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Chaos, self-organized criticality, and SETAR nonlinearity: An analysis of purchasing power parity between Canada and the United States

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Abstract

This paper uses monthly observations for the real exchange rate between Canada and the United States over the recent flexible exchange rate period (from January 1, 1973 to August 1, 2004) to test purchasing power parity between Canada and the United States using unit root and stationarity tests. Moreover, given the apparent random walk behavior in the real exchange rate, various tests from dynamical systems theory, such as for example, the Nychka et al. [Nychka DW, Ellner S, Ronald GA, McCaffrey D. Finding chaos in noisy systems. J Roy Stat Soc B 1992;54:399–426] chaos test, the Li [Li W. Absence of $1/f$ spectra in Dow Jones average. Int J Bifurcat Chaos 1991;1:583–97] self-organized criticality test, and the Hansen [Hansen, B.E. Inference when a nuisance parameter is not identified under the null hypothesis. Econometrica 1996;64:413–30] threshold effects test are used to distinguish between stochastic and deterministic origin for the real exchange rate.

1. Introduction

The theory of purchasing power parity has attracted a great deal of attention and has been explored extensively in the recent literature using recent advances in the field of applied econometrics. Empirical studies generally fail to find support for purchasing power parity, especially during the recent floating exchange rate period. In fact, the empirical consensus is that PPP does not hold over this period – see, for example, [1,25,32,16,13,40,45,9,42].

There are, however, other studies covering different groups of countries as well as studies covering periods of long duration or country pairs experiencing large differentials in price movements that report evidence of mean reversion towards PPP – see, for example, [14,11,15,34,38,23]. Also, studies using high-frequency (monthly) data over the recent

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floating exchange rate period report significant evidence favorable to purchasing power parity – see, for example, [37,8,19]. There are also recent studies that use panel methods, such as, for example, [18,31], as well as studies that consider the effect of transaction costs and nonlinear adjustments, such as, for example, [26], that report evidence favorable to PPP.

Purchasing power parity is a backward looking long-run relation and has been mostly formulated as a testable hypothesis in terms of cointegration. Taking a cointegration approach to testing long-run purchasing power parity typically requires the researcher to take a stance on a common order of integration for the individual price and exchange rate series. As a result, most of the literature ignores a recent important contribution to this topic by Ng and Perron [28] who show that we should be very wary of estimation and inference in nearly unbalanced nearly cointegrated systems. In this paper we test purchasing power parity between Canada and the United States by examining the time series properties of the real exchange rate. The finding that the real exchange rate is non-stationary contradicts the doctrine of purchasing power parity and also has implications for both estimation and hypothesis testing, both of which rely on asymptotic distribution theory. Moreover, we follow Serletis and Shahmoradi [43] and apply tests from dynamical systems theory to distinguish between deterministic and stochastic origin for the real exchange rate.

The paper is organized as follows. The next section describes the data and provides the necessary theoretical background. In Section 3 we investigate the univariate time series properties of the real exchange rate series and also follow the recent contributions by Whang and Linton [50] and Linton and Shintani [22] and construct the standard error for the Nychka et al. [29] dominant Lyapunov exponent for the real exchange rate series, thereby providing a statistical test for chaos. We also test for self-organized criticality and for threshold effects with the objective of identifying the nonlinear process that most affects the real exchange rate. The final section provides a brief summary and conclusion.

2. The real exchange rate

An approach to testing the hypothesis that the purchasing power parity relation is stationary would be to compute a linear combination of the purchasing power parity theory variables, such as, for example, the real exchange rate, and investigate its univariate time series properties using usual unit root testing procedures. The real exchange rate, E_t, can be calculated as

$$E_t = S_t \frac{P_t^*}{P_t} \tag{1}$$

where S_t denotes the nominal exchange rate (Canadian dollar value per US dollar), P_t the Canadian price level (in Canadian dollars), and P_t^* the US price level (in US dollars). Fig. 1 shows S_t and P_t^*/P_t on the primary axis and E_t on the secondary axis using monthly data from January 1, 1973 to August 1, 2004 – consumer price indices were used in the calculation of the relative price.

