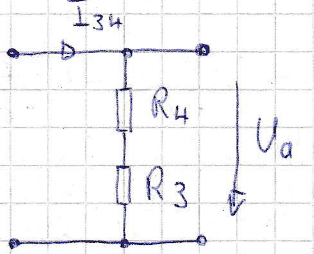
Regelungstechnik

Aufgabe 1.4.)

1.) $u_d = 0$ $\quad i_{einp} = 0$ $\quad i_{einn} = 0$



$$R_4 = (K-1) \cdot R_3$$

$$I_{34} = \frac{U_a}{R_3 + R_4}$$

$$U_{R3} = R_3 \cdot I_{34} = \frac{U_a}{R_3 + R_4} \cdot R_3$$

$$= \frac{R_3}{R_3 + (K-1) \cdot R_3} \cdot U_a = \frac{1}{1 + K - 1} \cdot U_a$$

$$= \frac{1}{K} \cdot U_a \Rightarrow U_a = U_{R3} \cdot K \Rightarrow U_{C2} \cdot K$$

$$u_d = 0 \Rightarrow U_{R3} = U_{C2}$$

Masche:

$$U_e - U_{R1} - U_{R2} - U_{C2} = 0$$

Knoten:

$$I_{R1} - I_{C1} - I_{R2} = 0$$

$$I_{R2} + I_{R5} - I_{C2} = 0$$

$$I_{R4} - I_{R3} = 0$$

$$\dot{U}_C = \frac{1}{C} \cdot i_C$$

$$\dot{U}_{C2} = \frac{1}{C_2} \cdot I_{C2} = \frac{1}{C_2} \cdot I_{R2} + \frac{1}{C_2} \cdot I_{R5}$$

$$I_{R2} = \frac{U_{R2}}{R_2} = \frac{(U_{C1} + U_a) - U_{C2}}{R_2}$$

$$I_{R5} = \frac{U_{R5}}{R_5} = \frac{U_S - U_{C2}}{R_5} \qquad R_5 = R_2$$

$$C_2 \cdot \dot{U}_{C2} = I_{R2} + I_{R5} = \frac{(U_{C1} + U_a) - U_{C2} + U_S - U_{C2}}{R_2}$$

$$\dot{U}_{C2} = \frac{U_{C1} + U_a - 2U_{C2} + U_S}{C_2 R_2} = \frac{U_{C1} + (K-2) \cdot U_{C2} + U_S}{C_2 R_2}$$

$$\dot{U}_{C1} = \frac{1}{C_1} I_{C1} = \frac{1}{C_1} \cdot I_{R1} - \frac{1}{C_1} \cdot I_{R2}$$

$$I_{R1} = \frac{U_e - (U_{C1} + U_a)}{R_1}$$

$$\dot{U}_{C1} = \frac{1}{C_1} \cdot \frac{U_e - (U_{C1} + U_a)}{R_1} - \frac{1}{C_1} \cdot \frac{(U_{C1} + U_a) - U_{C2}}{R_2}$$

$$= \frac{1}{C_1} \cdot \left(\frac{U_e - U_{C1} - K \cdot U_{C2}}{R_1} - \frac{U_{C1} + K \cdot U_{C2} - U_{C2}}{R_2} \right)$$

$$= \frac{1}{C_1} \cdot \left(\frac{U_e}{R_1} - \frac{U_{C1}}{R_1} - \frac{K \cdot U_{C2}}{R_1} - \frac{U_{C1}}{R_2} - \frac{K \cdot U_{C2}}{R_2} + \frac{U_{C2}}{R_2} \right)$$

$$= \frac{1}{C_1} \cdot \left(\frac{U_e}{R_1} - U_{C1} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - U_{C2} \cdot \left(\frac{K}{R_1} + \frac{K}{R_2} - \frac{1}{R_2} \right) \right)$$

$$= \frac{1}{C_1} \cdot \left(\frac{U_e}{R_1} - U_{C1} \cdot \left(\frac{R_1 + R_2}{R_1 \cdot R_2} \right) - U_{C2} \cdot \left(\frac{K \cdot R_2 + (K-1) \cdot R_1}{R_1 \cdot R_2} \right) \right)$$

$$\dot{U}_{C1} = \frac{1}{C_1 R_1} \cdot U_e - \frac{R_1 + R_2}{C_1 R_1 R_2} \cdot U_{C1} - \frac{K \cdot (R_1 + R_2) - R_1}{C_1 R_1 \cdot R_2} \cdot U_{C2}$$

