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Speculative bubbles and crashes in stock markets: an interacting-agent model of speculative activity

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Abstract

In this paper, we present an interacting-agent model of speculative activity explaining bubbles and crashes in stock markets. We describe stock markets through an infinite-range Ising model to formulate the tendency of traders getting influenced by the investment attitude of other traders. Bubbles and crashes are understood and described qualitatively and quantitatively in terms of the classical phase transitions. When the interactions among traders become stronger and reach some critical values, a second-order phase transition and critical behavior can be observed, and a bull market phase and a bear market phase appear. When the system stays at the bull market phase, speculative bubbles occur in the stock market. For a certain range of the investment environment (the external field), multistability and hysteresis phenomena are observed. When the investment environment reaches some critical values, the rapid changes (the first-order phase transitions) in the distribution of investment attitude are caused. The phase transition from a bull market phase to a bear market phase is considered as a stock market crash. Furthermore, we estimate the parameters of the model using the actual financial data. As an example of large crashes we analyze Japan crisis (the bubble and the subsequent crash in the Japanese stock market in 1987–1992), and show that the good quality of the fits, as well as the consistency of the parameter values are obtained from Japan crisis. The results of the empirical study demonstrate that Japan crisis can be explained quite naturally by the model that bubbles and crashes have their origin in the collective crowd behavior of many interacting agents.

Keywords: Speculative bubbles; Stock market crash; Phase transition; Mean field approximation; Japan crisis

1. Introduction

The booms and the market crashes in financial markets have been an object of study in economics and a history of economy for a long time. Economists [1] and

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economic historians [2–4] have often suggested the importance of psychological factors and irrational factors in explaining the historical financial euphoria. As Keynes [1], a famous economist and outstandingly successful investor, acutely pointed out in his book, *The General Theory of Employment*, *Interest and Money*, stock price changes have their origin in the collective crowd behavior of many interacting agents rather than the fundamental values which can be derived from the careful analysis of present conditions and future prospects of firms. In a recent paper published in the Economic Journal, Lux [5] modeled the idea explicitly and proposed a new theoretical model of bubbles and crashes which links market crashes to the phase transitions studied in statistical physics. He explained the emergence of bubbles and crashes as a self-organizing process of infection among heterogeneous traders. [1] In recent independent works, several groups of physicists [7–17] proposed and demonstrated empirically that large stock market crashes, such as the 1929 and the 1987 crashes, are analogous to critical points. They have claimed that the financial crashes can be predicted using the idea of log-periodic oscillations or by other methods inspired by the physics of critical phenomena. [2] In this paper, we present an interacting-agent model of speculative activity explaining bubbles and crashes in stock markets. We describe stock markets through an infinite-range Ising model to formulate the tendency of traders getting influenced by the investment attitude of other traders. Bubbles and crashes are understood and described qualitatively and quantitatively in terms of the classical phase transitions. [3] Although the interacting-agent hypothesis [21] is advocated as an alternative approach to the efficient market hypothesis (or rational expectation hypothesis) [22], little attention has been given to the point how probabilistic rules, that agents switch their investment attitude, are connected with their decision-making or their expectation formations. Our interacting-agent model follows the line of Lux [5], but differs from his work in the respect that we model speculative activity here from a viewpoint of traders' decision-making. The decision-making of interacting-agents will be formalized by the *minimum energy principle*, and the stationary probability distribution on traders' investment attitudes will be derived. Next, the stationary states of the system and the speculative dynamics are analyzed by using the mean field approximation. It is suggested that the mean field approximation can be considered as a mathematical formularization of *Keynes' beauty contest*. There are three basic stationary states in the system: a bull market equilibrium, a bear market equilibrium, and a fundamental equilibrium. We show that the variation of parameters like the bandwagon effect or the investment environment, which corresponds to the external field, can change the size of cluster of traders' investment attitude or make the system jump to another market phase. When the bandwagon effect reaches some critical value, a second-order phase transition and critical behavior can be observed. There is a symmetry breaking at the

[1] For a similar study see also Kaizoji [6].

[2] See also a critical review on this literature [18].

