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Self-organized criticality and partial synchronization in an evolving network

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Abstract

We describe an evolving network type model of stock market, and present numerical results. Recent works on Self-organized criticality in pulse coupled relaxation oscillators have shown SOC to be related to frustrated attempts of the system to synchronise. Other works have shown the emergence of a self-organized control parameter which feeds back onto an order parameter, causing avalanches which separate periods of stasis. We define an all-to-all network of connected spins which also have an attached fitness. We find a self-organized fitness threshold given by the mean fitness, which separates a solid type state from a gaseous type state. We also find a self-organized control parameter given by the fitness deviation. Approach of this parameter to zero causes 'avalanches' of partial synchronizations which occur on all sizes, separated by periods of stasis, and re-order the system.

1. Introduction

Recently much attention has been given to connections between self-organized criticality SOC and evolutionary phenomenon, particularly punctuated equilibrium, and to connections between SOC and synchronization. We describe a model which we hope draws some connection between these two ideas.

SOC has been proposed to describe out of equilibrium systems that are critical, that self-organize into a scale invariant critical state without tuning of a control parameter. [2–4] SOC has been applied to many different types of out of equilibrium systems as a unifying concept. Systems displaying SOC are [24] externally driven with a drive slower than any other characteristic time. These models are made critical by the choice of a threshold dynamics that forbids them to follow adiabatically the external drive. SOC states can show fractal time series characterised by scaling laws. Sand-pile [2–4] models are driven by noise, stick-slip models are driven by some continuous driving. Our model is similar to a stick-slip model.

Evolution SOC [32–34,21,22] type models have been proposed to explain punctuated equilibrium [35–37] and fractal phylogeny. Punctuated equilibrium is the phenomenon observed in the fossil record, where long periods of stasis are interrupted by sudden bursts of evolutionary change. Extinctions occur on all size scales. There have been five very large mass extinctions in the last 600 million years and a spectrum of smaller events. Of course these may have been caused by external shocks, meteorite collisions, etc. However SOC type dynamics would allow these extinctions to happen 'internally'. Kaufman and Johnsen [6] have modeled co-evolution, where agents live on a fitness landscape, [6–9] and walk around by random

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mutation, only moves to higher fitnesses are allowed. Once at a local maximum the walk would stop, except for the concept of co-evolution, where fitness landscapes of adaptive walkers are coupled. In this case moves by one agent deform the fitness landscapes of other agents and an agent may suddenly find itself not at a local maximum. Kauffman [6] and Langton [5] have linked this to the "Edge of Chaos" concept, similar to SOC [6]. Bak, Sneppen and Flyvbjerg [32,33] have taken a similar approach. They define a species as a barrier to increasing fitness. Bak and Sneppen [32] choose the least fit, randomly change the barrier and the barriers of its immediate neighbours. They also study a mean-field version [33]. The system is found to evolve to a critical state characterised by a power law distribution of size of events, and a self-organized fitness threshold. Sole and Manrubia (SM) [21,22] take a slightly different approach and describe an ecosystem by connections between species. A species is defined by its connections to other species, that is by its phenotype. This corresponds to a discrete Lotka–Volterra system, since species can be in either of two states, alive or dead. Here evolution occurs by randomly changing connections until the field on a particular agent becomes zero, the agent is then replaced, its connections are copied from another alive agent on the network. This may start an avalanche of extinctions. Here there is a feedback from a control parameter the mean-field, which goes to zero by equilibriation of the connections due to noise, and an ordering effect brought about by connection replication. They find extinctions characterised by a power law.

SOC has also been linked to periodic behaviour [24,26–31]. Large assemblies of oscillator units can spontaneously evolve to a state of large scale organization. Collective synchronization is the best known phenomenon of this kind. This effect has attracted much interest in biology for the study of large scale rhythms in populations of interacting elements [25]. Fireflies, where thousands flash in unison is the most cited example. Bottani [24], argues that there is a close relationship between SOC and synchronization. SOC appears related to a tendency to synchronization. SOC appears when a system is perturbed which otherwise should synchronise totally or partially, or which should be periodic. The Olami, Feder, Christensen model (OFC) [30] consists of oscillators on a square lattice that relax to zero when they exceed a given threshold and increment their neighbors by a pulse which is a constant $\alpha (\alpha \leqslant 1/4)$ times their value, this model shows SOC with open boundary conditions, but a very similar model, Feder and Feder (FF) [26], except that here the increment can be seen as a constant shows partial synchronization, not SOC [24,28,31]. Therefore the randomness of the initial conditions, which remains present in (OFC) changes the behaviour of the system from partial synchronization to SOC. If however a random noise is added to the increment in (FF) SOC reappears [24,31,28,29]. If the boundary conditions are periodic instead of open SOC disappears again. During activation, cycles of the oscillators participating in large avalanches are synchronized.