Taking logarithms of Eq. (1), the real exchange rate becomes a linear combination of the nominal exchange rate and the domestic and foreign price levels, as follows

$$\log E_t = \log S_t + \log P_t^* - \log P_t$$

Clearly, under long-run (absolute) purchasing power parity, the long-run equilibrium real exchange rate is equal to *1* (at every point in time), which would imply log $E_t = 0$. In the short-run, however, we expect deviations from purchasing power parity, coming from stochastic shocks, and the question at issue is whether these deviations are permanent or transitory.

A sufficient condition for a violation of absolute PPP is that the real exchange rate is characterized by a unit root. A number of approaches have been developed to test for unit roots. Nelson and Plosser [27], using augmented Dickey–Fuller (ADF) type regressions (see [10]), argue that most macroeconomic time series (including real exchange rates) have a unit root. Perron [33], however, has shown that conventional unit root tests are biased against rejecting a unit root where there is a break in a trend stationary process. Motivated by these considerations, Serletis and Zimonopoulos [45], using the methodology suggested by Perron and Vogelsang [34] and quarterly dollar-based and Deutschemark-based real exchange rates (over the period from 1957:1 to 1995:4) for 17 OECD countries show that the unit root hypothesis cannot be rejected even if allowance is made for the possibility of a one-time change in the mean of the series at an unknown point in time.

However, the (apparent) random walk behavior of the real exchange rate could be contrasted with chaotic dynamics. This is motivated by the notion that the real exchange rate follows a deterministic nonlinear process which generates output that mimics the output of stochastic systems. In other words, it is possible for the real exchange rate to appear to be random but not to be really random. In fact, Serletis and Gogas [41] test for chaos, using the Nychka et al. [29] test

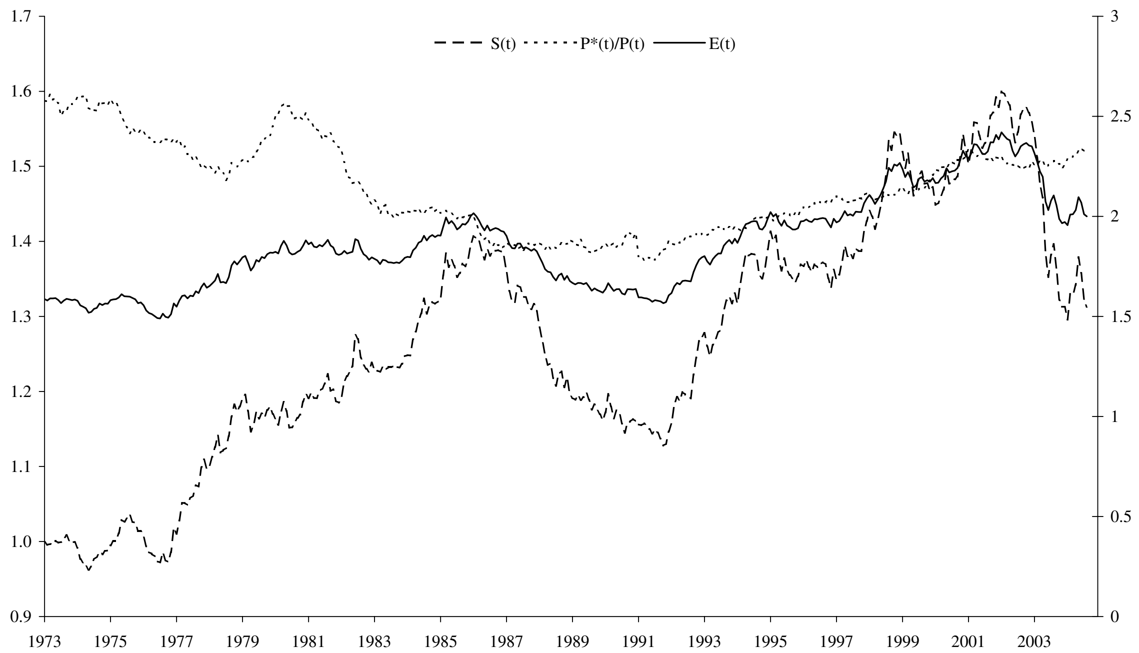

Fig. 1. The relative price and the nominal and real exchange rates.

