STATISTICAL METHODS IN FINANCE, PROBLEM SHEET 8
MSC IN MATHEMATICS AND FINANCE, 2023-2024

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Exercise 1 (Eigenvectors of covariance matrix)**.** Prove that the eigenvectors of a symmetric matrix are orthogonal, if their eigenvalues are different.

Exercise 2 (PCA ellipsoid)**.** Let $\mathbf{\Sigma}$ be symmetric positive definite. Consider the family of p-dimensional ellipsoids

$$\mathbf{x}^\top \mathbf{\Sigma}^{-1} \mathbf{x} = \text{const.}$$

Show that the principal components define principal axes of these ellipsoids.

Exercise 3 (Portfolio PCA)**.**

(i) Write a Python script that fetches historical daily stock price data from Yahoo finance for
`ticker_list = ['IBM', 'MSFT', 'FB', 'T', 'AMZN', 'AAPL', 'BA', 'TSLA', 'AEM', 'GOOGL']`
from `01/01/2013` onwards.

(ii) The goal is to calculate the contribution to the variance of the first three principal components of standard percentage returns.
- First perform the Eigenvalue decomposition of the covariance matrix
- Make sure $\Sigma \approx \Gamma \Lambda \Gamma^\top$ by calling `numpy.allclose`
- Calculate the contribution to the variance of each of the three components

(iii) Now use `sklearn.decomposition` library to perform the PCA. How do the approaches compare?

(iv) Plot the histogram of contributions and the cumulative contribution of these components

(v) From these principal components construct "statistical risk factors". These give us an idea of how much of the portfolio's returns comes from some *observable* statistical feature.

For fetching the data you can use the yahoo_fin package.

Exercise 4 (Eigenvalues of the correlation matrix)**.** It is often assumed that eigenvalues of correlation matrices lower than

$$\lambda_+ = \sigma^2 \left(1 + \sqrt{\frac{p}{n}}\right)^2$$

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are by a chance, and only the values higher than λ_+ are the significant common factors. Consider for instance a correlation matrix of a year long series (i.e. n =255 trading days) of p =50 stock returns. Which eigenvalues are statistically significant?

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