www.ietdl.org

Published in IET Image Processing
Received on 20th January 2014
Revised on 26th June 2014
Accepted on 31st July 2014
doi: 10.1049/iet-ipr.2014.0347

ISSN 1751-9659

Entropy maximisation histogram modification scheme for image enhancement

Zhao Wei, Huang Lidong, Wang Jun, Sun Zebin

School of Electronic and Information Engineering, Beihang University, Beijing 100191, People's Republic of China
E-mail: zhaowei203@buaa.edu.cn

Abstract: Contrast enhancement plays an important role in image processing applications. The global histogram equalisation (GHE)-based techniques are very popular for their simpleness. In the author's study, the authors originally divide the GHE techniques into two steps, that is, the pixel populations mergence (PPM) step and the grey-levels distribution (GLD) step. In the PPM step, the pixel populations of adjoining grey scales to be mapped to the same grey scale are merged firstly in input histogram. Then, the new grey scales are redistributed according to a corresponding transformation function in the GLD step. This division is meaningful because the entropy of enhanced image is only determined by pixel populations regardless of grey levels. Then, they prove the entropy of enhanced image is reduced because of mergence. Inspired by GHE, they propose a novel entropy maximisation histogram modification scheme, which also consists of PPM and GLD steps. However, the entropy is maximised, that is, the reduction of entropy is minimised under originally presented entropy maximisation rule in their PPM step. In the GLD step, they redistribute the grey scales in the merged histogram using a log-based distribution function to control the enhancement level. Experimental results demonstrate the proposed method is effective.

1 Introduction

The contrast enhancement, as a kind of significant processing technique for both images and videos, can effectively improve the image visual quality for human perception and automatic recognition. Many contrast enhancement techniques have been introduced, among which histogram equalisation (HE) is one of the most widely used method. Since the contrast gain is proportional to the height of the histogram, it tends to over enhance the image contrast if there are high peaks in the histogram, often resulting in a harsh and noisy appearance of the output image [1]. Numerous global HE (GHE) methods have been proposed for limiting the level of enhancement, most of which are obtained by modifications to HE [2–6]. For example, a bi-HE (BHE) was proposed to preserve the brightness. The BHE separates the input image's histogram into two parts based on its mean. The two histograms are then equalised independently [7]. To preserve the brightness and maximise the entropy, Wang and Ye proposed the brightness preserving HE with maximum entropy (BPHEME). The BPHEME obtained the target histogram by solving a constrained optimisation problem, which can maximise the entropy in the discrete case [8]. These techniques usually outperform the basic HE technique. However, they fail to emphasise details of the local regions because they use histogram information of the whole image.

To overcome aforementioned drawback, local HE (LHE)-based methods are developed. A natural extension of GHE is termed as adaptive histogram equalisation, which divides the input image into an array of subimages. Each subimage is histogram-equalised independently, and then the processed subimages are fused together by bi-linear interpolation [9]. The LHE-based methods generally require more computation and they not only highlight details in the image but also enhance noise. Besides traditional histogram-based methods, there are also unconventional approaches to solve the contrast enhancement problem. A histogram modification framework (HMF) proposed in [10] poses contrast enhancement as an optimisation that minimises a cost function. Some multiscale contrast enhancement techniques apply image decomposition before image enhancement to prevent artefacts [11, 12].

Among aforementioned image enhancement techniques, the GHE methods are very popular and competitive for its low computation cost. The GHE techniques enhance images using a single-valued and monotonically increasing transformation function to avoid image distortion. During our research, we find that all GHE methods can be divided into two steps, that is, the pixel populations mergence (PPM) step and the grey-levels distribution (GLD) step.

- In the PPM step, the pixel populations of grey scales mapped to the same grey scale are merged firstly.
- In the GLD step, the merged grey scales are redistributed according to a transformation function.

The originally proposed division is reasonable because PPM is only related to pixel population and GLD only influences the grey level of pixels. Considering the entropy is also only determined by pixel populations while regardless of their corresponding grey levels, we can

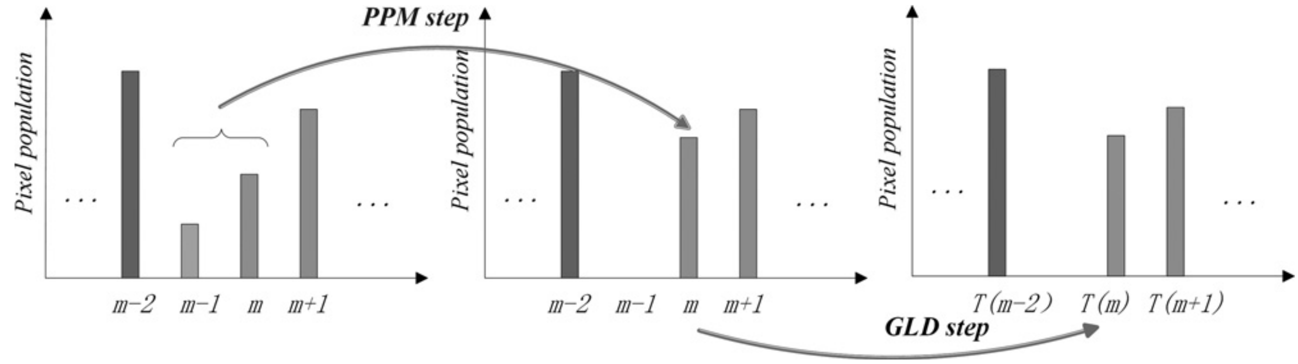

Fig. 1 *PPM and GLD steps of GHE techniques*

Horizontal and vertical axes are grey scales and corresponding pixel populations, respectively

maximise the entropy in the PPM step and then enhance image in the GLD step. Hence, we propose a novel entropy maximisation histogram modification (EMHM) scheme, which also consists of PPM and GLD steps. In the PPM step, the entropy of enhanced image is maximised using an originally developed mergence rule, that is, entropy maximisation rule (EMR). In the GLD step, we distribute new grey levels to the merged grey scales using a log-based distribution function (LDF) to alleviate over enhancement. The EMHM can not only maximise the entropy in the discrete case but also make the enhancement level controllable. What is more, the PPM operation in EMHM can reduce the redundancy of original image and control the number of grey scale with non-zero pixel populations (GNPP) of enhanced image, which is beneficial to image compression, especially in the application where the colour gradation of display device is limited.

