CS440 MP3 Report

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1. Naive Bayes Classifiers on Digit classification

1.1 Single pixels as features

- Briefly discuss your implementation, especially the choice of the smoothing constant.

  I build a dictionary to store the train data, the key of the dictionary is the classes (digits 0-10, in this problem), and the value of the dictionary is the time of occurrence of each digit for each class. Then I use the train dictionary to predict the digit shown in the test data by choosing the largest value of P(class)*P(Fij|class). The value is 1 if the pixel is foreground and the value is 0 if the pixel is background. For the smoothing constant, I wrote a loop to try each of numbers from 0.1 to 10 with step of 0.1. And I found that when smoothing constant is 0.1, the over-all accuracy is greatest, which is 0.9369369369369369.

- Report classification accuracy for each digit (note: this can be just the diagonal elements on the confusion matrix).

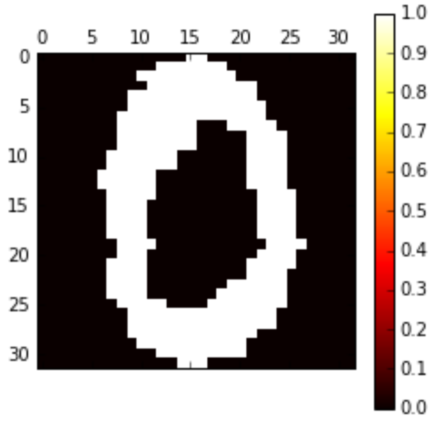
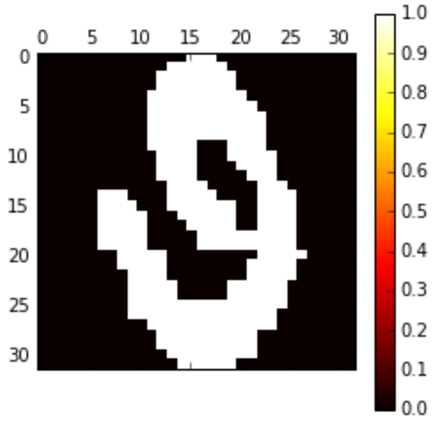
  The classification accuracy of 0 is: 0.97222222
  The classification accuracy of 1 is: 0.93333333
  The classification accuracy of 2 is: 0.85365854
  The classification accuracy of 3 is: 0.90909091
  The classification accuracy of 4 is: 0.88135593
  The classification accuracy of 5 is: 0.93103448
  The classification accuracy of 6 is: 0.97674419
  The classification accuracy of 7 is: 1
  The classification accuracy of 8 is: 1
  The classification accuracy of 9 is: 0.92857143

- Show the confusion matrix.

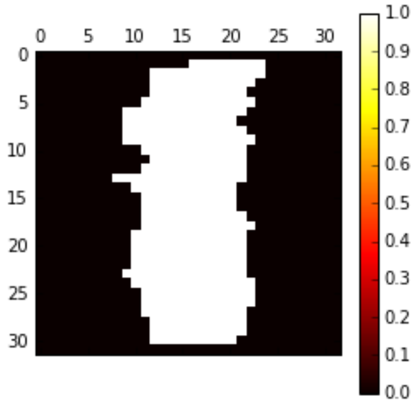
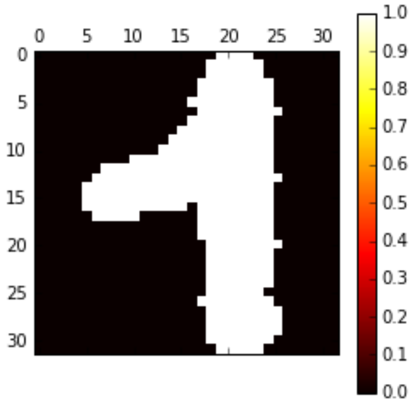
True\guess	0	1	2	3	4	5	6	7	8	9
0	0.972	0	0	0	0.028	0	0	0	0	0
1	0	0.933	0	0	0	0	0	0.022	0.022	0.022
2	0	0	0.854	0	0	0	0	0	0.122	0.024
3	0	0	0	0.909	0	0	0	0.030	0	0.061
4	0	0	0	0	0.881	0	0	0.068	0.051	0
5	0	0	0	0	0	0.931	0	0	0	0.069
6	0	0	0	0	0.023	0	0.977	0	0	0
7	0	0	0	0	0	0	0	1	0	0
8	0	0	0	0	0	0	0	0	1	0
9	0	0	0	0	0	0	0	0.024	0	0.929

- For each digit, show the test tokens from that class that have the highest and lowest posterior probabilities according to your classifier.

