Profile Analysis

Introduction

Profile analysis is the multivariate equivalent of repeated measures or mixed ANOVA. Profile analysis is most commonly used in two cases:

1) Comparing the same dependent variables between groups over several time-points.
2) When there are several measures of the same dependent variable (Ex. several different psychological tests that all measure depression).

Profile analysis uses plots of the data to visually compare across groups. Following this, specific equations can be used to test for the significance of the various patterns or effects.

Applying Profile Analysis

In profile analysis, the data are usually plotted with time points, observations, tests, etc. on the x-axis, with the response, score, etc. on the y-axis. These plots are then made into profiles—lines—representing the score across time points or tests for each group.

Profile analysis asks three basic questions about the data plots:

1) Are the groups **parallel** between time points or observations?
2) Are the groups at **equal levels** across time points or observations?
3) Do the profiles exhibit **flatness** across time points or observations?

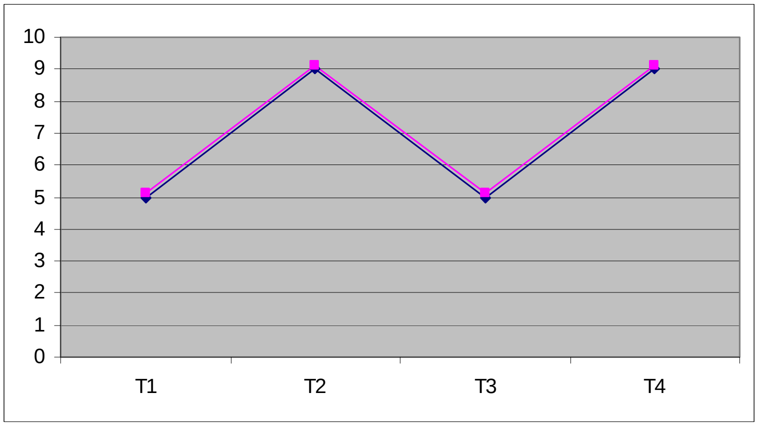
If the answer to any of these questions is no (i.e. that specific null hypothesis is rejected) then there is a significant effect. The type of effect depends on which of these null hypotheses is rejected.

Equal Levels

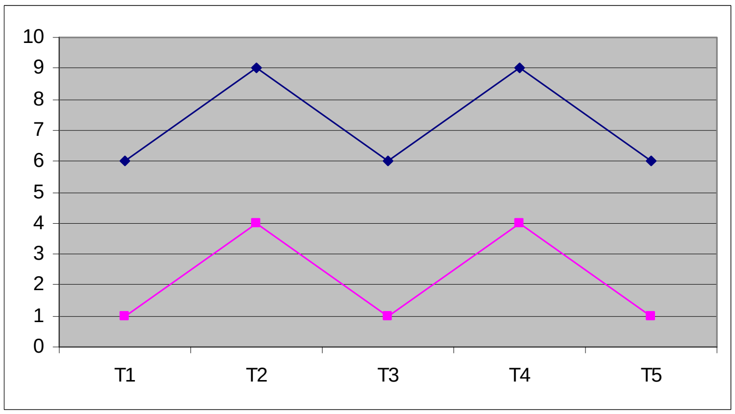
Whether or not the profiles have equal levels is the most straightforward test in profile analysis. The test is basically asking does one group score higher on average across all measures or time points?

To evaluate this, the grand mean of all time points or measures is calculated for each group. Since all of the time points or scores are collapsed into a group mean, this a univariate test. Essentially, this is equivalent to a between groups main effect.

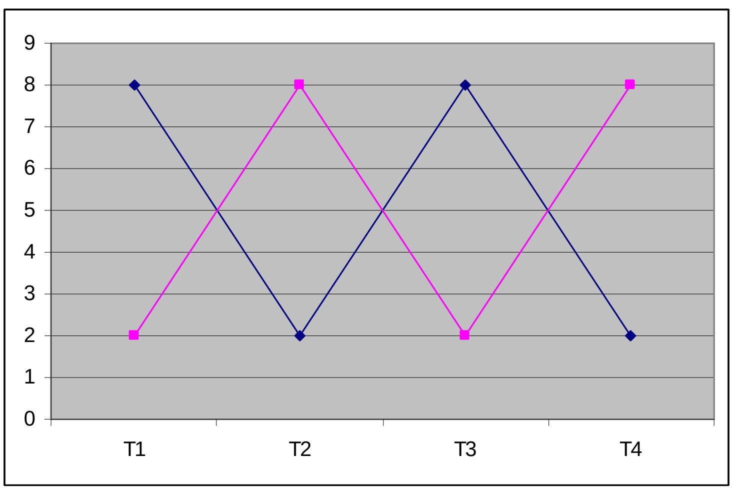
Here are a couple of graphs to help with visualization of equal levels:



Graph 1. Equal levels (coincident)—no between group main effect.



