Math 142
Homework Ten

November 6, 2013

1 Homework

Section 6.4: 1-8, 11-12, 17-22, 27-28, 31-38, 40-41

2 Extra Credit

Section 6.4: 42 (pretty hard)

41 There are three different triangles, and each triangle provides three equations. Instead of writing $\sin 60°$, $\sin A$, etc., I used w, x, and y for the sines of the angles. Since all of the sines cancel out in the end, this works out fine.

Here are the variables I used:

$$\begin{aligned}
w &= \sin ABC \\
x &= \sin BCA = \sin ACD = \sin BCD \\
y &= \sin CAD \\
z &= \sin CDA
\end{aligned}$$

$$\begin{aligned}
c &= BA \\
d &= AD
\end{aligned}$$

The equations you need are:

$$\begin{aligned}
\frac{z}{a} &= \frac{x}{c+d} \\
\frac{x}{d} &= \frac{y}{r} \\
\frac{z}{a} &= \frac{w}{r} \\
\frac{x}{c} &= \frac{y}{a} \\
\frac{w}{b} &= \frac{x}{c}
\end{aligned}$$

The procedure is:

- solve one of the equations for a variable you want to get rid of (anything other than a, b, or r).
- substitute the result into the other equations
- repeat until you end up with an equation with a, b, and r
- solve this equation for r

Get rid of w

$$\frac{w}{b} = \frac{x}{c}$$
$$w = \frac{xb}{c}$$

$$\frac{z}{a} = \frac{xb}{rc}$$

The equations now are:

$$\frac{z}{a} = \frac{x}{c+d}$$
$$\frac{x}{d} = \frac{y}{r}$$
$$\frac{x}{c} = \frac{y}{a}$$
$$\frac{z}{a} = \frac{xb}{rc}$$

Get rid of z:

$$\frac{z}{a} = \frac{xb}{rc}$$
$$z = \frac{abx}{rc}$$

$$\frac{z}{a} = \frac{x}{c+d}$$
$$\frac{abx}{arc} = \frac{x}{c+d}$$
$$\frac{b}{rc} = \frac{1}{c+d}$$

The equations now are:

$$\frac{b}{rc} = \frac{1}{c+d}$$
$$\frac{x}{d} = \frac{y}{r}$$
$$\frac{x}{c} = \frac{y}{a}$$

Get rid of d:

$$\frac{x}{d} = \frac{y}{r}$$
$$d = \frac{xr}{y}$$

$$\frac{b}{rc} = \frac{1}{c + {}^{xr}/_{y}}$$
$$\frac{b}{rc} = \frac{y}{yc + xr}$$

The equations now are:

$$\frac{x}{c} = \frac{y}{a}$$
$$\frac{b}{rc} = \frac{y}{yc + xr}$$

Get rid of y:

$$\frac{x}{c} = \frac{y}{a}$$
$$y = \frac{xa}{c}$$

$$\frac{b}{rc} = \frac{y}{yc + xr}$$
$$\frac{b}{rc} = \frac{xa}{c(xa + xr)}$$

x and c both cancel:

$$\frac{b}{rc} = \frac{xa}{c(xa + xr)}$$
$$\frac{b}{r} = \frac{xa}{x(a + r)}$$
$$\frac{b}{r} = \frac{a}{a + r}$$

solve for r:

$$\frac{b}{r} = \frac{a}{a + r}$$
$$b(a + r) = ra$$
$$ab + rb = ra$$
$$ra - rb = ab$$
$$r = \frac{ab}{a - b}$$

