

ECE183DA
Problem Set 0
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1. Linear Algebra

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 3 & 6 & 9 & 5 \\ 2 & 4 & 6 & 9 \end{bmatrix}$$

$$a) A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 3 & 6 & 9 & 5 \\ 2 & 4 & 6 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{rank} = 2$$

$$b) b_3 = 7$$

$$c) \left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 1 \\ 3 & 6 & 9 & 5 & 2 \\ 2 & 4 & 6 & 9 & 7 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 1 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 5 & 5 \end{array} \right] \Rightarrow x_4 = 1$$

$$x_1 + 2x_2 + 3x_3 = -1$$

Solution:

$$\begin{bmatrix} -1 - 2x_2 - 3x_3 \\ x_2 \\ x_3 \\ 1 \end{bmatrix}$$

\Rightarrow I was very rusty and needed to review but overall it was not too difficult

2. Differential Equations

$$\dot{y}(t) = z(t) + u(t) \quad (1)$$

$$\dot{z}(t) = 2y(t) + z(t) + 3u(t) \quad (2)$$

a) Write in state space

Laplace transform:

$$sY(s) = Z(s) + U(s)$$

$$sZ(s) = 2Y(s) + Z(s) + 3U(s)$$

$$\hookrightarrow (s-1)Z(s) = 2Y(s) + 3U(s)$$

$$Z(s) = \frac{2}{s-1}Y(s) + \frac{3}{s-1}U(s)$$

$$sY(s) = \frac{2}{s-1}Y(s) + \frac{3}{s-1}U(s) + U(s)$$

$$s(s-1)Y(s) = 2Y(s) + 3U(s) + (s-1)U(s)$$

$$(s^2 - s - 2)Y(s) = (s+2)U(s)$$

let $x_1 = y(t)$ $x_2 = z(t)$

$$\left. \begin{aligned} \dot{x}_1 &= x_2 + u \\ \dot{x}_2 &= 2x_1 + x_2 + 3u \end{aligned} \right\}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}}_A \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} + \underbrace{\begin{bmatrix} 1 \\ 3 \end{bmatrix}}_B u$$

b) Routh's test:

$$U(s) = \frac{s+2}{s^2-s-2} \leftarrow A(s)$$

$$\begin{array}{c|cc} 2 & 1 & -2 \\ 1 & -1 & 0 \\ 0 & -2 & \end{array} \left. \vphantom{\begin{array}{c|cc} 2 & 1 & -2 \\ 1 & -1 & 0 \\ 0 & -2 & \end{array}} \right\} \text{system is unstable due to sign changes in first column of Routh's table}$$

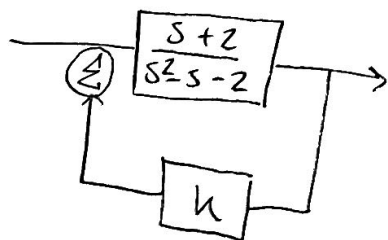
furthermore, the denominator polynomial of $s^2 - s - 2$

$$\frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} \rightarrow \begin{matrix} 1 \\ -2 \end{matrix}$$

poles are located at $s = -1$ and $s = 2$. The $s = 2$ pole causes the system to be unstable

c) before: $\frac{Y(s)}{U(s)} = \frac{s+2}{s^2-s-2} \Rightarrow U(s) \rightarrow \boxed{\frac{s+2}{s^2-s-2}} \rightarrow Y(s)$

now $u(t) = k \cdot y(t) \Rightarrow U(s) = k Y(s)$



Routh on next page
↑

$$\frac{\frac{s+2}{s^2-s-2}}{1 - \frac{k(s+2)}{s^2-s-2}} = \frac{1}{\frac{s^2-s-2}{s+2} - k} = \frac{1}{\frac{-ks+2k+s^2-s-2}{s+2}} = \frac{s+2}{s^2-s(k+1)-2(k+1)}$$

$$s+2$$

$$s^2 - s(k+1) - 2(k+1)$$

$$2 \begin{vmatrix} 1 & -2k-2 \\ -k-1 & 0 \\ 0 & \frac{2k^2-2}{-k-1} \end{vmatrix}$$