For each class, highest tokens are on the left and lowest token is on the right.
White pixels are 1 in the test file and black pixels are 0.
Highest token: lowest token:
0:

1:

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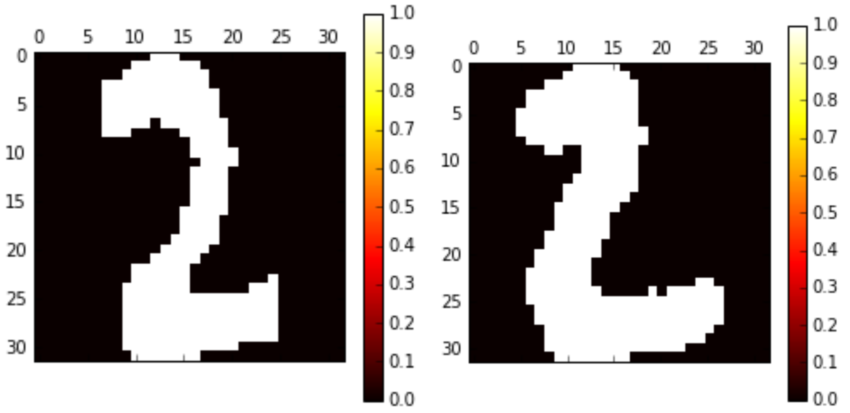
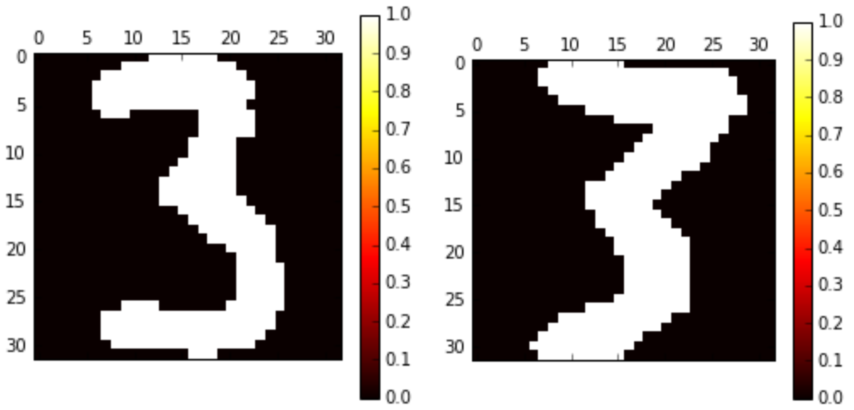

0 5 10 15 20 25 30
0 5 10 15 20 25 30
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.0
0 5 10 15 20 25 30
0 5 10 15 20 25 30
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.0

3:


0 5 10 15 20 25 30
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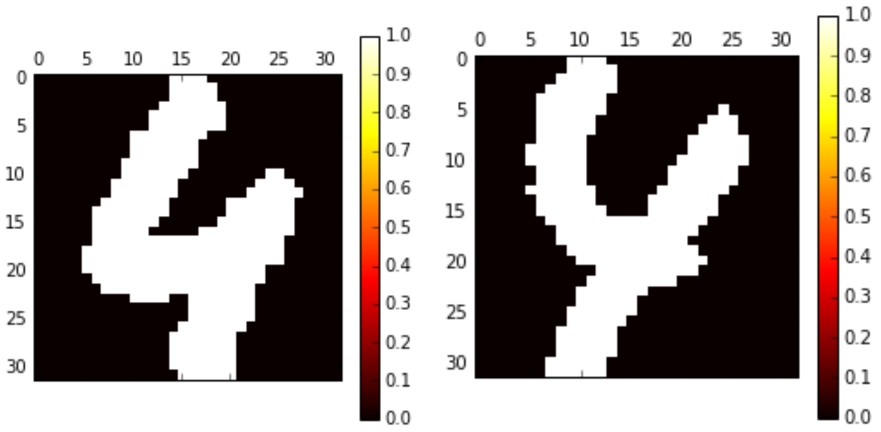
4:


0 5 10 15 20 25 30
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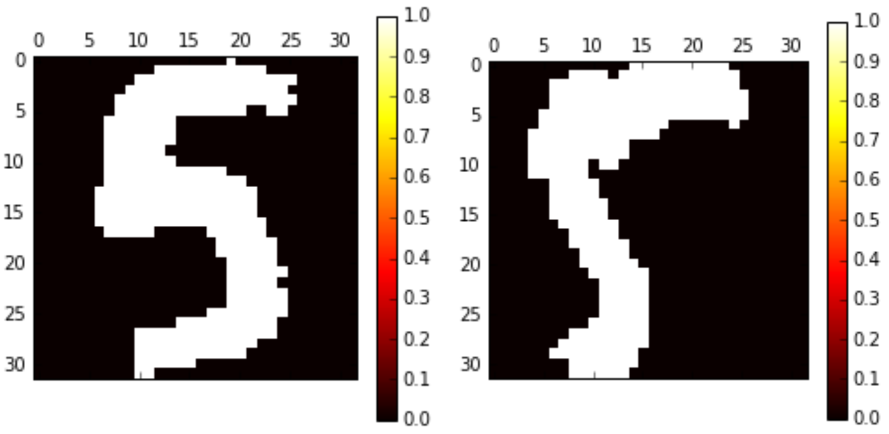
5: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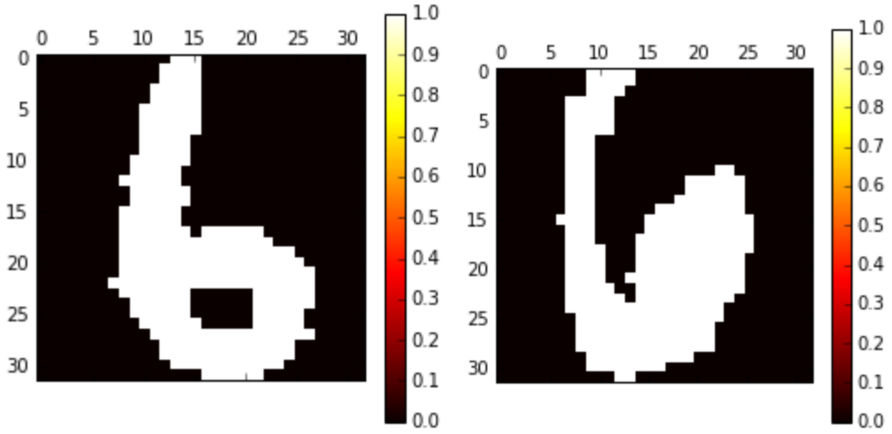
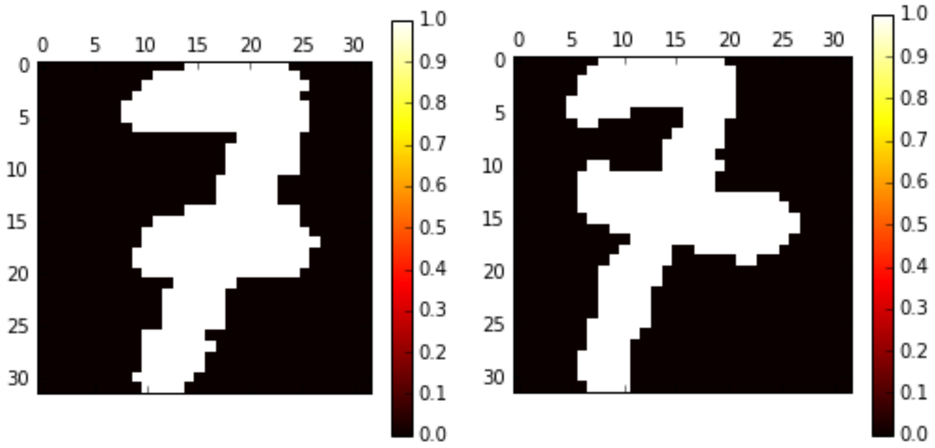
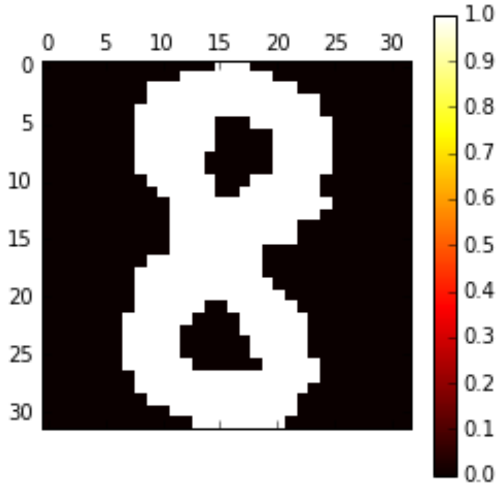
6:


0 5 10 15 20 25 30
0 5 10 15 20 25 30
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.0

7:


0 5 10 15 20 25 30
0 5 10 15 20 25 30
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.0

