

Math 142

Homework Ten

November 6, 2013

1 Homework

Section 6.4: 1-8, 11-12, 17-22, 27-28, 31-38, 40-41

2 Extra Credit

Section 6.4: 42 (pretty hard)

- 41** There are three different triangles, and each triangle provides three equations. Instead of writing $\sin 60$, $\sin A$, etc., I used w , x, y , for the sines of the angles. Since all of the sines cancel out in the end, this works out fine.

The equations you need are:

$$\begin{aligned}\frac{z}{a} &= \frac{x}{c+d} \\ \frac{x}{d} &= \frac{y}{r} \\ \frac{z}{a} &= \frac{w}{r} \\ \frac{x}{c} &= \frac{y}{a} \\ \frac{w}{b} &= \frac{x}{c}\end{aligned}$$

The procedure is:

- solve one of the equations for a variable you want to get rid of (anything other than a , b , or r).
- substitute the result into the other equations
- repeat until you end up with an equation with a , b , and r
- solve this equation for r

Get rid of w

$$\begin{aligned}\frac{w}{b} &= \frac{x}{c} \\ w &= \frac{xb}{c} \\ \frac{z}{a} &= \frac{xb}{rc}\end{aligned}$$

The equations now are:

$$\begin{aligned}\frac{z}{a} &= \frac{x}{c+d} \\ \frac{x}{d} &= \frac{y}{r} \\ \frac{x}{c} &= \frac{y}{a} \\ \frac{z}{a} &= \frac{xb}{rc}\end{aligned}$$

Get rid of z :

$$\begin{aligned}\frac{z}{a} &= \frac{xb}{rc} \\ z &= \frac{abx}{rc} \\ \frac{z}{a} &= \frac{x}{c+d} \\ \frac{abx}{arc} &= \frac{x}{c+d} \\ \frac{b}{rc} &= \frac{1}{c+d}\end{aligned}$$

The equations now are:

$$\begin{aligned}\frac{b}{rc} &= \frac{1}{c+d} \\ \frac{x}{d} &= \frac{y}{r} \\ \frac{x}{c} &= \frac{y}{a}\end{aligned}$$

Get rid of d :

$$\begin{aligned}\frac{x}{d} &= \frac{y}{r} \\ d &= \frac{xr}{y} \\ \frac{b}{rc} &= \frac{1}{c + \frac{xr}{y}} \\ \frac{b}{rc} &= \frac{y}{yc + xr}\end{aligned}$$

The equations now are:

$$\begin{aligned}\frac{x}{c} &= \frac{y}{a} \\ \frac{b}{rc} &= \frac{y}{yc + xr}\end{aligned}$$

Get rid of y :

$$\begin{aligned}\frac{x}{c} &= \frac{y}{a} \\ y &= \frac{xa}{c}\end{aligned}$$

$$\begin{aligned}\frac{b}{rc} &= \frac{y}{yc + xr} \\ \frac{b}{rc} &= \frac{xa}{c(xa + xr)}\end{aligned}$$

x and c both cancel:

$$\frac{b}{rc} = \frac{xa}{c(xa + xr)}$$

$$\frac{b}{r} = \frac{xa}{x(a + r)}$$

$$\frac{b}{r} = \frac{a}{a + r}$$

solve for r :

$$\frac{b}{r} = \frac{a}{a + r}$$

$$b(a + r) = ra$$

$$ab + rb = ra$$

$$ra - rb = ab$$

$$r = \frac{ab}{a - b}$$

3 Review

1. A bike with 26 inch diameter wheels is traveling at 15 mph. What is the RPM for the wheels?
2. Find an equation for this graph:

Solution:

$$f(t) = -2 \sin 2\pi t$$

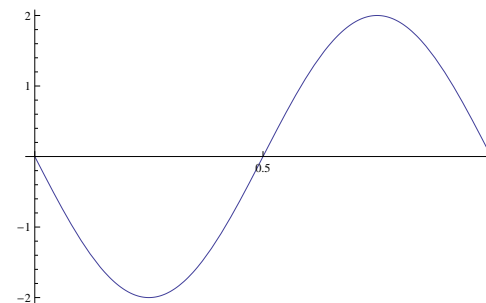


Figure 1: Find the Equation

4 Section 6.4

1

$$\frac{\sin 98.4^\circ}{376} = \frac{\sin 57^\circ}{x}$$

$$x \approx \boxed{318.8}$$

2

$$\frac{\sin 37.5^\circ}{17} = \frac{\sin 114.4^\circ}{x}$$

$$x \approx \boxed{25.43}$$

3

$$\frac{\sin 58^\circ}{26.7} = \frac{\sin 52^\circ}{x}$$

$$x \approx \boxed{24.81}$$

4

$$\frac{\sin 67^\circ}{80.2} = \frac{\sin \theta^\circ}{56.3}$$
$$\sin \theta \approx 0.6462$$
$$\theta \approx \boxed{40.25^\circ}$$

5

$$\frac{\sin 120^\circ}{45} = \frac{\sin \theta^\circ}{36}$$
$$\sin \theta \approx 0.6928$$
$$\theta \approx \boxed{43.85^\circ}$$

6

$$\frac{\sin 102^\circ}{185} = \frac{\sin 50^\circ}{x}$$
$$x \approx \boxed{144.9}$$

7

$$\frac{\sin 114^\circ}{65} = \frac{\sin 20^\circ}{b}$$
$$b \approx \boxed{24.33}$$

$$\frac{\sin 114^\circ}{65} = \frac{\sin 46^\circ}{a}$$
$$a \approx \boxed{51.18}$$

8

$$\frac{\sin 50^\circ}{2} = \frac{\sin 30^\circ}{a}$$
$$a \approx \boxed{1.305}$$

$$\frac{\sin 50^\circ}{2} = \frac{\sin 100^\circ}{c}$$
$$c \approx \boxed{2.571}$$

11

$$\frac{\sin 62^\circ}{230} = \frac{\sin 50^\circ}{a}$$
$$a \approx \boxed{199.5}$$

$$\frac{\sin 62^\circ}{230} = \frac{\sin 68^\circ}{b}$$
$$b \approx \boxed{231.5}$$

12

$$\frac{\sin 47^\circ}{50} = \frac{\sin 23^\circ}{a}$$
$$a \approx \boxed{63.39}$$

$$\frac{\sin 47^\circ}{50} = \frac{\sin 110^\circ}{b}$$
$$b \approx \boxed{64.24}$$

17

$$\frac{\sin 110^\circ}{28} = \frac{\sin B}{15}$$

$$B \approx \boxed{30^\circ}$$

$$C = 180^\circ - 110^\circ - 30^\circ$$

$$= \boxed{40^\circ}$$

$$\frac{\sin 110^\circ}{28} = \frac{\sin 40^\circ}{c}$$

$$c \approx \boxed{19.1}$$

18

$$\frac{\sin 37^\circ}{30} = \frac{\sin C}{40}$$

$$C \approx \boxed{53^\circ}$$

$$B = 180^\circ - 37^\circ - 53^\circ$$

$$= \boxed{90^\circ}$$

$$\frac{\sin 37^\circ}{30} = \frac{\sin 90^\circ}{c}$$

$$c \approx \boxed{49.8}$$

19

$$\frac{\sin 125^\circ}{20} = \frac{\sin C}{45}$$

$$\sin C \approx 1.84$$

No solution

20

$$\frac{\sin 38^\circ}{42} = \frac{\sin B}{45}$$

$$B \approx \boxed{41^\circ}$$

$$C = 180^\circ - 38^\circ - 41^\circ = \boxed{101^\circ}$$

$$\frac{\sin 38^\circ}{42} = \frac{\sin 101^\circ}{c}$$

$$c \approx \boxed{67.0}$$

21 solution 1:

$$\frac{\sin 25^\circ}{25} = \frac{\sin C}{30}$$

$$C \approx \boxed{30^\circ}$$

$$B = 180^\circ - 25^\circ - 30^\circ$$

$$= \boxed{125^\circ}$$

$$\frac{\sin 25^\circ}{25} = \frac{\sin 125^\circ}{b}$$

$$b \approx \boxed{48.4}$$

solution 2:

$$\frac{\sin 25^\circ}{25} = \frac{\sin C}{30}$$
$$C \approx \boxed{150^\circ}$$

$$B = 180^\circ - 25^\circ - 150^\circ$$
$$= \boxed{5^\circ}$$

$$\frac{\sin 25^\circ}{25} = \frac{\sin 5^\circ}{b}$$
$$b \approx \boxed{5.1}$$

22 solution 1:

$$\frac{\sin 30^\circ}{75} = \frac{\sin B}{100}$$
$$B \approx \boxed{42^\circ}$$

$$C = 180^\circ - 30^\circ - 42^\circ$$
$$= \boxed{108^\circ}$$

$$\frac{\sin 30^\circ}{75} = \frac{\sin 108^\circ}{c}$$
$$c \approx \boxed{142.7}$$

solution 2:

$$\frac{\sin 30^\circ}{75} = \frac{\sin B}{100}$$
$$B = 180^\circ - 42^\circ$$
$$\approx \boxed{138^\circ}$$

$$C = 180^\circ - 30^\circ - 138^\circ$$
$$= \boxed{12^\circ}$$

$$\frac{\sin 30^\circ}{75} = \frac{\sin 12^\circ}{c}$$
$$c \approx \boxed{31.2}$$

27 Find $\angle CDA$ and $\angle BDC$

$$\frac{\sin 30^\circ}{20} = \frac{\sin CDA}{28}$$
$$\angle CDA \approx 180^\circ - 44^\circ$$
$$= 136^\circ$$
$$\angle BDC = 44^\circ$$

Use the Law of Sines again to find $\angle CBD$

$$\frac{\sin 44^\circ}{20} = \frac{\sin CBD}{20}$$
$$\angle CBD \approx 44^\circ$$

The missing angles are the third angles in the two small triangles. You can find them by knowing that the sum of the angles in a

triangle is 180° :

$$\angle BCD = 180^\circ - 44^\circ - 44^\circ = \boxed{92^\circ}$$

$$\angle DCA = 180^\circ - 136^\circ - 30^\circ = \boxed{14^\circ}$$

- 28** Since the smaller triangle on the left is an isosceles triangle, $\angle BCD = \angle CBD = 25^\circ$

The third angle in this triangle is: $\angle BDC = 180^\circ - 25^\circ - 25^\circ = 130^\circ$

Use the Law of Sines to find BC :

$$\frac{\sin 130^\circ}{BC} = \frac{\sin 25^\circ}{12}$$

$$\angle BC \approx 21.8$$

The third angle in the large triangle is: $\angle BAC = 180^\circ - 25^\circ - 50^\circ = 105^\circ$.

Use the Law of Sines to find AB :

$$\frac{\sin 105^\circ}{21.8} = \frac{\sin 50^\circ}{AB}$$

$$\angle AB \approx 17.3$$

Now we can find the missing length:

$$DA = AB - BD = 17.3 - 12 = \boxed{5.3}$$

31 (a)

$$C = 180^\circ - 84.2^\circ - 93^\circ$$

$$= 2.8^\circ$$

$$\frac{\sin 84.2^\circ}{b} = \frac{\sin 2.8^\circ}{50}$$

$$b \approx \boxed{1018 \text{ mi}}$$

(b)

$$\sin 87^\circ = \frac{h}{1018}$$

$$h \approx \boxed{1017 \text{ mi}}$$

32 (a)

