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Overlapping portfolios, contagion, and financial stability

Fabio Caccioli [a,b,*], J. Doyne Farmer [d,c], Nick Foti [c], Daniel Rockmore [c,e,f]

[a] *Department of Computer Science, UCL, Gower Street, London, WC1E 6BT, UK*
[b] *Centre for Risk Studies, Cambridge Judge Business School, University of Cambridge, Trumpington St Cambridge CB21 AG, UK*
[c] *Santa Fe Institute, 1399 Hyde Park road, Santa Fe, NM 87501, USA*
[d] *Institute for New Economic Thinking at the Oxford Martin School and Mathematical Institute, Oxford University, Eagle House Walton Well Road Oxford OX2 6ED, UK*
[e] *Department of Computer Science, Dartmouth College, Hanover, NH 03755, USA*
[f] *Department of Mathematics, Dartmouth College, Hanover, NH 03755, USA*

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A B S T R A C T

We study the problem of interacting channels of contagion in financial networks. The first channel of contagion is counterparty failure risk; this is captured empirically using data for the Austrian interbank network. The second channel of contagion is overlapping portfolio exposures; this is studied using a stylized model. We perform stress tests according to different protocols. For the parameters we study neither channel of contagion results in large effects on its own. In contrast, when both channels are active at once, bankruptcies are much more common and have large systemic effects.

1. Introduction

The economic and financial crises of the early 21st century give strong indications that the high level of interconnectivity characterizing much of the contemporary economic system can amplify and propagate the stress originated in a specific economic sector or a specific financial institution to other sectors and other institutions (see for instance Babus and Allen, 2009).

Connections between financial institutions are of various kinds, ranging from common assets held on balance sheets of different institutions to direct linkages between institutions corresponding to specific transactions. While such connectivity can serve as a means of risk management or increased efficiency for these institutions, it can also provide channels for contagion, thereby creating potential sources of systemic risk.

Different types of connections between financial institutions are associated with different mechanisms of contagion. For instance, contagion can occur because of counterparty risk or liquidity hoarding in the case of interbank lending, or because of fire-sales in the case of overlapping portfolios. Most of the literature so far has been devoted to assessing the stability of

* Corresponding author at: Department of Computer Science, UCL, Gower Street, London, WC1E 6BT, UK. Tel.: +44 (0)20 7679 0365.
E-mail address: f.caccioli@ucl.ac.uk (F. Caccioli).

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banking networks with respect to single contagion mechanisms, and much less is known about the interaction between different mechanisms.

Upper (2011) discusses and categorizes much of the work on robustness and stability in financial markets. The basic object of study is a network of banks where edges encode financial exposure. Most of these papers (see for instance Staum, 2012) model shocks to the system using variants of an algorithm developed by Furfine (2003) (see also Watts, 2002). The three steps of this iterative algorithm are to (1) create an extinction of a single bank, (2) calculate consequent extinctions via exposure to the original bank above an equity threshold, and (3) spread the contagion to other banks that have exposure to the now extinct banks. Thus, Furfine's algorithm also provides a method for assessing risk associated with counterparty risk.

This basic threshold extinction model has been modified in various ways to include safety nets (Upper and Worms, 2004), risk management strategies (Elsinger et al., 2006), different illiquidity conditions (Furfine, 2003), probabilities of contagion (vs. thresholds) (Frisell et al., 2007; Sheldon and Maurer, 1998; Müller, 2006), and the integration of the topology of the banking network (Allen and Gale, 2000; Nier et al., 2007; Battiston et al., 2009; Drehmann and Tarashev, 2011). Of particular relevance is Müller's detailed analysis of the Swiss banking network (Müller, 2006), where, in addition to interbank exposures, the risk associated with the existence of credit lines between institutions is taken into account.

Banks interact not only through direct interbank connections, but also through indirect connections mediated by financial markets. When the portfolios of financial institutions overlap due to investment in common assets, contagion can occur because of fire-sales that cause common assets to be devalued, causing further asset sales and devaluations (Cifuentes et al., 2005; Huang et al., 2013; Caccioli et al., 2014; Corsi et al., 2013).

In this paper we build on previous work by studying multiple contagion channels. We consider contagion due to counterparty risk and overlapping portfolios, and we explicitly show that the major contribution to systemic risk comes from the interplay between the two. This finding may help in explaining the following paradox: empirical studies as well as recent theoretical developments suggest that, in realistic scenarios, networks of direct exposures are not an important source of systemic risk (Young and Glasserman, 2013; Upper, 2011). Yet, in the aftermath of Lehman Brother's bankruptcy there was a substantial dry-up of liquidity in interbank markets. Why should market participants avoid being connected to one another if such connections do not represent a large risk? This apparent contradiction can be explained in terms of the interaction between counterparty risk, associated with direct interbank exposure, and liquidity risk, associated with fire-sales and overlapping portfolios. Interbank lending stopped after the collapse of Lehman Brothers because market participants were aware that overlapping portfolios could deteriorate their positions and make them over-exposed to their counterparties. Our paper shows in an empirical context that the interaction between these contagion channels is very important, creating risks that may be much larger than any single channel of contagion alone. This reinforces results of Geanakoplos (2003, 2010) and Brunnermeier and Pedersen (2009) that market liquidity and funding liquidity are mutually reinforcing and their interaction can lead to liquidity spirals.

In this paper we focus on the Austrian interbank network, for which we perform stress tests according to different protocols. We also characterize the statistical properties of the balance sheets in this system and of the network of mutual exposures. The Austrian banking system has been previously studied by Boss et al. (2005) and Puhr et al. (2012). This work differs not only in the time period of study (which allows us to do some comparison of statistics over the different epochs), but more importantly, in that our primary focus is on the analysis of the interaction between different contagion pathways. The national nature of the data also places our work in the growing body of literature and results that are now being generated for individual national banking systems (e.g., e Santos and Cont, 2010; Iazzetta and Manna, 2009; Müller, 2006). Collectively, this kind of work can ultimately enable a lessons-learned approach to the study of network aspects of the stability of interbank systems as well as the role of regulation in this setting.

Our empirical investigation draws heavily on a growing corpus of theoretical and modeling efforts that aim to quantify the relation between network topology and contagion effects (see for instance Allen and Gale, 2000; Battiston et al., 2009; Nier et al., 2007; Drehmann and Tarashev, 2011; Gai et al., 2011). These kinds of network stress tests have their analogs in other complex systems, most notably, the *in silico* "knockout" experiments or "extinction analyses" that have been performed in a variety of network contexts (Albert et al., 2000) including metabolic networks (Jeong et al., 2000), protein networks (Jeong et al., 2001), food web models of ecosystems (see e.g., Section 4.6 of Dunne, 2006 and the many references therein as well as Allesina and Mercedes, 2009) and most recently, in macroeconomics as represented by the World Trade Web (WTW) (Foti et al., 2012). Of some relevance is the "robust yet fragile" (RYF) categorization of networks. This is shorthand for networks that are resilient under the failure of a random node, but experience a rapid degradation under a judicious targeting of nodes for failure. This has been shown to be related to power law degree distributions (Doyle et al., 2005; Barabási and Albert, 1999; Yook et al., 2002; Albert et al., 2000) and/or small world characteristics (Wang and Chen, 2002; Moore and Newman, 2000; Xia et al., 2010). In the case of the WTW, path length-related measures of robustness do not seem to be appropriate and the RYF characterization is generalized accordingly (Foti et al., 2012).