3 Review

1. A bike with 26 inch diameter wheels is traveling at 15 mph. What is the RPM for the wheels?

2. Find an equation for this graph:

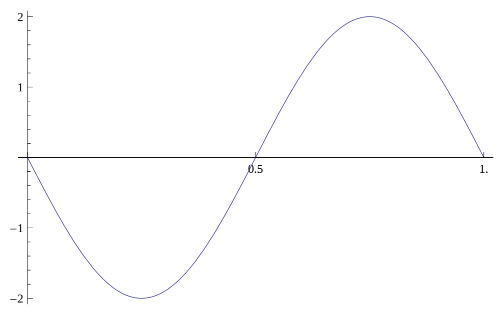

Figure 1: Find the Equation

Solution:

$$f(t) = -2 \sin 2\pi t$$

4 Section 6.4

1

$$\frac{\sin 98.4°}{376} = \frac{\sin 57°}{x}$$
$$x \approx \boxed{318.8}$$

2

$$\frac{\sin 37.5^\circ}{17} = \frac{\sin 114.4^\circ}{x}$$
$$x \approx \boxed{25.43}$$

3

$$\frac{\sin 58^\circ}{26.7} = \frac{\sin 52^\circ}{x}$$
$$x \approx \boxed{24.81}$$

4

$$\frac{\sin 67^\circ}{80.2} = \frac{\sin \theta^\circ}{56.3}$$
$$\sin\theta \approx 0.6462$$
$$\theta \approx \boxed{40.25^\circ}$$

5

$$\frac{\sin 120^\circ}{45} = \frac{\sin \theta^\circ}{36}$$
$$\sin\theta \approx 0.6928$$
$$\theta \approx \boxed{43.85^\circ}$$

6

$$\frac{\sin 102^\circ}{185} = \frac{\sin 50^\circ}{x}$$
$$x \approx \boxed{144.9}$$

7

$$\frac{\sin 114^\circ}{65} = \frac{\sin 20^\circ}{b}$$
$$b \approx \boxed{24.33}$$

$$\frac{\sin 114^\circ}{65} = \frac{\sin 46^\circ}{a}$$
$$a \approx \boxed{51.18}$$

8

$$\frac{\sin 50^\circ}{2} = \frac{\sin 30^\circ}{a}$$
$$a \approx \boxed{1.305}$$

$$\frac{\sin 50^\circ}{2} = \frac{\sin 100^\circ}{c}$$
$$c \approx \boxed{2.571}$$

11

$$\frac{\sin 62^\circ}{230} = \frac{\sin 50^\circ}{a}$$
$$a \approx \boxed{199.5}$$

$$\frac{\sin 62^\circ}{230} = \frac{\sin 68^\circ}{b}$$
$$b \approx \boxed{241.5}$$

12

$$\begin{aligned}\frac{\sin 47^\circ}{50} &= \frac{\sin 23^\circ}{a}\\ a &\approx \boxed{26.71}\end{aligned}$$

$$\begin{aligned}\frac{\sin 47^\circ}{50} &= \frac{\sin 110^\circ}{b}\\ b &\approx \boxed{64.24}\end{aligned}$$

17

$$\begin{aligned}\frac{\sin 110^\circ}{28} &= \frac{\sin B}{15}\\ B &\approx \boxed{30^\circ}\end{aligned}$$

$$\begin{aligned}C &= 180^\circ - 110^\circ - 30^\circ\\ &= \boxed{40^\circ}\end{aligned}$$

$$\begin{aligned}\frac{\sin 110^\circ}{28} &= \frac{\sin 40^\circ}{c}\\ c &\approx \boxed{19.1}\end{aligned}$$

18

$$\begin{aligned}\frac{\sin 37^\circ}{30} &= \frac{\sin C}{40}\\ C &\approx \boxed{53^\circ}\end{aligned}$$

$$\begin{aligned}B &= 180^\circ - 37^\circ - 53^\circ\\ &= \boxed{90^\circ}\end{aligned}$$

$$\begin{aligned}\frac{\sin 37^\circ}{30} &= \frac{\sin 90^\circ}{c}\\ c &\approx \boxed{49.8}\end{aligned}$$

19

$$\begin{aligned}\frac{\sin 125^\circ}{20} &= \frac{\sin C}{45}\\ \sin C &\approx 1.84\end{aligned}$$