8: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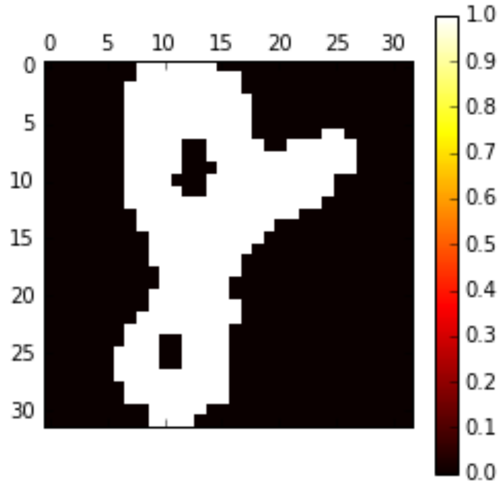
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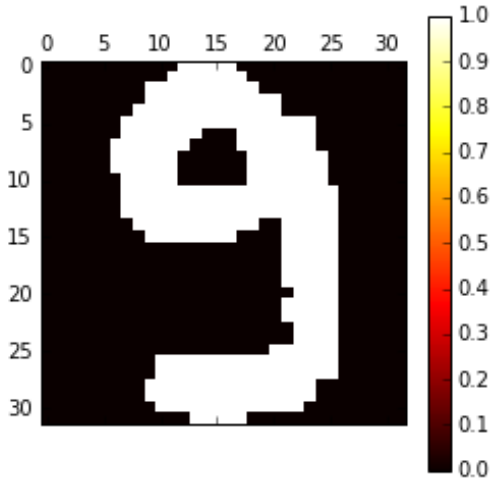
9: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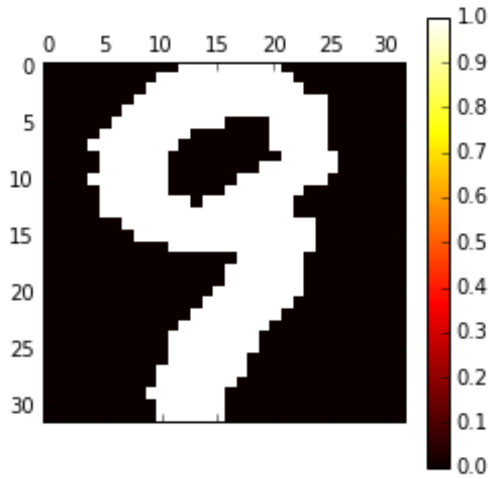
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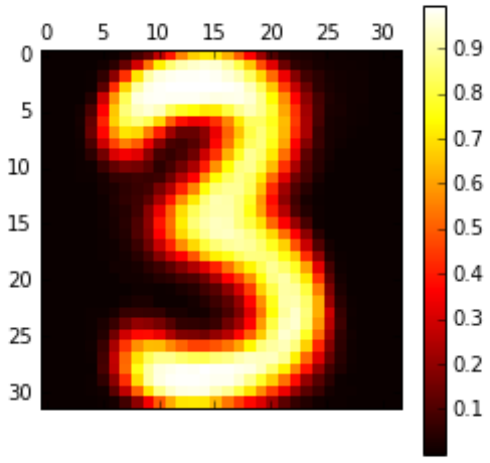
