



# Structuri de date și algoritmi

## MiniMax

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# Reminder backtracking



# Problemă: Varza, Capra și Lupul



**Lup**  
mănâncă  
**capra**

**Capra**  
mănâncă  
**varza**

Scop: mutarea  
unul câte unul pe  
mal opus fără să  
se mănânce



# Problemă: Varza, Capra și Lupul



**NU**

**Lupul  
mănâncă  
capra**





# Problemă: Varza, Capra și Lupul



**NU**

Capra  
mănâncă  
varza

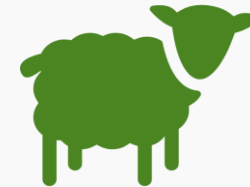


# Problemă: Varza, Capra și Lupul





# Problemă: Varza, Capra și Lupul





# Problemă: Varza, Capra și Lupul







# Problemă: Varza, Capra și Lupul





# Problemă: Varza, Capra și Lupul



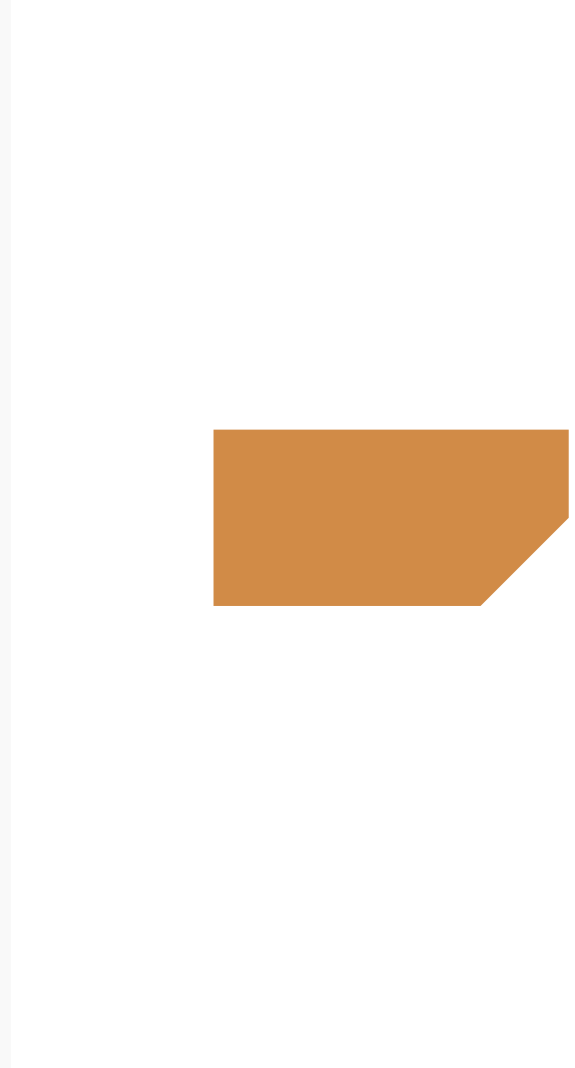


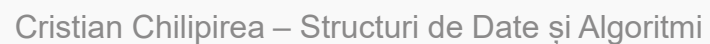
# Problemă: Varza, Capra și Lupul





# Problemă: Varza, Capra și Lupul







# Abordarea Dijsktra – varză capră și lup

<https://www.youtube.com/watch?v=0kXjl2e6qD0>

Scoate toate detaliile nesemnificative din problemă

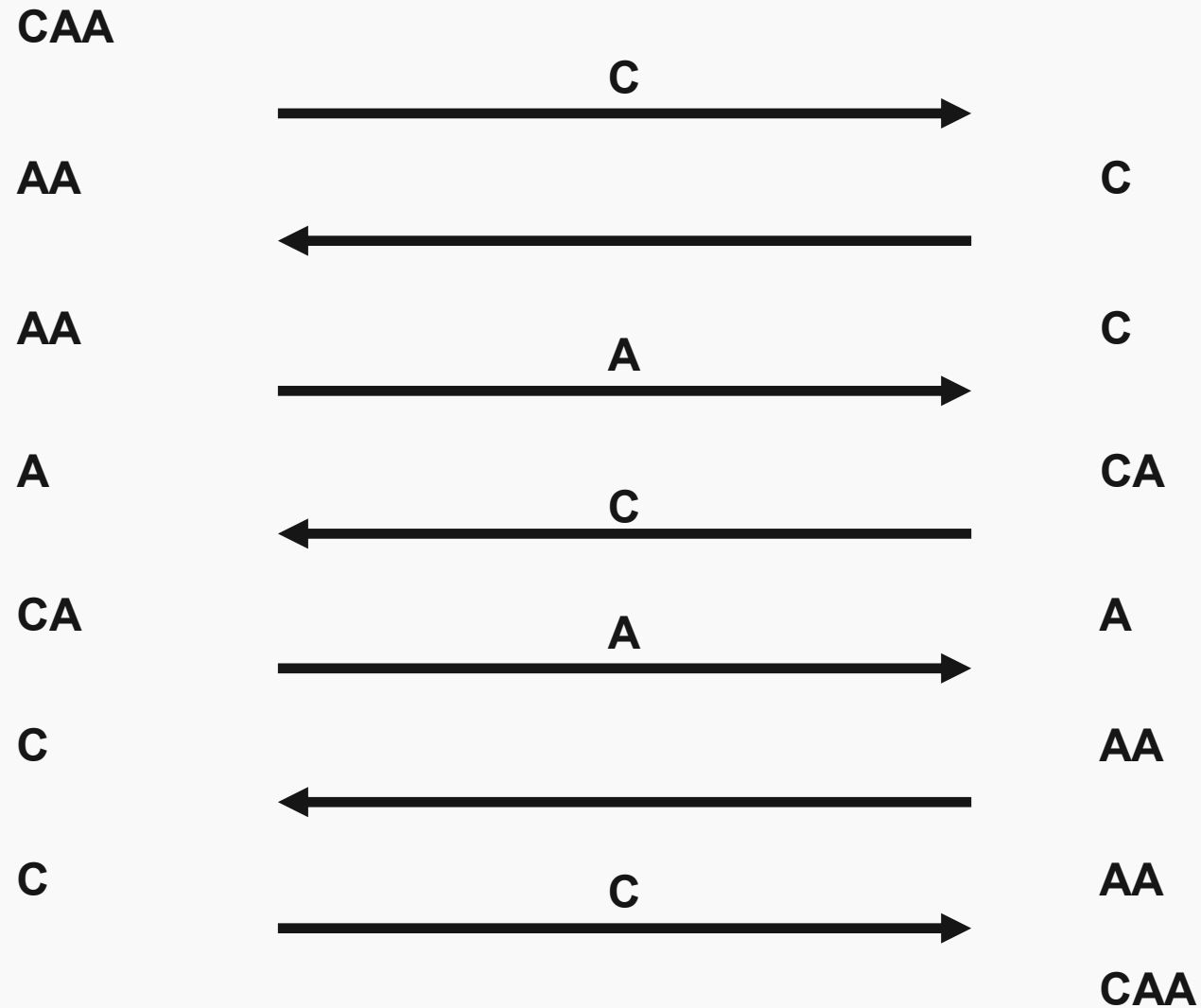
Contează dacă lupul mănâncă capra sau dacă invers capra mănâncă lupul?

Contează dacă capra mănâncă varză sau dacă invers lupul mănâncă capra?

Dacă NU atunci nu e diferență între lup și varză.