The model we study appears to link these two ideas with the emergence of avalanches of partial synchronization on all scales in a system of globally coupled spins when a control parameter becomes small. However our model was originally motivated as a model of the behaviour of traders in a financial market in the spirit of co-evolution.

Recent results have shown price time series to be fractal with Hurst exponent different from 0.5, [40] and with positive Lyapunov exponent [40,10–12,17–19], showing evidence of deterministic chaos and periodic behaviour. Chen [17] introduces time frequency analysis to the study of business cycles in noisy time series and finds convincing evidence of cycles. Large crashes have been supposed to be due to exogenous shocks, where information from fundamentals and macro-economics enters the market randomly. Indeed large crashes, bubbles and periods of slow growth are strongly reminiscent of punctuated equilibrium. In October 1987, when the large crash occurred on Black Monday, no new information entered the market. This was possibly an avalanche phenomenon, familiar from SOC. Since SOC states can produce fractal time series including avalanches on all scales, we suggest SOC type dynamics are present in financial market dynamics.

Various deterministic models [15,16,13,14,23] have been proposed to explain market movements. These divide investors into various types which exert different feedbacks on the market. In certain circumstances alternating and erratic switching between bull and bear markets are found. In a more evolutionary vein, [38] Palmer et al. describe a co-evolutionary model, where agents behavioural rules are represented by a gene, which subsequently mutates, the price and the fitness' are all internally defined emergent properties of the system, dependent on an agents performance, poorly performing rules are mutated. However there is no connection to SOC dynamics. Allen and Phang [20] describe a similar type of co-evolutionary stock market.

Finally we mention the dynamics of crowds [39]. Plummer illucidates the behaviour of financial markets from the viewpoint of self-reinforcing crowds. In all financial markets there are always generally two types of investor which form two crowds, these are the crowds which expects prices to rise, bulls, and the crowd which hopes prices will fall, bears. Bulls will own the share whereas bears will have borrowed the share and sold it, hoping to return it later keeping the profit. The dynamics of money markets are strongly influenced by the psychology of these two opposing crowds which behave in a globally correlated fashion. When a crowd is growing the price is moving in favour of the growing crowd, this gives a crowd a self-reinforcing feedback mechanism. Vaga's non-linear model [40] based on [41], which treats social phenomenon like an Ising system, includes explicitly a variable for the level of crowd behaviour, called the level of 'groupthink'.

2. Model

Our model is an internally defined financial market system. The dynamic, rather than being evolution in the sense of natural selection, is more like adaptation, however the fitness distribution reveals non-stationary evolution. In fact the species are not species but traders.

We model this with N spins, $s_i(t) = \pm 1$, Each spin i represents a agent on this market, with two states, depending on whether agent i is a bull or a bear, that is $s_i(t) = 1$ means the agent owns the share, and $s_i(t) = -1$ means the agent i does not own the share. The price $\Delta\, p(t)$ is simply given by, $\Delta\, p(0) = 0$, and

$$\Delta\, p(t) = \frac{1}{2}\sum_{i=1}^{N} s_i(t). \tag{1}$$

Although in a real market of course there would be a much external noise as new information entered the market randomly. This gives the price as an internally defined emergent property of the system. Excess demand increases the price as it should.

We have two dynamics, (1) trading and (2) evolution.

