Brightness Preserving Histogram Equalization with Maximum Entropy: A Variational Perspective

Chao Wang and Zhongfu Ye

Abstract — *Histogram equalization (HE) is a simple and effective image enhancing technique, however, it tends to change the mean brightness of the image to the middle level of the permitted range, and hence is not very suitable for consumer electronic products, where preserving the original brightness is essential to avoid annoying artifacts. This paper proposes a novel extension of histogram equalization, actually histogram specification, to overcome such drawback as HE. To maximize the entropy is the essential idea of HE to make the histogram as flat as possible. Following that, the essence of the proposed algorithm, named Brightness Preserving Histogram Equalization with Maximum Entropy (BPHEME), tries to find, by the variational approach, the target histogram that maximizes the entropy, under the constraints that the mean brightness is fixed, then transforms the original histogram to that target one using histogram specification. Comparing to the existing methods including HE, Brightness preserving Bi-Histogram Equalization (BBHE), equal area Dualistic Sub-Image Histogram Equalization (DSIHE), and Minimum Mean Brightness Error Bi-Histogram Equalization (MMBEBHE), experimental results show that BPHEME can not only enhance the image effectively, but also preserve the original brightness quite well, so that it is possible to be utilized in consumer electronic products*[1].

Index Terms —Image enhancement, histogram equalization, histogram specification, maximum entropy, variational approach, mean brightness preserving.

I. INTRODUCTION

Histogram is defined as the statistic probability distribution of each gray level in a digital image [1]. Histogram equalization (HE) is one of the well-known methods for enhancing the contrast of given images, making the result image have a uniform distribution of the gray levels [1]. It flattens and stretches the dynamic range of the image's histogram and results in overall contrast improvement. HE has been widely applied when the image needs enhancement, such as medical images enhancement [2]. However, in consumer electronics such as TV, HE is rarely employed because it may significantly change the brightness of an input image and cause undesirable artifacts. In theory, the mean brightness of its output image is always the middle gray level regardless of the input mean, because the "desired" histogram is flat. This is not a desirable property in some applications where brightness preservation is necessary.

Brightness preserving Bi-Histogram Equalization (BBHE) has been proposed to overcome that problem [3]. BBHE first separates the input image's histogram into two by its mean, and thus two non-overlapped ranges of the histogram are obtained. Next, it equalizes the two sub-histograms independently. It has been analyzed that BBHE can preserve the original brightness to a certain extent when the input histogram has a quasi-symmetrical distribution around its mean. Later, equal area Dualistic Sub-Image Histogram Equalization (DSIHE) has been proposed, it claims that if the separating level of histogram is the median of the input image's brightness, it will yield the maximum entropy after two independent sub-equalizations [4]. DSIHE will change the brightness to the middle level between the median level and the middle one of the input image.

Nevertheless, neither BBHE nor DSIHE could preserve the mean brightness. Then Minimum Mean Brightness Error Bi-Histogram Equalization (MMBEBHE) is proposed to preserve the brightness "optimally" [5]. MMBEBHE is to perform the separation based on the threshold level, which would yield minimum difference between input and output mean. This threshold level is essentially chosen by enumeration.

Another scheme, named Recursive Mean-Separate Histogram Equalization (RMSHE), has been proposed to preserve the brightness [6]. RMSHE uses the BBHE iteratively. First RMSHE separates the input histogram into two pieces, by the mean. Then, to each piece, it uses this operation many times to generate 2^n-pieces histograms. Finally, it equalizes each histogram piece independently. It is claimed theoretically that when the iteration level n grows larger, the output mean converges to the input mean, and thus yields good brightness preservation. Actually, when n grows to infinite, the output histogram is exactly the input histogram, and thus the input image will be output without any enhancement at all.

In the consumer electronics such as TV, the preservation of brightness is highly demanded. The aforementioned algorithms (HE, BBHE, DSIHE, MMBEBHE and RMSHE) preserve the brightness to some extent, however, they do not meet that desirable property quite well.

In this paper, a novel enhancement method is proposed which can yield the optimal equalization in the sense of entropy maximization, under the constraint of the mean

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Contributed Paper
Manuscript received October 9, 2005

brightness, called Brightness Preserving Histogram Equalization with Maximum Entropy (BPHEME).

BPHEME, together with the aforementioned algorithms, is essentially a kind of histogram specification [1] in general, except that different "ideal" histograms are employed in different algorithms. In the next section, histogram specification will be reviewed, and HE, BBHE, DSIHE, MMBEBHE will be quickly introduced as special cases of histogram specification. In Section III, the proposed algorithm, BPHEME, will be presented, which is "ideal" in the sense of maximum entropy with an invariant mean brightness. Section IV will list groups of experimental results to claim the performance of BPHEME, comparing with the ones of HE, BBHE, DSIHE and MMBEBHE. Section V makes some concluding remarks.

II. Histogram Specification

When histogram transformation method is considered, many applications require a desirable shape of histogram. We want to generate a processed image that has the specified desirable histogram, which is called Histogram Specification (HS) or histogram matching. Since a histogram can be viewed as the probability density function of the variable for gray levels, we introduce the histogram processing methods from a continuous view in the following parts. First this section covers the details of HS from a general perspective, actually a reprint of corresponding part in [1]. Then we regard HE, BBHE, DSIHE and MMBEBHE as the special cases of HS, give their corresponding desirable histograms.

A. General Histogram Specification

Let the gray levels r and z be continuous random variables with corresponding continuous probability density functions (PDFs) denoted by $p_r(r)$ and $p_z(z)$. In this notation, r and z denote the gray levels of the input and output (processed) images, respectively. We can estimate $p_r(r)$ from the given input image, while $p_z(z)$ is the specified PDF that we hope the output image to have.