The rest of this paper is organised as follows: Section 2 gives a detailed analysis of GHE technique and divides it into two steps, that is, PPM and GLD steps. Section 3 presents a novel image enhancement approach based on EMHM. Experiments and results are given in Section 4. Finally, our work is summarised in Section 5.

2 Detailed analysis of GHE technique

In this section, we give a detailed description of two divided steps of GHE techniques, that is, PPM and GLD steps, which is necessary to understand the later proposed EMHM. The impact of PPM operation on entropy of enhanced image is also included in this section.

2.1 Division of GHE technique

GHE techniques utilise the image histogram to obtain a single-indexed and monotonically increasing mapping function to modify the pixel values. Considering the HE is one of the most popular GHE techniques, we give the transformation function of HE as an example

$$x_k = \left\lfloor (N-1)\sum_{i=0}^{k} p_i + 0.5 \right\rfloor \quad (1)$$

For a b-bit image, $N = 2^b$. x_k is the mapped grey scale of intensity $k(k = 0, 1, \ldots, N-1)$ in the input image, p_i denotes the probability of intensity $i(i = 0, 1, \ldots, N-1)$ in the input image and $\lfloor a \rfloor$ is the floor operator, which returns the largest integer smaller than or equal to a. According to (1), we can implement the whole HE algorithm in following separated steps. At first, the pixels at adjoining grey scales, which mapped to the same grey scale because of the floor operation are merged. Then, the grey scales after mergence are mapped to different intensity values according to (1). We can generalise this division into other GHE techniques. That is to say, any GHE technique can be divided into PPM and GLD steps as shown in Fig. 1.

- *PPM:* The pixel populations of grey scales to be mapped to the same grey scale are merged firstly in the input histogram.
- *GLD:* The new GNPP in the merged histogram are redistributed according to transformation function T.

The division is useful because the entropy is only determined by pixel populations while regardless of their corresponding grey levels. Hence, we can control the entropy in the PPM step and then enhance image using transformation function in the GLD step. Moreover, the number of GNPP is controlled by mergence times in PPM step, which directly affects the compressibility of enhanced image.

2.2 Impact of pixel population mergence on entropy

As discussed above, the pixel populations in output histogram of GHE techniques are the emergence of those in input histogram. If p_m and p_n denote the probability of two grey levels m and $n(m, n \in [0, N-1])$ in the input histogram, hence, the difference between the entropy before and after merging pixels at these two levels can be expressed by

$$\begin{aligned} E_d = E_1 - E_2 = {} & -p_m \log(p_m) - p_n \log(p_n) \\ & + (p_m + p_n)\log(p_m + p_n) \end{aligned} \quad (2)$$

$$p_m = \frac{h_m}{H}, \quad p_n = \frac{h_n}{H} \quad (3)$$

where E_1 and E_2 are the entropy before and after merging, respectively, h_m and h_n are the pixel populations with grey levels m and n and H is the total amount of pixels in image. By plugging (3) into (2), the difference in (2) can be re-written as (see equation (4) on the bottom of the next page)

Since the logarithmic function is monotonically increasing and $h_m, h_n > 0$, $\log(h_m + h_n) > \log(h_m)$ and $\log(h_m + h_n) > \log(h_n)$.

Hence, $E_1 - E_2 > 0$, that is, $E_1 > E_2$, which means the entropy becomes smaller after each merging operation.

3 Proposed EMHM approach

Inspired by the division of GHE techniques, we propose a new EMHM method, which also consists of PPM and GLD. The pixel populations in histogram of input image are merged under a novel EMR in the PPM step. It is proved in theory that the proposed EMR can minimise the reduction of entropy in the PPM step. In the GLD step, the grey levels of merged histogram are redistributed using a LDF instead of a cumulative distribution function like that in HE to alleviate the effect of pixel populations to enhancement. Moreover, the level of contrast enhancement is adjustable in EMHM.

3.1 PPM step of EMHM

As proved above, the entropy of image decreases after mergence. Hence, the entropy of output image is maximised when the reduced entropy is minimised in each merging procedure. Let us further assume $p_n = kp_m (k \geq 1)$ and (2) can be re-written as (see (5))

and the derivative of E_d is given by

$$\frac{\partial E_d}{\partial k} = p_m(\log(1+k) - \log(k)) \geq 0 \tag{6}$$

According to (6), E_d is a monotonically increasing function with k. That is, the reduction of entropy E_d decreases as the decrease of k. Hence, merging p_n with p_m having a smaller k makes the entropy reduce less.

As proved above, the entropy becomes smaller after mergence and it reduces less when the merged pixel populations are more similar, that is, k is smaller. In fact, the pixels at a certain grey level can only be merged with either the left side or the right side. Hence, in order to maximise the entropy of the output image, we merge pixel population to the adjoining scale with more similar pixel population. We start the merging process from the grey scale with smallest pixel population because they are less important compared with those grey scales whose pixel populations are larger. The main steps of PPM with EMR in the proposed EMHM are as follows:

(a) Find the grey scale m with minimal pixel population in the input histogram.
(b) Merge the pixel population of m to that of the adjacent grey scale with more similar pixel population and set the pixel population of m to zero in the output histogram.
(c) Repeating (a) and (b) for T_m times, and T_m is the given mergence times.