The paper is organized as follows: in the next section we introduce the data-set considered here and provide a characterization of the statistical properties of balance sheets and of the topological properties of the interbank network. The properties of the Austrian banking system that we find are consistent with previous studies concerning other national interbank systems. This similarity suggests that the conclusions of our paper could be extended to interbank systems other than the specific one here considered. In Section 3 we present results of stress tests where contagion is due to counterparty risk. In Section 4 we consider what happens when we add a stylized model of overlapping portfolio risk. We explicitly show that, in certain regimes, the greatest contribution to systemic risk comes from the interaction between the two contagion mechanisms. We present our conclusions in Section 5.

2. The data

The data studied here[1] contain information on the balance sheets of Austrian banks. Data are available on a quarterly basis for the years 2006, 2007 and 2008 and consist of the following:

- interbank claims encoded in an exposure matrix L, whose entries L_{ij} represent the liabilities bank i has towards bank j;[2]
- total liabilities (including non-interbank liabilities) L_i^{tot};
- total assets (including non-interbank assets) A_i^{tot};
- total liquid assets A_i^{liq}, i.e., cash and reserves held with the central bank.

The number of banks for which information is provided changes slightly from quarter to quarter as detailed in the following table:

Year	1st quarter	2nd quarter	3rd quarter	4th quarter
2006	846	844	832	832
2007	834	834	825	825
2008	825	828	824	832

A similar data-set for the Austrian banking system concerning the period 2003–2006 has been previously studied by Boss et al. (2005).

Unless otherwise stated, we consider in the following subsections results obtained by aggregating data from all the quarters at our disposal. We have performed analysis also for subsets corresponding to single quarters and seen that the properties of the system are stable over time and consistent with that observed at the aggregate level.

2.1. Statistical properties of balance sheets

In Fig. 1 we show complementary cumulative distributions[3] for total assets and liquid assets. The two quantities display a similar pattern characterized by an initial regime with a flat distribution, a "typical" regime where the distribution roughly follows a power law distribution, and a subsequent cut-off regime.

As far as total assets are concerned (top panel of Fig. 1), in the small bank regime we have 3253 data-points with total assets smaller than (approximately) 60 million euro, while in the cutoff (large bank) regime 49 data-points have total assets greater than (approximately) 35 billion euro. The remaining 6687 data-points constitute the intermediate "typical" regime, that spans approximately three orders of magnitude. In this regime assets drop roughly as a power law with exponent $\simeq 0.75$. The bottom panel of Fig. 1 shows a similar structure for liquid assets. In this case the initial regime is made of banks with less than 5 million euro of liquid assets, while in the cutoff regime banks have more than 10 billion euro of liquid assets. The intermediate regime (ranging over three orders of magnitude) can again be characterized in terms of a power law, with an exponent $\simeq 0.67$. Note that banks in the "typical" regime for total assets are not necessarily in the "typical" regime for liquid assets (and vice versa). The inset of the figure presents a normalized histogram of the ratio between liquid assets and total assets. We see that the distribution is different from zero over almost its entire support, indicating that banks employ very heterogenous liquidity strategies.

An important parameter to characterize the investment strategy of financial institutions is the leverage, that is the ratio between total assets and equity. This measures the level of borrowing that banks use to finance their investments

$$\lambda_i = \frac{A_i^{tot}}{A_i^{tot} - L_i^{tot}}. \tag{1}$$

In Fig. 2 we present a scatter plot of total assets vs. leverage for the first quarter of 2006.[4] From the plot, two different groups of banks seem to emerge:

- *Region* I: A grouping of 765 banks with leverage higher than about 4.6 (blue dots).
- *Region* II: A grouping at the bottom of the plot consisting of 71 banks with leverage smaller than 4.6.[5]

[1] Our data was made available by the Oesterreichische Nationalbank. We would like to thank Claus Puhr and Martin Summer for their help in sharing and processing the data.

[2] Reporting data up until year-end 2007 excluded short-term interbank lending with a maturity of less than 1 month.

[3] We define the complementary cumulative distribution of a probability density function $f(x)$ as $F(y) = \int_y^\infty dx f(x)$, i.e., the probability that a random variable distributed as $f(x)$ is greater than y.

[4] We limit our study to the first quarter of 2006 for convenience. Analysis of other quarters produces similar plots.

[5] The data sample includes a number of special purpose banks (for instance pension funds or treasury departments of large corporates) as well as private banks. Neither of these engage in traditional lending business and have lower leverage. Nonetheless, since we do not have bank identifiers, we cannot say with certainty which institutions correspond to those in Region II.

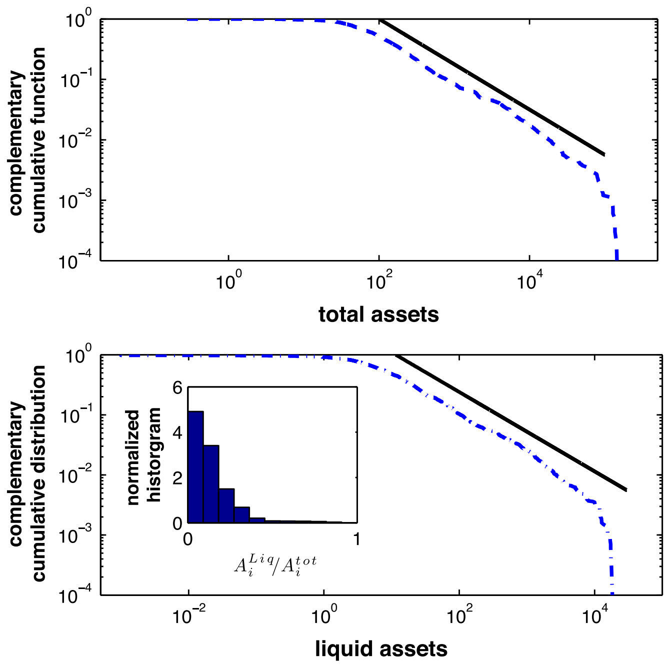

Fig. 1. Distribution of bank assets measured in millions of euro. *Upper panel*: complementary cumulative distribution of total assets (blue dashed line). Data are aggregated from all 12 quarters and are presented on a log-log scale. We observe three regimes: (i) banks with balance sheet size smaller than 60 M euro corresponding to the flat part of the distribution; (ii) typical banks with a balance sheet size between approximately 60 M and 35 B, for which the distribution can be approximated by power law behavior; (iii) big banks with balance sheet size greater than 35 B that characterize the cutoff of the distribution. The black solid line represents a power law of exponent 0.74 obtained from a least squares fit of banks in the "typical" regime. *Bottom panel*: complementary cumulative distribution of liquid assets. We observe again three regimes: (i) banks with small liquid assets (less than 1 M), for which the distribution is flat; (ii) banks with intermediate liquid assets (between 1 M and 1 B), for which the distribution is approximated by a power law; (iii) banks with large liquid assets (more than 1 B), in the cutoff of the distribution. The black solid line represents a power law of exponent 0.67 obtained from a least squares fit of banks in the "typical" regime. *Inset*: normalized histogram for the ratio between liquid assets and total assets, illustrating the dramatic cross-sectional difference in the liquidity. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