Let s be a random variable with the property

$$s = T(r) = \int_0^r p_r(w)dw \tag{1}$$

where T is essentially the cumulative operator and s is the cumulative histogram, i.e., the distribution function of the variable r. Suppose next that we define a random variable z with the property

$$G(z) = \int_0^z p_z(t)dt = s \tag{2}$$

It then follows from these two equations that $G(z)=T(r)$ and, therefore, that z must satisfy the condition

$$z = G^{-1}(s) = G^{-1}[T(r)] \tag{3}$$

That is the theoretical case. In application, we would like to map each given input gray level r to an output one, z, which yields the closest distance between the input and output cumulative histogram, i.e.,

$$z(r) = \arg\min_z |T(r) - G(z)|, \ \forall r \in S \tag{4}$$

where S is the support set of input gray level r. (4) is called SML (Single Mapping Law) in [7], and a general mapping law (GML) is proposed in [7], to improve the accuracy of HS. In this paper, we use SML, i.e., eq.(4), for its simplicity to realize.

B. Special Cases of Histogram Specification

HS can approximately yield a desirable histogram, however what is a desirable one? It is a problem deserving discussion. The aforementioned algorithms such as HE, BBHE, DSIHE and MMBEBHE, are all histogram specification essentially, see Fig.1 for the general shape of their desirable histograms. Without loss of generality, we assume that r has been normalized to the interval S=[0,1], with r=0 representing black and r=1 representing white.

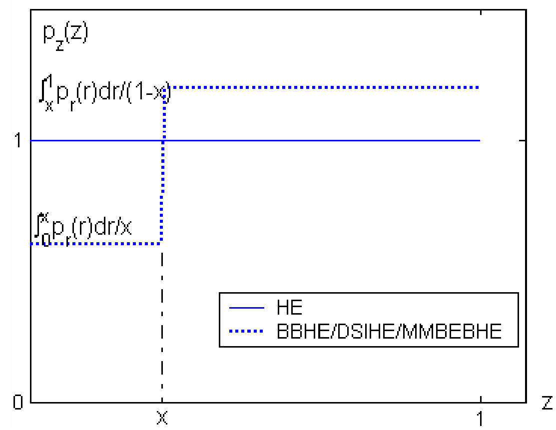

Fig.1 Desired histogram (PDF) of HE, BBHE, DSIHE and MMBEBHE. x is the separating gray level for BBHE, DSIHE and MMBEBHE.

1) Histogram Equalization

We focus our attention on the following transformation

$$z = T(r), \quad 0 \le r \le 1 \tag{5}$$

that maps to a level z for every pixel value r in the original image. $T(r)$ satisfies the following fundamental conditions:

- $T(r)$ is single-valued and monotonically increasing in the interval $0 \le r \le 1$;
- $0 \le T(r) \le 1$ for $0 \le r \le 1$.

If we choose $T(r) = \int_0^r p_r(w)dw$ like (1), it is not difficult to find that the PDF of the output gray level z follows a uniform distribution ranging from 0 to 1. Or, we may say that the determination for the variable z at a given pixel will provide us the maximum information, because z equals to any gray level with equal probability, thus, it contains the most uncertainty. HE can be regarded as a special case of HS when we choose the target output histogram as a uniform one, $p_z(z)=1$, $0 \le z \le 1$.

2) Brightness preserving Bi-Histogram Equalization

As mentioned before, the desired histogram of HE is uniform, and thus its desired mean brightness is 0.5, the middle level of S. So the mean brightness can not be preserved by HE.

BBHE [3] first separates the input histogram into two parts based on its mean brightness, and then equalizes the two sub-histograms independently. Thus there is a step in the histogram at the gray level $x_B = \mu_r = \int_0^1 r p_r(r) dr$. The desirable histogram of BBHE is a piecewise constant one as

$$p_z(z) = \begin{cases} \frac{1}{x_B} \int_0^{x_B} p_r(r) dr & , \quad 0 \le z < x_B \\ \frac{1}{1 - x_B} \int_{x_B}^1 p_r(r) dr & , \quad x_B \le z \le 1 \end{cases} \tag{6}$$

It is reported that BBHE can preserve the mean brightness, if the input histogram has a symmetrical distribution around its mean. However, that assumption is not the fact in many cases, and thus BBHE can not always preserve the brightness well.

3) Dualistic Sub-Image Histogram Equalization

DSIHE [4] is very similar to BBHE, except that the separating point x_D is selected as the median gray level of the input image, i.e., x_D satisfies

$$\int_0^{x_D} p_r(r) dr = 0.5 \tag{7}$$

For the applicable case, it may be modified as

$$x_D = \arg\min_x \left| \int_0^x p_r(r) dr - 0.5 \right| \tag{8}$$

The purpose of DSIHE is to find a separating point, based on which the desired histogram (PDF) can obtain a maximum entropy. And it has been proved that the brightness of the output image is

$$\mu_z = 0.5\,(x_D + 0.5) \tag{9}$$

It is clear that DSIHE always pulls the output brightness toward the input middle level from the input median level.

4) RMSHE and MMBEBHE

RMSHE [6] is to recursively implement BBHE to the histogram. Obviously, recursive separation on the histogram will divide the histogram into very small pieces. If the recursive level tends to infinite, equalization to each small piece will not change the whole histogram, and thus can preserve the mean brightness. So an infinite recursive level results in the same output image as the input one.

