TESTING THE AGENT-BASED MODEL WITH INSTANTANEOUS CONTRACT TRACING

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CONTENTS

1. Overview

This document outlines the tests that are performed on the individual-based-model model. The tests play multiple roles. First, they are regression tests to ensure that model is correctly coded and remains so as the code is developed. Second, we test the baseline parameter set of the model to ensure that it is consistent with our understanding of the disease. Third, we test the action of intervention strategies to ensure that the changes within the model are consistent with the strategy.

2. Tests on Demographics

2.1. **The proportion of simulated individuals within each age group.** This test is to check whether the simulated population matches the one it supposed to describe at the beginning of the simulations. The proportion of the population within each age group has to be about the figures listed in Table 1. First and last tests (with 10000 and 500000 people) have proportions of the UK population. Homogeneous, top-heavy (older), and bottom-heavy (younger) population pyramids are tested in second, third, and fourth trials, respectively.

TABLE 1. The parameters and values tested within each age group

parameter	values tested
n_total	{10000, 50000, 100000, 250000, 500000}
population_0_9	{10000 * 0.12, 50000 * 0.11, 100000 * 0.08, 250000 * 0.21, 500000 * 0.12}
population_10_19	{10000 * 0.11, 50000 * 0.11, 100000 * 0.09, 250000 * 0.14, 500000 * 0.11}
population_20_29	{10000 * 0.13, 50000 * 0.11, 100000 * 0.10, 250000 * 0.15, 500000 * 0.13}
population_30_39	{10000 * 0.13, 50000 * 0.11, 100000 * 0.12, 250000 * 0.16, 500000 * 0.13}
population_40_49	{10000 * 0.13, 50000 * 0.11, 100000 * 0.15, 250000 * 0.12, 500000 * 0.13}
population_50_59	{10000 * 0.13, 50000 * 0.11, 100000 * 0.13, 250000 * 0.11, 500000 * 0.13}
population_60_69	{10000 * 0.11, 50000 * 0.11, 100000 * 0.13, 250000 * 0.07, 500000 * 0.11}
population_70_79	{10000 * 0.08, 50000 * 0.11, 100000 * 0.13, 250000 * 0.03, 500000 * 0.08}
population_80	{10000 * 0.05, 50000 * 0.11, 100000 * 0.09, 250000 * 0.02, 500000 * 0.05}

2.2. **The size of households.** This test is to check whether the simulated numbers of people living in households of different sizes are proportional to the UK (or other possible) household sizes. The expected proportion of households of specific sizes are listed in Table 2.

TABLE 2. Different parameter sets tested for the proportion of households with $n = 1, 2, \ldots, 6$ people.

parameter	values tested
household_size_1	{ 0.30, 0.25, 0.33, 0.33 }
household_size_2	{ 0.35, 0.30, 0.37, 0.39 }
household_size_3	{ 0.15, 0.10, 0.18, 0.08 }
household_size_4	{ 0.13, 0.19, 0.10, 0.05 }
household_size_5	{ 0.04, 0.09, 0.02, 0.08 }
household_size_6	{ 0.02, 0.07, 0.01, 0.06 }

TABLE 3. The parameters tested and values tested in the Tests on Work-Place Networks

Test on Work-Place Networks	
mean_work_interactions_child	{10,6,4.63,0,2}
mean_work_interactions_adult	{7,10,3.25,0,6}
mean_work_interactions_elderly	{3,5,0.83,0.5,1}
daily_fraction_work	{0.5,0.43,0.83,0.5,1.0}
n_total	{10000,20000,20000,20000,20000}

3. TESTS ON INTERACTION NETWORK

3.1. **Interactions are 2-way.** This test looks through all the interactions in each person's interaction diary and checks to see whether their is corresponding interaction in the other person's diary. This test is repeated for populations of size 10,000, 20,000, 50,000, 100,000 and 250,000.

3.2. **Household Network.** This test checks that the number of interactions between individuals in a household is the expected number given the household size. In particular, people who live by themselves should have no interactions. This test is repeated for populations of size 10,000, 20,000, 40,000, 50,000 and 100,000.

3.3. **Tests on Work-Place Networks.** These tests are on the work-place networks and are run on a number of parameters (Table 3).

