



Can we predict crashes? The case of the Brazilian stock market

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ABSTRACT

In this study we analyze Brazilian stock prices to detect the development of bubbles and crashes in individual stocks using a log-periodic equation. We implement a genetic algorithm to calibrate the parameters of the model and we test the methodology for the most liquid stocks traded on the Brazilian Stock Market (Bovespa). In order to evaluate whether this approach is useful we employ nonparametric statistics and test whether returns after the predicted crash are negative and lower than returns before the crash. Empirical results are consistent with the prediction hypothesis, e.g., the method applied can be used to forecast the end of asset bubbles or large corrections in stock prices.

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1. Introduction

Large market crashes occurring simultaneously on various stock markets in the world can account for the disappearance of trillions of dollars in a very short period of time [1]. Market crashes can also provoke recessions, lead to failures in the financial system or consume years of savings and pensions instantaneously. Testing for the existence of log-periodic behavior and attempting to forecast crashes is thus important for financial regulators, risk and portfolio managers, policy makers and financial institutions.

Most works cover the major bubbles of the Dow Jones Industrial Average (DJIA), namely the October 1929 and October 1987 crashes, as well as the dot-com April 2000 crash of NASDAQ. Additionally, some works analyze crashes such as the Hong Kong October 1987 crash, the Russian crash of August 1997, the Japanese internet bubble, the South African bubbles from 2003 to 2006 and other weaker crashes in 1982, 1998 and 2001 in the DJIA. Other works also include real estate bubbles in the USA and South Africa¹.

Recent research claims that it is possible to forecast endogenous financial asset bubbles [3–11]. Indeed, an accurate prediction of crashes would be of valuable information to attempt to diminish their impact or to develop a successful way to deal with the crisis.

This paper contributes to the literature by testing whether it is possible to forecast large drawdowns in stock prices in the Brazilian equity market in the recent turbulent period. We employ intraday data and fit a power-law equation to most liquid stocks and forecast the critical time when one should expect asset prices to collapse. We make two important contributions. We show that a genetic algorithm can be employed to calibrate the parameters of the power-law equation and test whether

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¹ In an interesting paper Ref. [2] shows that interest rate spread can be considered as a statistical measure for the disparity in lenders' opinions about the future. This provides an operational definition of the uncertainty faced by economic agents after market crashes.

the method is useful in assessing the likelihood of crashes using nonparametric statistics. Empirical results suggest that this methodology is useful in the forecasting of financial crashes.

The remainder of the paper is structured as follows. The next section presents a brief literature review, whereas Section 3 discusses the methodology that will be employed in the paper. Section 4 provides an overview of the data set, while Section 5 presents empirical results. Finally, Section 6 concludes the paper.

2. Literature review

Since the seminal work of Sornette et al. [12] that found evidence of log-periodic structures in several crashes in a variety of markets, an abundance of research has found similar structure in financial markets. Sornette [6] argued that a bubble leading to a crash can be quantified in terms of a log-periodic power law in order to predict when a stock market will crash. Zhou and Sornette [9,13] considered that positive feedbacks, which are taken as the main causes for the formation of speculative bubbles, are well quantified by a log-periodic power law.

Ausloos and Ivanova [14,4] demonstrated, using the envelope of the Deutsche Aktien Index (DAX), that a log-periodic pattern exists before a crash. Their model considers huge fluctuations grouped around the crash date (increased volatility), collective effects leading to a bear market and a strong correlation found in fluctuations of different market indices before crashes. Johansen et al. [3] analyzed eight unrelated crashes from 1929 to 1998 and concluded that no major financial crash preceded by an extended bubble has occurred in the past two decades without exhibiting a log-periodic signature. Ausloos et al. [14] admitted that signals of crashes can be found in terms of power laws and oscillations to suggest remedies to control or avoid crashes. Suggestions are: reduce the number of orders, impose some delay in the orders, increase the number of actors on the market together with a decrease in exchanged volume.