BBHE and DSIHE belong to "two-piece-separating" histogram equalization algorithm, and so does HE in a broad sense. MMBEBHE [5] is another modification to two-piece-separating histogram equalization algorithm. MMBEBHE directly considers the gray level, based on which the input histogram is separated. It chooses the separating level that produces the minimum absolute mean brightness error (AMBE) to the original image. It really can preserve the brightness quite well. It is essentially an enumeration method though the authors of [5] provided a recursive method to compute AMBE. Further more, this method fixed the type of desirable histogram to a piecewise constant function with a step, this choice does not seem to have a convincing theoretical stand.

III. Brightness Preserving Histogram Equalization with Maximum Entropy

Let us re-focus our attention on HE. As mentioned in section II-B.1, HE is to make the output histogram as flat as possible. A superficial reason for HE relies on that a flat histogram makes all the gray levels uniform, and thus will cause a more comfortable perception. A further comprehension is that a uniform distribution limited to a given range gives the maximum information, measured by entropy! So we would like to find an "ideal" histogram (PDF) as a target one to perform the histogram specification. That ideal histogram preserves the mean brightness of the input image, and has the maximum entropy. That is the basic thought of our proposed method. In this section we quickly introduce the entropy of a continuous variable first, then we give the corresponding functional extremum problem and solve it.

A. Preliminary Information Theory [8]

Definition 1: Let X be a random variable with cumulative distribution function $F(x)$=Pr($X \le x$). If $F(x)$ is continuous, the random variable is said to be continuous. Let $f(x)=F'(x)$ when the derivative is defined. If $\int_{-\infty}^{+\infty} f(x) dx = 1$, then $f(x)$ is called the *probability density function* (*PDF*) for X. The set where $f(x)>0$ is called the *support set* of X.

Definition 2: The *differential entropy* $h(X)$ of a continuous random variable X with a density $f(x)$ is defined as

$$h(X) = -\int_S f(x) \log f(x) dx \tag{10}$$

where S is the support set of the random variable.

The differential entropy is a convex function over a convex set, and depends only on the probability density of the random variable, hence the differential entropy is sometimes written as $h(f)$ rather than $h(X)$.

$h(X)$ characterizes the randomness of the variable X, the more random (chaotic) the variable is, the more information it may provide, the more the differential entropy is.

Since we will not compare the randomness between continuous and discrete variables, we will not differentiate between a continuous variable's differential entropy and a discrete variable's entropy. So in the following parts, $h(X)$ is also named as the continuous variable's entropy.

B. BPHEME: Brightness Preserving Histogram Equalization with Maximum Entropy

After the entropy of a continuous variable has been defined, we can return to what we are discussing.

Since the preservation of the mean brightness is of high demands in consumer electronics such as TV, we may find an enhancement method with the mean brightness being constrained.

As has been said in the preface of this section, let an image be enhanced "optimally" means that the histogram (PDF) may have the most entropy. Thus an optimal brightness preserving enhancement method using histogram transformation may be to maximize the target histogram's entropy under the constraints of brightness. Mathematically speaking, we want to maximize $h(f)$ over all probability densities f subject to some constraints:

$$\max_f \{-\int_S f(s)\log f(s)ds\}, \text{s.t.} \begin{cases} f(s) \geq 0, s \in S \\ \int_S f(s)ds = 1 \\ \int_S sf(s)ds = \mu_r \end{cases} \quad (11)$$

where S=[0,1] is the normalized support set of gray level, and $\mu_r = \int_0^1 rp_r(r)dr$ is the mean brightness of the input image. Because a whole white or a whole black image is not the input image in general, we assume $\mu_r \in (0,1)$.

Since $h(f)$ has the convexity, we form the functional

$$J(f) = -\int_0^1 f(s)\ln f(s)ds + \lambda_1[\int_0^1 f(s)ds - 1] + \lambda_2[\int_0^1 sf(s)ds - \mu_r] \quad (12)$$

where λ_1, λ_2 are the undetermined parameters, and here we specify the logarithmical function as the nature one.

Using the variational approach, we can "differentiate" with respect to $f(s)$, the s^{th} component of f, to obtain

$$\frac{\partial J}{\partial f(s)} = -\ln f(s) - 1 + \lambda_1 + \lambda_2 s = 0, \quad s \in S \quad (13)$$

thus

$$f(s) = e^{\lambda_1 - 1} \cdot e^{\lambda_2 s}, \quad s \in S \quad (14)$$

Using the second and third constraints in (11), we may find

$$f(s) = \begin{cases} 1 & ,\text{if} \quad \mu_r = 0.5 \\ \dfrac{\lambda_2 e^{\lambda_2 s}}{e^{\lambda_2} - 1} & ,\text{if} \quad \mu_r \in (0,0.5)\cup(0.5,1) \end{cases} \quad \forall s \in S \quad (15)$$

In (15), λ_2 can be determined. When $\mu_r \neq 0.5$, λ_2 is the solution of equation

$$\mu_r = \frac{\lambda_2 e^{\lambda_2} - e^{\lambda_2} + 1}{\lambda_2(e^{\lambda_2} - 1)} \quad (16)$$

and λ_2=0 when μ_r=0.5. It is a single-valued function of μ_r with respect to λ_2, as shown in Fig.2. So given a $\mu_r \in (0,1)$, there exists the only λ_2 that generates the mean brightness as μ_r.