(for positivity of the dominant Lyapunov exponent), in the dollar-based real exchange rate series used by Serletis and Zimonopoulos [45], and find evidence of nonlinear chaotic dynamics in seven out of 15 real exchange rate series. This suggests that real exchange rate movements might not be really random and that it is perhaps possible to model (by means of differential/difference equations) the nonlinear chaotic generating mechanism and build a predictive model of real exchange rates – see [3] for some thoughts along these lines.

Another type of nonlinear process is self-organized criticality, recently discovered in physics by Bak et al. [6]. Self-organized criticality is a probabilistic process. It incorporates a dominant long-run trend toward greater sensitivity and a short-run catastrophic element, which is triggered by random shocks within the system. Unlike a chaotic system, however, prediction and control in a self-organized critical system is almost impossible, and the best way to control the system is by manipulating its rules, rather than by managing its outcomes. There is thus another issue, that of whether the real exchange rate is chaotic or self-organized critical.

3. Empirical evidence

3.1. Unit root and stationarity tests

A sufficient condition for a violation of absolute purchasing power parity is that the real exchange rate is characterized by a unit root. Thus we test for stochastic trends (unit roots) in the autoregressive representation of the logged real exchange rate, using four alternative testing procedures, to deal with anomalies that arise when the data are not very informative about whether or not there is a unit root. In particular, in the first three columns of panel A of Table 1 we report p-values (based on the response surface estimates given by MacKinnon [24]) for the augmented weighted symmetric (WS) unit root test (see [30]), the augmented Dickey–Fuller (ADF) test (see [10]), and the non-parametric $Z(t_{\hat{\alpha}})$ test of Phillips [35] and Phillips and Perron [36]. Moreover, given that unit root tests have low power against relevant alternatives, in the last two columns of panel A of Table 1 we present Kwiatkowski et al. [20] tests, known as KPSS tests, for level and trend stationarity.

As can be seen, the null hypothesis of a unit root can be rejected. Moreover, the t-statistic $\hat{\eta}_{\mu}$ that tests the null hypothesis of level stationarity is large relative to the 5% critical value of 0.463 given in Kwiatkowski et al. [20] and the t-statistic $\hat{\eta}_{\tau}$ that tests the null hypothesis of trend stationarity exceeds the 5% critical value of 0.146 (also given in [20]). Combining the results of our tests of the stationarity hypothesis with the results of our tests of the unit root

Table 1
Unit root and stationarity tests in E_t

WS	ADF	$Z(t_{\hat{\alpha}})$	$\hat{\eta}_\mu$	$\hat{\eta}_\tau$
Panel A. Log levels				
.223	.346	.654	3.359	0.450
Panel B. Logged first differences				
.002	.004	<.001	.109	.102

Note: Numbers in the WS, ADF, and $Z(t_{\hat{\alpha}})$ columns are tail areas of unit root tests. The 5% critical value for the KPSS $\hat{\eta}_\mu$ and $\hat{\eta}_\tau$ test statistics (given in Kwiatkowski et al. [20]) are 0.463 and 0.146.

hypothesis, we conclude that the (logged) real exchange rate series has at least one unit root. The null hypothesis of a second unit root is also tested in panel B of Table 1, using the same four testing procedures. Clearly, the real exchange rate series appears to be stationary in the logarithmic first differences, since the null hypothesis of a unit root is rejected and the null hypotheses of level and trend stationarity cannot be rejected.

This evidence is unfavorable to long-run absolute purchasing power parity. In what follows we use the logarithmic first difference of the real exchange rate to distinguish between deterministic and stochastic origin for the real exchange rate.

3.2. A statistical test for chaos

To contrast the apparent random walk behavior of the real exchange rate with nonlinear chaotic dynamics, we follow the recent contributions by Whang and Linton [50] and Linton and Shintani [22] and construct the standard error for the Nychka et al. [29] dominant Lyapunov exponent for the logarithmic first difference of the real exchange rate series, thereby providing a statistical test for chaos – see [44] for a detailed discussion of the methodology. We also follow Shintani and Linton [48] and Serletis and Shintani [44] and report both global and local Lyapunov exponents. As argued by Bailey [5], local Lyapunov exponents provide a more detailed description of the system's dynamics, in the sense that they can identify differences in short-term predictability among regions in the state space.