Graph 2. Unequal levels (non-coincident)—between group main effect.



Graph 3. Equal levels—no between group main effect. Although these profiles are not coincident, the *average* response for each group is the same, which is an important concept to remember here.

Mathematically, we are simply measuring the relative contributions of between-group and within-group contributions to the total sum of squared errors (the left side of the equation). This should look familiar, as it is the basis behind simple ANOVAs.

For *i* groups measured on *j* dependent variables:

$$\sum_i \sum_j (Y_{ij} - GM)^2 = np \sum_j \left(\overline{Y}_j - GM_{(y)} \right)^2 + p \sum_i \sum_j (Y_{ij} - \overline{Y}_j)^2$$

If the group “levels” are significantly different, then the equal levels null hypothesis is rejected.

Flatness

Flatness and parallelism are both multivariate tests which compare the multiple segments of the profile. A **segment** in this context is simply the difference in the response between time points or dependent variables. Therefore, the segment is equivalent to the slope of the line between two points on the x-axis.

The flatness null hypothesis is that the segments are 0, i.e. the slope of each line segment is zero and the profile is flat. This is evaluated independently for each group, making this a within-subjects test. If the line is not flat (any of the segments vary significantly from 0 then there is a within groups main effect of time-point, dependent variable, measure, etc.

MANOVA is used to test the difference of the zero-matrix and the segmented data for each group. Usually Hotelling’s T^2 is used here:

$$Hotelling's\ \mathrm{T}^2 = N(GM)' S_{wg}^{-1} (GM)$$

Where N is the number of segments, GM is the grand mean of segments, and S_{wg} is the within-group variance-covariance matrix.

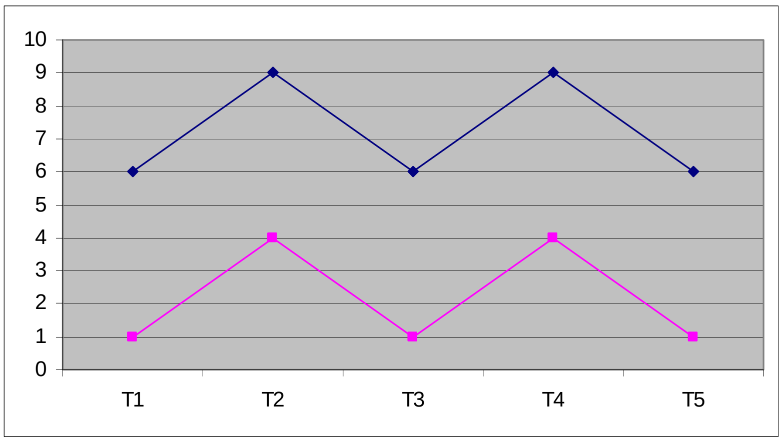
Wilks’ λ can then be calculated from T using the following equation:

$$\Lambda = \frac{1}{1 + T^2}$$

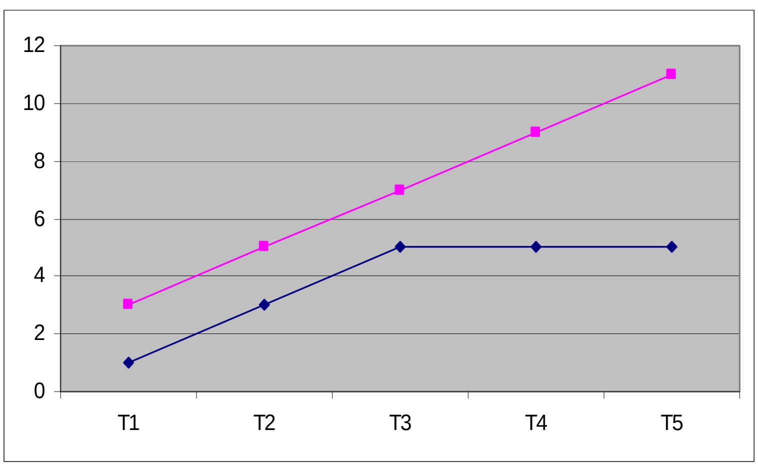
Parallelism

Parallelism is usually the main test of interest in profile analysis. The test for parallelism asks whether each segment is the same across all groups.

Here are some graphs to illustrate the concept of parallelism as it is used here:



Graph 1. Parallel—no within group/between group interaction



Graph 2. Non-parallel—within group/between group interaction.