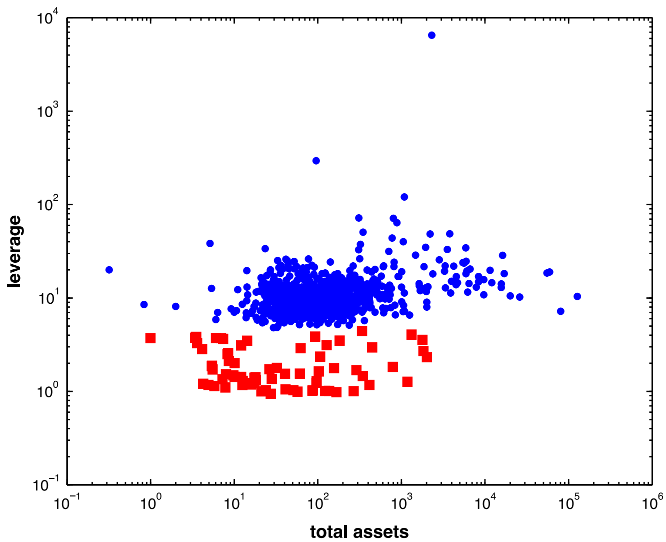

Fig. 2. Scatter plot of leverage vs. assets, in millions of euro. Banks can be roughly clustered into two different groups: red squares refer to banks with leverage smaller than 4.6 and blue dots to those with leverage greater than this. Data are for the first quarter of 2006. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

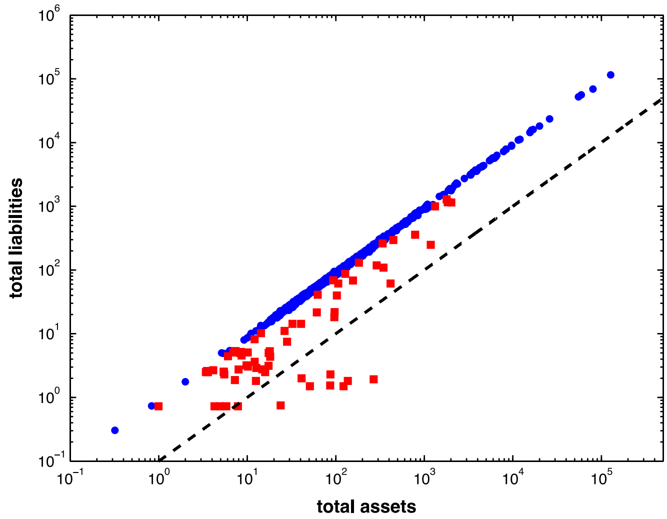

Fig. 3. Scatter plot of total assets vs. liabilities in log–log scale. The symbols and colors are as in Fig. 2. Banks with leverage higher than 4.6 show a rather precise linear relation between total assets and liabilities; this is much less clear for banks with smaller leverage. The black dashed line indicates a line of slope 1. Data refer to the first quarter of 2006. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

The usefulness of the naive grouping emerging from Fig. 2 becomes apparent in Fig. 3, where we present a scatter plot of total assets vs. total liabilities on a log–log scale. The relation between total assets and liabilities of a bank can be parametrized in terms of its leverage through the relation

$$\log L_i = \log\left(\frac{\lambda_i - 1}{\lambda_i}\right) + \log A_i. \tag{2}$$

Banks with the same leverage are therefore lying on the same straight line on the log–log plot, and different values of the leverage correspond to a different intercept. The value of the intercept is quite insensitive to changes in leverage: an infinite leverage corresponds to a value of zero, while a leverage of 4.6 to a value of about -0.1. Indeed we see from Fig. 3 that all banks with leverage greater than 4.6 are confined within a tiny strip of the plot, while only institutions with a significantly low leverage stand out.

2.2. Topological properties of the network

A network is the simplest representation of a collection of interacting objects. Our system is composed of banks that interact through interbank loans. Given the exposure matrix L_{ij}, we represent the interbank system in terms of a network of banks, where banks are nodes and a link between two banks represents the existence of a lending relationship at a given point in time. Links go from borrowers to lenders.

We focus here on the interpretation of some network metrics that are commonly used to describe financial networks, summarizing our results in Table 1 (for a more technical description see Appendix A). This characterization is important because results of complex network theory show that there is a relation between topology of networks and properties of dynamical processes taking place on them (see e.g. Barrat et al., 2008). Showing that the data-set considered in this study shares some of the properties previously observed for other interbank networks is then important as it suggests the generality of the contagion dynamics observed here.

- *Average degree*: The (total) degree of a node is the number of links to other nodes. A part of the degree reflects *in-degree* which is the number of incoming links for a given node, which in our case represents the number of banks a given bank lends to. The remainder is the *out-degree*, the number of outgoing links, which represents the number of banks a given bank borrows from. The average in- and out-degrees are simply the average of in- and out-degrees over all nodes, and represent the average diversification of interbank lending. The level of diversification plays an important role in determining the stability of interbank lending networks, as shown for instance in Allen and Gale (2000), Gai and Kapadia (2010) and Battiston et al. (2012). Empirical analyses performed on different interbank systems have shown that networks of interbank lending are sparse (Boss et al., 2005), i.e., that the average bank only lends to a small fraction of other banks. This is also true for the Austrian system, for which we measure an average degree of 26 for the period 2006–2008 (this counts both incoming and outgoing links – see Table 1).
- *Degree distribution*: The degree distribution is the probability distribution of degree across the nodes; the average degree is its first moment. Knowledge of the degree distribution makes it possible, for instance, to understand (and characterize)

Table 1
Network metrics.

