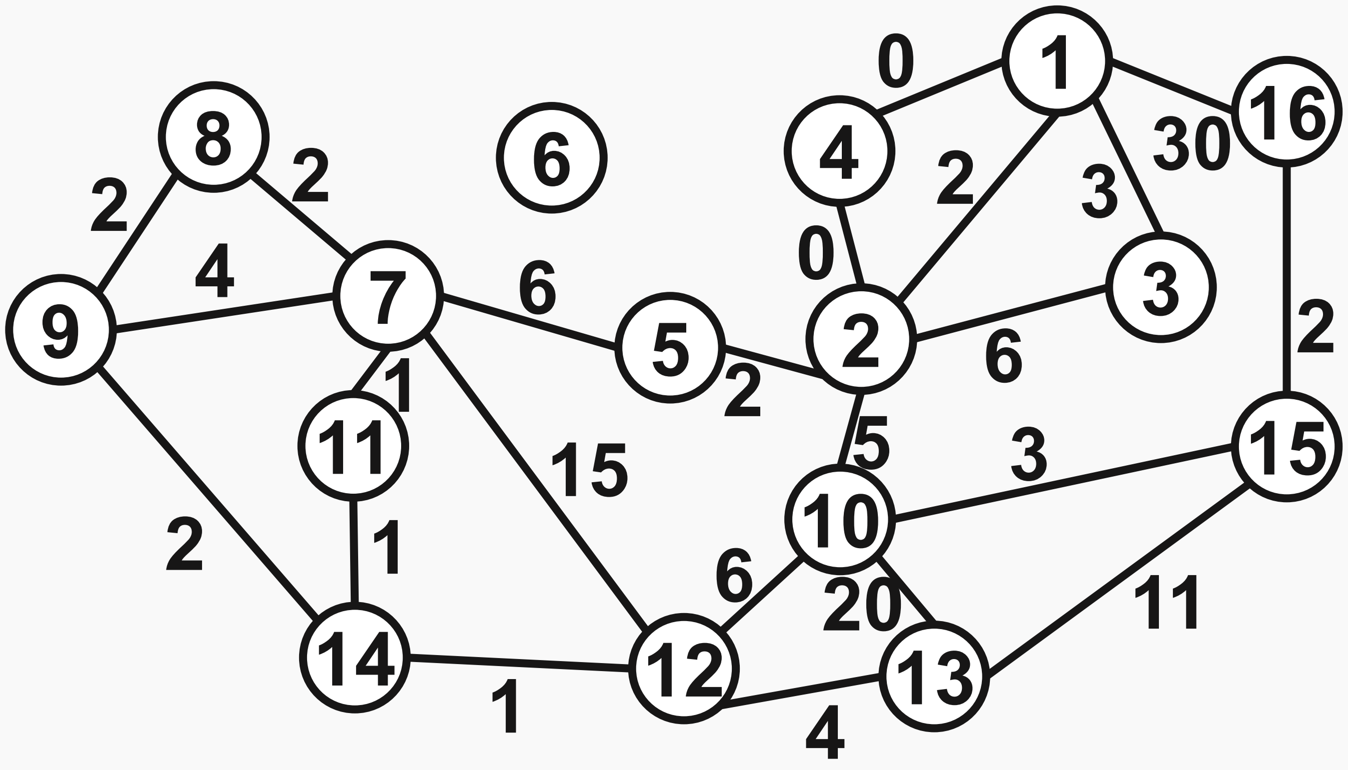
Structuri de date și algoritmi
Grafuri – Acoperire și Capacitate

Lect. Dr. Ing. Cristian Chilipirea – cristian.chilipirea@mta.ro

Arbori minimi de acoperire



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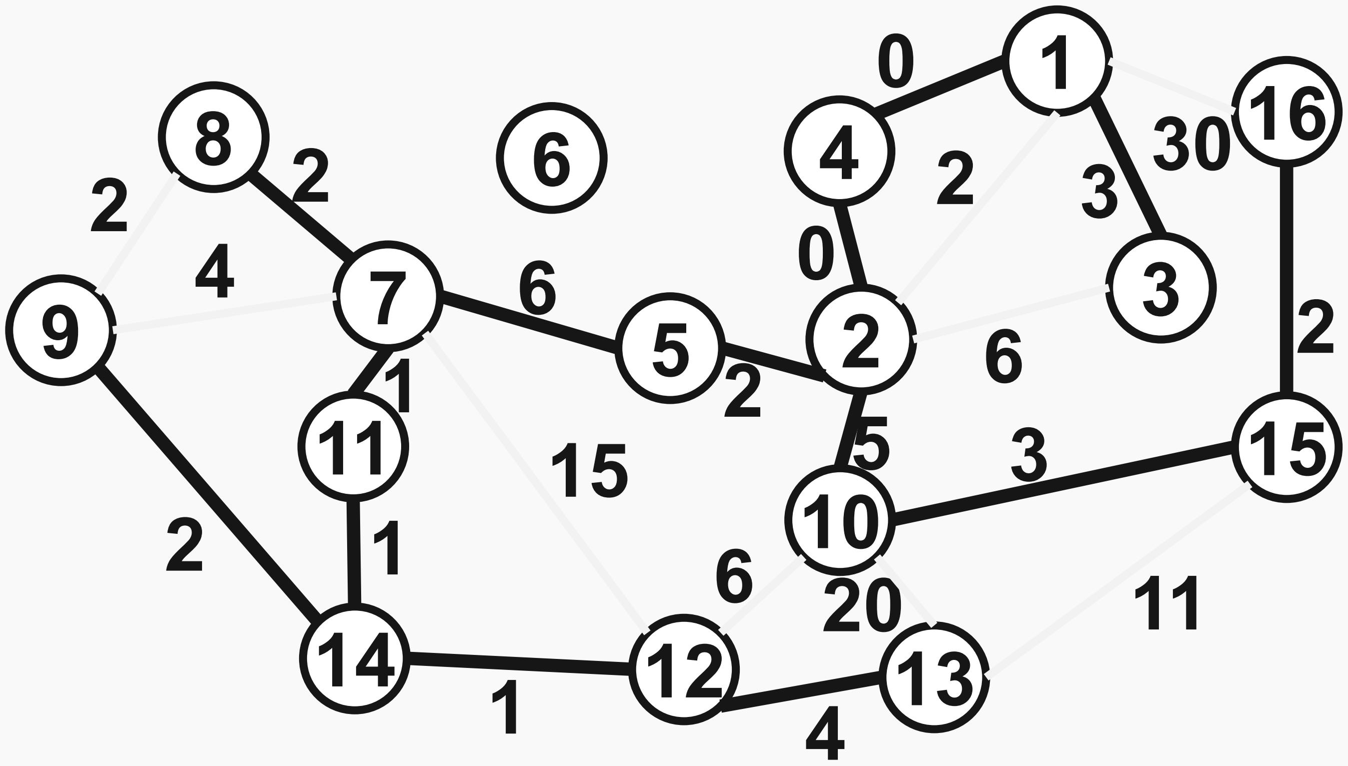
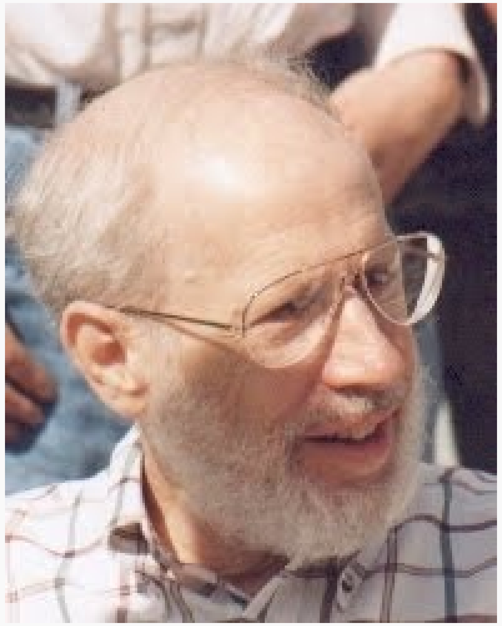
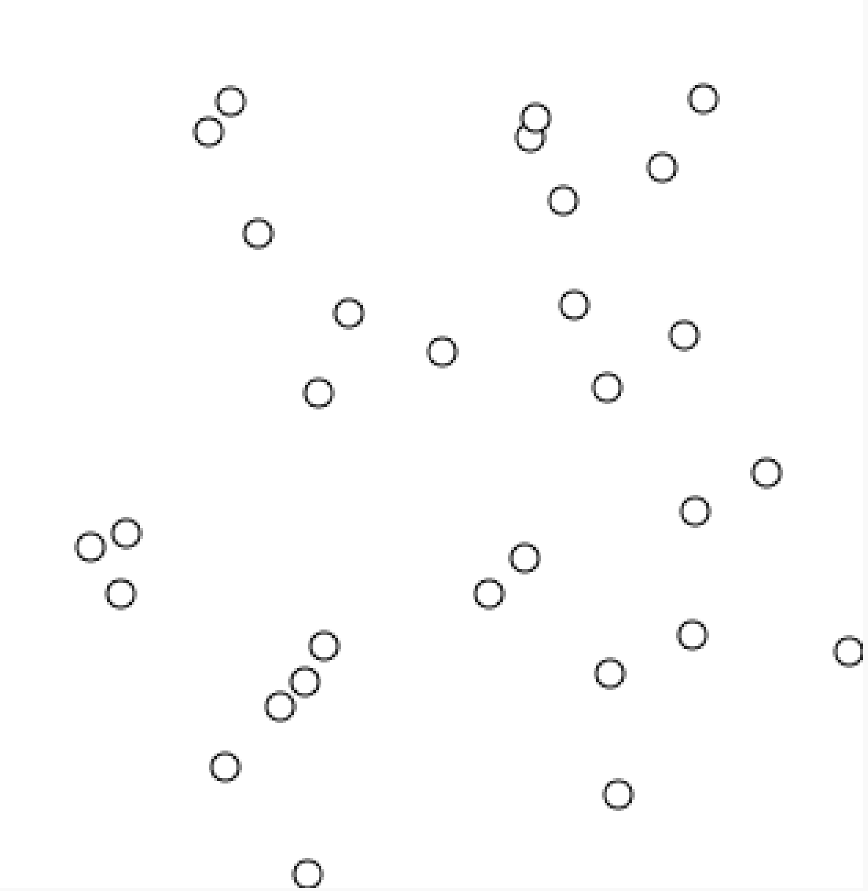


Algoritmul lui Kruskal (1956)

```
tree Kruskal(G) {
    sort(G.E); // sort by weight
    A = {};
    for each (node in G.V)
        Make_set(node);
    for each ((u, v) in G.E) {
        if (Find_set(u) != Find_set(v)) {
            A = A ∪ {(u, v)};
            Union(Find_set(u), Find_set(v));
        }
    }
    return A;
}
```



Algoritmul lui Kruskal



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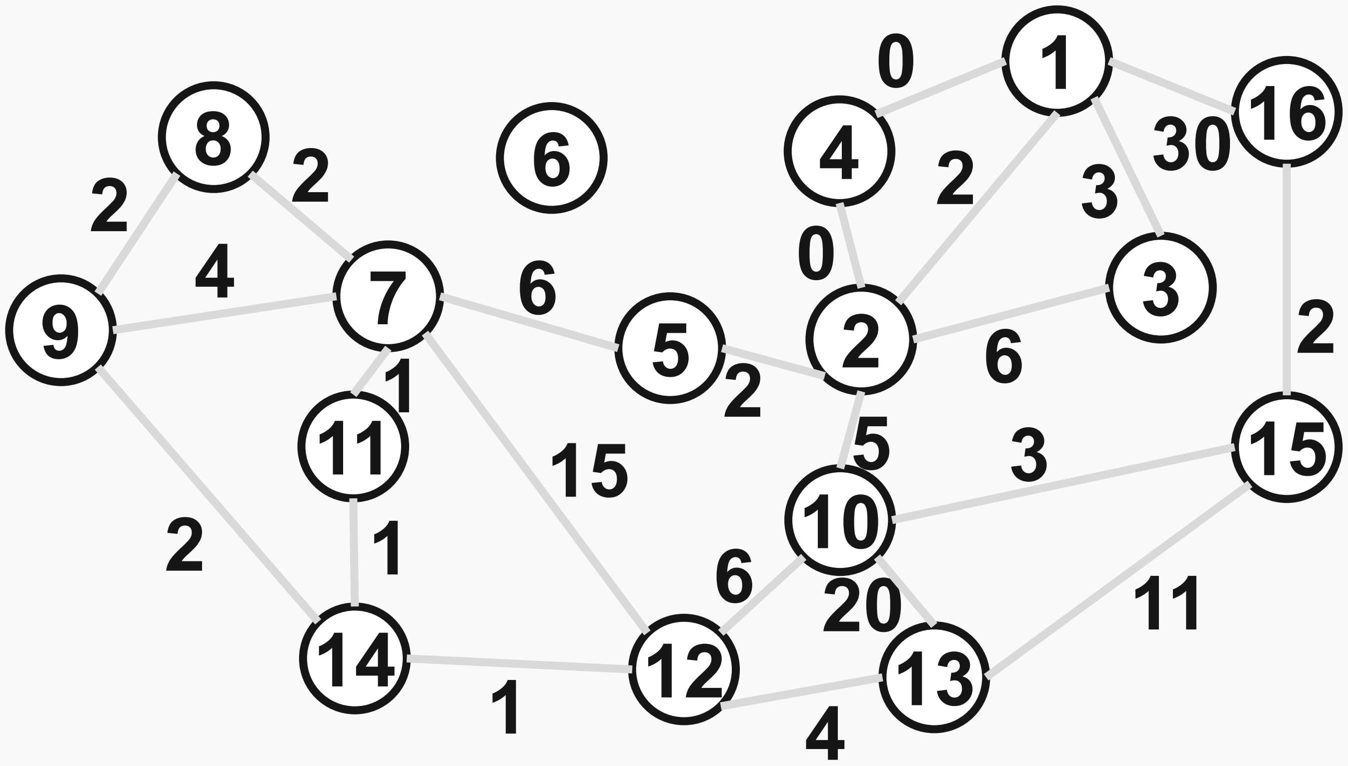

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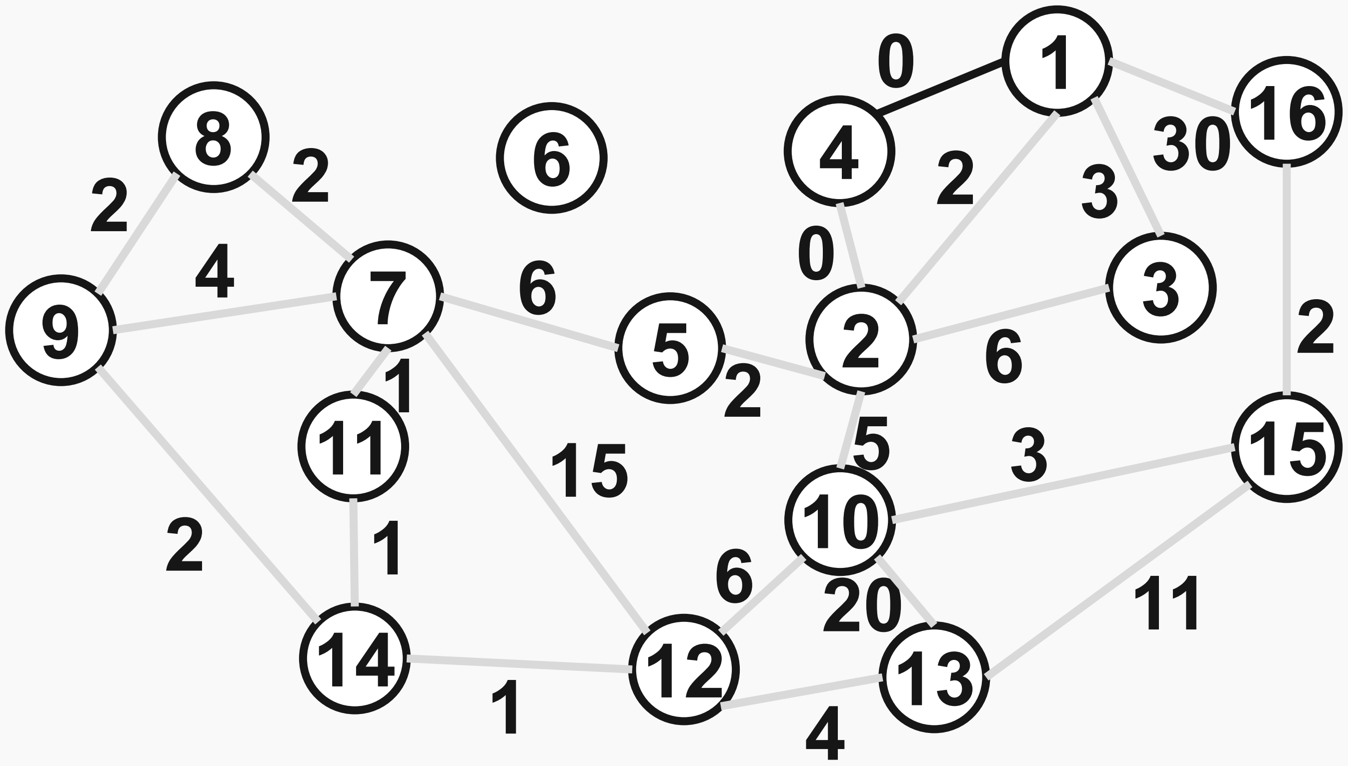
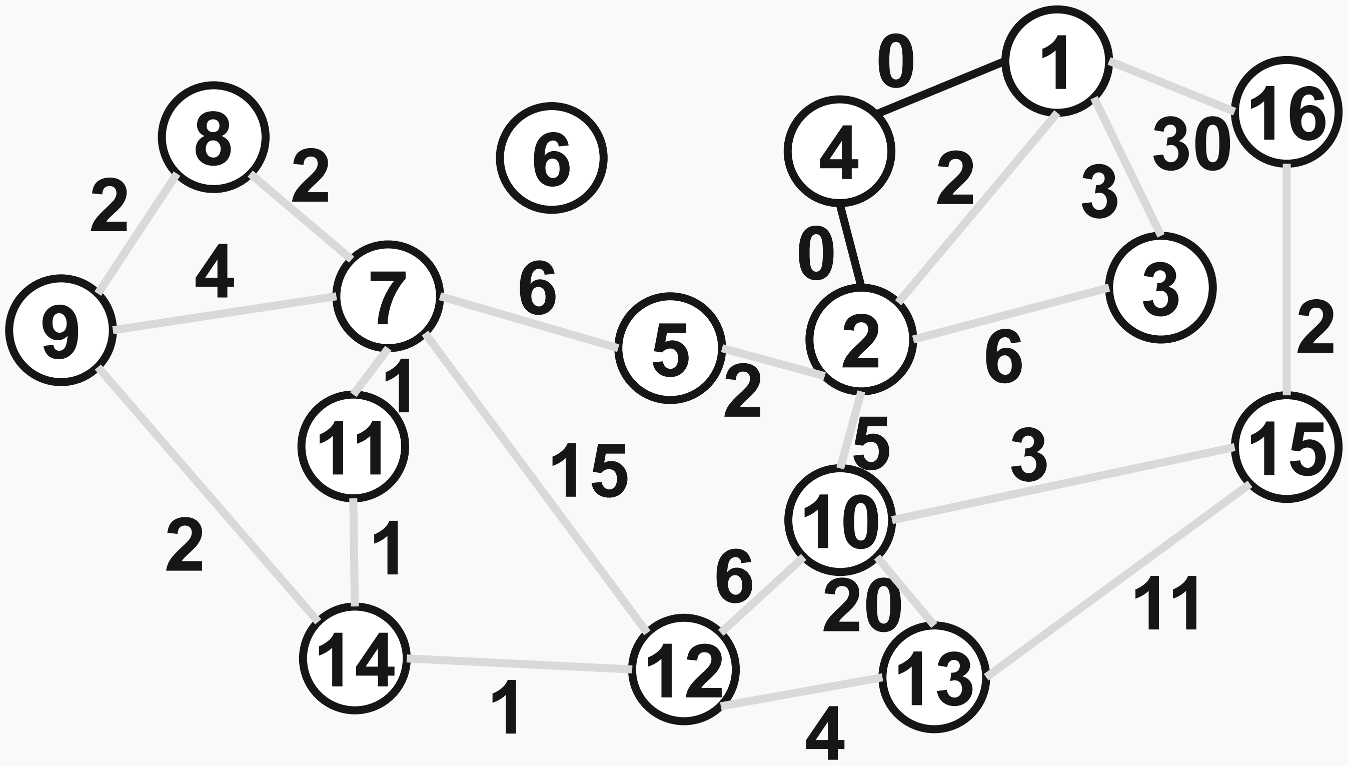


Algoritmul lui Kruskal



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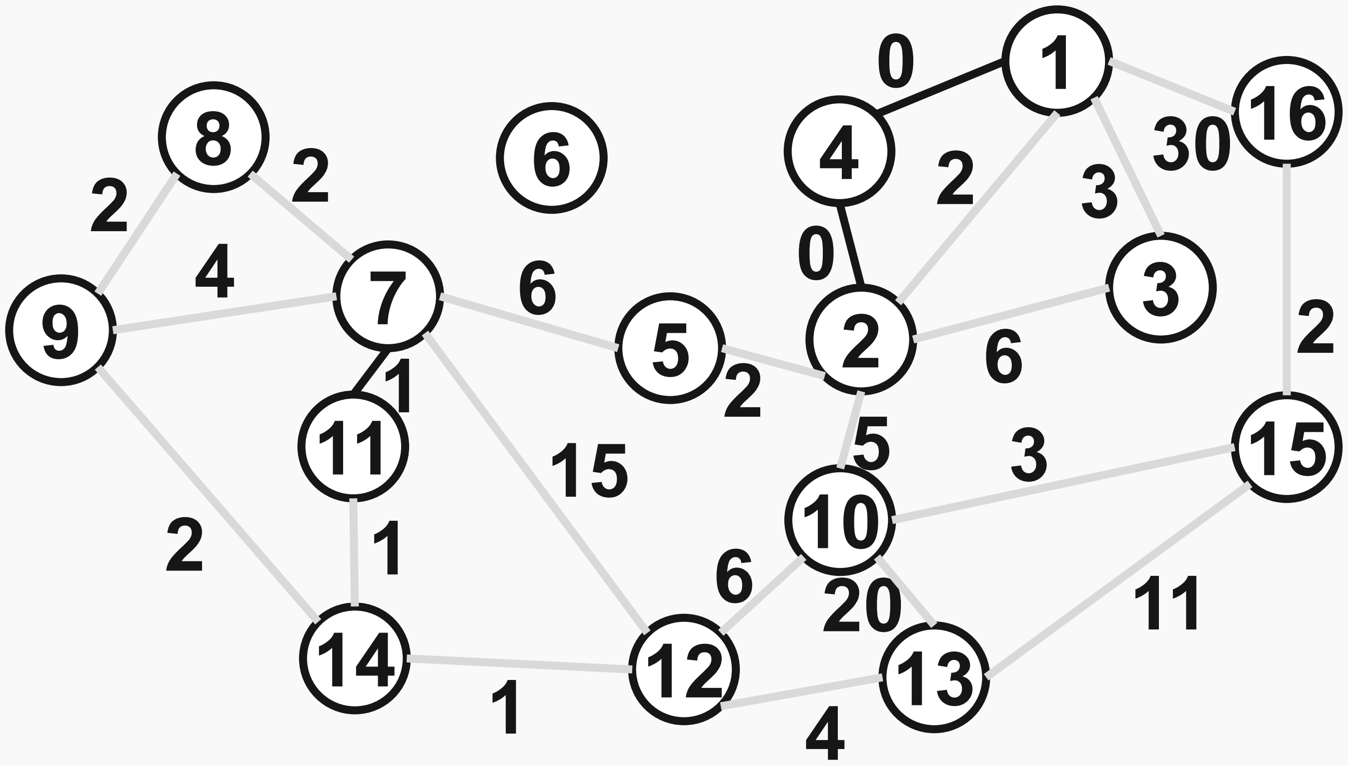

