Math 142
Homework Ten

November 6, 2013

1 Homework

Section 6.4: 1-8, 11-12, 17-22, 27-28, 31-38, 40-41

2 Extra Credit

Section 6.4: 42 (pretty hard)

41 There are three different triangles, and each triangle provides three equations. Instead of writing $\sin 60^\circ$, $\sin A$, etc., I used w, x, and y for the sines of the angles. Since all of the sines cancel out in the end, this works out fine.

The equations you need are:

$$\frac{z}{a} = \frac{x}{c+d}$$
$$\frac{x}{d} = \frac{y}{r}$$
$$\frac{z}{a} = \frac{w}{r}$$
$$\frac{x}{c} = \frac{y}{a}$$
$$\frac{w}{b} = \frac{x}{c}$$

The procedure is:

- solve one of the equations for a variable you want to get rid of (anything other than a, b, or r).
- substitute the result into the other equations
- repeat until you end up with an equation with a, b, and r
- solve this equation for r

Get rid of w

$$\frac{w}{b} = \frac{x}{c}$$
$$w = \frac{xb}{c}$$

$$\frac{z}{a} = \frac{xb}{rc}$$

The equations now are:

$$\frac{z}{a} = \frac{x}{c+d}$$
$$\frac{x}{d} = \frac{y}{r}$$
$$\frac{x}{c} = \frac{y}{a}$$
$$\frac{z}{a} = \frac{xb}{rc}$$

Get rid of z:

$$\frac{z}{a} = \frac{xb}{rc}$$
$$z = \frac{abx}{rc}$$

$$\frac{z}{a} = \frac{x}{c+d}$$
$$\frac{abx}{arc} = \frac{x}{c+d}$$
$$\frac{b}{rc} = \frac{1}{c+d}$$

The equations now are:

$$\frac{b}{rc} = \frac{1}{c+d}$$
$$\frac{x}{d} = \frac{y}{r}$$
$$\frac{x}{c} = \frac{y}{a}$$

Get rid of d:

$$\frac{x}{d} = \frac{y}{r}$$
$$d = \frac{xr}{y}$$

$$\frac{b}{rc} = \frac{1}{c + {}^{xr}/_{y}}$$
$$\frac{b}{rc} = \frac{y}{yc+xr}$$

The equations now are:

$$\frac{x}{c} = \frac{y}{a}$$
$$\frac{b}{rc} = \frac{y}{yc+xr}$$

Get rid of y:

$$\frac{x}{c} = \frac{y}{a}$$
$$y = \frac{xa}{c}$$

$$\frac{b}{rc} = \frac{y}{yc+xr}$$
$$\frac{b}{rc} = \frac{xa}{c(xa+xr)}$$

x and c both cancel:

$$\begin{aligned}\frac{b}{rc} &= \frac{xa}{c(xa+xr)}\\ \frac{b}{r} &= \frac{xa}{x(a+r)}\\ \frac{b}{r} &= \frac{a}{a+r}\end{aligned}$$

solve for r:

$$\begin{aligned}\frac{b}{r} &= \frac{a}{a+r}\\ b(a+r) &= ra\\ ab+rb &= ra\\ ra-rb &= ab\\ r &= \frac{ab}{a-b}\end{aligned}$$

3 Review

1. A bike with 26 inch diameter wheels is traveling at 15 mph. What is the RPM for the wheels?

2. Find an equation for this graph:

Solution:

$$f(t) = -2\sin 2\pi t$$

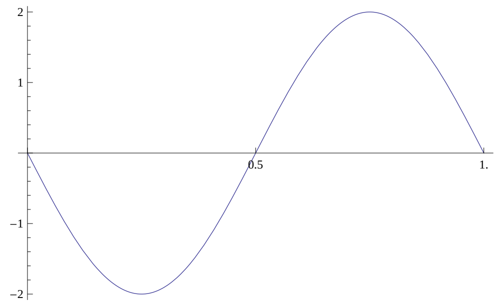

Figure 1: Find the Equation

4 Section 6.4

1

$$\begin{aligned}\frac{\sin 98.4°}{376} &= \frac{\sin 57°}{x}\\ x &\approx \boxed{318.8}\end{aligned}$$