In a way similar to (SM) [21,22] the agents are defined by their connections to all other agents, $w_{ij}(t)$, and they are updated using the standard neural-network type update rule [1], this is a discrete version of Lotka–Volterra for ecosystems.

$$s_i(t+1) = \Theta(h_i(t)), \tag{2}$$

where the field $h_i(t)$ is given by,

$$h_i(t) = \sum_{j=1}^{N} w_{ij}(t) s_j(t) \tag{3}$$

and

$$\Theta(x) = \begin{cases} +1 & x > 0, \\ -1 & x < 0. \end{cases} \tag{4}$$

This is a standard spin system update rule with no external field, and no temperature parameter for an all-to-all connected network but the connections are not symmetrical. There is no a priori reason why they should be. This rule means that an agent will behave like the crowd it belongs to, i.e. the crowd it is strongly positively connected to, and opposite to the crowd it is negatively connected to. Since an agent's state represents share ownership, spin flipping implies buying or selling, accordingly we define an agents fitness $F_i(t)$, by its total future potential to accumulate money, this is internally defined in the spirit of co-evolution, and therefore,

$$F_i(t+1) = \begin{cases} F_i(t) + \Delta\ p(t) & \text{if } s_i(t+1) = -1, s_i(t) = +1, \\ F_i(t) - \Delta\ p(t) & \text{if } s_i(t+1) = +1, s_i(t) = -1. \end{cases} \tag{5}$$

Since according to Plummer [39] an agent becomes fitter if it moves into the minority crowd since then the price will eventually move in its favour, the amount of agents being finite and growth of a crowd not forever sustainable. Spin update can be either synchronous or a synchronous, we tried both and obtained similar results in both cases.

Our second slow dynamic is evolution by competition. In this model as in (SM) evolution implies changing the connections. Since this is evolution the way we change the connections that is our evolutionary rule should be dependent on the fitness of the agent, and the rest of the fitnesses. That is the change in the phenotype of agent α here defined by the connection vector of agent α, $w_{\alpha j}$, should be dependent on its fitness $F_\alpha(t)$. $\Delta w_{\alpha j}(t) \sim f(F_\alpha(t), F_1(t), F_2(t), \ldots F_N(t))$, where f is some function. Here we agree with [6,32] and believe that fit agents should be stabilized and unfit destabilized. That is every 'generation' two traders are chosen at random, α and β. Say,

$$F_\alpha(t) > F_\beta(t) \tag{6}$$

then,

$$\begin{aligned} w_{\alpha j}(t+1) &= w_{\alpha j}(t) + n s_\alpha(t) s_j(t), \quad j = 1, \ldots, N, \\ w_{\beta j}(t+1) &= w_{\beta j}(t) - n s_\beta(t) s_j(t), \quad j = 1, \ldots N, \end{aligned} \tag{7}$$

where n is a parameter. This is subject to the constraints $-1/N \leqslant w_{ij}(t) \leqslant 1/N, i, j = 1, \ldots, N,$ and $w_{ii}(t) = 0, i = 1, \ldots, N$. An agents connections are just cut-off (we have also tried normalization of connections again with no change in the results). We believe an agent will become more correlated with the behaviour of like minded agents, and less correlated with oppositely minded agents if it is fit, the opposite will be true if it is unfit. This rule which is just hebbian learning [1] for fit agents, and 'un-hebbian' learning for unfit agents stabilizes the state of fit agents and destabilizes the state of unfit agents. In terms of the financial markets this is similar to 'taking a position', joining a bull or bear crowd. Indeed in these terms this is very reminiscent of the statement that 'a crowd has a mind of its own', that is the members of a crowd committed to a certain opinion or position do indeed behave in a globally correlated fashion.

In evolutionary terms this means that if we consider coupled fitness landscapes in co-evolution, [6], agents which are at local fitness maxima, corresponding to energy minima, stabilize their state further, by hebbian learning. That is we take the coupled fitness landscape scenario in a slightly different way, where instead of optimizing on a landscape by walking in upwards moves our agents at fitness maxima, that is energy minima, simply dig deeper energy minima by hebbian learning, while unfit do the opposite. This rather than co-evolution may be termed co-adaptation or co-learning.

In the results presented in this paper, we define the generation length as one time step, and chose two at random every time step, and update them according to condition (6) and Eq. (7). For initial conditions we randomly assigned spins either 1 or -1, with probability 1/2. Fitnesses were all set to zero and connections were chosen at random from the distribution $[-1/\mathrm{N}, 1/\mathrm{N}]$.