Johansen et al. [15] drew a link between crashes in the stock market and critical behavior of complex systems to show that critical behavior patterns can arrive in prices even when markets are rational. Also, evidence was presented to show that large financial crashes are outliers, that is, they do not belong to the distribution of drawdowns and have their origin in cooperative phenomena. Bothmer and Meister [16] derived a new restriction of the free variables on the model of Johansen et al. [15] allowing more accurate crash prediction.

Johansen and Sornette [17] proposed a general set of important guidelines for modeling financial markets: time asymmetry, robustness with respect to connectivity between agents, “bounded rationality” and a probabilistic description. Krawiecki and Hołyst [18] developed a bistable model for financial crashes and bubbles in which the effects of weak external forces can be amplified by stochastic resonance and then be assigned as the cause for a crash. Zhou and Sornette [19] extended the search for a non-parametric signature of log-periodicity. Two non-parametric techniques (q -analysis and the Hilbert transform) were applied to seven financial time series and results showed that both are consistent with the existence of a strong log-periodic signal. Johansen [20] developed a definition of a crash based on the historical volatility of a certain index, the few largest ϵ -drawdowns being considered a crash. (A drawdown is defined as a persistent decrease in the price over consecutive days.) Matsushita et al. [21] revisited the findings that crashes can be governed by log-periodic formulae and showed that, besides the one-harmonic and two-harmonic equations, a three-harmonic equation can fit to bubble and anti-bubble episodes and even outperform the others.

Johansen and Sornette [22] observe that endogenous financial crashes are preceded by a log-periodic power law (LPPL) and thus make the assignment that crashes that are not preceded by a LPPL are exogenous, that is, caused by a strong external burst of volatility. Endogenous crashes, which are characterized by preceding speculative bubble with a LPPL that became unsustainable, outnumber the exogenous ones in the 49 outliers analyzed. Sornette et al. [23] combined conditional probability calculations with a Multifractal Random Walk (MRW) model to distinguish an endogenous from an exogenous origin of volatility shocks.

The following works developed different approaches to test financial bubbles: Grech and Mazur [24] analyzed the behavior of the local Hurst exponent (α -exponent) in the Dow Jones Industrial Average (DJIA) index time-series data and found that it can significantly help to determine which mode a market is in at a given moment. A drop in the α value signals that a crash is going to happen; however, strong exterior parameters (that cause exogenous crashes) are not predicted by the local α behavior. Grech and Pamula [25] compared the local and global Hurst exponent approaches and argued that the local one better describes financial crash events both on developing and developed markets. Czarnecki et al. [26] showed that it is possible to identify changes in the local time-dependent Hurst exponent prior to crashes in the Warsaw Stock Exchange Index, thus allowing a sell signal to be generated before the crash happens.

Hart et al. [5] examined large endogenous crashes within a non-trivial generalization of the Grand Canonical Minority Game (GCMG) using a Markov chain description. The many paths the system may take allow one to predict the start and end of a crash as well as how the crash can be avoided: provoking a small response in the system today, one can protect it against larger responses in the future. Chakrabarti et al. [27] noticed a formal similarity between the two-fractal overlap model of earthquake time series and of the stock market to consider a comparison. They found that the features of the time series for the overlap of two cantor sets when one set moves with uniform relative velocity over the other looks similar to the time series of stock prices, and proposed an anticipation method based on these observations.

Agaev and Kuperin [7] found that in periods where critical events took place, a increase in regularity followed by small values of regularity for a long period of time was observed in the local Hoelder exponents. Therefore, a prediction can be made by observing this special behavior, which is defined as a crash pattern. Gnacinski and Makowiek [28] proposed another

type of log-periodic oscillation, where an exogenous crash initializes a log-periodic behavior of prices, but with a rising market. Fabretti and Ausloos [8] found that Recurrence Plot (RP) and Recurrence Quantification Analysis (RQA) techniques detect critical regimes preceding endogenous crashes so that a bubble can be recognized before it grows, permitting a warning with enough time before the crash. Kaizoji [10,11] used a power-law distribution to describe the upper tail of the complementary cumulative distribution function of the ensemble of the relative prices in the high value of the price. When the exponent of the power-law distribution approaches two, the bubble is about to burst, as Zipf's law for ensembles of asset prices indicates.