To test whether or not there is significant non-parallelism between groups, a MANOVA is used. The within-group variance comes from subtracting the segment matrix for each individual in the group from the group mean. The between groups variance is obtained by subtracting each group mean segment matrix from the grand mean segment matrix (see example). If the parallelism null hypothesis is rejected, there is a significant group by DV interaction effect.

Limitations

The data used in profile analysis must be on the same scale. This is not an issue for repeated measures since the same dependent variable is used at multiple time points. However, scales may differ if your profile analysis uses multiple DVs. In this case, a z-score or other transformation may be necessary. If responses are all on the same scale, no transformation is necessary.

Sample Size and Power

There must be more subjects in the smallest cell than the number of dependent variables as a rule of thumb. Small sample size can affect power and the homogeneity of variance/covariance test. However, missing data can be replaced.

Profile analysis is often used when univariate assumptions are not met. Profile analysis generally has more power than a corrected univariate test.

Assumptions

The assumptions made in profile analysis are similar to those made when using MANOVA.

Multivariate normality

- Not important if there are more subjects in the smallest cell than number of DVs and there are equal overall sample sizes
- Otherwise, check for skewness and kurtosis of DVs and perform transformation if needed
- All DVs should be checked for univariate and multivariate outliers

Homogeneity of Variance-Covariance matrices

- If sample sizes are equal, this is usually not an issue
- If sample sizes are unequal, then you need to test for homogeneity (ex. Box's M)

Linearity

- It is assumed that the DVs are linearly related to one another
- Scatter plots of the DVs can be used to assess linearity
- When DVs are normal and sample size is large this is not an issue

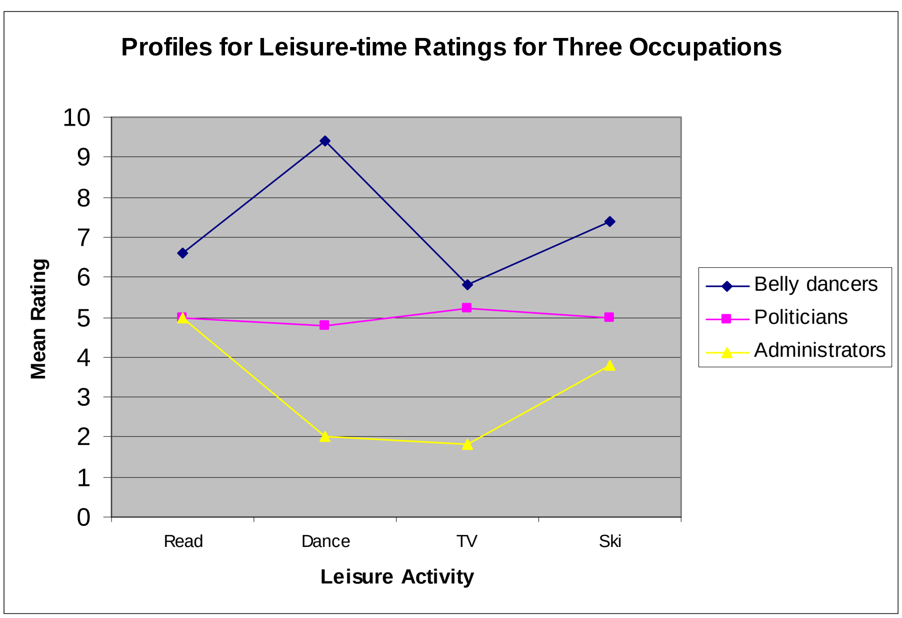
Example

The following example comes from Tabachnik & Fidell (1996).

Here is a data-set of leisure activity rankings for three different groups: politicians, administrators and belly-dancers:

Group		Read - Dance	Dance - TV	TV - Ski
		-3	4	1
		-1	4	-2
		-5	5	-3
		-4	4	-2
Belly dancers		-1	1	-2
	Mean BD	-2.8	3.6	-1.6
		0	0	0
		2	-1	2
		0	0	-1
		0	0	-1
Politicians		-1	-1	1
	Mean P	0.2	-0.4	0.2
		2	0	-1
		2	2	-4
		2	0	-3
		6	-1	-2
Administrators		3	0	0
	Mean A	3	0.2	-2
Grand Mean		0.13	1.13	-1.13

When you plot these data, you get the following profiles:



To test the equal levels hypothesis, you use the equation presented above. Applied to these data, the calculations look like this:

$$SS_{bg} = (5)(4)[(7.3-5.15)^2 + (5-5.15)^2 + (3.15-5.15)^2]$$

$$SS_{wg} = (4)[(7-7.3)^2 + (7.25-7.3)^2 + \cdots + (3.75-3.15)^2]$$

To prepare the data for multivariate analyses, you need to “segment” the data. For this example, you would get the following:

Group		Read - Dance	Dance - TV	TV - Ski
		-3	4	1
		-1	4	-2
		-5	5	-3
		-4	4	-2
Belly dancers		-1	1	-2
	Mean BD	-2.8	3.6	-1.6
		0	0	0
		2	-1	2
		0	0	-1
		0	0	-1
Politicians		-1	-1	1
	Mean P	0.2	-0.4	0.2
		2	0	-1
		2	2	-4
		2	0	-3
		6	-1	-2
Administrators		3	0	0
	Mean A	3	0.2	-2
Grand Mean		0.13	1.13	-1.13

This data can then be used to in a MANOVA to test for parallelism or flatness.

For example, to test for parallelism you would start by calculating the within group variance for the first belly dancer and it would look like this:

$$(Y_{111} - M_1) = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix} - \begin{bmatrix} -2.8 \\ 3.6 \\ -1.6 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.4 \\ 2.6 \end{bmatrix}$$

Then you would square this (variance²):

$$(Y_{111} - M_1)(Y_{111} - M_1)' = \begin{bmatrix} -0.2 \\ 0.4 \\ 2.6 \end{bmatrix} \begin{bmatrix} -0.2 & 0.4 & 2.6 \end{bmatrix}$$

We repeat this for all individuals to calculate within group SS and also repeat the process using group means to calculate between group SS.

Flatness is calculated in the same way, except that we are calculating the significance of the difference of each segment and zero. For example, if we wanted to test the overall flatness in this example we would start by subtracting the zero matrix from the segment matrix:

$$(GM - 0) = \begin{bmatrix} 0.13 \\ 1.13 \\ -1.13 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.13 \\ 1.13 \\ -1.13 \end{bmatrix}$$

Then we would use the customary equation for Hotelling's T^2:

$$Hotelling's\ T^2 = N(GM)'S_{wg}^{-1}(GM)$$

With this example, it would look something like this:

$$T^2 = (15)[.13 \quad 1.13 \quad -1.13]\begin{bmatrix} .05517 & .04738 & -.00119 \\ .04738 & .11520 & .01847 \\ -.00119 & .01847 & .04358 \end{bmatrix}\begin{bmatrix} .13 \\ 1.13 \\ -1.13 \end{bmatrix}$$

$$= 2.5825$$

And then the F statistic can easily be calculated as follows:

$$F = \frac{N-k-p+2}{p-1}(T^2)$$

$$F = \frac{15-3-4+2}{4-1}(2.5825) = 8.608$$

Where N is the total number of subjects, k is the number of groups, and p is the number of dependent variables.

SPSS: Profile Analysis

{Link to} Sample Data Set WAIS

*Scripts and more datasets for profile analysis using SAS can be found at:

http://psych.colorado.edu/.

The data set above is a good example of when and how to use a profile analysis. The rest of this page will give an overview of how to run a profile analysis using SPSS and the key outputs that are of interest. There are other websites that give examples of how to run a profile analysis in the same data set using SAS scripts.

Profile analysis datasets should be arranged so that the 'repeated measure' for all groups are found in the same column; the groups can be subdivided by a numbering scheme. Remember that the repeated measure can either be the same test administered over a series of time points or multiple different tests of the same measure.

A profile analysis can easily be accomplished using the repeated measures module under GLM in SPSS (**Analyze→General Linear Model→ Repeated Measure**). Define the number of levels in the within group factor by the number of subtests (or 'repeated measures'). The column defining the subject groups is the between subject factor. Under **plots** select the subtests to be on the horizontal axis and the groups column to be under individual lines; this will generate your profile plots. Additional statistics can be selected; descriptive statistics and homogeneity of variance test are important in order to ensure test assumptions are met.