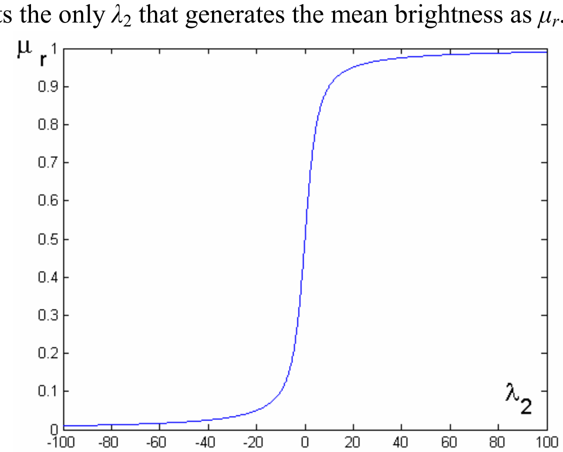

Fig.2 Relation between the mean brightness μ_r and the parameter λ_2.

Since $\mu_r(\lambda_2)$ is monotonically increasing, it is easy to use a Newton iteration [9] to solve the equation for a numerical solution.

Thus we have the cumulative histogram, or cumulative distribution function, $c(s)$ as following:

$$c(s) = \int_0^s f(t)dt = \begin{cases} s & ,\text{if} \quad \mu_r = 0.5 \\ \dfrac{e^{\lambda_2 s} - 1}{e^{\lambda_2} - 1} & ,\text{if} \quad \mu_r \in (0,0.5)\cup(0.5,1) \end{cases} \quad \forall s \in S \quad (17)$$

Had $f(s)$ or $c(s)$ been given, we can specify the input image's histogram under the instruction of $f(s)$ or $c(s)$, using histogram specification introduced in section II-A.

In applications, real-time processing is desired. It had better not cost too much time in solving (16). Fortunately, $\mu_r(\lambda_2)$ is monotonically increasing, and we can pre-list a table that designates a λ_2 to each μ_r. It is not necessary to discretize μ_r too delicately, since the following operation HS is implemented to a discrete case, and is an approximation in essence. What's more, Fig.2 shows the relation $\mu_r(\lambda_2)$ is anti-symmetrical about (μ_r, λ_2)=(0,0.5), so the memory cost can be further reduced by half. The pre-listing table will not cost too much computing time and memory at all.

C. Discussions on Discrete Version of BPHEME

The BPHEME scheme introduced in section III-B can be easily implemented to the discrete image. However, in this section we must clarify something related.

The expression "with maximum entropy" is only suitable for the continuous case. The histogram mapping methods, including all the aforementioned methods, are likely to merge some neighboring gray levels into a single one in the output image, and thus the processing of histogram mapping will decrease (or at most preserve) the discrete entropy of the histogram undoubtedly. So in a discrete case of BPHEME, theoretically speaking, BPHEME may not maximize the discrete entropy, the mean brightness preserving output image with maximum discrete entropy is certainly the input one, and thus the enhancement makes no sense. Though BPHEME does not maximize the discrete entropy, the continuous variable's differential entropy defined by (10) can still be maximized by BPHEME.

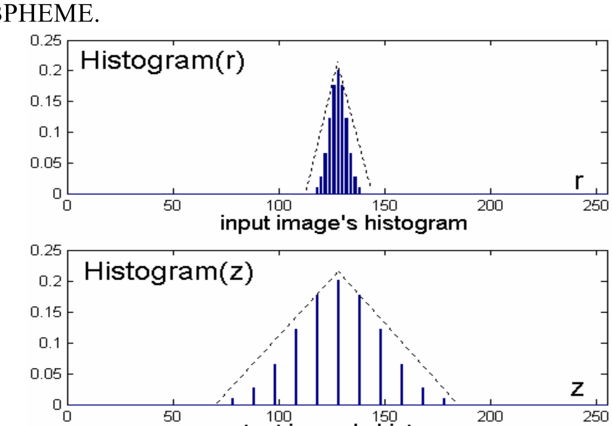

Fig.3 Equal discrete entropy may cause different image quality.

The formulations in section III-B, together with all HS methods, are all based on the assumption that the gray level be continuous and the histogram line can split into more than one, thus we can obtain exactly the same histogram as the desirable