[3] A similar idea has been developed in the Cont–Bouchaud model with an Ising modification [19] from another point of view. For a related study see also Ref. [20]. They study phase transitions in the social Ising models of opinion formation.

fundamental equilibrium, and two stable equilibria, the bull market equilibrium and bear market equilibrium appear. When the system stays in the bull market equilibrium, speculative bubble occurs in the stock market. For a certain range of the investment environment multistability and hysteresis phenomena are observed. When the investment environment reaches some critical values, the rapid changes (the first-order phase transitions) in the distribution of investment attitude are caused. The phase transition from a bull market phase to a bear market phase is considered as a stock market crash.

Then, we estimate the parameters of the interacting-agent model using an actual financial data. As an example of large crashes, we will analyze the Japan crisis (bubble and crash in Japanese stock market in 1987–1992). The estimated equation attempts to explain Japan crisis over six year period 1987–1992, and was constructed using monthly adjusted data for the first difference of TOPIX and the investment environment which is defined below. Results of estimation suggest that the traders in the Japanese stock market stayed the bull market equilibria, so that the speculative bubbles were caused by the strong bandwagon effect and the betterment of the investment environment in three year period 1987–1989, but a turn for the worse of the investment environment in 1990 gave cause to the first-order phase transition from a bull market phase to a bear market phase. We will demonstrate that the market-phase transition occurred in March 1990. In Section 2 we construct a model. In Section 3 we investigate the relationship between crashes and the phase transitions. We implement an empirical study of Japan crisis in Section 4. We give some concluding remarks in Section 5.

2. An interacting-agent model of speculative activity

We think of the stock market that large number of traders participate in trading. The stock market consists of N traders (members of a trader group). Traders are indexed by $j = 1, 2, \ldots, N$. We assume that each of them can share one of two investment attitudes, buyer or seller, and buy or sell a fixed amount of stock (q) in a period. x_i denotes the investment attitude of trader i at a period. The investment attitude x_i is defined as follows: if trader i is the buyer of the stock at a period, then $x_i = +1$. If trader i, in contrast, is the seller of the stock at a period, then $x_i = -1$.

2.1. Decision-making of traders

In the stock market the price changes are subject to the law of demand and supply, that the price rises when there is excess demand, and the price falls when there is excess supply. It seems natural to assume that the price raises if the number of the buyers exceeds the number of sellers because there may be excess demand, and the price falls if the number of seller exceeds the number of the buyer because there may be excess supply. Thus, a trader, who expects a certain exchange profit through trading, will predict every other traders' behavior, and will choose the same behavior as the other traders' behavior as thoroughly as possible. The decision-making of traders will be also influenced by changes of the firm's fundamental value, which can be derived from the analysis of present conditions and future prospects of the firm, and the return on the alternative asset (e.g. bonds). For the sake of simplicity of an empirical analysis we will use the ratio of ordinary profits to total capital, that is a typical measure of investment, as a proxy for changes of the fundamental value, and the long-term interest rate as a proxy for changes of the return on the alternative asset. Furthermore we define the investment environment as

the investment environment = ratio of ordinary profits to total capital, – long-term interest rate.

When the investment environment increases (decreases) a trader may think that now is the time for him to buy (sell) the stock. Formally, let us assume that the investment attitude of trader i is determined by the minimization of the following disagreement function $e_i(x)$:

$$e_i(x) = -\frac{1}{2}\sum_{j=1}^{N} a_{ij}x_ix_j - b_isx_i\,, \tag{1}$$

where a_{ij} denotes the strength of trader j's influence on trader i, and b_i denotes the strength of the reaction of trader i upon the change of the investment environment s which may be interpreted as an external field, and x denotes the vector of investment attitude $x = (x_1, x_2, \dots, x_N)$. The optimization problem that should be solved for every trader to achieve minimization of their disagreement functions $e_i(x)$ at the same time is formalized by

$$\min E(x) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}x_ix_j - \sum_{i=1}^{N} b_isx_i\,. \tag{2}$$