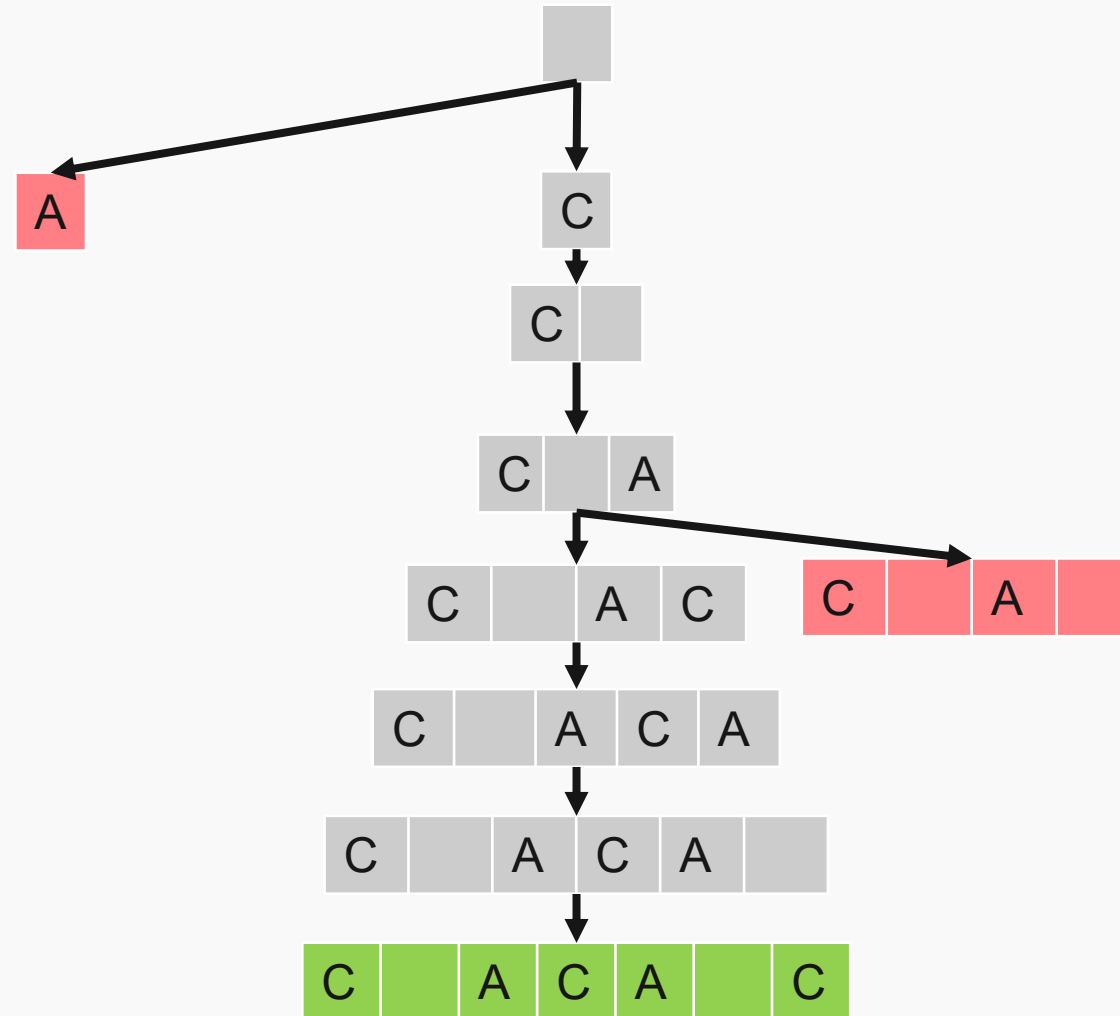


# Capră și restul





# Problemă: Varza, Capra și Lupul





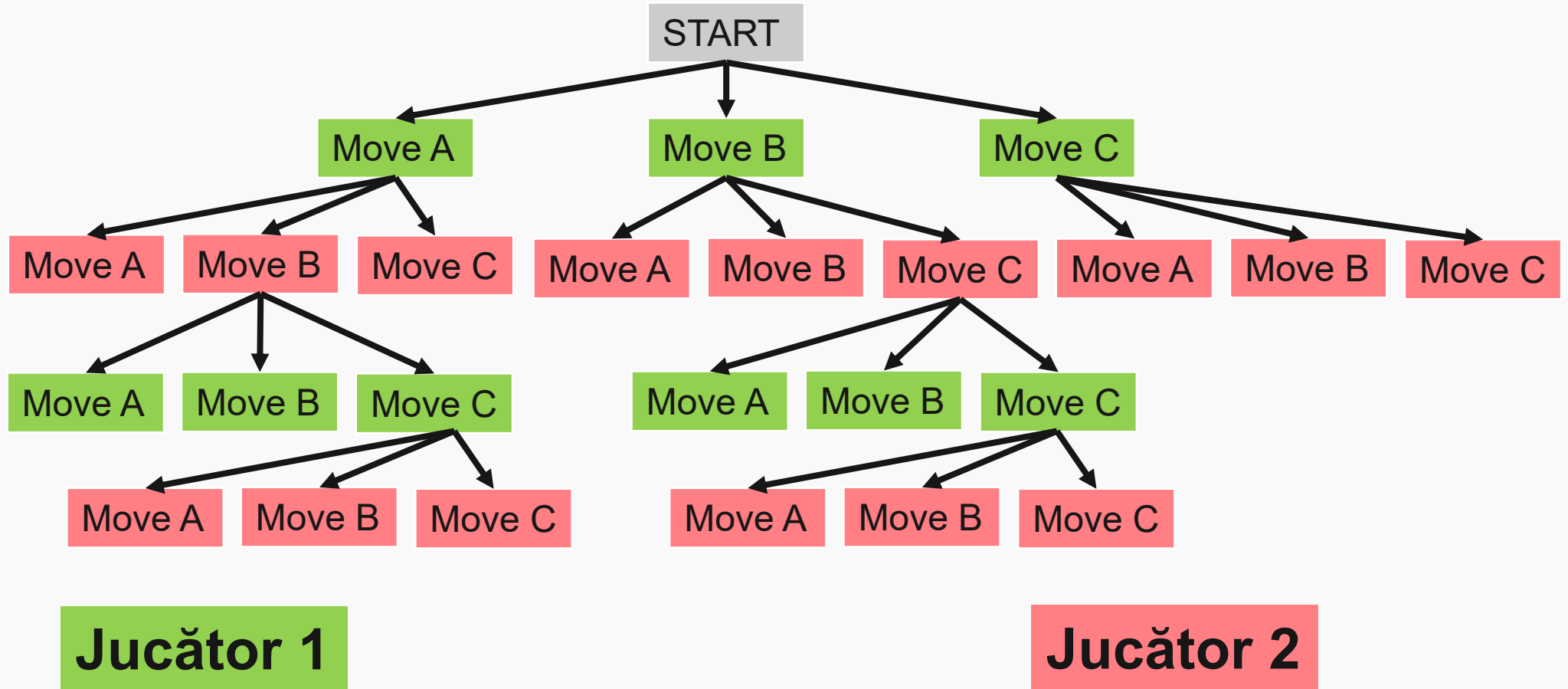


# MiniMax

- Ideea de la backtracking cu construcția arborelui de stări.
- Jocuri competitive. – Vrem algoritm “Inteligență artificială” care să câștige.
- Ce face adversarul?



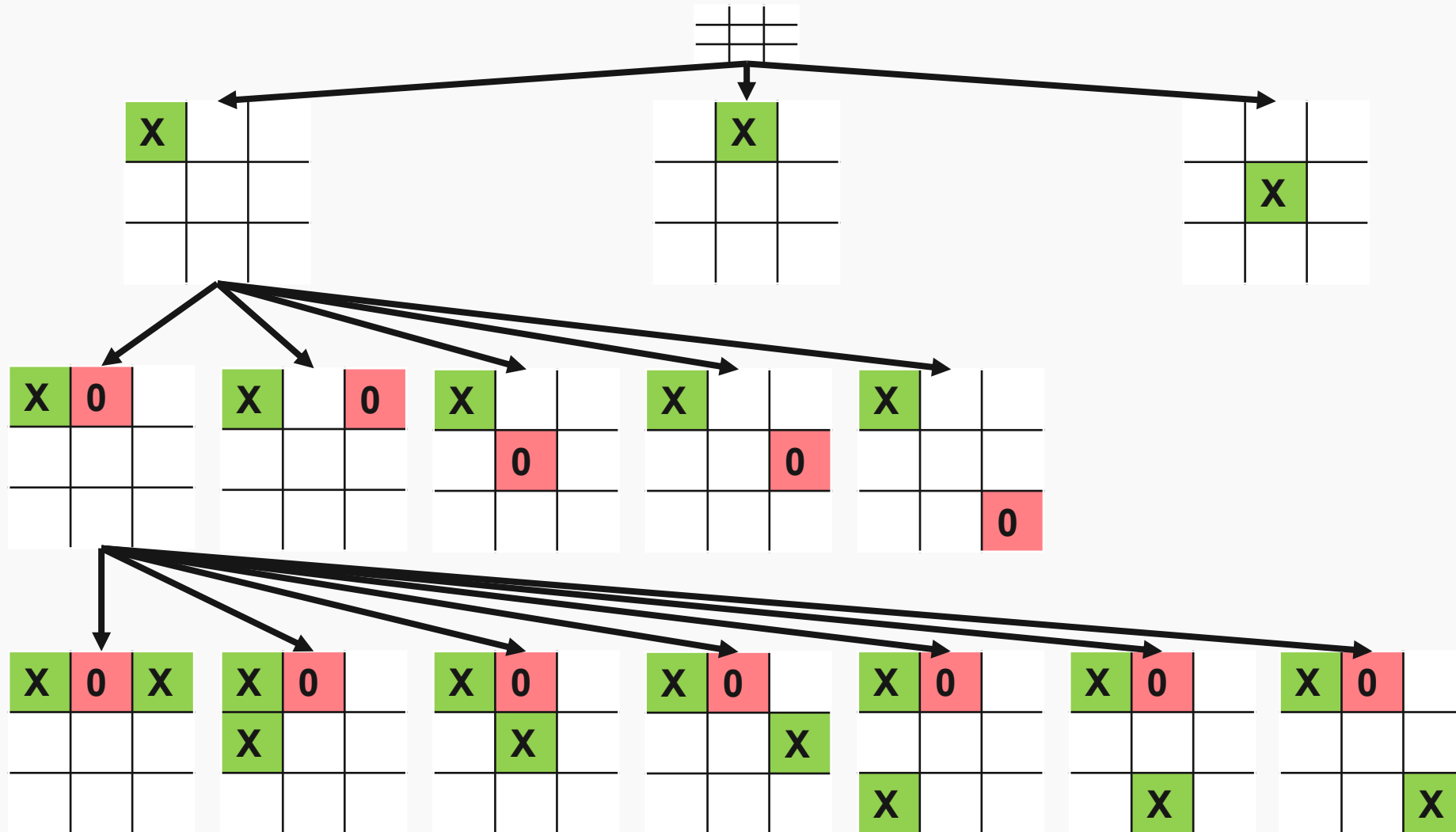
# MiniMax - Idee



Pe nivelele pare sunt mutări ale unui jucător. Pe nivele impare ale altui jucător.



# MiniMax – X și 0





# MiniMax

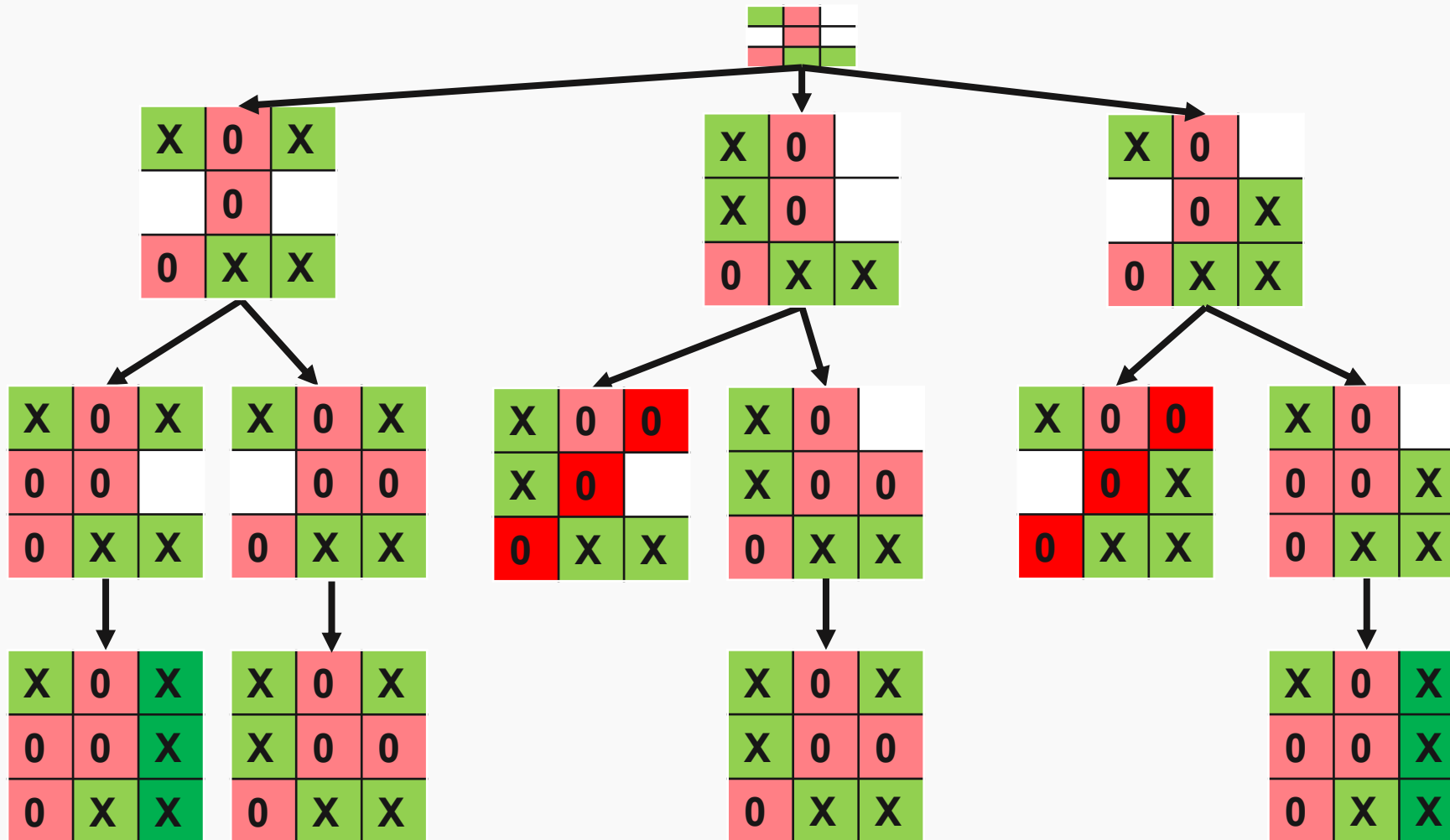
- Nu mai este suficient sa aplic backtracking.
- Chiar dacă găsesc un set de mutări prin care să câștig adversarul va alege mutări care să facă opusul.

