



Structuri de date și algoritmi

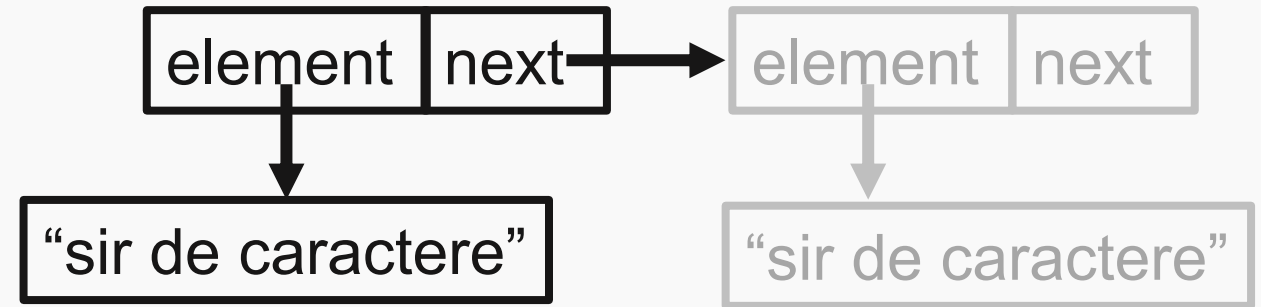
Liste, Stive, Cozi - 2

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Liste înlanțuite

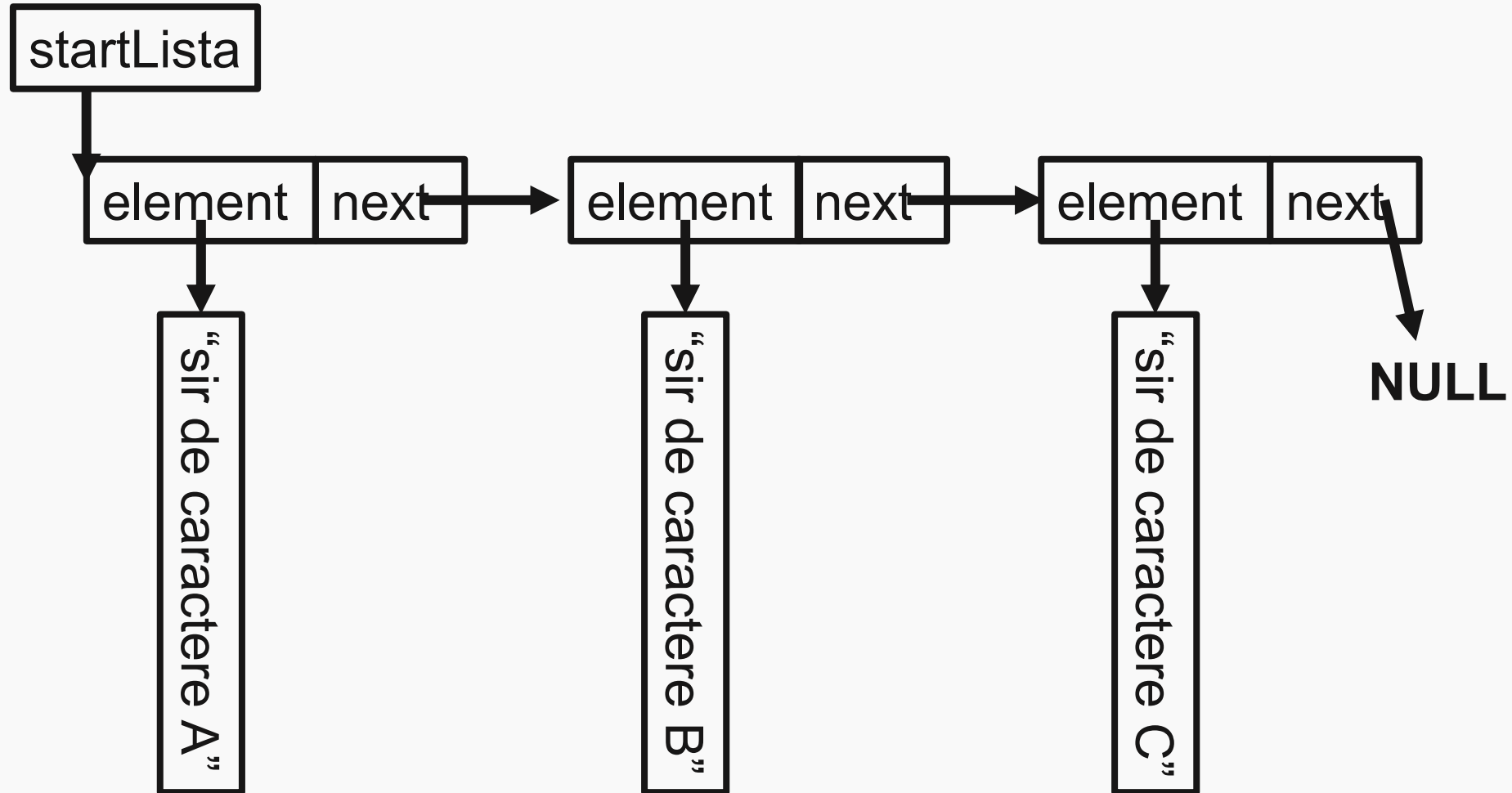
```
struct node {  
    char* element;  
    struct node* next;  
};
```



```
typedef struct node {  
    char* element;  
    struct node* next;  
}listNode;
```



Liste înlănțuite





Accesarea unui element din listă

```
listNode* getNode(listNode* listNode, int poz) {  
    for (int i = 0; i < poz; i++) {  
        listNode = listNode->next;  
        if (listNode->next == NULL)  
            break;  
    }  
    return listNode;  
}
```



Inserare element început de listă

```
void insertNodeHeadOfList(listNode** listStart, char* element) {  
    listNode* node = (listNode*)malloc(sizeof(listNode));  
    if (node == NULL) {  
        printf("ERROR: CAN NOT ALLOCATE RAM\n");  
        return;  
    }  
    node->next = *listStart;  
    node->element = element;  
    *listStart = node;  
}
```



Inserare element în listă la o poziție dată

```
void insertNodeInList(listNode** listStart, char* element, int poz) {  
    if (poz == 0) {  
        insertNodeHeadOfList(listStart, element);  
        return;  
    }  
    listNode* node = (listNode*)malloc(sizeof(listNode));  
    if (node == NULL) {  
        printf("ERROR: CAN NOT ALLOCATE RAM\n");  
        return;  
    }  
  
    listNode* aux = getNode(*listStart, poz - 1);  
  
    node->next = aux->next;  
    node->element = element;  
    aux->next = node;  
}
```



Ștergere element început de listă

```
void removeNodeHeadOfList(listNode** listStart) {  
    // free((*listStart)->element);  
    if (*listStart == NULL)  
        return;  
    listNode* aux = (*listStart);  
    *listStart = (*listStart)->next;  
    free(aux);  
}
```



Ștergere element din listă de la o poziție dată

```
void removeNodeFromList(listNode** listStart, int poz) {  
    if (poz == 0) {  
        removeNodeHeadOfList(listStart);  
        return;  
    }  
    listNode* aux = getNode(*listStart, poz-1);  
    if (aux->next != NULL) {  
        listNode* aux1 = aux->next;  
        aux->next = aux->next->next;  
        // free(aux1->element);  
        free(aux1);  
    }  
}
```




Complexități operații cu liste

- Accesarea unui element - $O(N)$
- Inserare element la capăt - $O(1)$
- Inserare element la o poziție - $O(N)$
 - ▣ Accesare + inserare
- Ștergere element la capăt - $O(1)$
- Ștergere element de la o poziție - $O(N)$
 - ▣ Accesare + ștergere



Liste vs Vectori

	Vector	Listă
Complexitate acces	$O(1)$	$O(N)$
Complexitate inserție	$O(N)$	$O(N)$
Complexitate inserție capete	$O(N)/O(1)$	$O(1)$
Complexitate ștergere	$O(N)$	$O(N)$
Complexitate ștergere capete	$O(N)/O(1)$	$O(1)$
Alocare spațiu	Doar la început	Oricând și oricât



De ce la liste avem inserție/ștergere $O(1)$ la ambele capete?



De ce la liste avem inserție/ștergere $O(1)$ la ambele capete?

Putem să avem pointeri HeadList și EndList



Vectori Alocați Dinamic



Vectori Alocați Dinamic

- O mărime de start – poate fi și un element.
- Când vectorul este plin și se încearcă un insert, se dublează dimensiunea sa.

1	2	3	4
---	---	---	---

push(**5**)

1	2	3	4	5			
---	---	---	---	---	--	--	--



Vectori Alocați Dinamic

Push()	Copiere	Mărime vector
1	0	1
2	1	2
3	2	4
4	0	4
5	4	8
6	0	8
7	0	8
8	0	8
9	8	16
10	0	16



Complexitate amortizată inserție vector dynamic (în capăt)

- 3 push 2 + 1 copieri
- 5 push 4 + 2 + 1 copieri
- 9 push 8 + 4 + 2 + 1 copieri
- ...
- $2^n + 1$ push $2^n + 2^{n-1} + 2^{n-2} + \dots + 2 + 1 = 2^{n+1} - 1$ copieri

$$\lim_{n \rightarrow \infty} \frac{2^{n+1} - 1}{2^{n+1}} = 2 = O(1)$$



Self-organizing lists



Self-organizing lists

- Premisă: Nu toate elementele sunt echiprobabile la căutare.
 - ▣ Unele elemente sunt accesate mult mai des decât altele.
- Dacă elementele sunt **echiprobabile** la căutare:
 - ▣ Probabilitatea de a găsi un element p_i este egală cu probabilitatea de a găsi orice alt element p_j

$$\sum_{i=0}^N p_i = 1 \Rightarrow p_i = \frac{1}{N}$$

- ▣ Timpul mediu de găsim a unui element este

$$\sum_{i=0}^N i * p_i = \frac{1}{N} O(N^2) = O(N)$$



Ordinea listei contează

- Presupunem că pentru un $i < j$ avem $p_i < p_j$
- $C = 1 p_1 + \dots + i \textcolor{teal}{p}_i + j \textcolor{red}{p}_j + \dots + N p_N$
- Dacă interschimbăm elementele de pe pozițiile i și j :
- $C' = 1 p_1 + \dots + i \textcolor{red}{p}_j + j \textcolor{teal}{p}_i + \dots + N p_N$

$$\begin{aligned} C' - C &= (i \textcolor{red}{p}_j + j \textcolor{teal}{p}_i) - (i \textcolor{teal}{p}_i + j \textcolor{red}{p}_j) = \\ &= i(\textcolor{red}{p}_j - \textcolor{teal}{p}_i) - j(\textcolor{teal}{p}_i - \textcolor{red}{p}_j) = (i - j)(\textcolor{red}{p}_j - \textcolor{teal}{p}_i) < 0 \Rightarrow C > C' \end{aligned}$$



Self-organizing lists

probabilități descrescătoare

$$p_i = \frac{1}{2^i}$$

$$\begin{aligned} C &= 1 p_1 + \dots + N p_N = 1 \frac{1}{2} + \dots + N \frac{1}{2^N} = \\ &= \sum_{i=1}^N \frac{i}{2^i} = 2 \quad (n \rightarrow \infty) \end{aligned}$$



Mecanisme de ordonare automată

- Sortare la intervale prestabilite de timp
- Când un element este accesat acesta se mută în capul listei *moveToFront*
- Când un element este accesat acesta își schimbă locul cu precedentul *Transpose*
- Numărăm fiecare accesare a unui element și schimbăm ordinea pentru a păstra ordonare dupa aceste valori.



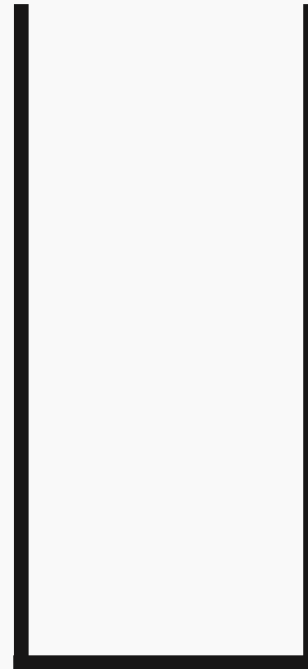
Stack - Stivă



Stack - Stivă

LIFO – Last In First Out

push(1)

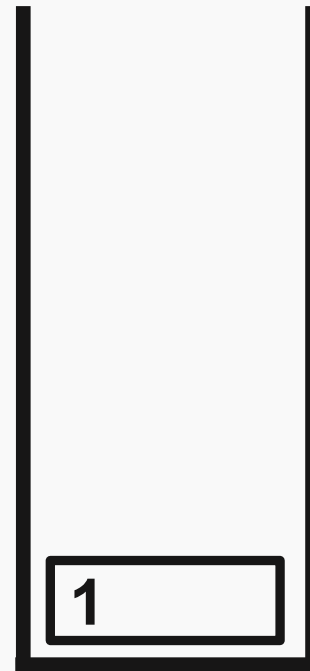




Stack - Stivă

LIFO – Last In First Out

push(2)

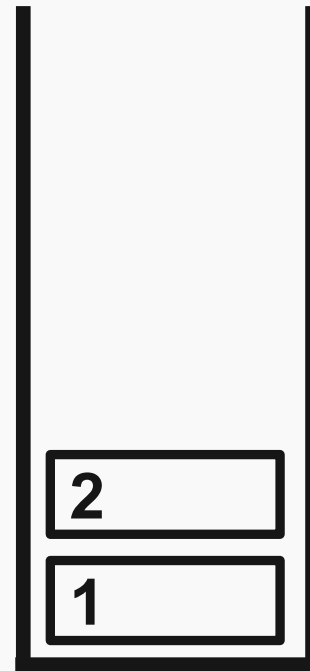




Stack - Stivă

LIFO – Last In First Out

push(3)

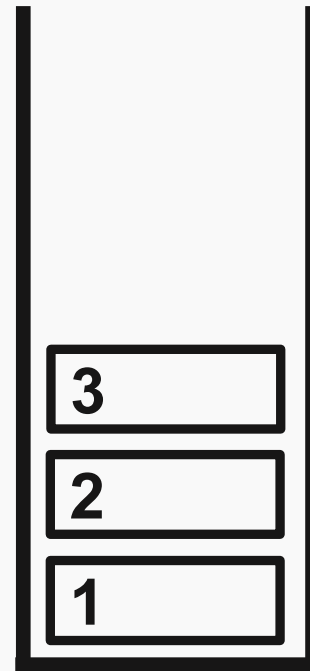




Stack - Stivă

LIFO – Last In First Out

push(4)

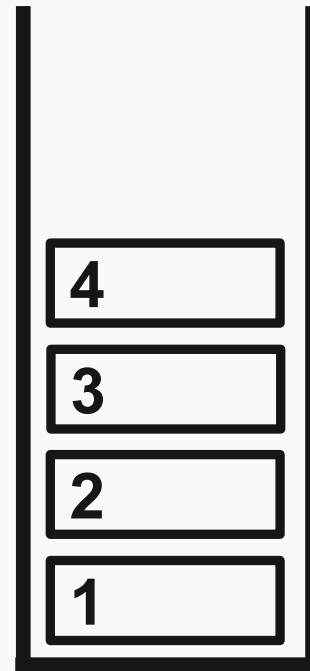




Stack - Stivă

LIFO – Last In First Out

push(5)

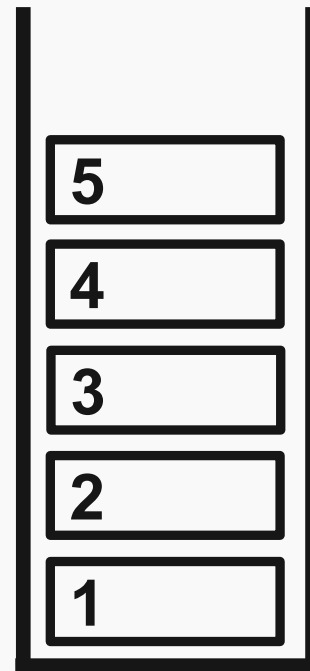




Stack - Stivă

LIFO – Last In First Out

pop()

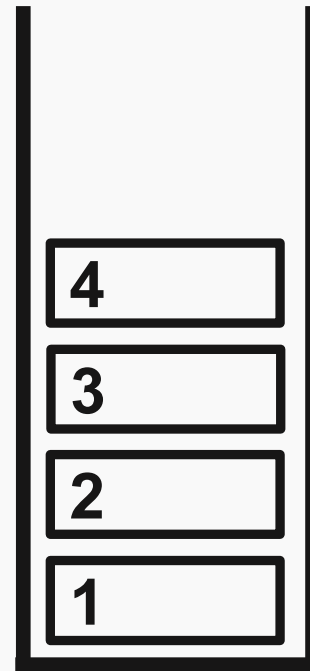




Stack - Stivă

LIFO – Last In First Out

5 pop()

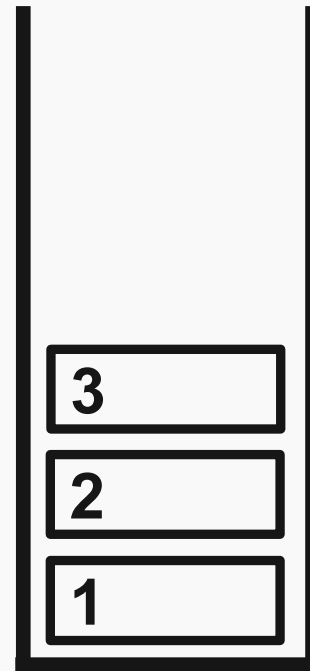




Stack - Stivă

LIFO – Last In First Out

4 pop()

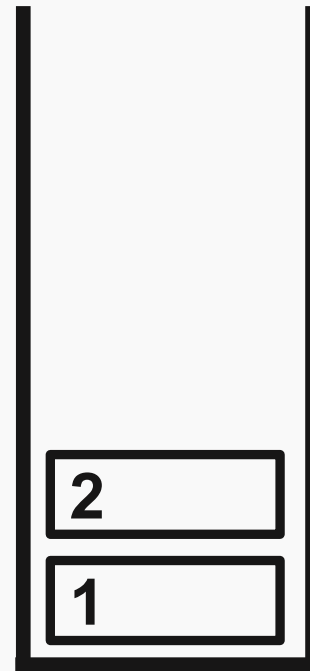




Stack - Stivă

LIFO – Last In First Out

3

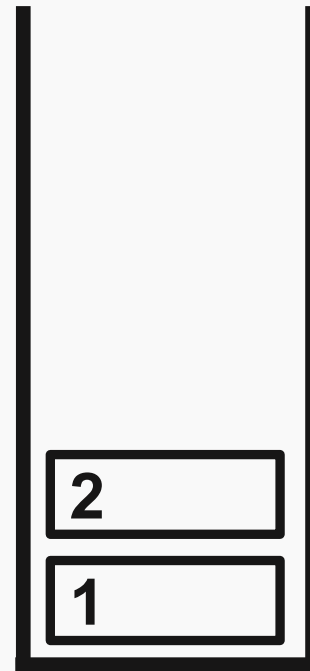




Stack - Stivă

LIFO – Last In First Out

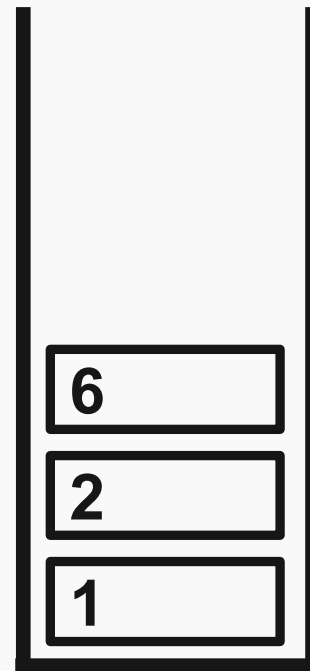
push(6)