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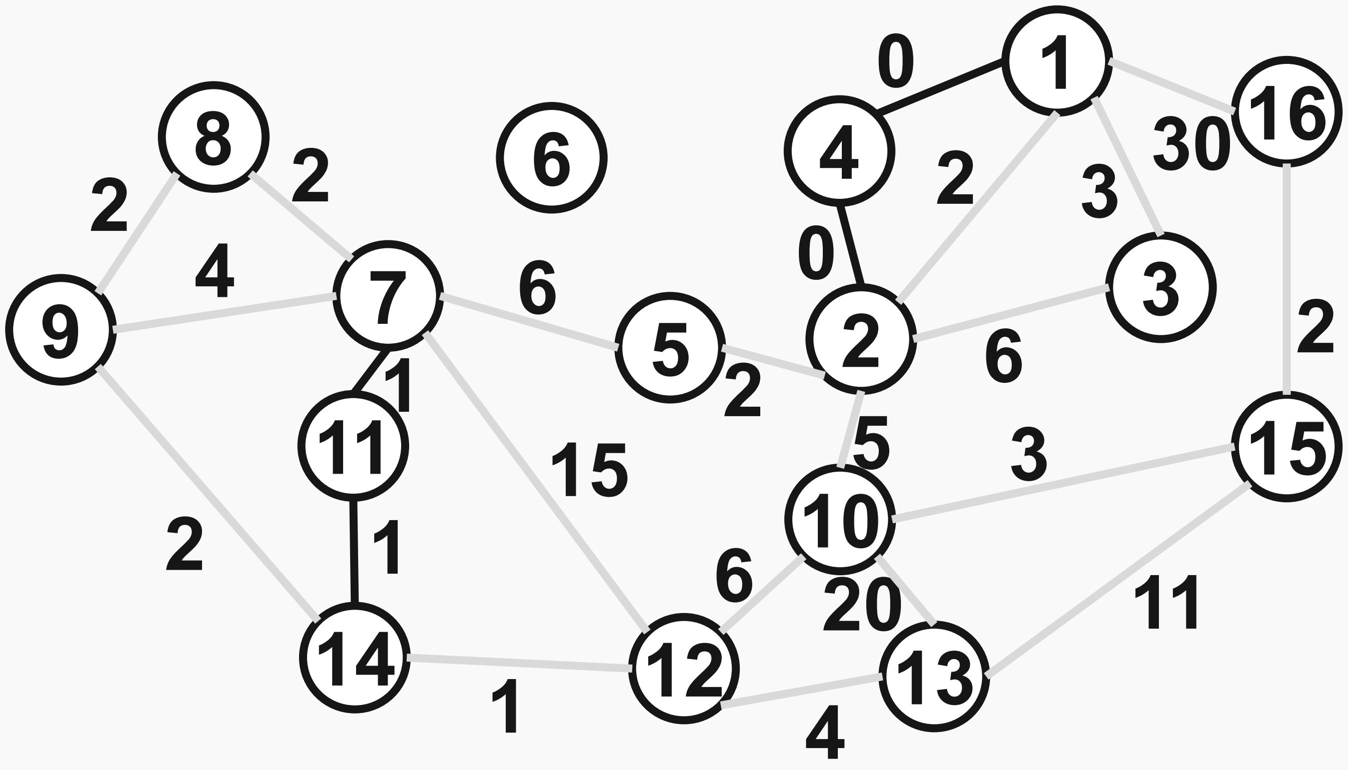

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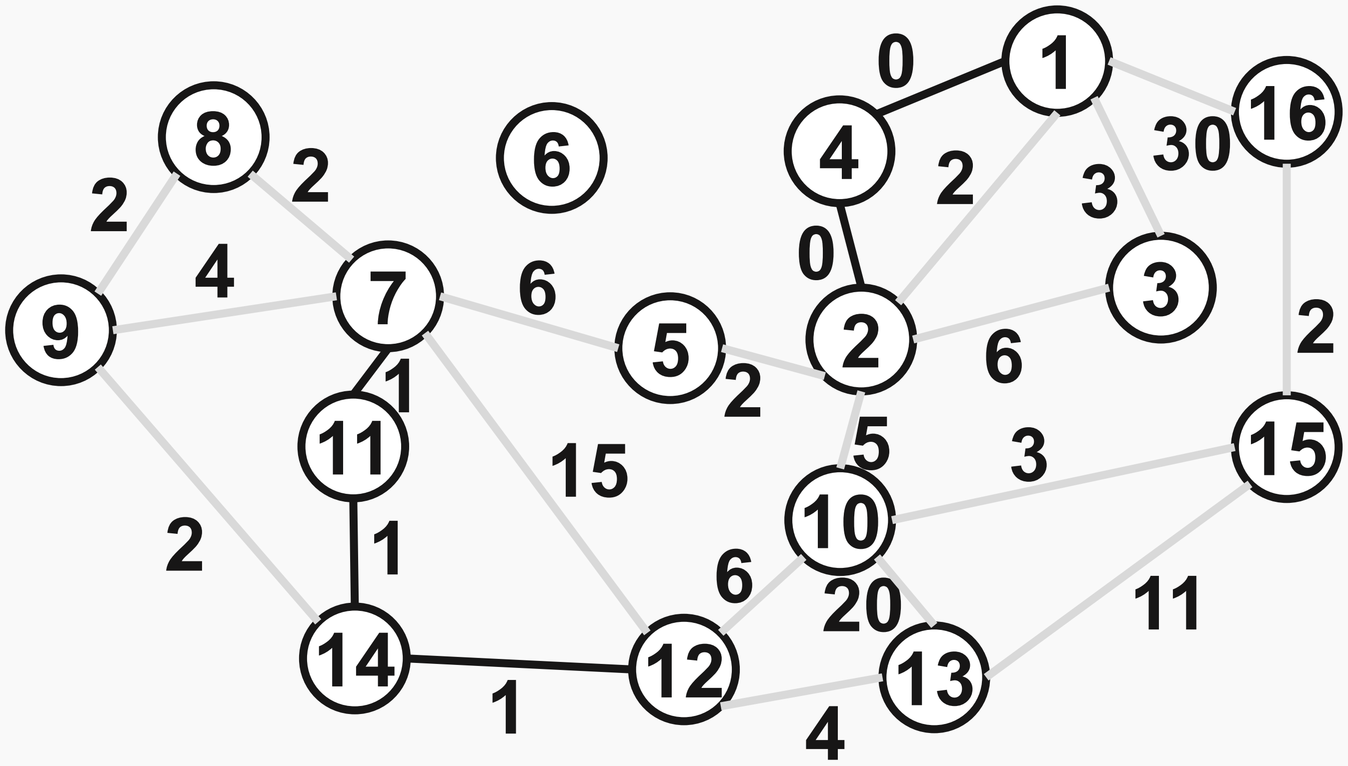

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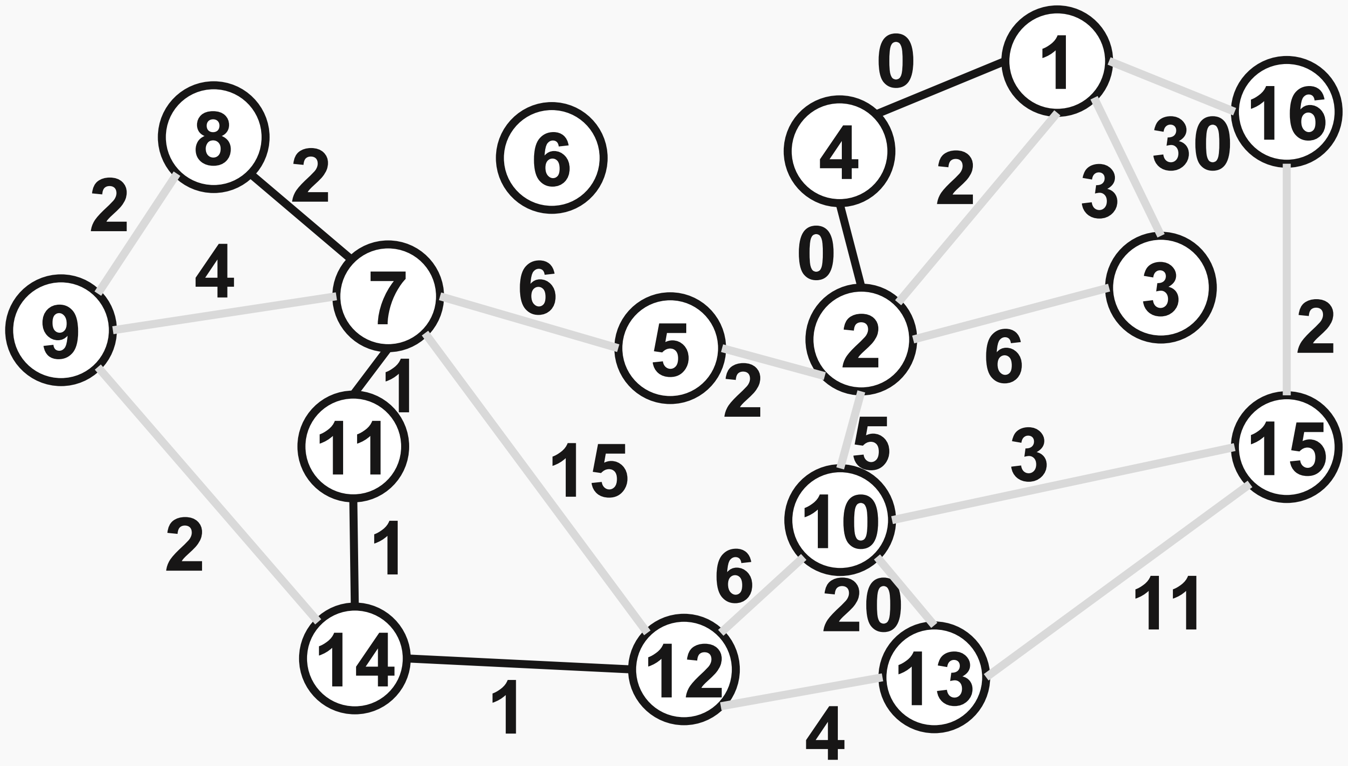

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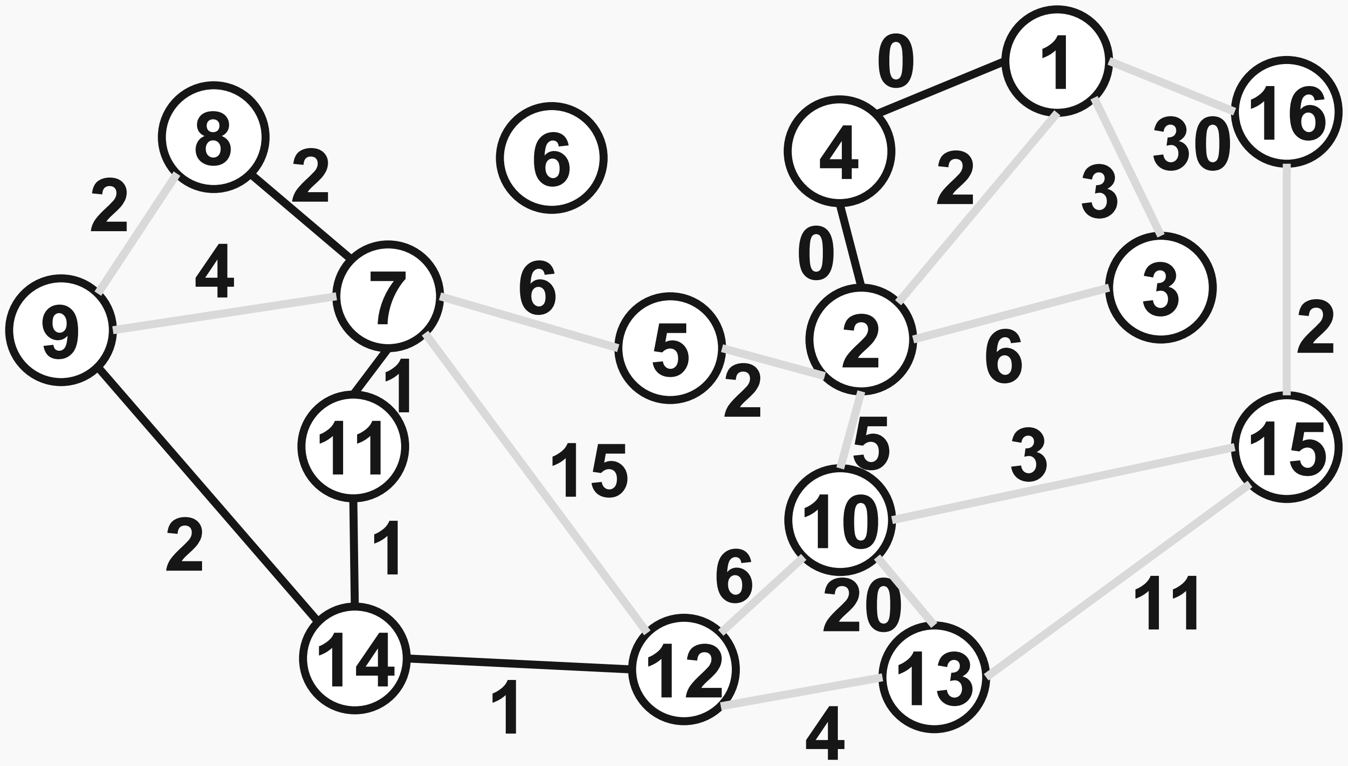

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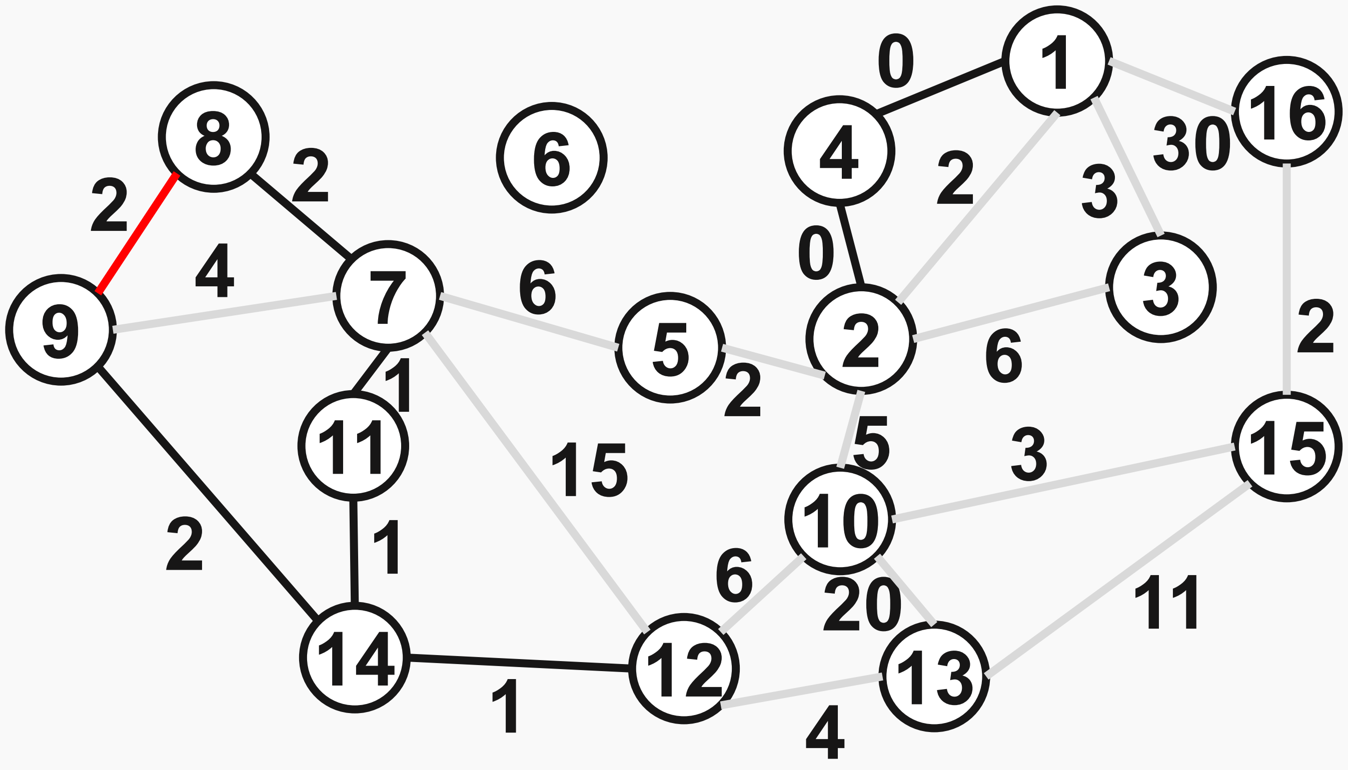

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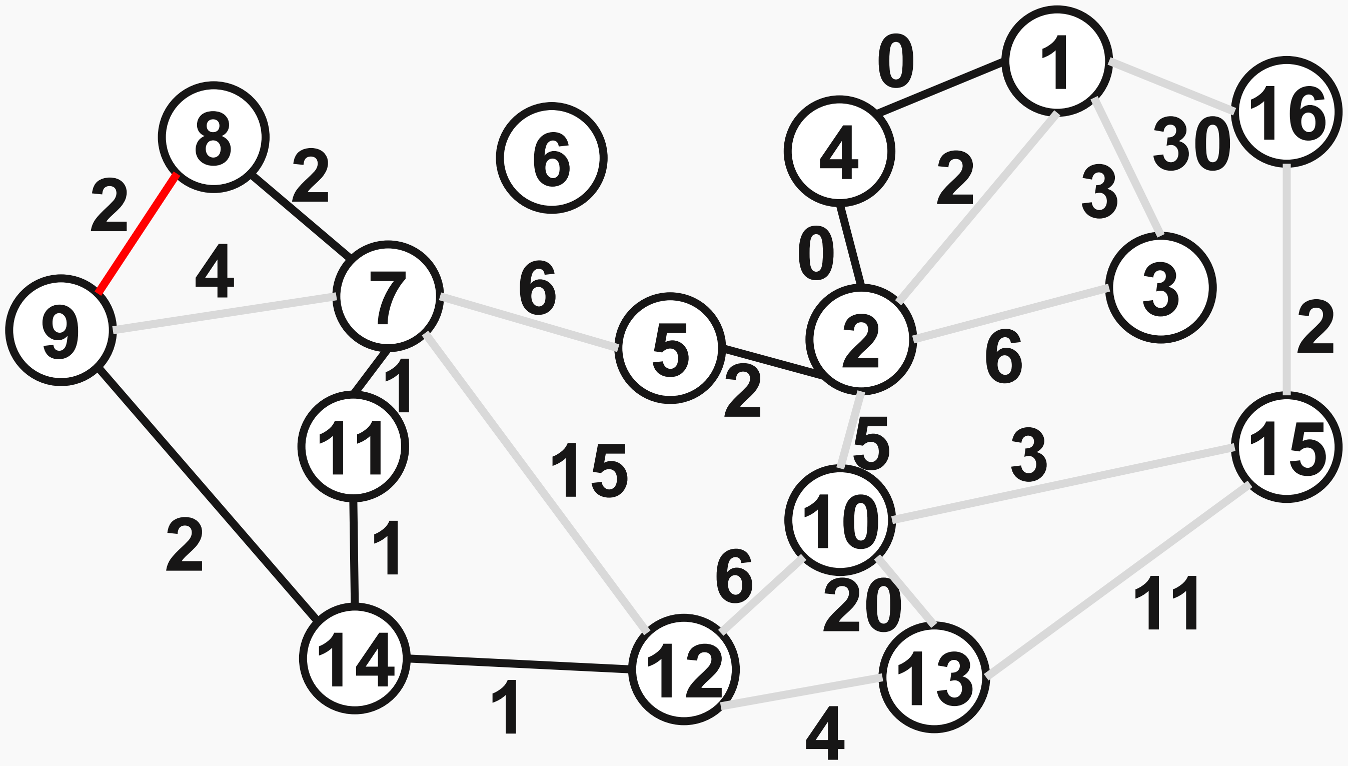

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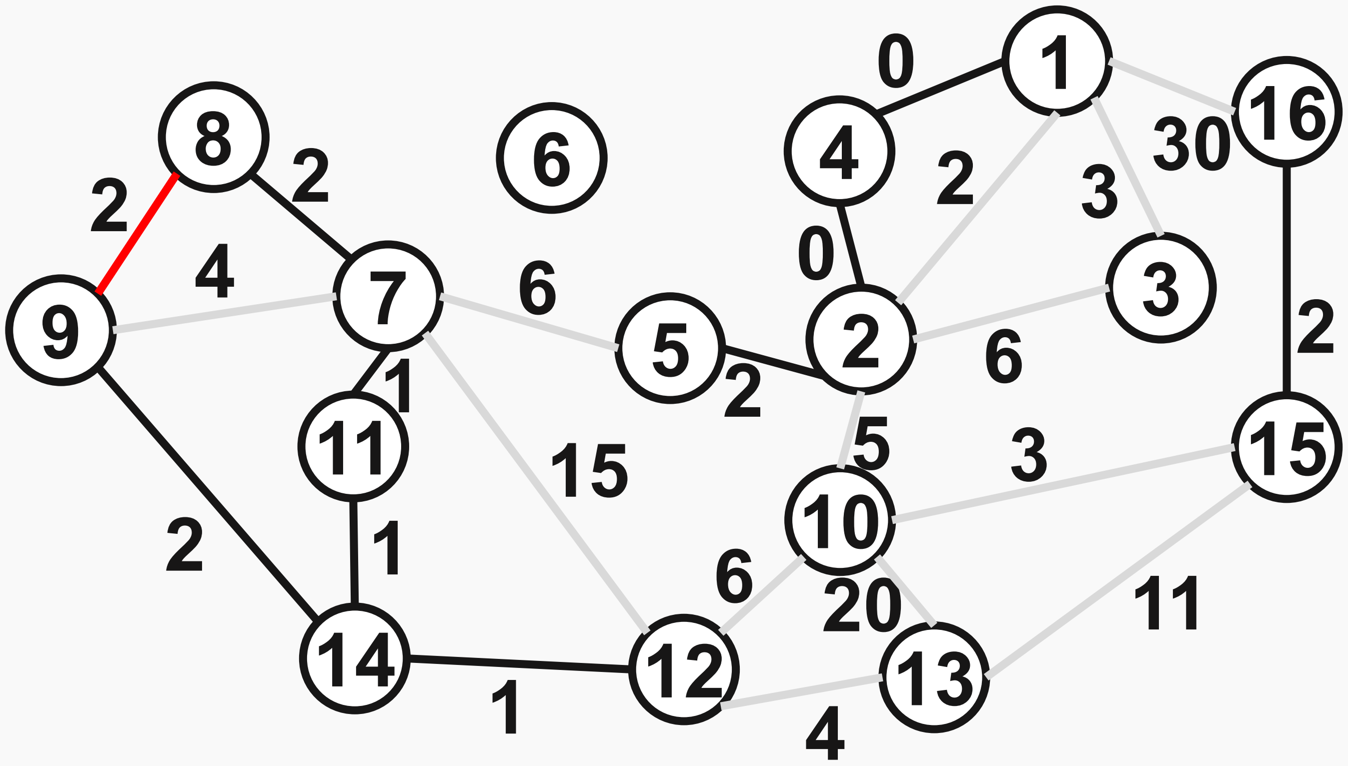

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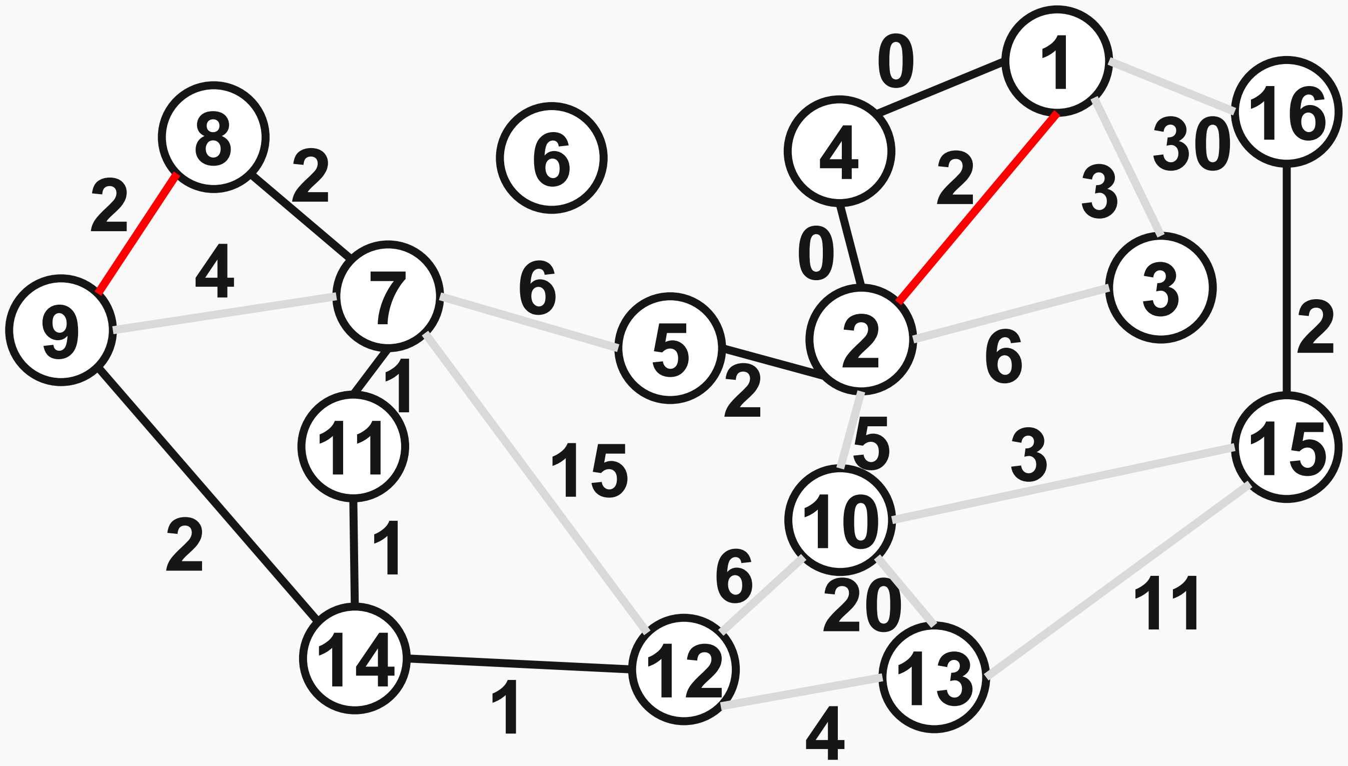