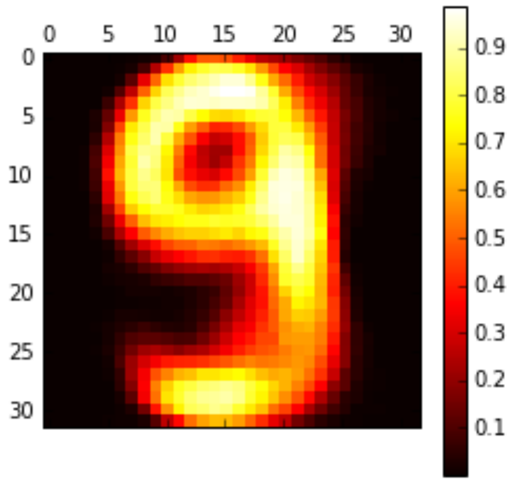
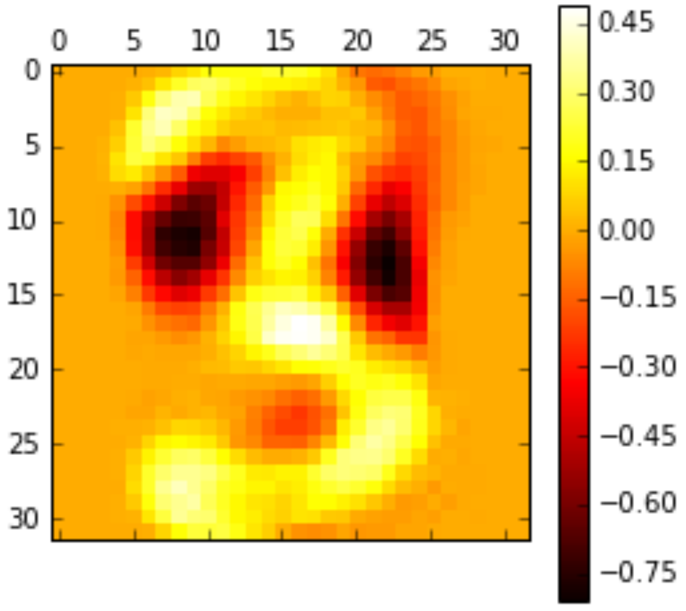
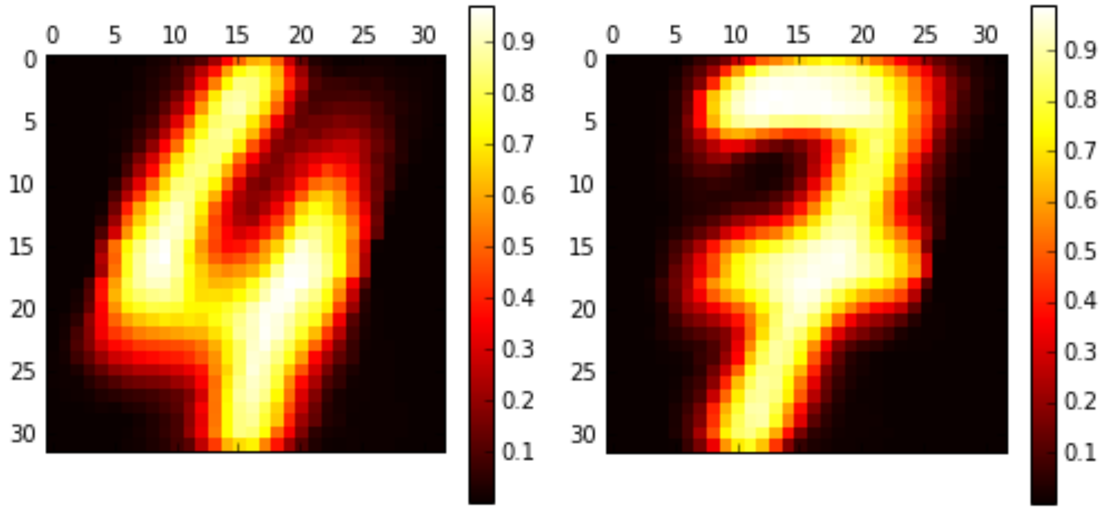
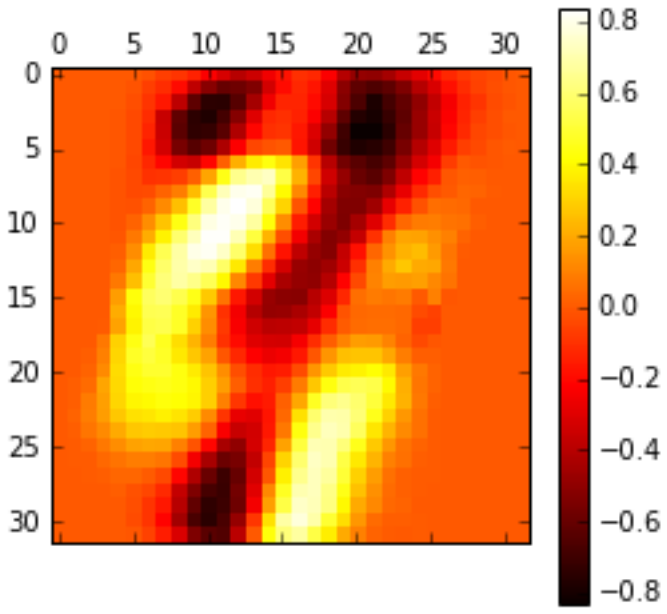
- Take four pairs of digit types that have the highest confusion rates, and for each pair, display feature likelihoods and odds ratio.