The PPM with EMR maximises the entropy of output image by minimise the reduction of entropy because of merging. Note that, the entropy is maximised in the discrete case. This is different from other entropy maximisation techniques such as HE and BPHEME, which make the entropy of enhanced image maximised in the continuous case while may be unfeasible in the discrete case. Moreover, the PPM operation can reduce the redundancy of output histogram because small pixel populations are merged. In addition, the number of GNPP is reduced and can be controlled by the mergence times T_m, which is beneficial to image compression, especially when the colour gradation of display device is limited.

We assume the grey scales in the range of $[m_i + 1, m_{i+1}]$ are merged to the same grey scale $n_i (i = 0, \ldots, L-1)$. Hence, the pixel populations of grey scales from $m_i + 1$ to $m_{i+1} (i = 0, \ldots, L-1)$ in $\boldsymbol{H}_{\text{input}}$ are merged to that of $n_i (i = 0, \ldots, L-1)$ in $\boldsymbol{H}_{\text{output}}$, that is

$$\sum_{m=m_i+1}^{m_{i+1}} \boldsymbol{H}_{\text{input}}(m) = \boldsymbol{H}_{\text{output}}(n_i), \quad (i = 0, \ldots, L-1) \tag{7}$$

where $\boldsymbol{H}_{\text{input}} = [H_{\text{input}}(1), H_{\text{input}}(2), \ldots, H_{\text{input}}(N)]^t$ and $\boldsymbol{H}_{\text{output}} = [H_{\text{output}}(n_0), H_{\text{output}}(n_1), \ldots, H_{\text{output}}(n_{L-1})^t]$ are, respectively, the input and the output histograms in the PPM step. We want to emphasise that L is the number of bins, or equivalently, the number of GNPP in $\boldsymbol{H}_{\text{output}}$, which is the difference between the intensity levels of input image N and the mergence times T_m, that is, $L = N - T_m$. We re-written (7) in vectors as

$$\boldsymbol{H}_{\text{output}} = \boldsymbol{Q}\boldsymbol{H}_{\text{input}} \tag{8}$$

where $\boldsymbol{Q} \in \mathbb{R}^{L \times N}$ is the mergence matrix, given by

$$\boldsymbol{Q} = \begin{bmatrix} Q_{1\times(m_1-m_0)} & 0 & \cdots & 0 \\ 0 & Q_{1\times(m_2-m_1)} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & Q_{1\times(m_L-m_{L-1})} \end{bmatrix}_{L\times N} \tag{9}$$

where $\boldsymbol{Q}_{1\times(m_{i+1}-m_i)} (i = 0, 1, \ldots, L-1)$ is a $1 \times (m_{i+1} - m_i)$ matrix of ones, $m_0 = 0$ and $m_L = N - 1$. The mergence matrix $\boldsymbol{Q}$, which will be used in the later GLD step can be obtained simultaneously with the merged histogram h_{output} after pixel populations merging operation. Fig. 2 is the illustration of PPM step. Fig. 2*a* is the original image. Fig. 2*b* is the entropy of output image after PPM operation with different mergence times T_m. As we can see from Fig. 2*b*, the entropy of output images decreases with the increase of T_m, which accords with the above proof.

$$\begin{aligned} E_d &= \left(\frac{h_m}{H} + \frac{h_n}{H}\right)\log\left(\frac{h_m}{H} + \frac{h_n}{H}\right) - \frac{h_m}{H}\log\left(\frac{h_m}{H}\right) - \frac{h_n}{H}\log\left(\frac{h_n}{H}\right) = \frac{\left\{(h_m + h_n)\log(h_m + h_n) - h_m\log(h_m) - h_n\log(h_n)\right\}}{H} \\ &= \frac{h_m\left(\log(h_m + h_n) - \log(h_m)\right) + h_n\left(\log(h_m + h_n) - \log(h_n)\right)}{H} \end{aligned} \tag{4}$$

$$\begin{aligned} E_d &= (1+k)p_m\log\left((1+k)p_m\right) - p_m\log(p_m) - kp_m\log(kp_m) = p_m\left\{(1+k)\left(\log(1+k) + \log(p_m)\right) - \log(p_m) - k\log(kp_m)\right\} \\ &= p_m\left\{(1+k)\log(1+k) - k\log(k)\right\} \end{aligned} \tag{5}$$

doi: 10.1049/iet-ipr.2014.0347

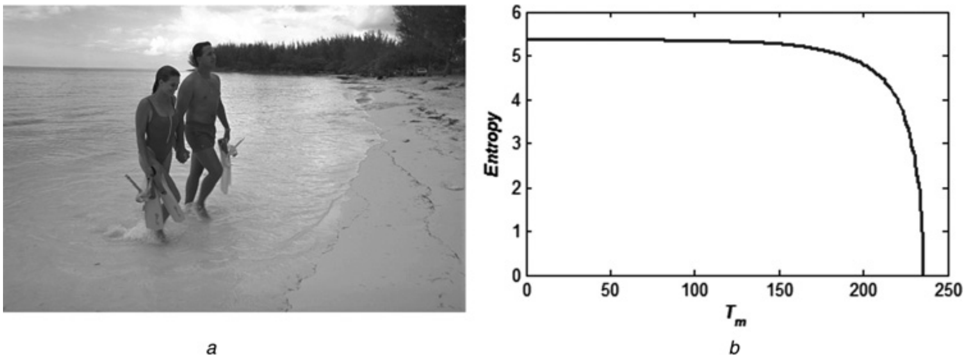

Fig. 2 *Illustration of PPM*
a Original image
b Entropy of output image with different mergence times

3.2 GLD step of EMER

The PPM proposed above can minimise the entropy reduction during mergence and control the amount of GNPP in enhanced image. However, it cannot overcome the main drawback of conventional HE, that is, the transformation function of HE is strongly affected by the pixel populations, which may result in a harsh and noisy appearance of the output image.