**La nivelele unde eu aleg, fac mutări care să **maximizeze** șansele de câștig.**

**La nivelele unde alege adversarul, acesta probabil va încerca să îmi **minimizeze** șansele de câștig. Deci așa presupun că va face.**

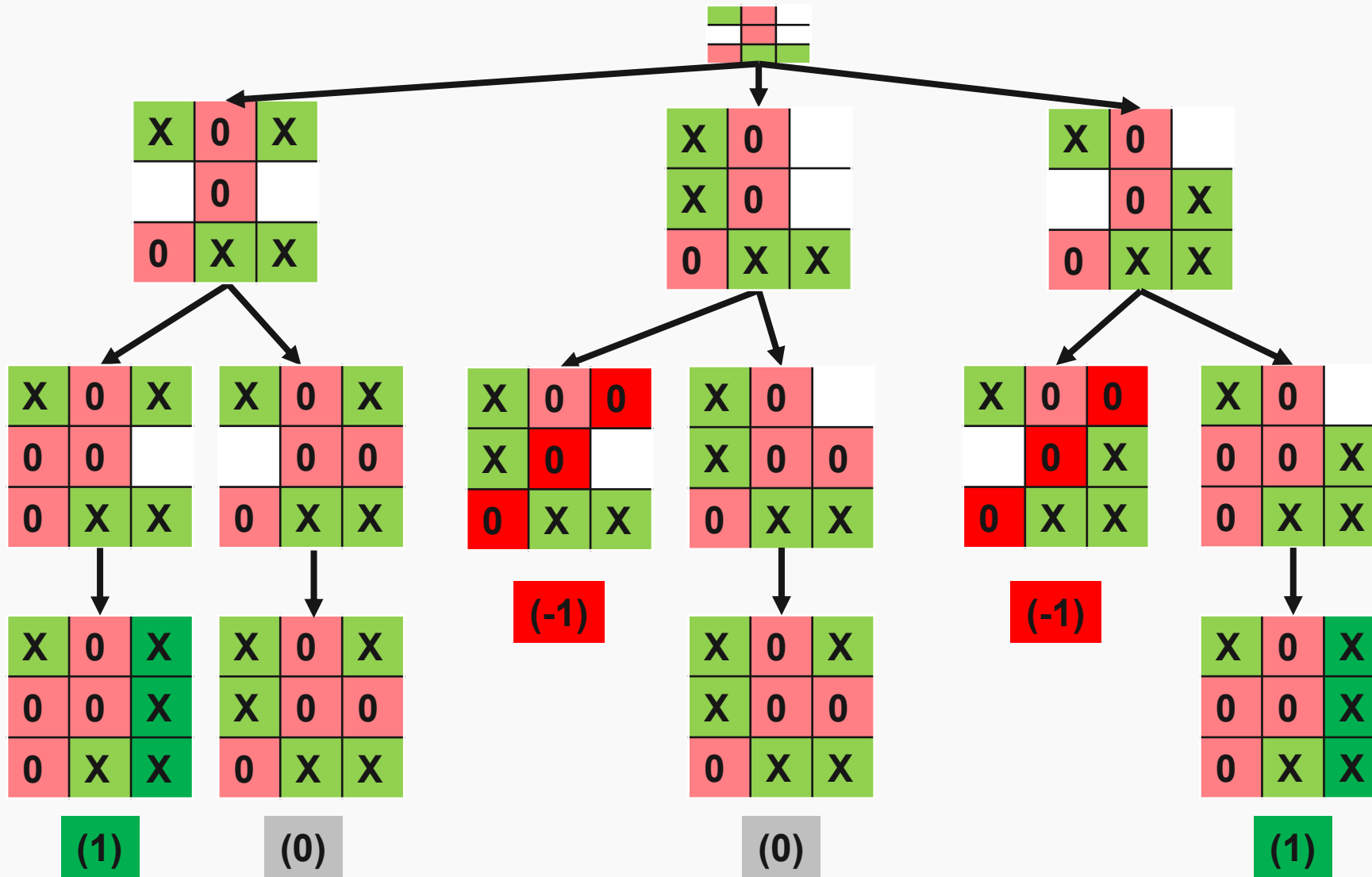


# MiniMax – X și 0



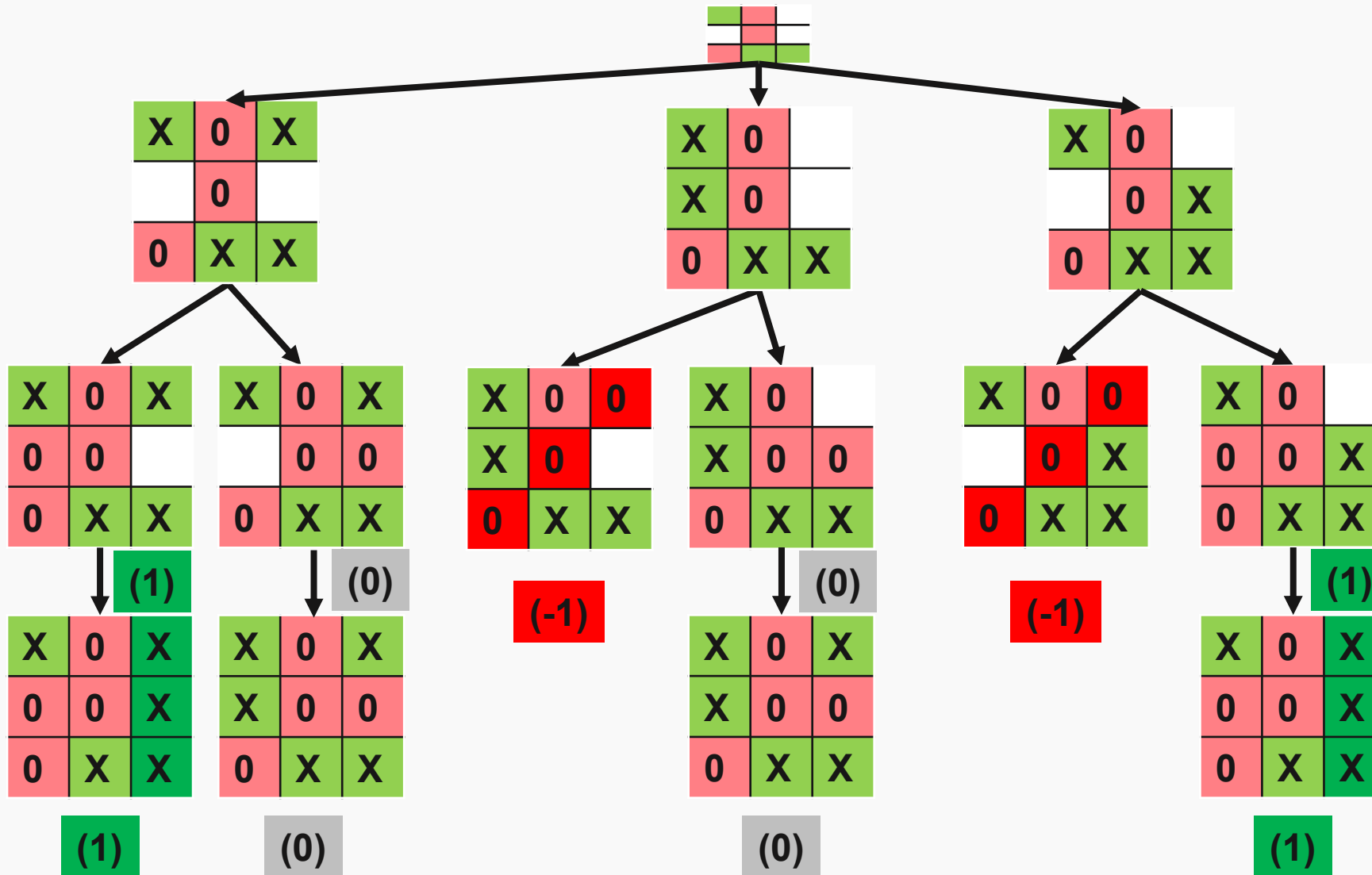


# MiniMax – X și 0



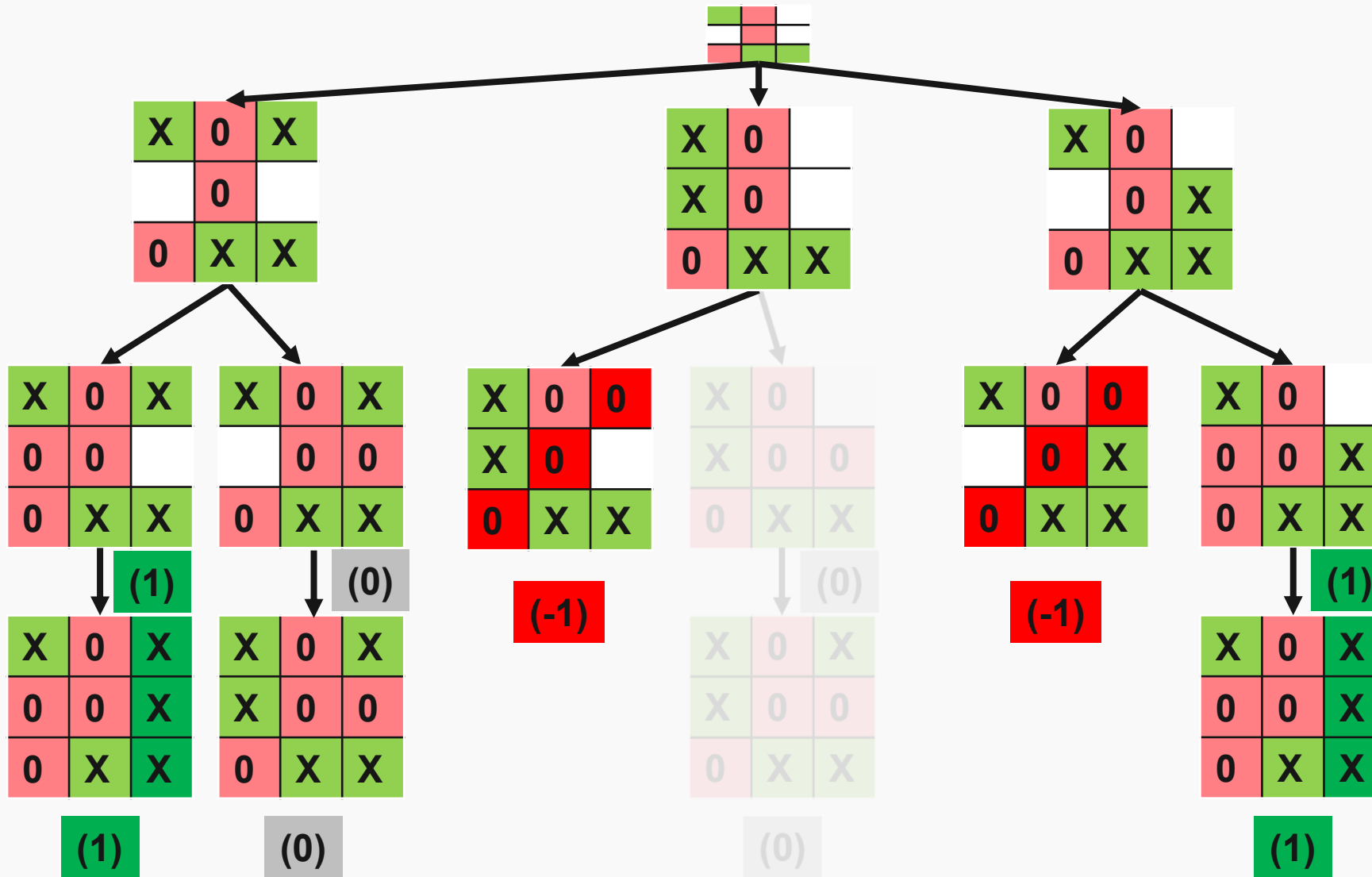