Within-Subjects Factors

Measure: MEASURE_1

factor1	Dependent Variable
1	Subtest1
2	Subtest2
3	subtest3
4	subtest4

Between-Subjects Factors

		N
Groups	1	12
	2	37

Descriptive Statistics

	Groups	Mean	Std. Deviation	N
Subtest1	1	8.75	3.251	12
	2	12.57	3.387	37
	Total	11.63	3.712	49
Subtest2	1	5.33	4.271	12
	2	9.57	3.476	37
	Total	8.53	4.078	49
subtest3	1	8.50	3.631	12
	2	11.49	3.330	37
	Total	10.76	3.609	49
subtest4	1	4.75	3.571	12
	2	7.97	1.922	37
	Total	7.18	2.766	49

Box's Test of Equality of Covariance Matrices[a]

Box's M	19.908
F	1.702
df1	10
df2	1918.162
Sig.	.075

Tests the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups.

a. Design: Intercept+Groups
Within Subjects Design: factor1

Tests of Between-Subjects Effects

Measure: MEASURE_1
Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	10762.604	1	10762.604	402.134	.000
Groups	460.726	1	460.726	17.215	.000
Error	1257.896	47	26.764		

The tests of between subject effects shows that there is a significant difference scores between the senile and non-senile groups (averaged across all subtests), this essentially suggests a difference in levels. It does not show what direction the difference is in or if the difference is the same across all tests. The same thing is true for the within-subject effects; there is a significant effect between the different subtests within one group. The statistic does not indicate if one or all of the subtests differ. Furthermore, SPSS automatically generates 'contrasts' by fitting lines (linear, quadratic, cubic functions) to data; this is usually not a very informative contrast. More often than not contrasts require reanalysis following a transformation of the dataset; contrasts may require a multivariate or univariate statistic.

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
factor1	Sphericity Assumed	453.464	3	151.155	29.411	.000
	Greenhouse-Geisser	453.464	2.611	173.653	29.411	.000
	Huynh-Feldt	453.464	2.838	159.809	29.411	.000
	Lower-bound	453.464	1.000	453.464	29.411	.000
factor1 * Groups	Sphericity Assumed	8.729	3	2.910	.566	.638
	Greenhouse-Geisser	8.729	2.611	3.343	.566	.615
	Huynh-Feldt	8.729	2.838	3.076	.566	.629
	Lower-bound	8.729	1.000	8.729	.566	.456
Error(factor1)	Sphericity Assumed	724.649	141	5.139		
	Greenhouse-Geisser	724.649	122.732	5.904		
	Huynh-Feldt	724.649	133.364	5.434		
	Lower-bound	724.649	47.000	15.418		

Tests of Within-Subjects Contrasts

Measure: MEASURE_1

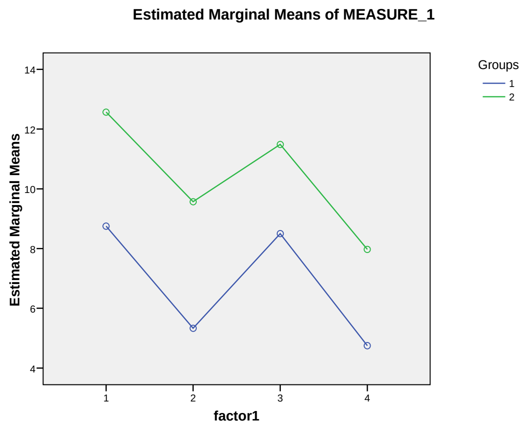
Source	factor1	Type III Sum of Squares	df	Mean Square	F	Sig.
factor1	Linear	194.098	1	194.098	33.065	.000
	Quadratic	1.625	1	1.625	.320	.575
	Cubic	257.741	1	257.741	57.748	.000
factor1 * Groups	Linear	4.164	1	4.164	.709	.404
	Quadratic	.074	1	.074	.014	.905
	Cubic	4.492	1	4.492	1.006	.321
Error(factor1)	Linear	275.900	47	5.870		
	Quadratic	238.977	47	5.085		
	Cubic	209.772	47	4.463		

Profile plots are the most informative output to look at after determining the data adequately meet the assumptions. Datasets that require the use of profile analysis are usually complicated enough that the between subject, within subject effects, and contrasts generated in the SPSS output give only limited information and do not direct you to the more subtle effects that are present.

In the profile below, there is a clear difference in level across all points, the senile patients scored lower (p=0.0001). The profile plot shows that the difference between the two groups lies primarily on the high scores of the non-senile group on subtest 1 and subtest 3 and the poor performance of the senile group on test 2 and 4. Contrasts may then be planned to help better describe this difference present between these subtests.

There is also a clear difference between different subtests within the same group. A look at the profile plot indicates that subtest 1and 3 and 2 and 4 may not be different from each other. Depending on the research question of interest the two subtests might be pooled or pulled out to test for difference using a simple contrast.

Profile plots are a great tool to help manage complicated multivariate data sets. The direct you to the most relevant contrasts and statistical tests to make when a simple test of group difference does not suffice.


Estimated Marginal Means of MEASURE_1
Estimated Marginal Means
14
12
10
8
6
4
1
2
3
4
factor1
Groups
1
2