To alleviate the over enhancement problem, we implement the GLD step, which redistributes the intensities of the left L grey scales in the output histogram of PPM according to a novel transformation function. Let us denote the transformation function of the left L grey scales in the output histogram as $T(l)$ ($l = 1, 2, \dots, L$), which is given by

$$\begin{gathered} T(l) = \sum_{j=0}^{l} \Delta_j, \quad l = 1, 2, \dots, L \\ \sum_{j=1}^{L} \Delta_j = N - 1, \quad \Delta_j \geq 1 \end{gathered} \tag{10}$$

where Δ_j is the intensity increment between the $(j-1)$th and jth grey scales after GLD. There are two constraints in (10). The equality constraint $\sum_{j=0}^{L} \Delta_j = N - 1$ ensures the dynamic range is fully exploited in the output image. The inequality constraint $\Delta_j \geq 1$ is to make the transformation function $T(l)$ monotonic and ensure the entropy maximised in the NPM step and unchanged in the GLD step. Without latter constraint, the intensity ordering of pixels may be reversed, which produces visually annoying artefacts in the output images. In addition, if $\Delta_j < 1$, the $(j-1)$th and jth grey scales may mapped to the same grey scale in the output image, which changes the histogram obtained in the GLD step and reduces the entropy accordingly.

To weaken the effect of pixel populations, we present a LDF. It is demonstrated in [11] that a logarithm function can successfully reduce the dynamic ranges of high-dynamic-range images while preserving the details. The mathematical expression of LDF is given as follows

$$\Delta_j = \log\left(\frac{h_{\text{output}}(j)}{\bar{h}} \times 10^{-m} + 1\right) \tag{11}$$

where $h_{\text{output}}(j)$ and $\bar{h}$, respectively, denote the jth grey scale's pixel population and the mean of pixel populations in the merged histogram and m is the parameter controlling the level of LDF. As m obtains larger, $h_{\text{output}}(j)/\bar{h} \times 10^{-m}$in (11) becomes a smaller number. Since $\log(1 + x) \cong x$ for a small x, we have

$$\Delta_j \cong \frac{h_{\text{output}}(j)}{\bar{h}} \times 10^{-m} \propto h_{\text{output}}(j) \tag{12}$$

It is obvious that the effect of pixel populations is less alleviated when m obtains larger. On the other hand, as m obtains smaller, $h_{\text{output}}(j)/\bar{h} \times 10^{-m}$in (11) becomes dominant. The intensity increment Δ_j can be approximated to

$$\begin{aligned} \Delta_j &\cong \log\left(\frac{h_{\text{output}}(j)}{\bar{h}} \times 10^{-m}\right) \\ &= \log(h_{\text{output}}(j)) - \log(\bar{h} \times 10^{m}) \cong -\log(\bar{h} \times 10^{m}) \end{aligned} \tag{13}$$

Since $h_{\text{output}}(j)$ is small compared with $\bar{h} \times 10^{m}$, Δ_j becomes a constant regardless of $h_{\text{output}}(j)$. Hence, the proposed LDF can alleviate the effect of pixel populations effectively and the alleviation degree is adjustable with the parameter m. More precisely, a smaller m results in stronger alleviation.

Finally, to satisfy the two constraints in (10), the intensity increment Δ_j is normalised as follows

$$\Delta_{nj} \cong 1 + \left[\frac{\Delta_j}{\sum_{j=1}^{L} \Delta_j} \times (N - L - 1)\right] \tag{14}$$

where Δ_{nj} is the normalised Δ_j. Hence, $T(l)$ in (10) can be re-written as (see equation (15) on the bottom of the next page)

The transformation function of the L GNPP in the output histogram of NPM is given above and we can obtain the mapped intensities $G = [G(1), G(2), \dots, G(N)]$ of the whole N grey scales in the original histogram as follows

$$\boldsymbol{G} = \boldsymbol{TQ} \tag{16}$$

where $\boldsymbol{T} = [T(1), T(2), \dots, T(L)]$. To simplify the complexity of calculation, we re-written (15) in vector form as

$$\boldsymbol{T} = \boldsymbol{C}\left(\boldsymbol{I} + \left[\frac{\boldsymbol{\Delta}}{\|\boldsymbol{\Delta}\|^1}(N - L - 1)\right]\right) \tag{17}$$