3.3.1. *Connections Exist.* Check that network connections exist on all work-place networks where the mean number of connections is greater than zero.

3.3.2. *Age Assignment.* Check that everybody is assigned to the correct work-place network for group. Adults can be on any of the network (i.e. teachers/carers).

3.3.3. *Mean Interactions.* For each network, check that the mean number of interactions each individual has is the number specified.

3.3.4. *Household Correlation Zero.* The work-place networks are constructed independently of the households, so the correlation of the household numbers of the individuals in each interaction should be zero.

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Table 4. The parameters tested and values tested in the Ratio of Adults in Work Networks

Ratio of Adults in Work Networks	
n_total	{40000, 40000, 40000, 40000, 40000}
child_network_adults	{0, 0.01, 0.1, 0.2, 0.45}
elderly_network_adults	{0, 0.01, 0.1, 0.2, 0.45}

Table 5. The parameters tested and values tested in the Tests on Random Network

Test on Random Network	
mean_random_interactions_child	{2,1,0,4,8}
mean_random_interactions_adult	{4,1,0,3,7}
mean_random_interactions_elderly	{3,1,0,2,6}
sd_random_interactions_child	{2,2,2,4,8}
sd_random_interactions_adult	{4,2,2,3,7}
sd_random_interactions_elderly	{3,2,2,2,6}
n_total	{50000,100000,40000,40000,10000}

3.3.5. *Ratio of Adults in Work Networks.* All workplace networks have adults. Whereas adult work networks have only adults, the ratio of adults to children (elderly) in children (elderly) workplace networks is determined by the parameter `child_network_adults` (`elderly_network_adults`). Tested ratios are listed in Table 4.

3.4. **Tests on Random Network.** These tests are on the random network and are run on a number of parameters (Table).

3.4.1. *Distribution of Interactions.* Checks the mean and standard deviation of the number of interactions of the number of connections in age-group is as specified in the parameters.

3.4.2. *Household Correlation Zero.* The random network are constructed independently of the households, so the correlation of the household numbers of the individuals in each interaction should be zero.

4. Tests on Infection Dynamics

4.1. **Exponential Growth on Homogeneous Random Network.** For a random network where everybody has the same number of the interactions in each time period, there is an analytical approximation for the rate of exponential growth at the start of ans epidemic. Whilst this is an over-simplified model, it allows us to formally test the part of the model which transmits the virus. Note, the construction of the interaction networks and disease dynamics are tested separately, which allows us to use simplified network to test the infection transmission code.

Consider the number of new infections at time t, N_t, and let the infectiousness of somebody who has had the disease for τ days to be $Rf(\tau)$. Then in a large random (i.e. unstructured)

TABLE 6. The parameters tested and values tested in the Exponential Growth on Homogeneous Random Network.

Exponential Growth on Homogeneous Random Network	
infectious_rate	$\{3.0,2.5,2.0,3.0,3.0\}$
mean_infectious_period	$\{6.0,6.0,6.0,9.0,8.0\}$
sd_infectious_period	$\{2.5,2.5,2.0,3.0,8.0\}$
mean_random_interactions_child	$\{5,5,5,5,5\}$
mean_random_interactions_adult	$\{5,5,5,5,5\}$
mean_random_interactions_elderly	$\{5,5,5,5,5\}$
random_interaction_distribution	0 (fixed)

network where each person has the same number connections and with a constant number of succeptibles

$$N_{t+1} = R \sum_{\tau=0}^{t-t_0} N_{t-\tau} f_D(\tau)$$

where t_0 is the start of the epidemic and $f_D(\tau)$ is a probability mass function. This is a linear equation and we look for solutions of the form $e^{\lambda t}$, so

$$e^{\lambda} = R \sum_{\tau=0}^{t-t_0} e^{-\lambda s} f_D(\tau)$$

Finally, taking the limit that the infection started a long-time ago we get

$$e^{\lambda} = R M_D(-\lambda) \tag{1}$$

where $M_D(\lambda)$ is the moment generating function of the distribution D. This equation can then be solved analytically to find the exponential growth rate at the early stage of an epidemic.