Thus this model is completely internally defined in a co-evolutionary sense. Fitnesses defined in terms of past behaviour feed onto an evolutionary dynamic which in turn define behaviour. This model is supposed be a model of the evolution of group behaviour, in particular as it applies to the dynamics of financial markets. We hoped clusterings would appear, where correlated trading behaviour would increase fitness at a higher rate than individual behaviour. Indeed this is what we see.

3. Results

Shown in Fig. 1 is the time series of fitness distribution, $F_i(t)$, for an $N = 100$ system. As can be seen the time evolution is that of punctuated equilibrium, we have periods of virtual stasis separated by sudden changes. The mean fitness, $F(t)$, given by

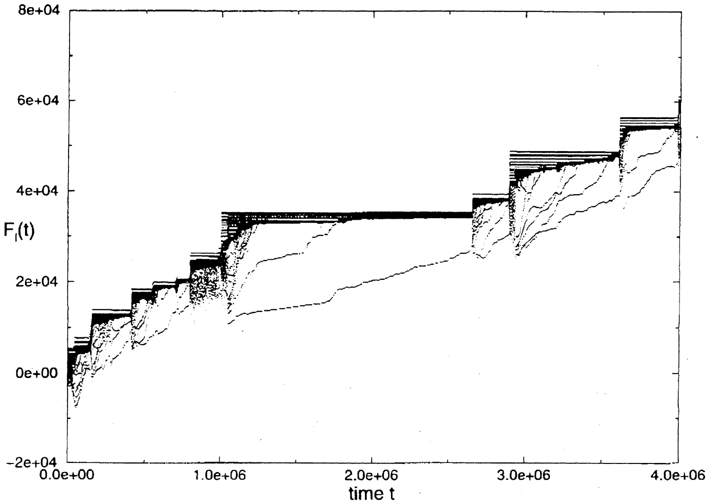

Fig. 1. Fitnesses $F_i(t)$ against time t, showing periods of relative stasis punctuated by sudden increases in mean-fitness and fitness deviation. $N = 100$ and the spin-update dynamic is synchronous.

$$F(t) = \frac{1}{N}\sum_{i=1}^{N} F_i(t) \tag{8}$$

increases slowly across long time lengths, and is interrupted by sudden jumps, in fact, these happen on all scales and look like a devils staircase. The price change $\Delta\ p(t)$, Fig. 2(c), which is simply the mean ownership, has periods of slow change separated by sudden rapid variations. This is very reminiscent of real price change time series which show sudden large deviations and high trading volume at these times. In fact the amount of frustrated spins at any time (not shown) is high at these times and low in the intervals of stasis.

We can rewrite Eq. (5) as

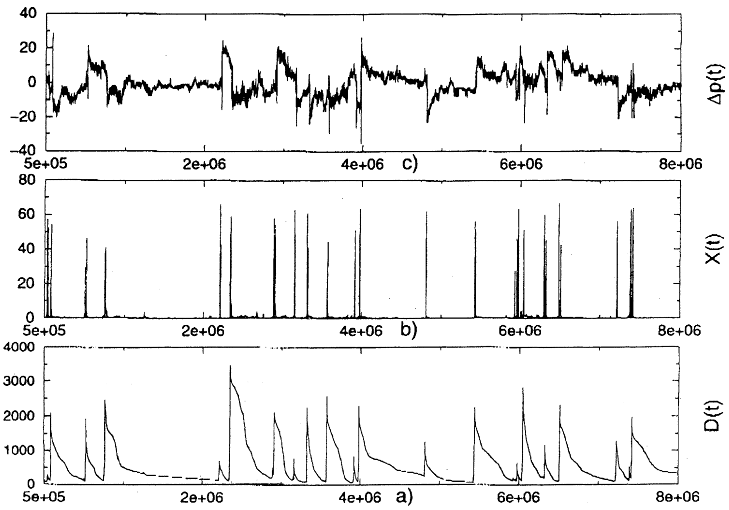

Fig. 2. (a) Fitness deviation $D(t)$ vs. time, (b) Susceptibility $X(t)$ vs. time (c) Price change $\Delta\ p(t)$ vs. time -sudden changes in price change $\Delta\ p(t)$ and clustering avalanches apparent from $X(t)$ appear when the control parameter $D(t)$ becomes small. This is a different time series to that shown in Fig. 1, here $N = 79$ and the spin update dynamics are synchronous.