# MiniMax – X și 0





# MiniMax – X și 0

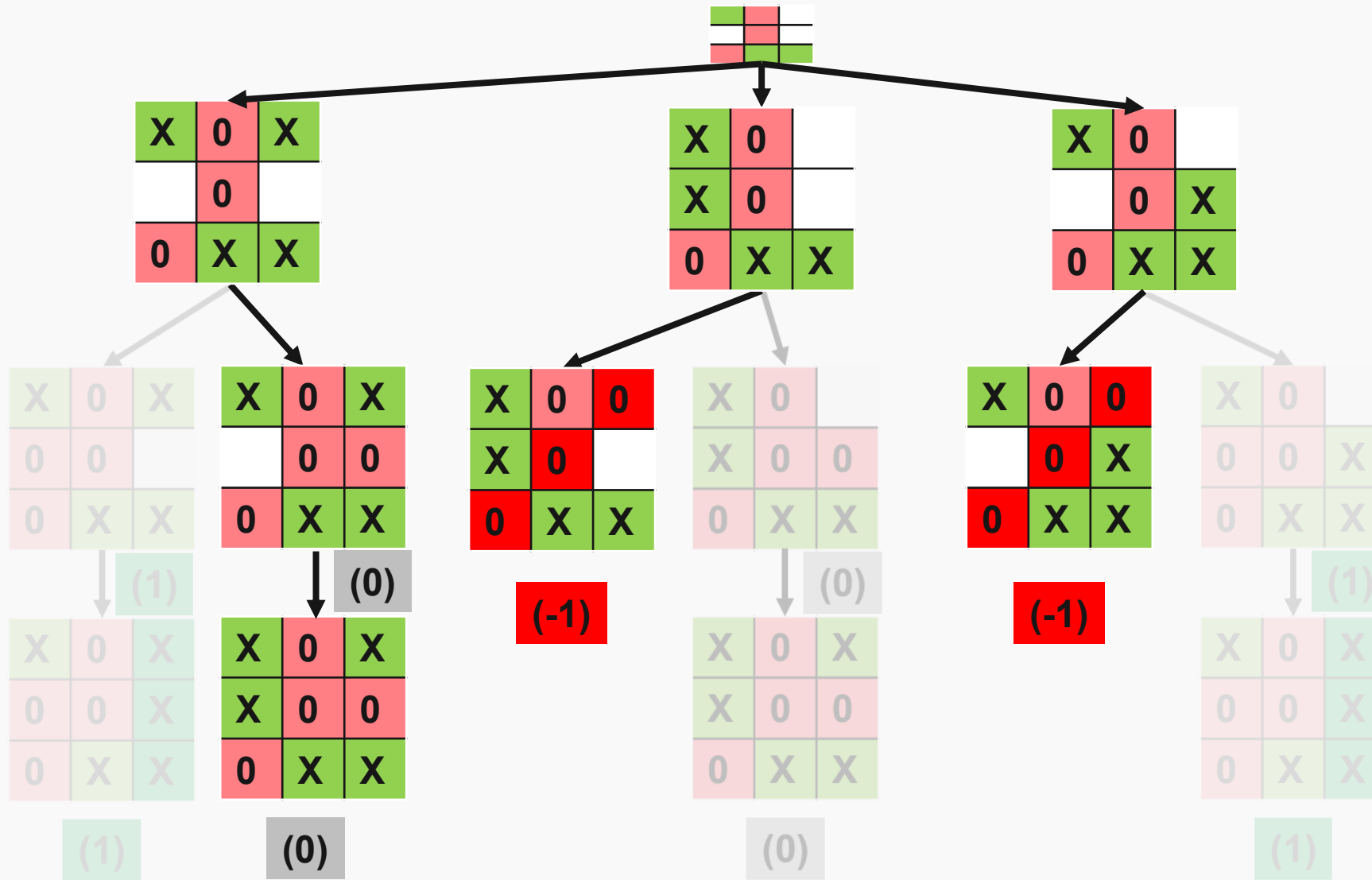






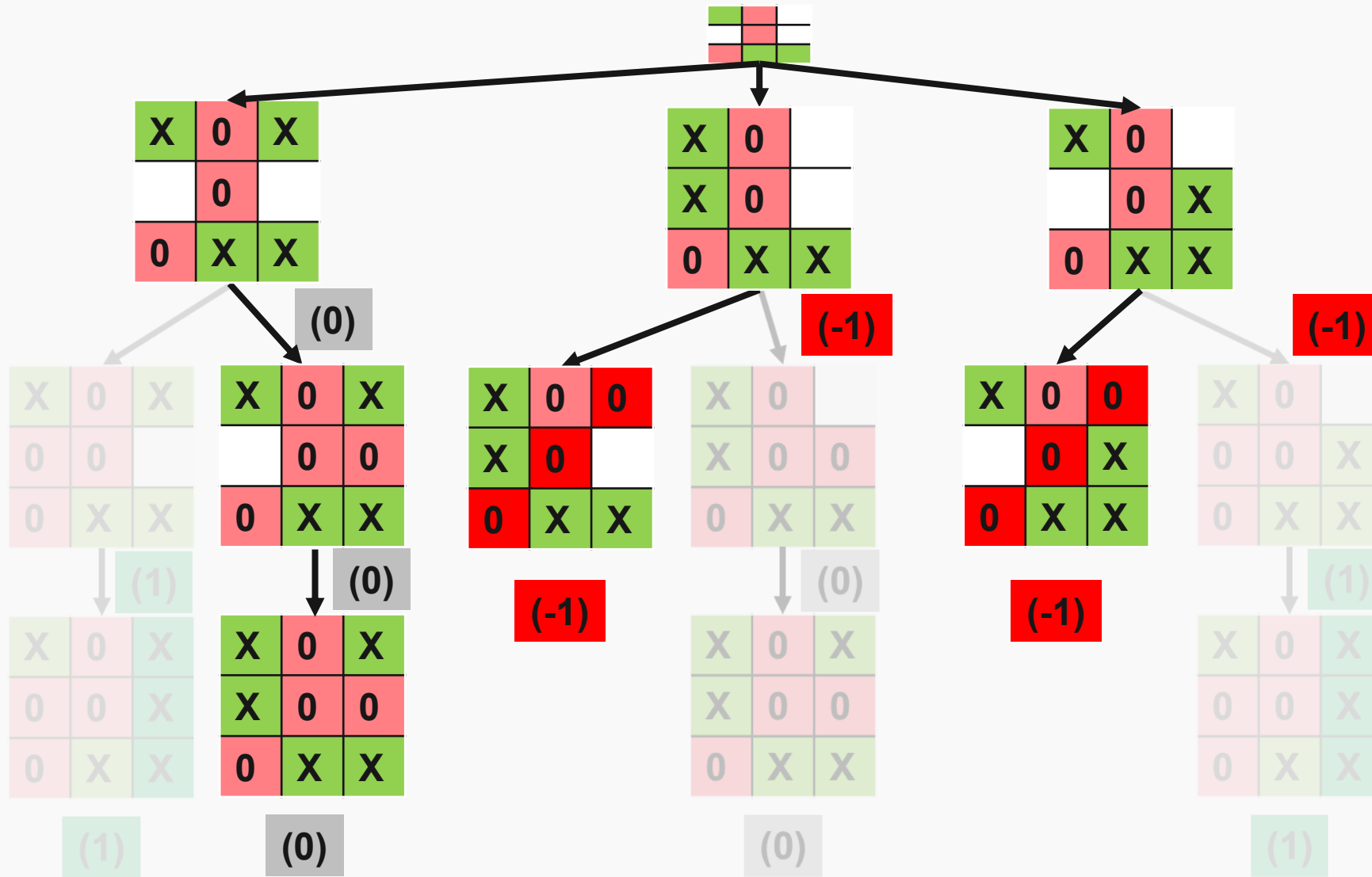


# MiniMax – X și 0



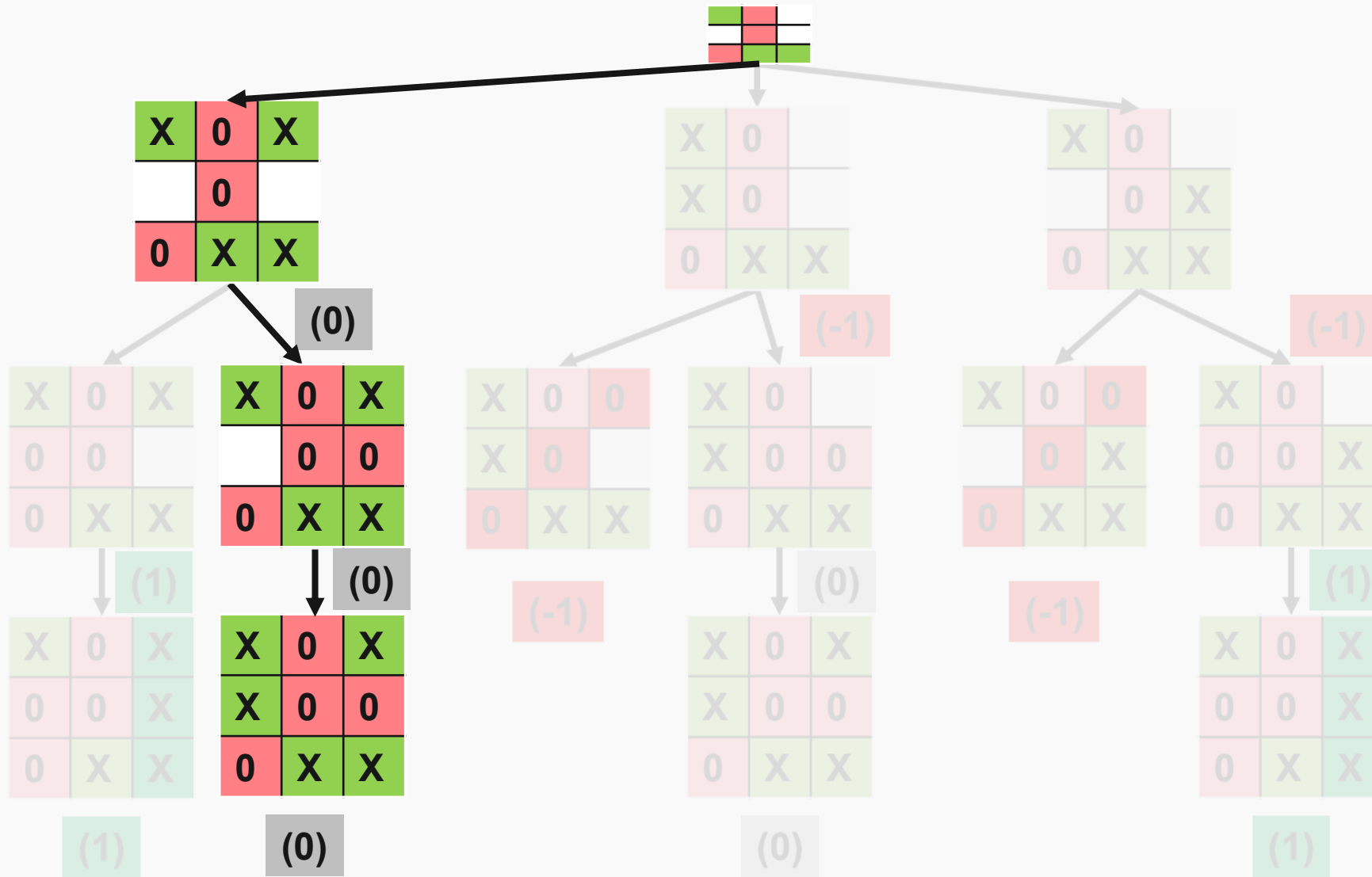


# MiniMax – X și 0





# MiniMax – X și 0





# MiniMax – Cât de mare este arborele?

- X și 0

Aproape

$9*8*7*6*5*4*3*2*1 = 9! = 362,880$  frunze

sub 1M, deci încapă în memorie.

- Dar șah?