Stack - Stivă

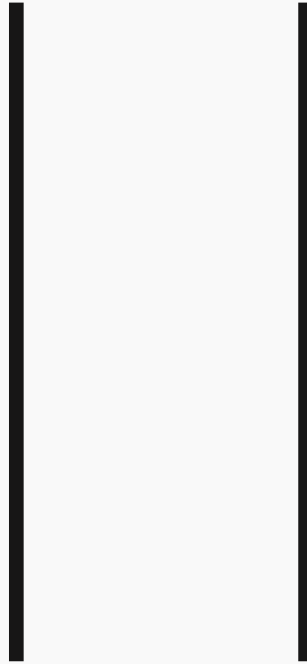
LIFO – Last In First Out





Queue - Coadă

FIFO – First In First Out

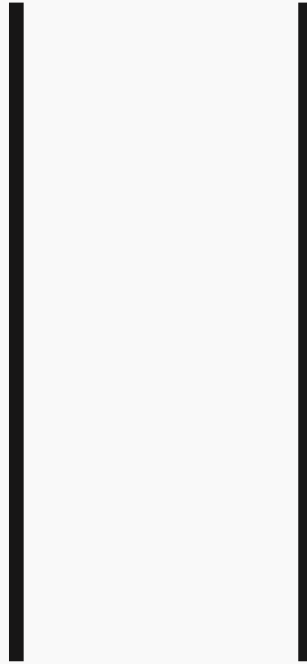




Queue - Coadă

FIFO – First In First Out

push(1)

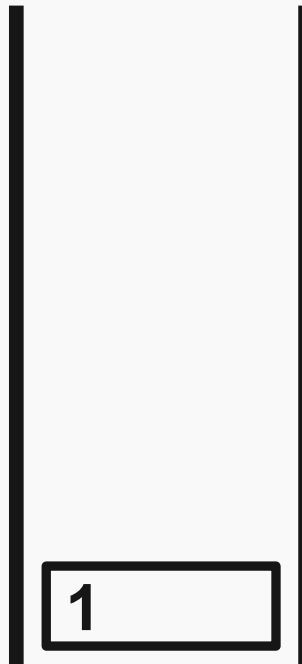




Queue - Coadă

FIFO – First In First Out

push()

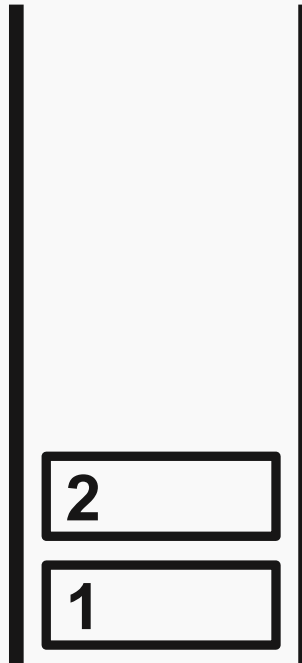




Queue - Coadă

FIFO – First In First Out

push(3)

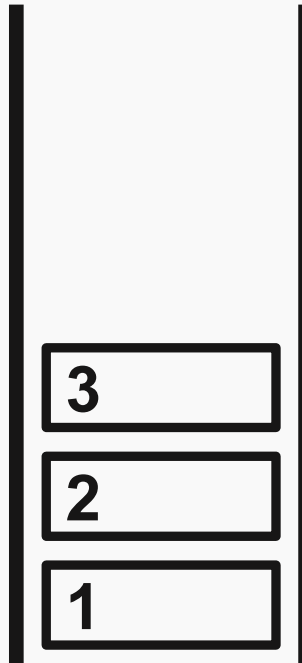




Queue - Coadă

FIFO – First In First Out

push()

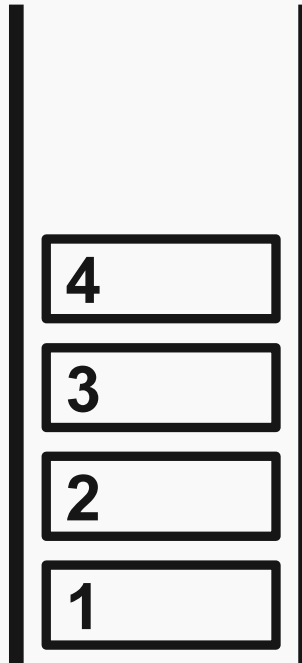




Queue - Coadă

FIFO – First In First Out

push(5)

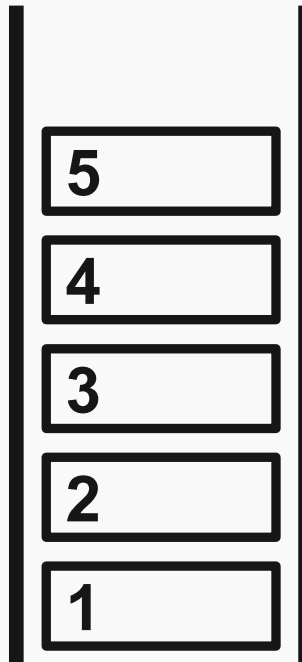




Queue - Coadă

FIFO – First In First Out

pop()

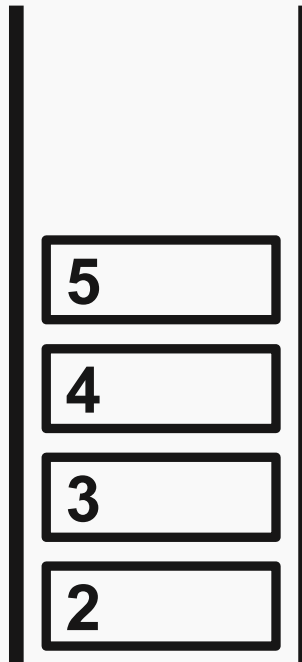




Queue - Coadă

FIFO – First In First Out

1 pop()

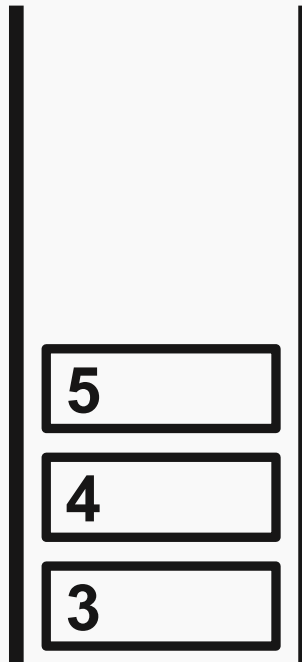




Queue - Coadă

FIFO – First In First Out

2 pop()

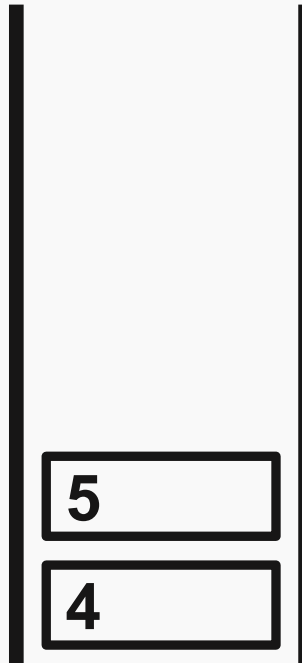




Queue - Coadă

FIFO – First In First Out

3

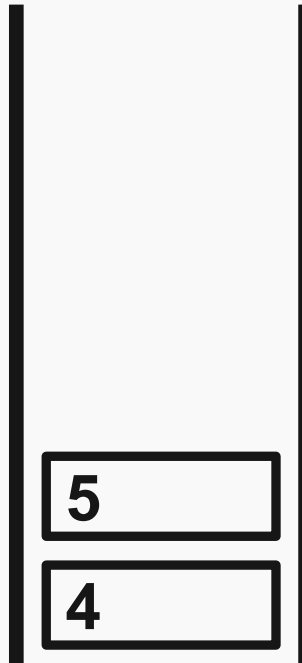




Queue - Coadă

FIFO – First In First Out

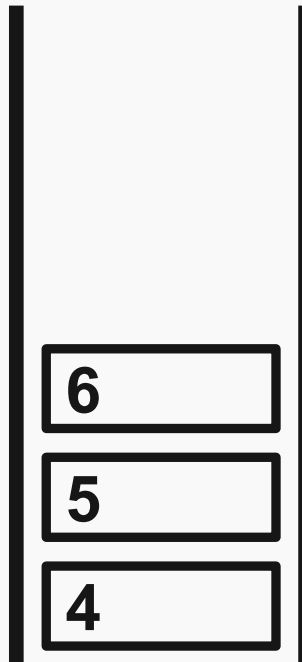
push()





Queue - Coadă

FIFO – First In First Out





Coadă de priorități



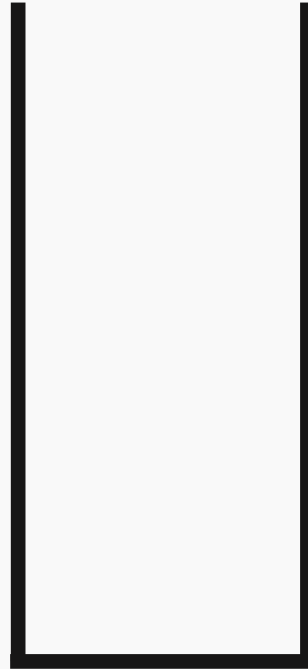


Inversarea unui șir



Inversarea unui șir

ABCDEF

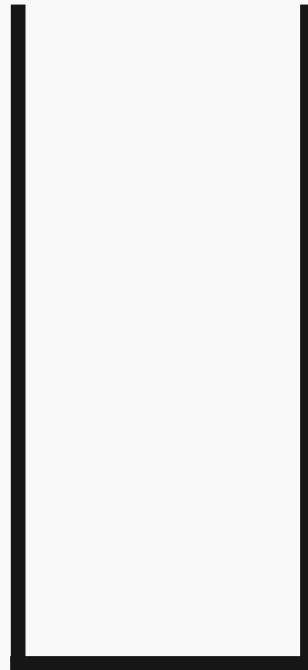




Inversarea unui șir

BCDEF

push(A)

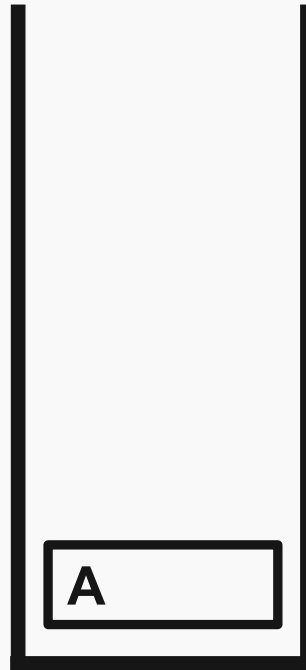




Inversarea unui șir

CDEF

push(**B**)

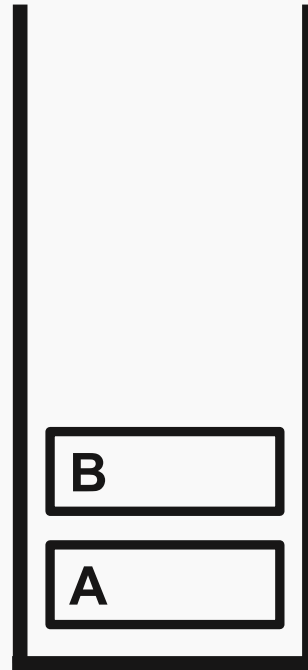




Inversarea unui şir

DEF

push(c)

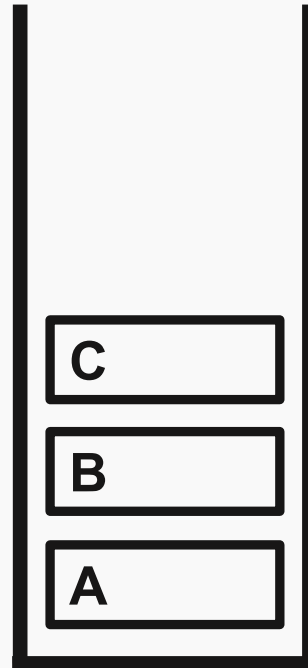




Inversarea unui șir

EF

push(D)

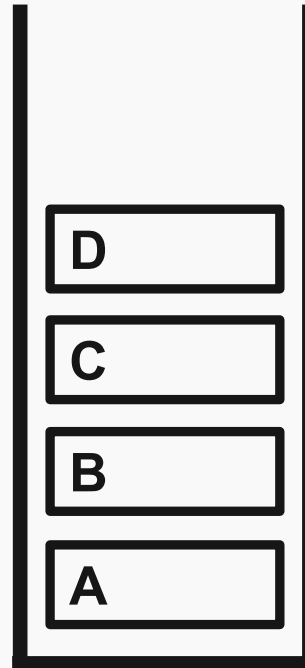




Inversarea unui șir

F

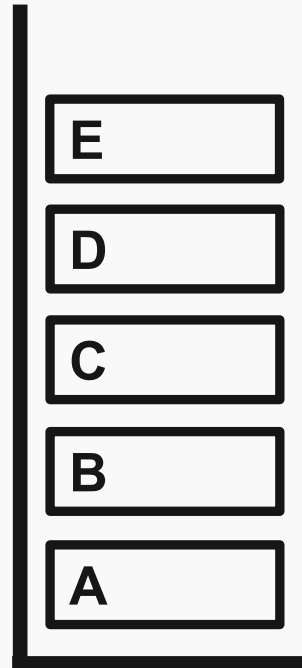
push(E)





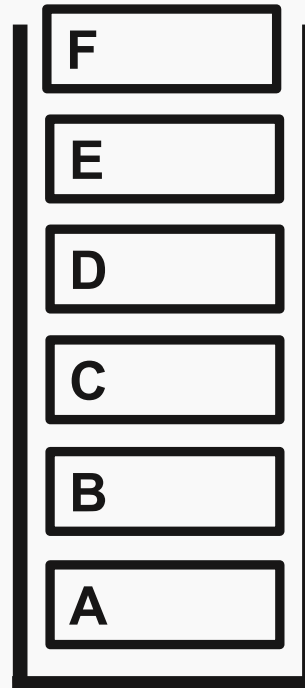
Inversarea unui șir

push(F)





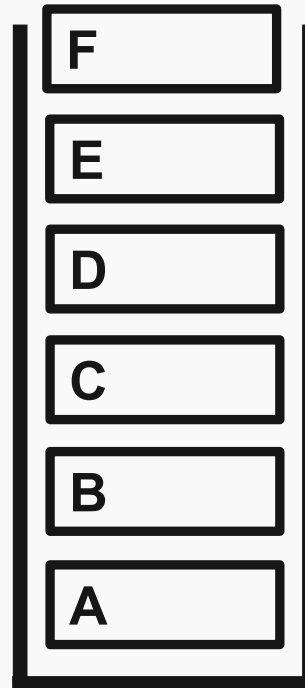
Inversarea unui șir





Inversarea unui șir

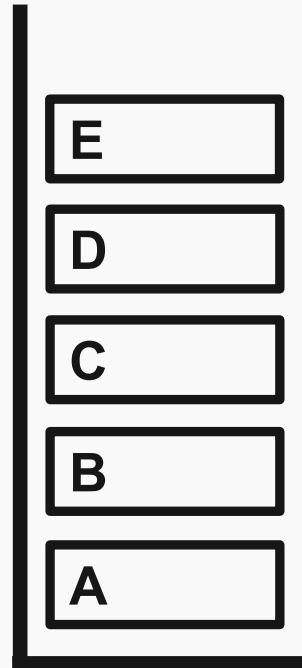
pop()





Inversarea unui șir

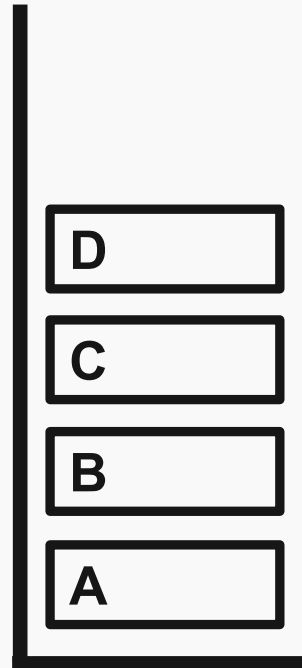
F pop()





Inversarea unui șir

E pop()

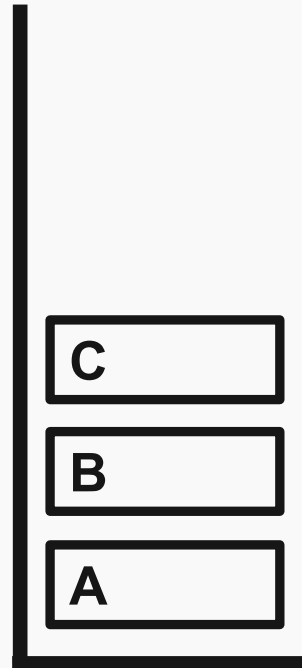


Afișare: F



Inversarea unui șir

D pop()

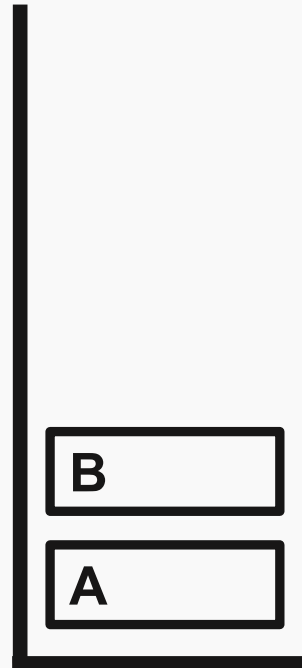


Afișare: FE



Inversarea unui şir

C pop()

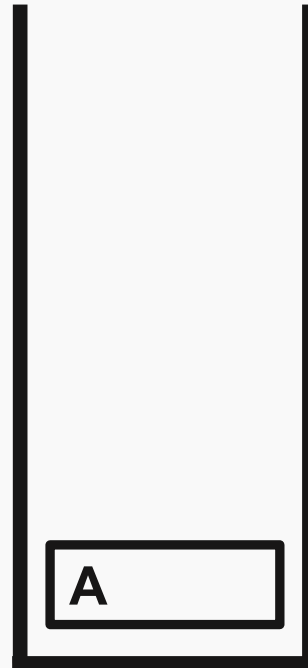


Afişare: FED



Inversarea unui șir

B pop()

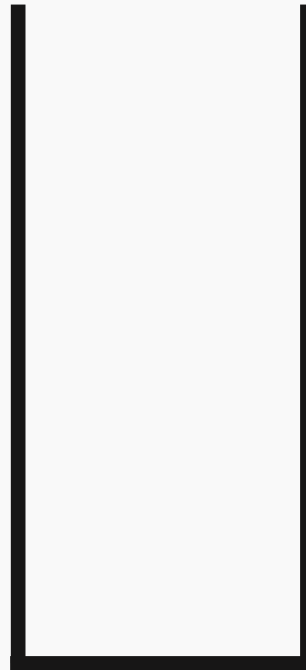


Afișare: FEDC



Inversarea unui şir

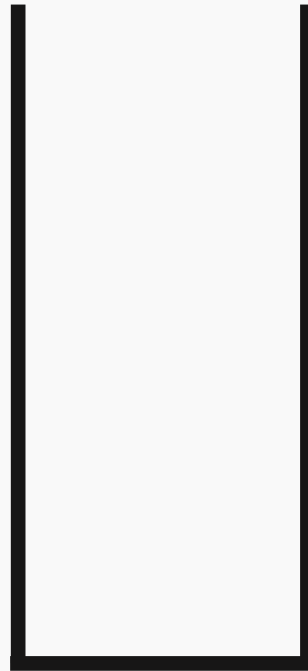
A



Afişare: FEDCB



Inversarea unui şir



Afişare: FEDCBA



Corectitudinea parantezelor unui expresii

1. $[(1+2)-3*(2+1)]$
 2. $\{[(())]\}$
 3. $[(()]$
 4. $[)()]$
 5. $[(1+)*2(3)]$
 6. $\{1+2+3+[2*3]*(1+2)\}$
 7. $\{1+2+3+[2*3*(1+2)]\}$
 8. $\{1+2+3+[2*3]*(1+2)))$
- Care expresii sunt corecte?