$$\Delta F_i(t) = -\Delta s_i(t)\Delta\, p(t)$$
$$= -\frac{1}{2}\Delta s_i(t)s_i(t) - \frac{1}{2}\Delta s_i \sum_{j\neq i}^{N} s_j(t), \quad (9)$$

$$\Delta F_i(t) = \frac{1}{2} - \frac{1}{2}\Delta s_i \sum_{j\neq i}^{N} s_j(t) \quad \Delta s_i(t) \neq 0, \quad (10)$$

where Δ represents a change at time t thereby we can immediately see that repeated trading (spin flipping) generates a rate of increase of 0.5, by a self effect, even if there is no correlation between Δs_i and s_j. This general increase in fitness provides a driving force in our model. The fitnesses are separated into a solid component of greater than mean fitness and a 'gaseous' type component with fitnesses less than the mean. The solid component do not change there fitness, but their fitness deviation, $D_i(t) = F_i(t) - F(t)$, decreases. The gaseous type, which are continually flipping, increase their fitnesses by Eq. (10). It is very easy to see why two components separate out. According to Eqs. (6) and (7) on average for large N we can say that the amount of times a certain agent i will be positively hebbian updated will be proportional to $|(F_i(t) - F(t))|$ and the same for negative hebbian updating we could therefore replace our competitive rule (6) by,

$$\Delta w_{ij}(t) = \frac{n}{N} s_i(t)s_j(t)(F_i(t) - F(t))\gamma_{ij}(t), \quad (11)$$

where here the Δ refers to a change due to the evolution dynamic and not to spin flipping, and $\gamma_{ij} = 1$ for $|w_{ij}| < 1/N$, or $\gamma_{ij} = 0$ for $|w_{ij}| = 1/N$. Indeed we have tried this model instead of our competitive dynamic (6), (7), and again we recover the same results. This is in fact a kind of mean-field version. (Actually the connection change will be proportional to the ordering of the fitnesses rather than their absolute value, but here the inter-fitness spacing are very regular so both models produce the same type of punctuated equilibrium behaviour). We see that due to the evolution dynamic a change due to learning, not including spin flips gives a change on the field of agent i like,

$$\Delta h_i(t) = \sum_{j=1}^{N} \Delta w_{ij}(t)s_j(t) = \gamma_i(t)\frac{n}{N} s_i(t)D_i(t), \quad (12)$$

and $\gamma_i = \sum_{j=1}^{N} \gamma_{ij}(t)$. Therefore if fitness deviation D_i is negative, the sign of $\Delta h_i(t)$ will be always opposite to the spin state $s_i(t)$ and the field $h_i(t)$ will be attracted to zero. In this gaseous state it will gradually increase its fitness F_i until fitness deviation $D_i(t)$ becomes positive. We therefore see a feedback between fitness and state i.e. solid or gas, agents continually change from the solid state to the gaseous state and vice-versa. We see a self-organized threshold [32,33] given be the mean-fitness $F(t)$ separating two areas of different behaviours. This $F(t)$ is indeed very similar to the threshold defined in stick-slip models. When a solid spin deviation $D_i(t)$ crosses zero from above, after some learning time necessary for the field to go to zero, it becomes gaseous until $D_i(t)$ becomes positive again. This increases $F(t)$ possibly causing other solid spins to become gaseous. If the overall fitness deviation $D(t)$,

$$D(t) = \frac{1}{N}\sum_{i=1}^{N} \mid F_i(t) - F(t) \mid \quad (13)$$

is small, see Fig. 2(a), the change produced in $F(t)$ by this may cause more positive $D_i(t)$ to become zero thus starting an avalanche. The stasis type state is characterized by a gradual decrease in the total fitness deviation $D(t)$, Fig. 2(a), since in this state $D_i(t)$ is usually opposite in sign to $\Delta D_i(t)$. Therefore $D(t)$, could be considered as a control parameter, [21,22]. In the small $D(t)$ state the fitness 'landscape' has become very flat.