Lyapunov exponent point estimates along with their t-statistics (in parentheses) are reported in Table 2 for the logged first difference of the real exchange rate. The results are presented for dimensions 1–6, with the optimal value of the number of hidden units (k) in the neural net being chosen by minimizing the BIC criterion. p-values for the null hypothesis H_0:$\lambda \geqslant 0$ are also reported in brackets. The Full column under each value of k shows the estimated largest Lyapunov exponent using the full sample. The Block column shows median values for the block estimation, with the number of blocks being set equal to 2.

The reported Lyapunov exponent point estimates in Table 2 are negative and in every case we reject the null hypothesis of chaotic behavior.

3.3. On self-organized criticality

As Bak et al. [7, p. 364] put it "[t]he temporal 'fingerprint' of the self-organized critical state is the presence of flicker noise or $1/f$ noise". Since noise can be classified according to its power spectrum, in this section we calculate the power spectrum of the logged real exchange rate series. In doing so, we follow Li [21] and calculate the power spectrum $P(f)$ using the following discrete Fourier transform

$$P(f) = N\|A(f)\|^2$$

where $\|A(f)\|$ is the module of the complex number

$$A(f) = \frac{1}{N}\sum_{j=1}^{N} x_j e^{\frac{i2\pi fj}{N}}$$

The power spectrum $P(f)$ can be modeled as a power law function, $P(f) = 1/f^{\alpha}$, where f is the frequency and α is a characteristic exponent. When plotted on a log–log scale, power laws appear as straight lines, since $\log(P(f)) = -\alpha \log(f)$ where $-\alpha$ is the slope. The power law indicates that there is 'scale invariance,' in the sense that no particular frequency is singled out and that the properties of any given frequency stand for all frequencies. If $\alpha = 0$, the series is white noise and the power spectrum is flat. If $\alpha = 2$, the time series is called $1/f^2$ noise. A random walk series (the best known nonstationary series) is exactly $1/f^2$ noise – see [21]. If $\alpha = 1$, the series is called $1/f$ noise or flicker noise.

Table 2
Lyapunov exponent estimates for logged first differences

NLAR lag (m)	Number of hidden units								
	$k=1$			$k=2$			$k=3$		
	BIC	Full	Block	BIC	Full	Block	BIC	Full	Block
1	−7.9203	−0.505	−0.468	−7.9650	−0.617	−0.530	−7.9655	−0.595	−0.521
		(−18.35)	(−10.30)		(−14.14)	(−9.99)		(−15.24)	(−10.42)
		[<0.001]	[<0.001]		[<0.001]	[<0.001]		[<0.001]	[<0.001]
2	−7.9831	−1.393	−1.387	−7.9825	−2.120	−2.137	−7.9968	−1.074	−1.101
		(−30.25)	(−17.51)		(−34.51)	(−28.16)		(−23.35)	(−18.15)
		[<0.001]	[<0.001]		[<0.001]	[<0.001]		[<0.001]	[<0.001]
3	−7.8960	−0.871	−0.858	−8.0091	−0.593	−0.579	−8.4787	−0.469	−0.465
		(−39.57)	(−21.88)		(−28.66)	(−16.26)		(−15.09)	(−0.503)
		[<0.001]	[<0.001]		[<0.001]	[<0.001]		[<0.001]	[<0.001]
4	−8.4828	−0.508	−0.528	−8.4973	−0.419	−0.421	−8.4804	−0.429	−0.401
		(−14.17)	(−8.45)		(−14.07)	(−7.90)		(−13.70)	(−7.18)
		[<0.001]	[<0.001]		[<0.001]	[<0.001]		[<0.001]	[<0.001]
5	−7.8979	−0.664	−0.656	−8.4196	−0.618	−0.613	−8.4475	−0.351	−0.338
		(−94.53)	(−31.44)		(−14.74)	(−8.38)		(−13.81)	(−8.66)
		[<0.001]	[<0.001]		[<0.001]	[<0.001]		[<0.001]	[<0.001]
6	−8.4345	−0.349	−0.349	−8.4489	−0.335	−0.327	−8.4039	−0.213	−0.214
		(−22.02)	(−11.26)		(−21.14)	(−10.94)		(−15.58)	(−9.01)
		[<0.001]	[<0.001]		[<0.001]	[<0.001]		[<0.001]	[<0.001]