Quantity	Mean	Standard deviation
Average degree	26	1
Number of hubs	45	2
Fraction of links of top 5% nodes	0.89	0.02
Assortativity	−0.62	0.03
Average local clustering	0.87	0.02

Summary statistics of network metrics. All quantities are computed averaging over the 12 quarters at our disposal. The average degree is the average number of links per node. The number of hubs is measured as the number of nodes with more than 100 connections. The fraction of links of the top 5% nodes is the number of connections involving at least one of the 5% mostly connected nodes divided by the total number of links in the network. The assortativity is measured as the correlation coefficient between degrees of neighboring nodes, while the local clustering is the average over all nodes of the fraction of a node's counterparties with an interbank relationship. The Austrian banking network between 2006 and 2008 shares the main stylized facts previously observed for other networks: sparseness, heavy tailed degree distribution, negative assortativity and large local clustering. More information can be found in Appendix A.

how homogenous or heterogeneous the system is in terms of diversification: do banks have a similar level of diversification or do there exist very well connected banks that act as intermediaries between other more poorly connected banks? The degree distribution is important because, for a fixed average diversification, different degree distributions lead to different contagion patterns, as shown, for instance, in Caccioli et al. (2012). Empirical analysis performed on different interbank systems has shown that networks of interbank lending are characterized by the presence of a small core of highly connected banks, called hubs (see for instance Craig and von Peter, 2014). This is also the case for the Austrian banking system, where there are only 45 banks that lend to more than 100 other banks at any given time, and about 89% of the links involve at least one of the 5% most connected nodes.[6]

- *Assortativity*: Network assortativity describes how banks tend to connect to one another. The assortativity is a measure of the correlation between the degrees of different banks: To what extent do banks tend to connect to other banks with similar/dissimilar degrees? Previous studies have highlighted the disassortative nature of the interbank system (Soramaki et al., 2007; Bech and Atalay, 2010; Iori et al., 2008), which we also observe in our analysis of the Austrian banking system. This indicates that poorly connected banks have a tendency to form links with highly connected banks, confirming that the network is characterized by a hub-and-spoke structure in which some banks play a key role.
- *Clustering coefficient*: The clustering of a node is the ratio between the number of existing and the number of possible connections. In other words, it is a measure of the probability that two banks with an interbank relationship have a common counterparty, and it is closely related to the counterparty risk externality recently described by Acharya and Bisin (2011) and Abbassi et al. (2013). Higher clustering corresponds to a bigger counterparty risk externality. For the Austrian interbank network we measured an average local clustering around 0.87, which is far higher than expected in a random network with the same average degree (about 0.03).

3. Stress tests: contagion through direct exposures

The first mechanism of contagion we consider is counterparty risk. If a given institution goes bankrupt its creditors face a loss and can in turn go bankrupt if this loss is greater than their equity. Note in this respect that interbank liabilities are often bidirectional, meaning that bank i borrows from bank j at the same time that j borrows from i. In the following we will consider for simplicity net exposures between banks, i.e., we define a matrix of net exposures L^{net} with entries $L_{ij}^{net} = \max\{0, L_{ij} - L_{ji}\}$, where L_{ij} is the amount of money that bank i borrowed from j. The implicit assumption is that if $L_{ij} > L_{ji}$ and bank i defaults on its obligations to j, j will cover part of the loss by not paying its debt to i. Let us define for convenience the capital (or equity) of bank i as the difference between its total assets and liabilities: $C_i = A_i^{tot} - L_i^{tot}$, where A_i^{tot} and L_i^{tot} represent total assets and liabilities of bank i. For each bank, we also introduce the state variable σ_i such that $\sigma_i = 1$ if i is bankrupt and zero otherwise. The protocol used to probe the stability of the system with respect to counterparty risk is the following:

1. A seed node q is selected and the corresponding bank is shut down. Here $\sigma_q = 1$, while $\sigma_i = 0$ for all other banks.
2. Banks revise their balance sheet. For each bank i if the following condition holds then the bank fails:
$$C_i < \sum_j L_{ji}^{net} \sigma_j, \tag{3}$$
where the right-hand side represents the loss of bank i, and we assume the extreme case where no money is recovered from failed banks. If i fails σ_i is set to one.
3. If there are new bankruptcies return to Step 2. Exit otherwise.

[6] The 5% as well as the 100 links thresholds have been chosen arbitrarily to give an intuition about the existence of a subset of highly connected hubs in the system. A comparison between the actual degree distribution and that of random networks can be found in Appendix A, Fig. A9.

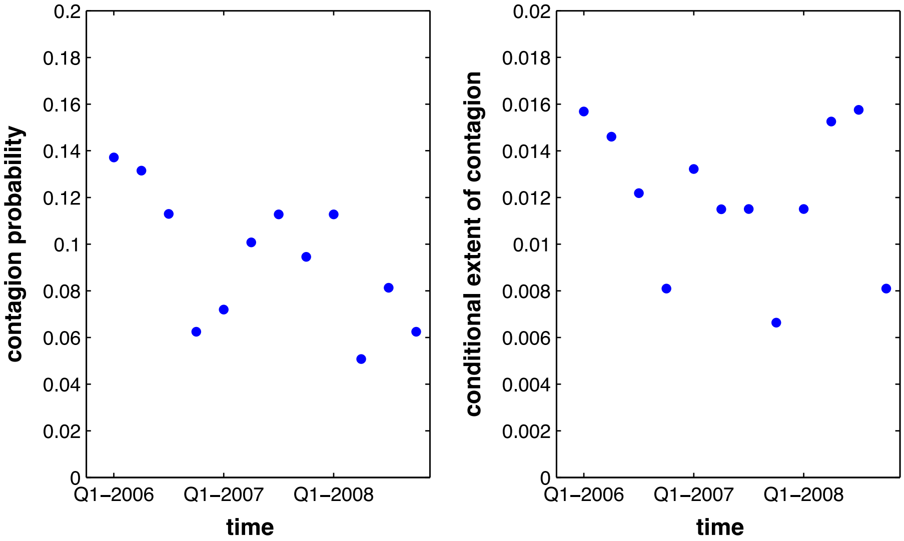

Fig. 4. Results of stress tests. *Left panel*: contagion probability due to counterparty risk for each quarter in 2006–2008. *Right panel*: conditional extent of contagion due to counterparty risk for 2006–2008. In every year only a small fraction of the banking system is affected by the initial shock. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

We define a *contagion event* as a situation where at least one bank goes bankrupt as a response to the initial perturbation.

Results are reported in Fig. 4 (blue dots) for the following two quantities:

(i) The *contagion probability*, defined as the probability of observing a contagion event.
(ii) The *conditional extent of contagion*, defined as the average fraction of banks affected by the initial shock if contagion occurs.

We do not restrict ourselves to first round effects, but rather run the default algorithms until a stationary state is reached where no new defaults occur between two consecutive iterations. We therefore also account for defaults that occur at any round of the process.