The work-place network was turned off by setting the number of interactions on it to 0 and the transmission on the household network was turned off. The distribution of the number of connections per person on the random network was set to be fixed (from negative-binomial) and the number of interactions independent of age. The disease dynamics was halted at symptomatic (to prevent recovery and hospitalisation) so that each person had the same infectiousness profile with time. Infection succeptibility was set to be the same for all age groups and asymptomatic infections were prevented. We then tested a range of values of R and the mean and width of the time-dependence of the infectious curve (Table 6) and validated the results was within 5% of that predicted by (1).

4.2. **Transmission Pairs.** This test looks at all the transmission events and makes sure that all expected types occur and no unexpected types occur. The checks are:
   (1) Transmission by individuals in all disease states (i.e. pre-symptomatic, asymptomatic, symptomatic, hospitalised, critical)
   (2) Transmission occurs only from people who have been infected for at least one time step and not longer than the tail of the infectious curve (defined as mean + 7 standard deviations)
   (3) Transmission occurs on all 3 types of networks

(4) Hospitalised people do not transmit on the household or work-place networks

4.3. Ratio of presymptomatic/symptomatic infections in the absence of interventions.
Ratio of presymptomatic to symptomatic individuals computed across all individuals should be one. Different overall population sizes and seeding infections have been tested, see Table 7 for the values. The equality of total presymptomatic and symptomatic individuals for corresponding age groups are checked as well.

TABLE 7. The parameters and values tested for the equality of the total presymptomatic and symptomatic infected individuals

parameter	values tested
n_total	{10000, 100000, 1000000, 10000000, 100000}
n_seed_infection	{1, 10, 100, 1000, 10000}

5. Tests on Disease Dynamics

5.1. Distribution of Times in Disease-States.
This tests looks at the distribution of times of disease progression (i.e. from pre-symptomatic to symptomatic) across the whole population during an uncontrolled epidemic. It calculates the mean and standard deviation of the transition times for the transitions between:

(1) pre-symptomatic to symptomatic
(2) symptomatic to hospitalised
(3) symptomatic to recovered
(4) hospitalised to critical
(5) hospitalised to recovered
(6) critical to death
(7) critical to recovered
(8) asymptomatic to recovered

and checks that the difference with the model parameters are within the confidence interval of the sampled values. We test 5 distinct parameters sets (Table 8).

5.2. Disease Outcome Proportions.
This test looks at the progression of the disease and tests whether the proportion of the population that have each outcome is consistent with the parameters. Many of the outcomes are age-dependent, so we test both the age-stratified and the overall outcomes. Specifically we test:

(1) proportion of asymptomatic vs all
(2) proportion of mild symptomatic vs all
(3) proportion of severe symptomatic vs all
(4) proportion of symptomatic requiring hospitalisation
(5) proportion of symptomatic requiring hospitalisation by age
(6) proportion of hospitalised requiring critical care
(7) proportion of hospitalised requiring critical care by age
(8) proportion of those in critical care who die
(9) proportion of those in critical care who die by age

TABLE 8. The parameters tested and values tested in the Distribution of Times in Disease-States

Distribution of Times in Disease-States	
`mean_time_to_symptoms`	{4.0,4.5,5.0,5.5,6.0}
`sd_time_to_symptoms`	{2.0,2.0,2.5,2.5,3.0}
`mean_time_to_hospital`	{1.0,1.2,1.4,1.6,1.8}
`mean_time_to_critical`	{1.0,1.2,1.41.6,2.0}
`mean_time_to_death`	{12.0,14.0,16.0,17.0,18.0}
`sd_time_to_death`	{5.0,5.5,6.0,5.0,6.0}
`mean_time_to_recover`	{20.0,18.0,16.0,14.0,12.0}
`sd_time_to_recover`	{8.0,7.0,7.0,6.0,6.0}
`mean_asymptomatic_to_recover`	{15.0,17.0,18.0,12.0,14.0}
`sd_asymptomatic_to_recover`	{5.0,5.0,7.0,4.0,5.0}

and checks that the difference with the model parameters are within the confidence interval of the sampled values. Note some of the age-stratified groups may contain no samples due to the small population and lack of disease progression for some age-groups. We test 5 distinct parameters sets (Table 9).