Now, let us assume that traders decision-making is subjected to a probabilistic rule. The summation over all possible configurations of agents' investment attitude $x = (x_1, \dots, x_N)$ is computationally explosive with size of the number of trader N. Therefore, under the circumstance that a large number of traders participates into trading, a probabilistic setting may be one of the best means to analyze the collective behavior of the many interacting traders. Let us introduce a random variable $x^k = (x_1^k, x_2^k, \dots, x_N^k)$, $k = 1, 2, \dots, K$. The state of the agents' investment attitude x^k occur with probability $P(x^k) = \mathrm{Prob}(x^k)$ with the requirement $0 < P(x^k) < 1$ and $\sum_{k=1}^{K} P(x^k) = 1$. We define the amount of uncertainty before the occurrence of the state x^k with probability $P(x^k)$ as the logarithmic function: $I(x^k) = -\log P(x^k)$. Under these assumptions the above optimization problem is formalized by

$$\begin{aligned} \min \quad & \langle E(x)\rangle = \sum_{k=1}^{N} P(x^k)E(x^k) \\ \text{s.t.} \quad & H = -\sum_{k=1}^{N} P(x^k)\log P(x^k), \quad \sum_{k=-N}^{N} P(x^k) = 1\,, \end{aligned} \tag{3}$$

where $E(x^k) = \frac{1}{2}\sum_{i=1}^{N} E_i(x^k)\, x^k$ is a state, and H is information entropy. $P(x^k)$ is the relative frequency of the occurrence of the state x^k. The well-known solutions of the above optimization problem is

$$P(x^k) = \frac{1}{Z}\exp(-\mu E(x^k)), \quad Z = \sum_{k=1}^{K}\exp(-\mu E(x^k)), \quad k = 1, 2, \ldots, K\,, \tag{4}$$

where the parameter μ may be interpreted as a *market temperature* describing a degree of randomness in the behavior of traders. The probability distribution $P(x^k)$ is called the *Boltzmann distribution*, where $P(x^k)$ is the probability that the traders' investment attitude is in the state k with the function $E(x^k)$, and Z is the partition function. We call the optimizing behavior of the traders with interaction among the other traders a *relative expectation formation.*

2.2. The volume of investment

The trading volume should depend upon the investment attitudes of all traders. Since traders are supposed to either buy or sell a fixed amount of stock (q) in a period, the aggregate excess demand for stock at a period is given by $q\sum_{i=1}^{N} x_i$.

2.3. The price adjustment processes

We assume the existence of a market-maker whose function is to adjust the price. If the excess demand qx is positive (negative), the market maker raises (reduces) the stock price for the next period. Precisely, the new price is calculated as the previous price y_t plus some fraction of the excess demand of the previous period according to $\Delta y_{t+1} = \lambda q \sum_{i=1}^{N} x_{it}$, where x_{it} denotes the investment attitude of trader i at period t, Δy_{t+1} the price change from the current period to the next period, i.e., $\Delta y_{t+1} = y_{t+1} - y_t$, and the parameter λ represents the speed of adjustment of the market price. At the equilibrium prices that clear the market there should exist an equal number of buyers or sellers, i.e., $\sum_{i=1}^{N} x_{it} = 0$. Using the Boltzman distribution (4) the mean value of the price changes $\Delta\bar{y}$ is given by $\Delta\bar{y} = \sum_{k=1}^{N} P(x^k) q \sum_{i=1}^{N} x_i^k$.

2.4. Mean field approximation: Keynes' beauty contest

We can derive analytically the stationary states of the traders' investment attitude using a mean-field approximation which is a well-known technique in statistical physics. Let us replace the discrete summation of the investment attitude with the mean-field variable, $\langle x\rangle = \langle (1/N)\sum_{i=1}^{N} x_i\rangle$. We additionally assume that the parameter a_{ij} is equal to $a_{ij} = a$ and $b_i = b$ for every trader. In this case, our interacting-agent model reduces to the Ising model with long-range interactions. Then the function (2) is approximated by $E(x) \approx -(1/2)\sum_{i=1}^{N}\sum_{j=1}^{N} aN\langle x\rangle x_i - \sum_{i=1}^{N} bsx_i$. Since the mean value of the stationary distribution is given by $\langle E(x)\rangle = -\,\partial \log Z/\partial\mu$, the mean field $\langle x\rangle$ is

$$\langle x\rangle = \tanh(\mu a N\langle x\rangle + \mu b s)\,. \tag{5}$$

The first term on the right-hand side of Eq. (5) represents that traders tend to adopt the same investment attitude as prediction of the average investment attitude and the second term represents the influence of the change of investment environment to traders' investment attitude. The first term may be interpreted as a mathematical formularization of *Keynes' beauty contest*. Keynes [1] argued that stock prices are not only determined by the firm's fundamental value, but in addition mass psychology and investors expectations influence the financial markets significantly. It was his opinion that the professional investors prefer to devote their energies not for estimating the fundamental values, but rather to analyze how the crowd of investors is likely to behave in the future. As a result, he said, most persons are "largely concerned, not with making superior long-term forecasts of the probable yield of an investment over its whole life, but with foreseeing changes in the conventional basis of valuation a short time ahead of the general public". Keynes used his famous *beauty contest* as a parable to stock markets. In order to predict the winner of a beauty contest, objective beauty is not much important, but knowledge or prediction of others' predictions of beauty is much more relevant. In Keynes view, the optimal strategy is not to pick those faces the player thinks the prettiest, but those the other players are likely to be about what the average opinion will be, or to proceed even further along this sequence.