3 & 9:

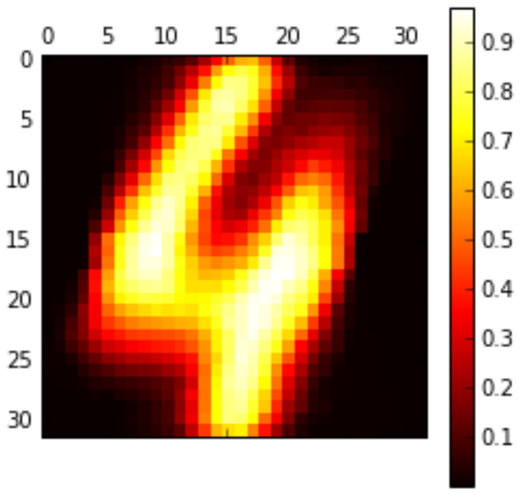




4 & 7:





4 & 8: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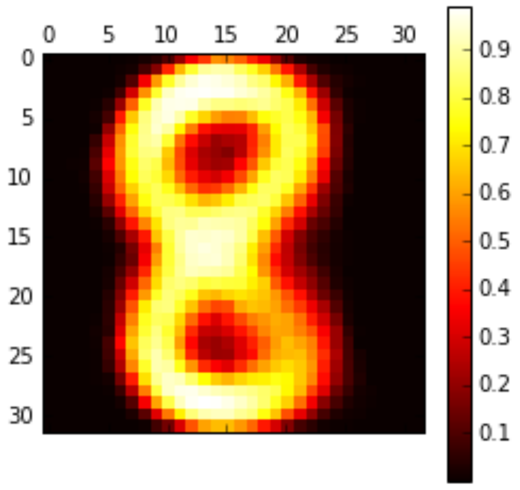
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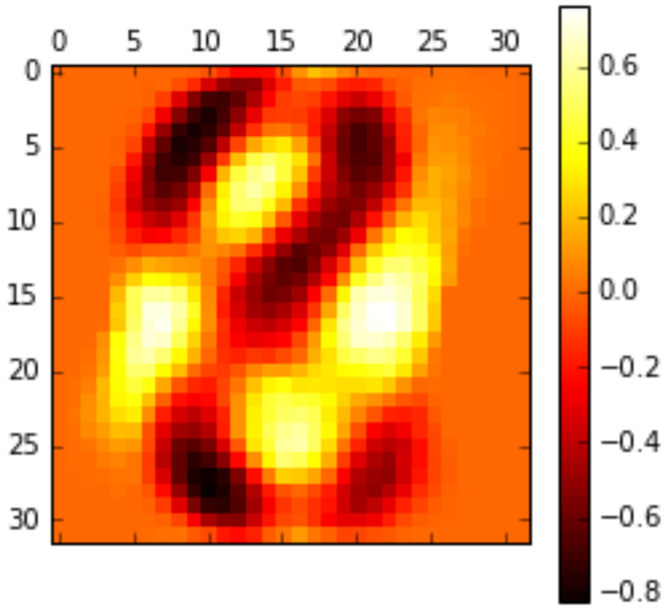
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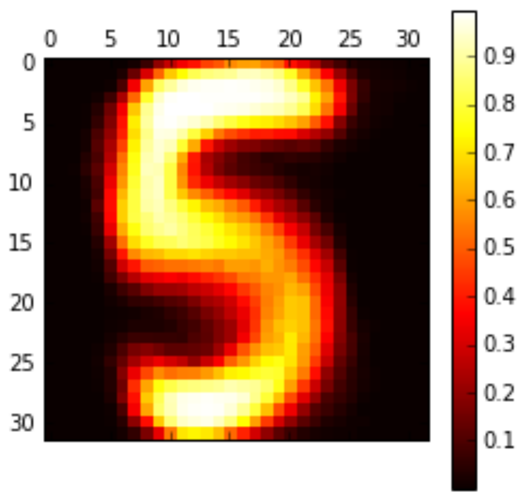
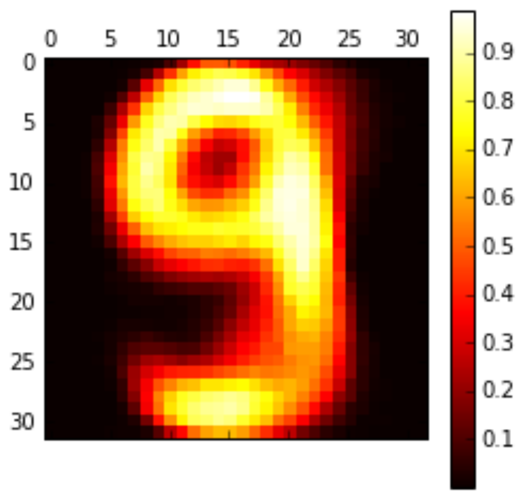
0.0
−0.2
−0.4
−0.6
−0.8

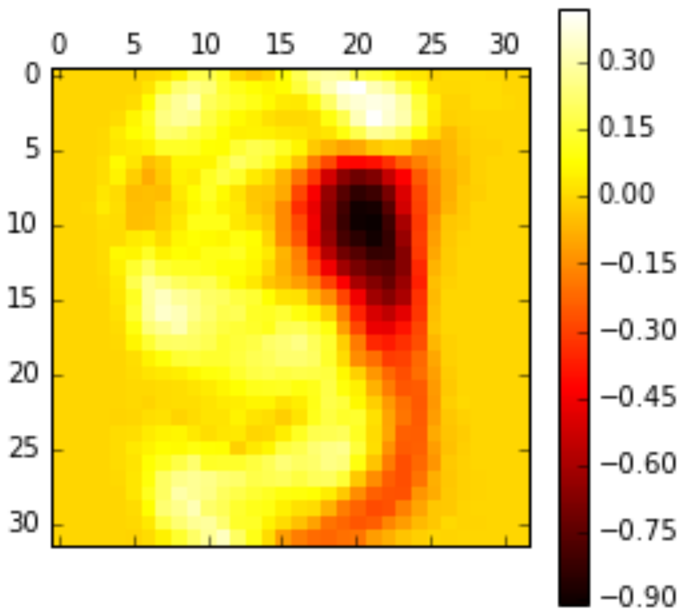
5 & 9:



0 5 10 15 20 25 30
0 5 10 15 20 25 30
0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1



0 5 10 15 20 25 30
0 5 10 15 20 25 30
0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1



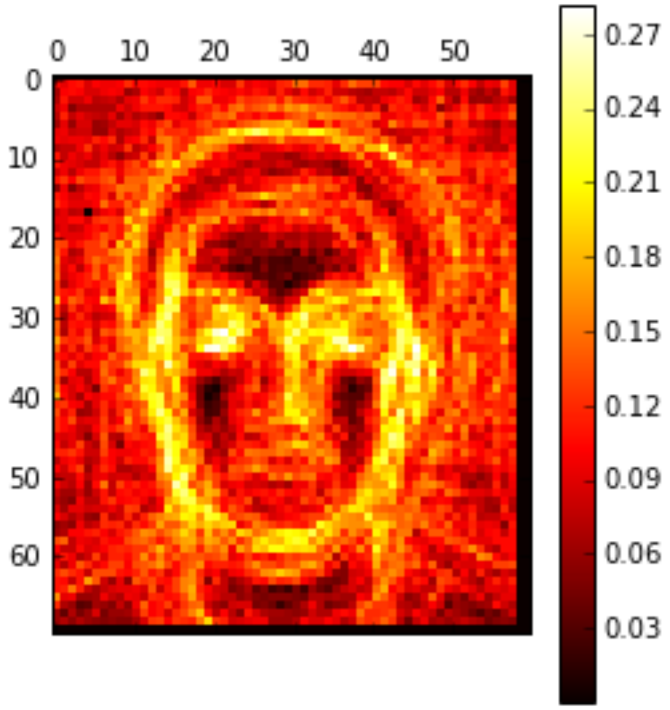
0 5 10 15 20 25 30
0 5 10 15 20 25 30
0.30 0.15 0.00 −0.15 −0.30 −0.45 −0.60 −0.75 −0.90