Corectitudinea parantezelor unui expresii

Parcurgem șirul de caractere

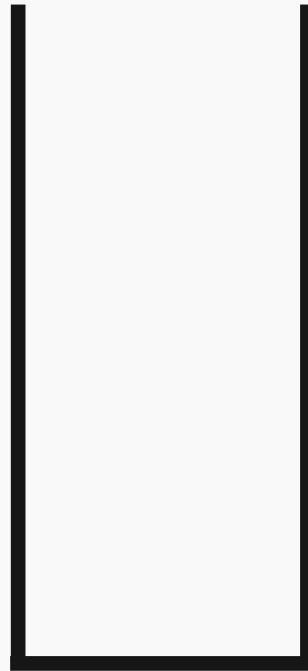
- Ignorăm orice caracter cu excepția [] () { }
- Pentru caracterele ([{ adăugăm pe acestea pe stivă
- Pentru caracterele)] } scoatem capul stivei și verificăm dacă se potrivește
 - **Dacă nu avem ce scoate expresia este greșită**
 - **Dacă nu expresia este greșită**

Când terminăm șirul știm că expresia este corectă



Corectitudinea parantezelor unui expresii

$[(1+2)-3*(2+1)]$

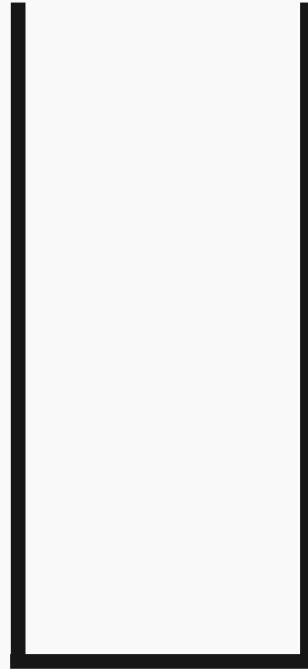




Corectitudinea parantezelor unui expresii

$(1+2)-3*(2+1)]$

push([])

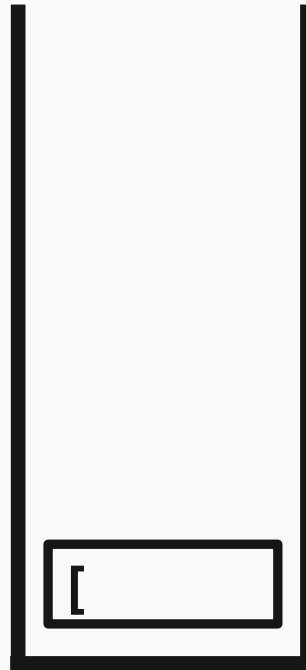




Corectitudinea parantezelor unui expresii

$1+2)-3*(2+1)]$

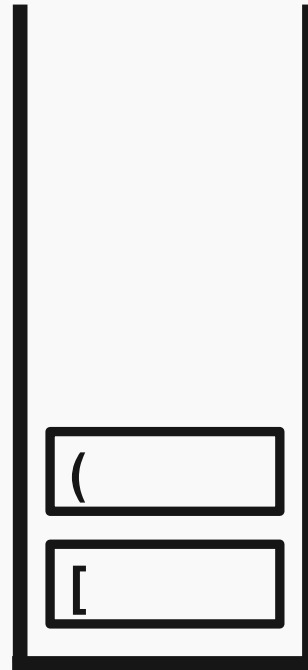
push(())





Corectitudinea parantezelor unui expresii

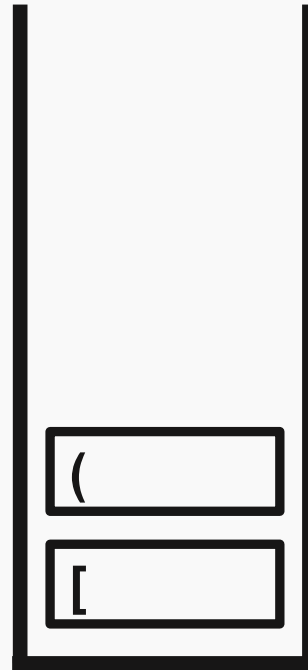
$1+2)-3*(2+1)]$





Corectitudinea parantezelor unui expresii

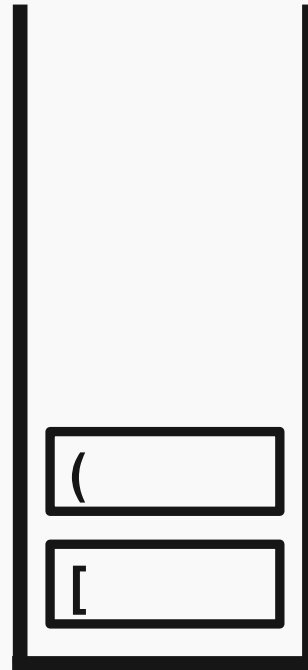
$+2)-3*(2+1)]$





Corectitudinea parantezelor unui expresii

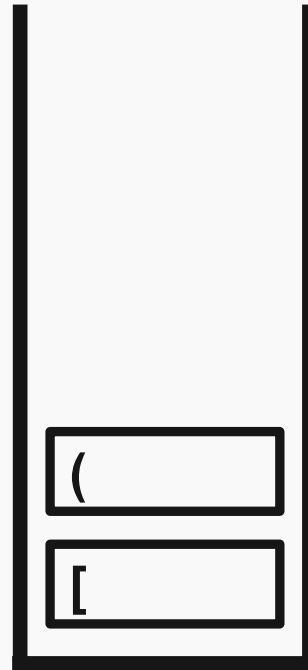
2)-3*(2+1)]





Corectitudinea parantezelor unui expresii

)-3*(2+1)]

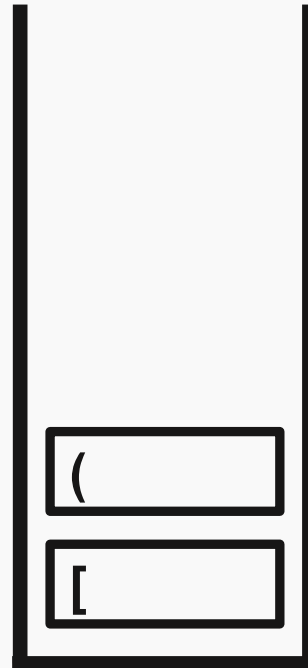




Corectitudinea parantezelor unui expresii

$-3*(2+1)]$

pop()
)

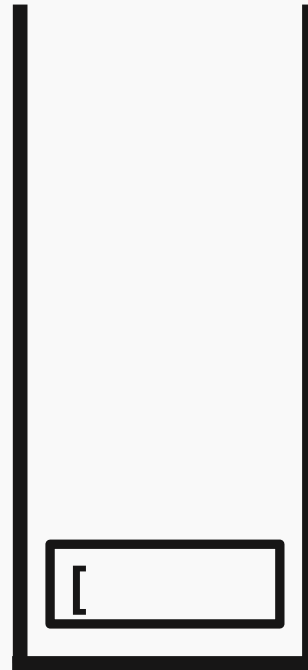




Corectitudinea parantezelor unui expresii

$-3*(2+1)]$

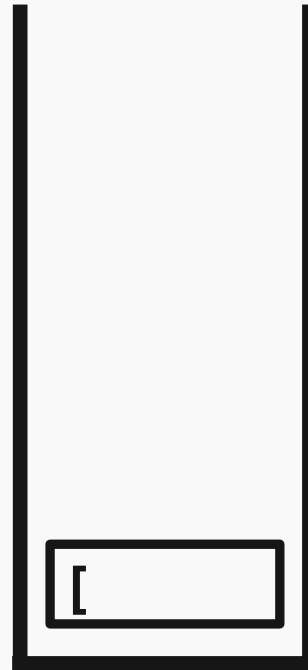
(pop()
)





Corectitudinea parantezelor unui expresii

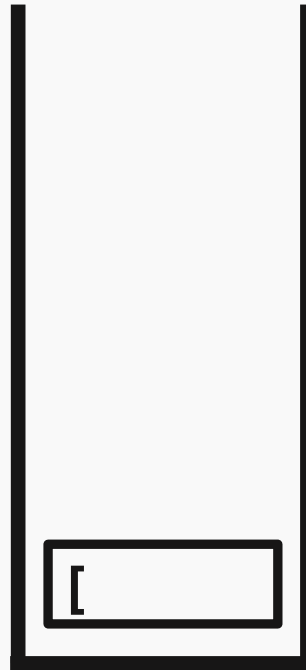
$3*(2+1)]$





Corectitudinea parantezelor unui expresii

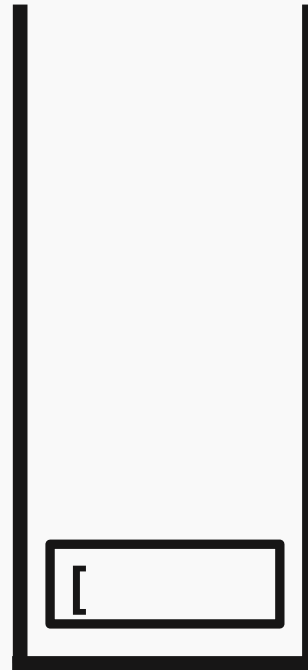
*(2+1)]





Corectitudinea parantezelor unui expresii

$(2+1)]$

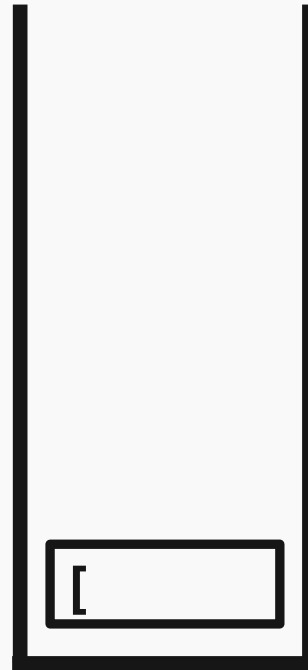




Corectitudinea parantezelor unui expresii

2+1)]

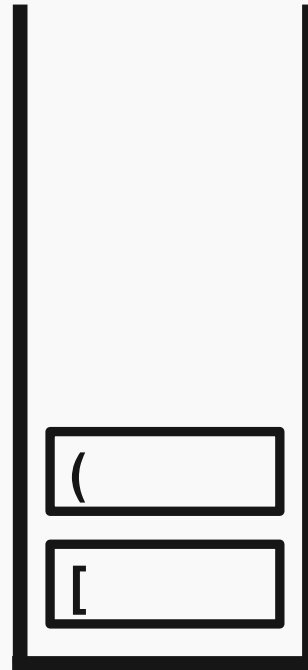
push(()





Corectitudinea parantezelor unui expresii

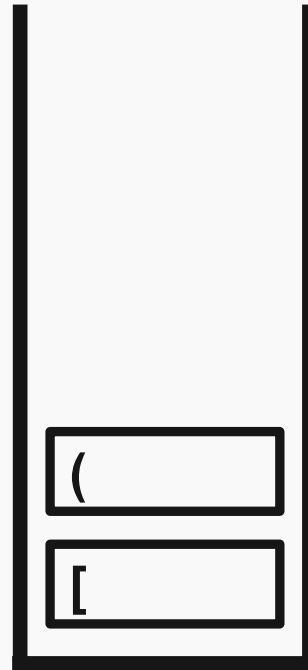
2+1)]





Corectitudinea parantezelor unui expresii

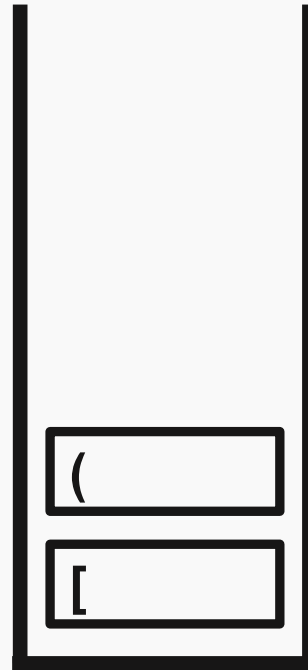
+1)]





Corectitudinea parantezelor unui expresii

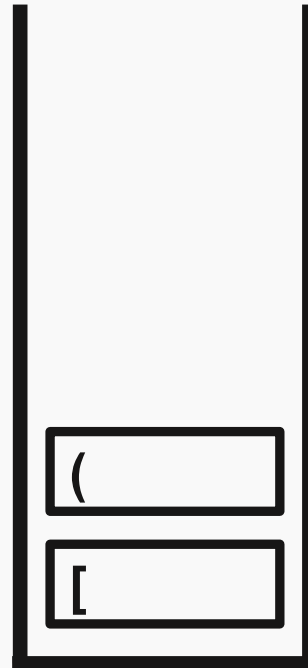
1)]





Corectitudinea parantezelor unui expresii

)]

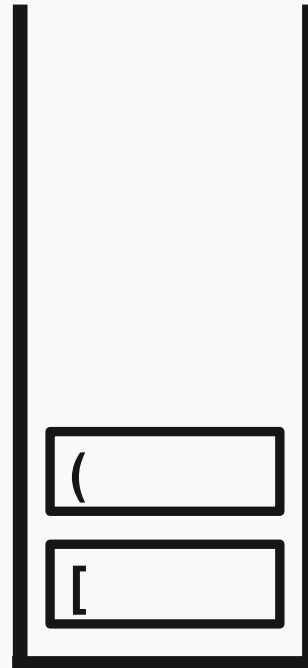




Corectitudinea parantezelor unui expresii

]

pop()
)

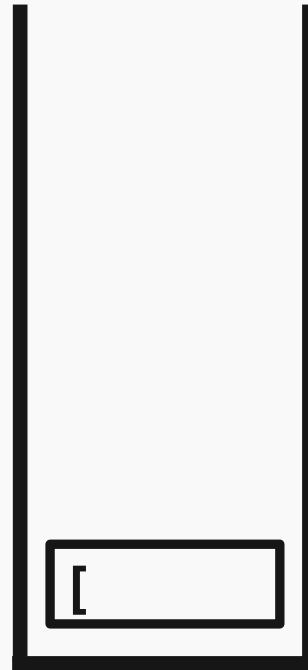




Corectitudinea parantezelor unui expresii

]

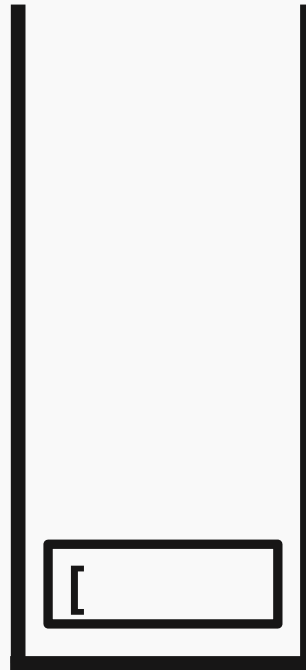
(pop()
)





Corectitudinea parantezelor unui expresii

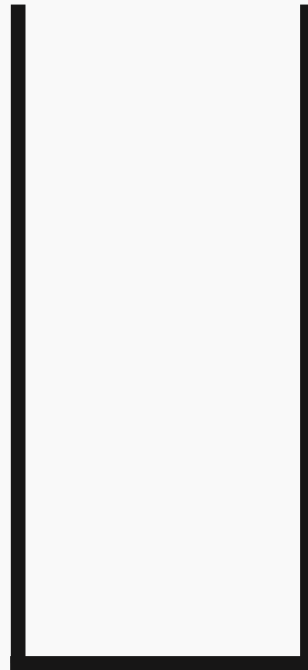
pop()
]





Corectitudinea parantezelor unui expresii

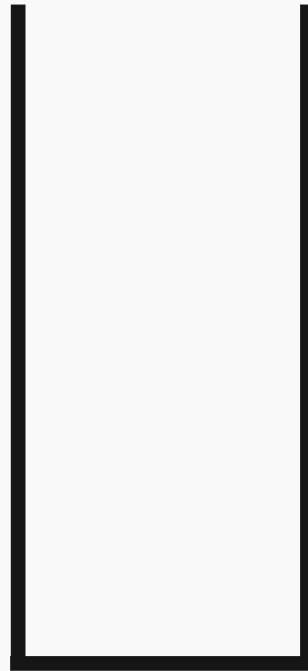
[pop()
]





Corectitudinea parantezelor unui expresii

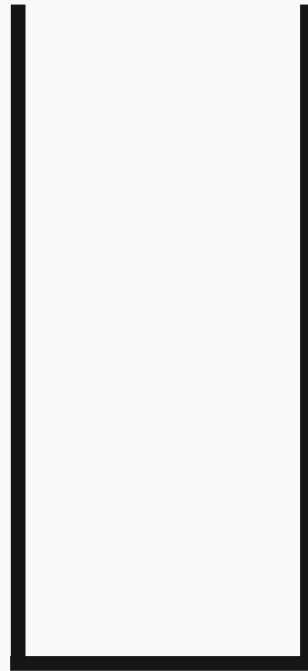
$[(1+2)-3*(2+1)]$ Corectă





Corectitudinea parantezelor unui expresii

[)(]

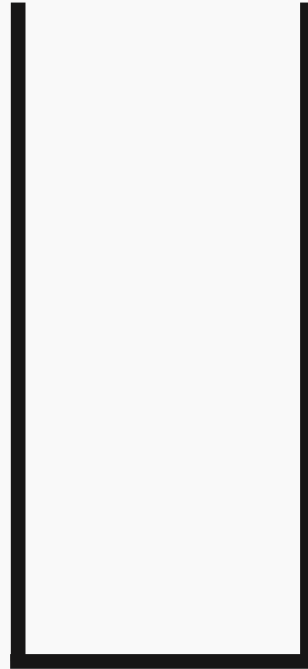




Corectitudinea parantezelor unui expresii

)([

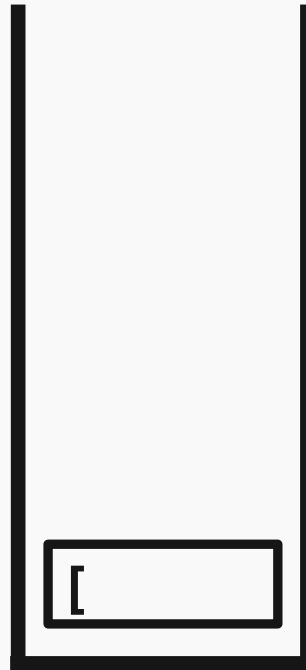
push([])





Corectitudinea parantezelor unui expresii

)([

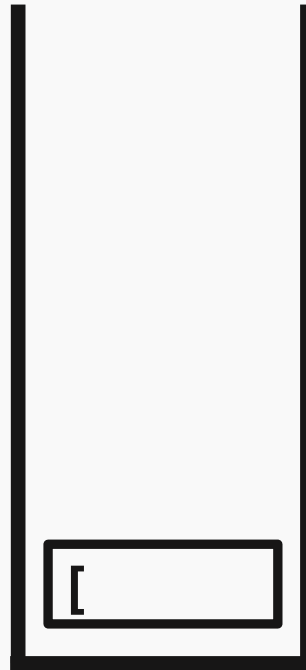




Corectitudinea parantezelor unui expresii

(]

pop()
)