2.5. Speculative price dynamics

We assume that the traders' investment attitude changes simultaneously (synchronous dynamics) in discrete time steps. We represent the dynamics of the investment attitude using a straightforward iteration of Eq. (5), such that

$$\langle x\rangle_{t+1} = \tanh(\alpha\langle x\rangle_t + \beta s_t), \quad \alpha = \mu a N,\ \beta = \mu b\,. \tag{6}$$

We call α the bandwagon coefficient and β the investment environment coefficient below because α may be interpreted as a parameter that denotes the strength that traders chase the price trend. Next, let us approximate the adjustment process of the price using the mean field $\langle x\rangle_t$. We can get the following speculative price dynamics:

$$\langle \Delta y\rangle_{t+1} = \lambda N\langle x\rangle_t\,. \tag{7}$$

Inserting the mean field of the price change, $\langle \Delta y\rangle_t$ into Eq. (6), the dynamics of the mean field of $\langle x\rangle_t$ can be rewritten as $\langle x\rangle_{t+1} = \tanh((\mu a/\lambda)\langle \Delta y\rangle_{t+1} + \mu b s_t)$. This equation shows that the traders result in basing their trading decisions on the analysis of the price trend $\langle \Delta y\rangle_{t+1}$ as well as the change of the investment environment s_t.

3. Speculative bubbles and crashes

The stationary states of the mean field $\langle x\rangle$ satisfy the following:

$$\langle x\rangle_t = \tanh(\alpha\langle x\rangle_t + \beta s) = ivf(\langle x\rangle_t, s)\,. \tag{8}$$

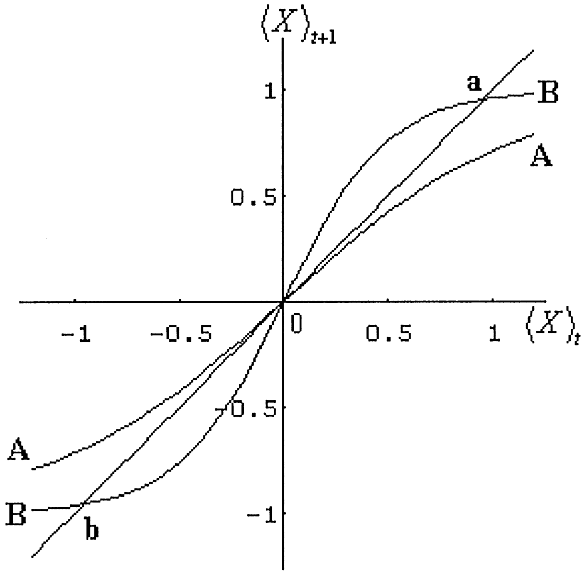

Fig. 1. The second-order phase transition. Numerical solution of Eq. (8) for $s=0$. The curve is the RHS of (8) plotted for different values of α: the AA line – $\alpha=0.9$; the BB line – $\alpha=2$.

Eq. (8) has a unique solution $\langle x\rangle$ for arbitrary βs, when α is less than 1. The stationary state $\langle x\rangle^*$ with $\beta s=0$ is called as the *fundamental equilibrium* which is stable and corresponds to the maximum of the stationary distribution of the investment attitude $P(x^*)$. In this stable equilibrium there is equal numbers of traders on average sharing both investment attitudes.

