Math 142
Homework Ten

November 6, 2013

1 Homework

Section 6.4: 1-8, 11-12, 17-22, 27-28, 31-38, 40-41

2 Extra Credit

Section 6.4: 42 (pretty hard)

41 There are three different triangles, and each triangle provides three equations. Instead of writing $\sin 60^\circ$, $\sin A$, etc., I used w, x, and y for the sines of the angles. Since all of the sines cancel out in the end, this works out fine.

Here are the variables I used:

$$\begin{aligned} w &= \sin ABC \\ x &= \sin BCA = \sin ACD = \sin BCD \\ y &= \sin CAD \\ z &= \sin CDA \end{aligned}$$

$$\begin{aligned} c &= BA \\ d &= AD \end{aligned}$$

The equations you need are:

$$\begin{aligned} \frac{z}{a} &= \frac{x}{c+d} \\ \frac{x}{d} &= \frac{y}{r} \\ \frac{z}{a} &= \frac{w}{r} \\ \frac{x}{c} &= \frac{y}{a} \\ \frac{w}{b} &= \frac{x}{c} \end{aligned}$$

The procedure is:

- solve one of the equations for a variable you want to get rid of (anything other than a, b, or r).
- substitute the result into the other equations
- repeat until you end up with an equation with a, b, and r
- solve this equation for r

Get rid of w

$$\frac{w}{b} = \frac{x}{c}$$
$$w = \frac{xb}{c}$$

$$\frac{z}{a} = \frac{xb}{rc}$$

The equations now are:

$$\frac{z}{a} = \frac{x}{c+d}$$
$$\frac{x}{d} = \frac{y}{r}$$
$$\frac{x}{c} = \frac{y}{a}$$
$$\frac{z}{a} = \frac{xb}{rc}$$

Get rid of z:

$$\frac{z}{a} = \frac{xb}{rc}$$
$$z = \frac{abx}{rc}$$

$$\frac{z}{a} = \frac{x}{c+d}$$
$$\frac{abx}{arc} = \frac{x}{c+d}$$
$$\frac{b}{rc} = \frac{1}{c+d}$$

The equations now are:

$$\frac{b}{rc} = \frac{1}{c+d}$$
$$\frac{x}{d} = \frac{y}{r}$$
$$\frac{x}{c} = \frac{y}{a}$$

Get rid of d:

$$\frac{x}{d} = \frac{y}{r}$$
$$d = \frac{xr}{y}$$

$$\frac{b}{rc} = \frac{1}{c + {}^{xr}/_{y}}$$
$$\frac{b}{rc} = \frac{y}{yc+xr}$$

The equations now are:

$$\frac{x}{c} = \frac{y}{a}$$
$$\frac{b}{rc} = \frac{y}{yc+xr}$$

Get rid of y:

$$\frac{x}{c} = \frac{y}{a}$$
$$y = \frac{xa}{c}$$

$$\frac{b}{rc} = \frac{y}{yc + xr}$$
$$\frac{b}{rc} = \frac{xa}{c(xa + xr)}$$

x and c both cancel:

$$\frac{b}{rc} = \frac{xa}{c(xa + xr)}$$
$$\frac{b}{r} = \frac{xa}{x(a + r)}$$
$$\frac{b}{r} = \frac{a}{a + r}$$

solve for r:

$$\frac{b}{r} = \frac{a}{a + r}$$
$$b(a + r) = ra$$
$$ab + rb = ra$$
$$ra - rb = ab$$
$$r = \frac{ab}{a - b}$$

3 Review

1. A bike with 26 inch diameter wheels is traveling at 15 mph. What is the RPM for the wheels?

Solution: Find the velocity in inches per minute:

$$v = \frac{15\,\text{miles}}{1\,\text{hr}} = 15,840\,\text{in/minute}$$

The circumference of the wheel is:

$$C = 2 \cdot \pi \cdot 13 \approx 81.68\,\text{in}$$

The RPM is:

$$\frac{15,840}{81.68} \approx 193.9\,\text{rpm}$$

2. Find an equation for this graph:

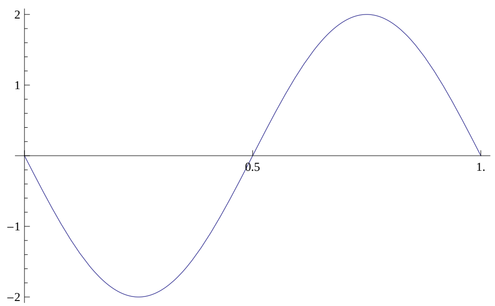

Figure 1: Find the Equation

Solution:

$$f(t) = -2 \sin 2\pi t$$

4 Section 6.4

1

$$\frac{\sin 98.4°}{376} = \frac{\sin 57°}{x}$$
$$x \approx \boxed{318.8}$$

2

$$\frac{\sin 37.5°}{17} = \frac{\sin 114.4°}{x}$$
$$x \approx \boxed{25.43}$$

3

$$\frac{\sin 58°}{26.7} = \frac{\sin 52°}{x}$$
$$x \approx \boxed{24.81}$$

4

$$\frac{\sin 67°}{80.2} = \frac{\sin \theta°}{56.3}$$
$$\sin \theta \approx 0.6462$$
$$\theta \approx \boxed{40.25°}$$

5

$$\frac{\sin 120°}{45} = \frac{\sin \theta°}{36}$$
$$\sin \theta \approx 0.6928$$
$$\theta \approx \boxed{43.85°}$$

6

$$\frac{\sin 102°}{185} = \frac{\sin 50°}{x}$$
$$x \approx \boxed{144.9}$$

7

$$\frac{\sin 114°}{65} = \frac{\sin 20°}{b}$$
$$b \approx \boxed{24.33}$$

$$\frac{\sin 114°}{65} = \frac{\sin 46°}{a}$$
$$a \approx \boxed{51.18}$$

8

$$\frac{\sin 50°}{2} = \frac{\sin 30°}{a}$$
$$a \approx \boxed{1.305}$$

$$\frac{\sin 50°}{2} = \frac{\sin 100°}{c}$$
$$c \approx \boxed{2.571}$$

11

$$\frac{\sin 62^\circ}{230} = \frac{\sin 50^\circ}{a}$$
$$a \approx \boxed{199.5}$$

$$\frac{\sin 62^\circ}{230} = \frac{\sin 68^\circ}{b}$$
$$b \approx \boxed{241.5}$$

12

$$\frac{\sin 47^\circ}{50} = \frac{\sin 23^\circ}{a}$$
$$a \approx \boxed{26.71}$$

$$\frac{\sin 47^\circ}{50} = \frac{\sin 110^\circ}{b}$$
$$b \approx \boxed{64.24}$$

17

$$\frac{\sin 110^\circ}{28} = \frac{\sin B}{15}$$
$$B \approx \boxed{30.2^\circ}$$

$$C = 180^\circ - 110^\circ - 30^\circ$$
$$= \boxed{39.8^\circ}$$

$$\frac{\sin 110^\circ}{28} = \frac{\sin 40^\circ}{c}$$
$$c \approx \boxed{19.1}$$

18 solution 1:

$$\frac{\sin 37^\circ}{30} = \frac{\sin C}{40}$$
$$C \approx \boxed{53.4^\circ}$$

$$B = 180^\circ - 37^\circ - 53.4^\circ$$
$$= \boxed{89.6^\circ}$$

$$\frac{\sin 37^\circ}{30} = \frac{\sin 89.6^\circ}{c}$$
$$c \approx \boxed{49.8}$$

solution 2:

$$C \approx 180^\circ - 53.4^\circ \boxed{126.6^\circ}$$

$$B = 180^\circ - 37^\circ - 126.6^\circ$$
$$= \boxed{16.4^\circ}$$

$$\frac{\sin 37^\circ}{30} = \frac{\sin 126.6^\circ}{c}$$
$$c \approx \boxed{14.1}$$