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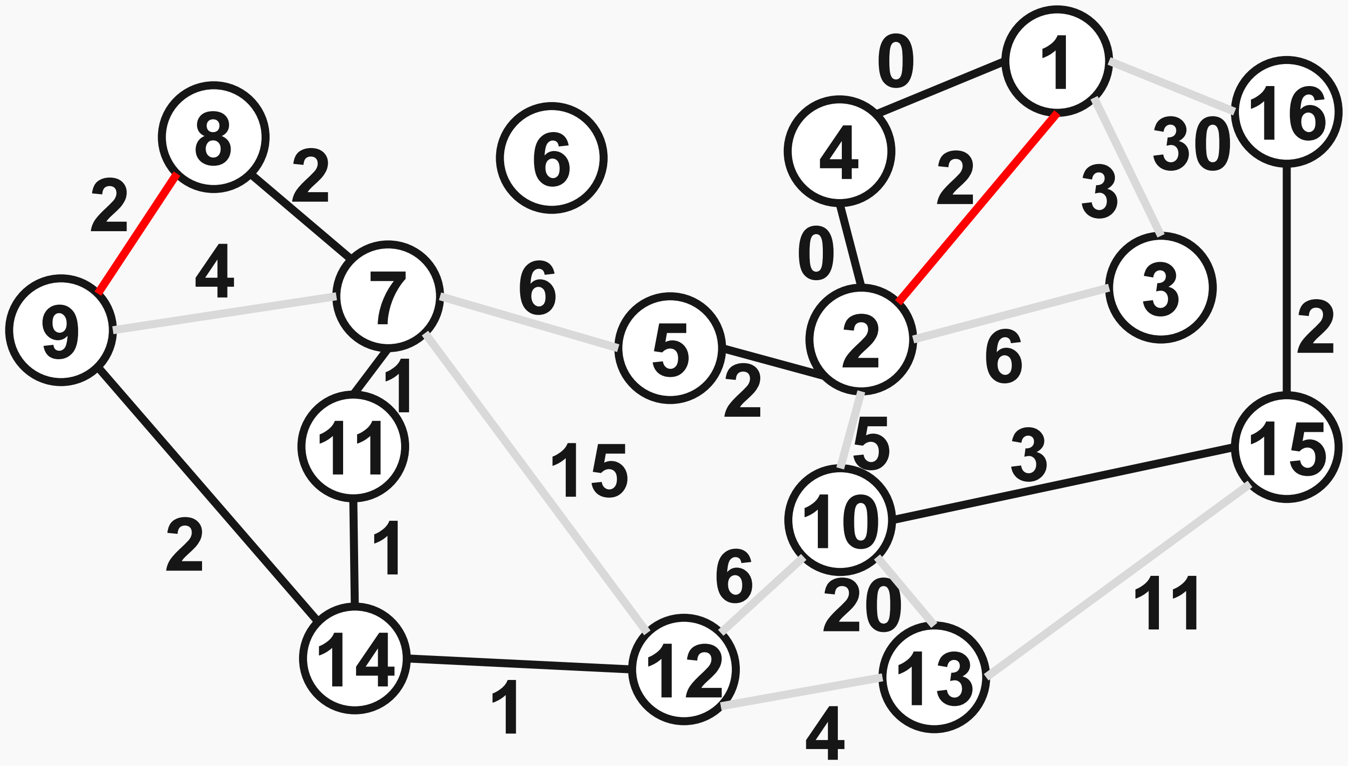

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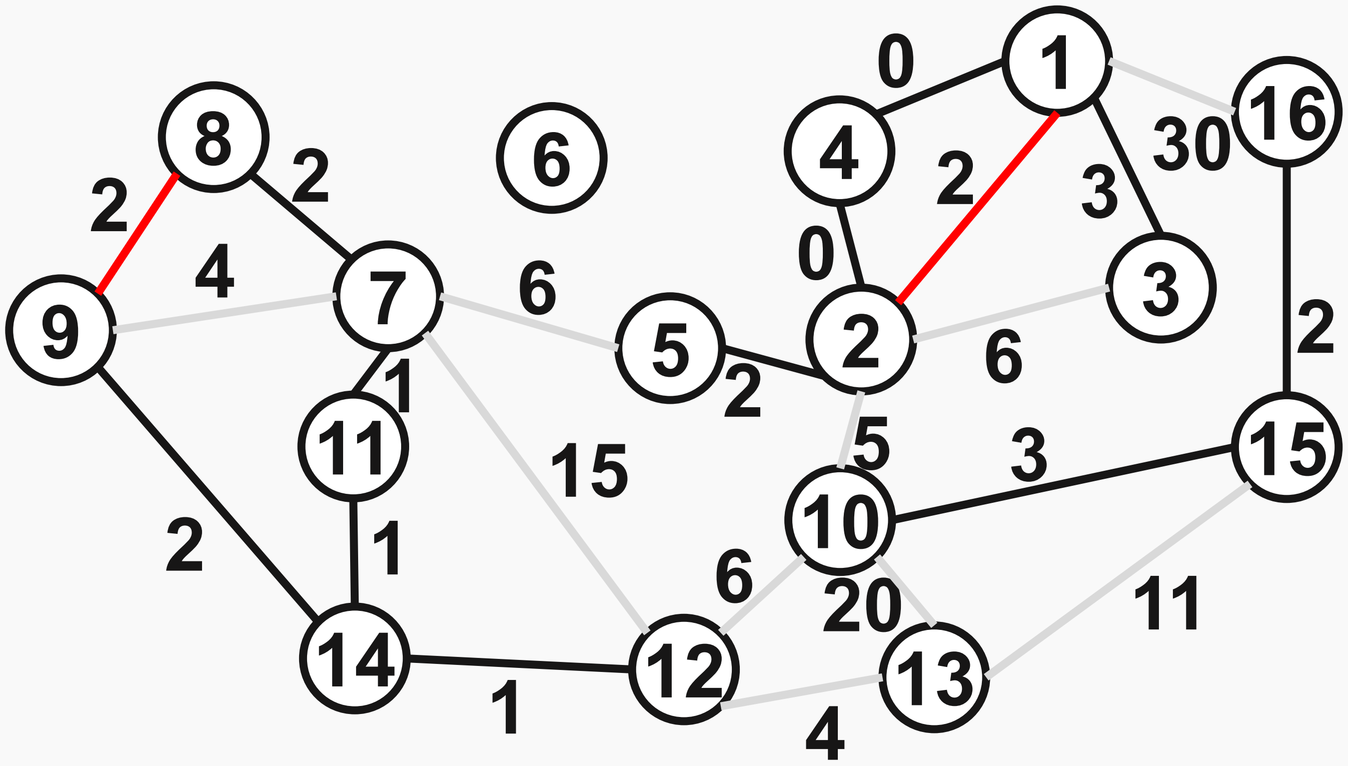

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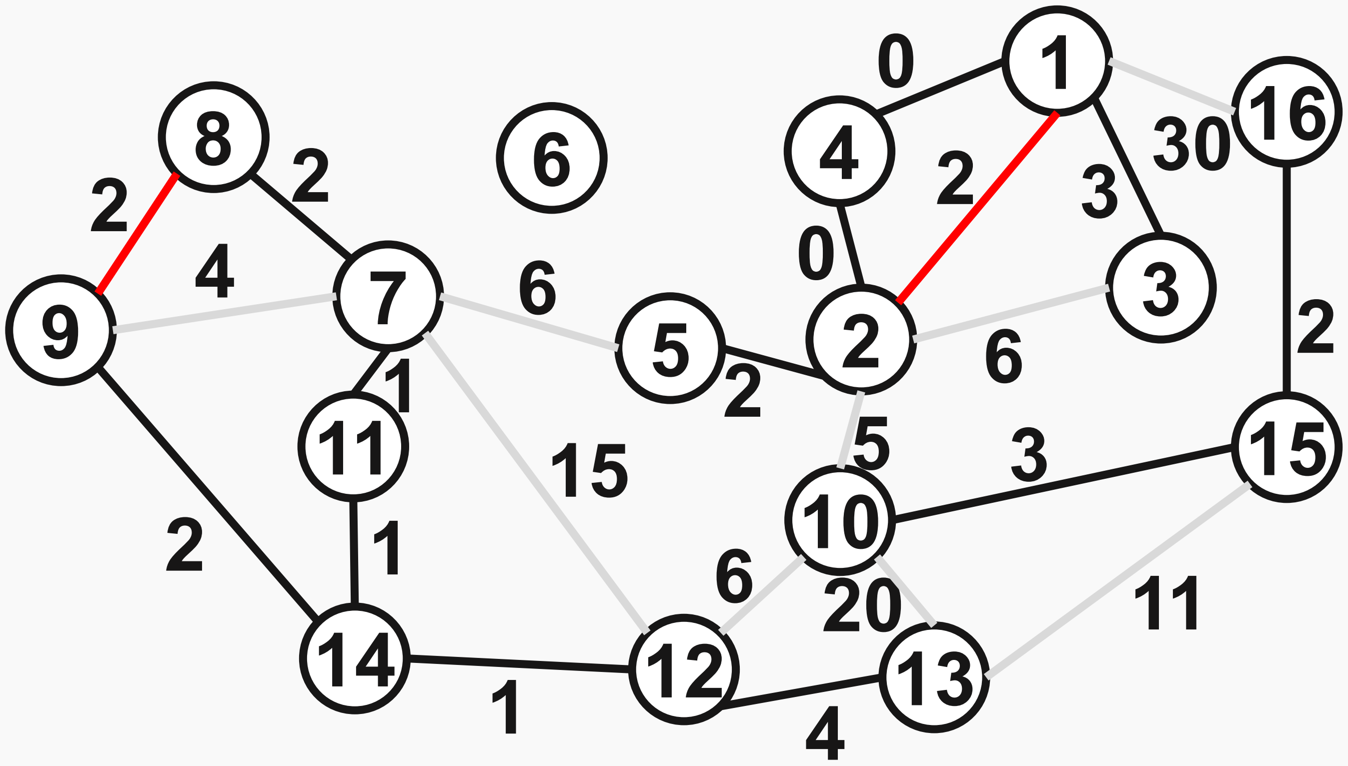

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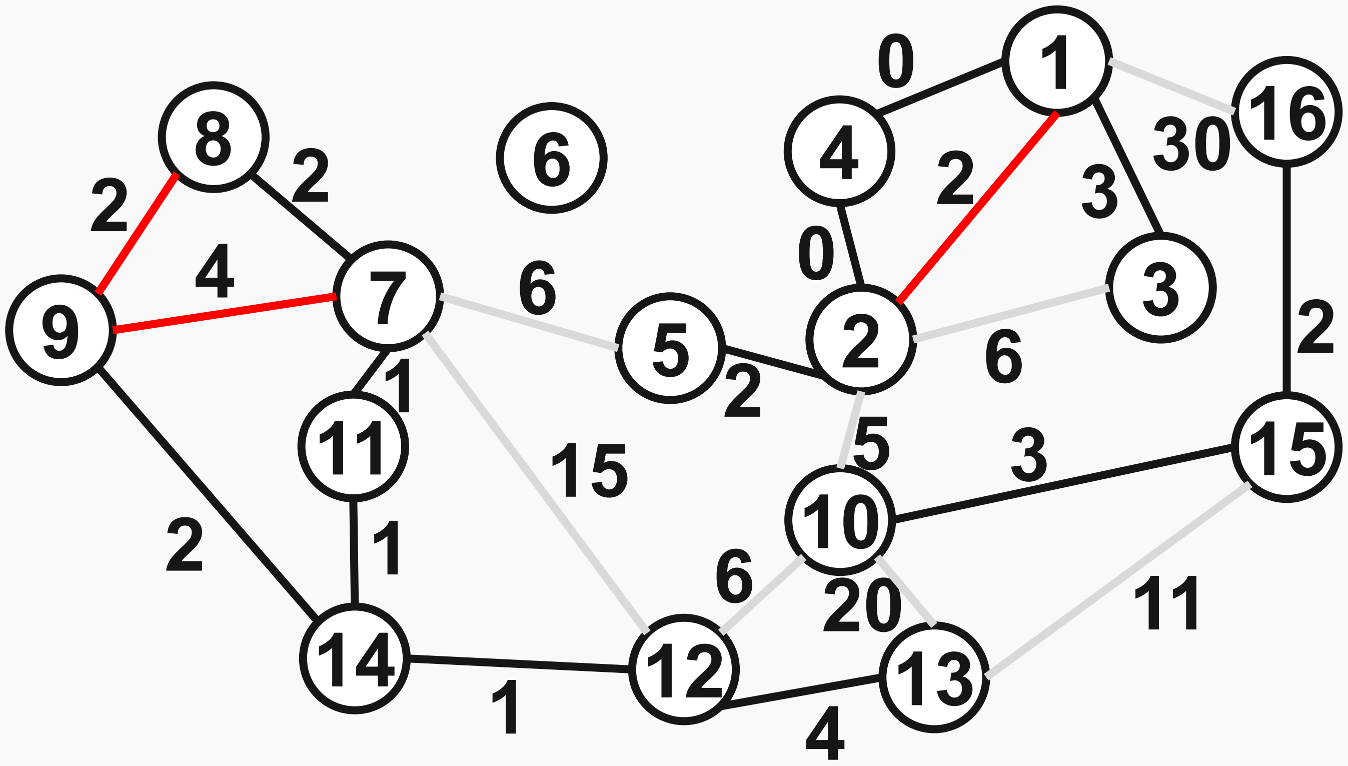

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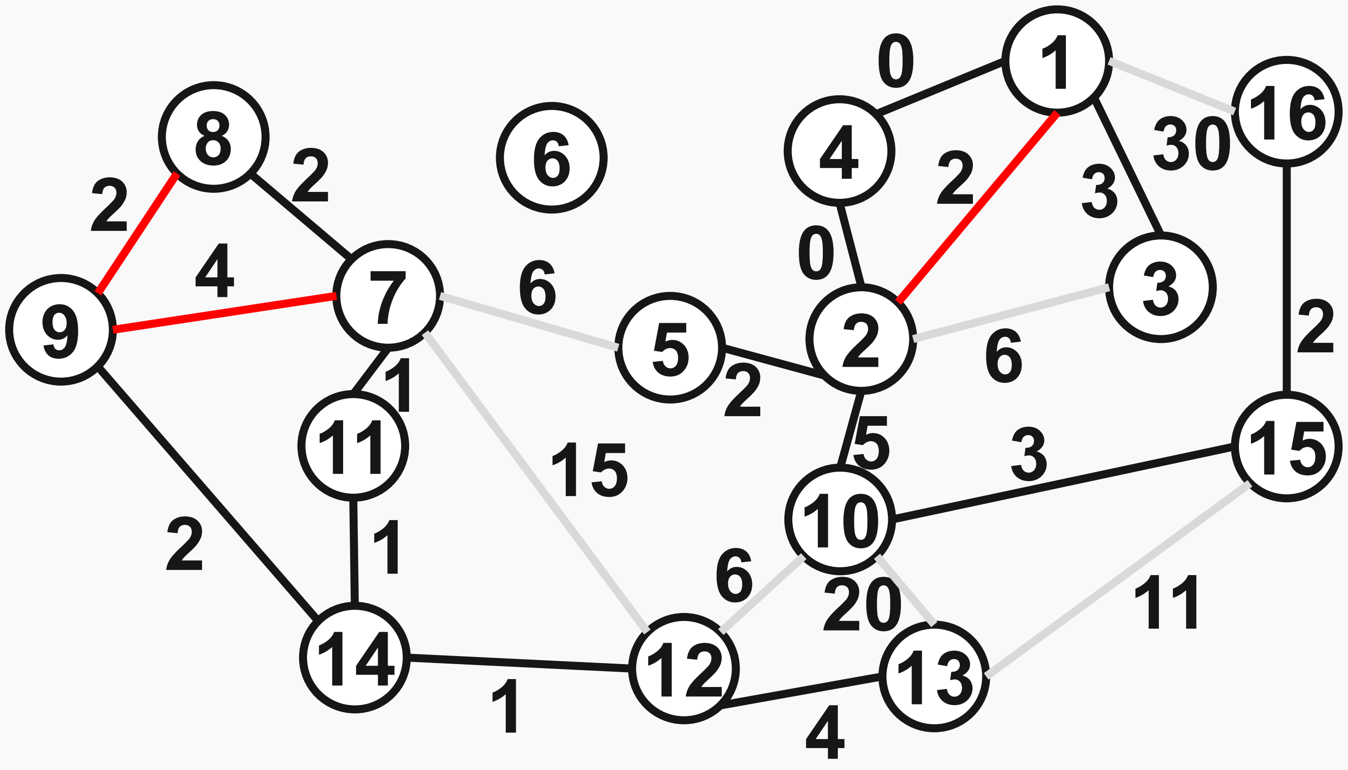

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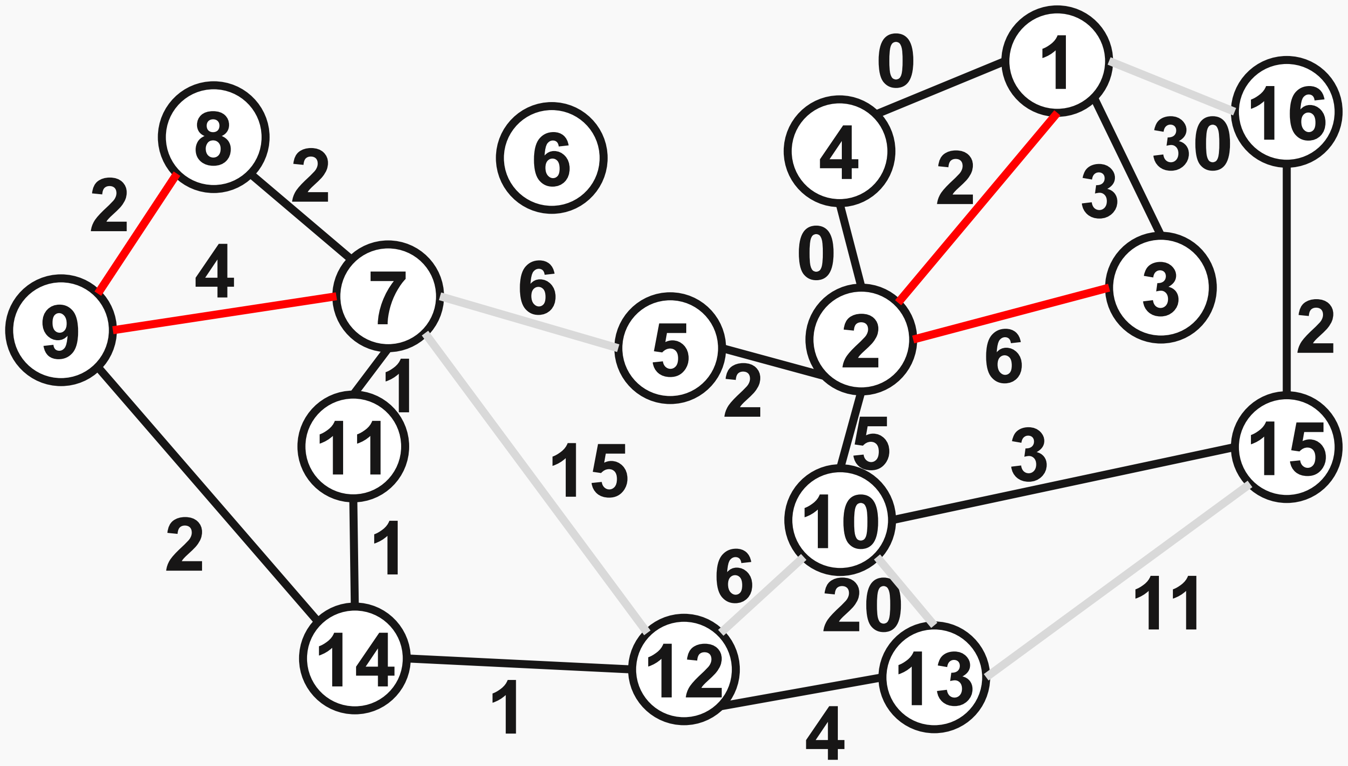

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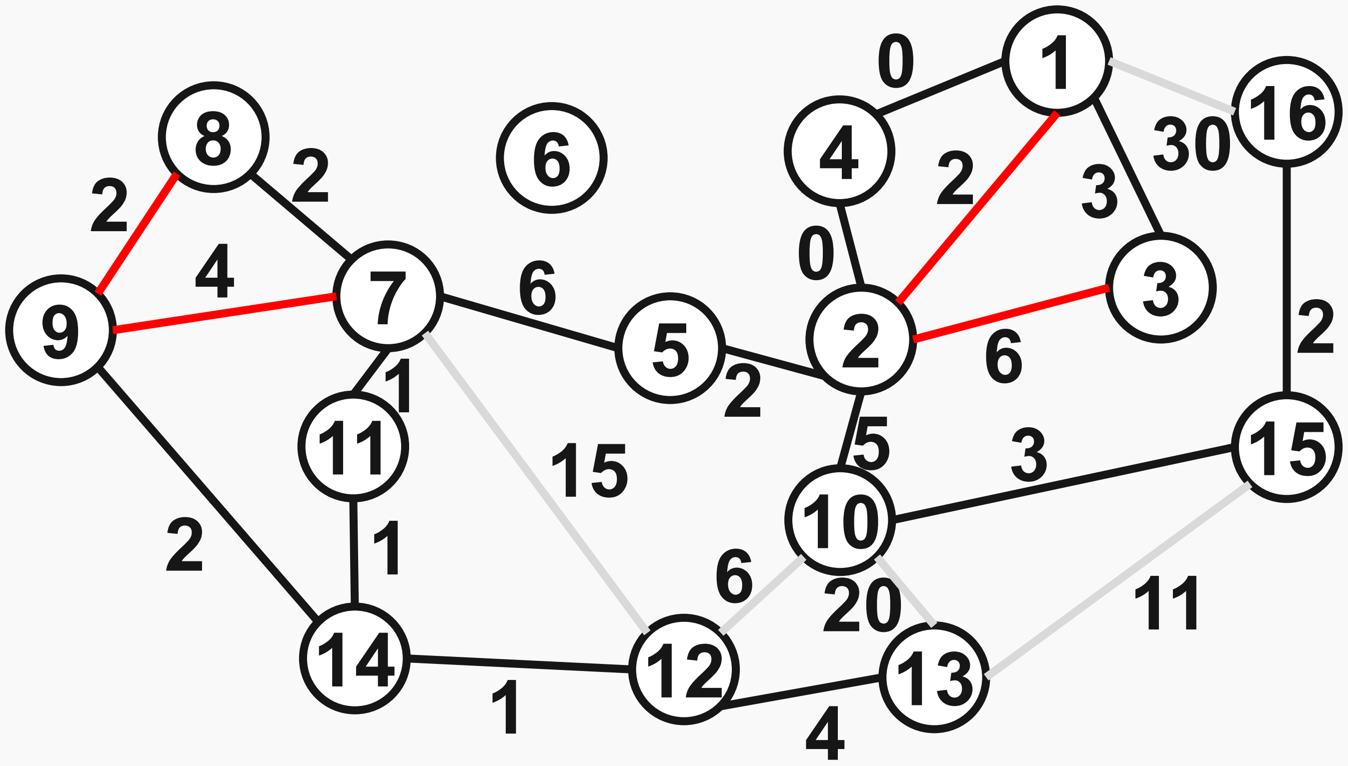

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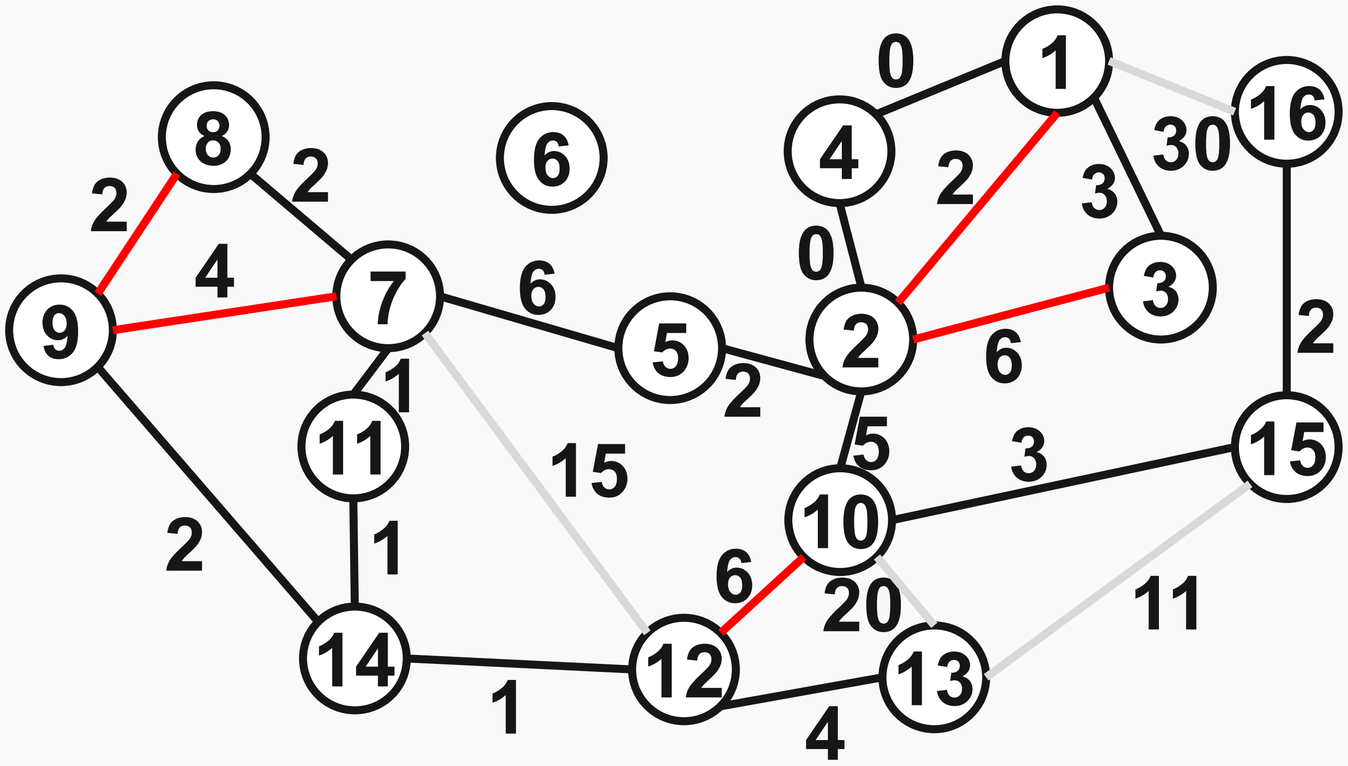

