is correlated with low debt in the near future because firms start de-leveraging during a recession to repair their balance sheet.

10.3. Understanding how the emergent properties emerge

Standard macroeconomic models mimic the autocorrelation of aggregate variables observed in real data by means of an exogenous autocorrelated shock, whose structure (usually an autoregressive process of order 1) shapes the autocorrelations of endogenous macro variables. In a MABM the autocorrelation of aggregate variables generated by artificial data is an

[42] As usual, the time series of unemployment and total debt are HP filtered in order to isolate the cyclical component.

[43] Time series of unemployment and credit are HP filtered, for the sake of comparison with statistics computed on artificial data.

[44] Notice, however, that the correlation bounces back after $t+5$ and reaches a new local maximum at $t+20$. Hence high unemployment today is correlated with high debt in the far future.

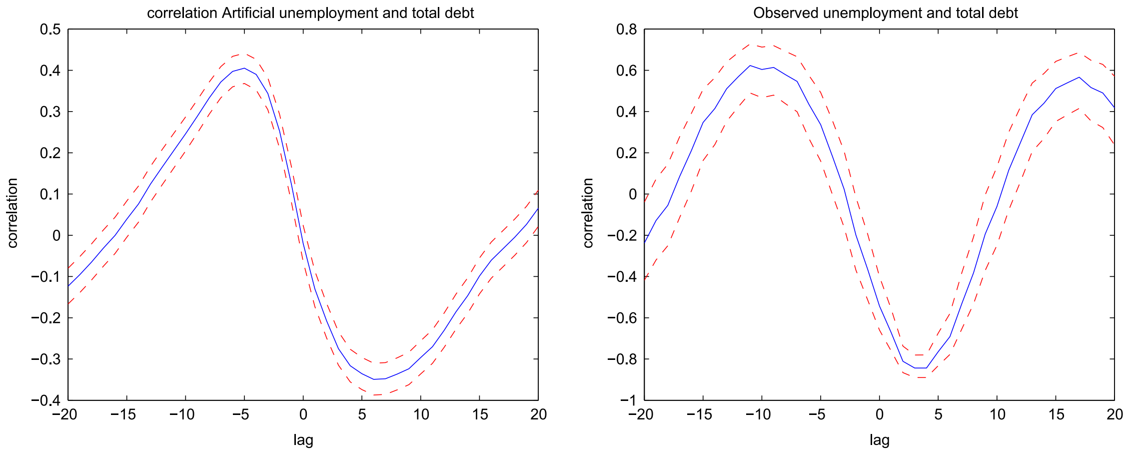

Fig. 9. *Left*: Artificial data. Correlation between the cycle component of the total debt lagged and the unemployment. *Right*: Observed data. Correlation between the cycle component of the total debt lagged and the unemployment.

Table 5
Adaptive behavior. The results on the presence of a crisis and on the autocorrelation in the GDP time series for different set ups (in brackets the 90% confidence interval). To generate deep crisis we need the full model. There is a crisis if at least one period among all simulated periods have more than 15% unemployment.

Model set up	Crisis	Autocorrelation (90% c.i.)
Full model	✓	0.6831 (0.6519, 0.7240)
No capital	✗	0.4728 (0.4314, 0.5085)
No financial	✗	0.6645 (0.6196, 0.7071)
No capital and no financial	✗	0.4114 (0.3678, 0.4563)

Table 6
Random behavior. The results on the presence of a crisis and on the autocorrelation in the GDP time series for different set ups (in brackets the 90% confidence interval). To generate deep crisis we need the full model. There is a crisis if at least one period among all simulated periods have more than 15% unemployment.

Model set up	Crisis	Autocorrelation (90% c.i.)
Full model	✗	0.0988 (0.0646, 0.1353)
No capital	✗	0.0647 (0.0362, 0.1115)
No financial	✗	−0.0080 (−0.0598, 0.0500)
No capital and no financial	✗	−0.0500 (−0.0961, −0.0089)

emergent property of the model, therefore it is not immediate to detect the mechanisms that are responsible for the emergence of these autocorrelations. In this section we try to shed some light on these mechanism.

In Tables 5 and 6 we report the effects of different setups of the model on the emergence of a crisis and on the autocorrelation of GDP. For each setup we run 20 simulations which last 3000 periods, discarding the first 1000.