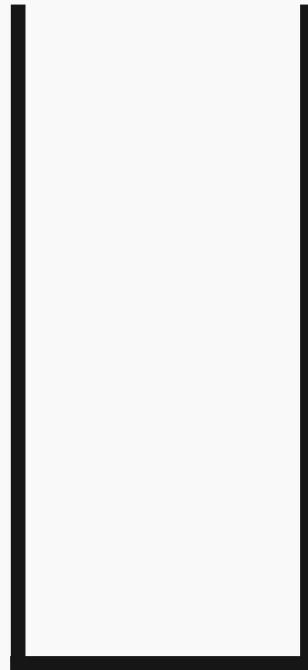




Corectitudinea parantezelor unui expresii

(] **Incorect**

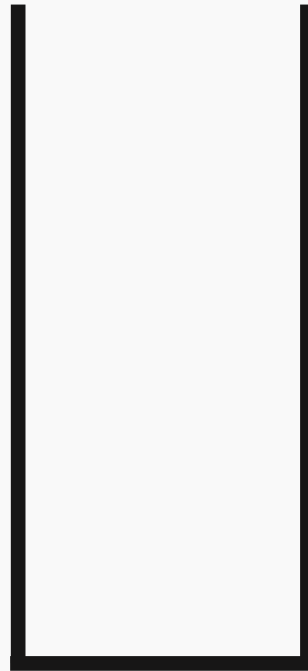
[pop()
)





Corectitudinea parantezelor unui expresii

`[]()` **Incorect**







Stiva programului

<http://pythontutor.com/c.html#mode=display>

Ce problemă observați?

```
#include<stdlib.h>
```

```
int fun1() {  
    return 1;  
}
```

```
void fun2() {  
    int d = 0;  
    fun1();  
    return;  
}
```

```
int main() {  
    int a = 5;  
    int v[4] = { 0,1,2,3 };  
    int* vv = (int*)malloc(5);  
    vv[3] = 5;  
    fun2();  
    a = fun1();  
    return 0;  
}
```




Stiva programului

<http://pythontutor.com/c.html#mode=display>

```
#include<stdlib.h>
```

```
int fun1() {  
    return 1;  
}
```

```
void fun2() {  
    int d = 0;  
    fun1();  
    return;  
}
```

```
int main() {  
    int a = 5;  
    int v[4] = { 0,1,2,3 };  
    int* vv = (int*)malloc(5 * sizeof(int));  
    vv[3] = 5;  
    fun2();  
    a = fun1();  
    return 0;  
}
```



Stiva programului

```
#include<stdio.h>
#define N 5
```

```
void fun1() {
    int v[N];
    int i;
    for (i = 0; i < N; i++) {
        v[i] = i;
        printf("fun1: %p %i\n", v + i, v[i]);
    }
}
```

```
void fun2() {
    int z[N];
    int i;
    for (i = 0; i < N; i++) {
        printf("fun2: %p %i\n", z + i, z[i]);
        z[i] = 0;
    }
}
```

Ce valori vor fi afișate de fun2?

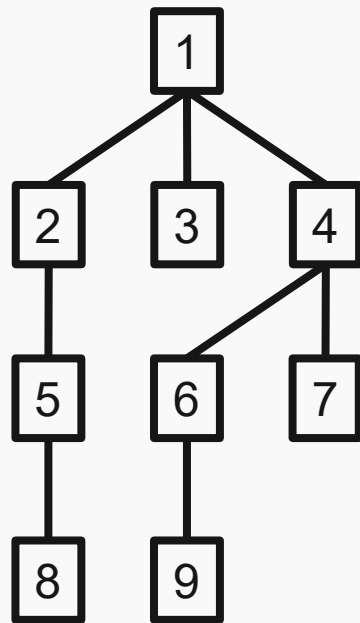
```
int main() {
    fun1();
    fun2();
    return 0;
}
```



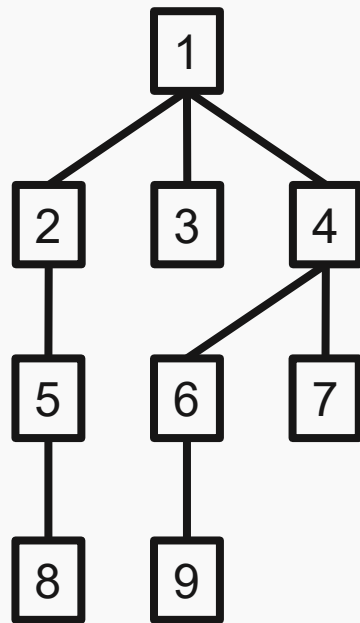
Implementare reală linked lists

https://gcc.gnu.org/onlinedocs/gcc-4.6.2/libstdc++/api/a00922_source.html





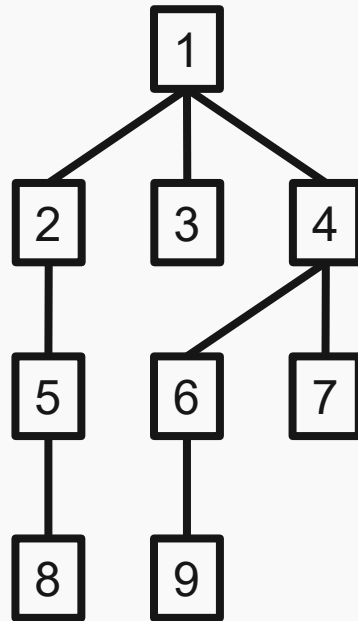
Care este rădăcina arborelui?



Care sunt frunzele arborelui?

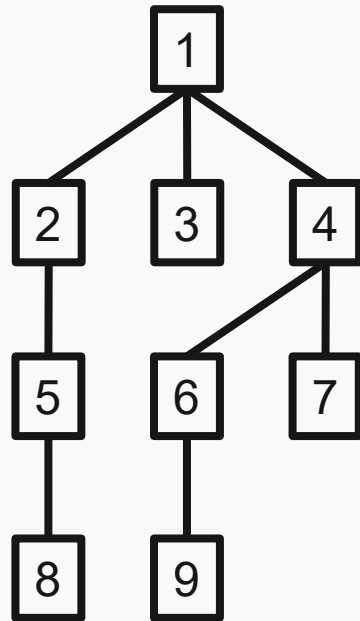


Afișarea unui arbore fără recursivitate





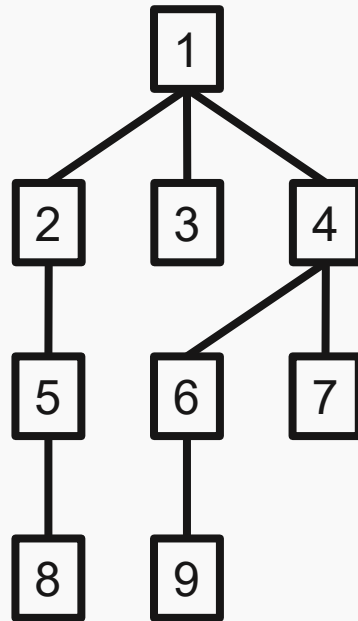
Afișarea unui arbore folosind o coadă



- Adăugăm rădăcina în coadă
- Până ce coada este goală
 - Scoatem un nod din coadă
 - Adăugăm toți copiii în coadă

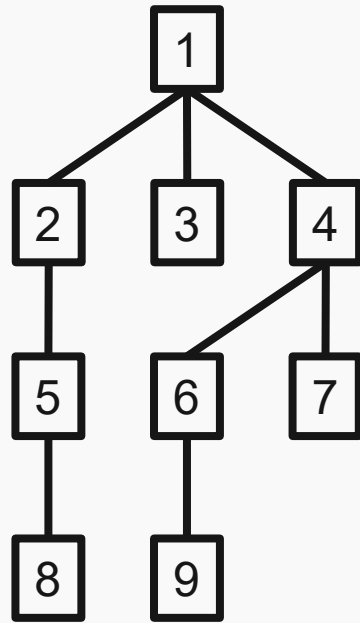


Afișarea unui arbore folosind o coadă

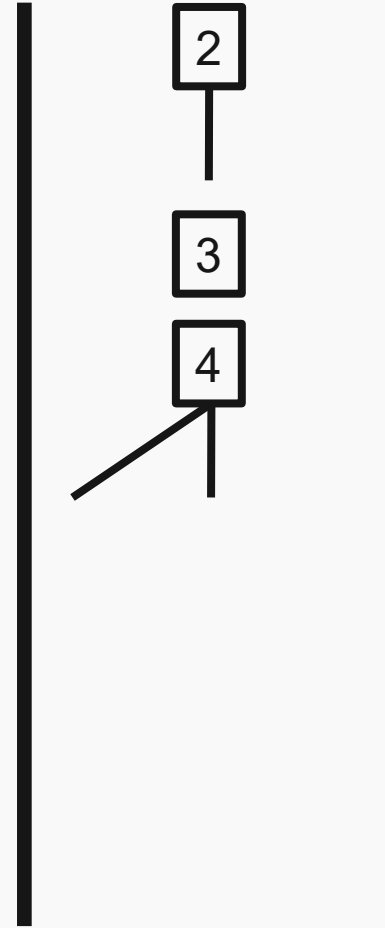




Afișarea unui arbore folosind o coadă

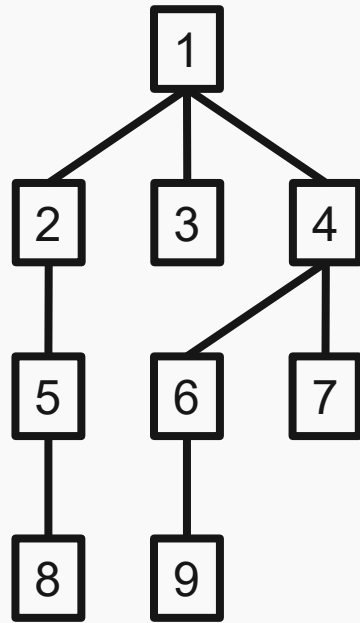


Afișare: 1



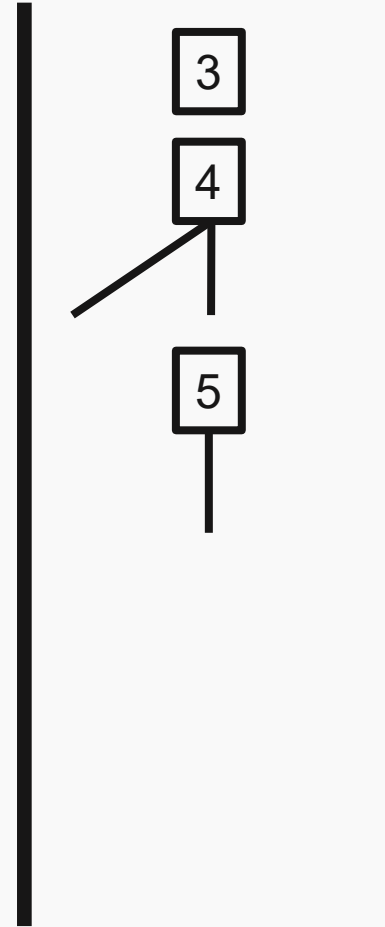


Afișarea unui arbore folosind o coadă



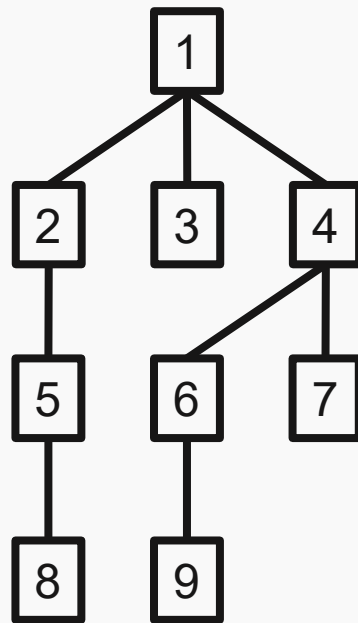
Afișare:

1	2
---	---



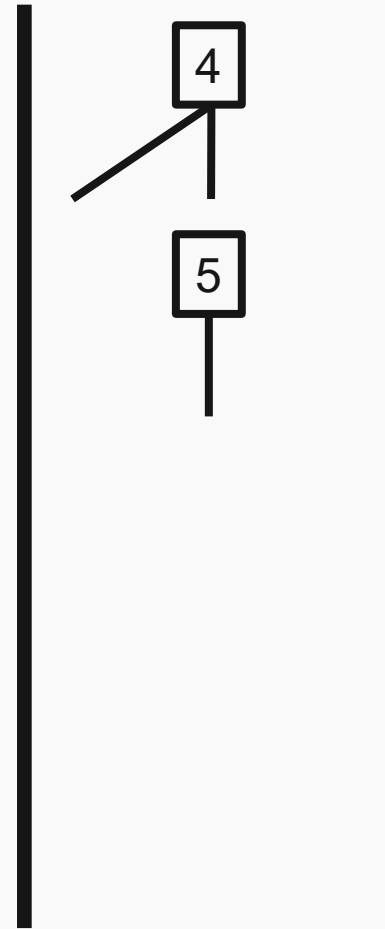


Afișarea unui arbore folosind o coadă



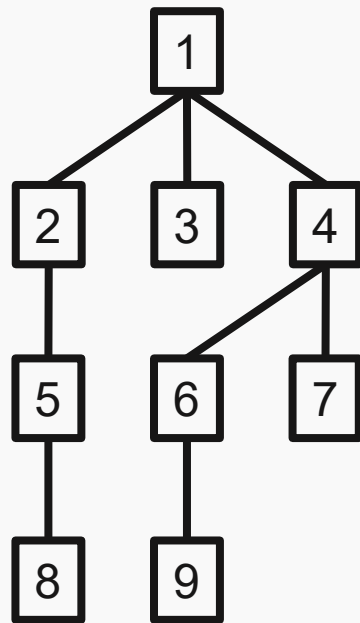
Afișare:

1	2	3
---	---	---



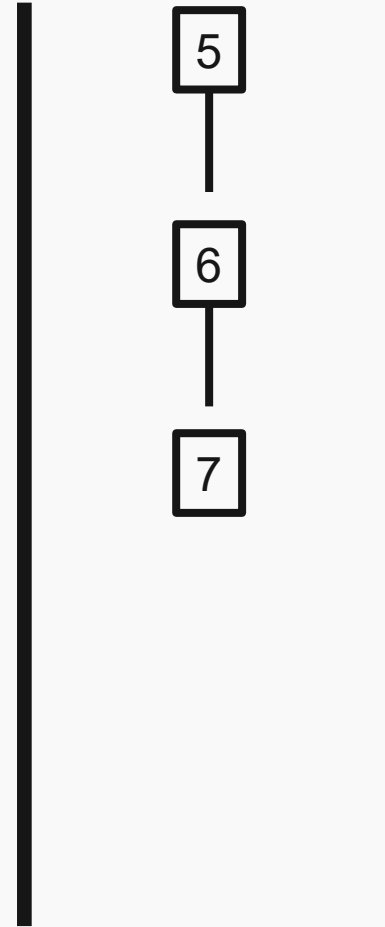


Afișarea unui arbore folosind o coadă



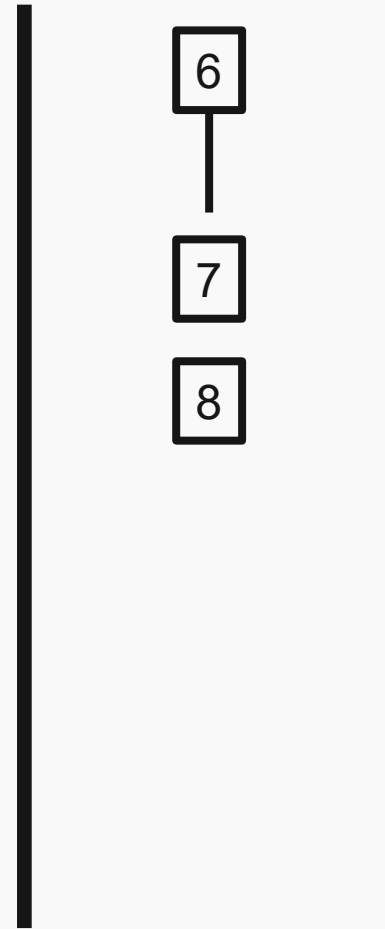
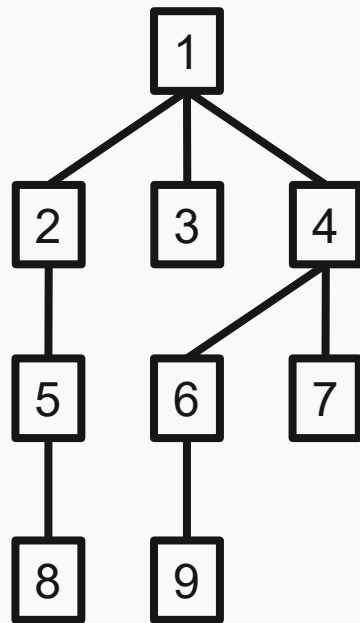
Afișare:

1	2	3	4
---	---	---	---





Afișarea unui arbore folosind o coadă

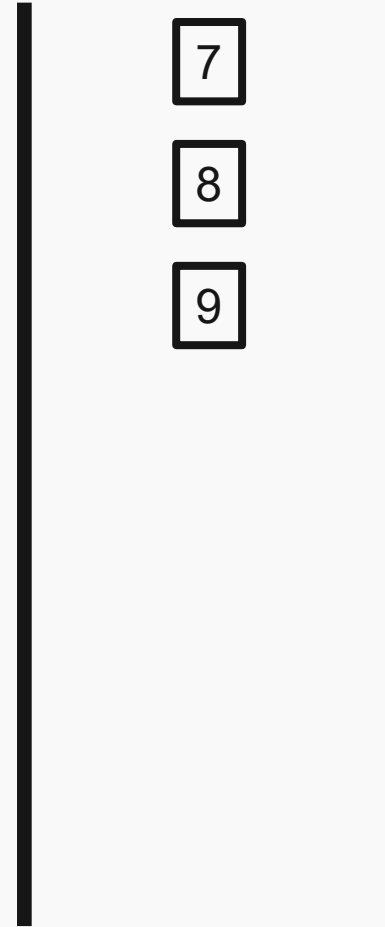
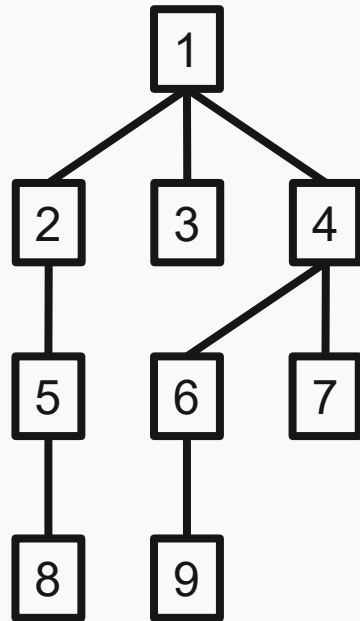


Afișare:

1	2	3	4	5
---	---	---	---	---



Afișarea unui arbore folosind o coadă

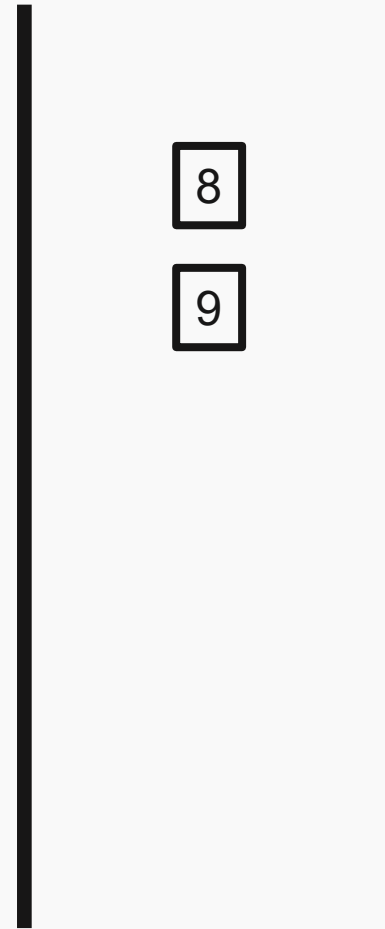
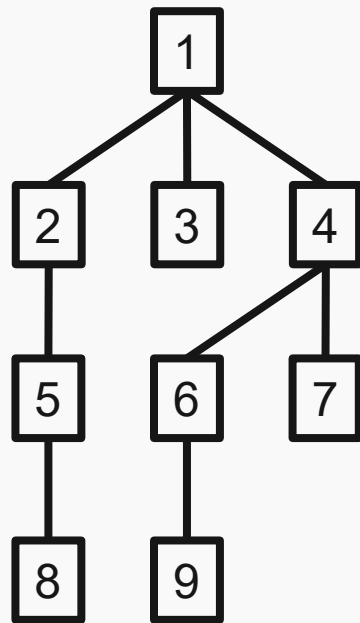


Afișare:

1	2	3	4	5	6
---	---	---	---	---	---



Afișarea unui arbore folosind o coadă

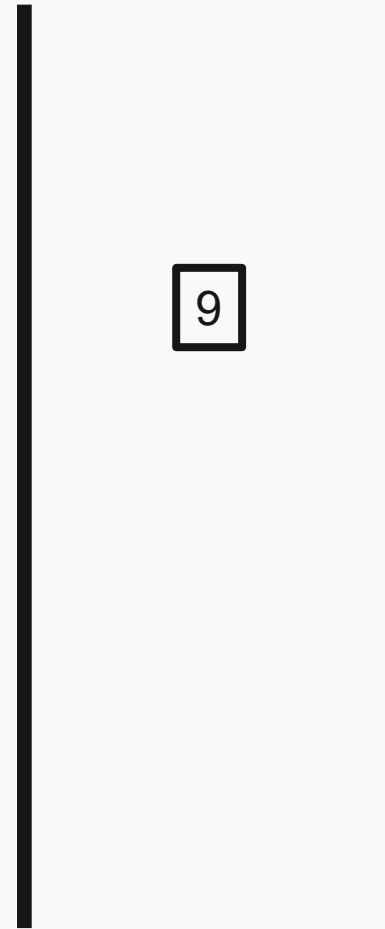
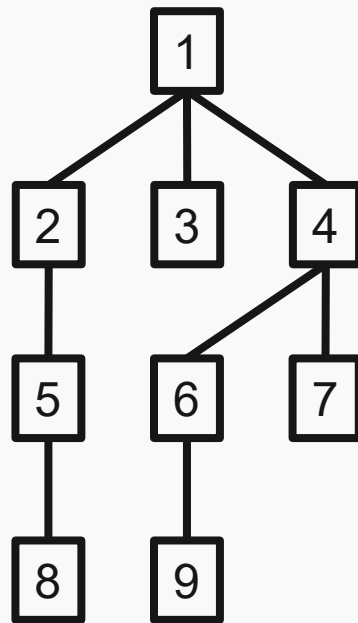


Afișare:

1	2	3	4	5	6	7
---	---	---	---	---	---	---



Afișarea unui arbore folosind o coadă

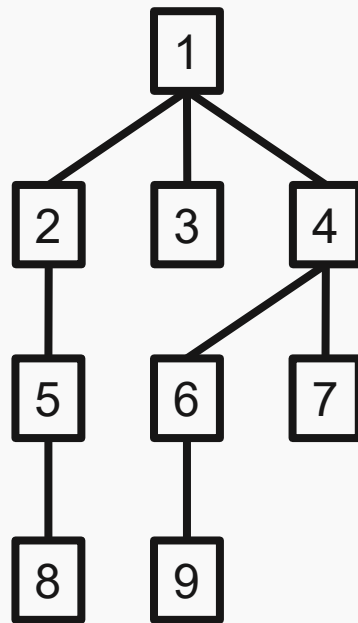


Afișare:

1	2	3	4	5	6	7	8
---	---	---	---	---	---	---	---



Afișarea unui arbore folosind o coadă

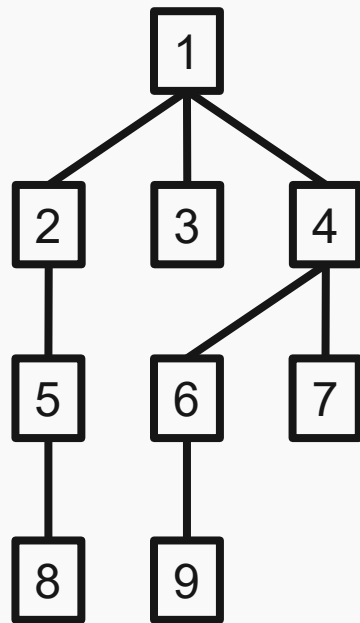


Afișare:

1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---

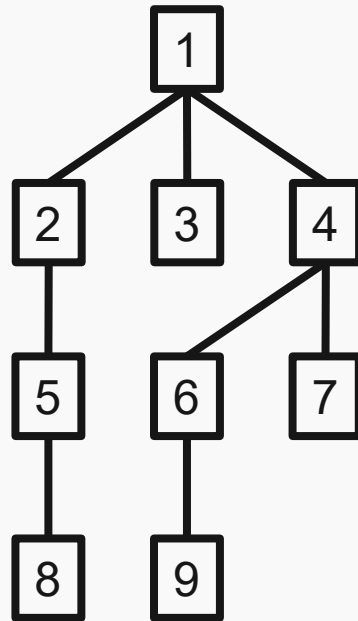


Afișarea unui arbore folosind o stivă





Afișarea unui arbore folosind o stivă

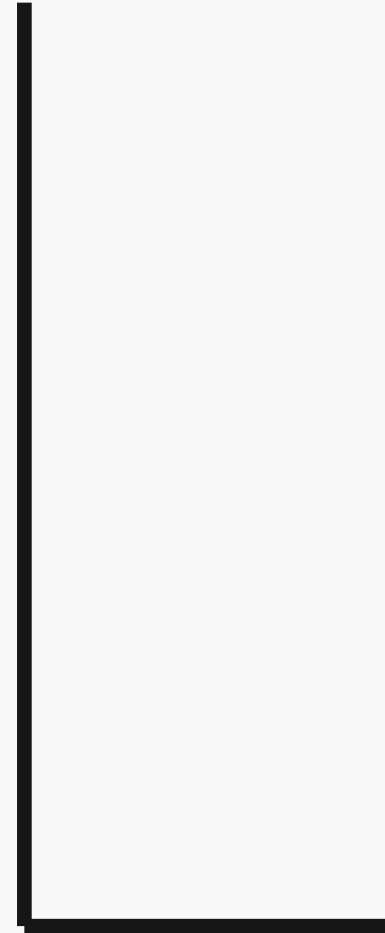
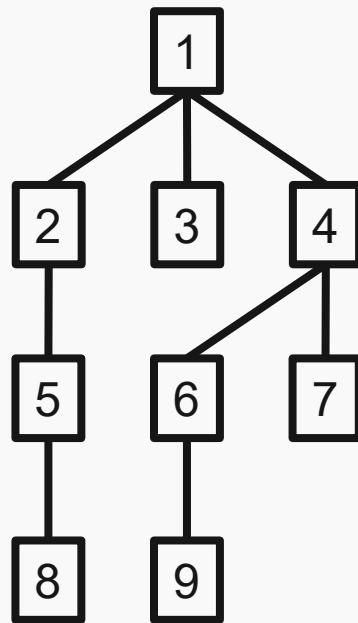


- Adăugăm rădăcina în stivă
- Până ce stiva este goală
 - Scoatem un nod din stivă
 - Adăugăm toți copiii în stivă

Cum va fi afișat arborele?



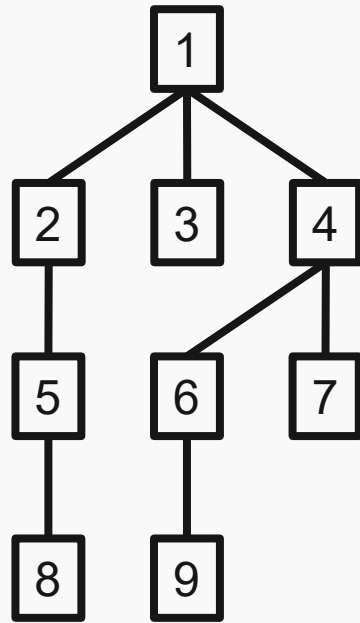
Afișarea unui arbore folosind o stivă



Afișare:



Afișarea unui arbore folosind o stivă

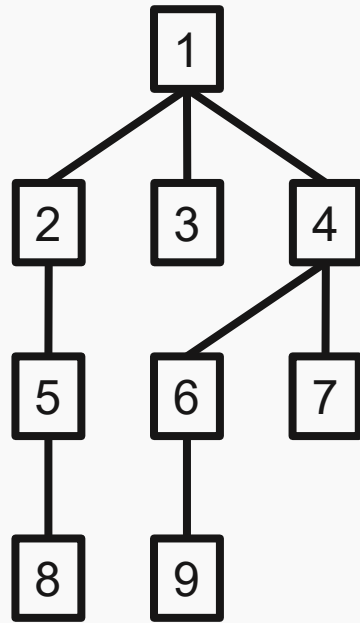


Afișare:

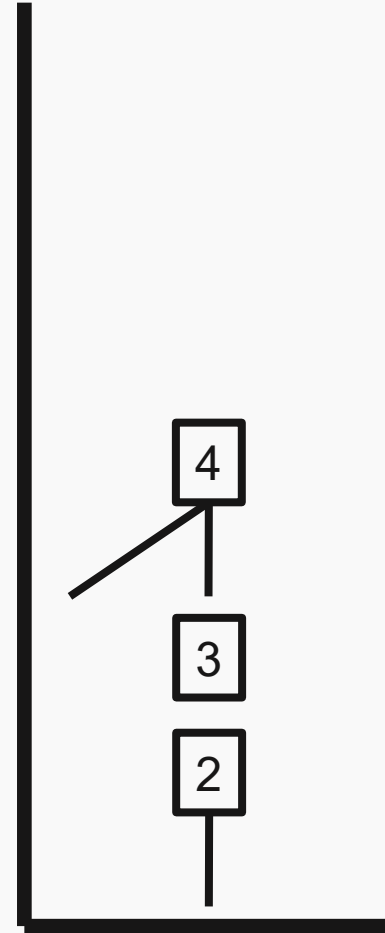




Afișarea unui arbore folosind o stivă

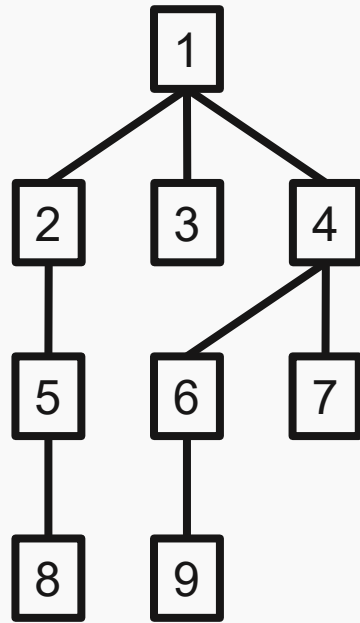


Afișare: 1



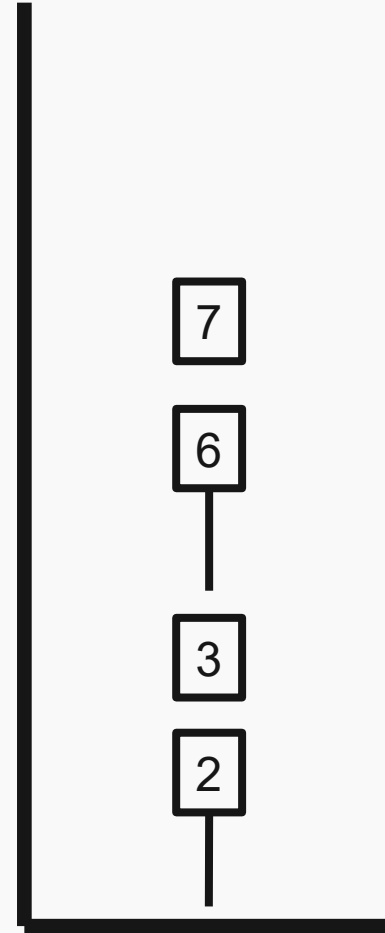


Afișarea unui arbore folosind o stivă



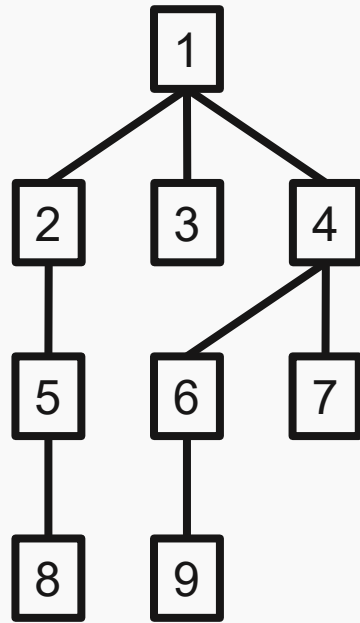
Afișare:

1	4
---	---



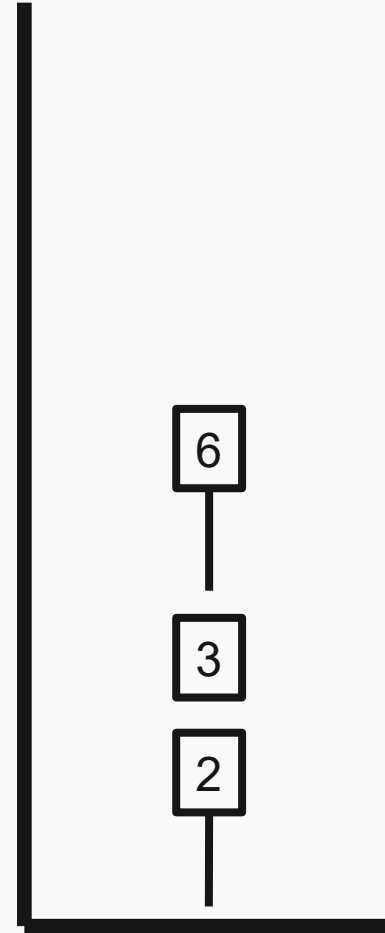


Afișarea unui arbore folosind o stivă



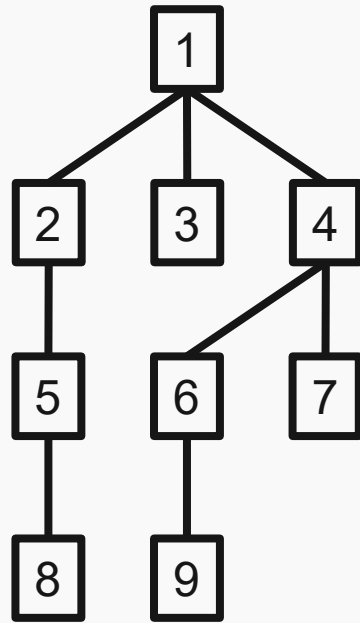
Afișare:

1	4	7
---	---	---



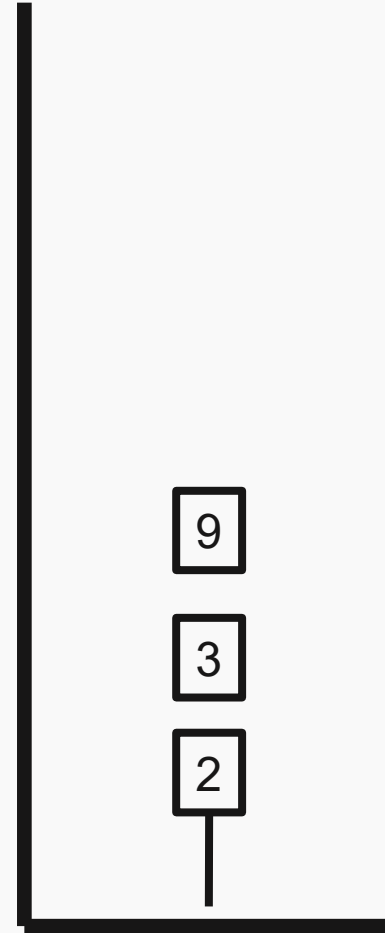


Afișarea unui arbore folosind o stivă



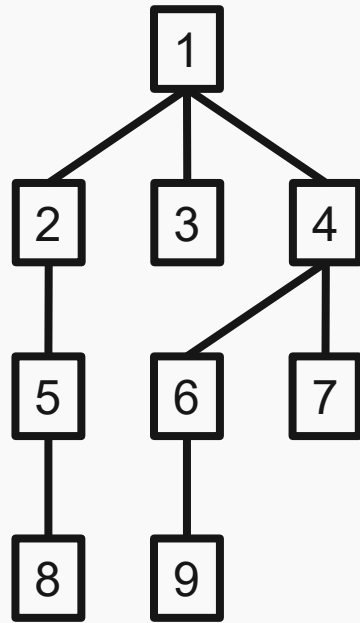
Afișare:

1	4	7	6
---	---	---	---



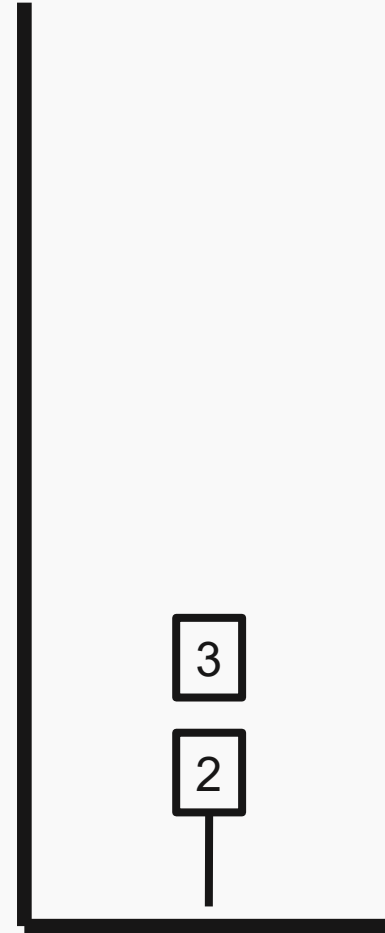


Afișarea unui arbore folosind o stivă



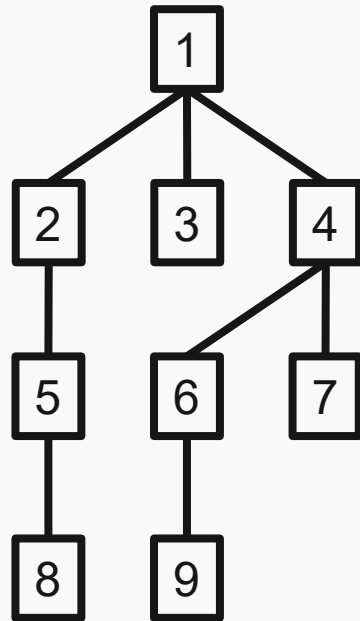
Afișare:

1	4	7	6	9
---	---	---	---	---



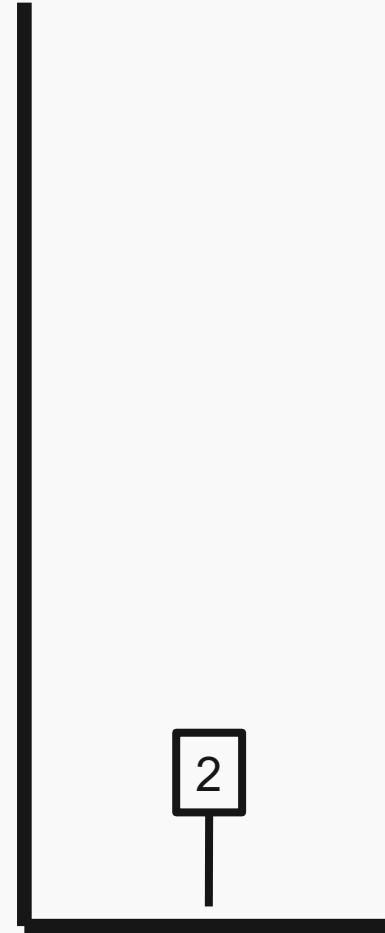


Afișarea unui arbore folosind o stivă



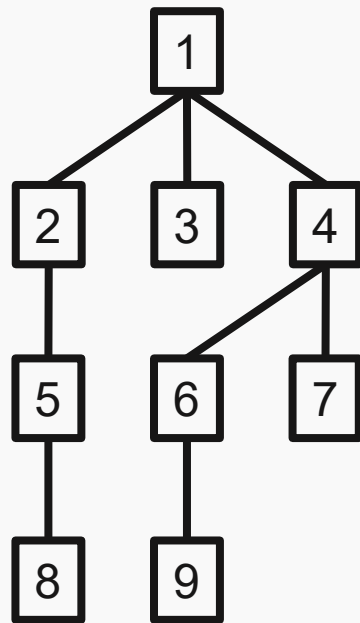
Afișare:

1	4	7	6	9	3
---	---	---	---	---	---





Afișarea unui arbore folosind o stivă

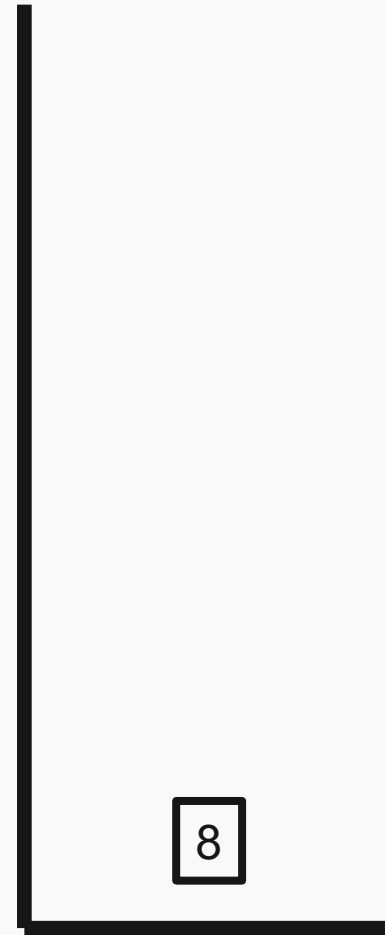
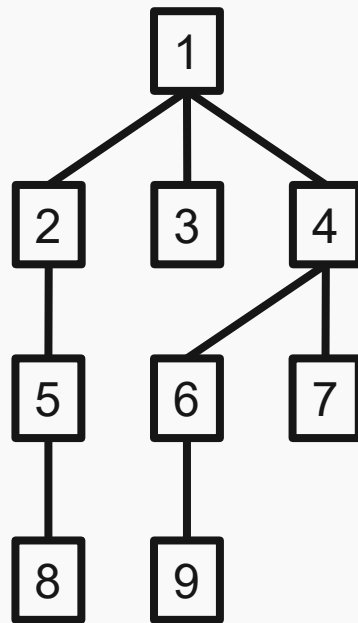


Afișare:

1	4	7	6	9	3	2
---	---	---	---	---	---	---



Afișarea unui arbore folosind o stivă

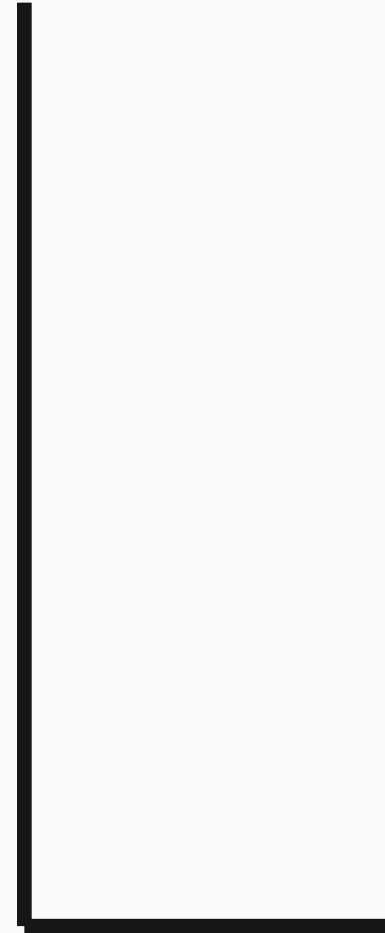
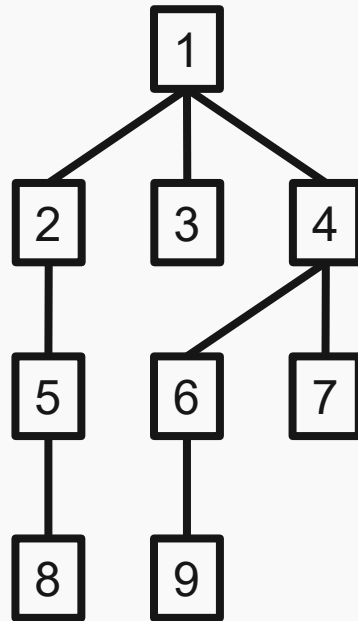


Afișare:

1	4	7	6	9	3	2	5
---	---	---	---	---	---	---	---



Afișarea unui arbore folosind o stivă



Afișare:

1	4	7	6	9	3	2	5	8
---	---	---	---	---	---	---	---	---



Skip list



Skip Lists

*Algorithms and
Data Structures*

*Jeffrey Vitter
Editor*

Skip Lists: A Probabilistic Alternative to Balanced Trees

Skip lists are data structures that use probabilistic balancing rather than strictly enforced balancing. As a result, the algorithms for insertion and deletion in skip lists are much simpler and significantly faster than equivalent algorithms for balanced trees.

William Pugh

Binary trees can be used for representing abstract data types such as dictionaries and ordered lists. They work well when the elements are inserted in a random order. Some sequences of operations, such as inserting the elements in order, produce degenerate data structures that perform very poorly. If it were possible to randomly permute the list of items to be inserted, trees would work well with high probability for any input

self-adjusting tree algorithms. Skip lists are also very space efficient. They can easily be configured to require an average of $1\frac{1}{2}$ pointers per element (or even less) and do not require balance or priority information to be stored with each node.

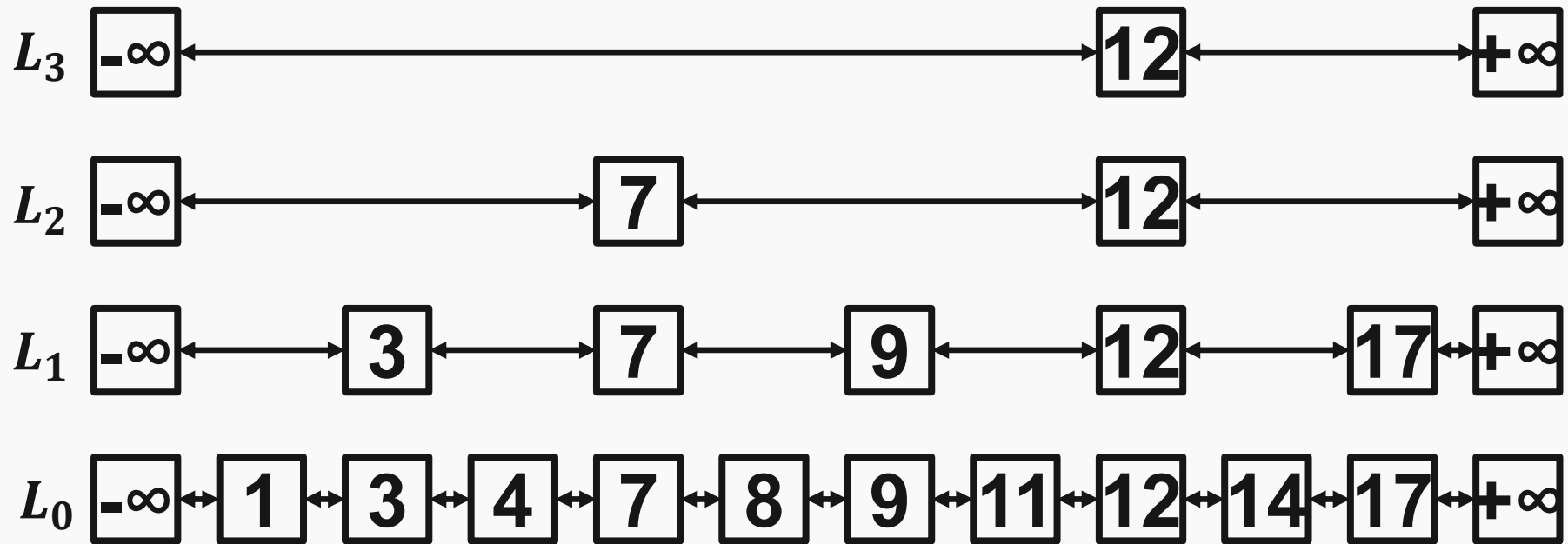
SKIP LISTS

We might need to examine every node of the list when





Skip list





Skip list - search

```
for (i = top; i >= 0; i--) { //Li
    while (searchValue > iterator->next->value) {
        iterator = iterator->next;
    }
    if (searchValue == iterator->value)
        return iterator;
    iterator = iterator->down;
}
```



Skip list - remove

```
search(node);  
removeFromAllLevels(node);
```



Skip list - insert

```
prevNode = search(node);  
insert(prevNode->next, node, L0);  
i = 1;  
while (random % 2 != 0) {  
    insert(node, Li)  
    i++;  
}
```



Skip list complexities

	Vector	Listă	Skip list
Complexitate acces	$O(1)$	$O(N)$	$O(\log_2(N))$
Complexitate inserție	$O(N)$	$O(N)$	$O(\log_2(N))$
Complexitate inserție capete	$O(N)/O(1)$	$O(1)$	$O(\log_2(N))$
Complexitate ștergere	$O(N)$	$O(N)$	$O(\log_2(N))$
Complexitate ștergere capete	$O(N)/O(1)$	$O(1)$	$O(\log_2(N))$



Bloom Filters



Bloom Filters

Space/Time Trade-offs in Hash Coding with Allowable Errors

BURTON H. BLOOM

Computer Usage Company, Newton Upper Falls, Mass.

In this paper trade-offs among certain computational factors in hash coding are analyzed. The paradigm problem considered is that of testing a series of messages one-by-one for membership in a given set of messages. Two new hash-coding methods are examined and compared with a particular conventional hash-coding method. The computational factors considered are the size of the hash area (space), the time required to identify a message as a nonmember of the given set (reject time), and an allowable error frequency.

The new methods are intended to reduce the amount of space required to contain the hash-coded information from that associated with conventional methods. The reduction in space is accomplished by exploiting the possibility that a

iteratively generating hash addresses of cells. The contents of these cells are then compared with the test messages. A match indicates the test message is a member of the set; an empty cell indicates the opposite. The reader is assumed to be familiar with this and similar conventional hash-coding methods [1, 2, 3].

The new hash-coding methods to be introduced are suggested for applications in which the great majority of messages to be tested will not belong to the given set. For these applications, it is appropriate to consider as a unit of time (called *reject time*) the average time required to classify a test message as a nonmember of the given set. Furthermore, since the contents of a cell can, in general, be recognized as not matching a test message by examining only part of the message, an appropriate assumption will be introduced concerning the time required to access individual bits of the hash area.

In addition to the two computational factors, reject time and space (i.e. hash area size), this paper considers a third computational factor, allowable fraction of errors. It will be shown that allowing a small number of test messages to be falsely identified as members of the given set will permit a much smaller hash area to be used without increasing the reject time. In some practical applications,



Silly Bloom Filters

5 0000_0101 OR
1 0000_0001 OR
3 0000_0011 OR
32 0010_0000 OR
33 0010_0001
=

????_????



Silly Bloom Filters

5 0000_0101 OR

1 0000_0001 OR

3 0000_0011 OR

32 0010_0000 OR

33 0010_0001

=

Rez 0010_0111



Silly Bloom Filters

Rez 0010_0111 **AND**
Număr inițial
 =
 ?? ?????_????



Silly Bloom Filters

Rez 0010_0111 **AND**
1 0000_0001
=
1 0000_0001



Silly Bloom Filters

Rez 0010_0111 **AND**
5 0000_0101
=
5 0000_0101



Silly Bloom Filters

Rez 0010_0111 **AND**
33 0010_0001
=
33 0010_0001



Silly Bloom Filters

Rez 0010_0111 **AND**
 x ?????_????
 =
 x ?????_????

Găsiți x diferit de numerele inițiale scris în zecimal și binar:



Confusion matrix

	Pozitive reale	Negative reale
Pozitive prezise	True pozitive	False positives
Negative prezise	False negatives	True negatives



Confusion matrix

		Fac parte din calcul inițial	NU fac parte din calcul inițial
		Pozitive reale	Negative reale
$x \& BF == x$	Pozitive prezise	True positives	False positives
$x \& BF != x$	Negative prezise	False negatives	True negatives

Ce poate întoarce un (silly) bloom filter testat pe multiple numere?



Confusion matrix – bloom filters

		Fac parte din calcul inițial	NU fac parte din calcul inițial
		Pozitive reale	Negative reale
$x \& BF == x$	Pozitive prezise	True positives	False positives
$x \& BF != x$	Negative prezise	False negatives	True negatives



Bloom filter - creare

- Un set de funcții hash ($h_1()$; $h_2()$; ... $h_k()$)
 - Funcțiile returnează valori între 1 și N
- Un șir de N biți initializați cu 0
- Un input x pe care dorim să îl adăugăm filtrului, setăm cu 1 pozițiile $h_i()$, oricare ar fi i .



Bloom filter - Verificare apartenență

Pentru un **y** pentru care dorim să știm dacă este în filtru.

- Aplicăm toate funcțiile **h_i()**
- Dacă pe toate pozițiile returnate găsim **1** în vectorul de biți putem considera că **y** aparține setului de numere cu care a fost creat filtrul. (Putem avea **false positives**)
- Dacă pe oricare poziție din cele returnate găsim **0** în vectorul de biți putem fi siguri că **y** nu a fost introdus în Bloom filter



