STATISTICAL METHODS IN FINANCE, PROBLEM SHEET 6
MSC IN MATHEMATICS AND FINANCE, 2023-2024

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Exercise 1 (Confidence interval)**.** Let $X_1, \ldots, X_n$ denote a random sample from a Gaussian random distribution with unknown mean μ and variance σ^2. Assume that observations yield $\sum_{i=1}^n x_i = b$.

(i) For any $\alpha \in (0, 1)$ determine the confidence interval for μ with $(1 - \alpha)\%$ confidence level.

(ii) Taking $\alpha = 5\%$, $n = 20$ and $b = 200$, plot the confidence interval as a function of σ. What are the values for $\sigma = 25\%$?

Exercise 2 (p-value, Lindley (1993))**.** Imagine an experiment consisting of a sequence of iid Bernoulli trials (X_n) with success probability p. However, you are only told the experiment resulted in the sequence 111110, where 0 denotes "failure" and 1 denotes "success". Say the null hypothesis is $p = \frac{1}{2}$.

(i) Let us assume $n = 6$ was fixed in advance. Calculate the p-value associated with the sequence.

(ii) Now instead we assume trials are repeated until failure under the same null hypothesis. What is the p-value in this case?

(iii) What can you conclude for tests in (i) and (ii) at significance levels $\alpha = 0.15$ and $\alpha = 0.05$?

Hint: Think of what the model for the entire sequence should be, given the assumptions.

Exercise 3 (Neyman-Pearson)**.** Consider $\mathcal{F} = \{\mathcal{N}(\mu, \theta^2), \theta > 0\}$, with $\mu \in \mathbb{R}$ known, and the hypotheses $\Theta_0 = (0, \sigma_0]$ and $\Theta_1 = (\sigma_0, +\infty)$, for some $\sigma_0 > 0$. Analyse the test defined by

$$\mathcal{R} := \left\{ \frac{\mathcal{L}_\theta(\mathcal{X}_n)}{\mathcal{L}_{\sigma_0}(\mathcal{X}_n)} > c \right\},$$

for some constant $c > 0$ to be determined, where $\mathcal{L}$ denote as usual the likelihood function.

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Exercise 4 (Hypothesis testing)**.** Consider an iid sample $X_1, \ldots, X_n$ distributed as an Exponential random variable with parameter $\lambda > 0$, and consider the test

$$\mathcal{H}_0 : \lambda = \lambda_0 \qquad \text{vs} \qquad \mathcal{H}_1 : \lambda = \lambda_1,$$

for some $\lambda_0, \lambda_1 > 0$ and some level $\alpha \in (0, 1)$.

(1) Determine the distribution of $\sum_{i=1}^n X_i$.
(2) Determine the joint distribution of $(X_1, \ldots, X_n)$.
(3) Using Neyman-Pearson's lemma, determine the optimal level c^*.

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