2

$$\begin{aligned}\frac{\sin 37.5°}{17} &= \frac{\sin 114.4°}{x}\\ x &\approx \boxed{25.43}\end{aligned}$$

3

$$\begin{aligned}\frac{\sin 58°}{26.7} &= \frac{\sin 52°}{x}\\ x &\approx \boxed{24.81}\end{aligned}$$

4

$$\begin{aligned}\frac{\sin 67^\circ}{80.2} &= \frac{\sin\theta^\circ}{56.3}\\ \sin\theta &\approx 0.6462\\ \theta &\approx \boxed{40.25^\circ}\end{aligned}$$

5

$$\begin{aligned}\frac{\sin 120^\circ}{45} &= \frac{\sin\theta^\circ}{36}\\ \sin\theta &\approx 0.6928\\ \theta &\approx \boxed{43.85^\circ}\end{aligned}$$

6

$$\begin{aligned}\frac{\sin 102^\circ}{185} &= \frac{\sin 50^\circ}{x}\\ x &\approx \boxed{144.9}\end{aligned}$$

7

$$\begin{aligned}\frac{\sin 114^\circ}{65} &= \frac{\sin 20^\circ}{b}\\ b &\approx \boxed{24.33}\end{aligned}$$

$$\begin{aligned}\frac{\sin 114^\circ}{65} &= \frac{\sin 46^\circ}{a}\\ a &\approx \boxed{51.18}\end{aligned}$$

8

$$\begin{aligned}\frac{\sin 50^\circ}{2} &= \frac{\sin 30^\circ}{a}\\ a &\approx \boxed{1.305}\end{aligned}$$

$$\begin{aligned}\frac{\sin 50^\circ}{2} &= \frac{\sin 100^\circ}{c}\\ c &\approx \boxed{2.571}\end{aligned}$$

11

$$\begin{aligned}\frac{\sin 62^\circ}{230} &= \frac{\sin 50^\circ}{a}\\ a &\approx \boxed{199.5}\end{aligned}$$

$$\begin{aligned}\frac{\sin 62^\circ}{230} &= \frac{\sin 68^\circ}{b}\\ b &\approx \boxed{231.5}\end{aligned}$$

12

$$\begin{aligned}\frac{\sin 47^\circ}{50} &= \frac{\sin 23^\circ}{a}\\ a &\approx \boxed{63.39}\end{aligned}$$

$$\begin{aligned}\frac{\sin 47^\circ}{50} &= \frac{\sin 110^\circ}{b}\\ b &\approx \boxed{64.24}\end{aligned}$$

17

$$\begin{aligned}\frac{\sin 110^\circ}{28} &= \frac{\sin B}{15}\\ B &\approx \boxed{30^\circ}\end{aligned}$$

$$\begin{aligned}C &= 180^\circ - 110^\circ - 30^\circ\\ &= \boxed{40^\circ}\end{aligned}$$

$$\begin{aligned}\frac{\sin 110^\circ}{28} &= \frac{\sin 40^\circ}{c}\\ c &\approx \boxed{19.1}\end{aligned}$$

18

$$\begin{aligned}\frac{\sin 37^\circ}{30} &= \frac{\sin C}{40}\\ C &\approx \boxed{53^\circ}\end{aligned}$$

$$\begin{aligned}B &= 180^\circ - 37^\circ - 53^\circ\\ &= \boxed{90^\circ}\end{aligned}$$

$$\begin{aligned}\frac{\sin 37^\circ}{30} &= \frac{\sin 90^\circ}{c}\\ c &\approx \boxed{49.8}\end{aligned}$$

19

$$\begin{aligned}\frac{\sin 125^\circ}{20} &= \frac{\sin C}{45}\\ \sin C &\approx 1.84\end{aligned}$$