The natural candidate to play the role of main driver of the autocorrelation is the interaction of adaptive micro-behaviors. Adaptive behavior is intuitively introducing memory in the individual choice variables, which is then transferred to the aggregate time series. Moreover, adaptive behavior generates two-way feedbacks between individual behavior and aggregate behavior. In order to assess the contribution of adaptive rules, in a first setup we replace them with purely *random behavior*: firms produce output equal to a constant plus an i.i.d. shock and sell at a price equal to a constant plus an i.i.d. shock. This implies the absence of any feedback between behaviors, and between the micro- and the macro-levels. The autocorrelation of GDP in the full model with adaptive behavior (i.e. the model discussed in the previous sections) is $+0.69$ and is only $+0.1$ in the model with random behavior.

What is the role of capital? In order to assess its role, we have to remove the autocorrelation built in the investment activity. To switch off this autocorrelation without changing the model too much, we still assume that C-firms use K-goods but capital is abundant and available for free. Without the autocorrelation built in capital adjustment, in the model with adaptive behavior the autocorrelation of GDP is $+0.47$.

What is the role of credit? To switch off the financial system we simply assume that the bank is considering all firms alike and risk free, and has infinite equity. Without credit constraint, in the model with adaptive behavior the autocorrelation of GDP is $+0.66$. The loss of autocorrelation is negligible, so the role of capital is more important in explaining the autocorrelation of GDP. Without capital and the financial system, the autocorrelation goes down to 0.41, close to the one without capital.

The autocorrelation is crucially determined by the adaptive behavior and enhanced by the presence of the capital sector. The emergence of a crisis, however, requires the presence of all the main features, i.e. adaptive behavior, Capital and Credit.

11. Conclusions

In this paper we have explored the emergent properties of a medium sized MABM with Capital and Credit. The interaction between the upstream and downstream segments of the corporate sector (consisting of K-firms and C-firms respectively) in a setting characterized by a durable and sticky input (capital), evolving financial conditions and limited access to credit are the main ingredients of the economic environment under scrutiny.

The upstream sector is relatively small (there are 50 K-firms) but exerts a crucial influence on the balance sheet and on the production costs of the relatively big downstream sector (consisting of 200 C-firms). GDP consists mainly of consumption goods, but the production of C-firms depends on households' demand and the availability of capital and credit, which in turn depend on the behavior of K-firms and banks.

In the long run GDP fluctuates around a mean which is not too far from potential output. A crisis, i.e. a dramatic recession followed by a long recovery, however, emerges only when the complex web of micro-behavioral interactions is properly taken into account. Behavioral interactions, in fact, are key in turning small changes of GDP into large contractions and expansions.

We assume that firms are unable to locate the demand functions they face and to observe the behavior of the competitors. To mitigate the risks of uncertainty, they follow adaptive behavioral rules, i.e. they use past information to decide on prices and quantities. This seemingly innocuous assumption has profound implications for the emerging properties of the models. Adaptive behavior in fact, may be conducive to macroeconomic feedbacks (through externalities) which amplify small exogenous idiosyncratic shocks and/or small changes in activity. Adaptive behavior, externalities and feedbacks are key for the emergence of a crisis and for the properties of the aggregate time series.

Experimental evidence has shown that human subjects faced with difficult choices in uncertain environments, tend to use simple heuristics to make decisions. Agents do not have full information, and even if they had full information they would not be able to process it. Conjecturing that they adopt simple adaptive heuristics is the most plausible assumption ex ante but also the most fruitful assumption ex post since they it is crucial in reproducing established stylized fact, e.g. the autocorrelation in the aggregate time series.

Future research will explore at least three sets of issues.

First of all, we would like to remove some of the most restrictive assumptions we made so far introducing, for instance, the storability of inventories of C-goods, diffuse ownership/shareholding (instead of a single owner for each firm), a different entry mechanism for new firms etc.

Second we want to model input–output relationships in a more granular representation of the corporate sector. Intermediate inputs with different technological properties (durability and stickiness) may play an important – and so far overlooked – role in the emergence of a crisis.

Last but not least we want to use this model for policy exercises. We already have a monetary policy parameter in the model (the risk free rate). The analysis of fiscal policy, on the contrary, requires a more sophisticated setting.

Bringing the model to the data will be key to assess the effects of policy moves in a quantitative framework. The econometric practice in agent based modeling is still in its infancy so that there is a long way to go before the model will be fully operative. In our opinion, however, the results obtained so far are promising and bode well for the future.

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