Cajueiro and Tabak [29] and Tabak [30] employed a different approach, known as bilinear unit roots, to test for the existence of bubbles in banking indices and for the Brazilian stock market, respectively. These authors found evidence of bubbles arising in approximately two-thirds of the markets analyzed.

Araújo and Louçã [31] investigated the dynamics of stocks in Standard and Poor's 500 index using a stochastic geometry technique, defining a new measure (the Index of the Market Structure) used to provide information on the intensity and the sectoral impact of the crises. Watanabe et al. [32] proposed a mathematical definition of bubbles and crashes by the use of exponential behaviors. When the exponential trend in the equation diverges upward it is a "bubble", when it is diverging downward it is called a "crash" and when it is converging it is a "convergence".

Overall, there is little doubt that asset bubbles generate economic instability and severe financial consequences, leading to recessions and crises. However, it is not a consensus that the optimal monetary policy requires the financial authorities to burst bubbles, going beyond the Central Bank's competencies. Some works [33–35] are of the view that the Central Bank can effectively avoid the economic distortions caused by bubbles and thus should tighten monetary policy in order to control these bubbles. Others [36–39] argue that a monetary tightening would create more damage to the economy than the bubble was generating, which means that the Central Bank should simply ignore asset prices in monetary policy making.

3. Methodology

In this paper, we estimate the log-periodic power law considered in [15]. Those authors suggested that the probability for the crash to happen in the next instant, if it had not happened yet, can be modeled by

$$h(t) \approx A + B(t_c - t)^\beta + C(t_c - t)^\beta \cos(\omega \log(t_c - t) + \phi) \quad (1)$$

where A , B , β , t_c , ω and ϕ are free parameters.

Furthermore, the evolution of the price before a crash and before a critical date is given by:

$$\log[p(t)] \approx \log[p_c] - \frac{\kappa}{\beta} \{B_0(t_c - t)^\beta + B_1(t_c - t)^\beta \cos[\omega \log(t_c - t) + \phi]\} \quad (2)$$

where B_0 , B_1 , p_c , κ , β , t_c , ω and ϕ are free parameters of the model.

An economic explanation for the existence of power-law behavior is the existence of positive feedback interactions, in which traders exchange information according to a hierarchical structure, which is intended to model the organization of market in the world. This hierarchy or discrete scale invariance, be it structurally built in or dynamically generated, has been recognized as the key ingredient for obtaining power-law behavior².

It is worth commenting that the estimation of the parameters of (2) is not trivial due to its nonlinear nature. In order to estimate this equation, we have proceeded using a methodology similar to [15]. Consider rewriting equation (2) as

$$\log[p(t)] = A + Bf(t) + Cg(t) \quad (3)$$

Following [15], we have slaved the parameters A , B and C , which are the solutions of the linear system

$$\sum_{i=1}^N \begin{bmatrix} \log(p(t_i)) \\ \log(p(t_i))f(t_i) \\ \log(p(t_i))g(t_i) \end{bmatrix} = \sum_{i=1}^N \begin{bmatrix} 1 & f(t_i) & g(t_i) \\ f(t_i) & f(t_i)^2 & f(t_i)g(t_i) \\ g(t_i) & f(t_i)g(t_i) & g(t_i)^2 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix}, \quad (4)$$

as a function of the other parameters t_c , β , ω and ϕ , which were estimated using a genetic algorithm³, where the fitness function is given by the inverse of the mean square error.

Therefore, we calibrate the model presented in equation (2) and estimate the predicted critical time. Then we assess the associated returns 5, 10 and 15 days after the predicted critical time. If the model is able to correctly predict large drops in stock prices then we should see a large percentage of stocks with negative returns after the critical time.

² A clear hierarchical structure ranges from an individual small investor or the various departments of the banks to the largest mutual funds or the "currency and trading-blocks".