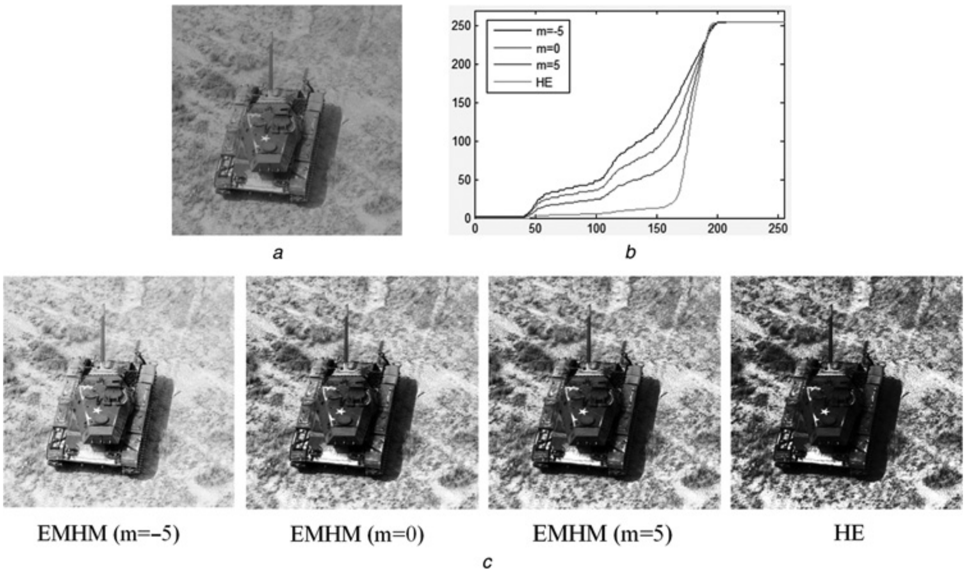

Fig. 3 *Illustration of GLD*

a Original image
b Transformation functions of conventional HE and EMHM with m equals to -5, 0 and 5, respectively
c Enhanced images with transform functions in (Fig. 3*b*)

where $\Delta = [\Delta_1, \Delta_2, \ldots, \Delta_{L-1}]$, I is a $L \times 1$ matrix of ones, $\|\Delta\|^1$ is the sum of all elements in Δ and $\boldsymbol{C} \in \mathbb{R}^{L \times N}$ is the cumulation matrix, that is,

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 1 & 0 \\ 1 & 1 & \cdots & 1 & 1 \end{bmatrix} \tag{18}$$

Substitute (18) and $L = N - T_m$ into (17), we have

$$\boldsymbol{G} = \boldsymbol{C}\left(\boldsymbol{I} + \left[\frac{\boldsymbol{\Delta}}{\|\Delta\|^1}(T_m - 1)\right]\right)\boldsymbol{Q} \tag{19}$$

The parameter m is the key parameter of transformation function $\boldsymbol{G}$, controlling the level of enhancement by modifying the intensity increment vector $\boldsymbol{\Delta}$ in (11). Fig. 3 gives an example of the GLD step. Fig. 3*a* is the original image, Fig. 3*b* illustrates how the proposed LHM scheme modifies the transformation function $\boldsymbol{G}$ according to parameter m and Fig. 3*c* shows the corresponding enhanced images of EMHM and conventional HE. In this test, the number of mergence times T_m is set to 100 in PPM. As we can see from Fig. 3*b*, the steep slope in the transformation function of HE is relaxed more with smaller parameter m, which means the transformation function $\boldsymbol{G}$ is less affected by the pixel populations. In the extreme case, when $m = -\infty$, the modified histogram becomes uniformly distributed. In the other extreme case, when $m = +\infty$, the histogram is not modified at all. Therefore, by controlling the single parameter m, LHM can obtain the transformation function, which varies between the identity function and the conventional HE result. It is observed from Fig. 3*c* that the HE overstretches the contrast and wipes out the details of tank in the original image because of the steep slope in the transformation function. On the other hand, the proposed EMHM method with $m = 0$ can enhance the background properly while preserve the details of the tank.

4 Experimental results

In the experiments, the proposed EMHM algorithm have been applied to process several grey and colour images and the performance of the method is compared with that of the conventional HE, BBHE [7], BPHEME [8], HMF [10], power-constrained contrast enhancement method (PCCE) [5] and image enhancement using the Gaussian mixture modelling (GMM) [4]. Figs. 5–8 show the six test images, where 'Brain', 'F-16' and 'Girl' are from the USC-SIPI database, 'Lighthouse' and 'Plane' are from the Kodak Lossless True Colour Image Suit and the others are downloaded from the Internet [13, 14].

4.1 Impacts of parameters T_m and m on image enhancement

As discussed above, the mergence times T_m and m in (19) are two key parameters of our proposed EMHM since they directly related to the enhancement results. Fig. 4 shows the images enhanced by EMHM with various combinations of T_m and m. As we can see from Fig. 4, the image is enhanced more sufficiently with larger T_m and m while it faces contrast overstretching problem when T_m and m are too large. To make the enhancement appropriately, we give the reference values of the two parameters as follows, with

$$T(l) = \sum_{j=0}^{l}\left(1 + \left[\frac{\log\left((h_{\text{output}}(j)/\bar{h}) \times 10^{-m} + 1\right)}{\sum_{j=1}^{L}\log\left((h_{\text{output}}(j)/\bar{h}) \times 10^{-m} + 1\right)} \times (N - L - 1)\right]\right) \tag{15}$$