6. TESTS ON INTERVENTIONS

6.1. **No quarantining.** This test checks no individuals are quarantined when all parameters related to quarantining are switched off.

6.2. **Quarantine Interactions.** This test checks that when people are in quarantine that they only have interactions on their household network and on the random network and that the number of interactions on the random network is that specified. We test 5 distinct parameters sets (Table 10).

6.3. **Quarantine on Symptoms.** This test checks the number of people who go in to quarantine on a particular day when the only reason to be quarantined is self-diagnosis of symptoms.

6.4. **Quarantine Household on Symptoms.** This tests checks that when somebody has started to self-quarantine because they have developed symptoms, that their entire household also enters quarantine (if this option is being used).

6.5. **Trace on Symptoms.** This test checks that when tracing occurs upon symptoms, that everybody who is traced has had an interaction with the index case. At the moment this test only considers tracing/interactions on the day that the index case developed symptoms (shall be extended, but requires interactions on multiple days to be written to file).

6.6. **Lock-down Transmission Rates.** This test checks that the effect of the lock-down on the first day is as expected. It runs the simulation twice for the same seed, with the lock-down being initiated at the final step in one of the runs. The test then checks that the number of transmission events that occur on the work and random networks decrease on lock-down by the same proportion by which the number of interactions is reduced (in the lock-down model). Further, for small changes in the household interaction rate on lock-down

TABLE 9. The parameters tested and values tested in the Distribution of Times in Disease-States

Disease Outcome Proportions	
fraction_asymptomatic_0_9	{0.05,0.15,0.35,0.70,0.70,0.18,0.10}
fraction_asymptomatic_10_19	{0.05,0.15,0.35,0.70,0.70,0.18,0.10}
fraction_asymptomatic_20_29	{0.05,0.15,0.35,0.60,0.40,0.18,0.10}
fraction_asymptomatic_30_39	{0.05,0.15,0.35,0.50,0.40,0.18,0.10}
fraction_asymptomatic_40_49	{0.05,0.15,0.35,0.40,0.40,0.18,0.10}
fraction_asymptomatic_50_59	{0.05,0.15,0.15,0.30,0.40,0.18,0.10}
fraction_asymptomatic_60_69	{0.05,0.15,0.15,0.20,0.30,0.18,0.10}
fraction_asymptomatic_70_79	{0.05,0.15,0.15,0.10,0.20,0.18,0.10}
fraction_asymptomatic_80	{0.05,0.15,0.15,0.10,0.20,0.18,0.10}
mild_fraction_0_9	{0.0,0.0,0.0,0.0,0.0,0.79,0.1}
mild_fraction_10_19	{0.0,0.0,0.0,0.0,0.0,0.79,0.1}
mild_fraction_20_29	{0.0,0.0,0.0,0.0,0.0,0.73,0.1}
mild_fraction_30_39	{0.0,0.0,0.0,0.0,0.0,0.68,0.1}
mild_fraction_40_49	{0.0,0.0,0.0,0.0,0.0,0.65,0.1}
mild_fraction_50_59	{0.0,0.0,0.0,0.0,0.0,0.59,0.1}
mild_fraction_60_69	{0.0,0.0,0.0,0.0,0.0,0.53,0.1}
mild_fraction_70_79	{0.0,0.0,0.0,0.0,0.0,0.41,0.1}
mild_fraction_80_	{0.0,0.0,0.0,0.0,0.0,0.27,0.1}
hospitalised_fraction_0_9	{0.05,0.05,0.05,0.02,0.2,0.03,0.20}
hospitalised_fraction_10_19	{0.1,0.1,0.05,0.02,0.2,0,08,0.20}
hospitalised_fraction_20_29	{0.1,0.1,0.05,0.02,0.2,0,11,0.20}
hospitalised_fraction_30_39	{0.2,0.2,0.2,0.1,0.2,0.19,0.20}
hospitalised_fraction_40_49	{0.2,0.2,0.2,0.15,0.2,0.24,0.20}
hospitalised_fraction_50_59	{0.2,0.2,0.2,0.2,0.2,0.36,0.20}
hospitalised_fraction_60_69	{0.3,0.3,0.3,0.25,0.2,0.46,0.20}
hospitalised_fraction_70_79	{0.3,0.3,0.8,0.3,0.2,0.48,0.20}
hospitalised_fraction_80_	{0.5,0.5,0.9,0.5,0.2,0.41,0.20}
critical_fraction_0_9	{0.05,0.05,0.05,0.02,0.2,0.05,0.20}
critical_fraction_10_19	{0.1,0.1,0.05,0.02,0.2,0.05,0.20}
critical_fraction_20_29	{0.1,0.1,0.05,0.02,0.2,0.05,0.20}
critical_fraction_30_39	{0.2,0.2,0.2,0.1,0.2,0.05,0.20}
critical_fraction_40_49	{0.2,0.2,0.2,0.15,0.2,0.063,0.20}
critical_fraction_50_59	{0.2,0.2,0.2,0.2,0.2,0.122,0.20}
critical_fraction_60_69	{0.3,0.3,0.3,0.25,0.2,0.274,0.20}
critical_fraction_70_79	{0.3,0.3,0.8,0.3,0.2,0.432,0.20}
critical_fraction_80_	{0.5,0.5,0.9,0.5,0.2,0.709,0.20}
fatality_fraction_0_9	{0.05,0.05,0.05,0.02,0.2,0.33,0.20}
fatality_fraction_10_19	{0.1,0.1,0.05,0.02,0.2,0.25,0.20}
fatality_fraction_20_29	{0.1,0.1,0.05,0.02,0.2,0.5,0.20}
fatality_fraction_30_39	{0.2,0.2,0.2,0.1,0.2,0.5,0.20}
fatality_fraction_40_49	{0.2,0.2,0.2,0.15,0.2,0.5,0.20}
fatality_fraction_50_59	{0.2,0.2,0.2,0.2,0.2,0.69,0.20}
fatality_fraction_60_69	{0.3,0.3,0.3,0.25,0.2,0.65,0.20}
fatality_fraction_70_79	{0.3,0.3,0.8,0.3,0.2,0.88,0.20}
fatality_fraction_80_	{0.5,0.5,0.9,0.5,0.2,1,0.20}