Note: Sample size $T = 380$. For the full sample estimation (Full), the largest Lyapunov exponent estimates are presented with t statistics in parentheses and p-values for $H_0{:}\lambda \geqslant 0$ in brackets. For the block estimation (Block), median values are presented; the number of blocks was set equal to 2. QS kernel with optimal bandwidth [2] is used for the heteroskedasticity and autocorrelation-consistent covariance estimation.

We present the power spectrum of the logarithmic first difference of the real exchange rate in Fig. 2. A least squares, best-fit line to all spectral components indicates behavior of the type $1/f^{\alpha}$, where $\alpha = 0.208$. This behavior is consistent with the evidence reported in the previous two sections.

3.4. Evidence of SETAR nonlinearity

A synthesis of the linear econometric approach with the nonlinear disequilibrium approach to economic fluctuations has recently been provided by a class of nonlinear time series models known as (switching regression or) threshold autoregressive (TAR) models – see Tong [49] for a review of such models. These models involve interesting nonlinearities, with switches in parameter values according to the region in which the recent past of the series lies. Moreover, the fact that the switching (transition) variable is a lag of the dependent variable makes these models capable of characterizing different forms of asymmetric behavior.

In this section we explore the presence of such nonlinearities in the real exchange rate using the most popular TAR model, the self-exciting threshold autoregressive (SETAR) model. A two-regime version of this model for the logarithmic first difference of the real exchange rate can be written as

$$x_t = \alpha_0 + \alpha_1 x_{t-1} + \cdots + \alpha_p x_{t-p} + (\beta_0 + \beta_1 x_{t-1} + \cdots + \beta_p x_{t-p})\{x_{t-d} \leqslant \kappa\} + \varsigma_t, \tag{2}$$

where $p \geqslant 1$ is the autoregressive order (or delay parameter) and κ is the threshold parameter. According to (2), x_t is generated by one of two distinct autoregressive models depending on the level of one lagged variable, $x_{t-\mathrm{d}}$; the model could be generalized to depend on the levels of more than one lagged variable and/or to have more than two distinct regimes.

Following Potter [39] and Hansen [17], we estimated the model using least squares, allowing the threshold parameter to vary from the 15th to the 85th percentile of the empirical distribution of x_t and the delay parameter from 1 to 12. Our estimates are $\hat{\kappa} = .005$, $\hat{p} = 2$, and (with heteroskedastic-consistent standard errors in parentheses):