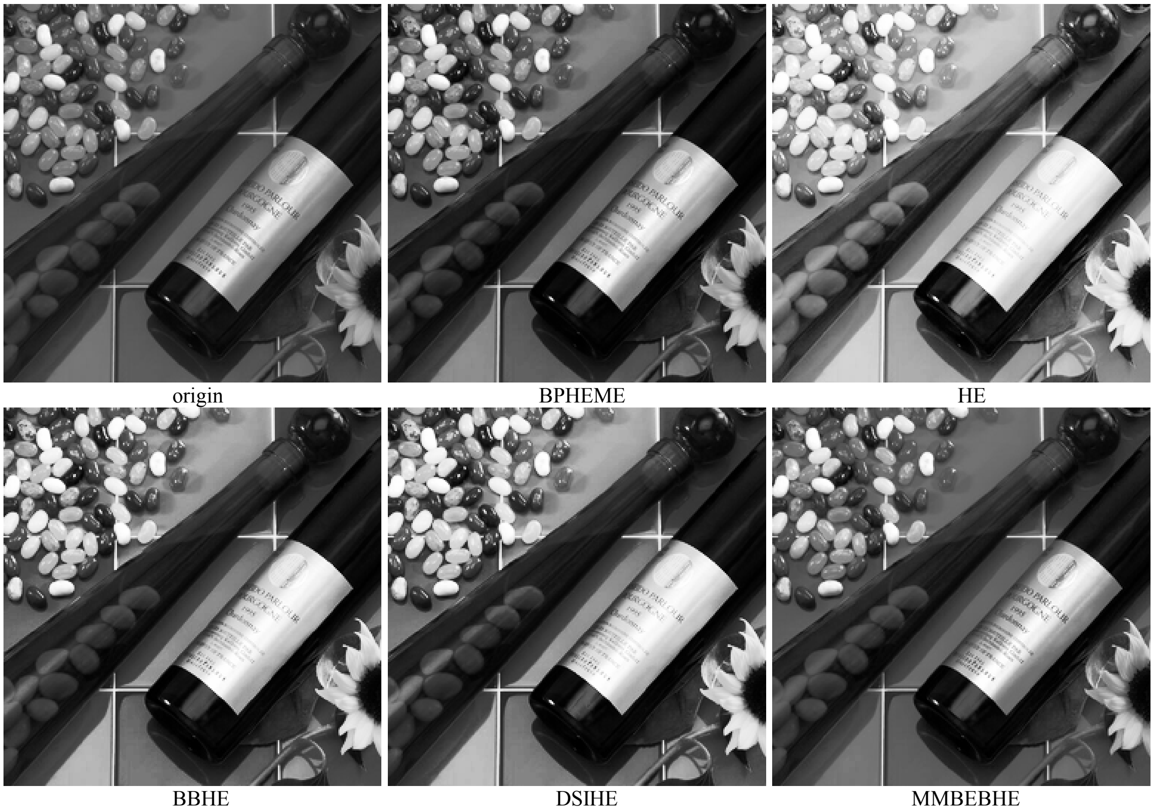

Fig.4 Enhancement for *bottle* based on HE, BBHE, DSIHE, MMBEBHE and BPHEME.

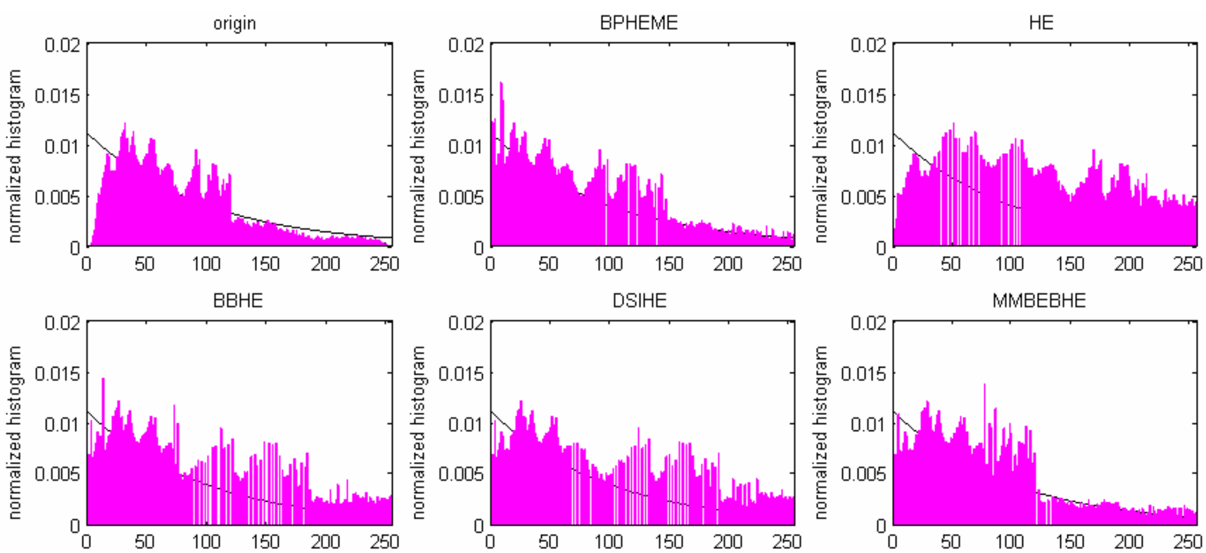

Fig.5 Histograms of each enhanced image *bottle* based on aforementioned algorithms in Fig.4. The line plot in each sub-figure is the desirable histogram of BPHEME.

one. The whole contrast is much more interpretable using continuous variable's differential entropy than using the discrete variable entropy, as shown in Fig.3. The input image has the gray levels in a small interval, and the output image is to stretch the gray levels in a larger interval. Thus they have the same discrete entropy. However, the output is obviously "enhanced" due to its larger dynamic range than the input. In a continuous view, see the dotted line in Fig.3, the histogram of the output image is more flat and the gray level of a pixel is more uncertain than the input image, so the differential entropy of the output is larger than the input, and thus it may be an enhanced version of the input image.

IV. Experimental Results

A. Experiments of HE, BBHE, DSIHE MMBEBHE and BPHEME

In this section, we present some experimental results of our proposed method, together with HE, BBHE, DSIHE and MMBEBHE for comparison.

The source image for the first experiment is *bottle*. The source image, together with the results based on HE, BBHE, DSIHE, MMBEBHE and BPHEME, is shown in Fig.4. From Fig.4, we can see that the results based on HE, BBHE and DSIHE seem to have larger contrast than the ones based on MMBEBHE and BPHEME. However, the mean brightness of HE, BBHE and DSIHE deviate very much from the original image's. The mean brightness of *bottle*, BPHEME, HE, BBHE, DSIHE and MMBEBHE are 77.27, 77.93, 128.37, 93.23, 96.70 and 81.04 respectively, as listed in Table I. BPHEME preserves the mean brightness very well. The histogram of the six images are shown in Fig.5, and the desirable histogram of BPHEME is superimposed on each one denoted by the line plot. Fig.5 shows clearly that BPHEME is in well accordance with the desirable one, and MMBEBHE can also preserve the mean brightness well to some extent. Different output histograms match the desired one of BPHEME differently, and result in different brightness preservation in different algorithms.