Quarantine Interactions	
`quarantined_daily_interactions`	{0,1,2,3}
`self_quarantine_fraction`	{1.0,0.75,0.5,0.25}
`seasonal_flu_rate`	{0,0.0005,0.001,0.005}

it checks the number of transmissions in households increases in line with this (note that due to saturation effects this is only true for small changes).

6.7. **App Users.** This test checks that the proportion of app users by age is the same as that specified by the parameters. It further checks that in the absence of asking household members to quarantine as well, that all contact tracing originates from somebody with the app and only ask people to quarantine who have the app.

7. MONOTONICITY TESTS

7.1. **Monotonicity of transmissions within networks.** For each type of network, changes in the relative transmission rate for the network should lead to corresponding changes in the proportion of transmissions within that network. We check whether a change of a relative transmission rate within a network, with fixed relative transmission rates of the other networks, results in corresponding change in a proportion of transmissions within that network. For all the sets tested see Table 11. The test compares the proportions of transmissions that occur in a network from outputs of the consecutive runs for each set. For example, the first set `relative_transmission_household` = $\{0, 0.5, 1, 1.5, 2, 10, 100\}$, `relative_transmission_workplace` = $\{1\}$, `relative_transmission_random` = $\{1\}$ requires a comparison of household transmissions proportion when the subset is $\{0, 1, 1\}$ with when it is $\{0.5, 1, 1\}$, and so on.

TABLE 11. The parameters and values tested for the monotonicity of the network transmissions

parameter	values tested
`relative_transmission_household`	{0, 0.5, 1, 1.5, 2, 10, 100}, {1 (fixed)}, {1 (fixed)}, {1 (fixed)}
`relative_transmission_workplace`	{1 (fixed)}, {0, 0.5, 1, 1.5, 2, 10, 100}, {1 (fixed)}, {1.1, 1, 0, 0.1, 0.1, 0.1, 0.3}
`relative_transmission_random`	{1 (fixed)}, {1 (fixed)}, {0, 0.5, 1, 1.5, 2, 10, 100}, {1 (fixed)}

7.2. **Inverse proportionality of total infections and fraction asymptomatic.** An increase in fraction of asymptomatic infections should decrease the overall number of infections (and *vice versa*) due to the reduced infectiousness of asymptomatically infected individuals. We compare the relation of total infections from consecutive runs with the relation of parameter values fed into the consecutive runs, and check whether the inverse proportionality holds. Tested values of parameters are listed in Table 12.