The probability of contagion fluctuates in a range of roughly 5–14%. The conditional extent of contagion also fluctuates, but remains small, never rising above 1.6%.[7]

The robustness of the system does not come as a surprise, as banks have a simple way of reducing the risk associated with direct interbank exposures. In this case a prudent bank manager will make sure that their bank is not exposed to a single other institution by more than the equity of their own bank (in some countries, like Germany and Austria, this is enforced by law). It is trivial to see that if most of the banks adopt this simple measure, for which no information is required by banks other than their own balance sheet, all domino effects are damped at the start. Given this consideration, are networks of direct interbank exposures important at all? Our answer is yes, they can become important as an amplification mechanism when other contagion channels are in place. To make this point, in the next section we consider the case of banks with common asset holdings, and show that network effects can in certain regimes greatly amplify the effect of a sudden devaluation of the common assets.

4. Stress tests: contagion due to overlapping portfolios

We have considered so far the network of interbank claims as a channel of stress propagation in the system. Now we want to account for a different mechanism of contagion, namely the existence of overlaps between bank portfolios, as studied in Cifuentes et al. (2005), Caccioli et al. (2014), Huang et al. (2013), and Corsi et al. (2013). As a first step, we consider the effect of banks sharing a common asset in their balance sheets, as done for instance by Cifuentes et al. (2005). For simplicity, we assume that all banks have a fraction c of a single common asset in their balance sheet, so that if A_i^{tot} is the total assets of bank i, cA_i^{tot} is the amount of that asset held by i. The parameter c is then a measure of the overlap of banks' balance sheets. The resulting system is a simple example of a multiplex network, i.e., a set of nodes connected by different types of links (see for instance Yağan and Gligor (2012) for an application of multiplex networks to contagion dynamics). In our setting banks are connected through both interbank lending and overlapping portfolios. Because we do not have data on the portfolios of Austrian banks, we consider the simple model above, with all banks connected to a unique common

[7] If we instead consider the share of total assets associated with defaulted banks we obtain a similar pattern but with a slightly higher conditional extent of contagion (around 4.8%).

asset. The analysis can be generalized to more complex structures where multiple asset classes are considered (Huang et al., 2013; Caccioli et al., 2014; Corsi et al., 2013), but for our purposes here a single asset model is sufficient to make the key point.

4.1. Exogenous shock to a common asset

We now ask what happens if the price of the common asset drops from a reference value 1 to $1-\phi$. This causes the value of the assets of bank i to fall, i.e.

$$A_i^{tot} \to A_i^{tot}(1-c) + A_i^{tot}c(1-\phi) = A_i^{tot}(1-c\phi). \tag{4}$$

As a result of the depreciation a bank will go bankrupt if its liabilities exceed the new value of its assets.[8]

We now establish a baseline for the effect of overlapping portfolio risk on its own. The blue solid curve in Fig. 5 shows the fraction of banks going underwater as a function of $c\phi$. To establish a reference point, note that $c\phi \simeq 0.05$ results in only a few percent of the banks in the system failing.

We now study the effect of interbank risk in tandem with common portfolio exposure, using the simple model above. The simulation is done as follows:

1. The common asset is depreciated.
2. Some banks go down because their liabilities exceed the new value of their assets.
3. These banks cause their creditors new losses, and contagion propagates due to counterparty risk.

The result of simulating the effect of common portfolio exposures together with counterparty risk is shown by a red dashed line in Fig. 5. Not surprisingly, the addition of counterparty risk increases the extent of contagion; the fraction of bankruptcies is consistently higher than before, and the functional dependence of the fraction of bankruptcies vs. $c\phi$ is similar. However, for small values of $c\phi$ the amplification of risk is dramatic. To emphasize this point, in the inset of the figure we zoom in for small values of $c\phi$ and plot the ratio of the fraction of bankruptcies when we also account for counterparty risk to that of depreciation of the common asset by itself, i.e., we plot the ratio of the red dashed curve to the blue curve. When $c\phi < 0.05$ the extent of contagion is increased by a large factor, ranging from roughly 3 to 10.

This amplification mechanism can be understood intuitively as follows: as we mentioned earlier the risk of default due to counterparty risk by itself can be effectively controlled by banks if they avoid being directly exposed to other institutions by an amount bigger than their own equity. This prudent measure can become ineffective, however, if the equity of banks is reduced by the devaluation of a common asset, as in this case banks may suddenly find themselves exposed to other institutions by more than their capital buffer. Therefore, contagion due to counterparty risk sets in if the devaluation of the common asset is big enough. Note that, if the common asset is highly devalued, the network of interbank exposures does not amplify contagion simply because most of the banks are already driven out of business as a result of the initial shock affecting their balance sheets. This is why the curve plotted in the inset of Fig. 5 goes to one for high values of $c\phi$.

4.2. Fire-sales and endogenous devaluation of a common asset

In the previous section we assumed that the shock to asset prices came from outside the system. We now consider a different scenario, in which the exogenous shock is to the balance sheet of a single bank. Due to the common asset, the shock is amplified through a fire sale of the assets of the failed bank, and by subsequent fire sales of the banks that fail as the shock propagates.

The stress protocol we consider is the following:

1. A bank i is selected for bankruptcy.
2. The interbank liabilities of bank i are not repaid and its portfolio of illiquid assets is sold.
3. The sale induces a devaluation of the common asset proportional to the relative size of bank i, i.e., $\theta A_i^{tot}/\sum_j A_j^{tot}$, where the parameter θ is related to the liquidity of the common asset.[9]
4. All banks suffer a loss that is due to the devaluation of the common asset. Banks with direct interbank exposures to i take a further hit due to counterparty risk.
5. If new banks are bankrupt, return to Step 2. Exit otherwise.

[8] The implementation of overlapping portfolios here through banks investing a share of their total assets in the same common asset is equivalent to the introduction of a common haircut on the size of banks' balance sheets. This translates into a leverage determined haircut on bank capital. We opt for an interpretation in terms of overlapping portfolios because it applies to more general settings with a granular structure, such as those described in Huang et al. (2013), Caccioli et al. (2014), and Corsi et al. (2013).

[9] Under a linear market-impact function, the loss suffered by bank k if bank i liquidates its portfolio is $\theta c A_k^{tot} A_i^{tot}/\sum_j A_j^{tot}$. Therefore the dynamics depends on the fraction of common asset held by banks and on its liquidity only through the product θc.