doi: 10.1049/iet-ipr.2014.0347

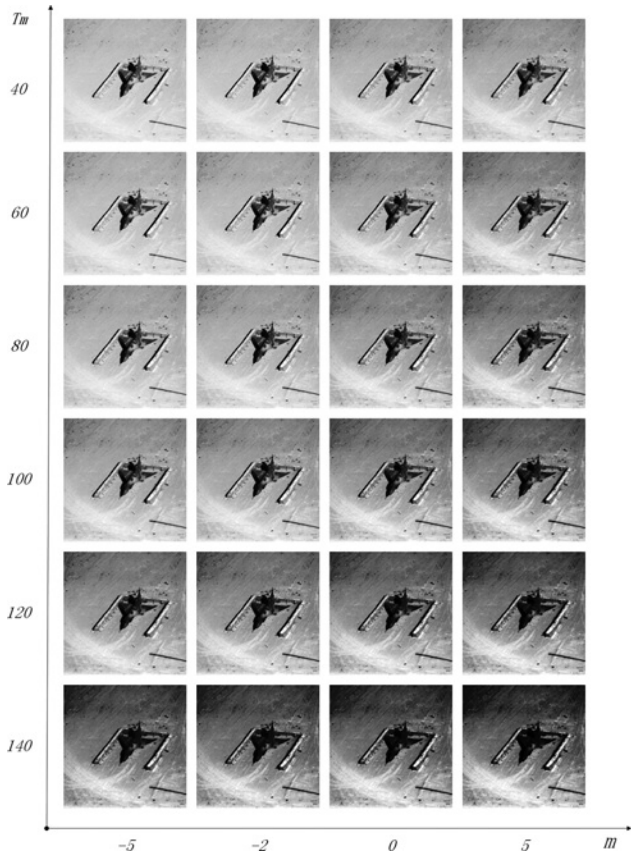

Fig. 4 *Enhanced images by EMHM with different T_m and m*

which high subjective quality can be achieved in extensive experiments.

- The reference value of T_m. The impact of pixel populations on enhancement is alleviated more with higher T_m because the pixels with smaller populations are merged first in the PPM step of EMHM. However, an extreme high T_m results in low entropy and brings artefacts in the enhanced image. Considering an image with more GNPP has more redundancy, the reference value of T_m is proportional to the amount of GNPP in the original image, that is

$$T_m = k * L_o \tag{20}$$

where $0 \le k < 1$ and L_o is the amount of GNPP in the input histogram.
- The reference value of m. The m in LDF is another parameter, which alleviates the effect of pixel populations. According to (11), a steeper input histogram requires a smaller m. In our paper, the reference value of parameter m is given by

$$m = \frac{1}{2\sqrt{\sum_{i=0}^{N} (h_i' - \overline{h_i'})^2}} - c \tag{21}$$

where $h_i' = (h_i - h_{\min})/(h_{\max} - h_{\min})$, $h_i (i = 0, 1, \ldots, N)$ denotes the number of pixels with intensity i, $\overline{h_i'}$ is the mean of h_i' and $3 \le c \le 8$. As we can see from (21), m is inversely proportional to the standard deviation of pixel populations in the input histogram, which weakens the effect of pixel populations automatically.

After extensive experiments, we suggest $k = 0.6$ in (20) and $c = 5$ in (21), which produce good subjective quality in most cases. Note that, the two parameters k and c can be changed in real practice. Table 1 lists the reference and optimal values of T_m and m for six test images in Figs 5–10. The optimal values are obtained using an exhaustive searching method to achieve the best subjective quality. The intervals of T_m and m are 5 and 0.25, respectively. As we can see from Table 1, the reference values of both T_m and m obtained by (20) and (21) are very close to the corresponding optimal values, which means calculating T_m and m using (20) and (21) is a reliable choice. The reference values of T_m and m in Table 1 are adopted in the following experiments.

4.2 Subjective assessment

(1) *Grey-scale images.* Figs. 5–7 show the test images and their corresponding enhanced versions of the conventional HE, BBHE [7], BPHEME [8], HMF [10], PCCE [5], GMM [4] and the proposed EMHM. In the view of visual quality, from Figs. 5–7, it is shown that EMHM performs better