$$\det \begin{vmatrix} a_0 & a_2 \\ b_0 & b_2 \end{vmatrix}$$

$$= -(2k-2)(-k-1)$$

$$= -[-2k^2 + 2k - 2k + 2]$$

$$= 2k^2 - 2$$

$$\Rightarrow -k-1 > 0 \Rightarrow k < -1$$

$$\frac{2k^2-2}{-k-1} > 0 \Rightarrow 2k^2-2 < 0$$

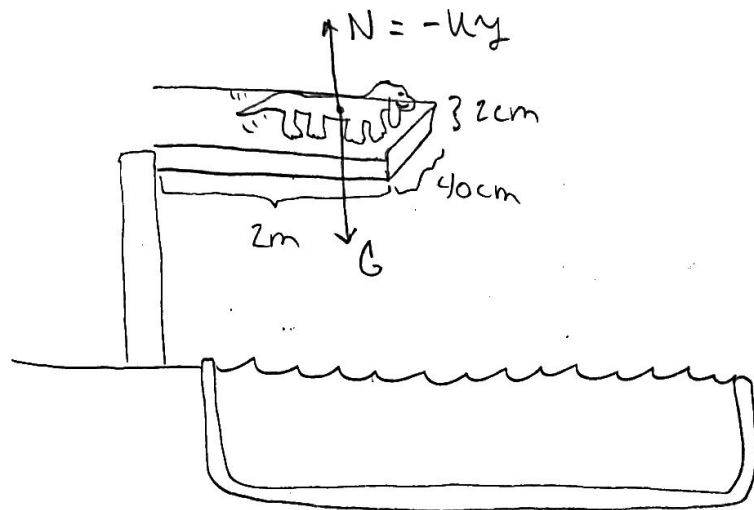
$$k^2 < 1 \Rightarrow k < \pm 1$$

$$\therefore \boxed{k < -1}$$

\Rightarrow This problem was conceptually difficult so I had to review a lot from ECE141 (controls)

3 Physics.

a) 



b) $y = -10\text{cm}$

Conservation of energy \Rightarrow potential turns into kinetic

$$\frac{1}{2}ky^2 = mgh \Rightarrow h = \frac{ky^2}{2mg}$$

$h = 0.5\text{m}$ above the diving board

$$g = 10\text{ m/s}^2$$

$$d = \frac{1}{2}at^2 + v_0t^0$$

$$d = 5t^2 \Rightarrow 0.5\text{m} = 5t^2$$

$$0.1 = t^2 \Rightarrow t = 0.316\text{s}$$

$$v(t) = at + v_0t^0 \Rightarrow v(0.316) = 10(0.316)$$

$$v(0.316) = 3.16\text{ m/s}$$

c) $t' = 2.02 \text{ cm}$

⇒ Pretty simple physics problem

$\frac{1}{2} k y^2 = m g h'$ → Easy

$h' = \frac{k y^2}{2 m g} \Rightarrow \boxed{h' = 0.515 \text{ m}}$

4. Probability

Flip 8 times and get T T T H T T H T

a) $\boxed{\frac{1}{2^8}}$

b) $\frac{1}{4^2} \times \left(\frac{3}{4}\right)^6 = \frac{3^6}{4^8} = \boxed{\frac{3^6}{2^{16}}}$

c) $\boxed{\frac{1}{4}}$

d) $P(\text{Coin is biased} \mid \text{sequence})$

$= \frac{P(\text{sequence} \mid \text{coin is biased}) \times P(\text{coin is biased})}{P(\text{sequence})}$

$\frac{\left(\frac{3}{4}\right)^6 \times \left(\frac{1}{4}\right)^2 \times \frac{1}{4}}{\left(\frac{3}{4}\right)^6 \times \left(\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^2}$

$\frac{1}{2}$

$P(\text{sequence})$

$\boxed{\frac{\frac{3^6}{2^{18}}}{\frac{3^8}{2^{16}} + \frac{1}{2^8}}}$

⇒ Easy probability problem