For the aforementioned five algorithms, we cut the same part of the results, together with the original image, as shown in Fig.6, which is focusing on the the text of the bottle mark. We can see that the last two characters "NE" in HE, BBHE and DSIHE are not very clear to recognize, however the same characters can be easily recognized in the results based on MMBEBHE and the proposed BPHEME.

Fig.6 Corresponding piece of the images inFig.4, showing the text on the bottle.

More examples are presented. The source images are *F16*, *Einstein*, *house* and *girl*. These four groups of experiments are shown as Fig.7-Fig.10. The five algorithms are implemented to these sources respectively.

The mean brightness of all these results are computed and listed in Table I. As to each source image, it can be found that BPHEME reaches the minimum absolute mean brightness error (AMBE), and MMBEBHE is the runnerup. AMBE is defined as

$$\mathrm{AMBE}(X,R)=|E(X)-E(R)| \tag{18}$$

where $E(.)$ denotes the expectation (i.e., mean), R is the referenced image, and here R represents the original image.

We define the mean AMBE (MAMBE) as

$$\mathrm{MAMBE}=\frac{1}{N}\sum_{i=1}^{N}\mathrm{AMBE}(X_i,R_i) \tag{19}$$

Since we have five source images here, N=5. MAMBE generally represents the brightness preserving ability of an algorithm. Using this measurement, the MAMBE of HE, BBHE, DSIHE, MMBEBHE and BPHEME are respectively 39.75, 14.95, 17.24, 1.16 and 0.33. That is to say, BPHEME

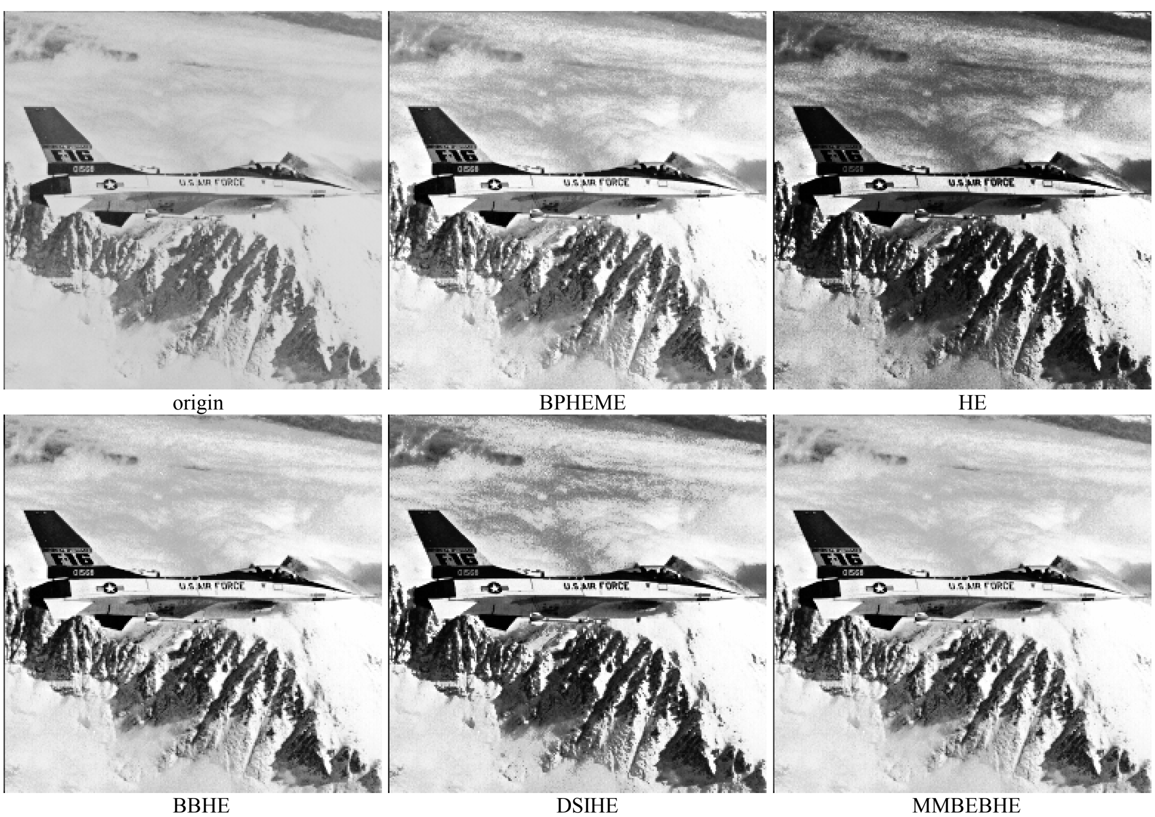

Fig.7 Enhancement for *F16* based on HE, BBHE, DSIHE, MMBEBHE and BPHEME.

