ECE183DA
Problem Set Ø
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1. Linear Algebra

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 3 & 6 & 9 & 5 \\ 2 & 4 & 6 & 9 \end{bmatrix}$$

a)

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 3 & 6 & 9 & 5 \\ 2 & 4 & 6 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{rank} = 2}$

b) $\boxed{b_3 = 7}$

c)

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 1 \\ 3 & 6 & 9 & 5 & 2 \\ 2 & 4 & 6 & 9 & 7 \end{array}\right] \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 5 & 5 \end{bmatrix} \Rightarrow x_4 = 1$$

$$x_1 + 2x_2 + 3x_3 = -1$$

Solution: $\begin{bmatrix} -1 - 2x_2 - 3x_3 \\ x_2 \\ x_3 \\ 1 \end{bmatrix}$

$\Rightarrow$ I was very rusty and needed to review but overall it was not too difficult

2. Differential Equations

$$\dot{y}(t) = z(t) + u(t) \qquad (1)$$

$$\dot{z}(t) = 2y(t) + z(t) + 3u(t) \qquad (2)$$

a) Write in state space

Laplace transform:

$$sY(s) = Z(s) + U(s)$$

$$sZ(s) = 2Y(s) + Z(s) + 3U(s)$$

$$\hookrightarrow (s-1)Z(s) = 2Y(s) + 3U(s)$$

$$Z(s) = \frac{2}{s-1}Y(s) + \frac{3}{s-1}U(s)$$

$$sY(s) = \frac{2}{s-1}Y(s) + \frac{3}{s-1}U(s) + U(s)$$

$$s(s-1)Y(s) = 2Y(s) + 3U(s) + (s-1)U(s)$$

$$(s^2 - s - 2)Y(s) = (s+2)U(s)$$

let $x_1 = y(t)$ $\quad x_2 = z(t)$

$$\left.\begin{aligned} \dot{x}_1 &= x_2 + u \\ \dot{x}_2 &= 2x_1 + x_2 + 3u \end{aligned}\right\}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 3 \end{bmatrix}}_{B} u$$

b) Routh's test:

$$M(s) = \frac{s+2}{s^2 - s - 2} \leftarrow A(s)$$

$$\left.\begin{array}{c|cc} 2 & 1 & -2 \\ 1 & -1 & 0 \\ 0 & -2 & \end{array}\right\}$$ system is <u>unstable</u> due to sign changes in first column of Routh's table

Furthermore, the denominator polynomial of

$s^2 - s - 2$

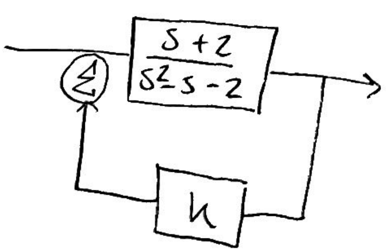
$$\frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} \begin{array}{l} \nearrow 1 \\ \searrow -2 \end{array}$$

poles are located at $s = -1$ and $s = 2$

The $s = 2$ pole causes the system to be unstable

c) before: $\frac{Y(s)}{U(s)} = \frac{s+2}{s^2 - s - 2} \Rightarrow U(s) \rightarrow \boxed{\frac{s+2}{s^2-s-2}} \rightarrow Y(s)$

now $u(t) = k\,y(t) \Rightarrow U(s) = k\,Y(s)$



Routh on next page ↑

$$\frac{\frac{s+2}{s^2-s-2}}{1 - \frac{k(s+2)}{s^2-s-2}} = \frac{1}{\frac{s^2-s-2}{s+2} - k} = \frac{1}{\frac{-ks-2k+s^2-s-2}{s+2}} = \frac{s+2}{s^2 - s(k+1) - 2(k+1)}$$

$$\frac{s+2}{s^2 - s(k+1) - 2(k+1)}$$

$$\det\begin{vmatrix} a_0 & a_2 \\ b_0 & b_1 \end{vmatrix}$$

$$\begin{array}{c|cc} 2 & 1 & -2k-2 \\ 1 & -k-1 & 0 \\ 0 & \frac{2k^2-2}{-k-1} & \end{array}$$