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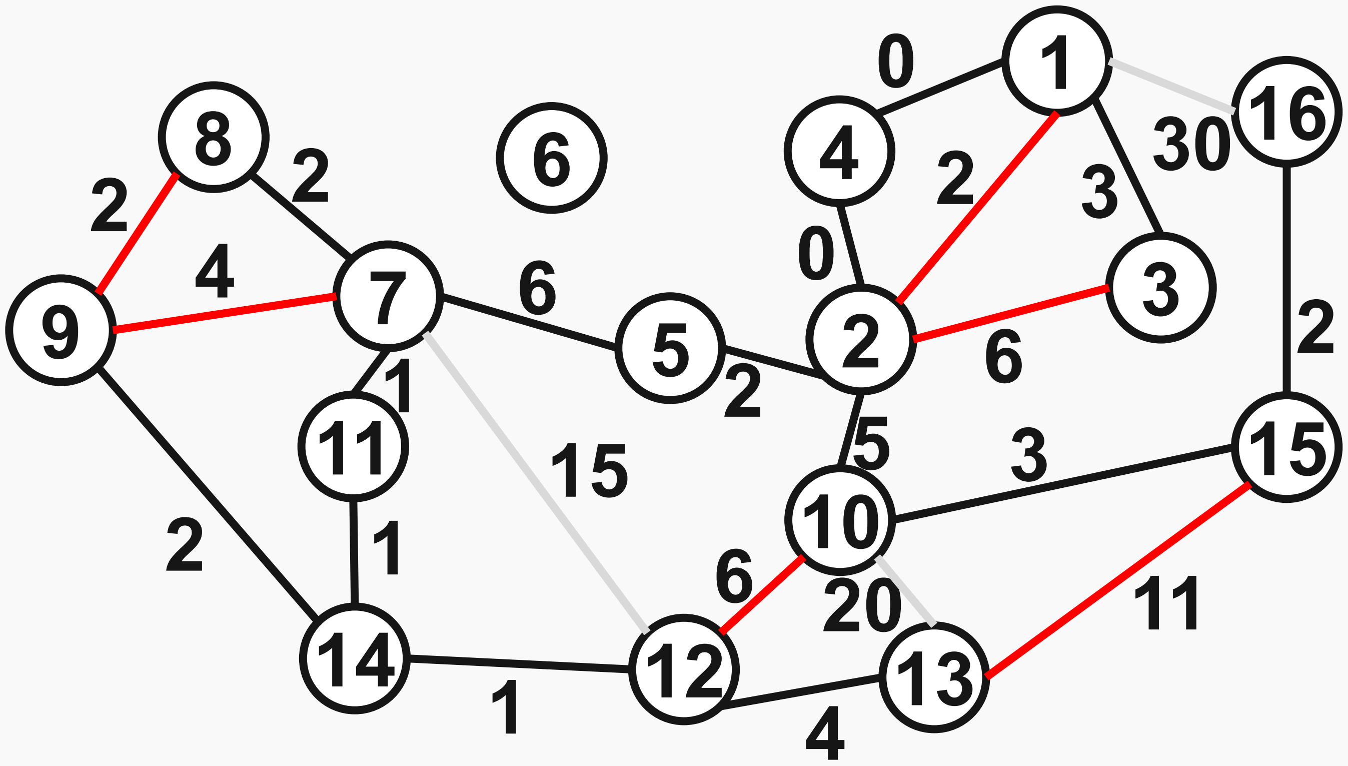

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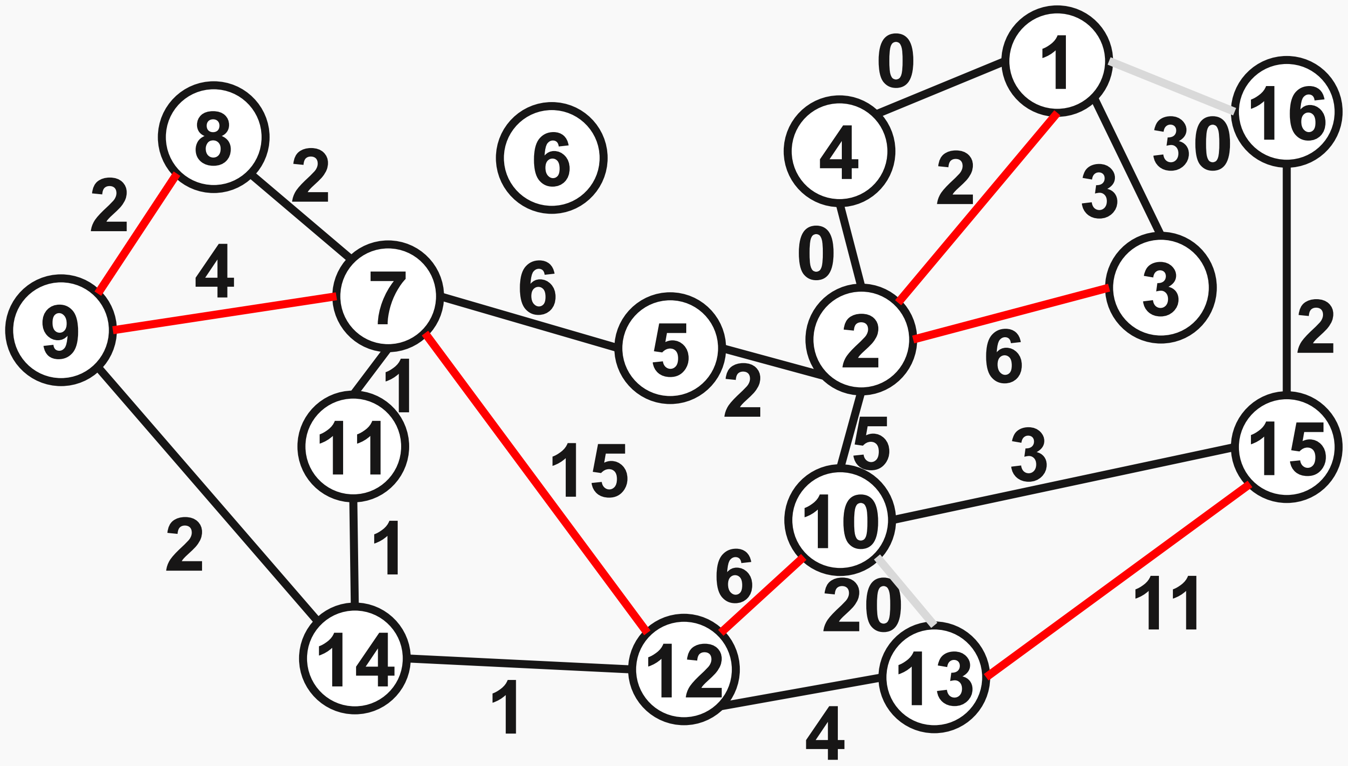

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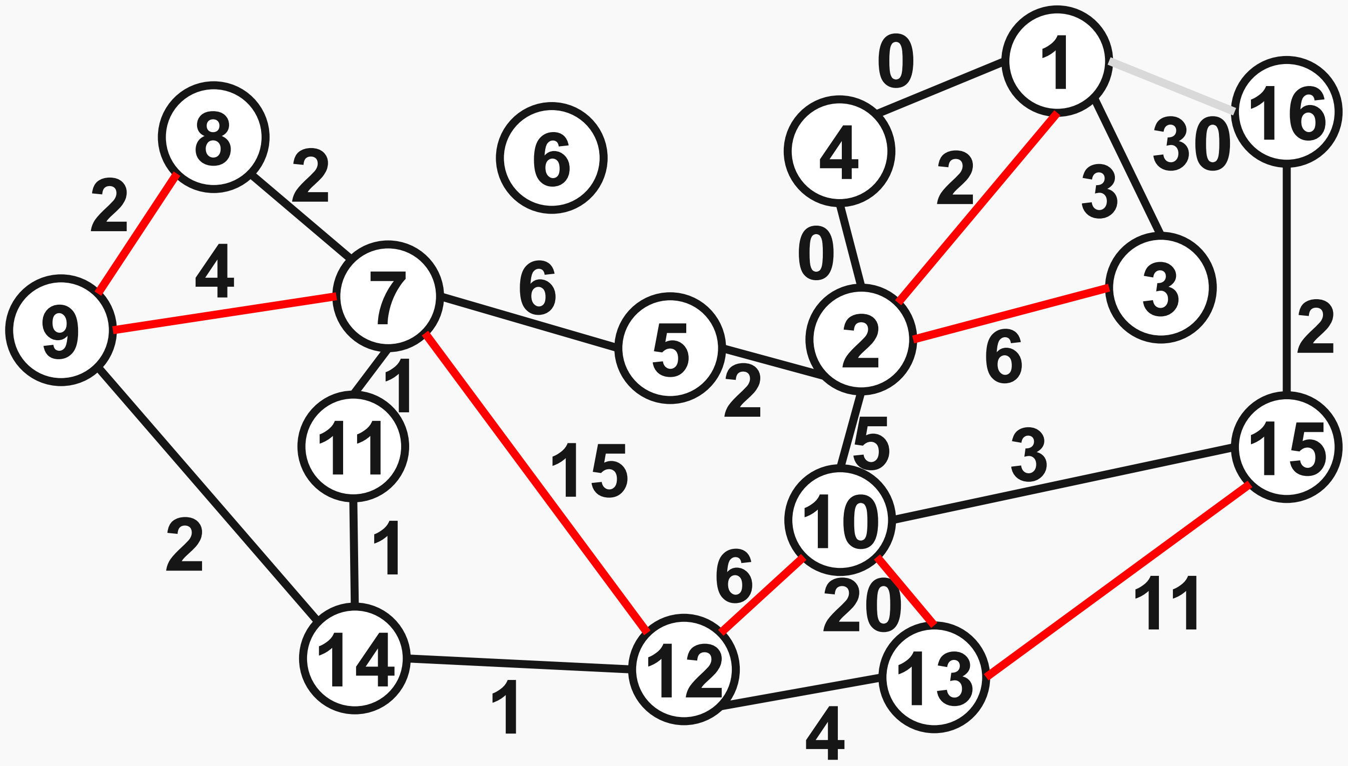

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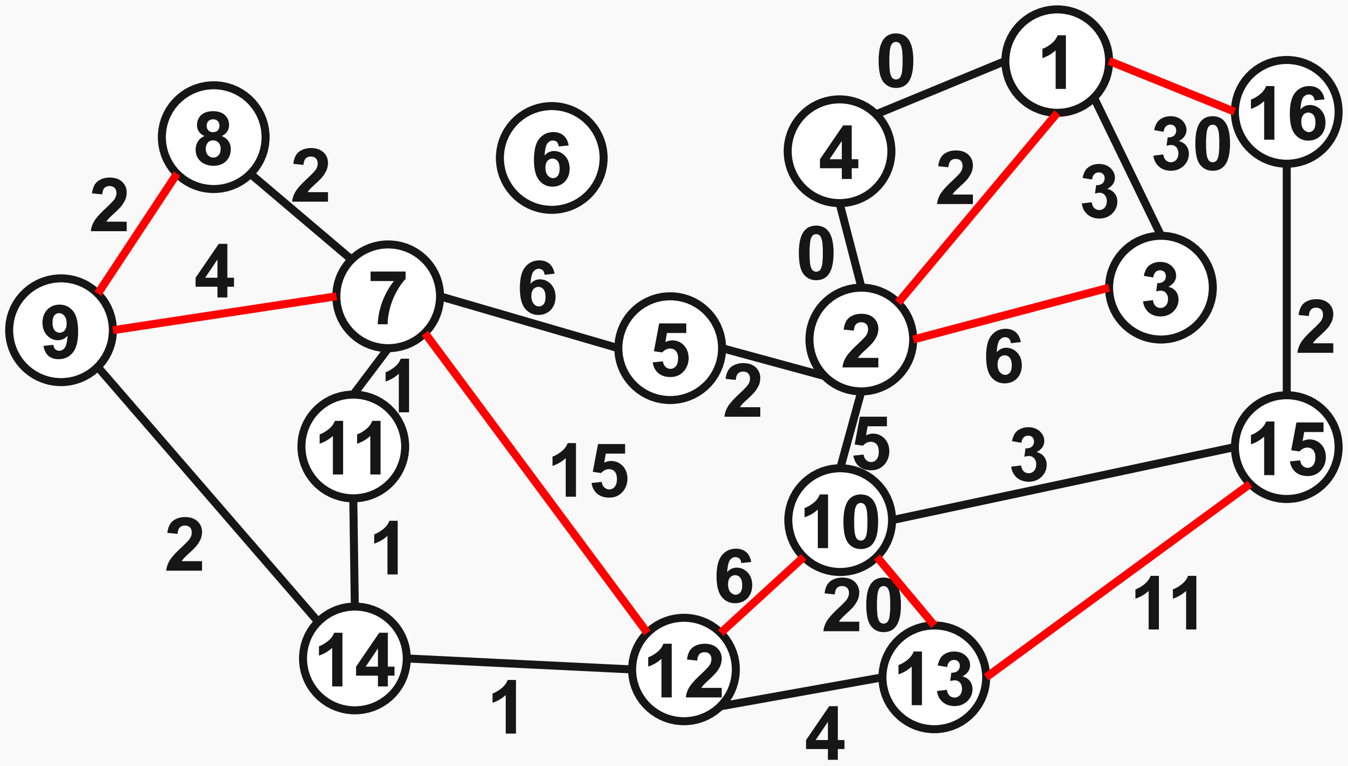

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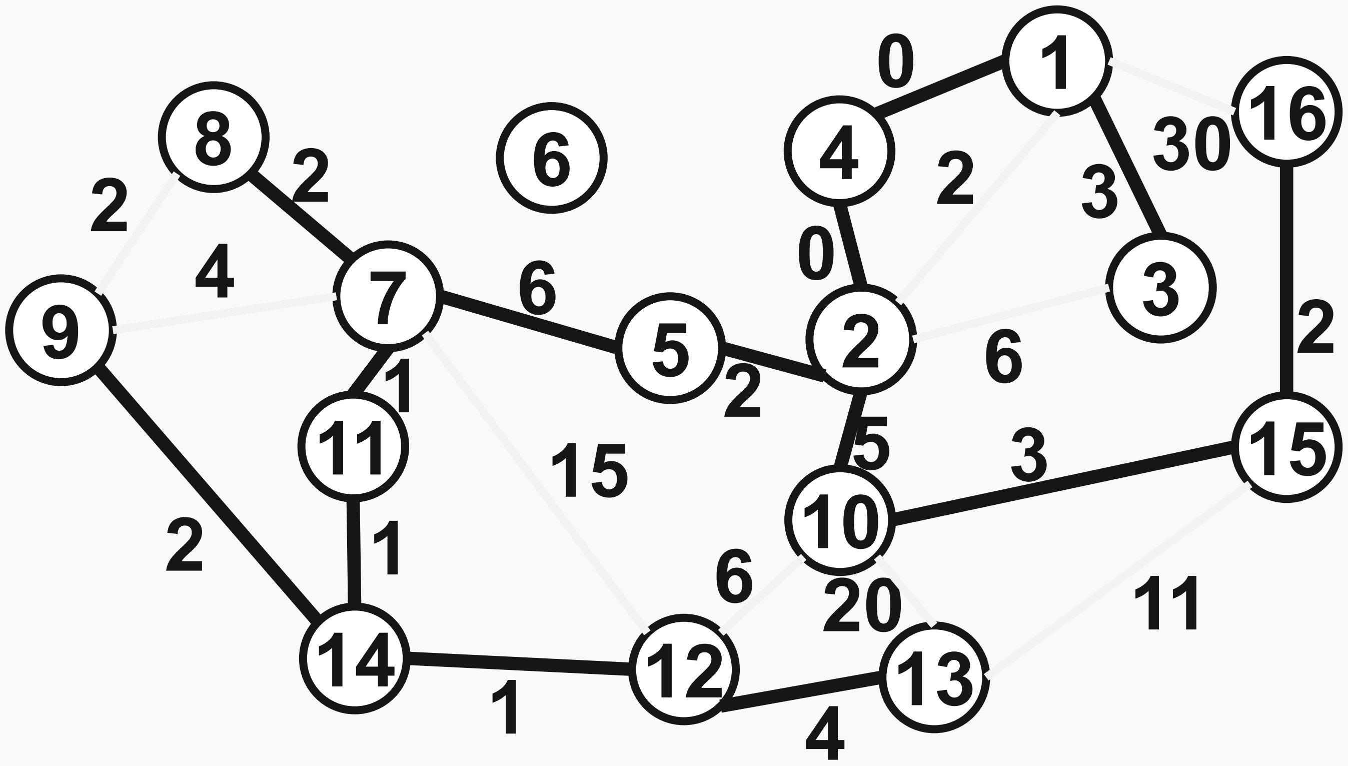