No solution

20

$$\begin{aligned}\frac{\sin 38^\circ}{42} &= \frac{\sin B}{45}\\ B &\approx \boxed{41^\circ}\end{aligned}$$

$$\begin{aligned}C &= 180^\circ - 38^\circ - 41^\circ\\ &= \boxed{101^\circ}\end{aligned}$$

$$\begin{aligned}\frac{\sin 38^\circ}{42} &= \frac{\sin 101^\circ}{c}\\ c &\approx \boxed{67.0}\end{aligned}$$

21 solution 1:

$$\begin{aligned}\frac{\sin 25^\circ}{25} &= \frac{\sin C}{30}\\ C &\approx \boxed{30^\circ}\end{aligned}$$

$$\begin{aligned}B &= 180^\circ - 25^\circ - 30^\circ\\ &= \boxed{125^\circ}\end{aligned}$$

$$\begin{aligned}\frac{\sin 25^\circ}{25} &= \frac{\sin 125}{b}\\ b &\approx \boxed{48.4}\end{aligned}$$

solution 2:

$$\frac{\sin 25^\circ}{25} = \frac{\sin C}{30}$$
$$C \approx \boxed{150^\circ}$$

$$B = 180^\circ - 25^\circ - 150^\circ$$
$$= \boxed{5^\circ}$$

$$\frac{\sin 25^\circ}{25} = \frac{\sin 5^\circ}{b}$$
$$b \approx \boxed{5.1}$$

22 solution 1:

$$\frac{\sin 30^\circ}{75} = \frac{\sin B}{100}$$
$$B \approx \boxed{42^\circ}$$

$$C = 180^\circ - 30^\circ - 42^\circ$$
$$= \boxed{108^\circ}$$

$$\frac{\sin 30^\circ}{75} = \frac{\sin 108^\circ}{c}$$
$$c \approx \boxed{142.7}$$

solution 2:

$$\frac{\sin 30^\circ}{75} = \frac{\sin B}{100}$$
$$B 180^\circ - 42^\circ$$
$$\approx \boxed{138^\circ}$$

$$C = 180^\circ - 30^\circ - 138^\circ$$
$$= \boxed{12^\circ}$$

$$\frac{\sin 30^\circ}{75} = \frac{\sin 12^\circ}{c}$$
$$c \approx \boxed{31.2}$$

27 Find $\angle CDA$ and $\angle BDC$

$$\frac{\sin 30^\circ}{20} = \frac{\sin CDA}{28}$$
$$\angle CDA \approx 180^\circ - 44^\circ$$
$$= 136^\circ$$
$$\angle BDC = 44^\circ$$

Use the Law of Sines again to find $\angle CBD$

$$\frac{\sin 44^\circ}{20} = \frac{\sin CBD}{20}$$
$$\angle CBD \approx 44^\circ$$

The missing angles are the third angles in the two small triangles. You can find them by knowing that the sum of the angles in a triangle is 180°:

$$\angle BCD = 180° - 44° - 44° = \boxed{92°}$$
$$\angle DCA = 180° - 136° - 30° = \boxed{14°}$$

28 Since the smaller triangle on the left is an isosceles triangle, $\angle BCD = \angle CBD = 25°$

The third angle in this triangle is: $\angle BDC = 180° - 25° - 25° = 130°$

Use the Law of Sines to find BC:

$$\frac{\sin 130°}{BC} = \frac{\sin 25°}{12}$$
$$\angle BC \approx 21.8$$

The third angle in the large triangle is: $\angle BAC = 180° - 25° - 50° = 105°$.

Use the Law of Sines to find AB:

$$\frac{\sin 105°}{21.8} = \frac{\sin 50°}{AB}$$
$$\angle AB \approx 17.3$$

Now we can find the missing length:

$$DA = AB - BD = 17.3 - 12 = \boxed{5.3}$$

31 (a)

$$\begin{aligned} C &= 180° - 84.2° - 93° \\ &= 2.8° \end{aligned}$$

$$\frac{\sin 84.2°}{b} = \frac{\sin 2.8°}{50}$$
$$b \approx \boxed{1018\,\text{mi}}$$