$V_0 = U_e \quad V_1 = U_{C1} + U_a \quad V_2 = U_{C2} \quad V_3 = U_a$

$V_4 = U_{R3} \quad V_5 = U_S$

$$I_{C1} = C \cdot \dot{U}_{C1} = C \cdot \frac{d}{dt}(V_1 - V_3)$$

$$I_{C2} = C \cdot \dot{U}_{C2} = C \cdot \frac{d}{dt}(V_2)$$

$$I_{R1} = \frac{V_0 - V_1}{R_1} \qquad I_{R2} = \frac{V_1 - V_2}{R_2} \qquad I_{R3} = \frac{V_4}{R_3}$$

$$I_{R4} = \frac{V_3 - V_4}{R_4} \qquad I_{R5} = \frac{V_5 - V_2}{R_2}$$

$$\begin{bmatrix} \dot{U}_{C1} \\ \dot{U}_{C2} \end{bmatrix} = \begin{bmatrix} -\frac{R_1 + R_2}{C_1 R_1 R_2} & -\frac{K \cdot (R_1 + R_2) - R_1}{C_1 R_1 R_2} \\ \frac{1}{C_2 R_2} & \frac{K - 2}{C_2 R_2} \end{bmatrix} \cdot \begin{bmatrix} U_{C1} \\ U_{C2} \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1 R_1} & 0 \\ 0 & \frac{1}{C_2 R_2} \end{bmatrix} \cdot \begin{bmatrix} U_e \\ U_S \end{bmatrix}$$

$$U_a = \begin{bmatrix} 0 & K \end{bmatrix} \cdot \begin{bmatrix} U_{C1} \\ U_{C2} \end{bmatrix}$$

Aufgabe 1.4)

2.) Linearisieren des Dglsystems 1. Ordnung um die Ruhelage $x_R = [U_{C1,R} \quad U_{C2,R}]^T$

$u_R = U_{e,R} \quad d_R = U_{S,R}$

$$\Delta\dot{x} = A \cdot \Delta x + b_u \cdot \Delta u + b_d \, \Delta d$$

$$\Delta y = c^T \Delta x + d_u \cdot \Delta u + d_d \cdot \Delta d$$

$$A_{Lin} = \frac{\partial}{\partial x_R} f(x_R, u, d) = \begin{bmatrix} -\cancel{\frac{R_1+R_2}{C_1 R_1 R_2}} \frac{\partial}{\partial U_{C1}} \dot{U}_{C1} & -\frac{K(R_1+R_2)-R_1}{C_1 R_1 R_2} \\ \frac{1}{C_2 R_2} & \frac{K-2}{C_2 R_2} \end{bmatrix}$$

$$b_{u\,Lin} = \frac{\partial}{\partial u_R} f(x, u_R, d) = \begin{bmatrix} \frac{1}{C_1 R_1} \\ 0 \end{bmatrix}$$

$$b_{d\,Lin} = \frac{\partial}{\partial d_R} f(x, u, d_R) = \begin{bmatrix} 0 \\ \frac{1}{C_2 R_2} \end{bmatrix}$$

$$c^T_{Lin} = \frac{\partial}{\partial x_R} h(x_R, u, d) = [0 \quad K]$$

$$d_{u\,Lin} = \frac{\partial}{\partial u_R} h(x_R, u_R, d) = 0$$

$$d_{d\,Lin} = \frac{\partial}{\partial d_R} h(x, u, d_R) = 0$$

$$A^{[1,1]}_{Lin} = \frac{\partial}{\partial U_{C1}} \dot{U}_{C1} \qquad i = \frac{dQ}{dt} \qquad Q(U)$$

$$= \frac{\partial}{\partial U_{C1}} \left(\frac{1}{C_1} \cdot \underbrace{\left(\frac{U_e}{R_1} - U_{C1} \cdot \frac{R_1+R_2}{R_1 \cdot R_2} - U_{C2} \cdot \frac{K \cdot (R_1+R_2) - R_1}{R_1 \cdot R_2} \right)}_{\hat{=}\, \dot{U}_{C1} \cdot C_1} \right)$$

Produktregel:

$$= \frac{C_1'}{C_1^2} \cdot \dot{U}_{C1} \cdot \cancel{C_1} + \frac{1}{C_1} \cdot \left(-\frac{R_1+R_2}{R_1 \cdot R_2} \right) = \frac{1}{C_1} \left(C_1' \cdot \dot{U}_{C1} - \frac{R_1+R_2}{R_1 \cdot R_2} \right)$$

Aufgabe 1.4.)