³ The genetic algorithm is based on the idea of natural evolution of species (see Refs. [40–43]). This concept is used to find optimal solutions in big spaces. It is effective in reference to a population of individuals in which each individual is a potential solution in the search space for the problem under consideration. The quality of any possible solution is measured using the so-called fitness function, a real function that associates each solution to a real number. The evolution of the population of individuals is achieved through a set of operations known as crossover and mutation, which actuate directly on

Table 1

The returns for each one of the 21 stock prices in our sample, 1, 5, 10 and 15 days before and after the critical time.

STOCK	– 15 days	– 10 days	– 5 days	– 1 day	1 day	5 days	10 days	15 days
VIVO4	0.087	0.083	0.069	–0.010	–0.058	–0.044	–0.067	–0.139
GGBR4	0.160	0.110	0.067	–0.009	–0.029	–0.062	–0.008	–0.064
GOLL4	–0.079	–0.071	–0.152	–0.024	0.003	–0.006	–0.082	–0.146
ITSA4	0.147	0.037	0.035	0.007	–0.002	–0.023	–0.036	–0.055
KLBN4	–0.015	–0.023	–0.010	0.005	–0.005	–0.030	–0.103	–0.070
LAME4	0.092	0.004	–0.048	0.014	–0.031	0.008	–0.038	–0.124
LREN3	0.145	0.117	0.024	–0.006	–0.006	–0.018	0.053	0.004
GOAU4	0.275	0.132	0.099	0.020	–0.033	–0.051	0.023	–0.024
NETC4	–0.064	–0.043	–0.024	–0.008	–0.004	0.063	–0.051	–0.042
BNCA3	–0.007	0.055	–0.083	–0.007	0.015	0.062	0.025	0.030
PCAR4	0.193	0.155	0.141	0.017	0.000	–0.023	–0.029	–0.032
PETR3	0.019	–0.006	–0.025	–0.007	0.035	–0.052	–0.043	–0.102
PETR4	0.030	0.005	–0.011	0.009	0.010	–0.072	–0.062	–0.118
CSNA3	0.160	0.034	–0.100	0.021	–0.009	0.127	0.075	0.108
CRUZ3	–0.059	–0.043	–0.031	–0.016	–0.017	0.030	0.033	–0.042
TAMM4	0.019	–0.135	–0.015	–0.036	–0.016	–0.040	–0.161	–0.061
TLPP4	0.008	–0.011	–0.081	–0.007	0.015	–0.018	–0.045	–0.017
TCSL4	0.067	0.034	0.118	0.029	0.017	0.034	–0.179	–0.221
UBBR11	0.109	0.109	–0.006	0.002	–0.008	–0.070	–0.099	–0.107
USIM5	0.177	0.255	0.103	0.014	0.002	–0.005	0.119	0.122
VALE3	0.194	0.132	0.047	0.007	–0.004	–0.040	–0.105	–0.062

4. Data

In this paper we analyze the indices from the 21 companies with greatest liquidity in the Brazilian Stock Market Bovespa from the period of 2 January 2008 to 24 March 2008. The intraday series consist of 4358 observations on average, with intervals of 5 minutes among them. To fit the model, we considered the first 2000 observations of each stock.

5. Empirical results

We estimate the date of the crash for each stock in our sample, using intraday observations. Then, we calculate the changes in stock prices 1, 5, 10 and 15 days after the crash. The method is said to perform well if a large percentage of stock prices display large negative returns within this short period of time. We also compare price dynamics before and after the predicted crash. If a crash has happened then evidence would favor increases in prices before the critical time and large falls after the critical time. We test this assumption, comparing median returns before and after the predicted crash. Figs. 1 and 2 present the vertical lines indicating the rupture points and fits of prices for four stocks, whereas Table 3 presents the calibrated parameters.

According to Table 1, about 61.9% of the stocks analyzed showed a decrease in their asset prices 1 day after the critical time, 66% after 5 days, 71% after 10 days and 80% after 15 days. The decreases were considerably higher (equal to or above 5%) in approximately 47% of the stocks after 10 days and 57% after 15 days. We can see that median returns increase on average by 9% 15 days before the predicted crash, whereas they decrease by 6% 15 days after the critical time. This confirms that the method applied here has some potential for prediction, and thus can be used to forecast drops in asset prices.