Complexitate?

```
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            A = A ∪ {(u, v)};
            Union(Find_set(u), Find_set(v));
        }
    }
    return A;
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```

Complexitate?

$$O(E\log(E))$$

Flux maxim

Graf capacitate

- În general valoarea de pe muchie reprezintă o distanță.

  - O distanță mai mare face muchia mai greu de parcurs.

- Valoarea muchiei poate reprezenta o capacitate.

  - Similar apei/curentului, cu cât capacitatea e mai mare cu atât e mai ușor de parcurs.

- Șoselele au și distanță și capacitate (număr benzi/viteză max)

Flux maxim Algoritmul Ford-Fulkerson

- c capacitate muchie
- f flow muchie. Capacitate folosită
- $c_f(u, v) = c(u, v) - f(u, v)$ diferența de capacitate
- G_f graful cu muchii c_f

FORD-FULKERSON (G, s, t)
for each edge $(u, v) \in G.E$
    $(u, v).f = 0$
while there exists a path p from s to t in the residual network G_f
    $c_f(p) = \min\{c_f(u, v): (u, v) \ is \ in \ p\}$
    for each edge (u, v) in p
        if $(u, v) \in G.E$
            $(u, v).f = (u, v).f + c_f(p)$
        else
            $(v, u).f = (v, u).f - c_f(p)$

Flux maxim Algoritmul Ford-Fulkerson (1956)

- c capacitate muchie