Moving in the parameter space (α, β) and starting from different configurations one has several possible scenarios of market-phase transitions. To begin with, let us consider the case that the investment environment s is equal to zero. In theory, when the bandwagon coefficient α is increased starting from the phase at $0<\alpha<1$, there is a symmetry breaking at critical point, $\alpha=1$, and two different market phases appear, that is, the *bear market* and the *bull market*. In analogy to the physical systems, the transitions may be called the second-order phase transition. Fig. 1 illustrates the two graphical solutions of Eq. (8) for $\alpha>1$ and $s=0$ and for $0<\alpha<1$ and $s=0$. The figure shows that for $0<\alpha<1$ and $s=0$ the fundamental equilibrium is unique and stable, but for $\alpha>1$ and $s=0$ the fundamental equilibrium 0 becomes unstable, and the two new equilibria, the bull market equilibrium a and the bear market equilibrium b are stable. At the bull market equilibrium more than half number of traders are buyers, so that the speculative bubbles occur in the stock market. Then, let us consider the effect of changes of the investment environment s in the case with a weak bandwagon coefficient, $0<\alpha<1$ and a positive investment environment coefficient, $\beta>0$. Fig. 2 shows that as the investment environment s changes for the better $(s>0)$, the system shifts to the bull market phase. By contrast, as the investment environment s changes

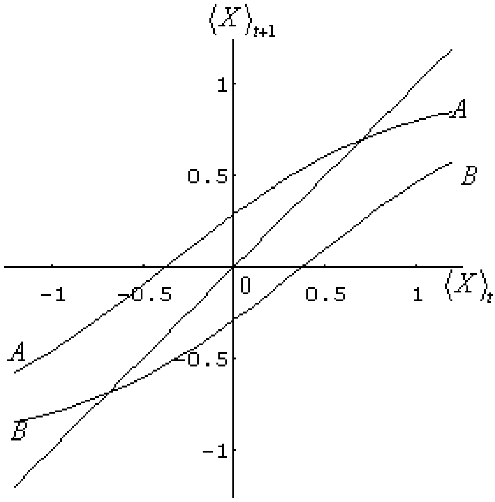

Fig. 2. Numerical solution of Eq. (8) for $\alpha = 0.8$ and $\beta = 1$. $f(\langle x \rangle_t, s)$ is the RHS of Eq. (8) plotted for different values of α: the AA line – $s = 0.3$; the BB line – $s = -0.3$.

for the worse ($s < 0$), the system shifts to the bear market phase. Therefore, when the bandwagon effect is weak, the stock price go up or down slowly according to the rises or falls in the investment environment.

Finally, let us consider the effect of changes of βs in the case with a strong bandwagon coefficient $\alpha > 1$. In this case, multistability and hysteresis phenomena in the distribution of investment attitude, as well as market-phase transitions are observed. The system has three equilibria, when $\alpha > 1$ and $|s| < s^*$, where s^* is determined by the equation

$$\cosh^2[\beta s \pm \sqrt{\alpha(\alpha - 1)}] = \alpha . \tag{9}$$

Two maximum of the stationary distribution on the investment attitude are found at $\langle x \rangle^-$ and $\langle x \rangle^+$and one minimum at $\langle x \rangle^*$. For $|s| = s^*$ and $\alpha > 1$, two of the three equilibria coincide at

$$\langle x \rangle_c = \sqrt{(\alpha - 1)/\alpha} . \tag{10}$$

For $|s| > s^*$ and $\alpha > 1$ the two of three equilibria vanish and only one equilibrium remains. In analogy to physical systems the transitions may be called first-order phase transitions. Fig. 3 illustrates the effect of changes in the investment environment in the case with a strong bandwagon effect $\alpha > 1$. In this case, speculative bubbles and market crashes occur. When the investment environment changes for the better (worse), the curve that denotes $f(\langle x \rangle_t, s)$, shifts upward (downward). As the investment environment s keeps on rising, and when it reach a critical value, that is, $|s| = s^*$, the bear market equilibrium and the fundamental equilibrium vanish, and the bull market equilibrium becomes an unique equilibrium of the system, so that the speculative bubble occurs in the stock market. Even though the investment environment changes for the

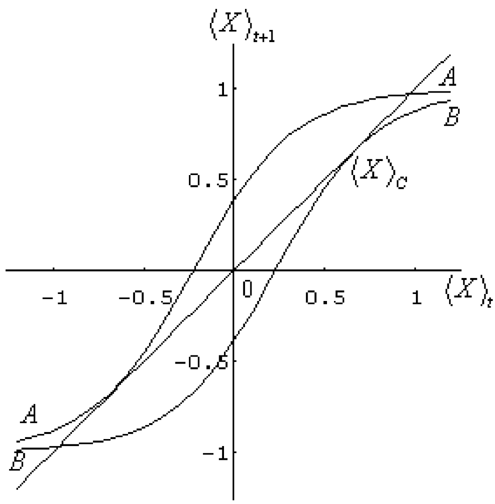

Fig. 3. The first-order transition: speculative bubble and crash. Numerical solution of Eq. (8) for $\alpha = 1.8$ and $\beta = 1$. The curves is $f(\langle x \rangle_t, s)$ plotted for different values of the investment environment: the AA line – $s = 0.41$; the BB line – $s = -0.41$.