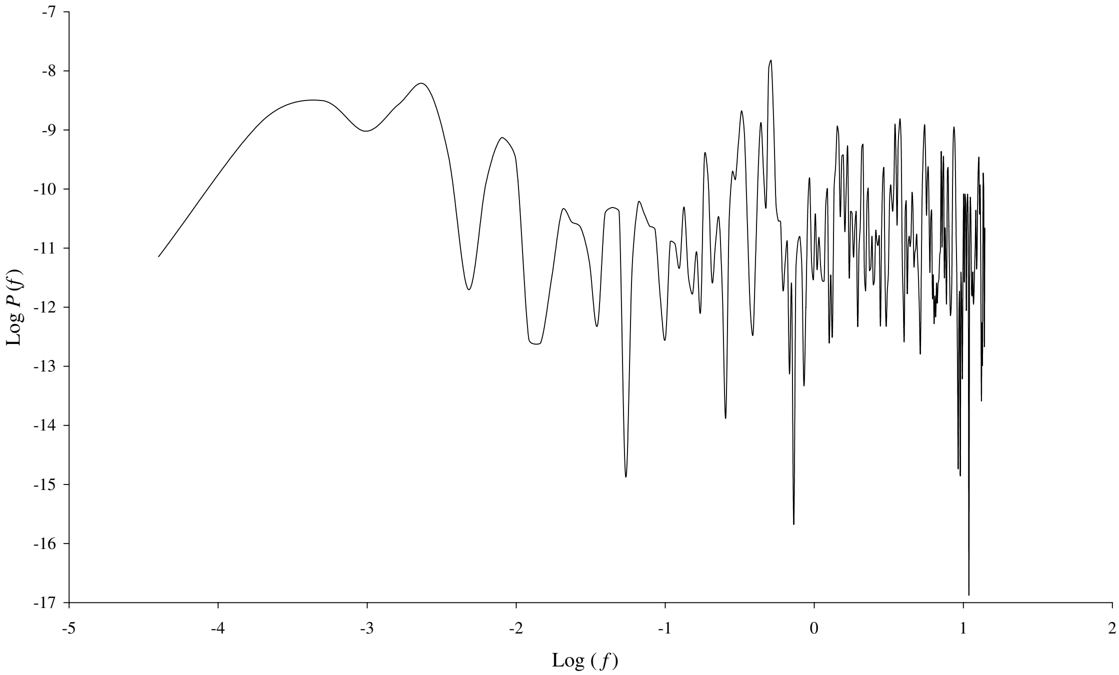

Fig. 2. The power spectrum of the logged first difference of $E(t)$.

Regime A – $x_{t-2} \leqslant 0.005$:

$$x_t = \underset{(.0009)}{.0008} + \underset{(.063)}{.257}\, x_{t-1} - \underset{(.133)}{.047}\, x_{t-2} - \underset{(.062)}{.012}\, x_{t-8} - \underset{(.081)}{.063}\, x_{t-9} + \underset{(.065)}{.207}\, x_{t-10} + \underset{(.078)}{.075}\, x_{t-11} - \underset{(.081)}{.014}\, x_{t-12}$$

$$\hat{\sigma}_1^2 = .0001$$

Regime B – $x_{t-2} > 0.005$:

$$x_t = -\underset{(.002)}{.003} + \underset{(.090)}{.035}\, x_{t-1} + \underset{(.206)}{.176}\, x_{t-2} + \underset{(.084)}{.460}\, x_{t-8} + \underset{(.102)}{.026}\, x_{t-9} + \underset{(.079)}{.108}\, x_{t-10} + \underset{(.084)}{.122}\, x_{t-11} - \underset{(.092)}{.026}\, x_{t-12}$$

$$\hat{\sigma}_1^2 = .0002$$

We test the null hypothesis of no threshold effect (single regime), using the LM-based test statistics used by Hansen [17], and report the results in Table 3 (with p-values in parentheses).

If the homoskedasticity hypothesis is maintained the SupLM, ExpLM, and AveLM are far from the rejection region. Since the point estimates of the error variance in the two regimes suggest that there is no error heteroskedasticity, the results implied by the standard models suffice for our inference. Thus we cannot reject the null hypothesis of no threshold effects, providing evidence against TAR nonlinearity in the real exchange rate.

Table 3
Tests for threshold effects

SupLM	ExpLM	AveLM	SupLM^h	ExpLM^h	AveLM^h
28.7 (.100)	10.2 (.100)	11.6 (.100)	32.4 (.000)	11.3 (.000)	9.8 (.100)

4. Conclusion

We have used monthly observations for the real exchange rate between Canada and the United States, over the recent flexible exchange rate period (from January 1, 1973 to August 1, 2004), and applied a variety of tests to distinguish between stochastic and deterministic origin for the series. We have found evidence against purchasing power parity, although traditional explanations of purchasing power parity would favor acceptance of purchasing power parity for the bilateral Canada–US relation (at least more so than for any other bilateral relation that could be analyzed). We have also applied tests from dynamical systems theory to distinguish between deterministic and stochastic origin for the real exchange rate and found evidence against nonlinear chaotic dynamics, self-organized criticality, and threshold autoregressive type nonlinearity in the real exchange rate. There is thus the issue of properly assessing the relations between our models and the real world for further significant contributions regarding exchange rate movements – see, for example, [4,46,47] for some suggestions along these lines.

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