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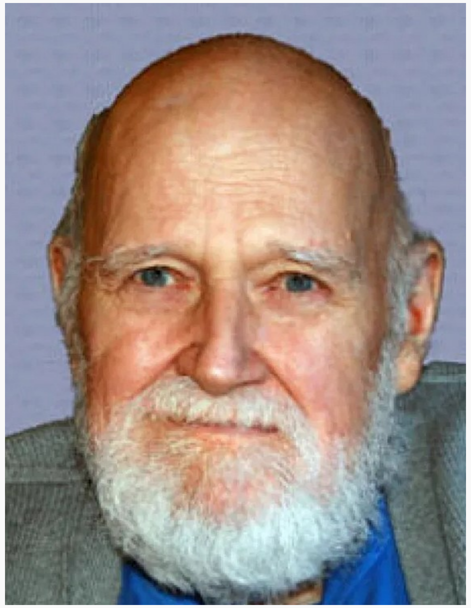
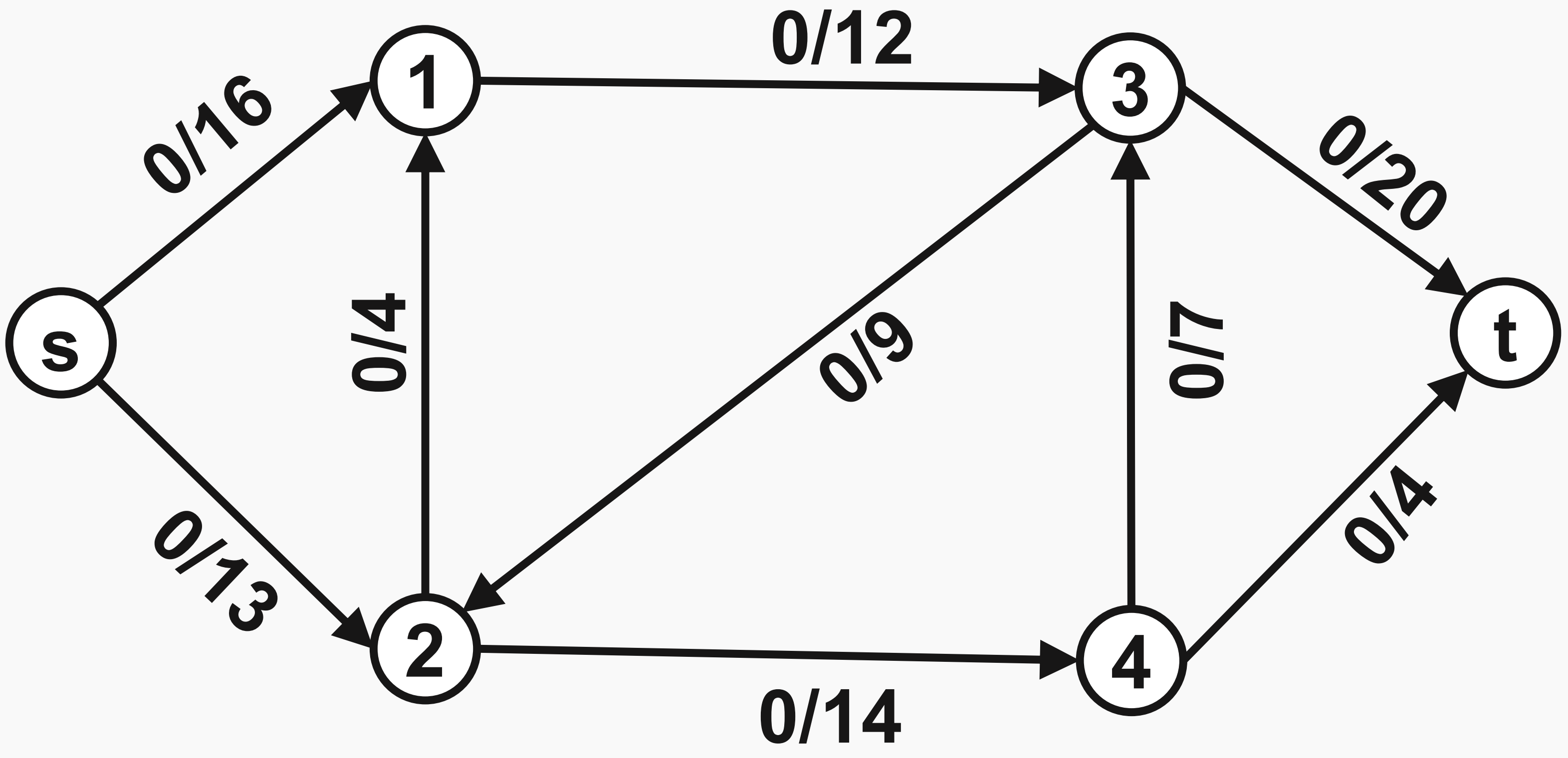


Algoritmul Ford-Fulkerson



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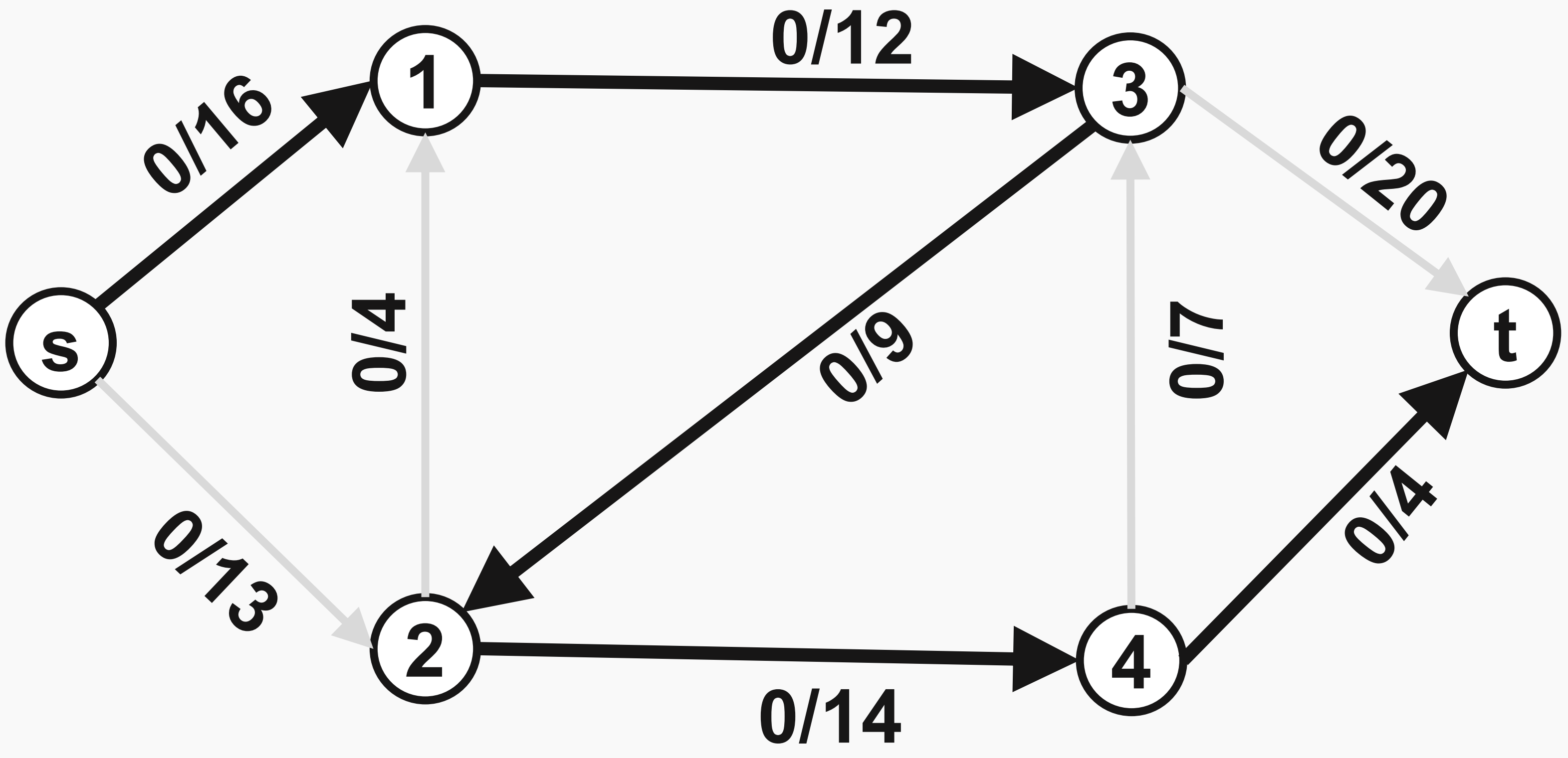

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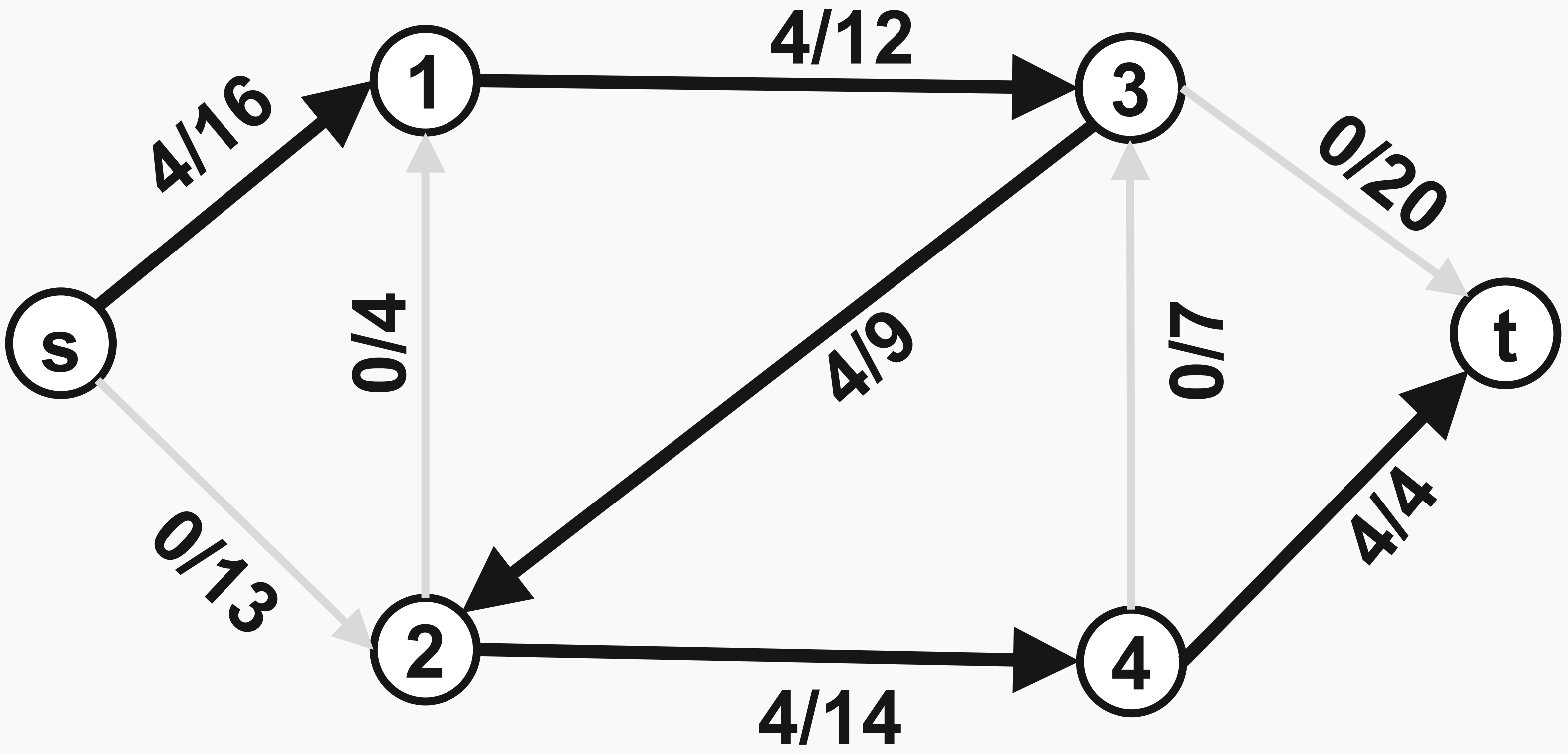

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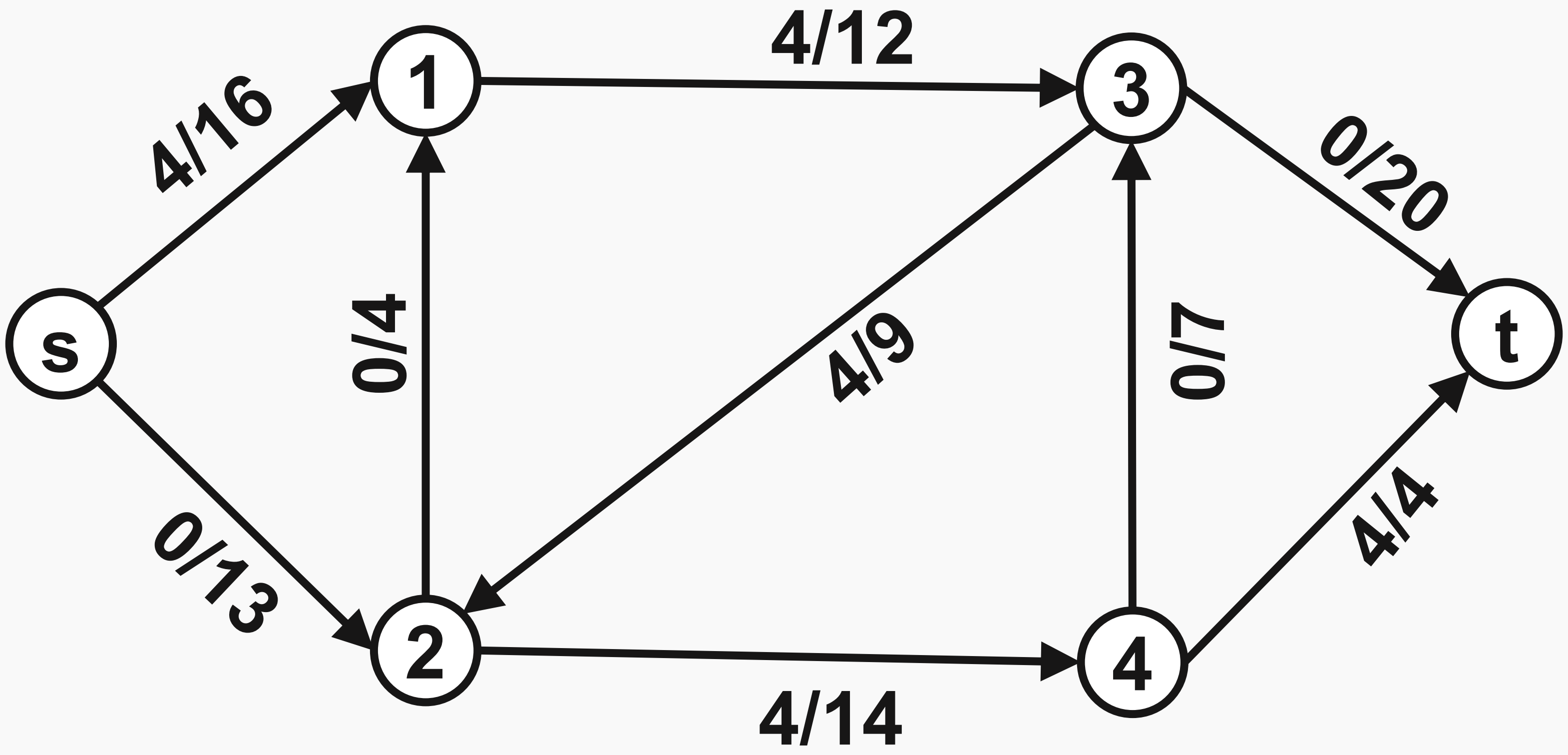

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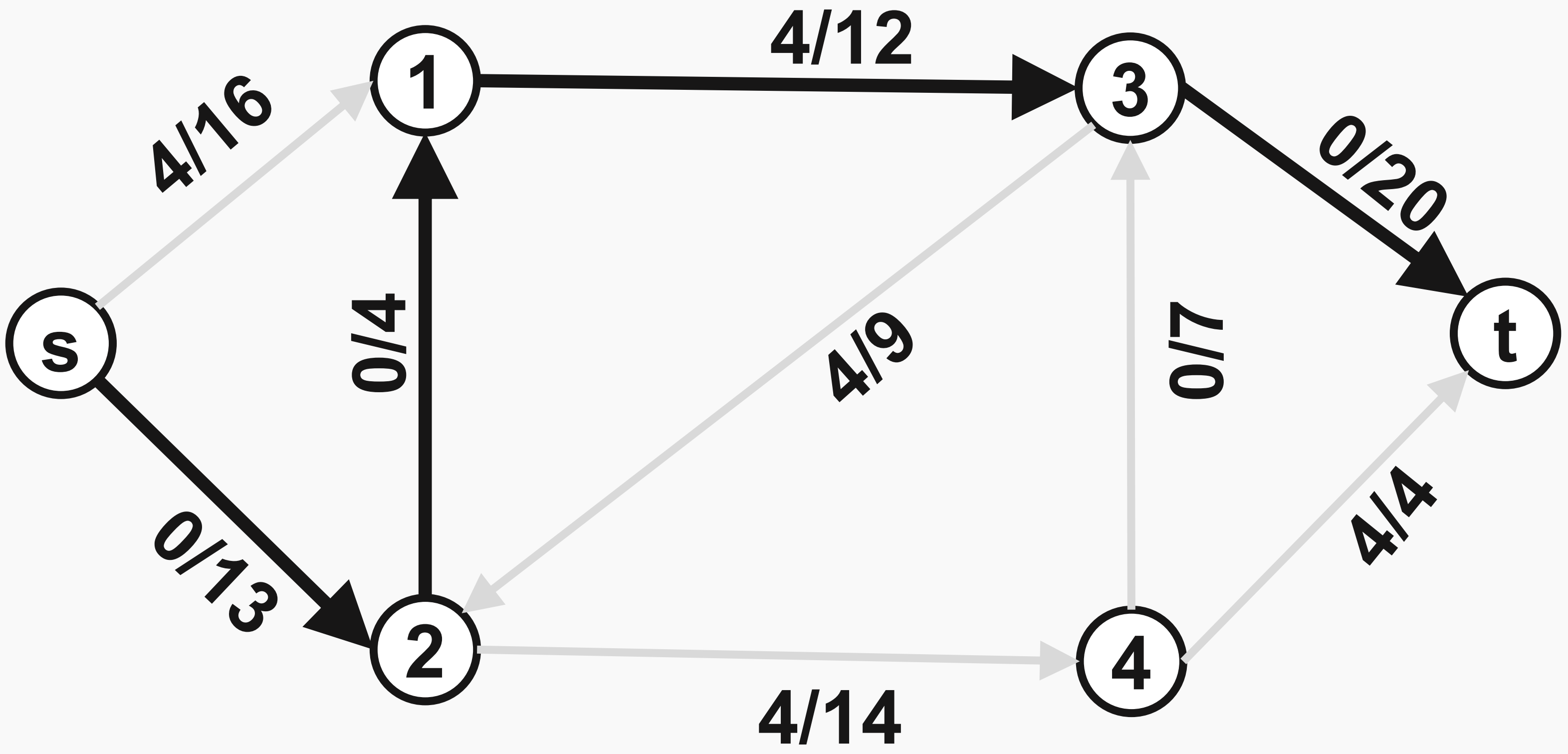

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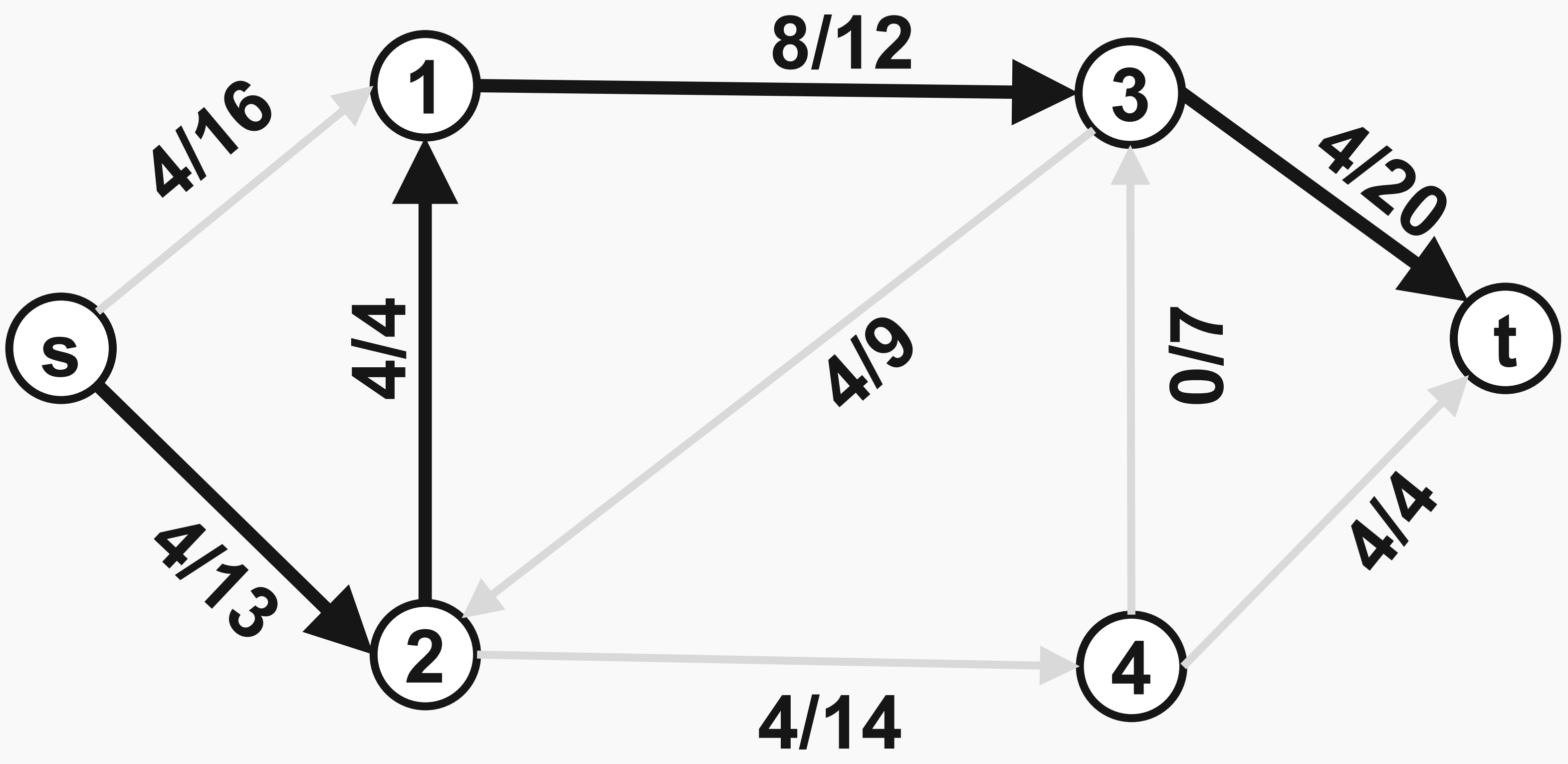

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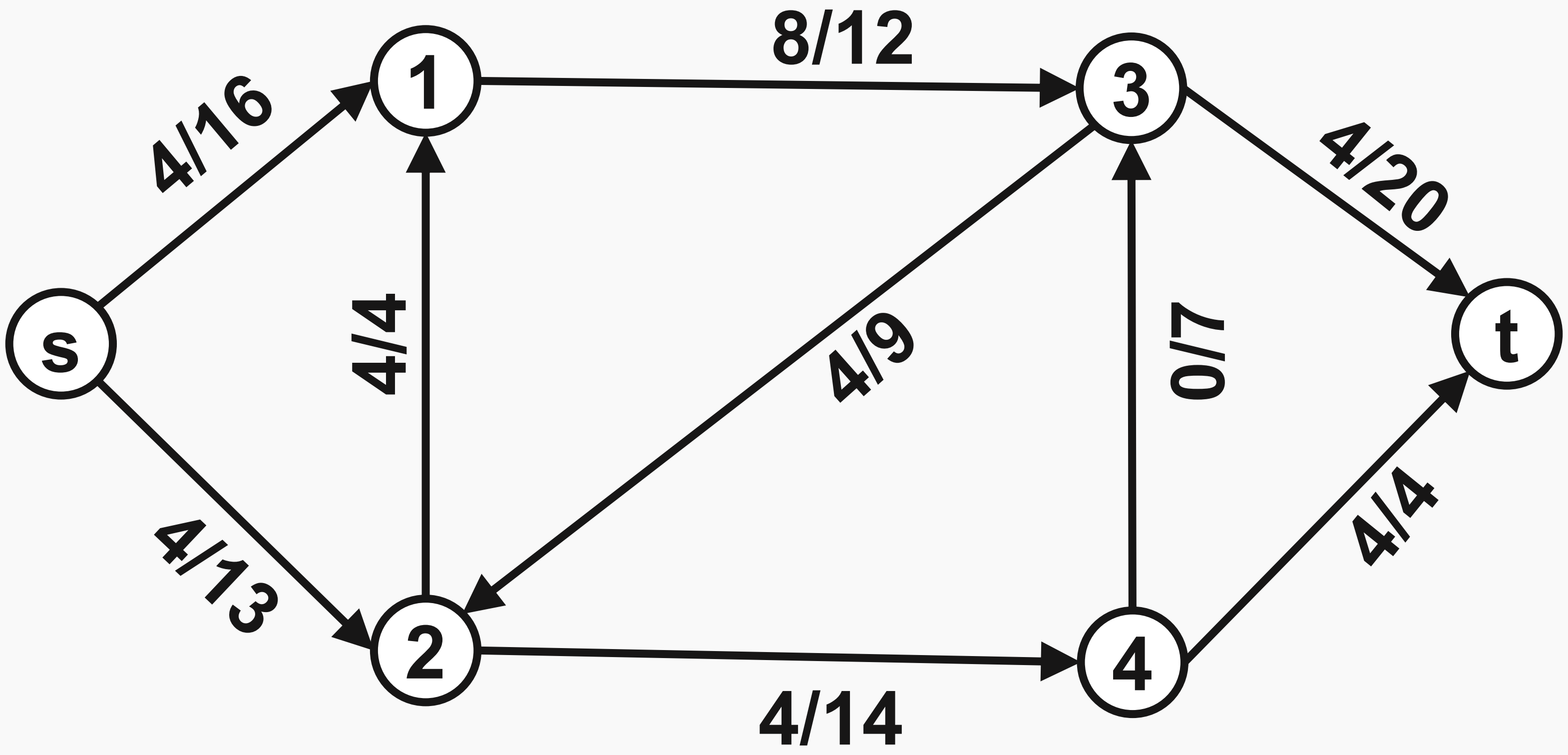

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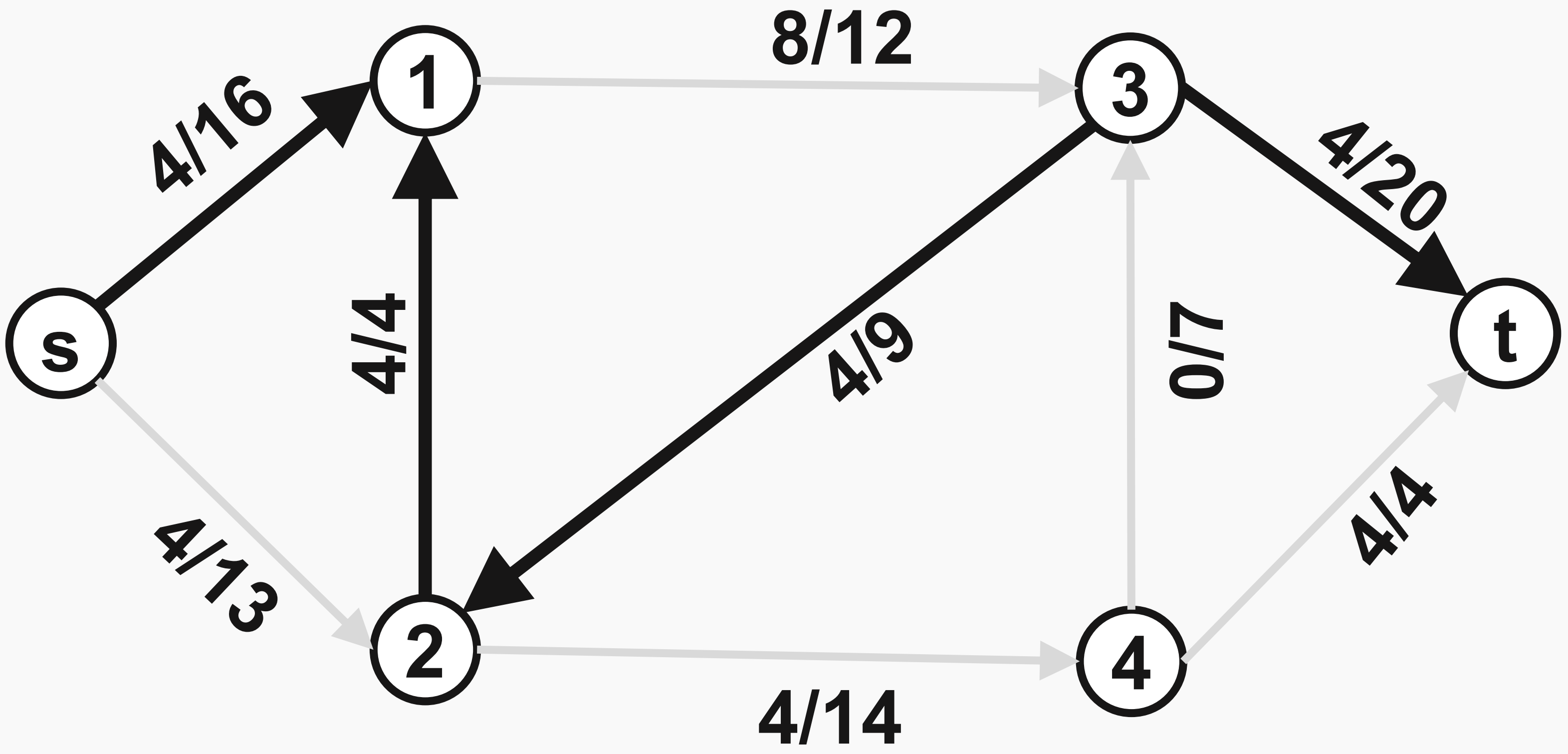

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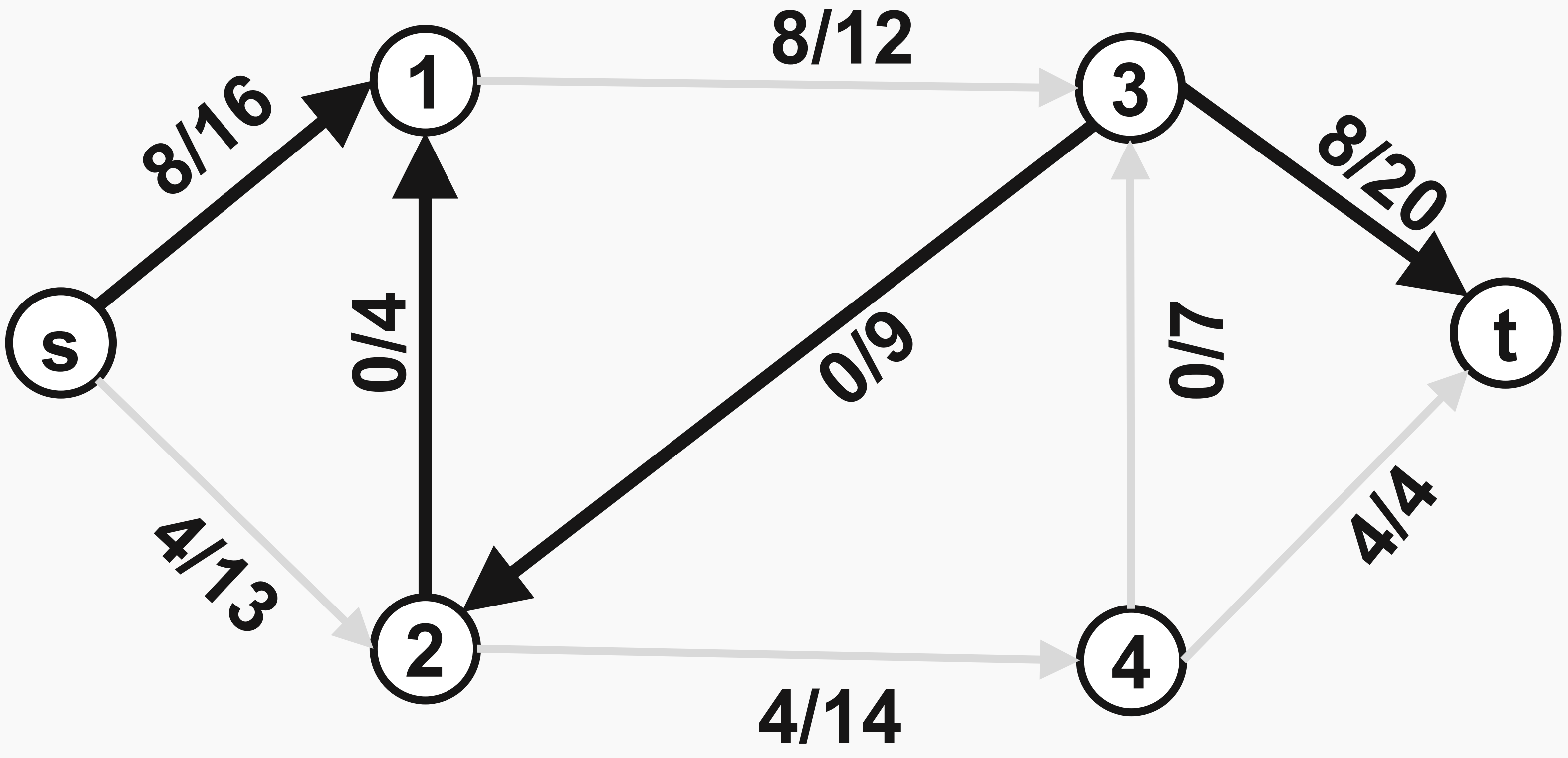

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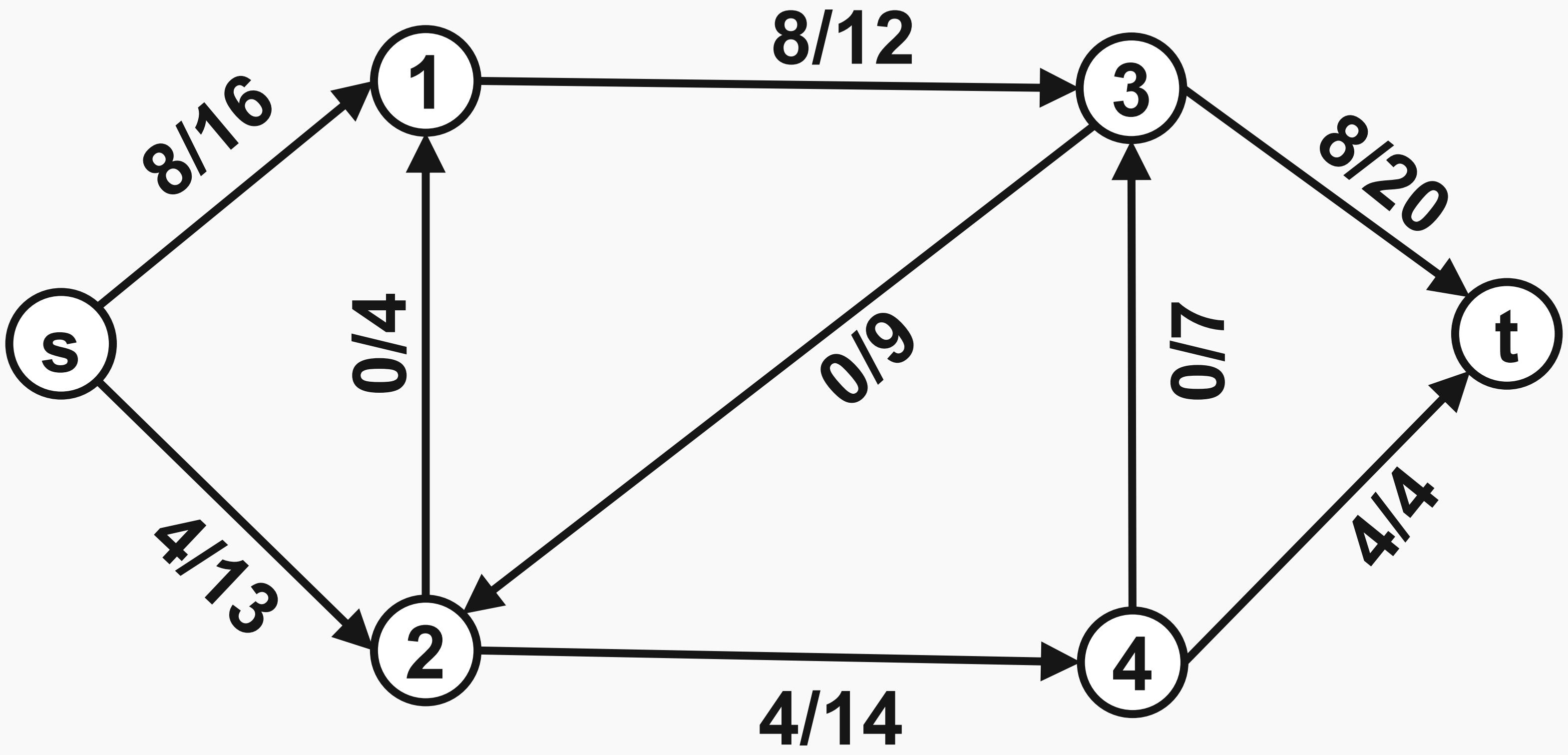

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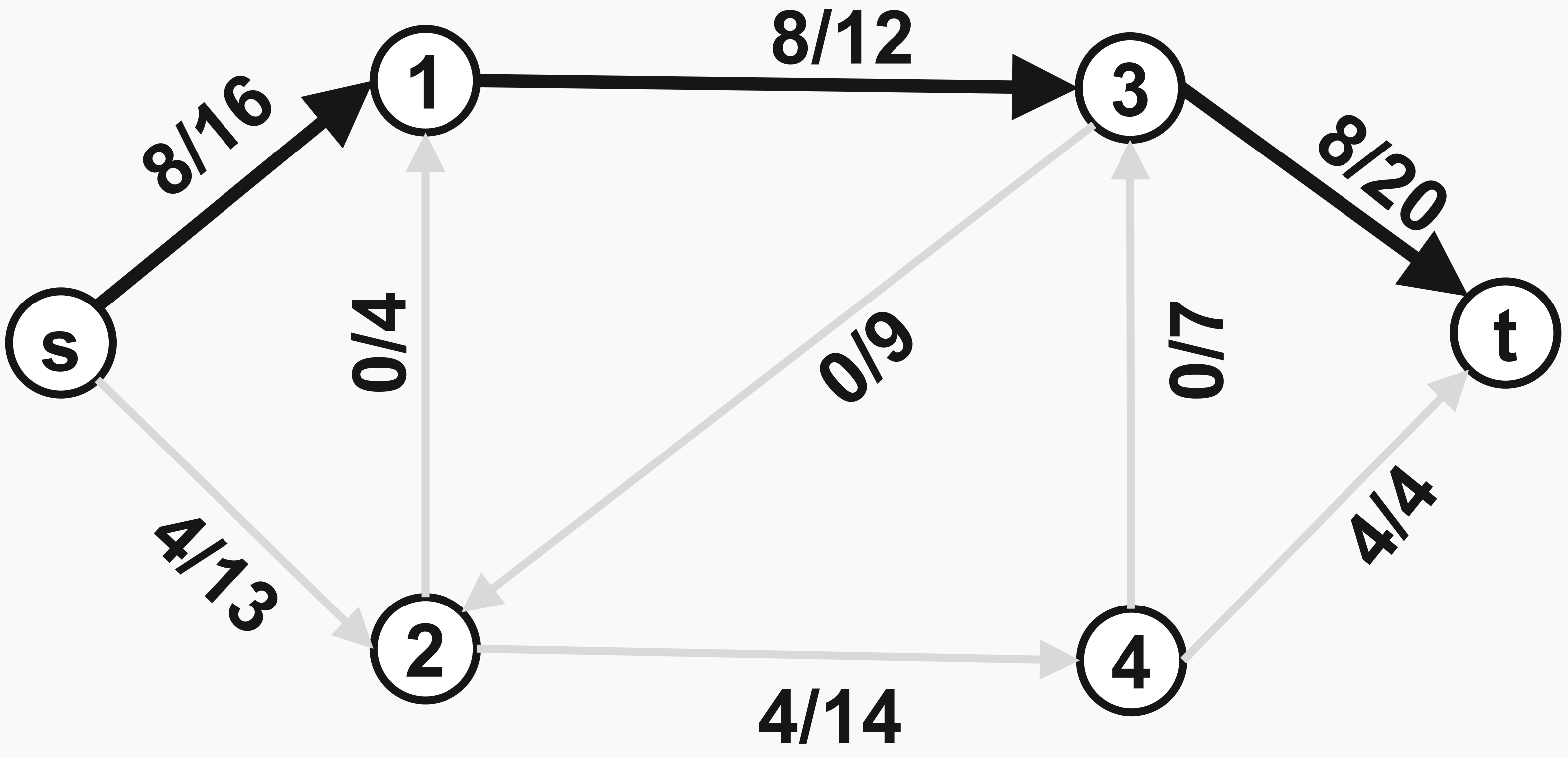

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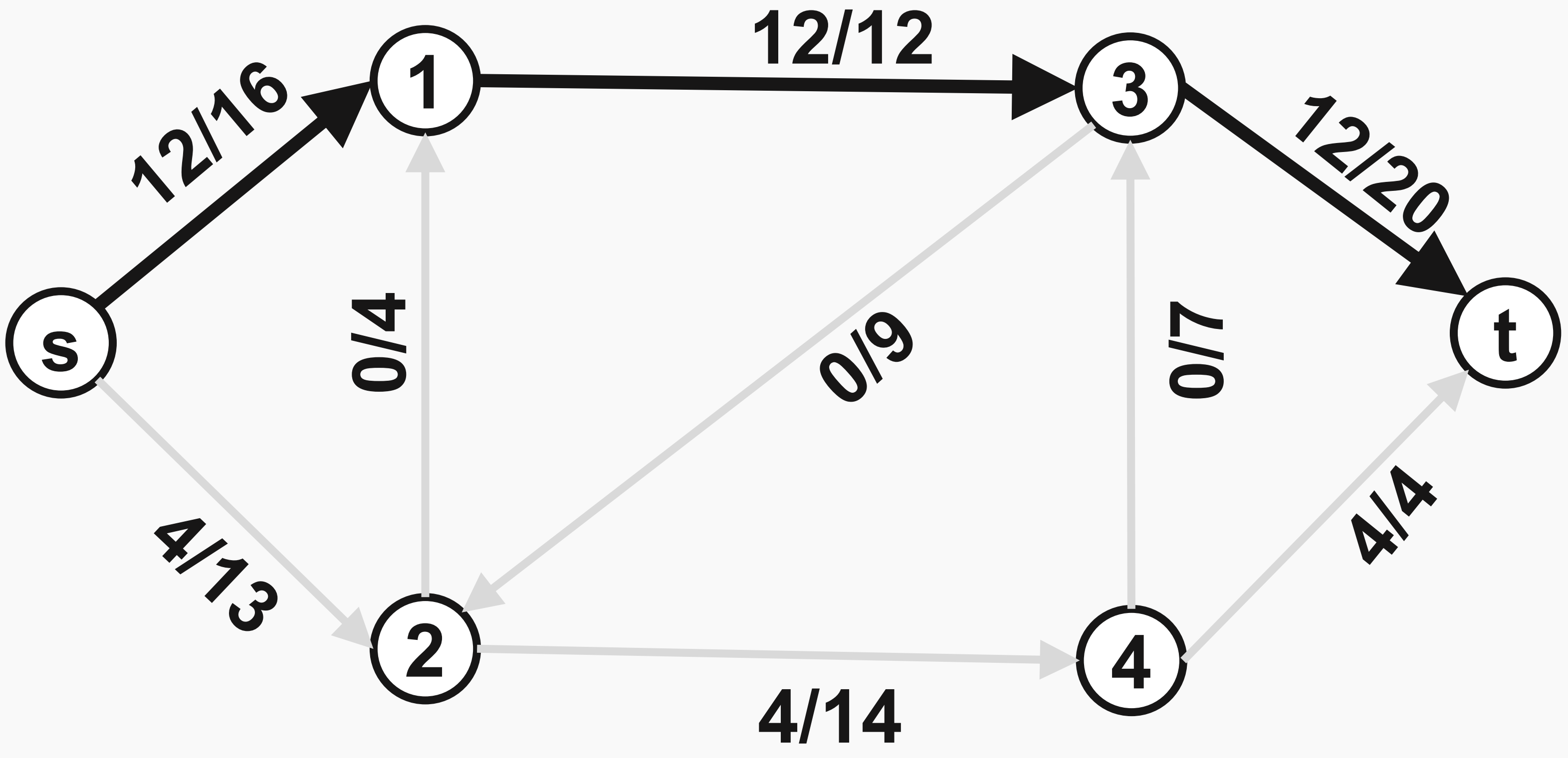