worse, the hysteresis phenomena are observed in a range of the investment environment that can be calculated by solving Eq. (9). In other words the speculative bubble continues in a range of the investment environment. However, when the negative impact of the investment environment reaches a critical value, that is, $|s| = s^*$, the bull market vanish. Further decrease of s causes the market-phase transition from a bull market to a bear market. In a bear market on average more than half the number of traders are sellers, so that the stock price continues to fall on average. Thus, this market-phase transition may be considered as the stock market crash.

4. Empirical analysis: Japan crisis

Perhaps, the most spectacular boom and bust of the late 20th century involved the Japan's stock markets in 1987–1992. Stock prices increased from 1982 to 1989. At their peak in December 1989, Japanese stocks had a total market value of about 4 trillion, almost 1.5 times the value of all US equities and close to 45% of the world's equity market capitalization. The fall was almost as extreme as the US stock-market crash from the end of 1929 to mid-1932. The Japanese (Nikkei) stock-market index reached a high of almost 40 000 yen in the end of 1989. By mid-August 1992, the index had declined to 14 309 yen, a drop of about 63%.

In this section, we will estimate the parameter vector (α, β) for the Japan's stock market in 1987–1992. We use monthly adjusted data for the first difference of Tokyo Stock Price Index (TOPIX) and the investment environment over the period 1987–1992. In order to get the mean value of the stock price changes, TOPIX is adjusted by the

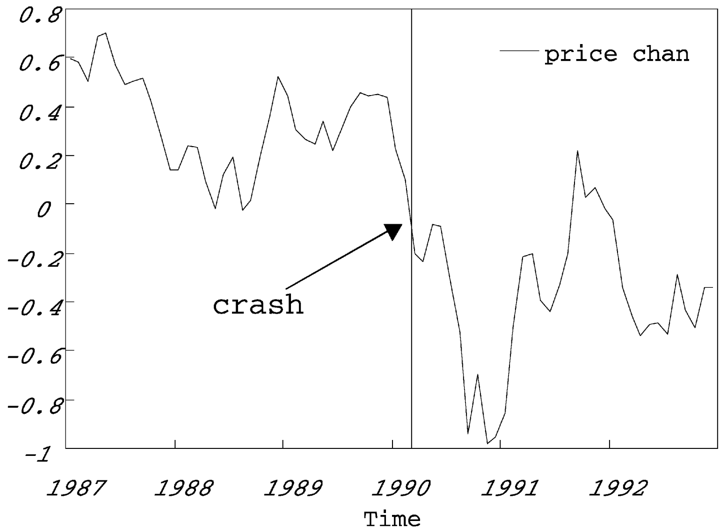

Fig. 4. The adjusted stock price change, $\langle \Delta p \rangle$: January 1987–December 1992.

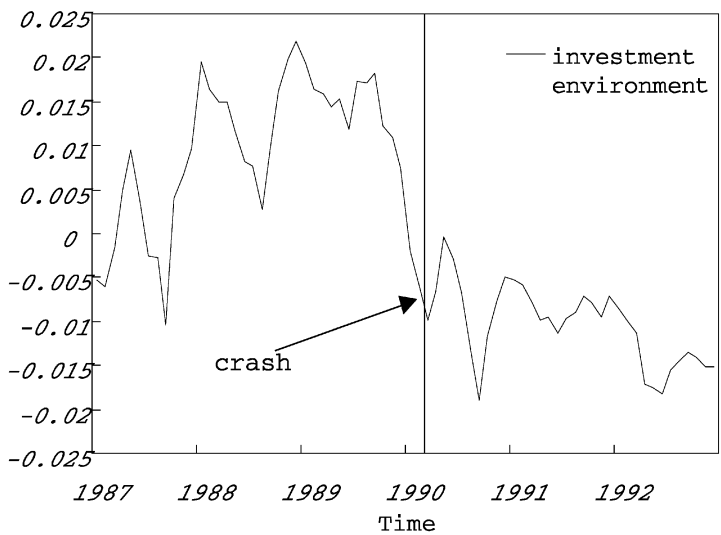

Fig. 5. The Japan's investment environment s_t: January 1987–December 1992.