(b)

$$\sin 87° = \frac{h}{1018}$$
$$h \approx \boxed{1017\,\text{mi}}$$

32 (a)

$$\begin{aligned} C &= 180° - 32° - 48° \\ &= 100° \end{aligned}$$

$$\frac{\sin 100°}{5} = \frac{\sin 48°}{b}$$
$$b \approx \boxed{3.77\,\text{mi}}$$

(b)

$$\sin 32° = \frac{h}{3.77}$$
$$h \approx \boxed{2\,\text{mi}}$$

33

$$\begin{aligned} C &= 180^\circ - 82^\circ - 52^\circ \\ &= 46^\circ \end{aligned}$$

$$\begin{aligned} \frac{\sin 46^\circ}{200} &= \frac{\sin 52^\circ}{b} \\ b &\approx \boxed{219\,\text{ft}} \end{aligned}$$

34 Find the other two angles:

$$\begin{aligned} \frac{\sin B}{312} &= \frac{\sin 48.6^\circ}{527} \\ B &\approx 26.4^\circ \\ C &= 180^\circ - 48.6^\circ - 26.4^\circ \\ &= 105^\circ \end{aligned}$$

Find AB

$$\begin{aligned} \frac{\sin 48.6^\circ}{527} &= \frac{\sin 105^\circ}{c} \\ c &\approx \boxed{679\,\text{ft}} \end{aligned}$$

35 Find the third angle:

$$\begin{aligned} C &= 180^\circ - (90^\circ - 5.6^\circ) - 29.5^\circ \\ &= 66.1^\circ \end{aligned}$$

Find the height

$$\begin{aligned} \frac{\sin 66.4^\circ}{105} &= \frac{\sin 29.2^\circ}{h} \\ h &\approx \boxed{56\,\text{m}} \end{aligned}$$

36

$$\begin{aligned} \frac{\sin C}{165} &= \frac{\sin 67^\circ}{180} \\ C &\approx 57.54^\circ \end{aligned}$$

$$\begin{aligned} B &= 180^\circ - 67^\circ - 57.54^\circ \\ &= 55.46^\circ \end{aligned}$$

$$\begin{aligned} \frac{\sin 55.46^\circ}{b} &= \frac{\sin 67^\circ}{180} \\ b &\approx \boxed{161\,\text{ft}} \end{aligned}$$

37 Find the angle at the top of the tree:

$$\begin{aligned} A &= 180^\circ - 52^\circ - 90^\circ \\ &= 38^\circ \end{aligned}$$

Find the height:

$$\begin{aligned} \frac{\sin 38^\circ}{215} &= \frac{\sin 30^\circ}{h} \\ h &\approx \boxed{175\,\text{ft}} \end{aligned}$$

38 Find the angle at the top of the tower:

$$\begin{aligned} A &= 180^\circ - 58^\circ - 12^\circ - 90^\circ \\ &= 20^\circ \end{aligned}$$

Find the remaining angle:

$$\begin{aligned} B &= 180^\circ - 20^\circ - 12^\circ \\ &= 148^\circ \end{aligned}$$

Find the length of the wire:

$$\frac{\sin 20^\circ}{100} = \frac{\sin 148^\circ}{a}$$
$$a \approx \boxed{155\,\mathrm{m}}$$

40 Find the angle at the top of the tower:

$$\frac{\sin 8^\circ}{30} = \frac{\sin A}{120}$$
$$A \approx 33.83^\circ$$

The angle of inclination of the hill completes a right triangle:

$$\begin{aligned} B &= 90^\circ - 33.83^\circ - 8^\circ \\ &= \boxed{48.17^\circ} \end{aligned}$$

41 Find the angle between Sun-Earth-Venus

$$\frac{\sin 39.4^\circ}{0.723} = \frac{\sin A}{1}$$
$$A \approx 118.61^\circ$$

Find the remaining angle:

$$\begin{aligned} B &= 180^\circ - 118.61^\circ - 39.4^\circ \\ &= 22^\circ \end{aligned}$$

Find the distance:

$$\frac{\sin 39.4^\circ}{0.723} = \frac{\sin 22^\circ}{d}$$
$$A \approx \boxed{0.4267\,\mathrm{AU}}$$