The parameter n in the time series shown here is set to $n = 0.00005$ however we believe the SOC behaviour is robust to this parameter, providing it remains small. Although we have no theory for this at

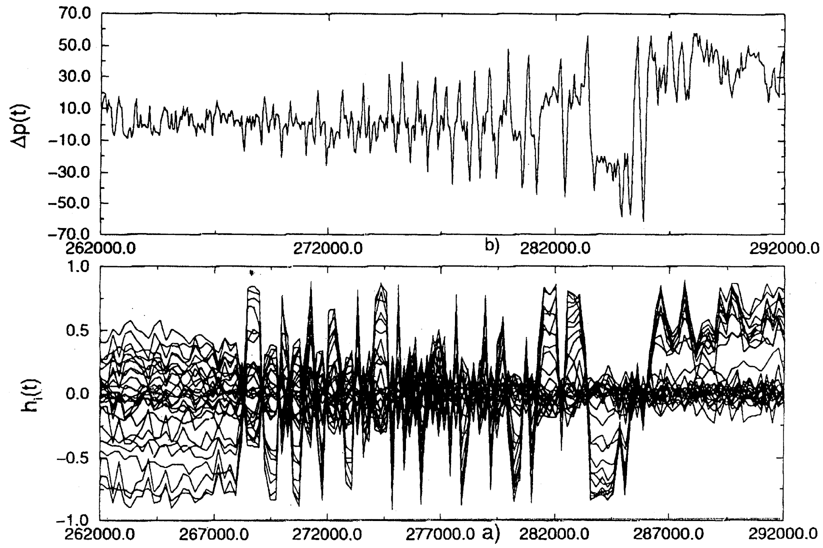

Fig. 3. (a) Fields $h_i(t)$ vs. time, showing global synchronization, (b) corresponding price change time series $\Delta\ p(t)$, the system ends up in a much more ordered state. This time series is an $N = 200$ asynchronous spin update system, not all the fields are shown.

present, extremely long time series computations show SOC behaviour to not be a transient, although for n large, that is for $n = 1$ we have seen the appearance of clusters of flipping spins which flip synchronously, with no solidification after an SOC type transient state. These results will be reported in a more extensive paper to be published. In fact smaller n which controls the learning rate allows the field h_i to more accurately reflect the fitness deviation D_i. For larger n a single comparison and update given by (6) and (7) may cause the connections to go to there cutoffs immediately without sampling the fitness space. However as can be seen from Fig. 3(a), which is taken from an asynchronous spin update system, our avalanches are actually the formation of two clusters, which behave in the same way as two single spins flipping out of phase. When changes in $F(t)$ cause large amounts of fields to simultaneously cross zero, a large avalanche occurs, these spins flipping simultaneously feedback onto the price change $\Delta\ p(t)$. The group in the majority flip roughly in phase with the price change $\Delta\ p(t)$ Fig. 3(b), thereby increasing their fitnesses, whereas the minority group, flip out of phase and become less fit. Eventually the minority group becomes attracted to zero field, because they have large negative $D_i(t)$, while the fit group solidify into a highly correlated group with like spins and large positive $D_i(t)$ therefore re-ordering the system, and taking $D(t)$ far from zero. This partial synchronization which in fact occurs on all scales if we magnify the mean-fitness (not shown) we believe provides an interesting example of synchronization studied in relation to SOC [24,28,31]. The feedback from our control parameter $D(t)$ links this to the work of Sole and Manrubia [21,22], and to the self-organized fitness threshold of Bak, Sneppen and Flyvbjerg [32,33].

Also during an avalanche the price change $\Delta\ p(t)$ moves far from zero. Spin randomization occurs after this because spins condensing into the solid state do not have a preferred orientation. During stasis periods the price change $\Delta\ p(t)$ follows a random walk with some trend towards zero. Vaga distinguishes market movements and describes uncorrelated periods where the price may follow an random walk, interspersed with highly correlated group-think periods, [40] remarkably similar to our results.