TABLE 12. The parameters and values tested for the inverse proportionality of the total infections and fraction of asymptomatic infections

parameter	values tested
fraction_asymptomatic_0_9	{0, 0.2, 0.5, 0.5, 1, 0.1}
fraction_asymptomatic_10_19	{0, 0.2, 0.5, 0.5, 1, 0.1}
fraction_asymptomatic_20_29	{0, 0.2, 0.5, 0.5, 1, 0.1}
fraction_asymptomatic_30_39	{0, 0.2, 0.5, 0.5, 1, 0.1}
fraction_asymptomatic_40_49	{0, 0.2, 0.5, 0.5, 1, 0.1}
fraction_asymptomatic_50_59	{0, 0.2, 0.5, 0.5, 1, 0.1}
fraction_asymptomatic_60_69	{0, 0.2, 0.5, 0.5, 1, 0.1}
fraction_asymptomatic_70_79	{0, 0.2, 0.5, 0.5, 1, 0.1}
fraction_asymptomatic_80	{0, 0.2, 0.5, 0.5, 1, 0.1}

7.3. **Proportionality of the total infections and the asymptomatic infectiousness factor.** Increasing the factor of infectiousness of asymptomatic people should increase the overall number of infections (and *vice versa*). We compare the relation of total infections from consecutive runs with the relation of parameter values fed into the consecutive runs, and check whether the proportionality holds. Tested values of parameters are listed in Table 13.

TABLE 13. The parameter and its values tested for the proportionality of the total infections and the asymptomatic infectiousness factor

parameter	values tested
asymptomatic_infectious_factor	{0.1, 0.25, 0.5, 0.5, 1, 0.1}

7.4. **Proportionality of the total infections and the mild infectiousness factor.** Increasing the factor of infectiousness of mildly symptomatic people should increase the overall number of infections, and *vice versa*, decreasing the factor should lead to a decay in number of mildly symptomatic individuals. We compare the relation of total infections from consecutive runs with the relation of parameter values fed into the consecutive runs, and check whether the proportionality holds. Tested values of parameters are listed in Table 14.

TABLE 14. The parameter and its values tested for the proportionality of the total infections and the mild infectiousness factor. The proportions of mildly symptomatic individuals are kept to be 0.8 for all age groups.

parameter	values tested
mild_infectious_factor	{0.1, 0.25, 0.5, 0.5, 1, 0.1}

7.5. **Proportionality of the infections within each age group and the relative susceptibility.** Increasing the relative susceptibility of an age group should increase the number of infections within that age group (and decreasing the former should lead to a decay in the latter). We compare the relation of infections from consecutive runs with the relation of parameter values fed into the consecutive runs, and check whether the proportionality holds. Tested values of parameters are listed in Table 15.

TABLE 15. The parameters and their values tested for the proportionality of the total infections and the relative susceptibility. Here X stands for $\{0, 0.5, 1, 1.5, 2\}$ and x stands for $\{1, 1, 1, 1, 1\}$.

parameter	values tested
relative_susceptibility_0_9	$X, x, x, x, x, x, x, x, x$
relative_susceptibility_10_19	$x, X, x, x, x, x, x, x, x$
relative_susceptibility_20_29	$x, x, X, x, x, x, x, x, x$
relative_susceptibility_30_39	$x, x, x, X, x, x, x, x, x$
relative_susceptibility_40_49	$x, x, x, x, X, x, x, x, x$
relative_susceptibility_50_59	$x, x, x, x, x, X, x, x, x$
relative_susceptibility_60_69	$x, x, x, x, x, x, X, x, x$
relative_susceptibility_70_79	$x, x, x, x, x, x, x, X, x$
relative_susceptibility_80	$x, x, x, x, x, x, x, x, X$

8. TESTING INFRASTRUCTURE

The test are implemented in Python using pytest.