Table 2 shows median values and standard deviations for 5, 10 and 15 days before the critical time (B5, B10 and B15) as well as for 5, 10 and 15 days after the critical time (A5, A10 and A15). We compare median returns before and after critical time and employ a non-parametric test to test for the significance of the results. Empirical results suggest that the median returns are positive before the critical time and negative afterwards.

The results show that there were positive returns in the stocks analyzed before the critical time, whereas after it the returns were negative, which corroborates the previous assumption that prices must go up before the critical time (the period of bubble development) and right after that begin to decrease. The fact that median returns 5 days before the critical time are negative suggests that there is some uncertainty about the exact moment of the predicted fall in stock prices.

We also compared the standard deviations of the stocks in our sample. The standard deviation of cross-sectional returns falls after the critical time if we consider the window of 5 days, and is statistically significant. This suggests that within a very short time window the cross-sectional standard deviation falls, which could be evidence of herding within this market [13].

a binary representation of each solution — a vector formed by zeros and ones. Crossover is an operation associated only with the most adapted individuals. It combines two different solutions to form a new solution. Mutation, in its turn, is a stochastic rule that changes one element (0 or 1) in the binary representation of an individual. The population evolves through the selection of the best individuals of the population using the fitness function as the discriminatory rule. Ultimately, if the best solution in each step is always kept in the population this heuristic method is able to ensure the convergence to the best solution.

Table 2

A comparison of returns calculated before and after the estimated critical time. We present a comparison of median returns using the Wilcoxon–Mann–Whitney (W) test. We perform these tests using 1, 5, 10 and 15 days as benchmarks. B1, B5, B10 and B15 stand for returns calculated 1, 5, 10 and 15 days before critical time, whereas A1, A5, A10 and A15 stand for returns calculated 1, 5, 10 and 15 days after critical time. We also test for a change in variance of returns before and critical time (F-test).

	W	F test
1 day	0.91	1.622
5 days	0.99	2.36**
10 days	2.97*	1.51
15 days	3.92*	1.46
Variable	Median	Standard deviation
B1	0.00175	0.016
B5	−0.01	0.076
B10	0.03	0.091
B15	0.09	0.097
A1	−0.004	0.020
A5	−0.02	0.050
A10	−0.04	0.074
A15	−0.06	0.080

* Indicates statistical significance at the 1% level.

** Indicates statistical significance at the 10% level.



(a) VIVO4.



(b) ITSA4.



(c) PETR3.



(d) VALE3.

Fig. 1. Vertical lines indicate the calculated rupture points t_c .

6. Conclusions

The analysis presented here uses the log-periodic power-law model of Sornette to fit intraday data from the Brazilian stock market and predict a “critical time”, which would mean the end of a bubble or a large drawdown in stock prices. The results have suggested evidence that it is possible to forecast these changes in the behavior of asset prices.