Bloom filter – inițializare

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---



Bloom filter – inițializare

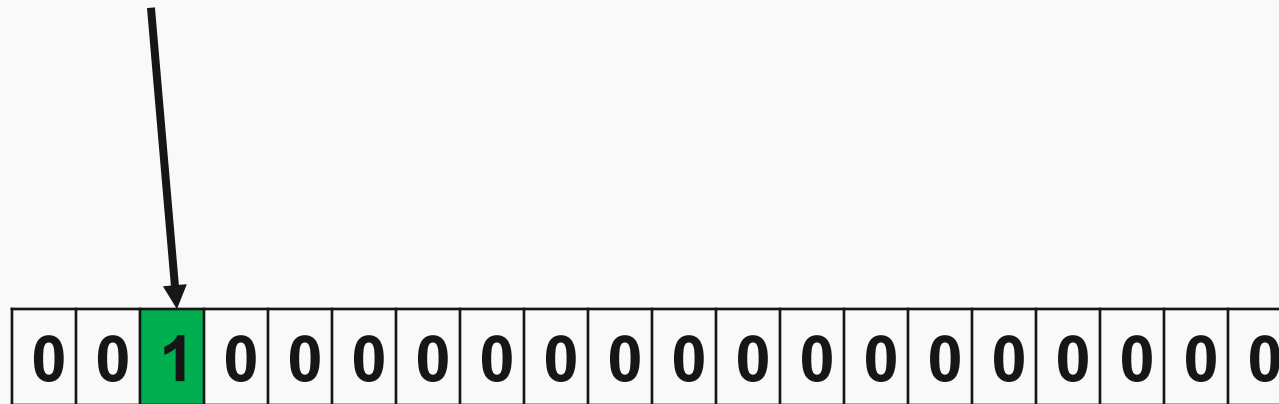
X_1

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---



Bloom filter – inițializare

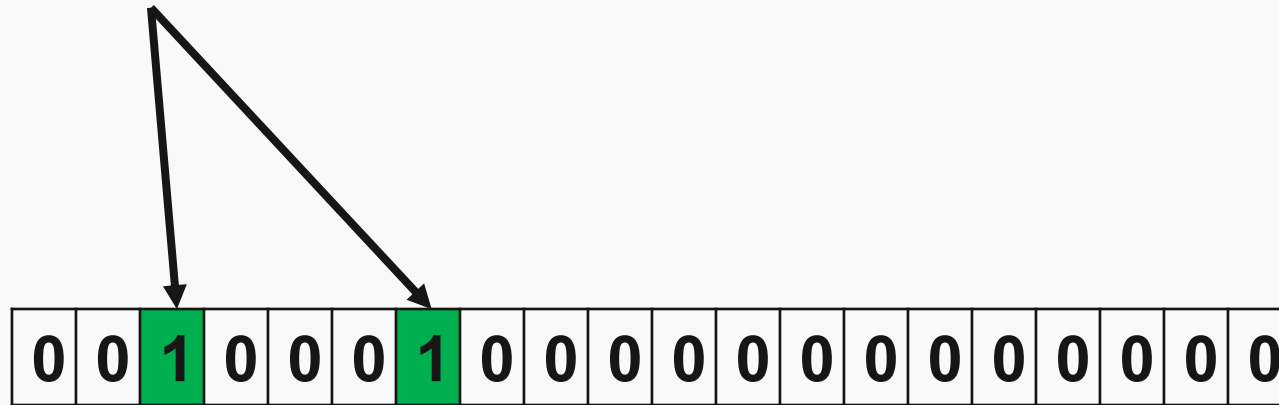
$$h_1(X_1) = 3$$





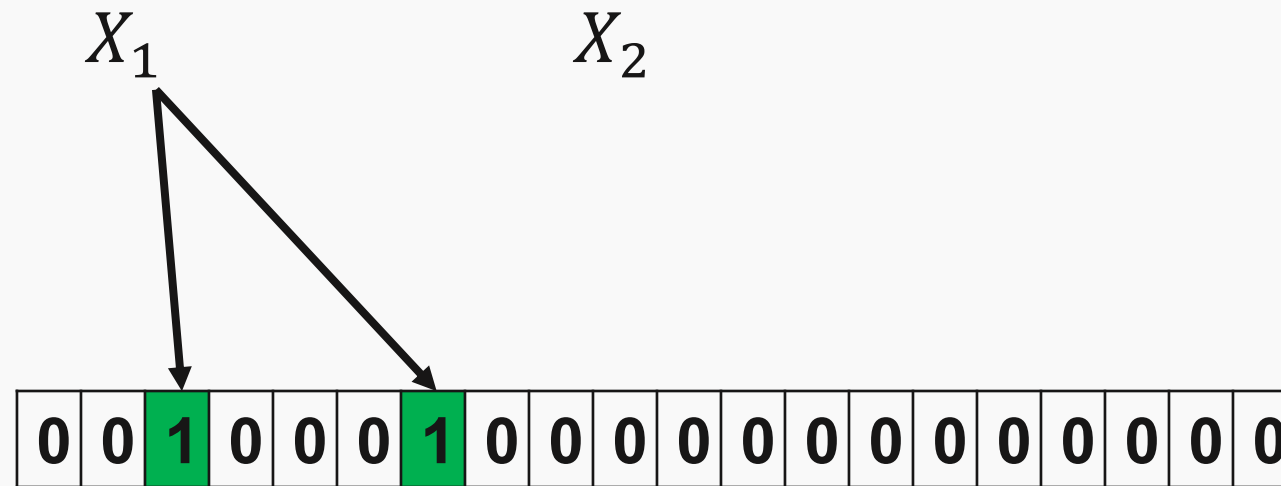
Bloom filter – inițializare

$$h_2(X_1) = 7$$



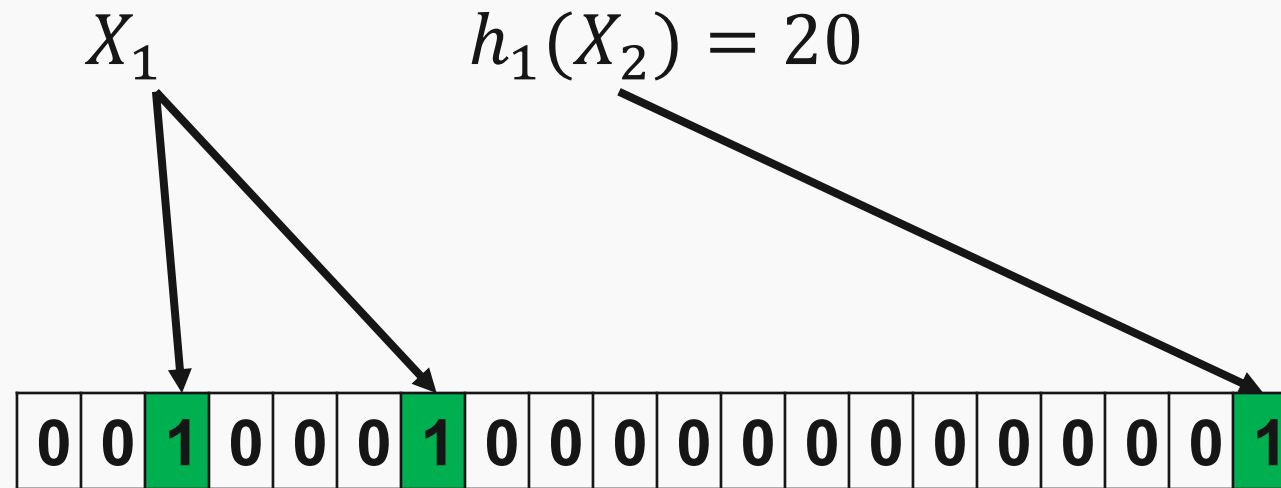


Bloom filter – inițializare



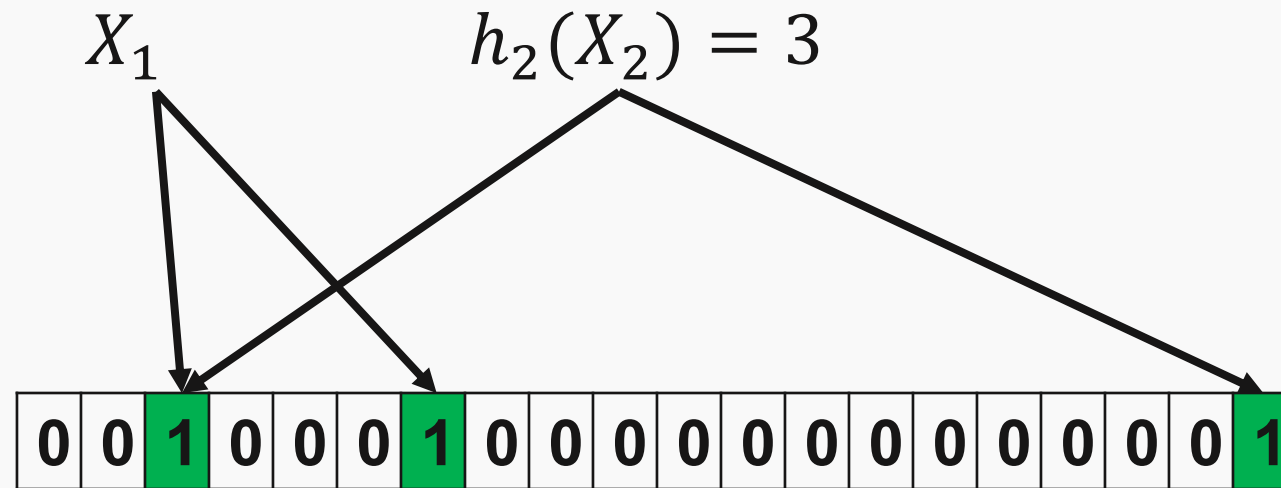


Bloom filter – inițializare



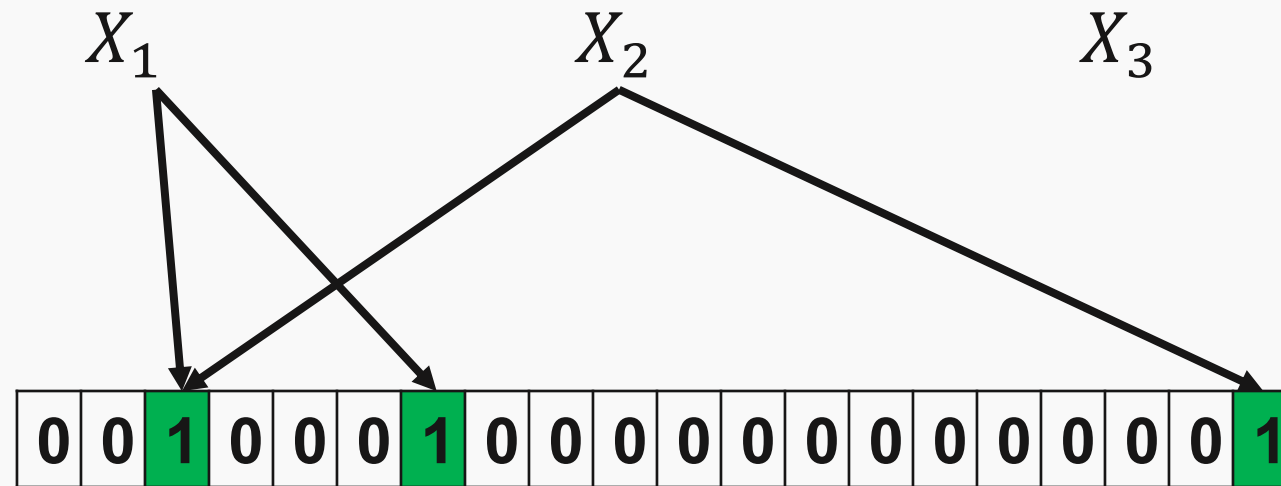


Bloom filter – inițializare



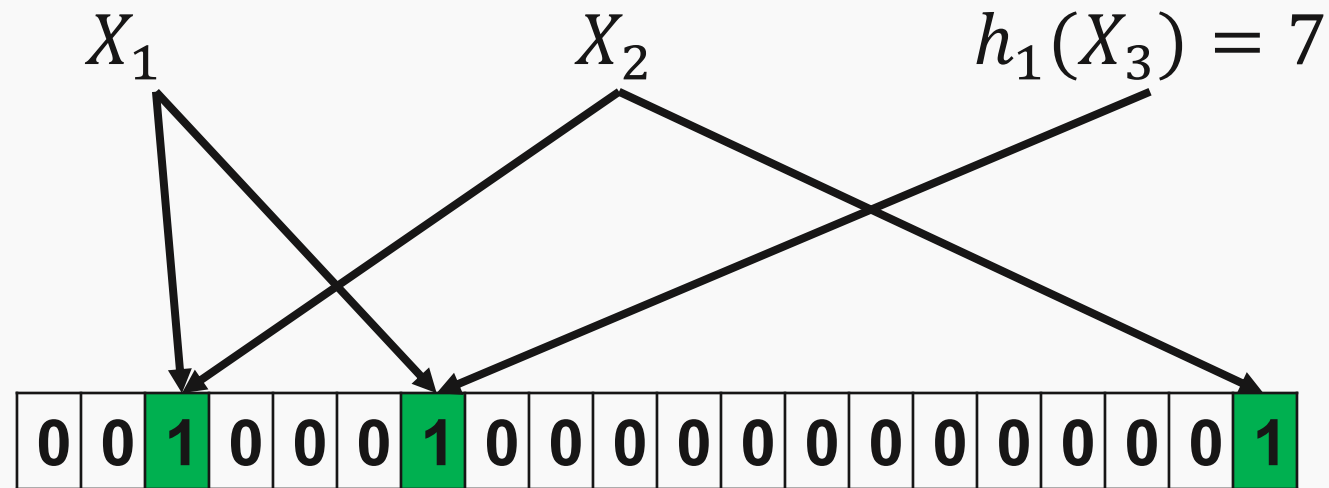


Bloom filter – inițializare



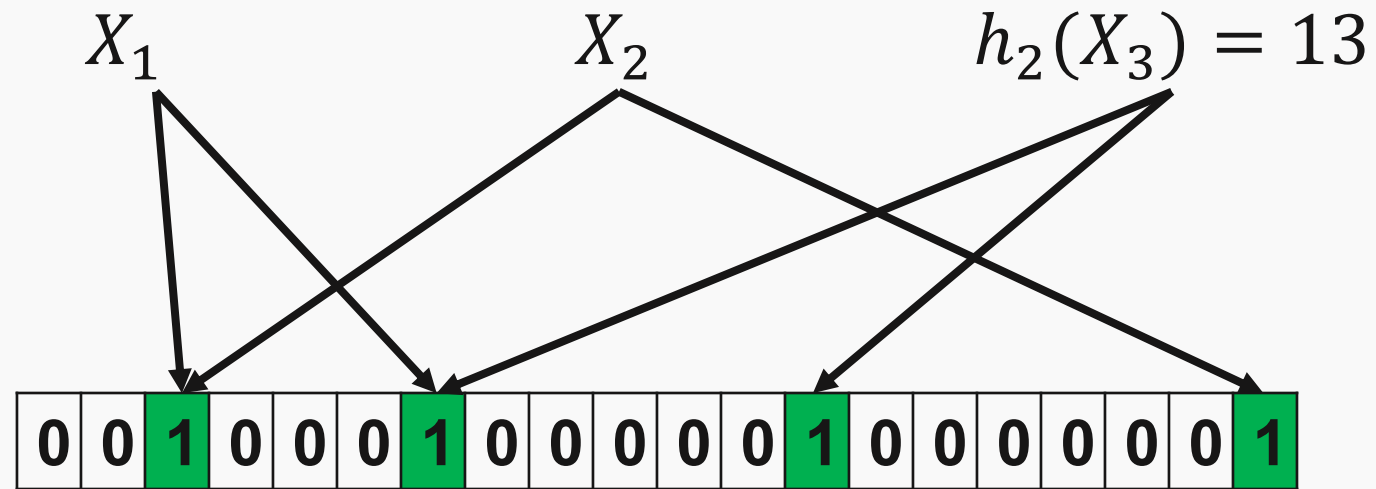


Bloom filter – inițializare



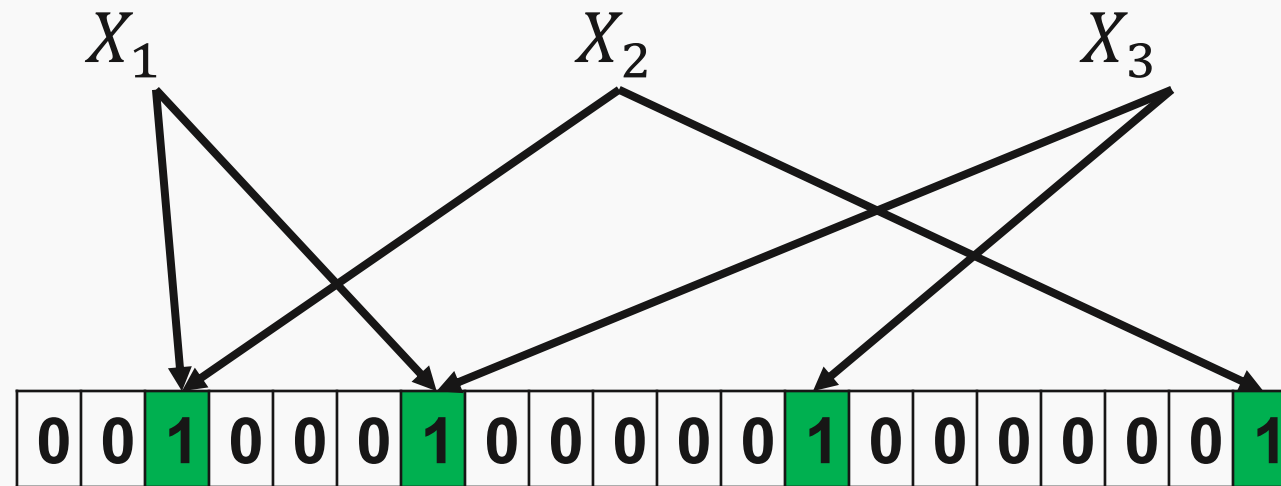


Bloom filter – inițializare



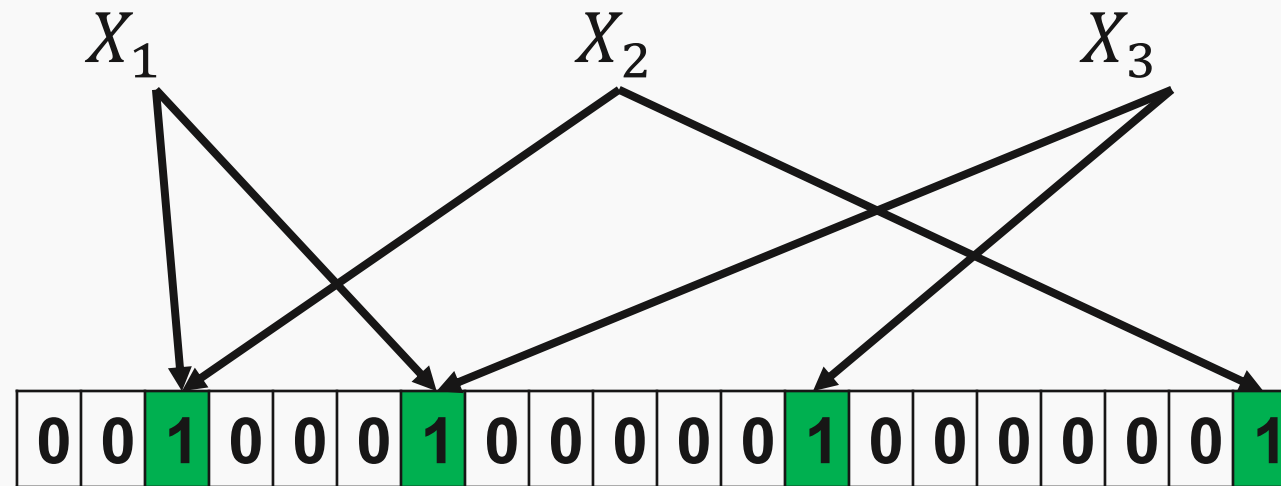


Bloom filter – inițializare





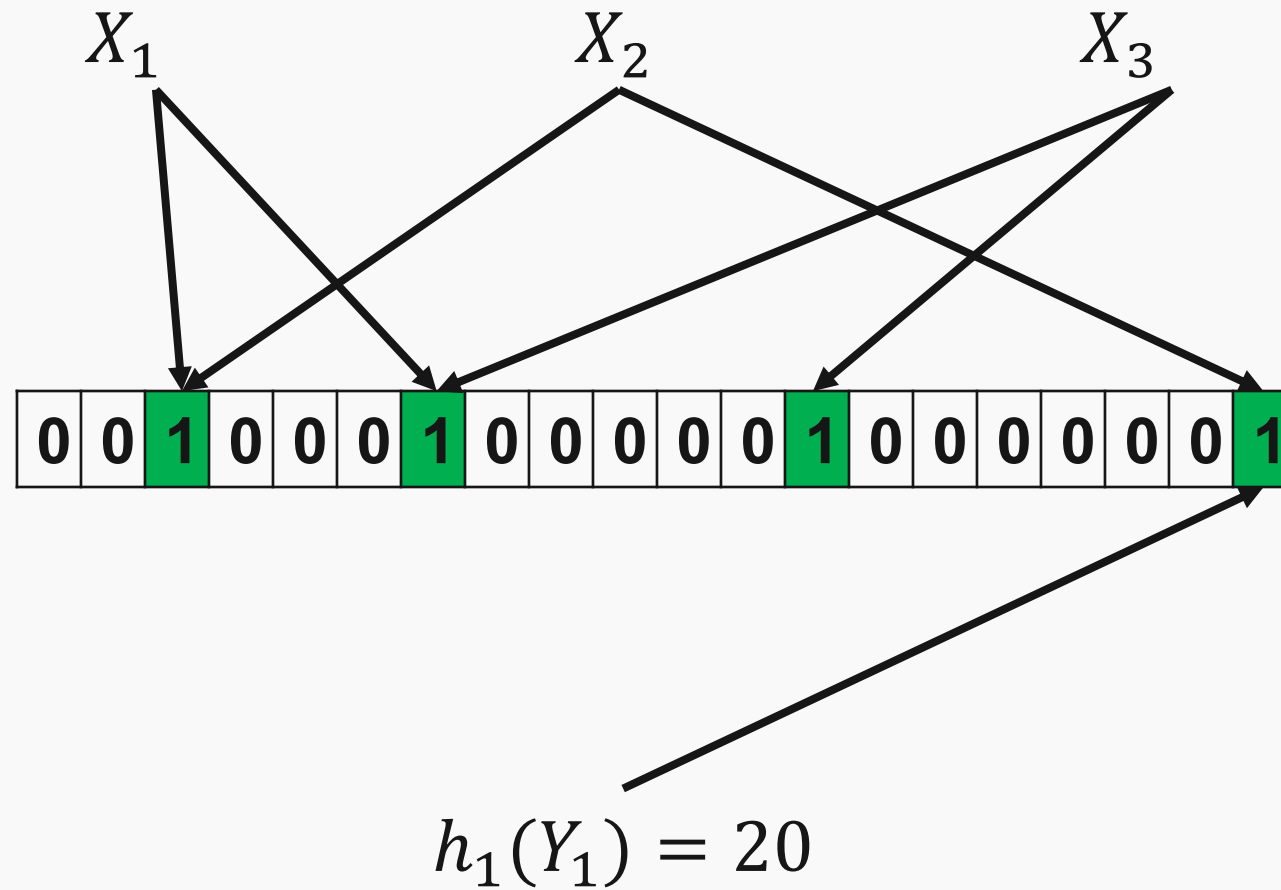
Bloom filter – folosire



$$Y_1 = X_2$$

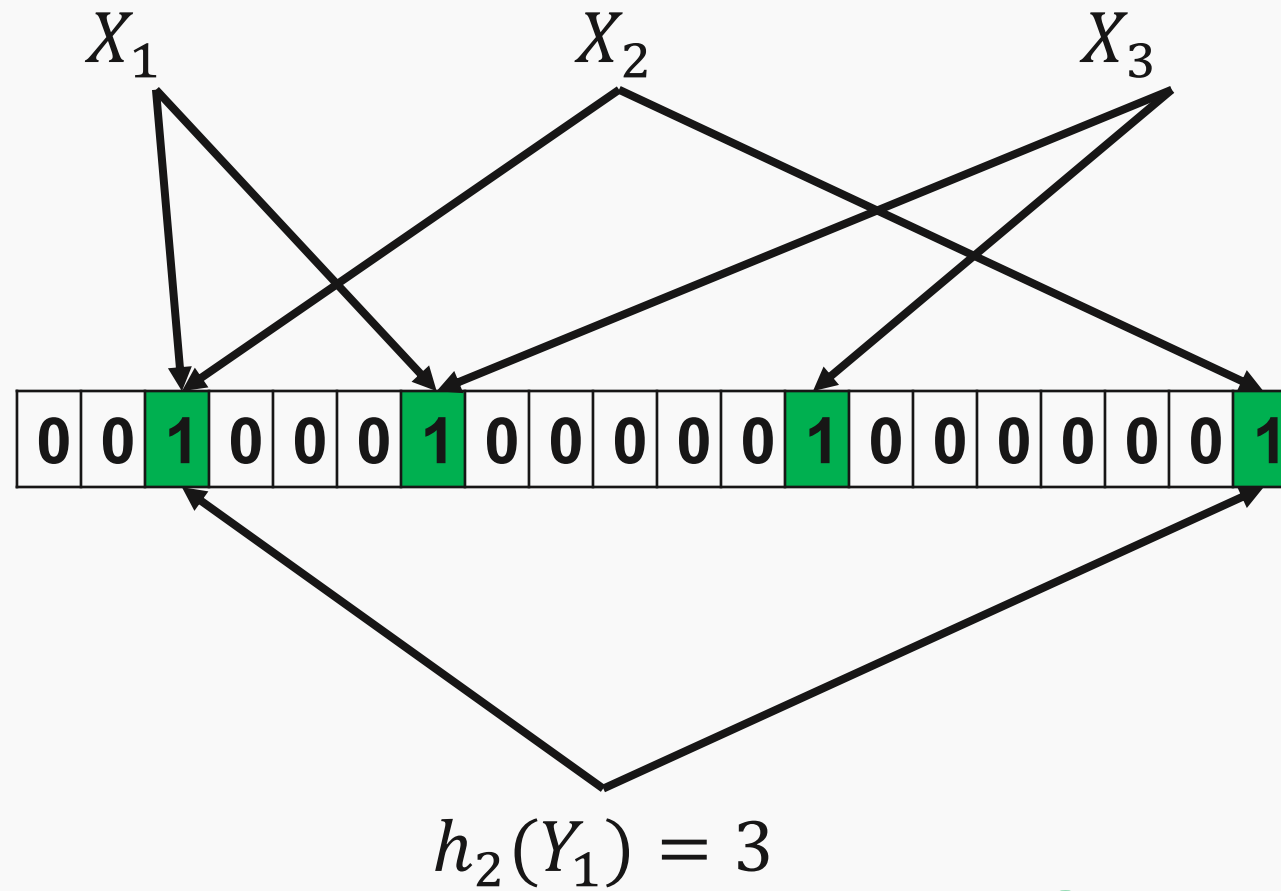


Bloom filter – folosire





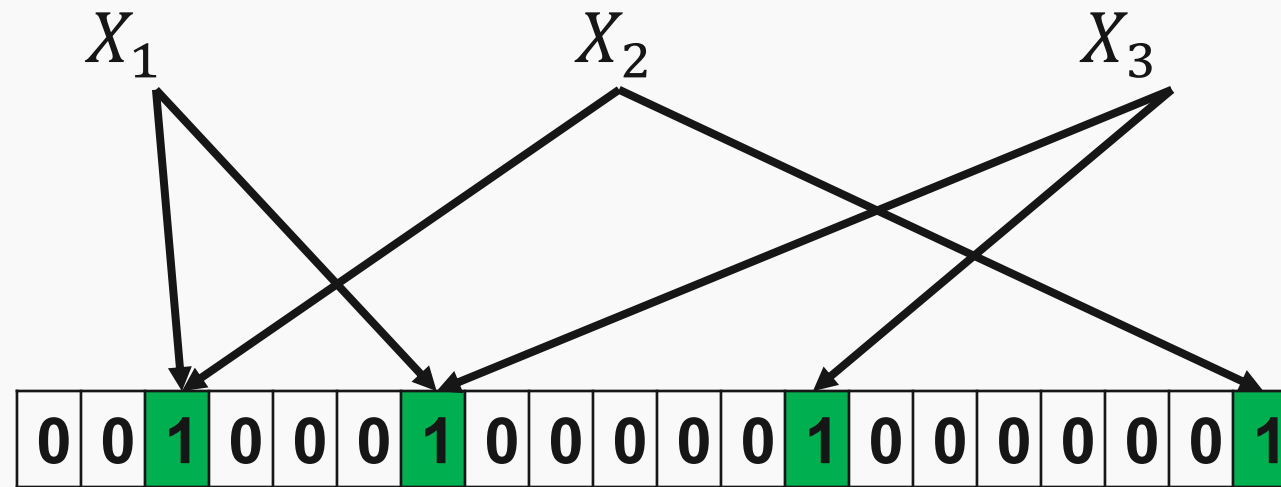
Bloom filter – folosire



OK



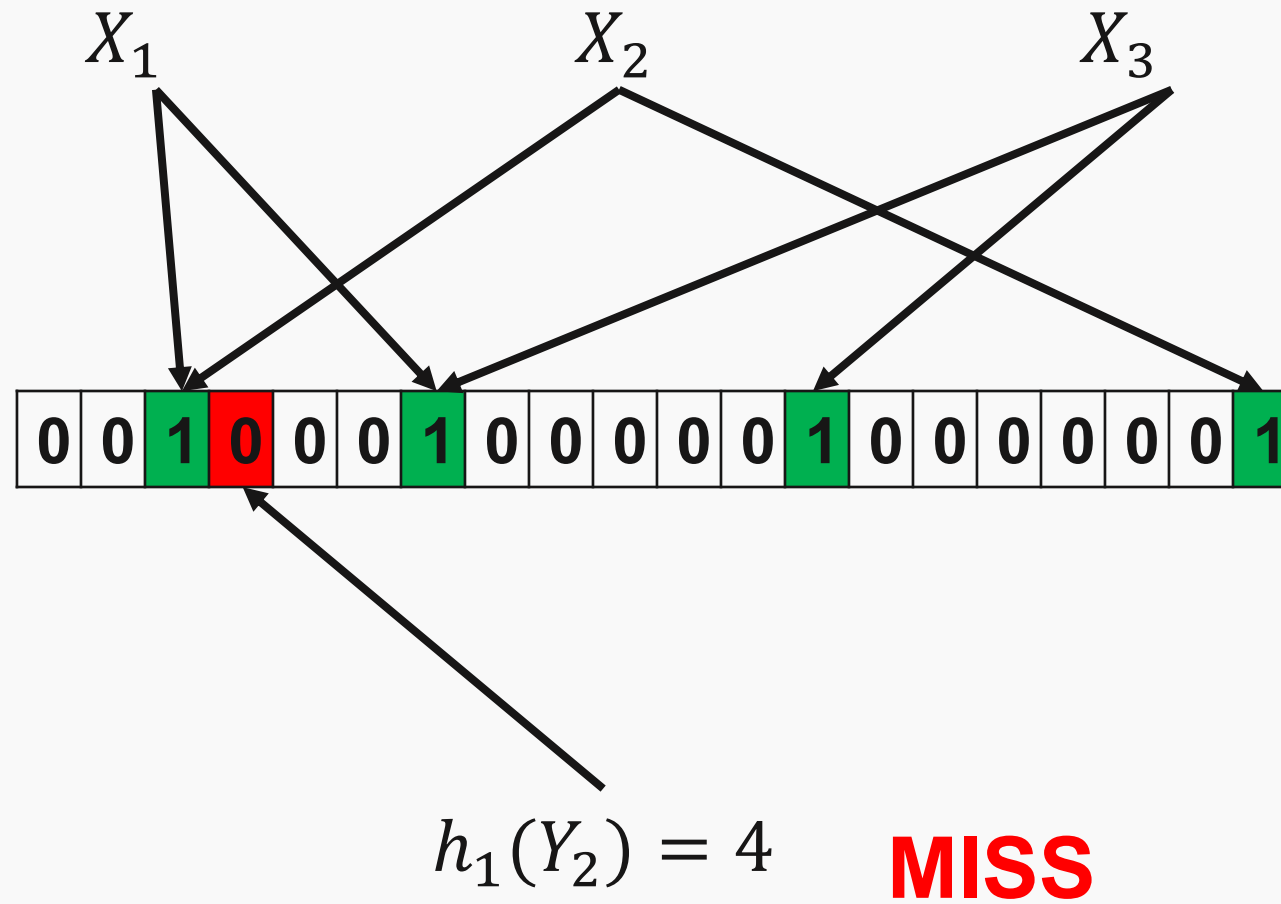
Bloom filter – folosire



Y_2

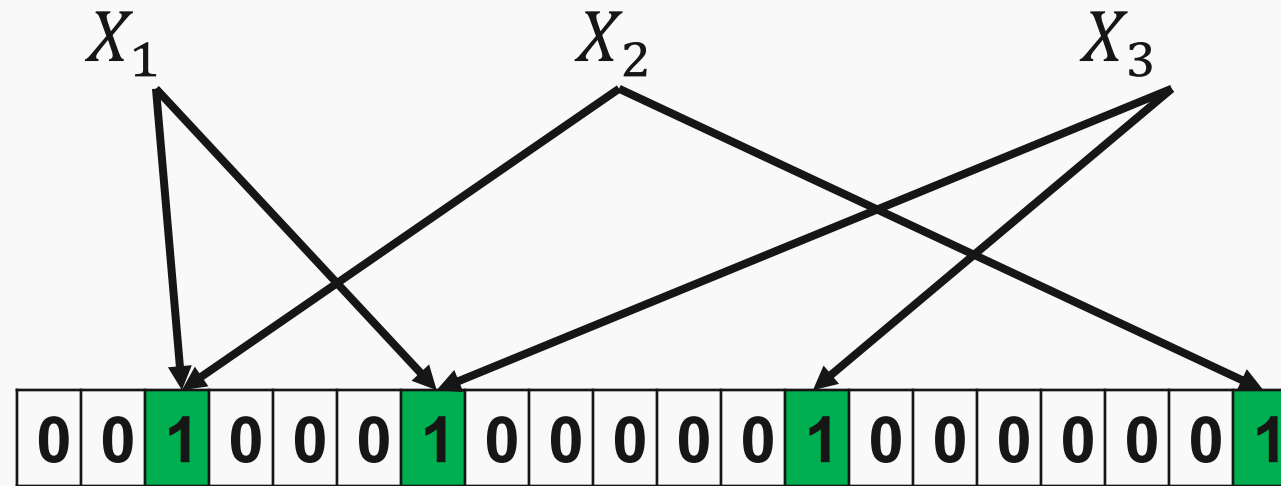