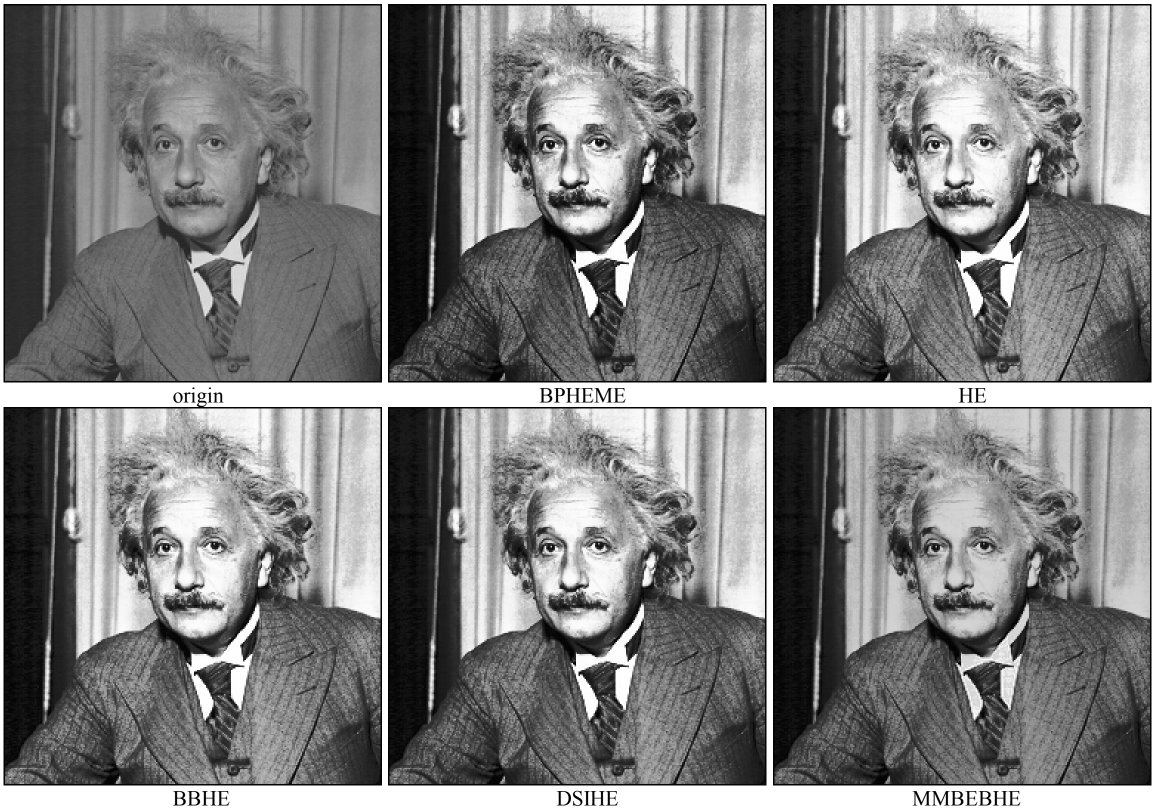

Fig.8 Enhancement for *Einstein* based on HE, BBHE, DSIHE, MMBEBHE and BPHEME.

Fig.9 Enhancement for *house* based on HE, BBHE, DSIHE, MMBEBHE and BPHEME.

outperforms the existing histogram processing methods in the sense that BPHEME can preserve the mean brightness quite well, which is very suitable for consumer electronics such as TV.

In the view of visual quality, from Fig.4, Fig.7-Fig.10, it is shown that BPHEME does enhance the original image, and it is at least not worse than the existing methods.

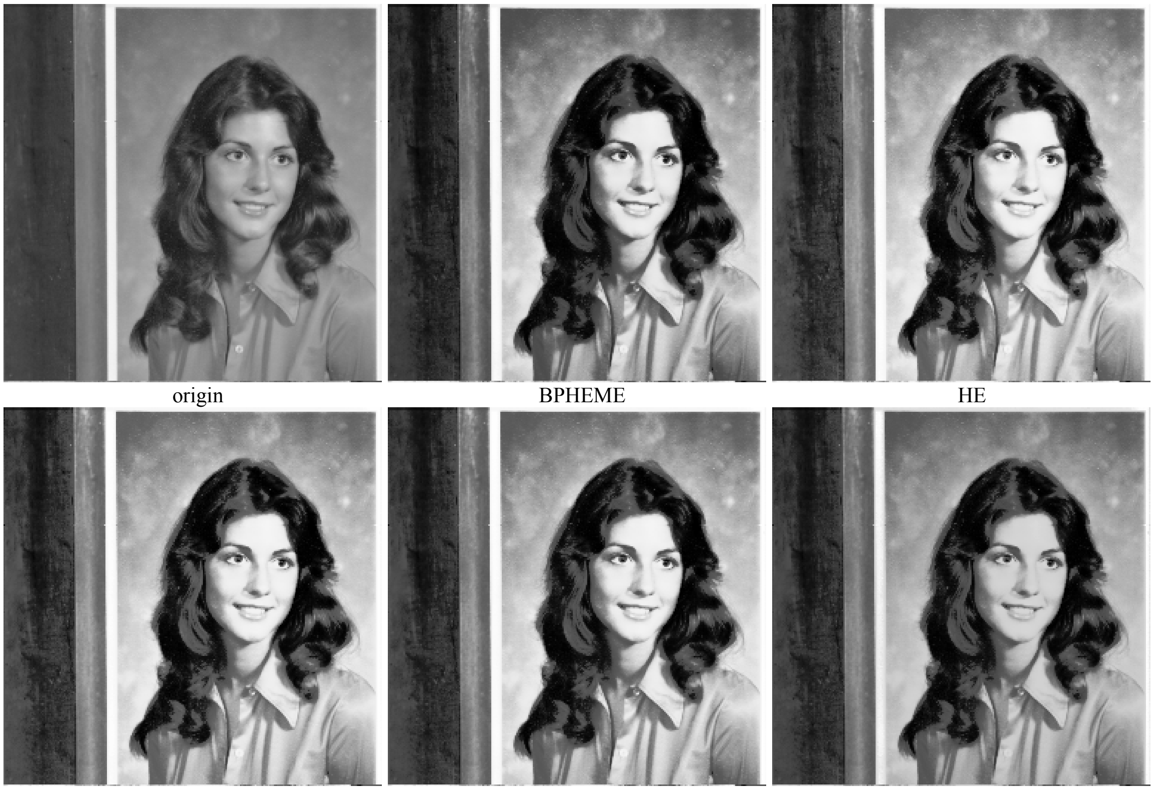

Fig.10 Enhancement for *girl* based on HE, BBHE, DSIHE, MMBEBHE and BPHEME.