Fig. 2(b) shows the quantity, $X(t)$, similar to the susceptibility

$$X(t) = \frac{1}{N} \sum_{ij} |\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle| \tag{14}$$

here $\langle \ldots \rangle$ denotes averaging over a small time length T in the time series shown, that was $T = 8N$. It is only non-zero when synchronized flipping occurs, and therefore characterizes our avalanches quite well. It shows

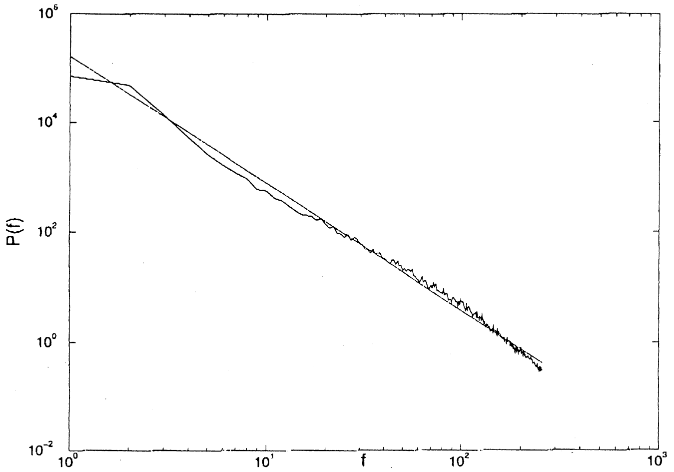

Fig. 4. Power spectrum for fitness deviation $D(t)$, the exponent $a = -2.3 \pm 0.01$.

sudden large peaks and spectrum of smaller ones. Indeed different magnifications, that is different T show the same pattern. This avalanche measure however is different to usual ones in SOC systems, where usually the driving is stopped until the avalanche has completely relaxed.

Finally shown in Fig. 4 is the power spectrum for $D(t)$. Power law behaviour $P(f) \propto f^{-a}$ is evident, the exponent is $a = -2.3 \pm 0.01$. This exponent is identical to the one found by Sole and Manrubia [21,22], and similar to Raup's data [35]. This is evidence of a critical state characterised by avalanches on all scales.

4. Conclusion

This model is very different to standard SOC type models in various respects. Firstly and most importantly SOC models are defined with two dynamics, driving and avalanching. Avalanching is supposed to occur instantaneously on the time scale of driving. In our model we do not halt the driving while an 'avalanche' is occurring, however the driving produced by general increase in fitness is sufficiently slow near an avalanche to consider avalanches as instantaneous. $X(t)$ (Fig. 2(b)) shows clustering avalanches to be short compared to stasis periods. Secondly there is a time delay between when a agent crosses the threshold $F(t)$ and when it changes its behaviour, from flipping to solid and vice-versa. This is due to the finite learning time, necessary to bring the field to zero. Despite these differences we believe this model, because it displays power law behaviour and because it is robust to changes in definition, sheds some light on the relationship of SOC behaviour to synchronization. The punctuated equilibrium behaviour and partial synchronizations are stable to changes in evolution dynamic from competitive to mean-field, to changes in the learning rate parameter n and to changing the spin update dynamic from synchronous to asynchronous. The partial synchronizations which occur when the control parameter $D(t)$ approaches zero show SOC and synchronization to be two sides of the same coin [24]. Remarkably in the asynchronous trading update rule, we still obtain synchronized trading.

Many other models are possible, in particular we should couple changes in price Δp to the learning dynamic so that agents stabilize their state when the price is moving in their favour, and others destabilize their state. This would be more realistic. Many different connection update rules are also possible. Dead unfit agents may simply restart their connections in new random configurations, etc. In a subsequent paper where we study these dynamics in a much simplified mean-field version of this system we are able to take a much more theoretical approach.

Uncannily like a real market [39] we see separation of two crowds committed to opposing positions, when these crowd have become fully grown, i.e. almost all traders are members of one or the other, the two crowds effectively become two single agents, which trade in a highly correlated fashion. After a short bubble/crash, characterized by sudden price changes and high volatility the winning crowd, separates out as a new highly fit group of like spins, while the losers separate not-committed to either position. This indeed is a metaphore for life. Although this model is a highly stylized type of stock market, many obvious extensions are possible, it is intended as an example of possible SOC behaviour in crowd dynamics and large scale human behaviour in general. Remarkably we think it actually captures some of the types of behaviour of real markets. Indeed human history shows revolutions and large scale upheavals occurring in short bursts on all scales separated by periods of stasis.

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