## MiniMax – Cât de mare este arborele?

- În general nu putem analiza tot arborele. Este prea mare.
- Soluția: mergem până la o adâncime și încercăm să estimăm cât de avantajos sau dezavantajos este acea configurație a jocului.
- Estimăm -> Euristică (heuristic)



# MiniMax - Algoritm

```
int miniMax(gameNode node, int level) {  
    if (level == maxDepth || isEndGame(node))  
        return heuristic(node);  
  
    if (level % 2 == maximizingPlayer) {  
        int value = -∞;  
        for each (child of node)  
            value = max(value, miniMax(child, level + 1));  
        return value;  
    } else {  
        int value = +∞;  
        for each (child of node)  
            value = min(value, miniMax(child, level + 1));  
        return value;  
    }  
}
```



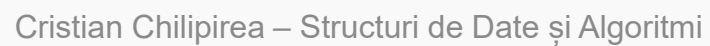
# Apelare MiniMax

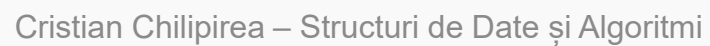
```
for each (child of node)  
    score[child] = miniMax(child, 0);
```

Mutarea este acel child pentru care score e maxim



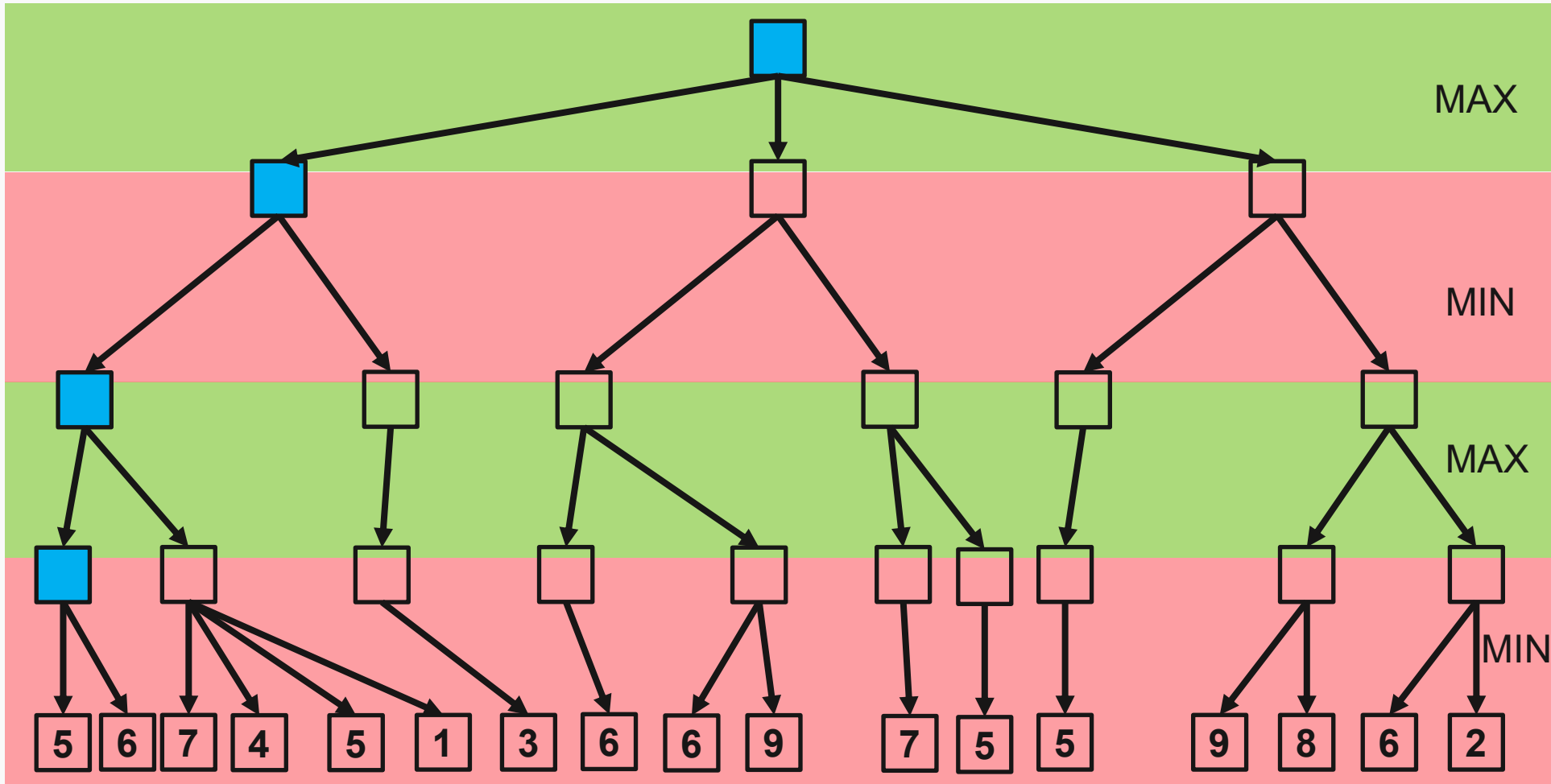






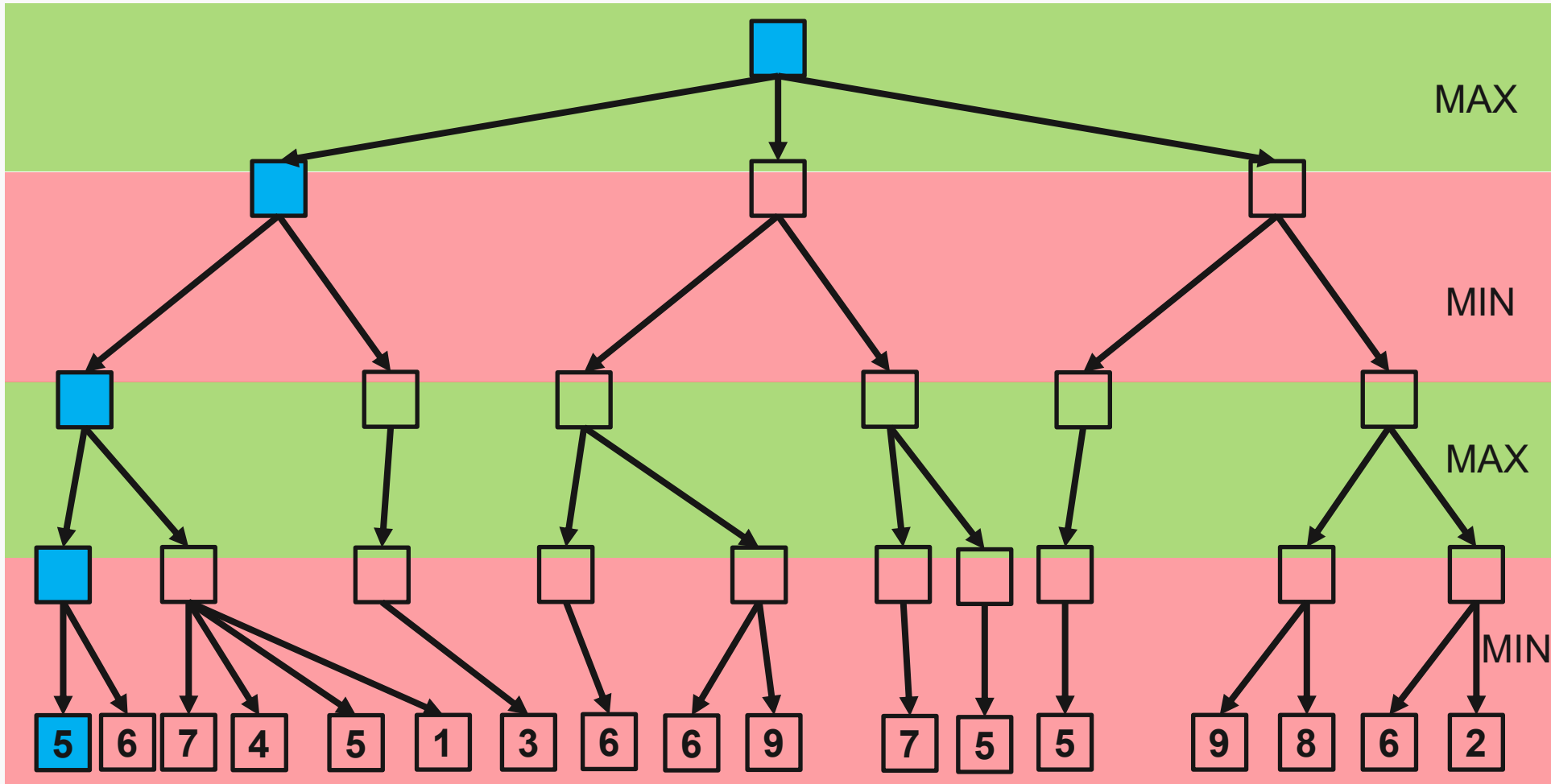


# AlphaBeta Prunning Idee



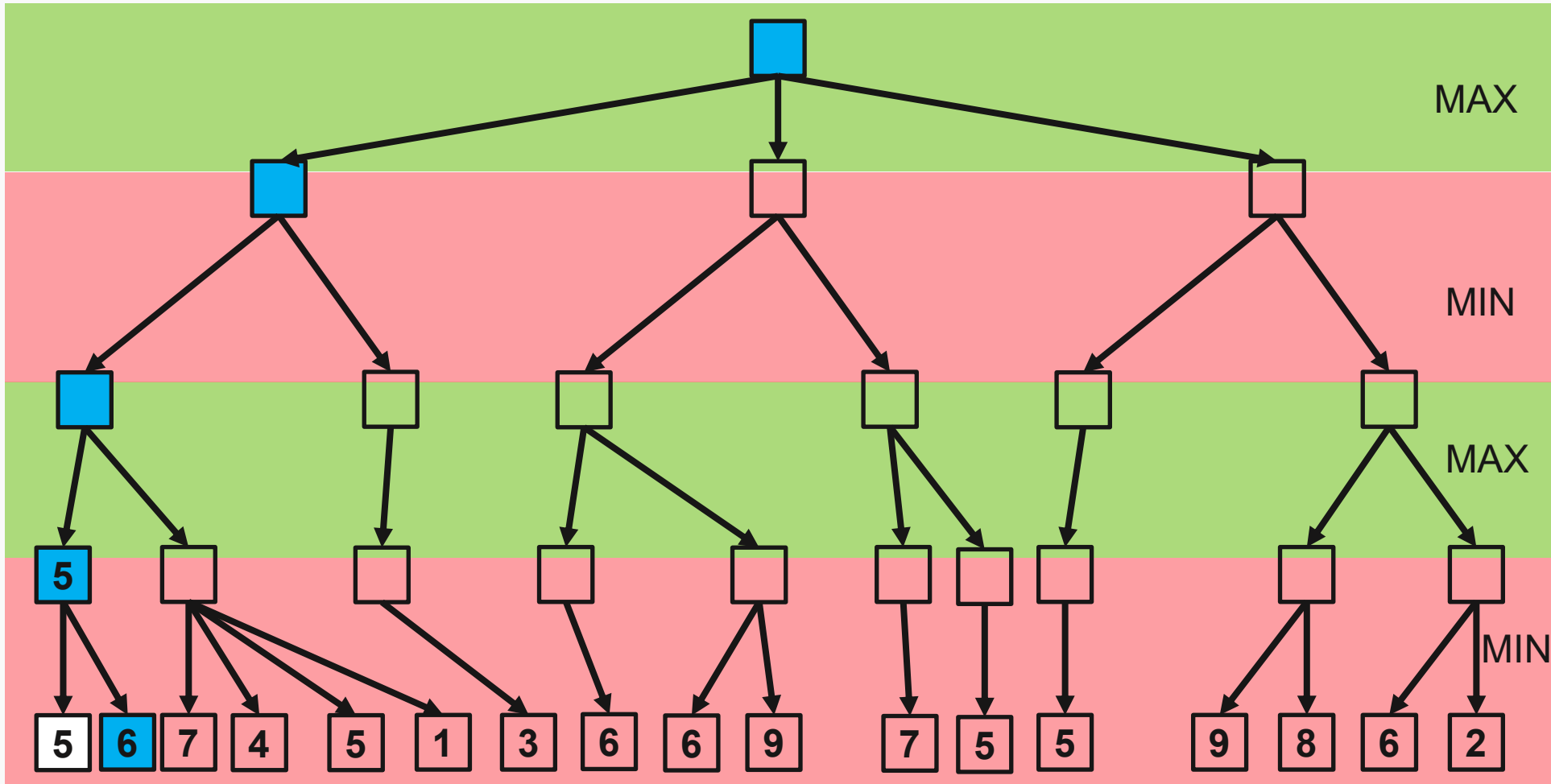


# AlphaBeta Prunning Idee





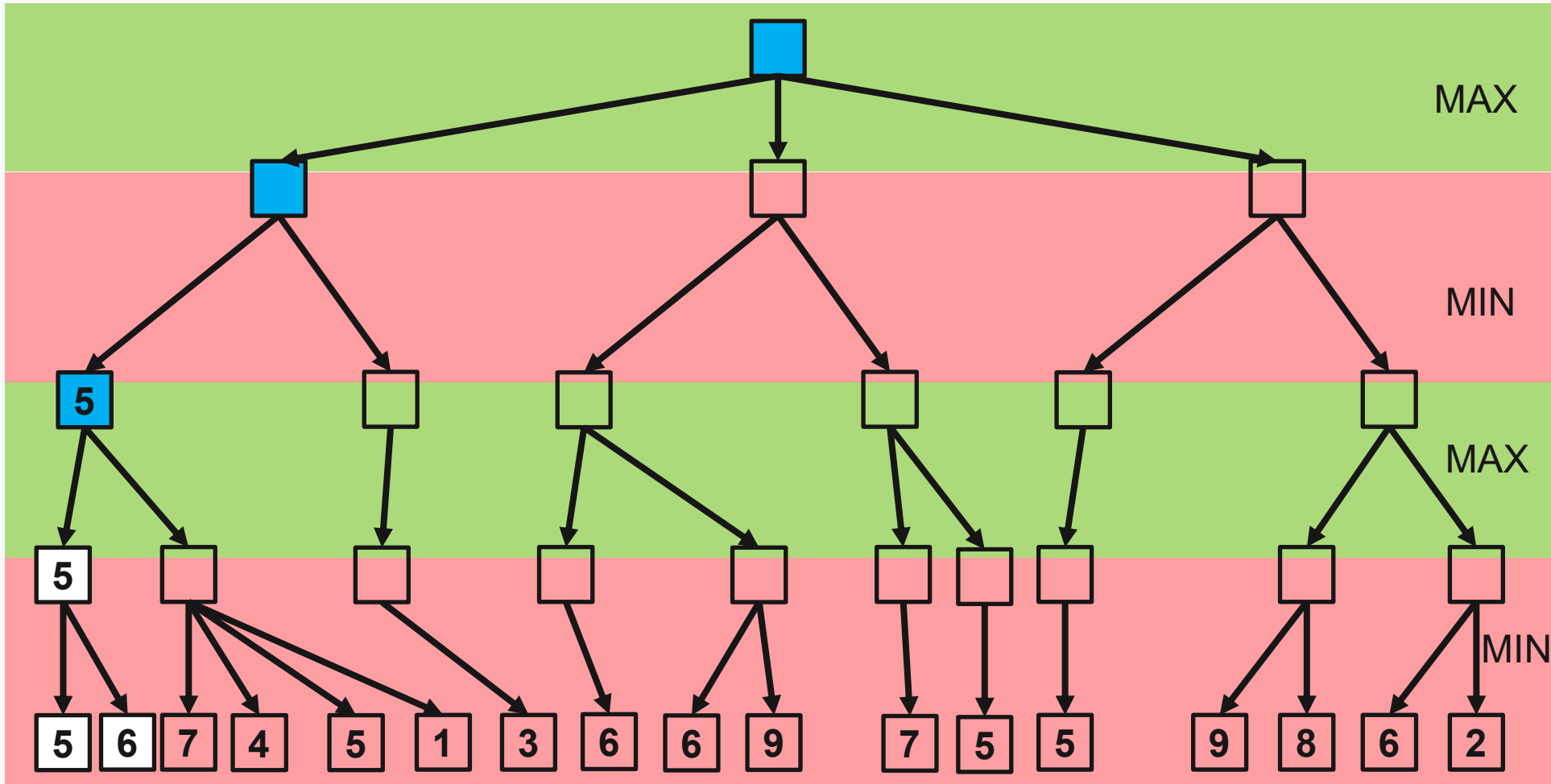
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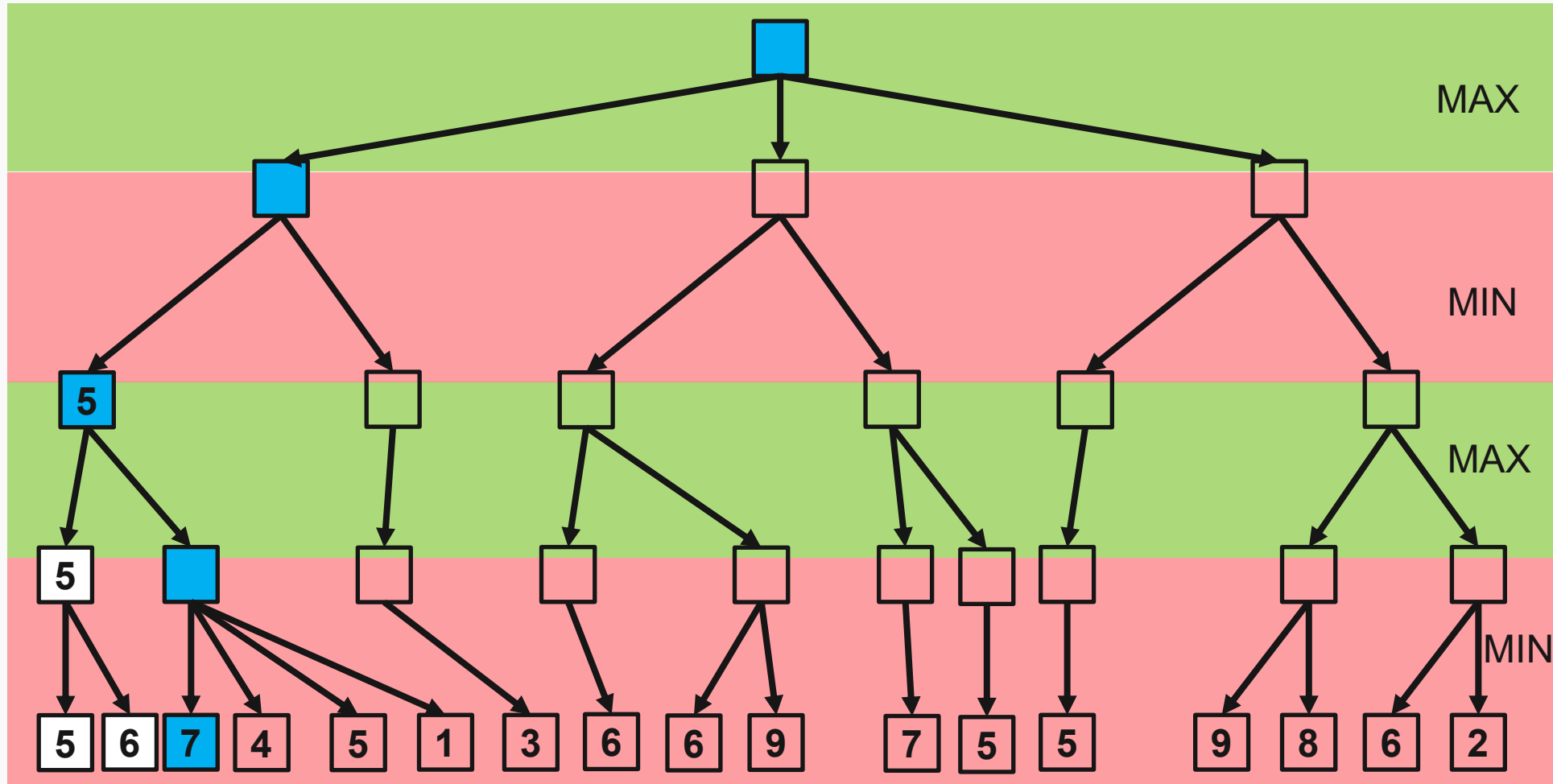


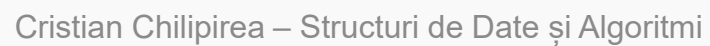


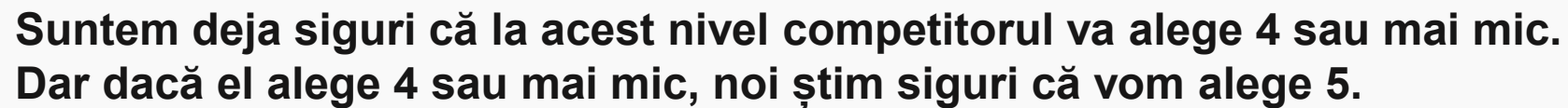




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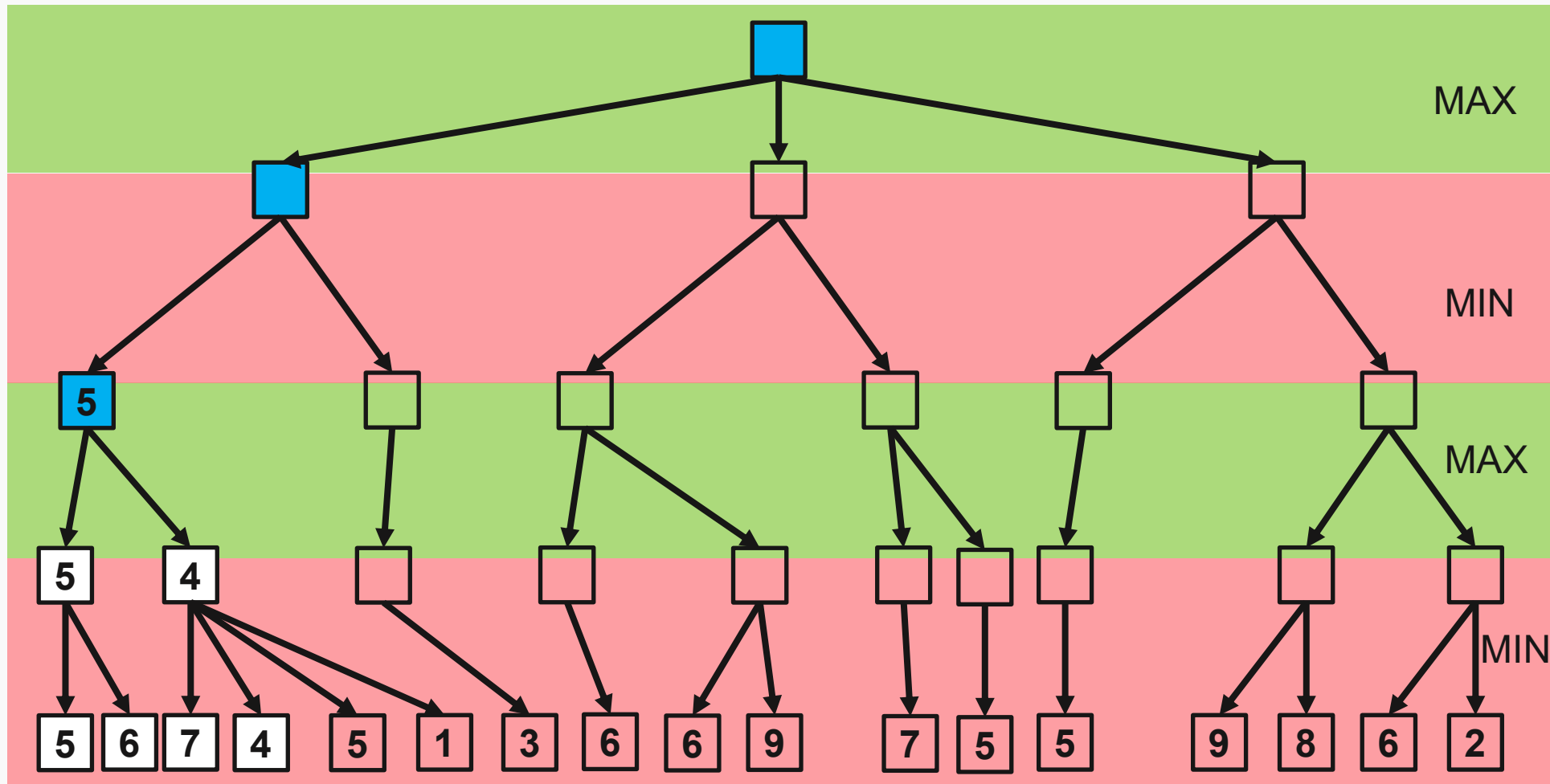