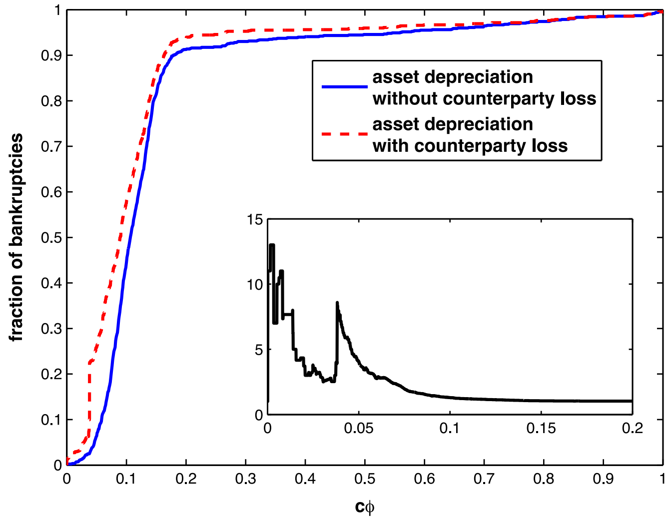

Fig. 5. Result of stress tests. Main panel: fraction of bankruptcies in the system due to the depreciation of the common asset in absence (blue solid line) or presence (red dashed line) of contagion via counterparty risk. Inset: ratio of the two quantities plotted in the main panel. The network introduces a channel of contagion that can significantly increase the number of failures if $c\phi < 0.05$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

In Fig. 6 we plot the probability and conditional extent of contagion as a function of θc, where θ is a liquidity parameter and c is the fraction of total assets invested by each bank in the common asset.[10] The results are for the first quarter of 2006. Each panel shows two lines. Blue solid lines refer to the case in which the network of direct interbank exposures is not accounted for, while red dashed lines show what happens when interbank exposures are included. The presence of the network increases both the probability and the conditional extent of contagion. The contagion probability gradually increases from about 0.14 to 0.49 as c increases from 0 to 1. More interestingly, we observe that when interbank exposures are included there is a sudden jump in the conditional extent of contagion for $c \simeq 0.3$.[11]

What causes the sudden jump in the conditional extent of contagion when interbank exposures are included? The key fact is that overlapping portfolio risk is fundamentally global in its scope, whereas interbank risk is more localized. To better understand this point, in Fig. 7 we show histograms of the fraction of bankruptcies observed in our simulations under the different stress protocols, i.e., counterparty risk alone, overlapping portfolio risk alone, and both of them together. Counterparty risk alone never causes systemwide cascades. Overlapping portfolio risk does cause systemwide cascades, but only rarely (12 cases out of 846). In contrast, when both of them act at once, the frequency of system-wide cascades is greatly amplified. In the latter case, out of 846 experiments corresponding to the initial failure of one bank, 413 cases yielded a cascade affecting roughly 99% of the system.

4.3. Dynamics of cascading failure

Although the Furfine/Watts algorithm for counterparty failure is somewhat artificial,[12] it can nonetheless be used to investigate interesting differences in the dynamics of failure between the channels of contagion. This is done by simply counting the number of banks that fail at each time step relative to the initial shock. The results, normalized to the total number of bankruptcies observed, are shown in Fig. 8 and a summary is given in Table 2.

The first result is that the dynamics are not long-lived: in all three cases the algorithm always halts within four steps. Nonetheless, there are interesting differences. For counterparty failure and overlapping portfolios alone the number of failures decays over time. Interestingly, when both channels of contagion are included, a dramatic climax is reached in the second time step, during which almost all the failures occur.

The difference in the time development of the two channels of contagion observed here may be an artifact of the simplifying assumption we have made here of a single overlapping asset. In a more complicated and more realistic scenario

[10] Note that the total size of the effect will vary depending on what fraction of the assets are held by the banks as opposed to other agents and how liquid those assets are. For example, the behavior of the system for different values of θ and constant c can be inferred from Fig. 6.

[11] Similar patterns would be observed if we were to consider the fraction of assets held by failed banks.

[12] The Furfine/Watts algorithm organizes failures into discrete turns, and it also assumes that banks do not respond to external conditions by adjusting their portfolios at all until they fail, at which points all their assets are sold. The meaning of "time" in iterating the algorithm is thus somewhat artificial.

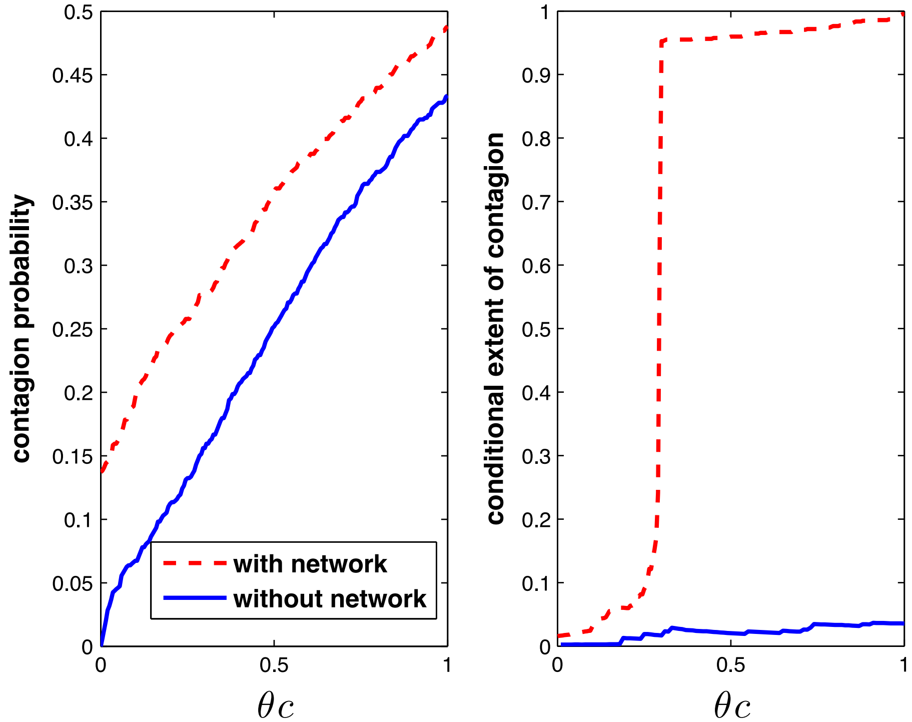

Fig. 6. Results of stress tests. *Left panel*: contagion probability due to liquidation of the common asset in absence (blue solid line) or presence (red dashed line) of contagion due to counterparty risk. *Right panel*: conditional extent of contagion due to liquidation of the common asset in absence (blue solid line) or presence (red dashed line) of contagion due to counterparty risk. The presence of a network of direct interbank exposures can substantially amplify contagion due to liquidation of common assets. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