Bloom filter – folosire





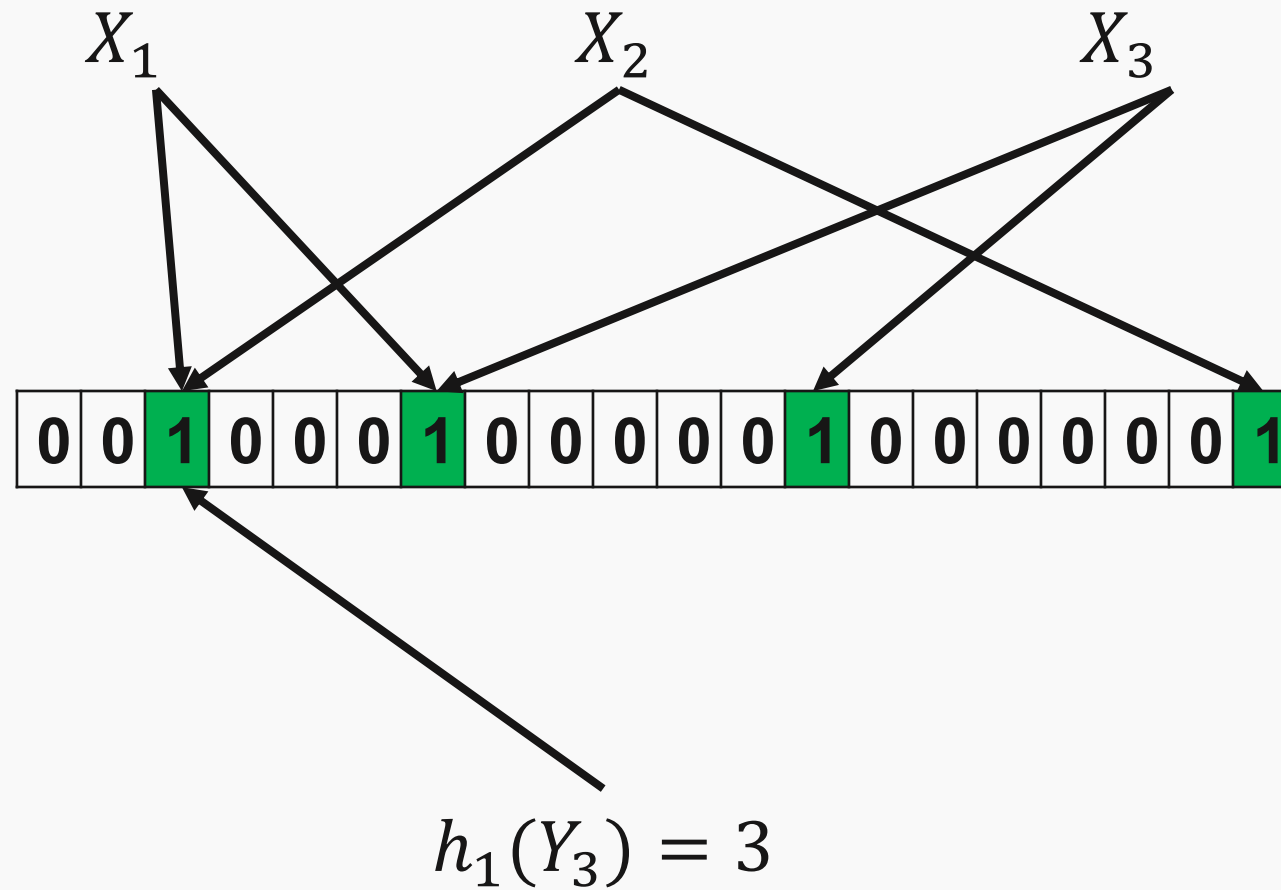
Bloom filter – folosire



$$Y_3! = X_1 \quad Y_3! = X_2 \quad Y_3! = X_3$$

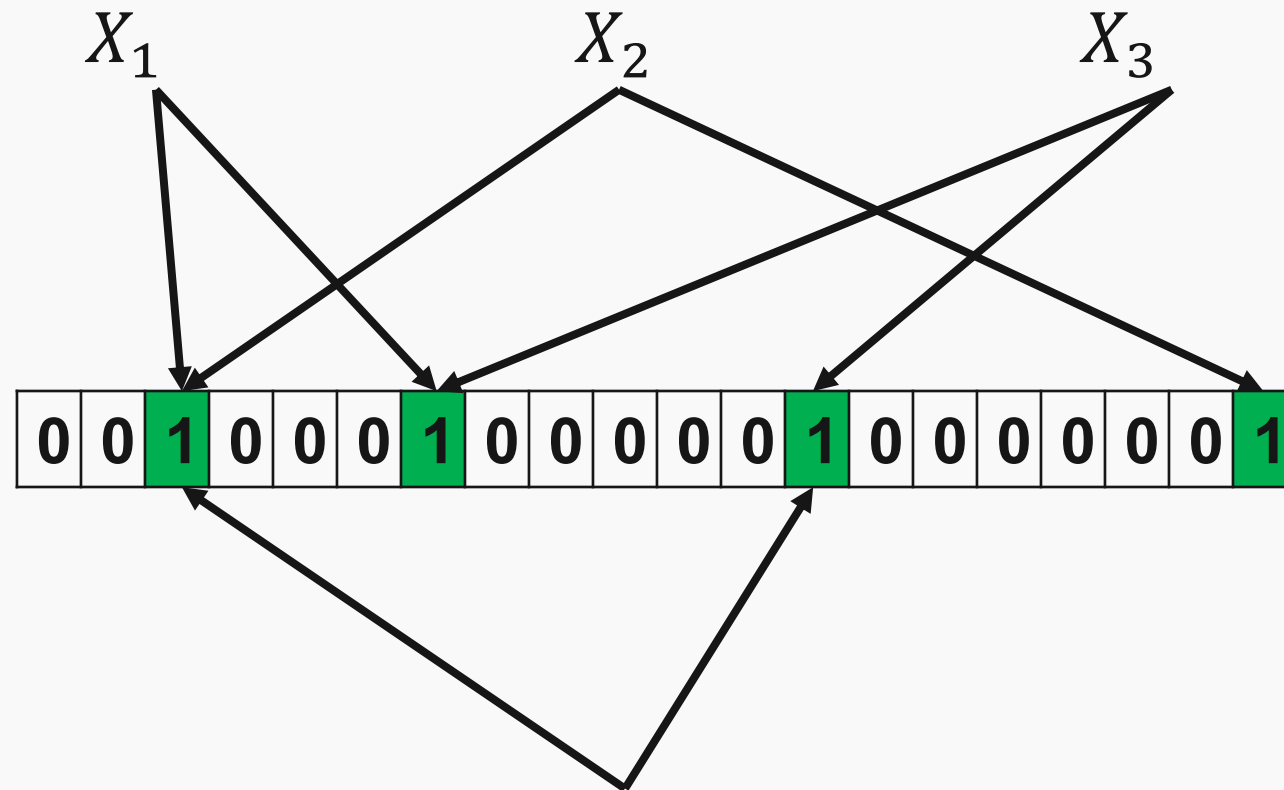


Bloom filter – folosire





Bloom filter – folosire



$h_2(Y_3) = 13$ **FALSE OK**



Bloom filter – Performanță

- Probabilitate de false positive = $\left(\frac{\#1}{N}\right)^k$
- Procent de biți egali cu **0**:

$$\left(1 - \frac{1}{N}\right)^{d*k} = \left(1 - \frac{1}{N}\right)^{\frac{1}{N}N*d*k} = e^{-\frac{d*k}{N}}$$

- Probabilitate de false positive:

$$\left(1 - e^{-\frac{d*k}{N}}\right)^k$$

d numărul de valori din filtru
k numărul de funcții de hash
N numărul de biți



Bloom filter

■ Avantaje

- ❑ Verificare extrem de rapidă (chiar mai rapid decât la hash map)
 - La hash map am avea $O(1)$ dar constanta poate fi mare, unele cazuri $O(N)$
- ❑ Memorie extrem de puțină (foarte bun pentru sisteme distribuite)

■ Dezavantaj

- ❑ Nu poate fi folosit mereu deoarece este aproximativ (**false positives**)

■ Folosire:

- ❑ Baze de date distribuite (se filtrează cererile)
- ❑ Distribuirea liste imense (gen URL-uri nesigure)
- ❑ Web Crawling – listă cu URL-uri deja crawluite
- ❑ Sistem recomandări – bloom filter pentru fiecare utilizator verifică dacă a primit deja o recomandare