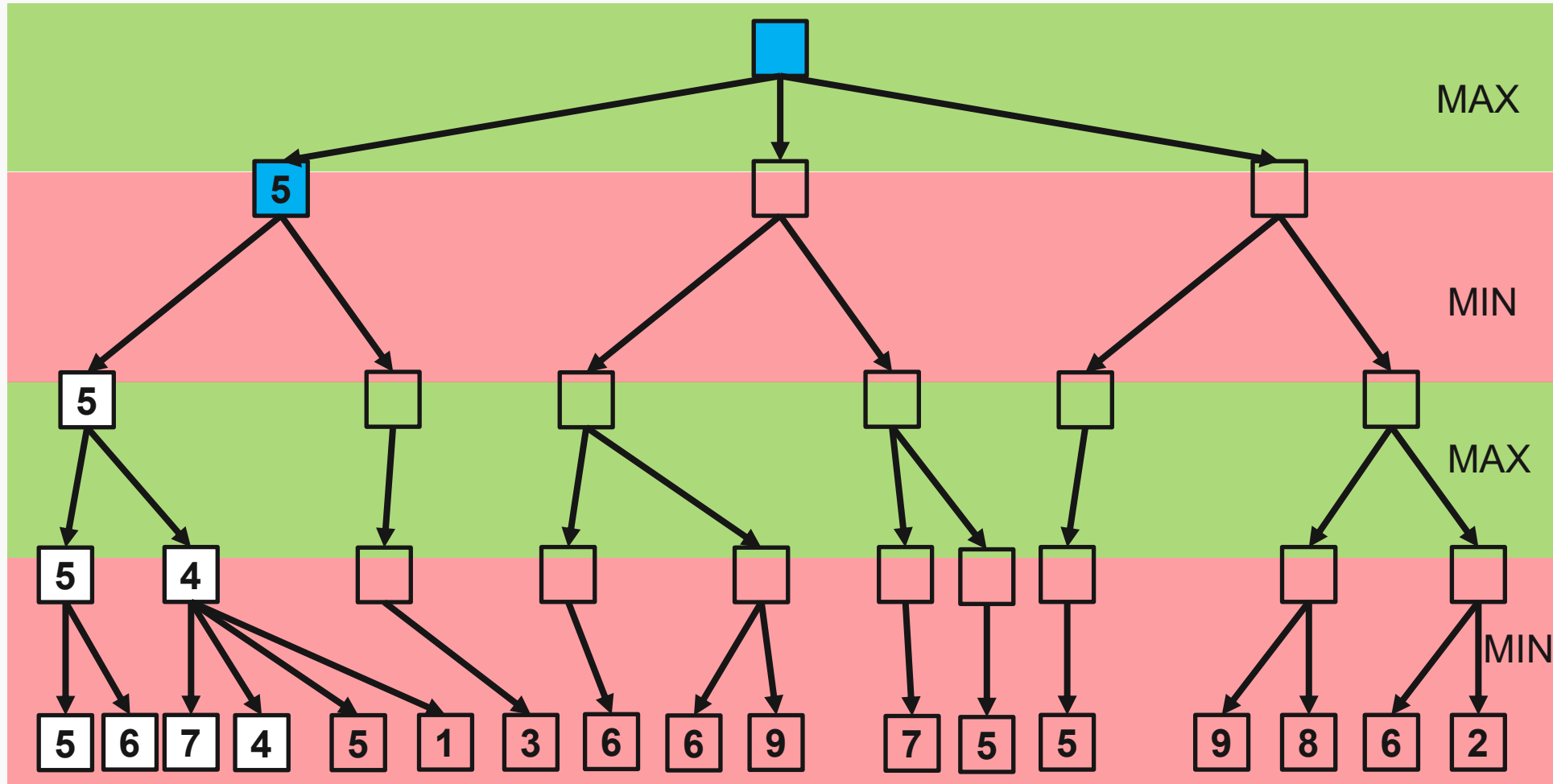


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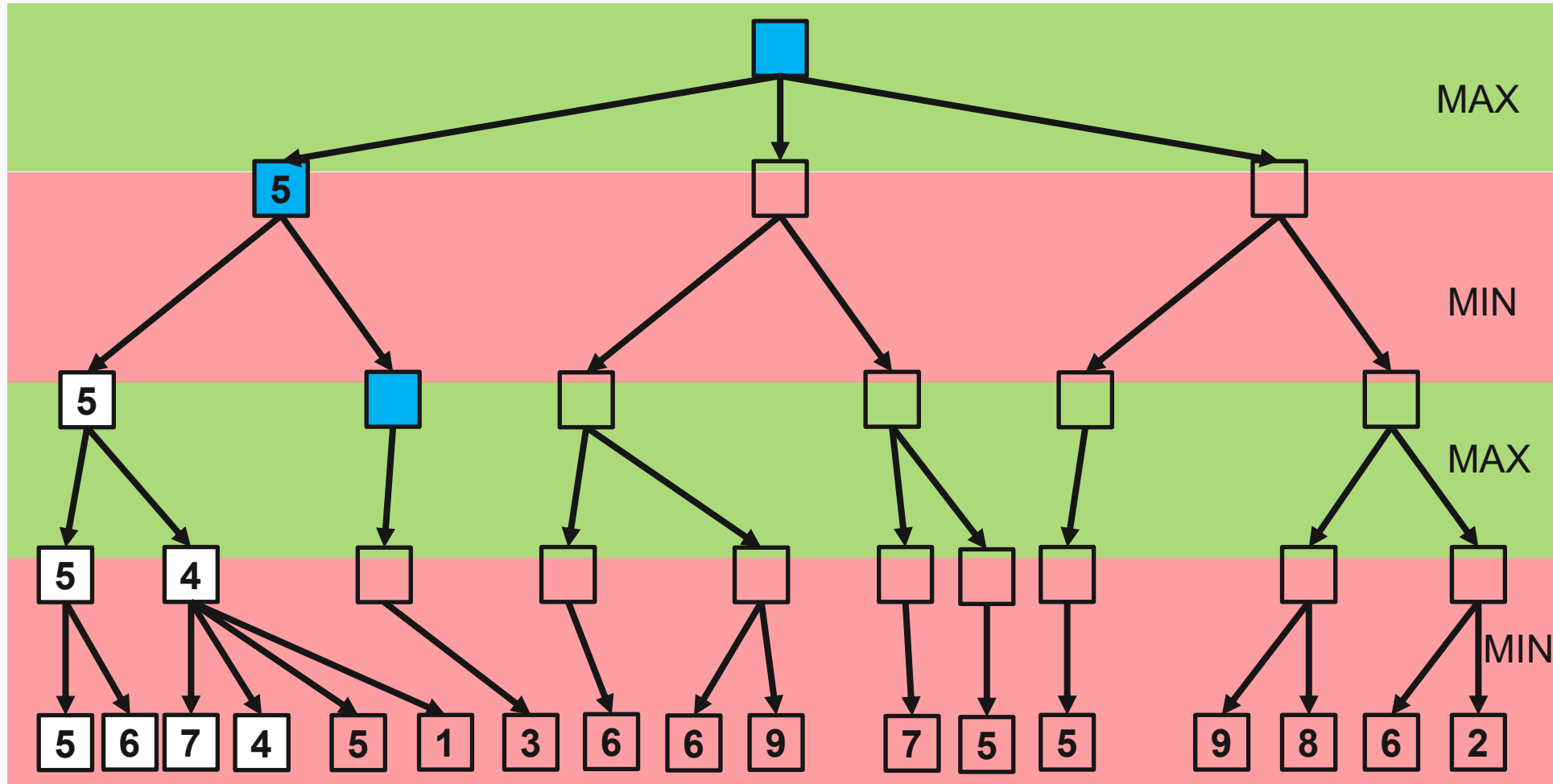


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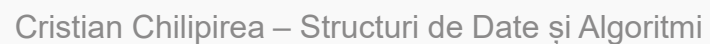
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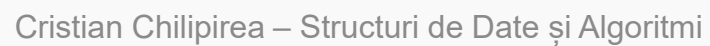