No solution

20

$$\begin{aligned}\frac{\sin 38^\circ}{42} &= \frac{\sin B}{45}\\ B &\approx \boxed{41^\circ}\end{aligned}$$

$$\begin{aligned}C &= 180^\circ - 38^\circ - 41^\circ\\ &= \boxed{101^\circ}\end{aligned}$$

$$\begin{aligned}\frac{\sin 38^\circ}{42} &= \frac{\sin 101^\circ}{c}\\ c &\approx \boxed{67.0}\end{aligned}$$

21 solution 1:

$$\frac{\sin 25^\circ}{25} = \frac{\sin C}{30}$$
$$C \approx \boxed{30.5^\circ}$$

$$A \approx 180^\circ - 25^\circ - 30.5^\circ$$
$$= \boxed{124.5^\circ}$$

$$\frac{\sin 25^\circ}{25} = \frac{\sin 124.5}{a}$$
$$a \approx \boxed{48.8}$$

solution 2:

$$C \approx 180^\circ - 30.5^\circ \boxed{149.5^\circ}$$

$$B = 180^\circ - 25^\circ - 149.5^\circ$$
$$= \boxed{5.5^\circ}$$

$$\frac{\sin 25^\circ}{25} = \frac{\sin 5.5^\circ}{a}$$
$$a \approx \boxed{5.7}$$

22 solution 1:

$$\frac{\sin 30^\circ}{75} = \frac{\sin B}{100}$$
$$B \approx \boxed{41.8^\circ}$$

$$C = 180^\circ - 30^\circ - 41.8^\circ$$
$$= \boxed{108.2^\circ}$$

$$\frac{\sin 30^\circ}{75} = \frac{\sin 108.2^\circ}{c}$$
$$c \approx \boxed{142.5}$$

solution 2:

$$B \approx 180^\circ - 41.8^\circ$$
$$\approx \boxed{138.2^\circ}$$

$$C \approx 180^\circ - 30^\circ - 138.2^\circ$$
$$= \boxed{11.8^\circ}$$

$$\frac{\sin 30^\circ}{75} = \frac{\sin 11.8^\circ}{c}$$
$$c \approx \boxed{30.6}$$

27 Find $\angle ABC$:

$$\frac{\sin 30^\circ}{20} = \frac{\sin ABC}{28}$$
$$\angle ABC \approx 44.4^\circ$$

Since $\triangle BCD$ is isosceles:

$$\angle BDC = \angle ABC \approx 44.4^\circ$$

The missing angles are the third angles in the two small triangles. You can find them by knowing that the sum of the angles in a triangle is 180°:

$$\angle BCD \approx 180^\circ - 44.4^\circ - 44.4^\circ = \boxed{91.2^\circ}$$
$$\angle DCA \approx 180^\circ - 135.6^\circ - 30^\circ = \boxed{14.4^\circ}$$

28 Since the smaller triangle on the left is an isosceles triangle:

$$\angle BCD = \angle CBD = 25^\circ$$

The third angle in this triangle is:

$$\angle BDC = 180^\circ - 25^\circ - 25^\circ = 130^\circ$$

Use the Law of Sines to find BC:

$$\frac{\sin 130^\circ}{BC} = \frac{\sin 25^\circ}{12}$$
$$\angle BC \approx 21.8$$

The third angle in the large triangle is:

$$\angle BAC = 180^\circ - 25^\circ - 50^\circ = 105^\circ$$

Use the Law of Sines to find AB:

$$\frac{\sin 105^\circ}{21.8} = \frac{\sin 50^\circ}{AB}$$
$$AB \approx 17.3$$

Find the missing length:

$$DA = AB - BD \approx 17.3 - 12 = \boxed{5.3}$$

31 (a)

$$\begin{aligned} C &= 180^\circ - 84.2^\circ - 93^\circ \\ &= 2.8^\circ \end{aligned}$$

$$\frac{\sin 84.2^\circ}{b} = \frac{\sin 2.8^\circ}{50}$$
$$b \approx \boxed{1018\,\text{mi}}$$