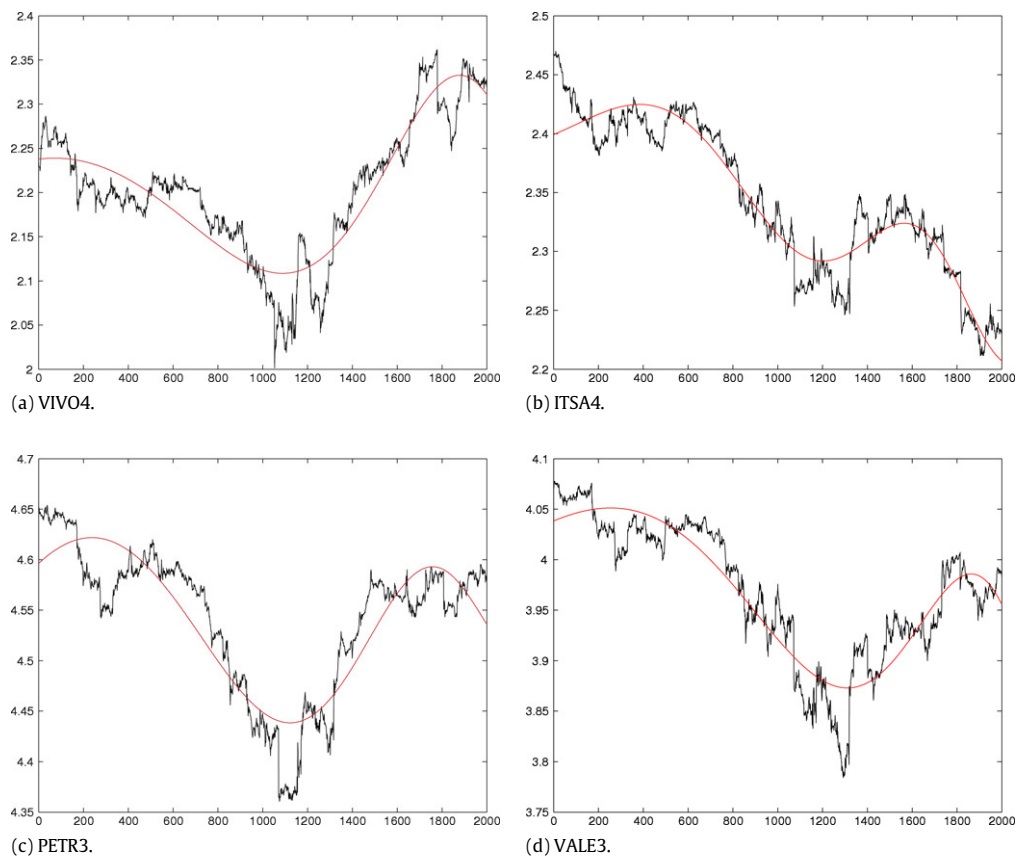


Fig. 2. Fits of the prices of some stocks.

Table 3

The calibrated parameters, namely, A , B , β , ω , ϕ and t_c for each stock.

STOCK	A	B	β	ω	ϕ	t_c
VIVO4	2080.5868	−2077.6501	0	6.5758	3.8682	3008.3989
GGBR4	3.372	0.0584	0.2734	8.815	2.8039	3143.4557
GOLL4	3.1858	0.0207	0.3893	6.4178	2.848	2731.343
ITSA4	1.3318	0.2389	0.1865	12.1046	4.4079	3298.9256
KLBN4	1.7447	0.0001	0.8071	7.0785	3.2841	2872.5789
LAME4	−5209.9215	5211.2225	0	6.3218	3.0149	2880.4001
LREN3	2.9413	0.0002	1	4.7703	3.1732	3328.9153
GOAU4	3.8698	0.0059	0.4925	8.7306	3.008	3183.6325
NETC4	−604.1523	606.6461	0.0001	8.4471	3.0243	3087.8484
BNCA3	3.1453	0	1	9.3499	2.4938	2365.4771
PCAR4	66.6151	−63.0979	0.0002	6.75	2.9193	2908.3955
PETR3	4.5668	−0.0001	0.894	7.6128	2.7234	2942.9552
PETR4	4.3634	−0.0001	0.6555	8.0002	2.6354	3032.0143
CSNA3	−5.2332	2.4977	0.1799	10.5982	0.0001	3125.249
CRUZ3	291.1555	−286.909	0.0002	3.2939	1.9075	2375.3793
TAMM4	3.5318	0.0001	0.9314	8.3999	2.553	3079.0923
TLPP4	3.9255	−0.0001	1	9.8306	6.2796	2530.7104
TCSL4	2.1389	−0.1153	0.1369	8.2396	2.0514	2867.7712
UBBR11	−1117.5653	1119.7971	0.0001	15.3751	1.7926	3293.6139
USIM5	5.3477	−0.5844	0.0743	2.7008	0.9679	2153.8467
VALE3	−594.2641	597.6847	0.0001	6.8218	2.8617	2928.425

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