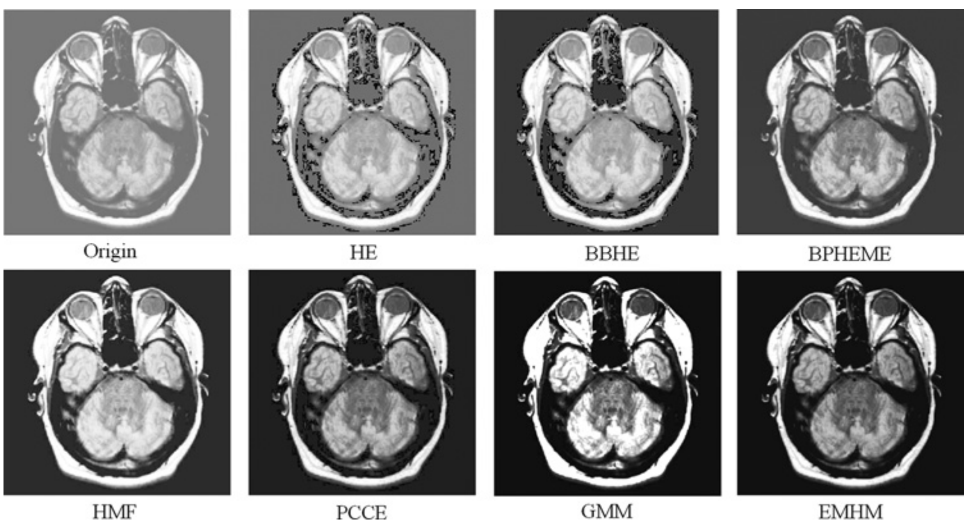

Fig. 5 *Enhancement results of brain*

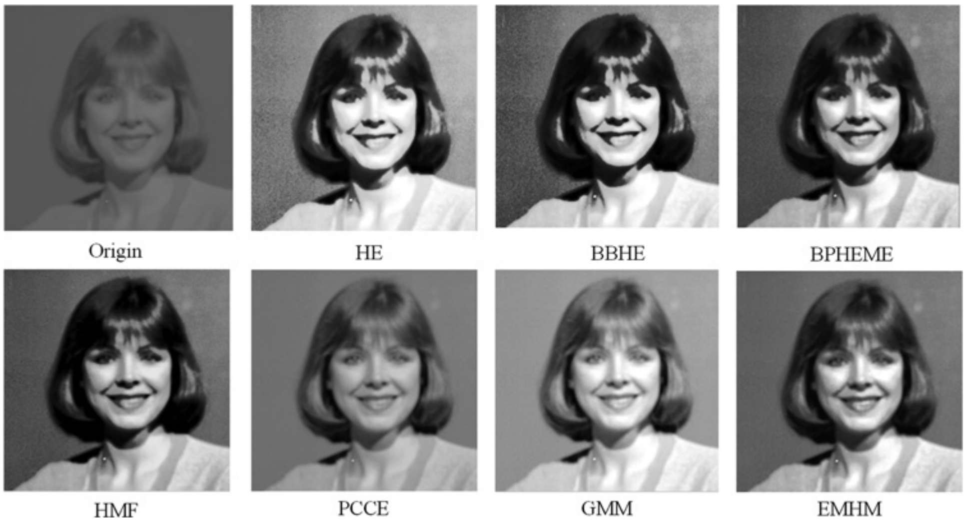

Fig. 6 *Enhancement results of face*

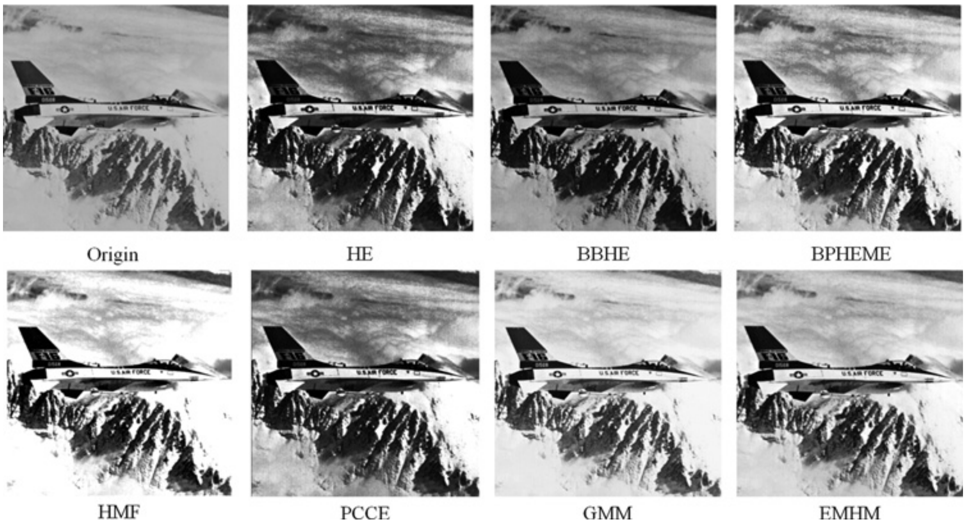

Fig. 7 *Enhancement results of F-16*

Fig. 8 *Enhancement results of girl*

doi: 10.1049/iet-ipr.2014.0347

Table 1 Reference and optimal values of T_m and m

Image	Brain	Face	F-16	Girl	Lighthouse	Plane
reference value of T_m	103	89	142	108	150	126
optimal value of T_m	105	85	140	105	155	125
reference value of m	2.41	0.12	−2.41	1.17	−2.17	2.1
optimal value of m	2.25	0	−2.25	1.0	−2.0	2.0

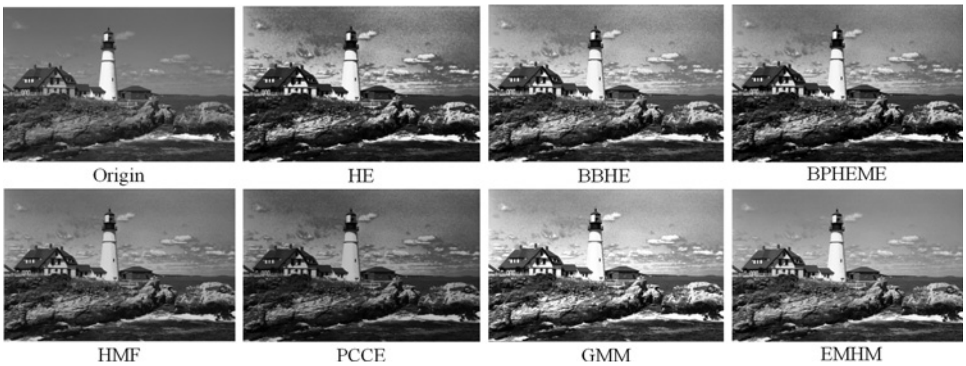

Fig. 9 *Enhancement results of lighthouse*

Fig. 10 *Enhancement results of plane*

than the other six techniques. The HE, BBHE and BPHEME usually result in excessive contrast enhancement, which may amplify noise and create visual artefacts. The over enhancement phenomenon is lessened by HMF, PCCE and GMM. However, it is not alleviated completely. Both HMF and GMM lose details in some bright parts of original images, which can be seen clearly in Figs. 5 and 7. The PCCE creates artefacts in Fig. 5 and makes the snow areas too dark in Fig. 7. The proposed algorithm, on the other hand, enhances the images appropriately and thus yields high subjective quality. As we can see from Fig. 6, the amplified noises in the enhanced images of HE, BBHE and BPHEME are avoided in that of EMHM. Moreover, the contrast is improved more by our EMHM compared with HMF, PCCE and GMM. This is because the number of GNPP is adjustable and the reduced entropy because of mergence is minimised in the PPM step of our EMHM. Moreover, the level of enhancement can be controlled effectively by the proposed LDF in the GLD step of EMHM.