use of a simple centered 12 point moving average, and are normalized into the range of -1–$+1$ by dividing by the maximum value of the absolute value of the price changes. The normalized price change is defined as $\langle \Delta p \rangle_t$. Figs. 4 and 5 show the normalized stock price changes and the investment environment, respectively. This normalized stock price change $\langle \Delta p \rangle_t$ is used as a proxy of the mean field $\langle x \rangle_t$. Substituting $\langle \Delta p \rangle_t$ for Eq. (8), we get $f(\langle \Delta p \rangle_t, s_t) = \tanh(\alpha \langle \Delta p \rangle_t + \beta s_t)$.

4.1. Model estimation: the gradient-descent algorithm

The bandwagon coefficient α and the investment environment coefficient β should be estimated using any one of the estimation technique. Since we cannot get the analytical solution because the function $f(\langle\Delta p\rangle_t, s_t)$ is nonlinear, we use the *gradient-descent algorithm* for the parameter estimation. [4] The gradient-descent algorithm is a stable and robust procedure for minimizing the following one-step-prediction error function:

$$E(\alpha_k, \beta_k) = \tfrac{1}{2}\sum_{t=1}^{n}[\langle\Delta p\rangle_t - f(\langle\Delta p\rangle_{t-1}, s_{t-1})]^2 . \tag{11}$$

More specifically, the gradient-descent algorithm changes the parameter vector (α_k, β_k) to satisfy the following condition:

$$\Delta E(\alpha_k, \beta_k) = \frac{\partial E(\alpha_k, \beta_k)}{\partial \alpha_k}\Delta\alpha_k + \frac{\partial E(\alpha_k, \beta_k)}{\partial \beta_k}\Delta\beta_k < 0 , \tag{12}$$

where $\Delta E(\alpha_k, \beta_k) = E(\alpha_k, \beta_k) - E(\alpha_{k-1}, \beta_{k-1})$, $\Delta\alpha_k = \alpha_k - \alpha_{k-1}$, and $\Delta\beta_k = \beta_k - \beta_{k-1}$. To accomplish this the gradient-descent algorithm adjusts each parameter α_k and β_k by amounts $\Delta\alpha_k$ and $\Delta\beta_k$ proportional to the negative of the gradient of $E(\alpha_k, \beta_k)$ at the current location:

$$\alpha_{k+1} = \alpha_k - \eta\frac{\partial E(\alpha_k, \beta_k)}{\partial \alpha_k}, \qquad \beta_{k+1} = \beta_k - \eta\frac{\partial E(\alpha_k, \beta_k)}{\partial \beta_k} \tag{13}$$

where η is a learning rate. The gradient-descent rule necessarily decreases the error with a small value of η. If the error function (11), thus, has a single minimum at $E(\alpha_k, \beta_k) = 0$, then the parameters (α_k, β_k) approaches the optimal values with enough iterations. The error function $f(\alpha_k, \beta_k)$, however, is nonlinear, and hence it is possible that the error function (11) may have *local minima* besides the global minimum at $E(\alpha_k, \beta_k) = 0$. In this case, the gradient-descent rule may become stuck at such a local minimum. To check convergence of the gradient-descent method to the global minimum we estimate the parameters for a variety of alternative start up parameters (α_0, β_0) with $\eta = 0.01$. As a consequence, the gradient-descent rule estimated the same values of the parameters, $(\alpha^*, \beta^*) = (1.04, 0.5)$ with enough iterations.

4.2. Results

The fit of the estimated equation can be seen graphically in Fig. 6, which compares the actual and forecasted series over one period, respectively. The correlation coefficient that indicates goodness of fit is 0.994. The capacity of the model to forecast adequately in comparison with the competing models is an important element in the evaluation of its overall performance. Let us consider the linear regression model

[4] The gradient-descent method is often used for training multilayer feedforward networks. It is called the back-propagation learning algorithm which is one of the most important historical developments in neural networks [23].