$$C = 180^\circ - 32^\circ - 48^\circ$$

$$= 100^\circ$$

$$\frac{\sin 100^\circ}{5} = \frac{\sin 48^\circ}{b}$$

$$b \approx \boxed{3.77 \text{ mi}}$$

(b)

$$\sin 32^\circ = \frac{h}{3.77}$$

$$h \approx \boxed{2 \text{ mi}}$$

33

$$\begin{aligned} C &= 180^\circ - 82^\circ - 52^\circ \\ &= 46^\circ \end{aligned}$$

$$\begin{aligned} \frac{\sin 46^\circ}{200} &= \frac{\sin 52^\circ}{b} \\ b &\approx \boxed{219 \text{ ft}} \end{aligned}$$

34 Find the other two angles:

$$\begin{aligned} \frac{\sin B}{312} &= \frac{\sin 48.6^\circ}{527} \\ B &\approx 26.4^\circ \\ C &= 180^\circ - 48.6^\circ - 26.4^\circ \\ &= 105^\circ \end{aligned}$$

Find AB

$$\begin{aligned} \frac{\sin 48.6^\circ}{527} &= \frac{\sin 105^\circ}{c} \\ c &\approx \boxed{679 \text{ ft}} \end{aligned}$$

35 Find the third angle:

$$\begin{aligned} C &= 180^\circ - (90^\circ - 5.6^\circ) - 29.5^\circ \\ &= 66.1^\circ \end{aligned}$$

Find the height

$$\begin{aligned} \frac{\sin 66.4^\circ}{105} &= \frac{\sin 29.2^\circ}{h} \\ h &\approx \boxed{56 \text{ m}} \end{aligned}$$

36

$$\begin{aligned} \frac{\sin C}{165} &= \frac{\sin 67^\circ}{180} \\ C &\approx 57.54^\circ \end{aligned}$$

$$\begin{aligned} B &= 180^\circ - 67^\circ - 57.54^\circ \\ &= 55.46^\circ \end{aligned}$$

$$\begin{aligned} \frac{\sin 55.46^\circ}{b} &= \frac{\sin 67^\circ}{180} \\ b &\approx \boxed{161 \text{ ft}} \end{aligned}$$

37 Find the angle at the top of the tree:

$$\begin{aligned} A &= 180^\circ - 52^\circ - 90^\circ \\ &= 38^\circ \end{aligned}$$

Find the height:

$$\begin{aligned} \frac{\sin 38^\circ}{215} &= \frac{\sin 30^\circ}{h} \\ h &\approx \boxed{175 \text{ ft}} \end{aligned}$$

38 Find the angle at the top of the tower:

$$\begin{aligned} A &= 180^\circ - 58^\circ - 12^\circ - 90^\circ \\ &= 20^\circ \end{aligned}$$

Find the remaining angle:

$$\begin{aligned} B &= 180^\circ - 20^\circ - 12^\circ \\ &= 148^\circ \end{aligned}$$

Find the length of the wire:

$$\frac{\sin 20^\circ}{100} = \frac{\sin 148^\circ}{a}$$
$$a \approx \boxed{155 \text{ m}}$$

40 Find the angle at the top of the tower:

$$\frac{\sin 8^\circ}{30} = \frac{\sin A}{120}$$
$$A \approx 33.83^\circ$$

The angle of inclination of the hill completes a right triangle:

$$B = 90^\circ - 33.83^\circ - 8^\circ$$
$$= \boxed{48.17^\circ}$$

41 Find the angle between Sun-Earth-Venus

$$\frac{\sin 39.4^\circ}{0.723} = \frac{\sin A}{1}$$
$$A \approx 118.61^\circ$$

Find the remaining angle:

$$B = 180^\circ - 118.61^\circ - 39.4^\circ$$
$$= 22^\circ$$

Find the distance:

$$\frac{\sin 39.4^\circ}{0.723} = \frac{\sin 22^\circ}{d}$$
$$A \approx \boxed{0.4267 \text{ AU}}$$