(b)

$$\sin 87^\circ = \frac{h}{1018}$$
$$h \approx \boxed{1017\,\text{mi}}$$

32 (a)

$$\begin{aligned} C &= 180° - 32° - 48° \\ &= 100° \end{aligned}$$

$$\begin{aligned} \frac{\sin 100°}{5} &= \frac{\sin 48°}{b} \\ b &\approx \boxed{3.77\,\text{mi}} \end{aligned}$$

(b)

$$\begin{aligned} \sin 32° &= \frac{h}{3.77} \\ h &\approx \boxed{2\,\text{mi}} \end{aligned}$$

33

$$\begin{aligned} C &= 180° - 82° - 52° \\ &= 46° \end{aligned}$$

$$\begin{aligned} \frac{\sin 46°}{200} &= \frac{\sin 52°}{b} \\ b &\approx \boxed{219\,\text{ft}} \end{aligned}$$

34 Find the other two angles:

$$\begin{aligned} \frac{\sin B}{312} &= \frac{\sin 48.6°}{527} \\ B &\approx 26.4° \\ C &= 180° - 48.6° - 26.4° \\ &= 105° \end{aligned}$$

Find AB

$$\begin{aligned} \frac{\sin 48.6°}{527} &= \frac{\sin 105°}{c} \\ c &\approx \boxed{679\,\text{ft}} \end{aligned}$$

35 Find the third angle:

$$\begin{aligned} C &= 180° - (90° - 5.6°) - 29.5° \\ &= 66.1° \end{aligned}$$

Find the height

$$\begin{aligned} \frac{\sin 66.4°}{105} &= \frac{\sin 29.2°}{h} \\ h &\approx \boxed{56\,\text{m}} \end{aligned}$$

36

$$\frac{\sin C}{165} = \frac{\sin 67^\circ}{180}$$
$$C \approx 57.54^\circ$$

$$B \approx 180^\circ - 67^\circ - 57.54^\circ = 55.46^\circ$$

$$\frac{\sin 55.46^\circ}{b} = \frac{\sin 67^\circ}{180}$$
$$b \approx \boxed{161\,\text{ft}}$$

37 Find the angle at the top of the tree:

$$A = 180^\circ - 52^\circ - 90^\circ = 38^\circ$$

Find the height:

$$\frac{\sin 38^\circ}{215} = \frac{\sin 30^\circ}{h}$$
$$h \approx \boxed{175\,\text{ft}}$$

38 Find the angle at the top of the tower:

$$A = 180^\circ - 58^\circ - 12^\circ - 90^\circ = 20^\circ$$

Find the remaining angle:

$$B = 180^\circ - 20^\circ - 12^\circ = 148^\circ$$

Find the length of the wire:

$$\frac{\sin 20^\circ}{100} = \frac{\sin 148^\circ}{a}$$
$$a \approx \boxed{155\,\text{m}}$$

40 Find the angle at the top of the tower:

$$\frac{\sin 8^\circ}{30} = \frac{\sin A}{120}$$
$$A \approx 33.83^\circ$$

The angle of inclination of the hill completes a right triangle:

$$B = 90^\circ - 33.83^\circ - 8^\circ = \boxed{48.17^\circ}$$

41 Find the angle between Sun-Earth-Venus

$$\frac{\sin 39.4^\circ}{0.723} = \frac{\sin A}{1}$$
$$A \approx 118.61^\circ$$

Find the remaining angle:

$$B \approx 180^\circ - 118.61^\circ - 39.4^\circ = 22^\circ$$

Find the distance:

$$\frac{\sin 39.4^\circ}{0.723} = \frac{\sin 22^\circ}{d}$$
$$A \approx \boxed{0.4267\,\text{AU}}$$