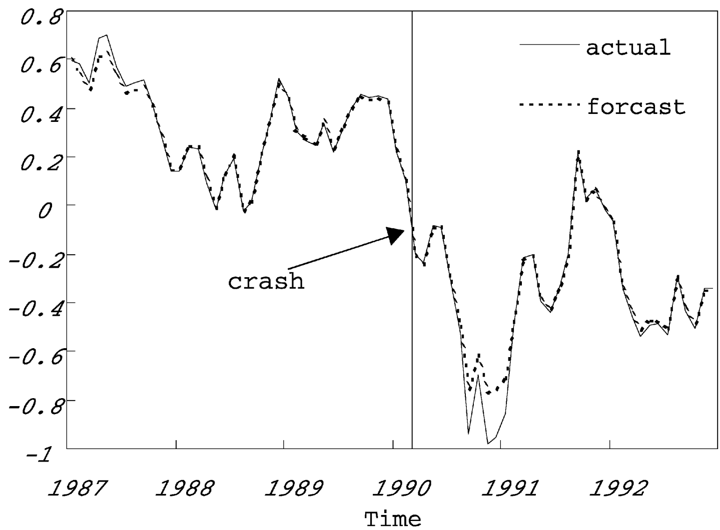

Fig. 6. One-step-forecast of the stock price changes: January 1987–December 1992.

$\langle \Delta y \rangle_{t+1} = c\langle \Delta y \rangle_t + \mathrm{d}s_t$. The regression results are $\langle \Delta p \rangle_{t+1} = 0.855\langle \Delta p \rangle_t + 3.81s_t$. The correlation coefficient of the linear regression model is equal to 0.93, and is lower than that of the interacting-agent model. Root-mean-squared errors (RMSEs) of the interacting-agent model is equal to 0.05, and that of the linear regression model is equal to 0.148 over the period 1987–1992. The interacting-agent model has a smaller RMSE than that of the linear regression model. By the traditional RMSE criterion the interacting-agent model is, thus, superior to the linear regression model in terms of their forecasting performance. In conclusion, we can say that the model not only explains the Japan crisis but also correctly forecasts the changing stock price over the period 1987–1992. Given that the set of parameters (α, β) is equal to $(1.04, 0.5)$, the critical point of the investment environment at which the phase transition are caused from the bull market to the bear market are calculated from the theoretical results in the preceding section. One can say that the phase transition from the bull market to the bear market occurs when the investment environment is below -0.007 under $(\alpha, \beta) = (1.04, 0.5)$. By contrast, the phase transition from the bear market to the bull market occurs when the investment environment is beyond 0.007 under $(\alpha, \beta) = (1.04, 0.5)$. From theoretical viewpoint one may say that bursting speculative bubbles will begin when the investment environment is below -0.007, and the process of the collapse of the stock market continues till the investment environment is beyond 0.007.

In the real world the investment environment became below -0.007 in March 1990 for the first time and continued to be negative values since then over the period 1987–1992. The fall of the actual price in the Japan's stock market began at March 1990, and the stock prices continued to fall over the period 1990–1992. On these grounds we have come to the conclusion that the theory and practice are in a perfect harmony (see Table 1).

Table 1
The stock market crash in the Japanese stock market

Period	$\langle \Delta p \rangle$	Predicted value	s_t
1989.09	0.4591	0.4487	0.0182
1989.10	0.4468	0.4361	0.0122
1989.11	0.4531	0.4408	0.011
1989.12	0.4369	0.4258	0.0075
1990.01	0.2272	0.2294	−0.002
1990.02	0.0985	0.0986	−0.0055
1990.03	−0.2013	−0.2097	−0.0099
1990.04	−0.2354	−0.2415	−0.0066
1990.05	−0.0845	−0.0871	−0.0004
1990.06	−0.0883	−0.0924	−0.0029
1990.07	−0.3136	−0.3159	−0.0067
1990.08	−0.5245	−0.4992	−0.0136

5. Conclusion

This paper presents an interacting-agent model of speculative activity explaining the bubbles and crashes in stock markets in terms of a mean field approximation. We show theoretically that these phenomena are easy to understand as the market-phase transitions. Bubbles and crashes are corresponding to the second-order transitions and the first-order transitions, respectively. Furthermore, we estimate the parameters of the model for the Japanese stock market in 1987–1992. The empirical results demonstrate that the theory and practice are in a perfect harmony. This fact justifies our model that bubbles and crashes have their origin in the collective crowd behavior of many interacting agents.

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