(2) *Colour images*. Contrast enhancement can be easily extended to colour images. The simplest way is to apply the method to luminance component and to preserve the chrominance components. Some examples using colour images are given in Figs. 8–10. In Fig. 8, the contrast of original image is over enhanced by HE, BBHE and BPHEME, which amplifies noise in the backgrounds. The amplified noises are reduced by HMF, PCCE and GMM while still obvious. Our EMHM depresses the noise effectively and produces a high quality enhanced result by subjective evaluation. The clouds in Fig. 9 become darker in images enhanced by all the methods except GMM and our proposed EMHM. However, GMM loses details in the

Table 2 Number of GNPP of enhanced images in Figs. 5–10

Method/image	Brain	Face	F-16	Girl	Lighthouse	Plane
original value	172	149	237	180	250	211
HE	70	58	99	75	102	86
BBHE	87	97	119	101	158	139
BPHEME	99	112	122	123	147	132
HMF	79	85	115	99	131	110
PCCE	82	95	120	107	144	120
GMM	76	87	108	94	121	99
EMHM	68	59	94	72	100	84

Table 3 Entropy of enhanced images

Image	Brain	Face	F-16	Girl	Lighthouse	Plane
origin	4.96	5.40	6.71	6.53	6.93	5.78
HE/EMHM	3.68/4.22	4.88/5.22	5.71/6.24	5.83/6.01	5.82/6.18	5.24/5.32
BBHE/EMHM	4.61/4.72	5.25/5.36	6.51/6.59	6.21/6.33	6.07/6.60	5.40/5.51
BPHEME/EMHM	4.70/4.80	5.30/5.41	6.45/6.62	6.46/6.39	6.32/6.73	5.43/5.59
HMF/EMHM	3.78/4.90	5.11/5.38	6.23/6.67	6.01/6.40	6.13/6.79	5.37/5.61
PCCE/EMHM	3.81/4.71	4.99/5.32	5.81/6.52	5.88/6.27	6.11/6.70	5.21/5.58
GMM/EMHM	4.11/4.68	5.17/5.25	6.41/6.61	6.10/6.31	6.21/6.76	5.39/5.50
EMHM	4.20	5.21	6.32	6.11	6.27	5.40

Table 4 PSNR of enhanced images

Image	HE	BBHE	BPHEME	HMF	PCCE	GMM	EMHM
brain	13.31	18.45	18.51	19.28	19.41	19.37	19.74
face	12.09	11.38	13.02	15.28	15.21	16.64	18.34
F-16	13.48	20.06	20.75	21.48	21.52	21.11	22.24
girl	13.00	13.36	14.20	16.05	22.14	22.75	26.76
lighthouse	16.15	15.70	18.83	19.93	27.32	25.10	31.94
plane	13.01	19.28	21.11	25.07	28.41	26.66	33.62

bright parts of original image such as the lighthouse. In Fig. 10, the skies in HE, BBHE and BPHEME enhanced images look very unnatural. Unnatural look of sky is lessened by using HMF and PCCE while graininess in the sky still exists in the region close to the plane. The GMM avoids the graininess while it does not enhance the grass sufficiently. Our EMHM enhances the image sufficiently without creating graininess in the sky.

4.3 *Objective assessment*

To compare these image enhancement methods quantitatively, three objective evaluation indices, that is, the entropy, the number of GNPP and the peak signal-to-noise ratio (PSNR) [1] are used. The entropy is adopted to measure the content of an image, and a higher value indicates an image with richer details. The number of GNPP, that is, the number of GNPP can measure the compressibility of enhanced image, and a lower value indicates the image with less redundant information and means it require less storage space after compression. A higher PSNR value indicates the enhanced image with less noise. The colour image's entropy, the number of GNPP and PSNR are those of luminance component because we process only the luminance component without modifying the chrominance components.

Table 2 lists the number of GNPP for images in Figs. 5–10. The number of GNPP is reduced for all GHE techniques. The number of GNPP of images enhanced by HE and proposed EMHM are similar and they are significantly smaller than those of images enhanced by the other five methods. Moreover, the number of GNPP of image enhanced by proposed EMHM is controllable. An image with lower number of GNPP means it requires less storage space, which is beneficial to image compression, especially when the colour gradation of display device is limited.

Table 3 shows the entropy of the six test images enhanced by HE, BBHE, BPHEME, HMF, PCCE, GMM and proposed EMHM. It is observed from Table 2 that the mergence times are different for all seven GHE techniques. The comparison of entropy is meaningful when the mergence time is the same. Then, we calculated the entropy of images enhanced by EMHM with the same mergence time as that of other six GHE techniques. The entropy of our EMHM enhanced image is listed behind that of HE, BBHE, BPHEME, HMF, PCCE and GMM enhanced images, respectively, in the second to seventh rows of Table 3. The last row lists the entropy of enhanced images using our EMHM with mergence times calculated by (20). As we can see from Table 3, the entropy of images enhanced by the seven GHE techniques is reduced compared with that of original image. Moreover, the entropy of images enhanced by HE, BBHE, BPHEME, HMF, PCCE and GMM are smaller than that processed by our EMHM with the same T_m. This is because the entropy becomes smaller after mergence and EMHM minimises the reduction of entropy using EMR in the discrete case. Our EMR is more effective than HE and BPHEME because both of them just make the entropy maximised in the continuous case.

The PSNR of six test images enhanced by the seven image enhancement methods are listed in Table 4. Comparison of PSNR values shows that the proposed method outperforms the other techniques for all images. This is because the proposed EMHM can alleviate the over enhancement effectively and thus avoid amplified noise and image artefacts.

5 Conclusion

This paper proposes a novel EMHM scheme, which consists of PPM and GLD steps. The histogram of input image is merged with proposed EMR in the PPM step, which can minimise the reduction of entropy because of mergence and compress the number of GNPP in output histogram. In the GLD step, the new grey levels are redistributed using a LDF, which can alleviate the contrast overstretching. Extensive experimental results show that the proposed EMHM algorithm is effective.

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