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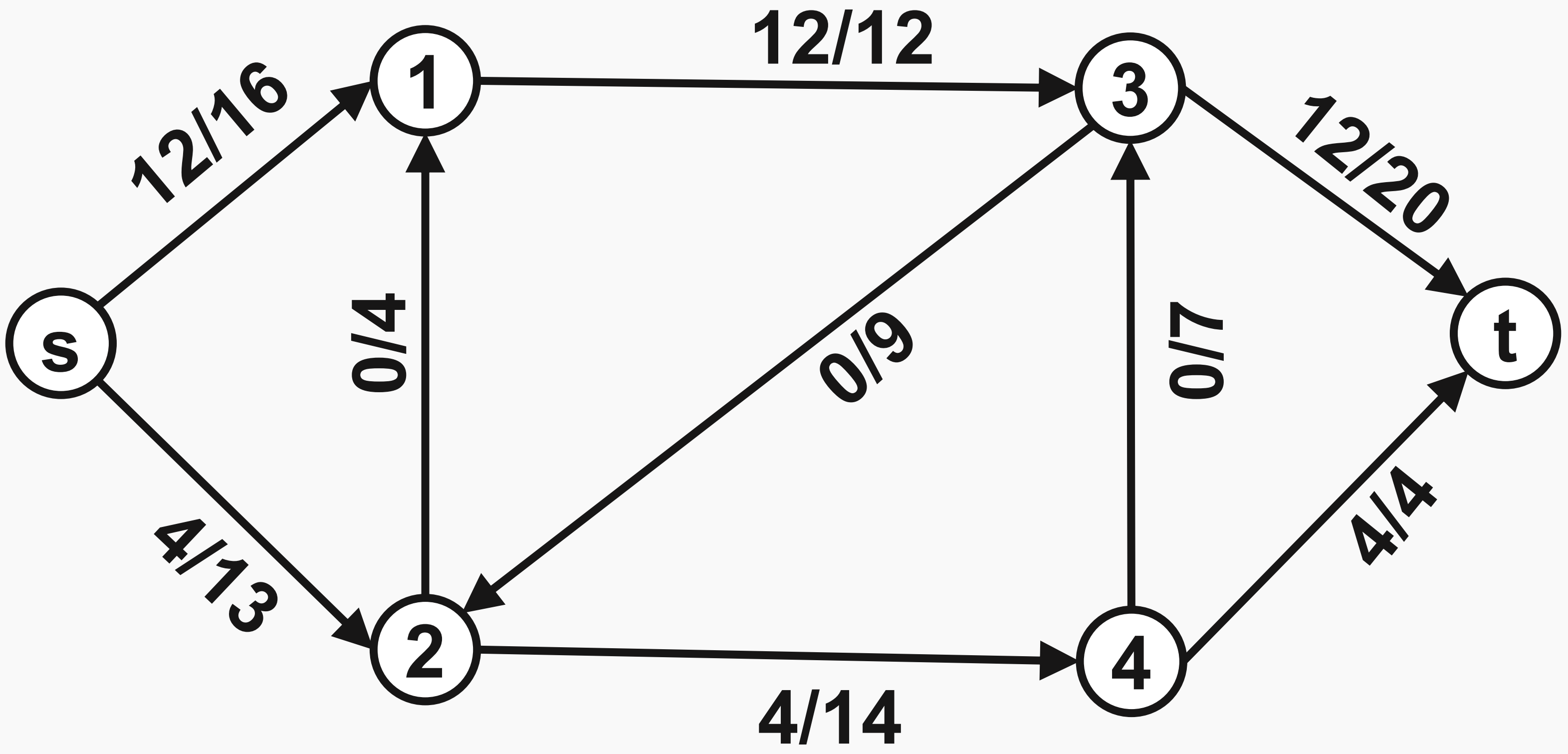

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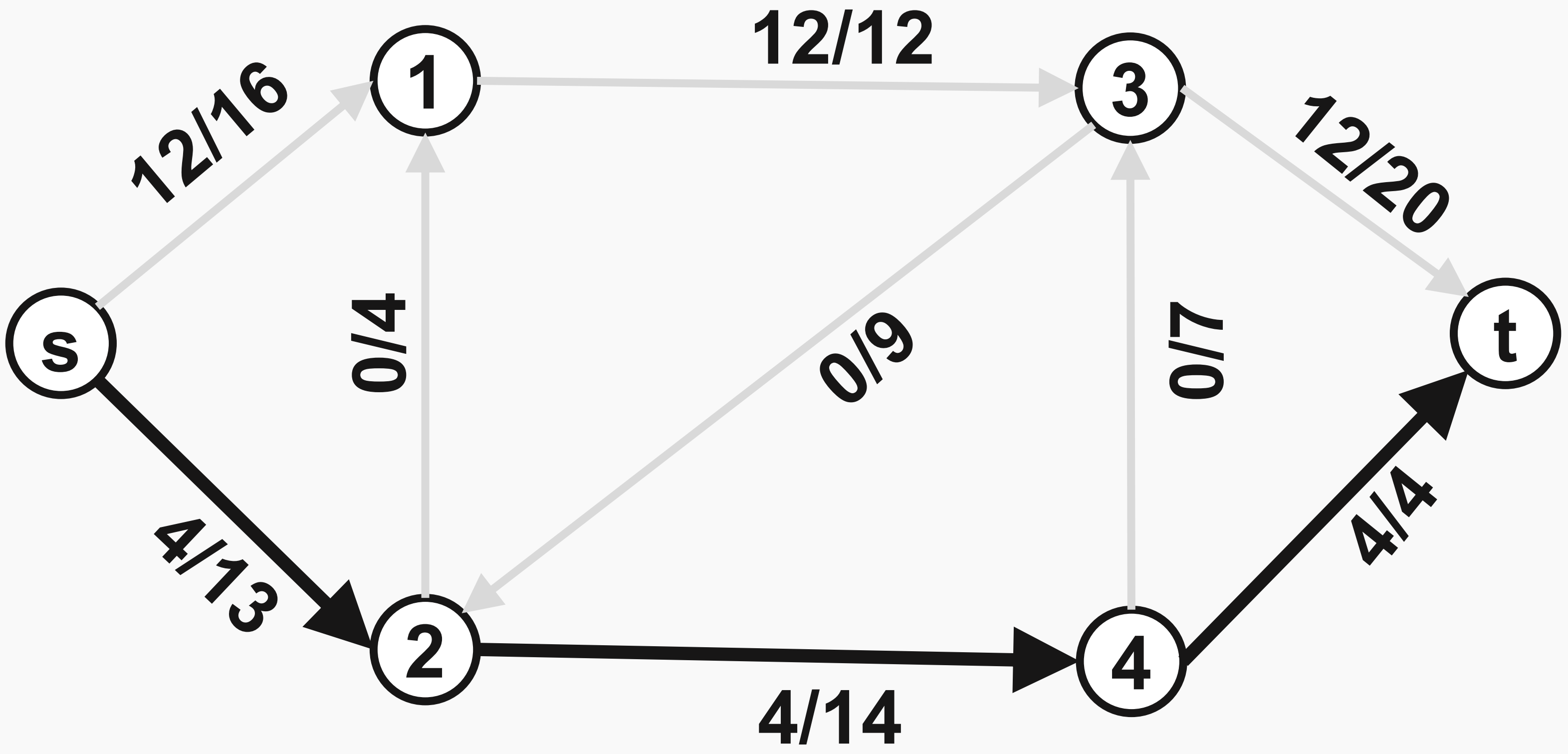

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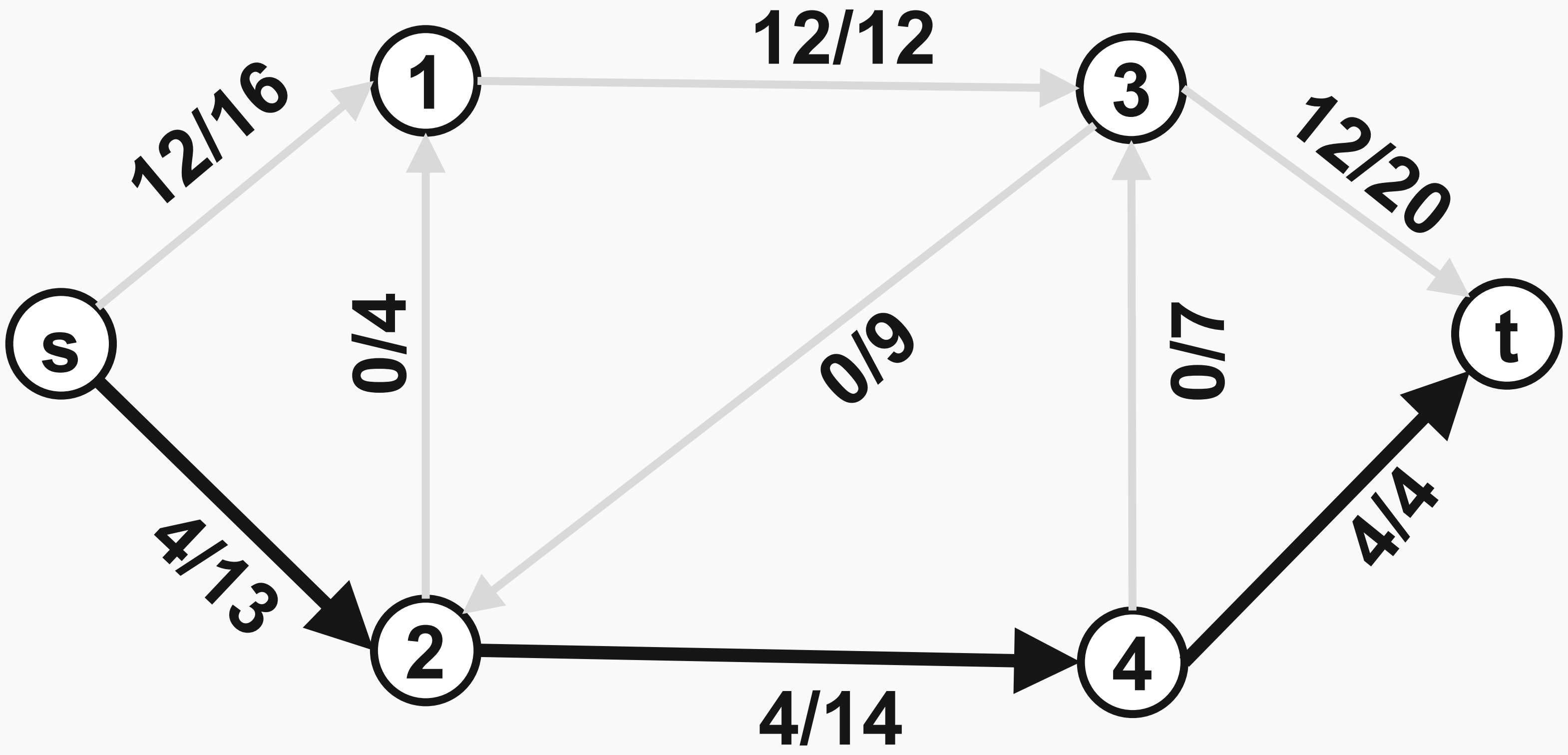

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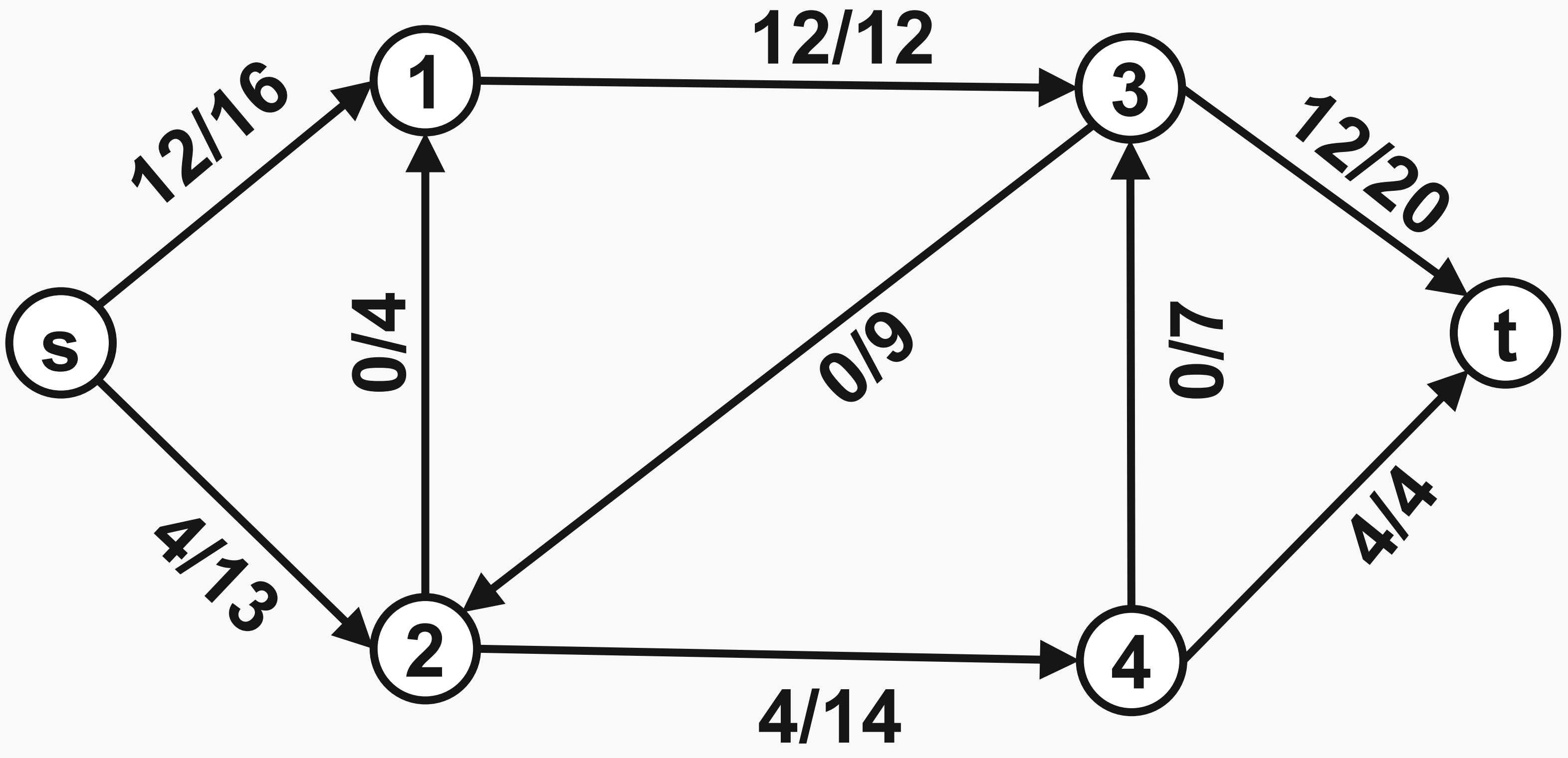

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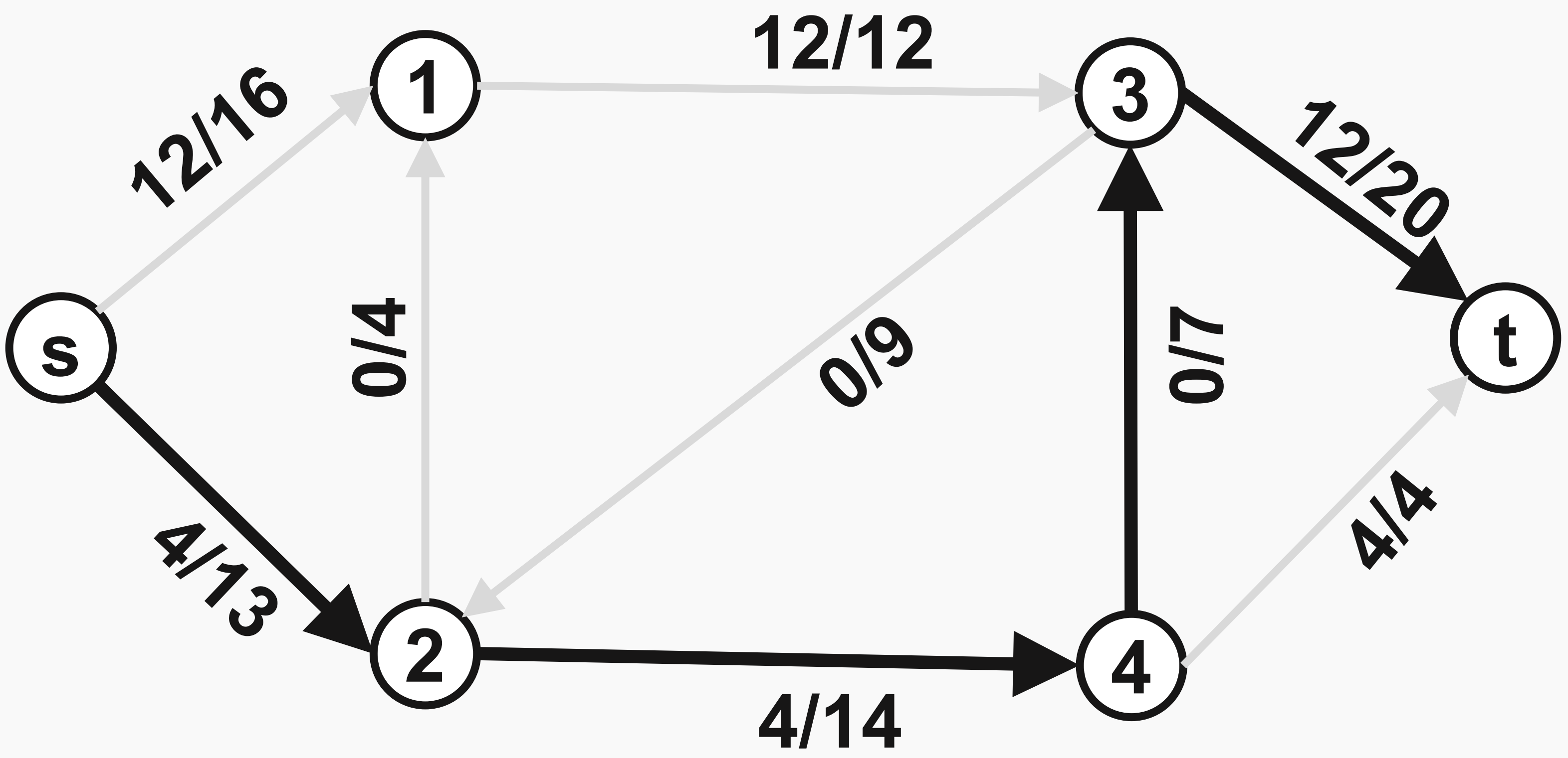

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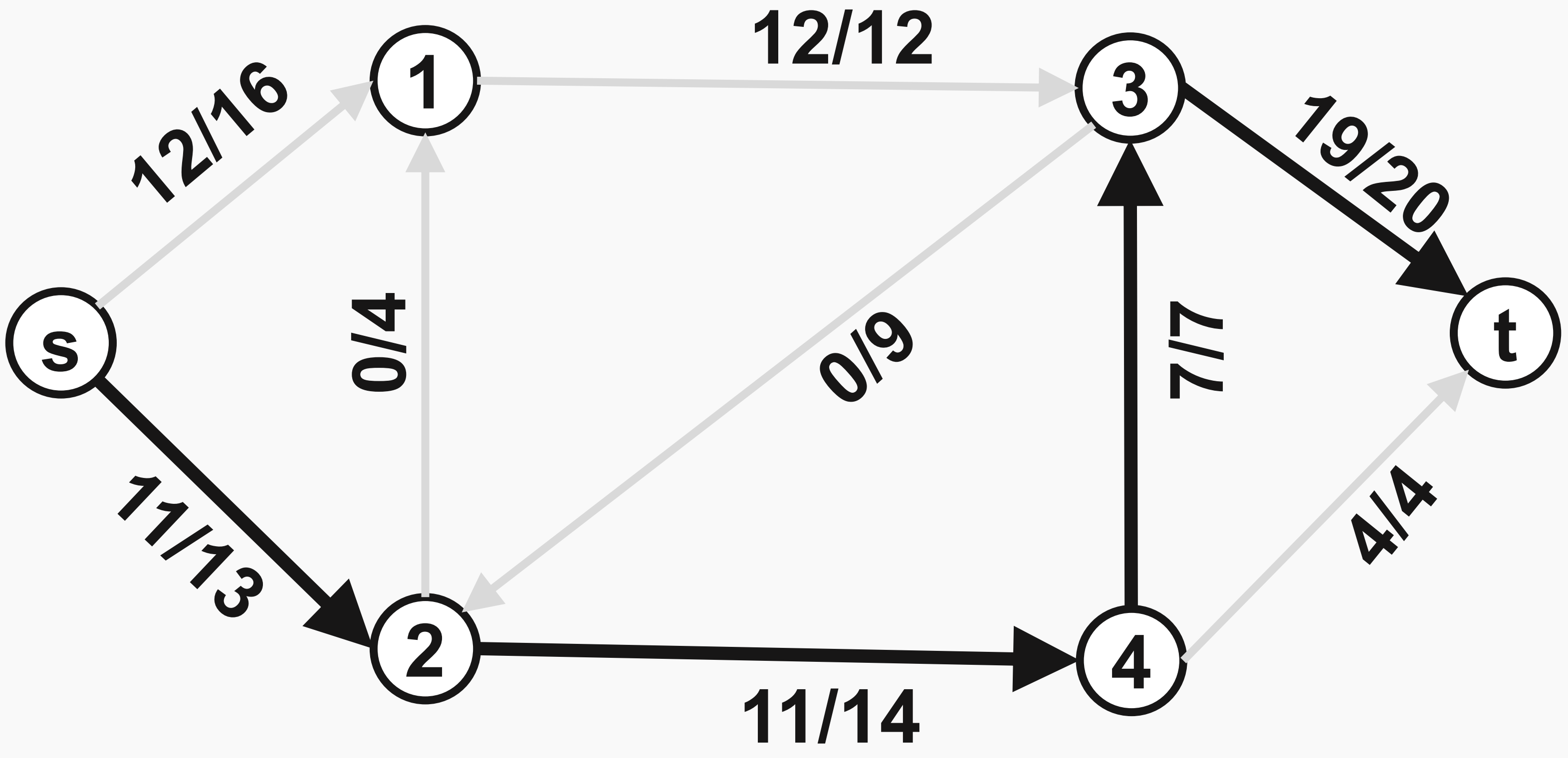

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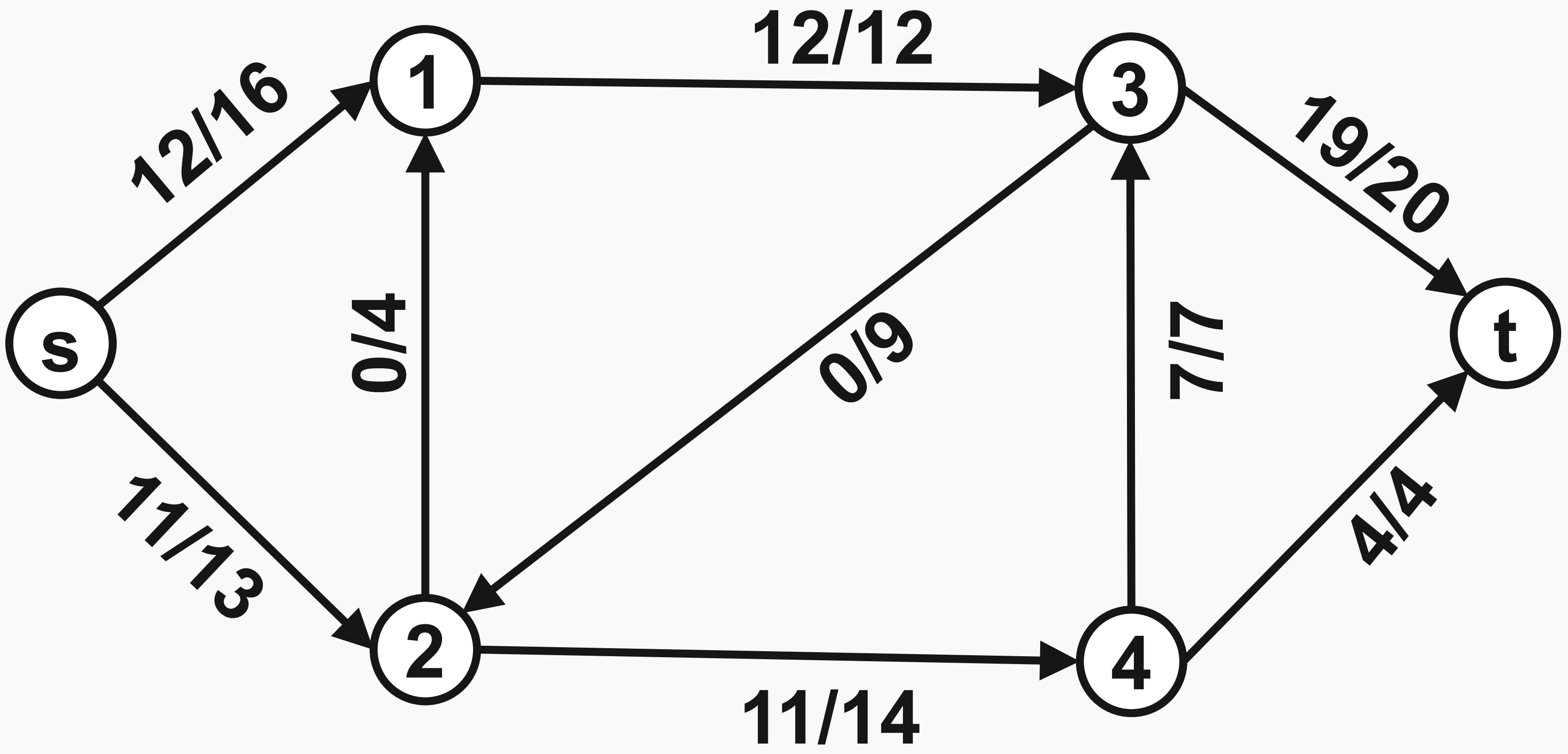

Complexitate?

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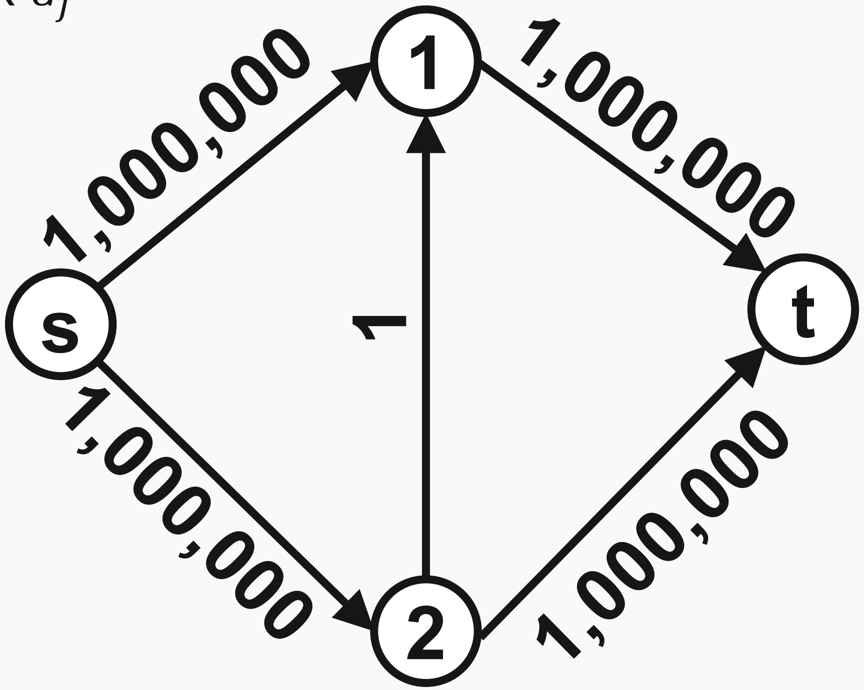

Complexitate?

Dacă alegem p random

$$O(E|f^*|)$$

FORD-FULKERSON (G, s, t)
for each edge $(u, v) \in G.E$
$\quad (u, v).f = 0$
while there exists a path p from s to t in the residual network G_f
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$\quad$ **for** each edge (u, v) in p
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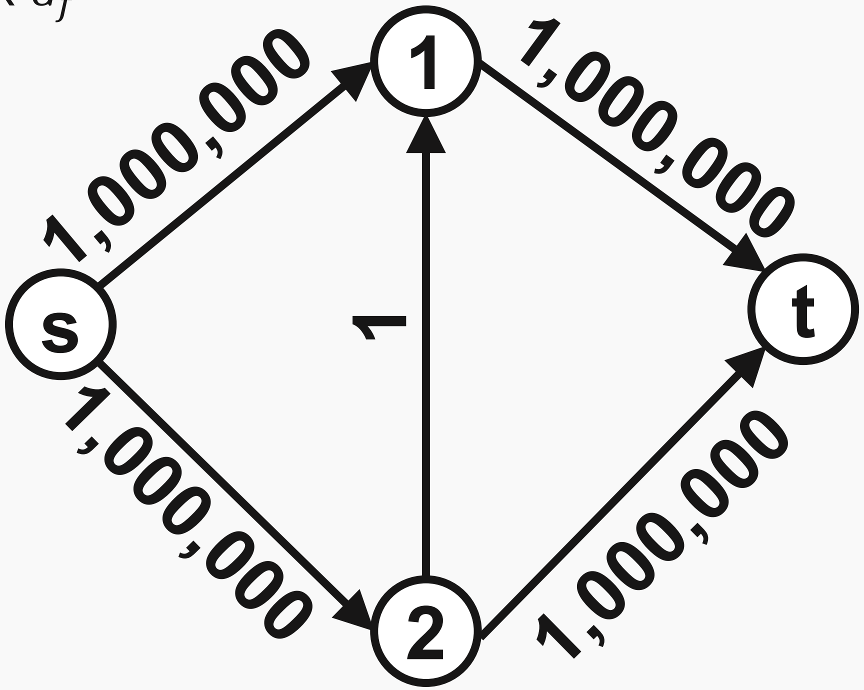
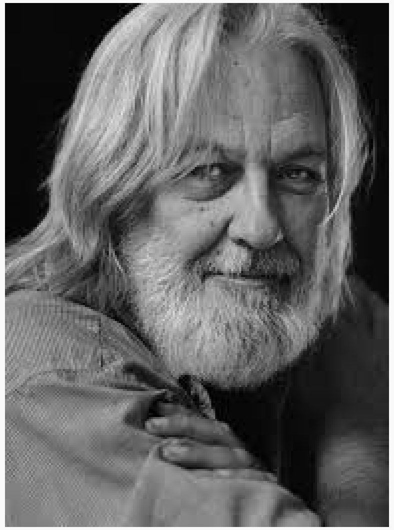


Algoritmul Edmonds–Karp (1972)