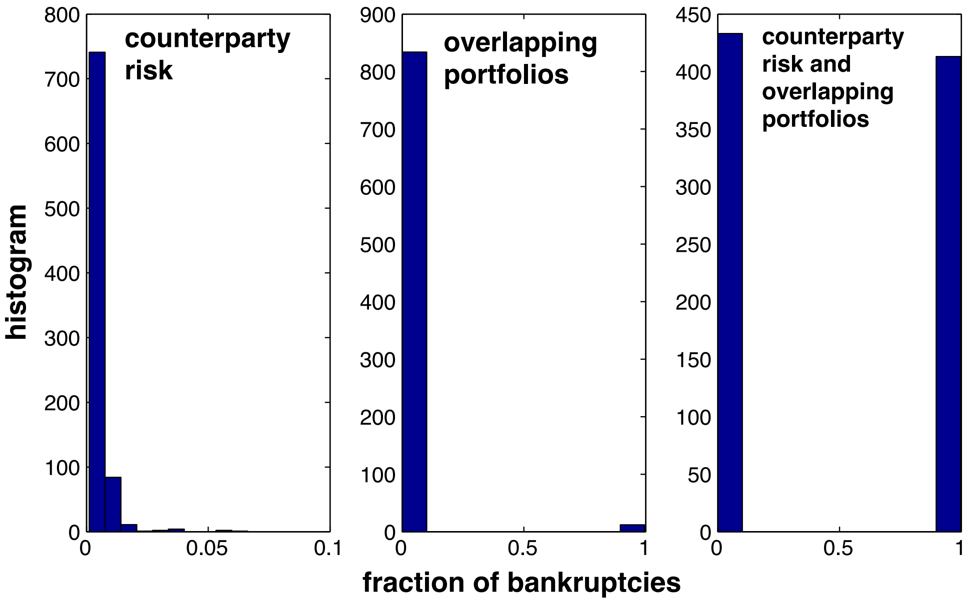

Fig. 7. Histograms of the fraction of bankruptcies observed in simulations according to different stress protocols. *Left panel*: contagion due to counterparty risk alone. *Middle panel*: contagion due to overlapping portfolios alone ($\theta c = 1$). *Right panel*: contagion due to both counterparty risk and overlapping portfolios ($\theta c = 1$). Overlapping portfolios allow for the existence of large cascades, but the probability of observing systemwide cascades is small unless counterparty-risk is also accounted for.

in which the overlaps have a richer network structure, such as that studied in Caccioli et al. (2014), one might expect to observe richer dynamics in which the contagion propagates more gradually. This may also be related to the global, “all or nothing” nature of the contagion observed in the previous section.

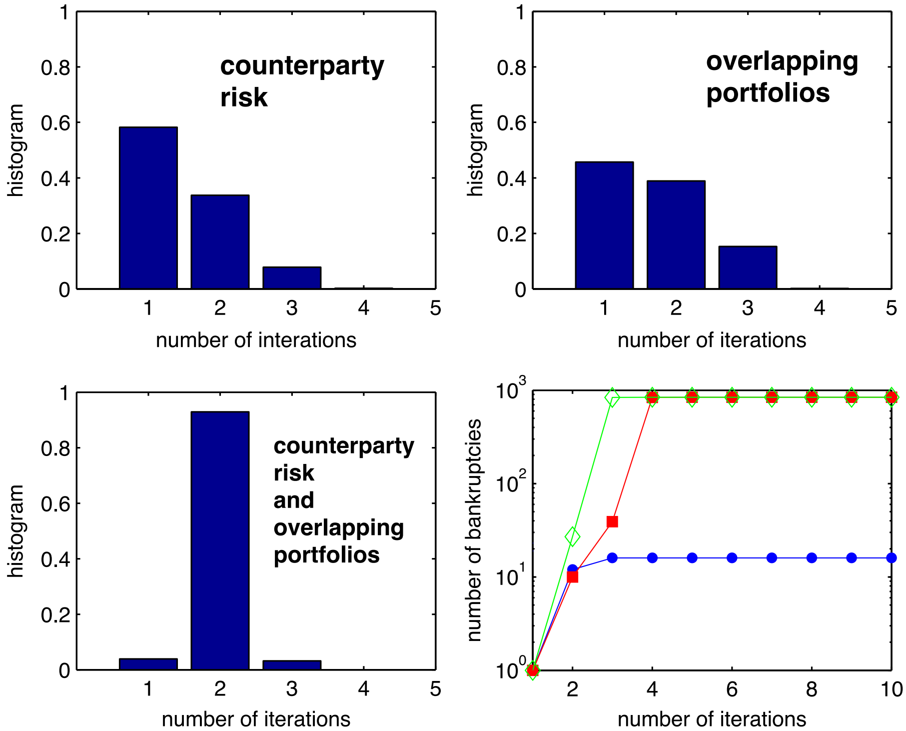

Fig. 8. *Top left panel*: distribution of times of default for the case of counterparty risk as a contagion mechanism. *Top right panel*: distribution of times of default for the case of overlapping portfolios as a contagion mechanism. *Bottom left panel*: distribution of times of default when both contagion mechanisms are in place. *Bottom right panel*: the number of bankruptcies as a function of the number of iterations for single instances. Blue dots: counterparty risk. Red squares: overlapping portfolios. Green diamonds: counterparty risk and overlapping portfolios. The system always reach the absorbing state after a few iterations: contagion is always fast. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Table 2

Contagion mechanism	Mean	Standard deviation
Counterparty risk	1.5	0.6
Overlapping portfolios ($\theta c = 1$)	1.7	0.7
Overlapping portfolios ($\theta c = 1$) and counterparty risk	2.0	0.3

Summary statistics for the time of default. Stress quickly spread throughout the system.

5. Conclusion

This paper focuses on assessing the importance of different channels of financial contagion and of their mutual interaction. Recent empirical and theoretical work suggests that, in real interbank networks, direct interbank exposures by themselves do not contribute significantly to financial contagion. The point we address here is that of understanding whether contagion due to direct exposures can nonetheless become relevant in interaction with other contagion mechanisms. We find that contagion due to counterparty risk can strongly amplify the stress induced by the presence of common asset holdings in bank balance sheets.

In this paper we considered data for the Austrian banking system in the period 2006–2008. In the first part of the paper we provided a statistical characterization of the system, in particular for its balance sheet and network properties. We found that there is a power law regime in the distribution of banks' size and that the network of interbank exposures is characterized by a heavy-tailed distribution of node degree and by negative correlations between degrees of neighboring nodes. Such properties have been shown to be relevant in terms of financial contagion and have been observed in the past for other data-sets of national banking systems. This fact suggests that the generic properties of the results of the stress tests presented in this paper apply more broadly than to just the Austrian system.

We performed stress tests of the Austrian banking system according to different stress protocols, and studied two different contagion mechanisms, counterparty risk and overlapping portfolios. The system is fairly stable if counterparty risk is the only contagion mechanism. However, when counterparty risk is combined with overlapping portfolio risk, it can strongly amplify the contagion, resulting in much larger cascading failures than would be observed otherwise.

Furthermore, we have shown that external asset shocks are not required to trigger the occurrence of large cascades. The failure of a single bank can trigger large effects through the resulting fire-sale of its assets. Once again, interbank exposures strongly amplify the effect.