$$= -(2k-2)(-k-1)$$
$$= -[-2k^2+2k-2k+2]$$
$$= 2k^2-2.$$

$\Rightarrow$ $-k-1 > 0 \Rightarrow k < -1$

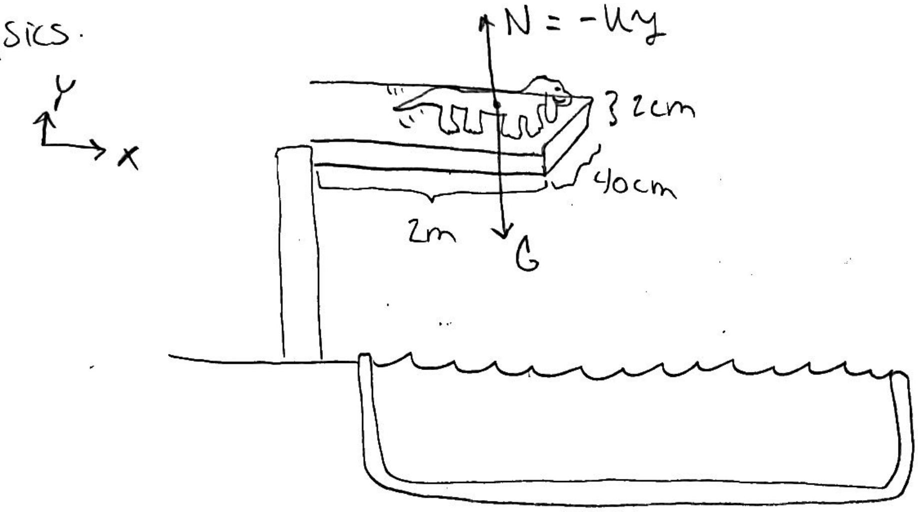
$$\frac{2k^2-2}{-k-1} > 0 \Rightarrow 2k^2-2 < 0$$
$$k^2 < 1 \Rightarrow k < \pm 1$$

$\therefore$ $\boxed{k < -1}$

$\Rightarrow$ This problem was conceptually diffizult so I had to review a lot from ECE141 (controls)

3 Physics

a)



b) $y = -10cm$

Conservation of energy $\Rightarrow$ potential turns into kinetic

$\frac{1}{2}ky^2 = mgh \Rightarrow h = \frac{ky^2}{2mg}$

$\boxed{h = 0.5m \text{ above the diving board}}$

$g = 10 \, m/s^2$ $\qquad d = \frac{1}{2}at^2 + \cancel{v_0 t}^{\,0}$

$d = 5t^2 \Rightarrow 0.5m = 5t^2$

$0.1 = t^2 \Rightarrow t = 0.316s$

$v(t) = at + \cancel{v_0}^{\,0} t \Rightarrow v(0.316) = 10(0.316)$

$\boxed{v(0.316) = 3.16 \, m/s}$

c) $t' = 2.02\,cm$

⇒ Pretty simple physics problem

→ Easy

$$\frac{1}{2} k y^2 = mgh'$$

$$h' = \frac{k y^2}{2mg} \Rightarrow \boxed{h' = 0.515\,m}$$

4. <u>Probability</u>

Flip 8 times and get T T T H T T H T

a) $\boxed{\dfrac{1}{2^8}}$

b) $\dfrac{1}{4^2} \times \left(\dfrac{3}{4}\right)^6 = \dfrac{3^6}{4^8} = \boxed{\dfrac{3^6}{2^{16}}}$

c) $\boxed{\dfrac{1}{4}}$

d) P(coin is biased | sequence)

$$= \frac{P(\text{sequence} \mid \text{coin is biased}) \times P(\text{coin is biased})}{P(\text{sequence})}$$

$$\frac{\left(\frac{3}{4}\right)^6 \times \left(\frac{1}{4}\right)^2 \times \frac{1}{4}}{\frac{\left(\frac{3}{4}\right)^6 \times \left(\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^2}{2}} = \boxed{\frac{\frac{3^6}{2^{18}}}{\frac{\frac{3^8}{2^{16}} + \frac{1}{2^8}}{2}}}$$

⇒ Easy probability problem

```python
import numpy as np
import matplotlib.pyplot as plt

def seqSim(biased, numFlips):
    seq = []
    pHeads = 0
    if biased:
        pHeads = 0.25
    else:
        pHeads = 0.75

    pBiased = 0
    flips = 0
    biasProbs = []
    while (numFlips > 0):
        r = np.random.rand()
        if r <= pHeads:
            seq.append("H")
        else:
            seq.append("T")
        flips += 1

        numH = seq.count("H")
        numT = seq.count("T")

        pBiased = (np.power(0.75, numT) * np.power(0.25, numH) * 0.25)/((np.power(0.75, numT) * np.power(0.25, numH) + np.power(0.5,
flips))/2)
        biasProbs.append(pBiased)
        numFlips -= 1

    return(biasProbs)


fairSim1 = seqSim(False, 100)
fairSim2 = seqSim(False, 100)
fairSim3 = seqSim(False, 100)
fairSim4 = seqSim(False, 100)
fairSim5 = seqSim(False, 100)

biasSim1 = seqSim(True, 100)
biasSim2 = seqSim(True, 100)
biasSim3 = seqSim(True, 100)
biasSim4 = seqSim(True, 100)
biasSim5 = seqSim(True, 100)

x = range(100)
plt.figure()
plt.plot(x, fairSim1)
plt.plot(x, fairSim2)
plt.plot(x, fairSim3)
plt.plot(x, fairSim4)
plt.plot(x, fairSim5)
plt.xlabel('Number of Flips')
plt.ylabel('Probability of Biased Coin Picked')
plt.title('Probability of Biased Coin Picked using a Fair Coin')
plt.show()

plt.figure()
plt.plot(x, biasSim1)
plt.plot(x, biasSim2)
plt.plot(x, biasSim3)
plt.plot(x, biasSim4)
plt.plot(x, biasSim5)
plt.xlabel('Number of Flips')
plt.ylabel('Probability of Biased Coin Picked')
plt.title('Probability of Biased Coin Picked using a Biased Coin')
plt.show()
```