- Dacă alegem p folosind BFS



$$O(VE^2)$$

Există și algoritmi cu complexitate mai bună: $O(V^3)$ și $O(VE)$

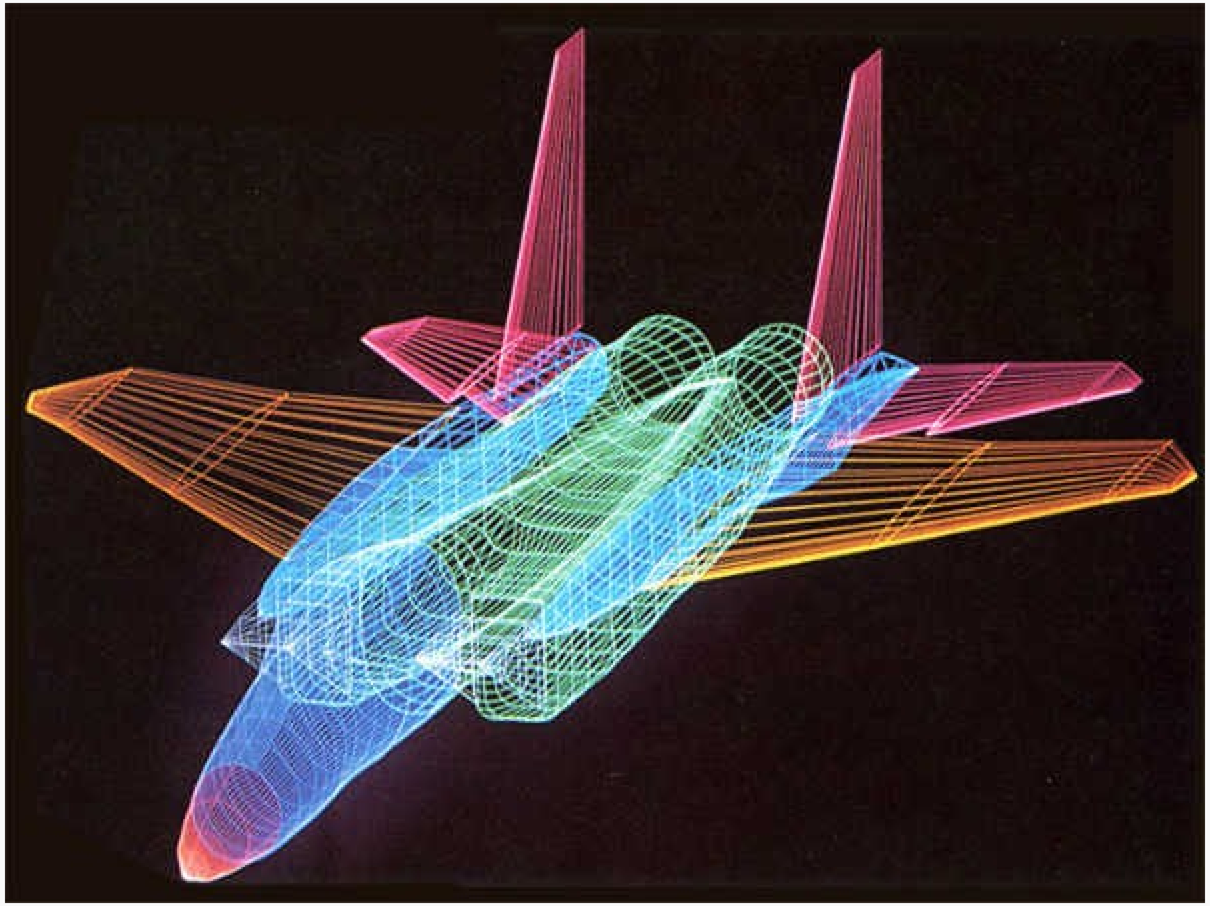
Alte considerente grafuri

- Se pot schimba în timp.

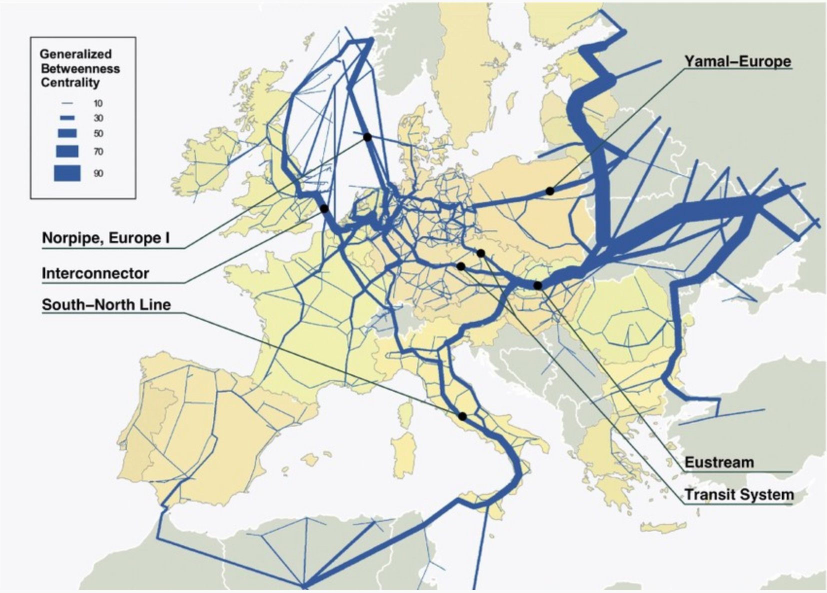
- LineGraph – Pentru un graf non-direcțional muchiile devin noduri și nodurile muchii.

Use case grafuri – Granițe

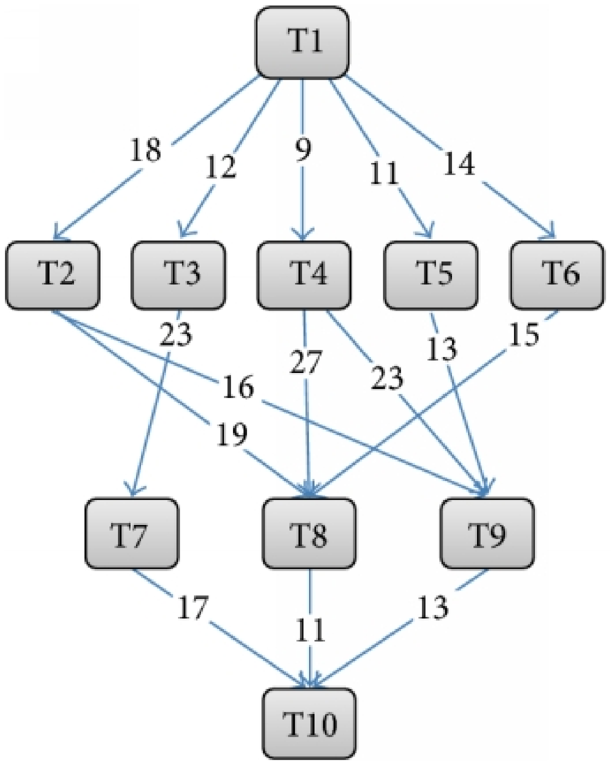
Use case grafuri – Grafică calculator



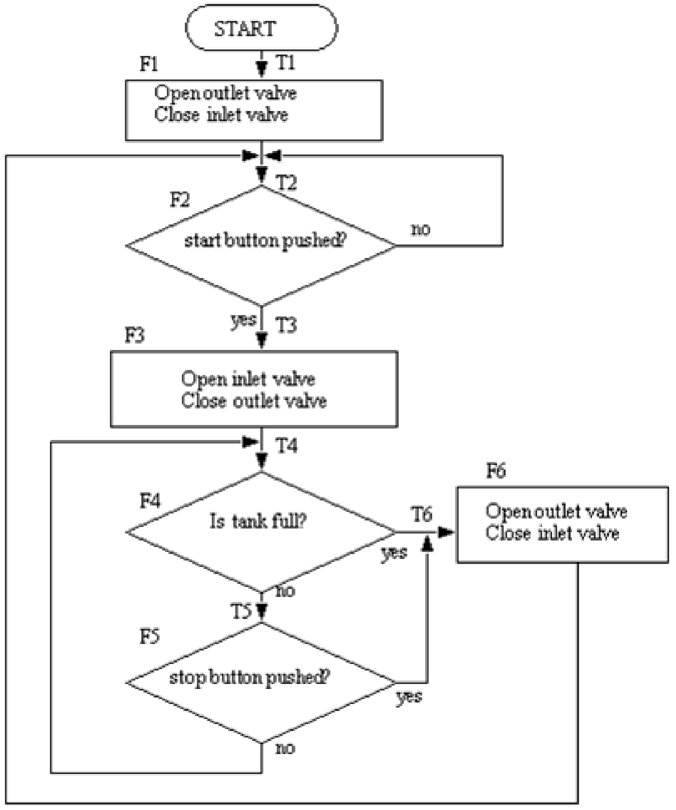
Use caser grafuri – utilități



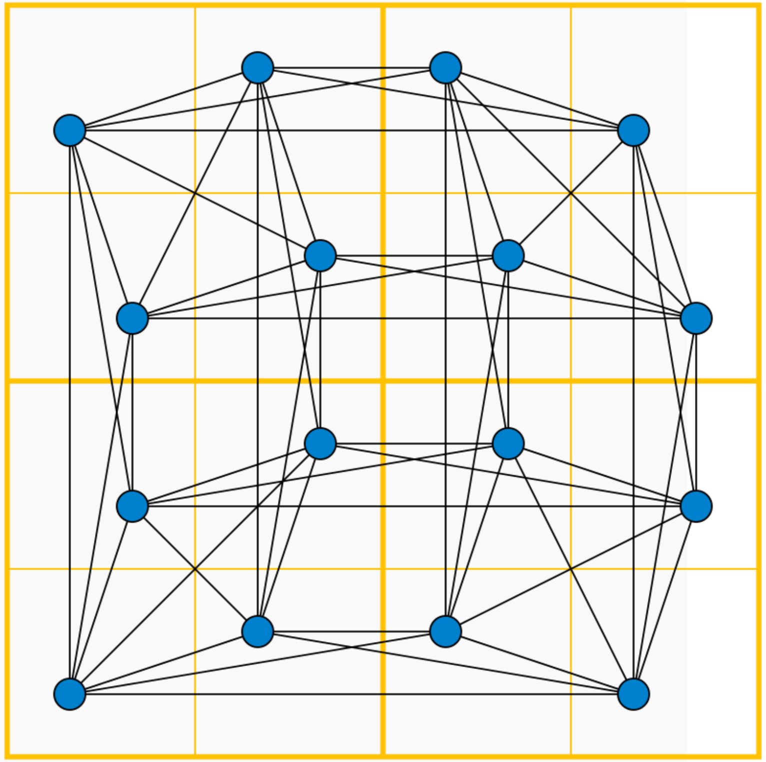
Use case grafuri – Code

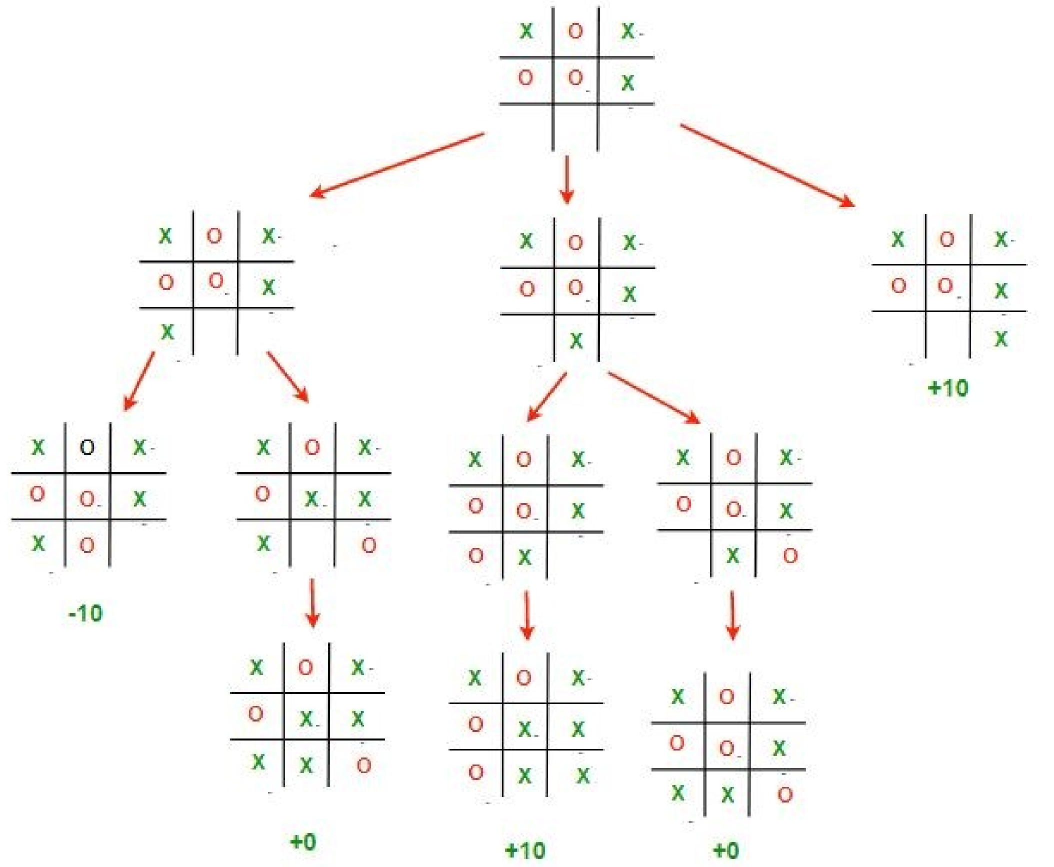


Task	P_1	P_2	P_3
T1	14	16	9
T2	13	19	18
T3	11	13	19
T4	13	8	7
T5	12	13	10
T6	13	16	9
T7	7	15	11
T8	5	11	14
T9	18	12	20
T10	21	7	16

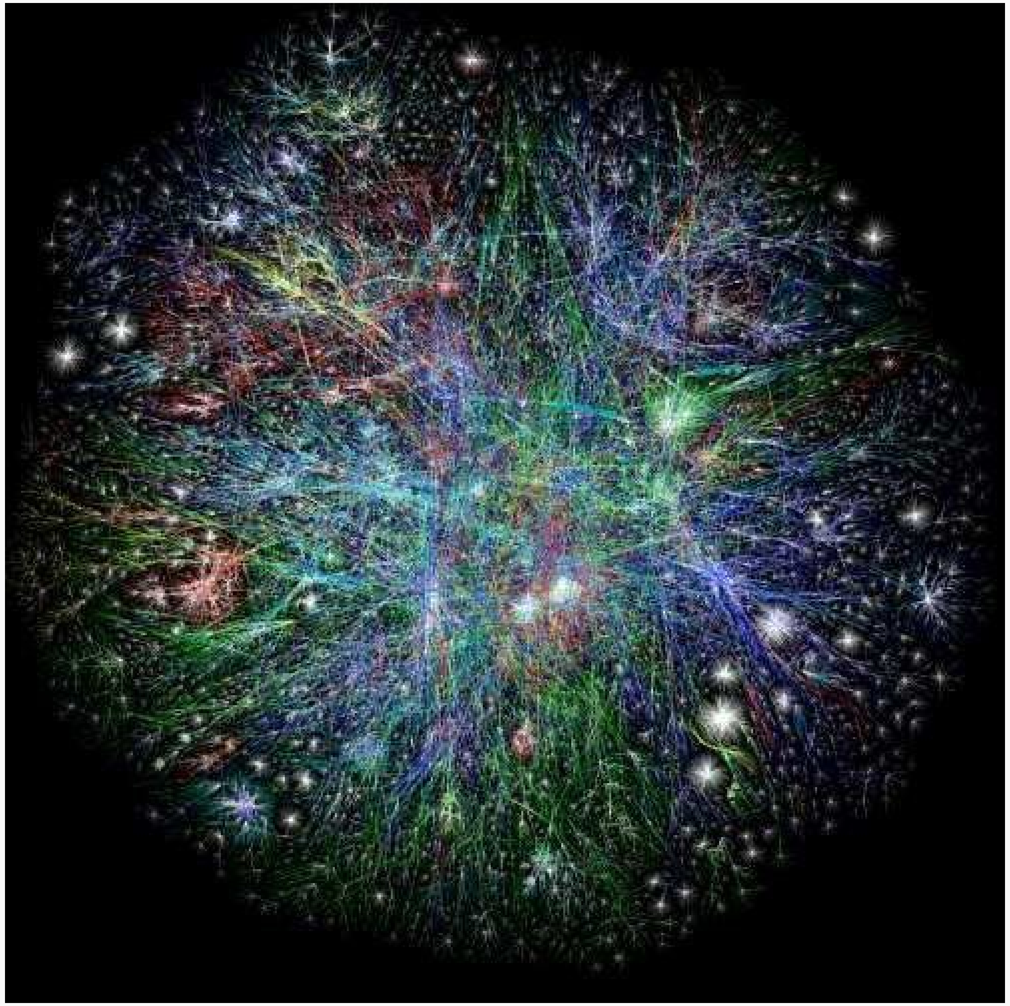


Use case grafuri – Reprezentare Jocuri

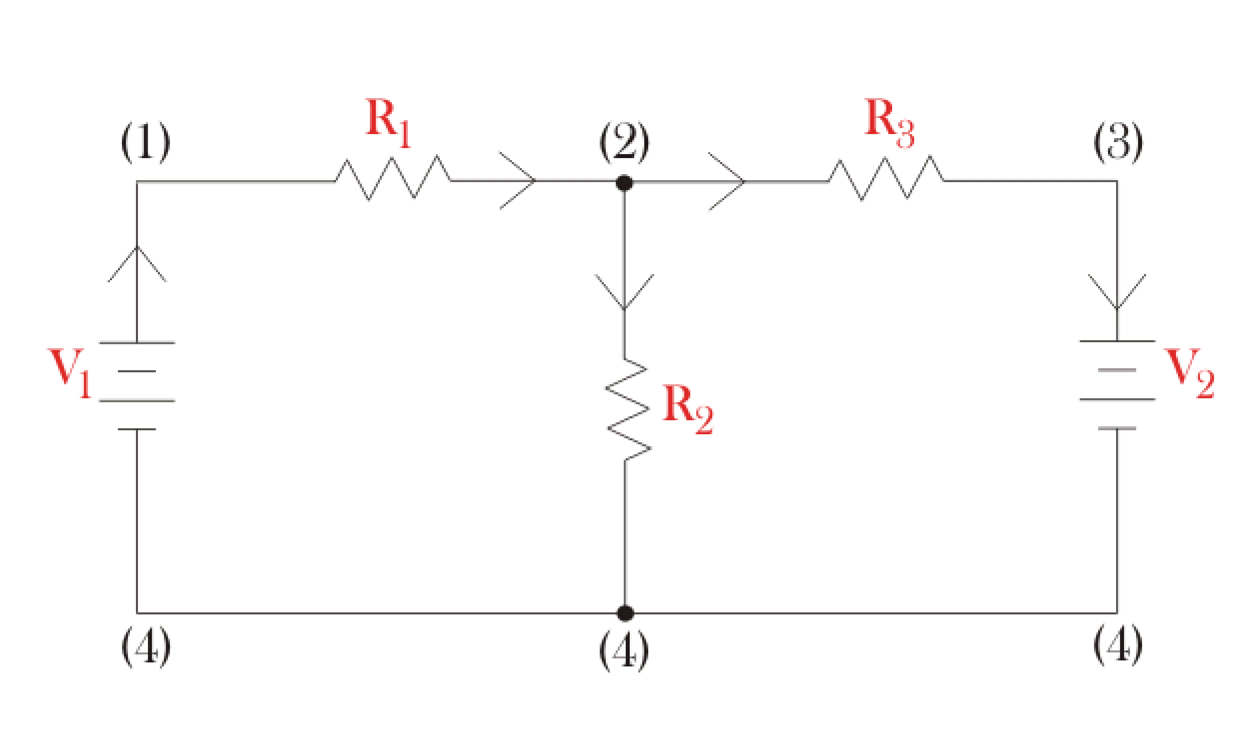




Use case grafuri - Internet



Use case grafuri – Circuite electrice



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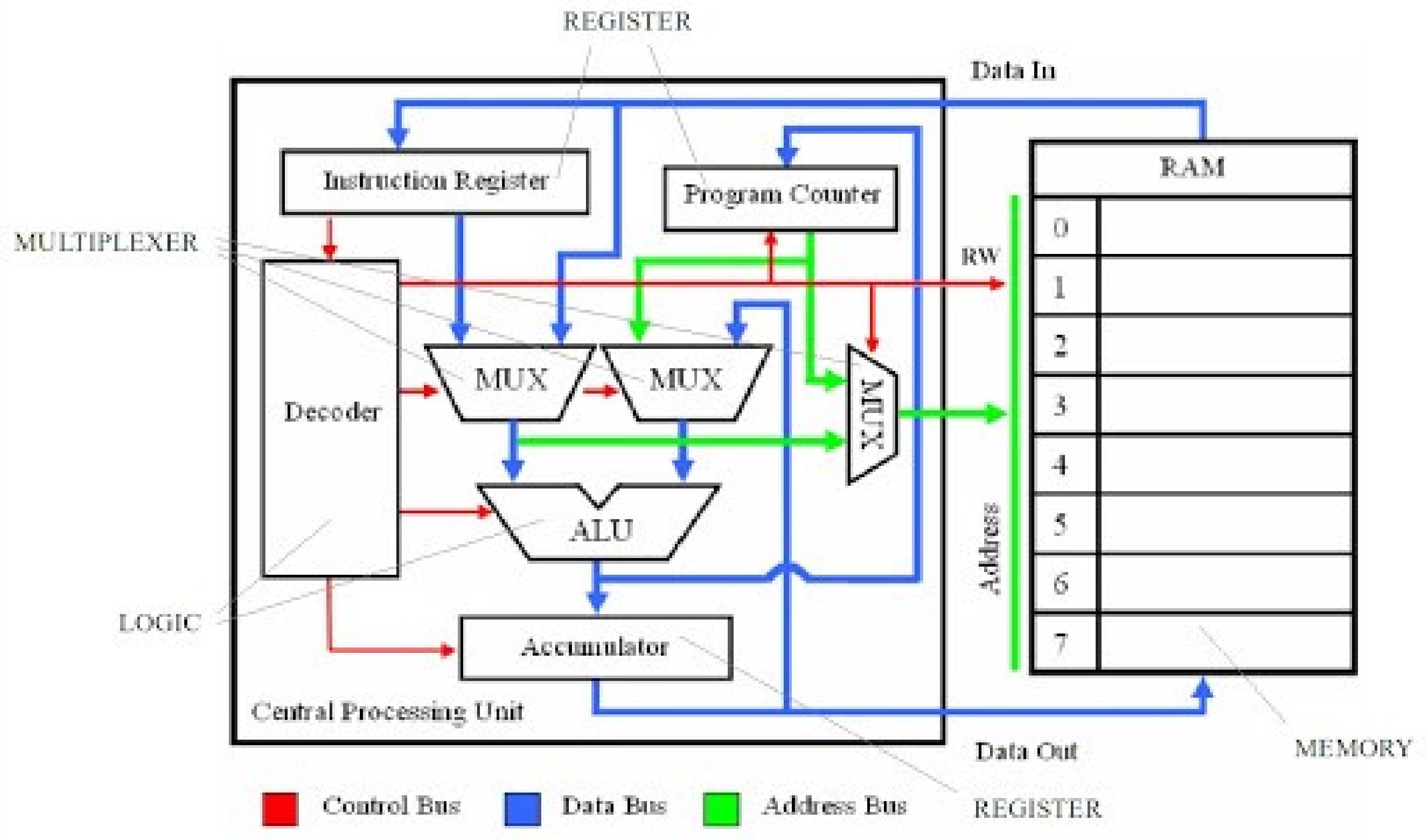
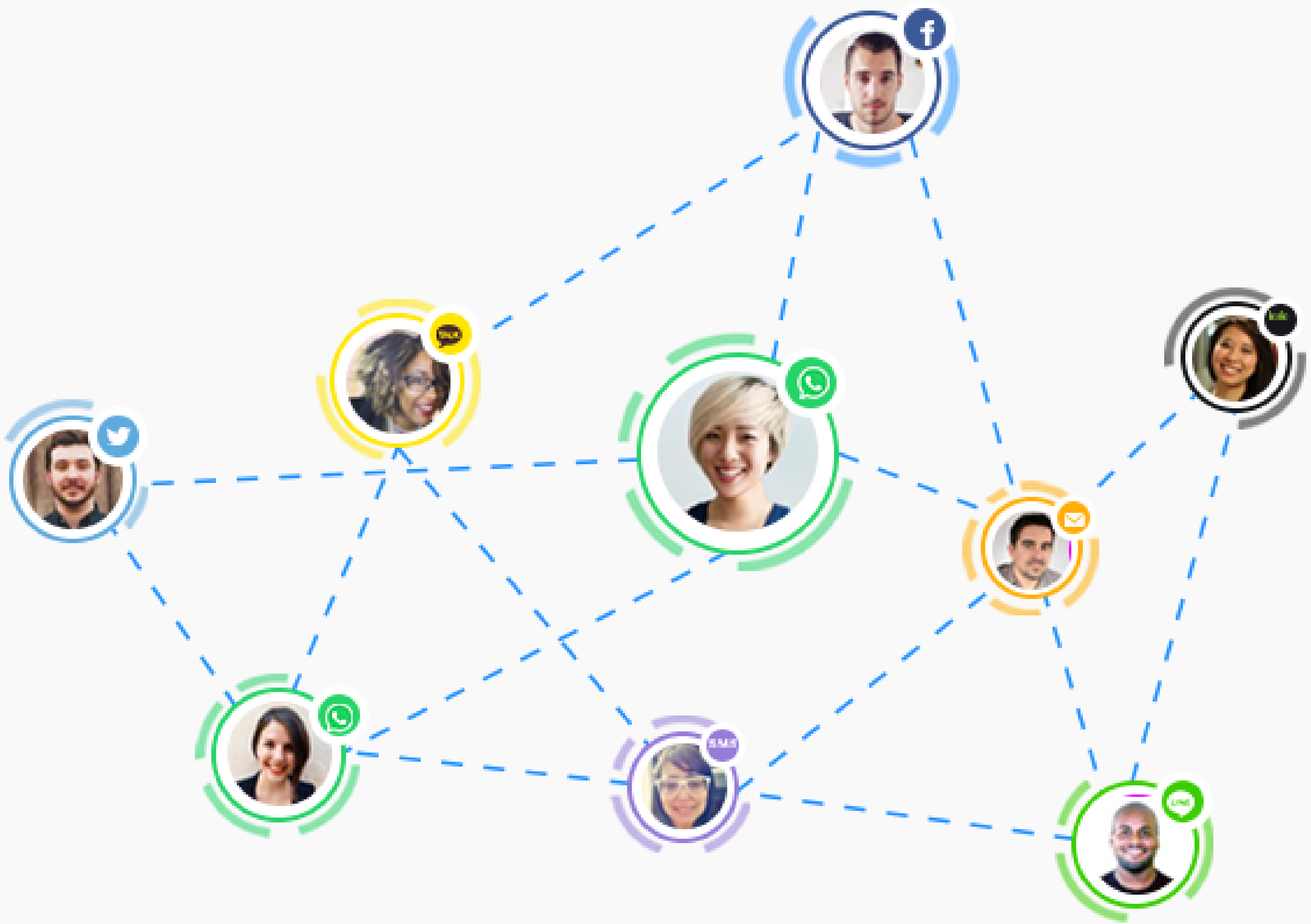
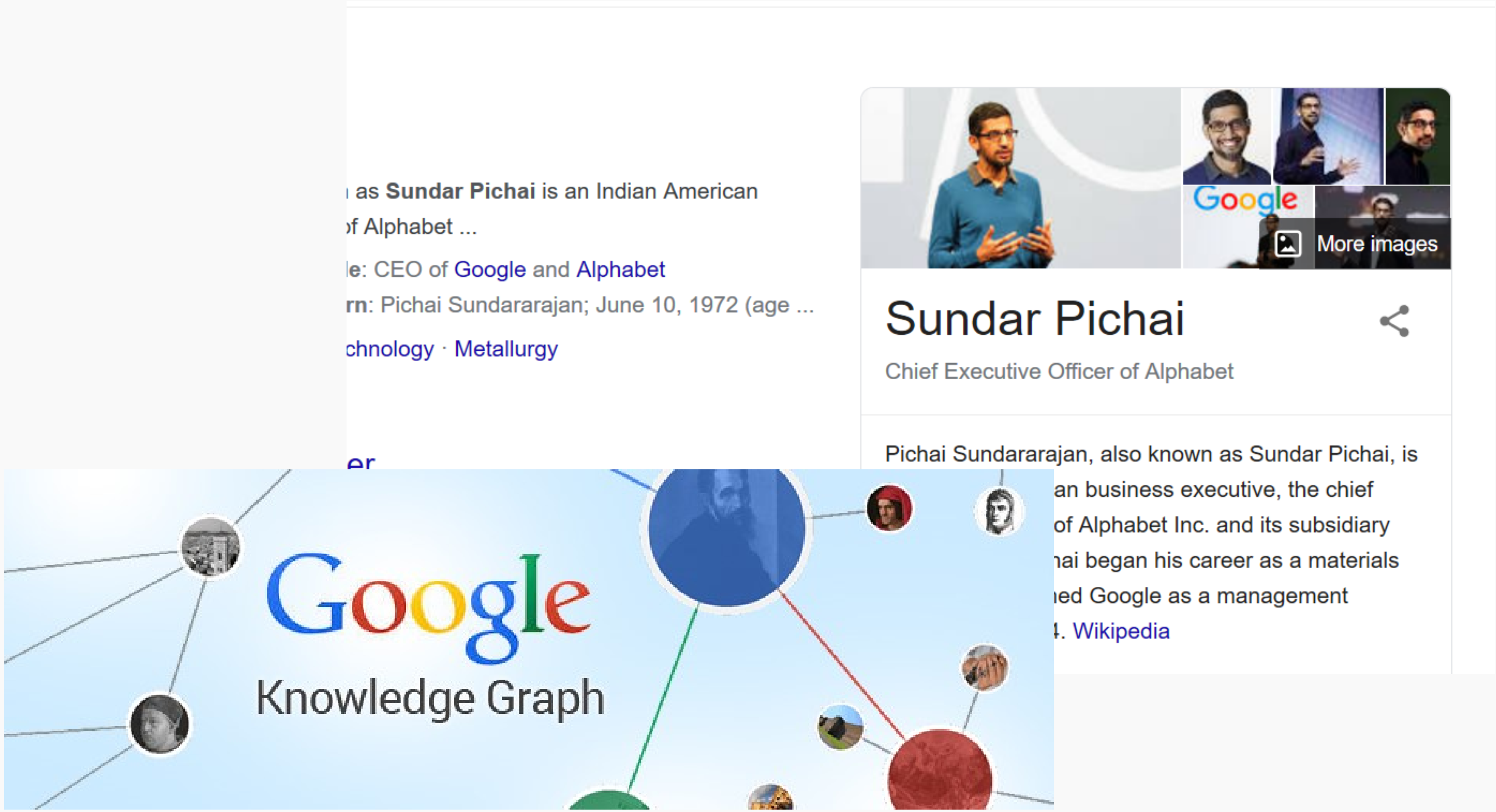


Use case grafuri – Grafuri sociale



Use case-uri – Knowledge graph



Use case-uri – Organigrame