This paper only begins to address the problem of inter-related channels of contagion. A more complete study would replace the over-simplified model of a single common asset used here with a richer network model of overlapping portfolio exposures, such as that used in Caccioli et al. (2014). This would be particularly interesting if it were possible to gather empirical data for both the interbank exposures and the portfolios of individual banks. An even more realistic extension would treat the dynamics of the buying and selling of assets prior to bank failure.

Acknowledgments

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Appendix A. Topological properties of the network

Given the exposure matrix L_{ij}, we can characterize the topological properties of the network by introducing binary directed and undirected adjacency matrices for the network. Each of these matrices will use the notation X. A directed liability matrix X^{lia} can be built in such a way that $X_{ij}^{lia} = 1$ if bank i borrowed money from bank j. This is just the exposure matrix thresholded at zero.

A complementary asset matrix X^{asset} can be built by assigning $X_{ij}^{asset} = 1$ if bank i lent money to bank j. Thus, $(X^{asset})' = X^{lia}$ where the superscript prime denotes transpose. Finally, an undirected matrix X can be built such that $X_{ij} = X_{ji} = 1$ whenever a relation exists between banks i and j, i.e.

$$X_{ij} = \max\{X_{ij}^{lia}, X_{ij}^{asset}\}.$$

Previous empirical work has shown that interbank networks are characterized by heavy-tailed degree distributions and negative degree correlations (Boss et al., 2005; Iazzetta and Manna, 2009; Degryse and Nguyen, 2004), and theoretical work has also been carried on to understand their impact on contagion dynamics (Caccioli et al., 2012; Georg, 2013; Lenzu and Tedeschi, 2012). We find that the same topological properties apply to the data-set considered here.

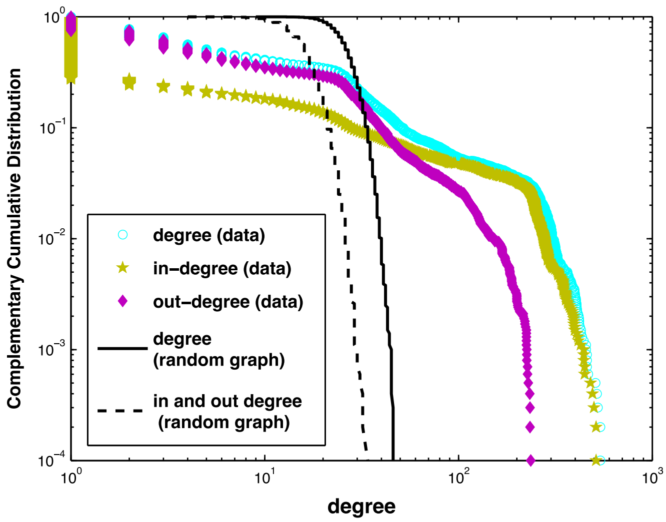

Fig. A9. Degree distribution. Cyan circle: complementary cumulative distributions of node degrees, computed from the undirected adjacency matrix X as $k_i = \sum_j X_{ij}$. Yellow stars: complementary probability distributions of node in-degrees, computed from the directed matrix X^{asset} as $k_i^{in} = \sum_j X_{ij}^{asset}$. Magenta diamonds: complementary cumulative distributions of node out-degrees, computed from the directed matrix X^{lia} as $k_i^{out} = \sum_j X_{ij}^{lia}$. Black solid line: complementary cumulative distribution of node degrees for Erdős–Renyi random graphs with the same average degree. Black dashed line: complementary cumulative distribution of node degrees for directed Erdős–Renyi random graphs with the same in/out average degree. Data are aggregated from all 12 matrices. The degree distribution of the real system appears to be heavy-tailed. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

In Fig. A9 we show the complementary cumulative distributions of in-degrees ($\sum_j X_{ij}^{asset}$), out-degrees ($\sum_j X_{ij}^{lia}$) and degrees ($\sum_j X_{ij}$). There is no obvious power law, but in all the three cases the distribution is characterized by a heavy tail, as can be observed by the comparison with the corresponding distributions obtained for Erdős–Renyi random graphs[13] with the same average degree. The presence of heavy tails in the degree distribution is usually associated with a higher overall robustness of the network with respect to the failure of a random node, but with a higher fragility with respect to "targeted failures" of highly connected nodes (Albert et al., 2000; Caccioli et al., 2012).

The level of correlations can be measured through the usual degree assortativity (cf. Newman, 2010) that quantifies the extent to which nodes of a given degree link to one another

$$r = \frac{\langle kk' \rangle_l - \langle (k+k')/2 \rangle_l^2}{\langle (k^2+k'^2)/2 \rangle_l - \langle (k+k')/2 \rangle_l^2}. \tag{A.1}$$

Here $\langle \cdots \rangle_l$ denotes the average over all links and k, k' the degrees of two nodes connected through a link.

We measured the assortativity for each of the 12 undirected adjacency matrices, obtaining an average assortativity (averaged over the 12 quarters) of $r_{av} \simeq -0.62 \pm 0.03$. The negative value indicates that in the interbank network nodes of low degree tend to be connected with nodes of high degree, a structure characteristic of networks that are effectively of a "hub and spoke" topology. As for the heavy-tailed nature of the degree distribution, the presence of correlations among degrees affects the probability of cascades triggered by the failure of a (random) bank (Caccioli et al., 2014) with respect to uncorrelated Erdős–Renyi random graphs.

Clustering coefficients (see Newman, 2010) measure the tendency of neighbors of a given node to be linked to each other. The propensity of these networks to form triangles (directed or not) is a natural proxy for the financial interdependence of the institutions. The local clustering coefficient C_i for a node i is defined as

$$C_i = \frac{\text{number of links between neighbors of } i}{\text{number of possible links between neighbors of } i}, \tag{A.2}$$

and gives a measure of how the neighborhood of i is close to be a clique.

The *average local clustering* is then measured as[14]

$$\overline{C} = \frac{1}{N}\sum_i C_i.$$

When we also average over the 12 quarters at our disposal, we find that $\overline{C} = 0.87 \pm 0.02$, where the error has been computed as the standard deviation over the different quarters. This value can be compared to that of Erdős–Renyi random networks of the same average degree, which have $\overline{C} = 0.032 \pm 0.002$.

A comparison with a null model in which we randomly rewire links while preserving the in- and out-degrees of each of the nodes (often called the "configuration model"), shows that the degree sequence is enough to reproduce both the negative correlations and the high local clustering observed in the data.

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[13] Erdős–Renyi random graphs are constructed by fixing a probability p of connecting any two nodes and performing the independent coin-tosses edge-by-edge. Note that in this case for a graph with n nodes, the expected degree is $p(n-1)$, so given a graph with average degree c, the appropriate wiring parameter is $p = c/(n-1)$. Similar networks can be generated for the directed case by assigning random directions to links.

[14] Notice that the average clustering coefficient puts more weights on low connected nodes, and the observed high value may be driven by the hub and spoke structure of the network.

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