19

$$\frac{\sin 125^\circ}{20} = \frac{\sin C}{45}$$
$$\sin C \approx 1.84$$

No solution

20 solution 1:

$$\frac{\sin 38^\circ}{42} = \frac{\sin B}{45}$$
$$B \approx \boxed{41.3^\circ}$$

$$C = 180^\circ - 38^\circ - 41.3^\circ$$
$$= \boxed{100.7^\circ}$$

$$\frac{\sin 38^\circ}{42} = \frac{\sin 100.7^\circ}{a}$$
$$a \approx \boxed{67.0}$$

solution 2:

$$B \approx 180^\circ - 41.3^\circ = 138.7^\circ$$

$$C = 180^\circ - 38^\circ - 138.7^\circ$$
$$= \boxed{3.3^\circ}$$

$$\frac{\sin 38^\circ}{42} = \frac{\sin 3.3^\circ}{a}$$
$$a \approx \boxed{3.9}$$

21 solution 1:

$$\frac{\sin 25^\circ}{25} = \frac{\sin C}{30}$$
$$C \approx \boxed{30.5^\circ}$$

$$A \approx 180^\circ - 25^\circ - 30.5^\circ$$
$$= \boxed{124.5^\circ}$$

$$\frac{\sin 25^\circ}{25} = \frac{\sin 124.5}{a}$$
$$a \approx \boxed{48.8}$$

solution 2:

$$C \approx 180^\circ - 30.5^\circ \boxed{149.5^\circ}$$

$$B = 180^\circ - 25^\circ - 149.5^\circ$$
$$= \boxed{5.5^\circ}$$

$$\frac{\sin 25^\circ}{25} = \frac{\sin 5.5^\circ}{a}$$
$$a \approx \boxed{5.7}$$

22 solution 1:

$$\begin{aligned}\frac{\sin 30°}{75} &= \frac{\sin B}{100} \\ B &\approx \boxed{41.8°}\end{aligned}$$

$$\begin{aligned}C &= 180° - 30° - 41.8° \\ &= \boxed{108.2°}\end{aligned}$$

$$\begin{aligned}\frac{\sin 30°}{75} &= \frac{\sin 108.2°}{c} \\ c &\approx \boxed{142.5}\end{aligned}$$

solution 2:

$$\begin{aligned}B &\approx 180° - 41.8° \\ &\approx \boxed{138.2°}\end{aligned}$$

$$\begin{aligned}C &\approx 180° - 30° - 138.2° \\ &= \boxed{11.8°}\end{aligned}$$

$$\begin{aligned}\frac{\sin 30°}{75} &= \frac{\sin 11.8°}{c} \\ c &\approx \boxed{30.6}\end{aligned}$$

27 Find $\angle ABC$:

$$\begin{aligned}\frac{\sin 30°}{20} &= \frac{\sin ABC}{28} \\ \angle ABC &\approx 44.4°\end{aligned}$$

Since $\triangle BCD$ is isosceles:

$$\angle BDC = \angle ABC \approx 44.4°$$

The missing angles are the third angles in the two small triangles. You can find them by knowing that the sum of the angles in a triangle is 180°:

$$\angle BCD \approx 180° - 44.4° - 44.4° = \boxed{91.2°}$$

$$\angle DCA \approx 180° - 135.6° - 30° = \boxed{14.4°}$$

28 Since the smaller triangle on the left is an isosceles triangle:

$$\angle BCD = \angle CBD = 25°$$

The third angle in this triangle is:

$$\angle BDC = 180° - 25° - 25° = 130°$$

Use the Law of Sines to find BC:

$$\begin{aligned}\frac{\sin 130°}{BC} &= \frac{\sin 25°}{12} \\ \angle BC &\approx 21.8\end{aligned}$$

The third angle in the large triangle is:

$$\angle BAC = 180° - 25° - 50° = 105°$$

Use the Law of Sines to find AB:

$$\begin{aligned}\frac{\sin 105^\circ}{21.8} &= \frac{\sin 50^\circ}{AB} \\ AB &\approx 17.3\end{aligned}$$

Find the missing length:

$$DA = AB - BD \approx 17.3 - 12 = \boxed{5.3}$$

31 (a)

$$\begin{aligned}C &= 180^\circ - 84.2^\circ - 93^\circ \\ &= 2.8^\circ\end{aligned}$$

$$\begin{aligned}\frac{\sin 84.2^\circ}{b} &= \frac{\sin 2.8^\circ}{50} \\ b &\approx \boxed{1018\,\text{mi}}\end{aligned}$$