1.2 Pixel groups as features

- For this part, we implement the Naïve Bayes Classifier Model using the given training data. When testing, we use P(class)*P(Gij|class), where Gij is the pixel group. (G1,1 = (P1,1, P1,2, P2,1, P2,2)). For this one, I also choose 0.1 as the Laplace smoothing constant.
- Accuracies:
  - We find that the accuracy with overlapping pixel is higher than the disjoint ones. But both of them have higher accuracy than single pixel.
- Running time:
  - In case of disjoint square, the Running Time (4*4)<Running Time(4*2)=Running Time(2*4)<Running Time(2*2). The running time decreases as the size of patches getting larger.
  - In case of overlapping square, the Running Time(4*4)>Running Time(3*3)>Running Time(4*2)>Running Time(3*2)>Running Time(2*2). The running time increases as the size of patches getting larger.

1.3 Face classification (Extra Credit)

I applied the Naïve Bayes classifier with single pixels as features (same as part 1.1). I took 0.1 as the smoothing constant, and here is the likelihood maps for the face (when class=1):



I wrote a loop to find the optimized smoothing constant and found that when smoothing constant is 1.1, the accuracy is 90%. The confusion matrix is following:

Real/guess	Non-face	face
Non-face	0.8831	0.1169
face	0.0685	0.9315

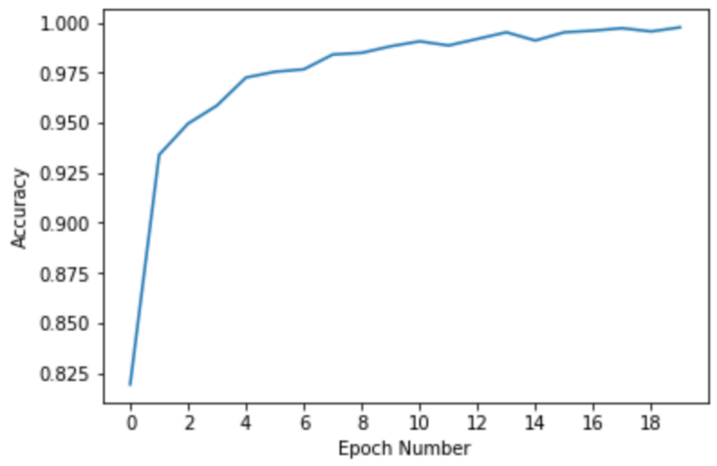
2. Alternative models for Digit classification

2.1 Digit classification with perceptrons

Implementation of perceptrons:

The idea about perceptrons is about weights. So it is really important to build up a weight vector from training data. The first step we did is zero initialize the weight vector to shape(10,1025) since we need leave a space for bias. If the prediction is wrong, then we update the correct weight by doing weight[i] + alpha*features[i], alpha is the learning rate decay function which equals to 1/(1+1*epoch), the learning rate is set to 1 for convenience right now. We also update the wrong weight by doing weight[i] - alpha*features[i] for punishment. After that, we start to do it over 20 epochs by looping through fixed order of data.

After trained by 20 epochs with biased, zero-initialized weight vector, applied learning rate decay function and fixed order of training examples, we got the training curve of over the epochs:



Then we applied this perceptron to the tests, and get the following accuracy on test set:

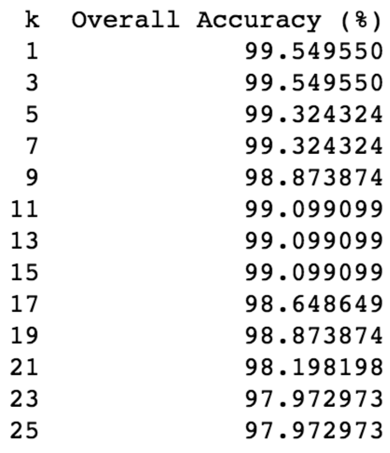
```
Accurate Rate is: 0.9617117117117117
```

Confusion matrix:

N = 444 Predict/Actual	0	1	2	3	4	5	6	7	8	9
0	36	0	1	0	0	0	1	0	0	0
1	0	43	0	0	1	0	0	0	1	0
2	0	0	37	0	0	0	0	0	0	1
3	0	0	0	33	0	0	0	0	0	2
4	0	0	0	0	56	0	0	0	0	0
5	0	0	0	0	0	58	0	0	1	0
6	0	0	0	0	0	0	42	0	0	0
7	0	2	0	0	0	0	0	47	0	1
8	0	0	3	0	2	0	0	0	38	1
9	0	0	0	0	0	0	0	0	0	37