3.) Ruhelagen bestimmen zu $x_R = [U_{C1,R} \quad U_{C2,R}]^T$

$u_R = U_{e,R} \qquad d_R = U_{S,R}$

x_R ist eine Ruhelage ~~wenn~~ des Systems

$\dot{x} = f(x)$, wenn $f(x_R) = 0$

$$\frac{1}{C_1 R_1} \cdot U_{e,R} - \frac{R_1+R_2}{C_1 R_1 \cdot R_2} \cdot U_{C1,R} - \frac{K\cdot(R_1+R_2)-R_1}{C_1 R_1 R_2} \cdot U_{C2,R} = 0$$

$$U_{e,R} - \frac{R_1+R_2}{R_2} \cdot U_{C1,R} - \frac{K\cdot(R_1+R_2)-R_1}{R_2} \cdot U_{C2,R} = 0$$

$$\Rightarrow U_{C1,R} = \frac{U_{e,R} - \frac{K\cdot(R_1+R_2)-R_1}{R_2} \cdot U_{C2,R}}{\frac{R_1+R_2}{R_2}}$$

$$U_{C1,R} = \frac{R_2 \cdot U_{e,R} - (K\cdot(R_1+R_2)-R_1)\cdot U_{C2,R}}{R_1+R_2}$$

$$\frac{U_{C1,R} + (K-2)\cdot U_{C2,R} + U_{S,R}}{C_2 R_2} = 0$$

$$U_{C1,R} = -(K-2)\cdot U_{C2,R} - U_{S,R}$$

$$\Rightarrow -(K-2)\cdot U_{C2,R} - U_{S,R} = \frac{R_2 \cdot U_{e,R} - (K\cdot(R_1+R_2)-R_1)\cdot U_{C2,R}}{R_1+R_2}$$

$$-(K-2)\cdot U_{C2,R}\cdot(R_1+R_2) - U_{S,R}\cdot(R_1+R_2) = R_2\cdot U_{e,R} - (K\cdot(R_1+R_2)-R_1)\cdot U_{C2,R}$$

$$-U_{C2,R}\cdot(K-2)\cdot(R_1+R_2) + U_{C2,R}\cdot(K\cdot(R_1+R_2)-R_1) = R_2\cdot U_{e,R} + (R_1+R_2)\cdot U_{S,R}$$

$$U_{C2,R}\cdot(K\cdot(R_1+R_2) - R_1 - (K-2)\cdot(R_1+R_2)) = R_2\cdot U_{e,R} + (R_1+R_2)\cdot U_{S,R}$$

$$U_{C2,R}\cdot(-R_1 + 2R_1 + 2R_2) = R_2\cdot U_{e,R} + (R_1+R_2)\cdot U_{S,R}$$

$$U_{C2,R} = \frac{R_2\cdot U_{e,R} + (R_1+R_2)\cdot U_{S,R}}{R_1+2R_2}$$

$$\cancel{U_{C1,R} = \frac{R_2\cdot U_{e,R} - (K\cdot(R_1+R_2)-R_1)\cdot \frac{R_2\cdot U_{e,R} + (R_1+R_2)\cdot U_{S,R}}{R_1+2R_2}}{R_1+R_2}}$$

Aufgabe 1.4.)

3.) Fortsetzung

$$U_{C1,R} = -(K-2)\cdot U_{C2,R} - U_{S,R}$$

$$U_{C1,R} = -(K-2)\cdot \frac{R_2 \cdot U_{e,R} + (R_1+R_2)\cdot U_{S,R}}{R_1 + 2R_2} - U_{S,R}$$

4.)

$$Q_1(U_{C1}) = \left(C_{1,ref} + k_{C1}\left(\frac{U_{C1}}{2} - U_{C1,ref}\right)\right) U_{C1}$$
$$= C_{1,ref}\cdot U_{C1} + \frac{k_{C1}}{2}\cdot U_{C1}^2 - k_{C1}\cdot U_{C1,ref}\cdot U_{C1}$$
$$= U_{C1}\cdot(C_{1ref} - k_{C1}\cdot U_{C1,ref}) + U_{C1}^2 \cdot \frac{k_{C1}}{2}$$

$$C_1 = Q_1'(U_{C1}) = C_{1ref} - k_{C1}\cdot U_{C1,ref} + k_{C1}\cdot U_{C1}$$
$$Q_1''(U_{C1}) = k_{C1}$$

5.) $P-T_2$ mit $G(s) = \frac{V}{1 + 2\xi(sT) + (sT)^2}$

$$G(s) = \frac{1371000}{s^2 + 1600s + 995900}$$
$$= \frac{\frac{1371000}{995900}}{1 + \frac{1600}{995900}s + \frac{1}{995900}s^2}$$

$$V = \frac{1371000}{995900}$$

$$T = \sqrt{\frac{1}{995900}}$$

$$\xi = \frac{1600}{995900\cdot 2T}$$