TABLE I
MEAN BRIGHTNESS OF THE EXPERIMENTAL RESULTS

Image	Origin	BPHEME	HE	BBHE	DSIHE	MMBEBHE
Bottle	77.27	**77.93**	128.37	93.23	96.70	81.04
F16	179.19	**179.32**	129.40	179.68	162.81	179.62
Einstein	107.75	**108.00**	128.83	126.51	119.24	108.31
house	67.54	**67.62**	128.94	91.63	95.64	67.98
girl	113.13	**113.68**	128.52	128.58	123.93	113.72

TABLE II
DISCRETE ENTROPY OF THE EXPERIMENTAL RESULTS (:BITS)

Image	Origin	BPHEME	HE	BBHE	DSIHE	MMBEBHE
Bottle	7.45	7.33	7.20	7.26	7.28	7.30
F16	6.67	6.57	6.43	6.58	6.55	6.57
Einstein	6.88	6.71	6.75	6.74	6.73	6.70
house	6.92	6.62	6.74	6.68	6.68	6.61
girl	7.24	7.03	7.02	7.03	7.03	7.02

Though the discrete entropy does not necessarily represent the enhancement quality, it can depict the richness of details to some extent. In Table II, we presents the discrete entropy of each results, which is defined as

$$\mathrm{ENT}(q) = -\sum_{i=0}^{255} q(i)\log_2 q(i) \quad \text{(bits)} \qquad (20)$$

where $q(i)$ is the normalized probability of the gray level i.

In Table II, we can see that for each source image, the resultant discrete entropy based on BPHEME is equal to, or even slightly higher than MMBEBHE. When the source images *Einstein* and *house* are considered, the discrete entropy of BPHEME falls behind HE, BBHE and DSIHE, however, for these two test images, HE, BBHE and DSIHE lost the mean brightness seriously.

B. *Potential Application of BPHEME*

The theory and experimental results show that BPHEME preserves the mean brightness exactly, the minor error comes from the discretizaion error and the non-cleavability of the histogram line. In applications, *maybe* there is some threshold of mean brightness change for people's perception to bear. That is, when enhancement is implemented, the mean brightness is permitted to vary in a small interval $[\mu_r - \delta, \mu_r+\delta]$. This problem can also be solved using BPHEME with minor modification. We modify the target mean brightness μ as

$$\mu = \arg\min_{\mu}\{|\mu - 0.5| : \mu_r - \delta \le \mu \le \mu_r + \delta\}$$
$$= \begin{cases} 0.5 & ,\text{if} \quad 0.5 \in [\mu_r - \delta, \mu_r + \delta] \\ \mu_r + \delta & ,\text{if} \quad \mu_r + \delta < 0.5 \\ \mu_r - \delta & ,\text{if} \quad \mu_r - \delta > 0.5 \end{cases} \tag{21}$$

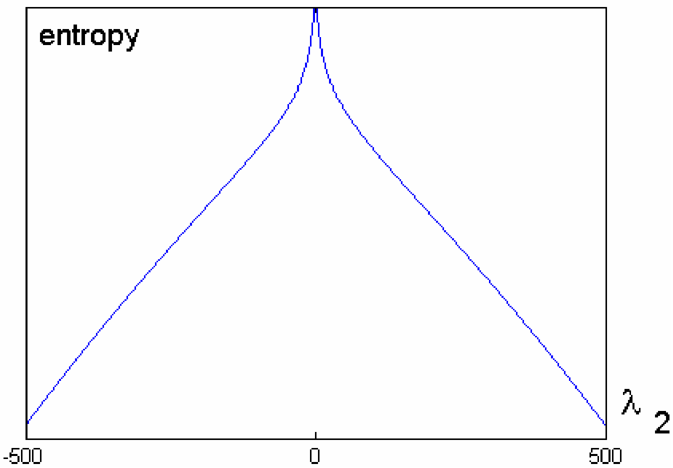

Fig.11 Relation between the entropy and the parameter λ_2.

To choose the target mean brightness μ like (21) is reasonable. Because the mean brightness is monotonically increasing with λ_2 (see Fig.2), and the relation between entropy and λ_2 is shown as Fig.11, the target brightness chosen as eq.(21) can ensure the maximum output entropy when mean brightness is constrained within a small interval.

V. CONCLUSION

In this paper, we present a novel case of histogram specification, which can preserve the mean brightness with maximum entropy (BPHEME), in a continuous view. BPHEME tries, using variational approach, to find the optimal histogram, which has the maximum differential entropy under the mean brightness constraint, and then implements the histogram specification under the instruction of that desired histogram. Experimental results show that BPHEME can enhance the image quite well when preserving the mean brightness, which is very suitable for consumer electronics such as TV. BPHEME may find its potential applications considering the tolerant threshold for the human visual systems.

ACKNOWLEDGEMENT

The authors would like to thank Mr. Liangliang Cao (Chinese University of Hong Kong) for occasional and useful discussion.

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