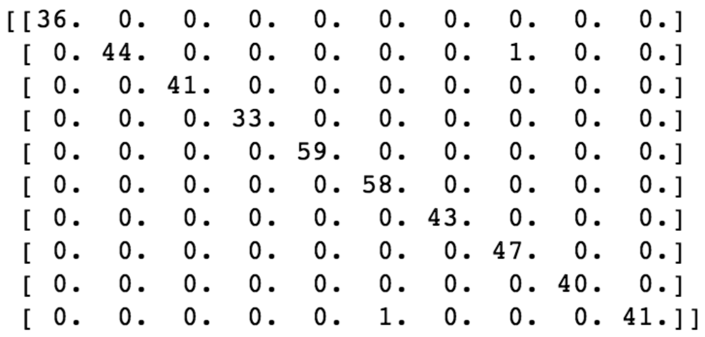
2.2 Digit classification with nearest neighbor

The team's choice of similarity function is by evaluating the Euclidean distance between any two vectors. It yields the same output as the Manhattan distance, but is more efficient and thus quick to compute. Based on the team's research, Euclidean distance is also the mostly widely used similarity function in the implementation of nearest neighbor classification. The overall accuracy decreases as n increases, as shown by the following figure. Based on the team's speculation, a large k introduces bias, which may explain the relatively poor performance of large k's. However, small k's are prone to outliers in the training datasets. Luckily, the good performance of the model when k is small indicates the overall quality of the dataset is very good.



```
 k  Overall Accuracy (%)
 1             99.549550
 3             99.549550
 5             99.324324
 7             99.324324
 9             98.873874
11             99.099099
13             99.099099
15             99.099099
17             98.648649
19             98.873874
21             98.198198
23             97.972973
25             97.972973
```

Since both k=1 and k=3 yields the highest accuracies and a model with k=1 is more efficient. The team constructs the confusion matrix with k=1. It is shown below.



```
[[36.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0. 44.  0.  0.  0.  0.  0.  1.  0.  0.]
 [ 0.  0. 41.  0.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0. 33.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0. 59.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0. 58.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0. 43.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0. 47.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0. 40.  0.]
 [ 0.  0.  0.  0.  0.  1.  0.  0.  0. 41.]]
```

To account for the fluctuation in the running time of individual queries, the team takes the total running time of all 444 pieces of data in the test sets and divide it by 444. The running time for a single query is about 196 µs. It is very efficient, since we utilized Euclidean distance as our similarity function. To further improve its performance, we may consider eliminating choices of potential labels if the distance between the query in the test dataset and several points in the training set far exceeds the distance with the current nearest neighbor. This process of elimination saves running costs from evaluating labels of tiny possibilities.

The nearest-neighbor accuracy is almost 100%, while the Naïve Bayes is about 93% and the perceptron is about 96% (see above). It is obvious that nearest-neighbor algorithm has much better accuracy than Naïve Bayes algorithm and perceptron learning.

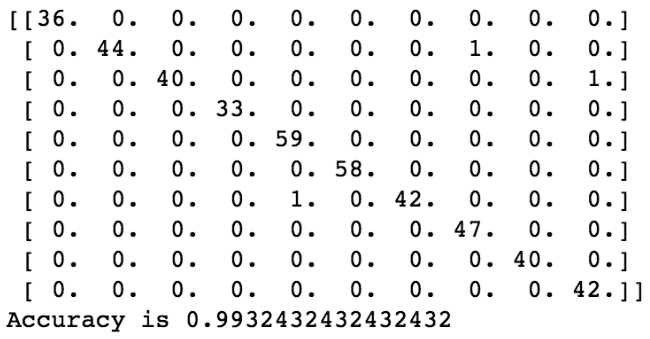
2.3 Extra Credit

2.3.3 Other learning algorithms

The team referenced the svm classifier (by scikit-learn) tutorial which could be found at the following URL.

http://scikit-learn.org/stable/auto_examples/classification/plot_digits_classification.html

The team used it to classify the digits supplied by this assignment. The svm model with default parameters yields an accuracy of less than 97% accuracy on the test dataset. However, after experimenting with different gamma values, the team found that a gamma=0.01 yields only 3 errors out of the 444 data, which is an accuracy of 99.32%. The confusion matrix is shown in the following figure.



```
[[36.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
 [ 0. 44.  0.  0.  0.  0.  0.  1.  0.  0.]
 [ 0.  0. 40.  0.  0.  0.  0.  0.  0.  1.]
 [ 0.  0.  0. 33.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0. 59.  0.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  0. 58.  0.  0.  0.  0.]
 [ 0.  0.  0.  0.  1.  0. 42.  0.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0. 47.  0.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0. 40.  0.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  0. 42.]]
Accuracy is 0.9932432432432432
```