(b)

$$\begin{aligned}\sin 87^\circ &= \frac{h}{1018} \\ h &\approx \boxed{1017\,\text{mi}}\end{aligned}$$

32 (a)

$$\begin{aligned}C &= 180^\circ - 32^\circ - 48^\circ \\ &= 100^\circ\end{aligned}$$

$$\begin{aligned}\frac{\sin 100^\circ}{5} &= \frac{\sin 48^\circ}{b} \\ b &\approx \boxed{3.77\,\text{mi}}\end{aligned}$$

(b)

$$\begin{aligned}\sin 32^\circ &= \frac{h}{3.77} \\ h &\approx \boxed{2\,\text{mi}}\end{aligned}$$

33

$$\begin{aligned}C &= 180^\circ - 82^\circ - 52^\circ \\ &= 46^\circ\end{aligned}$$

$$\begin{aligned}\frac{\sin 46^\circ}{200} &= \frac{\sin 52^\circ}{b} \\ b &\approx \boxed{219\,\text{ft}}\end{aligned}$$

34 Find the other two angles:

$$\begin{aligned}\frac{\sin B}{312} &= \frac{\sin 48.6^\circ}{527}\\ B &\approx 26.4^\circ\\ C &= 180^\circ - 48.6^\circ - 26.4^\circ\\ &= 105^\circ\end{aligned}$$

Find AB

$$\begin{aligned}\frac{\sin 48.6^\circ}{527} &= \frac{\sin 105^\circ}{c}\\ c &\approx \boxed{679\,\text{ft}}\end{aligned}$$

35 Find the third angle:

$$\begin{aligned}C &= 180^\circ - (90^\circ - 5.6^\circ) - 29.5^\circ\\ &= 66.1^\circ\end{aligned}$$

Find the height

$$\begin{aligned}\frac{\sin 66.4^\circ}{105} &= \frac{\sin 29.2^\circ}{h}\\ h &\approx \boxed{56\,\text{m}}\end{aligned}$$

36

$$\begin{aligned}\frac{\sin C}{165} &= \frac{\sin 67^\circ}{180}\\ C &\approx 57.54^\circ\end{aligned}$$

$$B \approx 180^\circ - 67^\circ - 57.54^\circ = 55.46^\circ$$

$$\begin{aligned}\frac{\sin 55.46^\circ}{b} &= \frac{\sin 67^\circ}{180}\\ b &\approx \boxed{161\,\text{ft}}\end{aligned}$$

37 Find the angle at the top of the tree:

$$A = 180^\circ - 52^\circ - 90^\circ = 38^\circ$$

Find the height:

$$\begin{aligned}\frac{\sin 38^\circ}{215} &= \frac{\sin 30^\circ}{h}\\ h &\approx \boxed{175\,\text{ft}}\end{aligned}$$

38 Find the angle at the top of the tower:

$$A = 180^\circ - 58^\circ - 12^\circ - 90^\circ = 20^\circ$$

Find the remaining angle:

$$B = 180^\circ - 20^\circ - 12^\circ = 148^\circ$$

Find the length of the wire:

$$\begin{aligned}\frac{\sin 20^\circ}{100} &= \frac{\sin 148^\circ}{a}\\ a &\approx \boxed{155\,\text{m}}\end{aligned}$$

40 Find the angle at the top of the tower:

$$\frac{\sin 8^\circ}{30} = \frac{\sin A}{120}$$
$$A \approx 33.83^\circ$$

The angle of inclination of the hill completes a right triangle:

$$B = 90^\circ - 33.83^\circ - 8^\circ = \boxed{48.17^\circ}$$

41 Find the angle between Sun-Earth-Venus

$$\frac{\sin 39.4^\circ}{0.723} = \frac{\sin A}{1}$$
$$A \approx 118.61^\circ$$

Find the remaining angle:

$$B \approx 180^\circ - 118.61^\circ - 39.4^\circ = 22^\circ$$

Find the distance:

$$\frac{\sin 39.4^\circ}{0.723} = \frac{\sin 22^\circ}{d}$$
$$A \approx \boxed{0.4267\,\text{AU}}$$