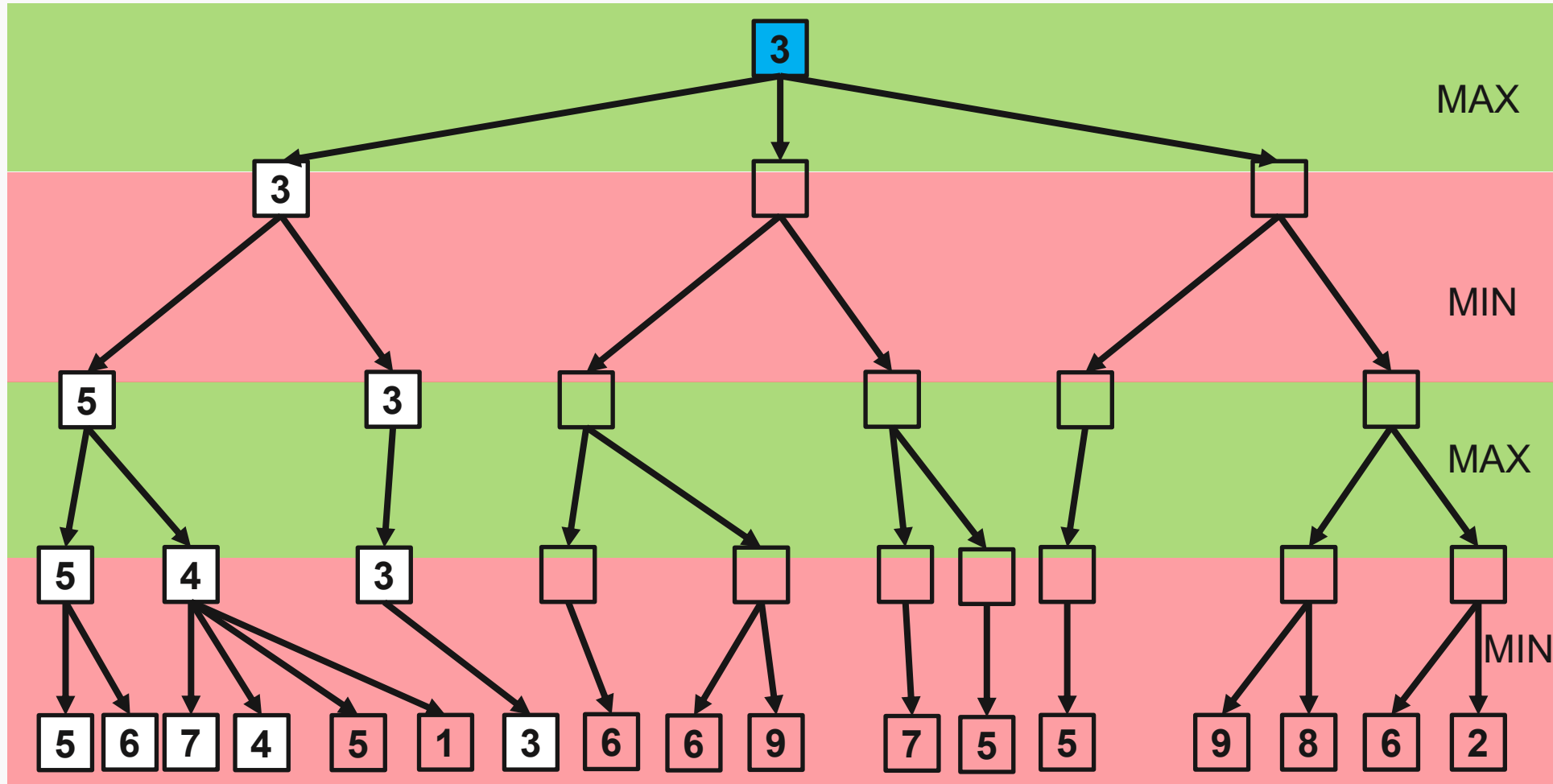






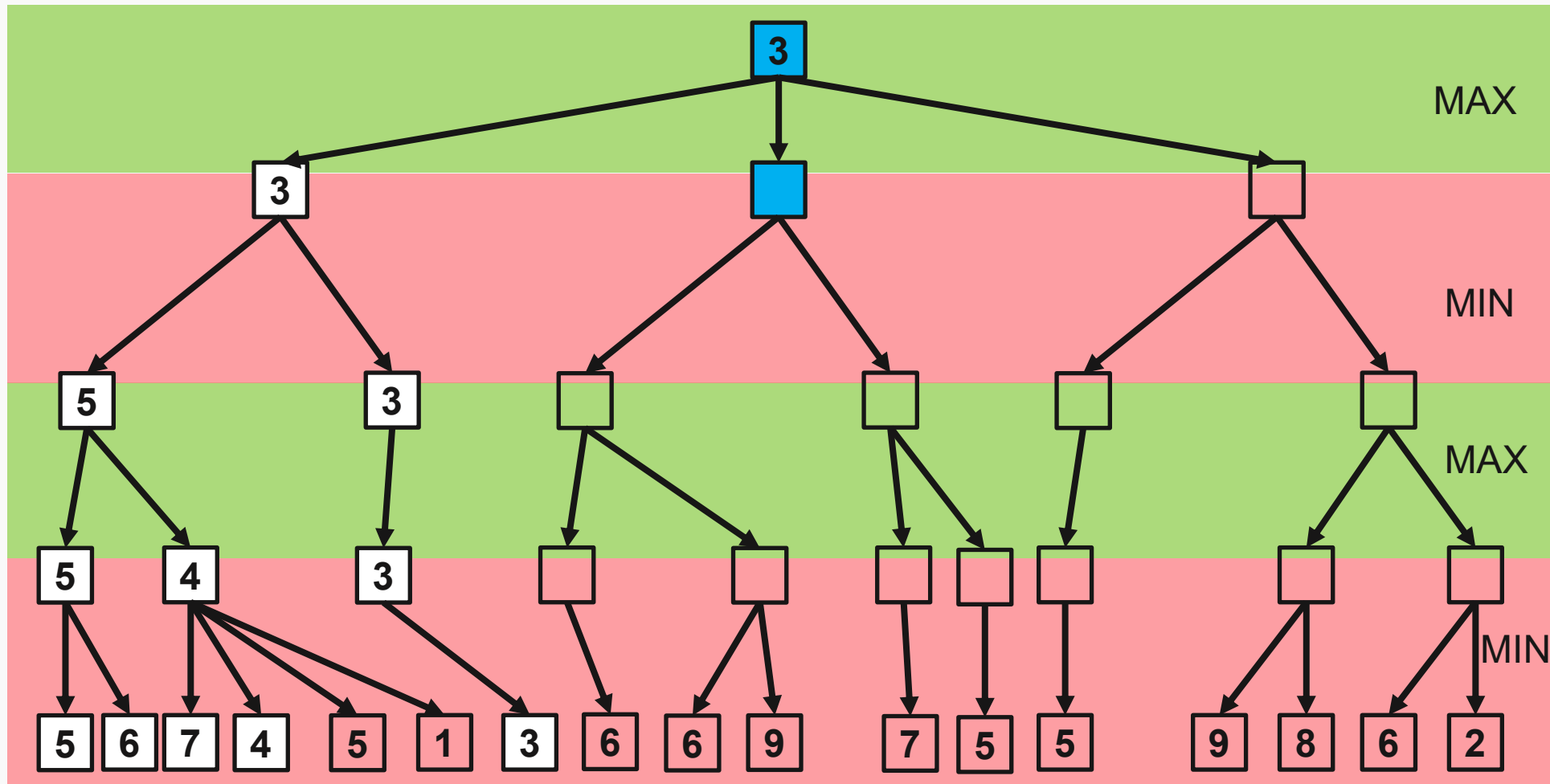


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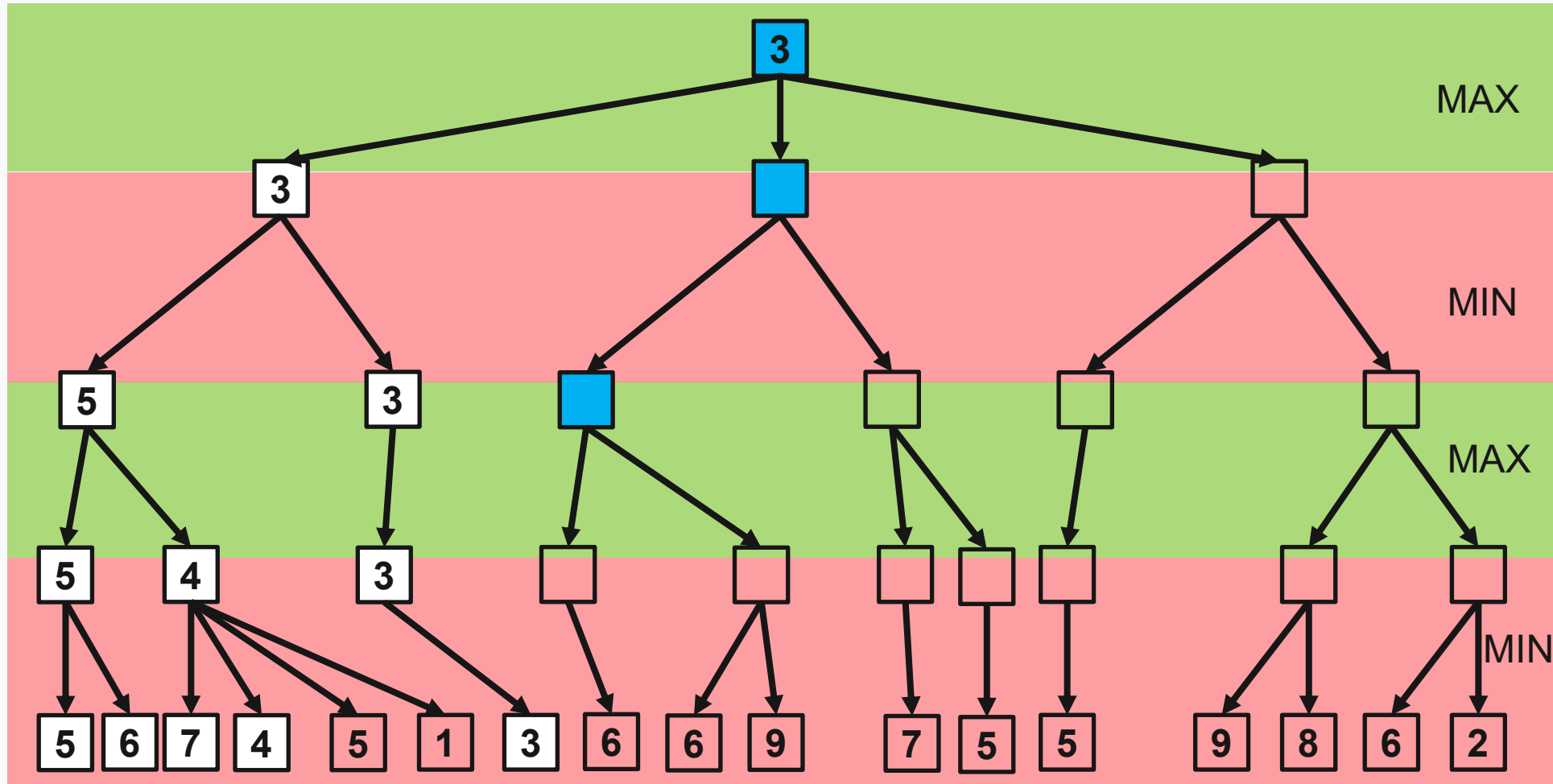


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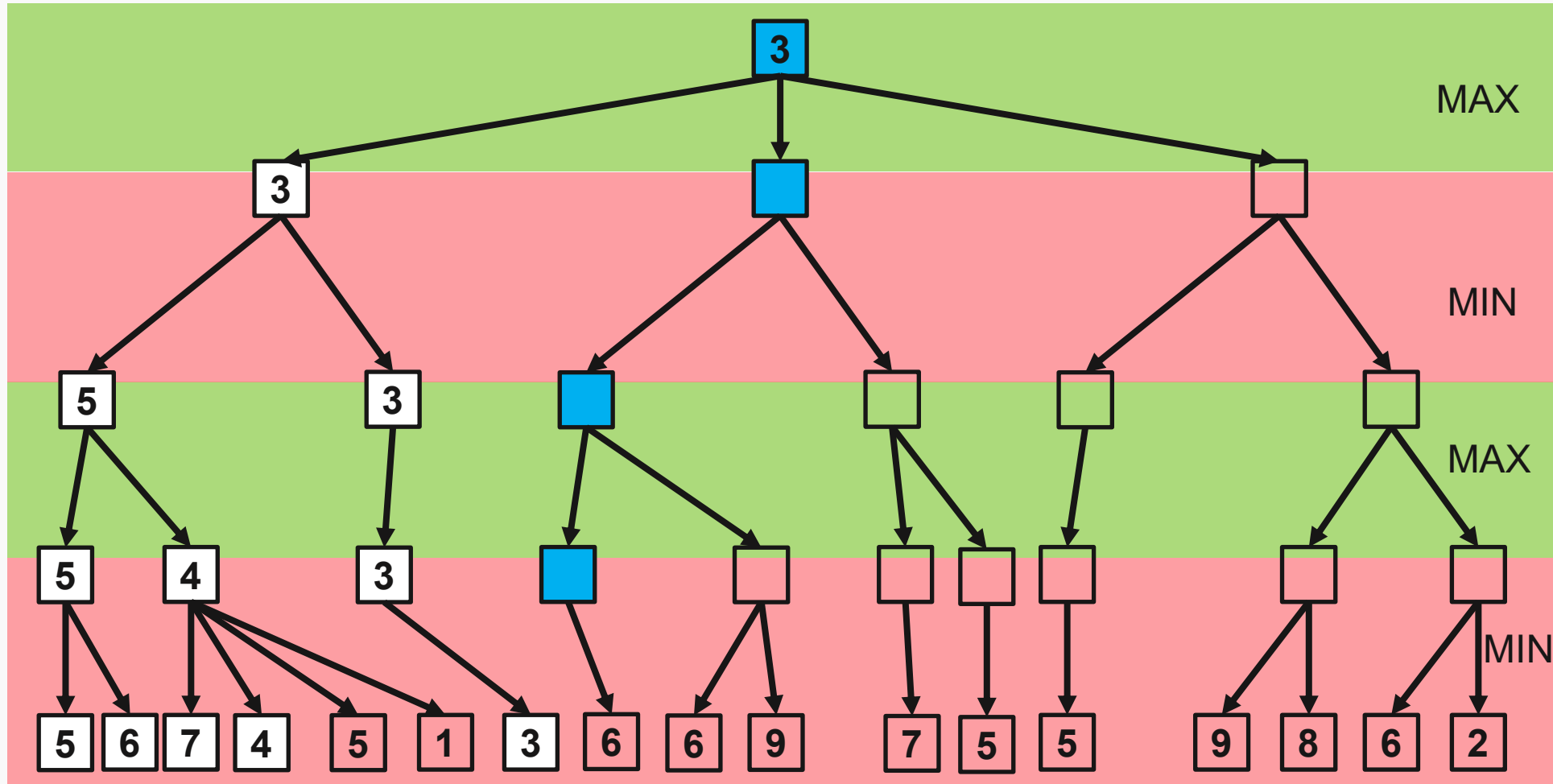


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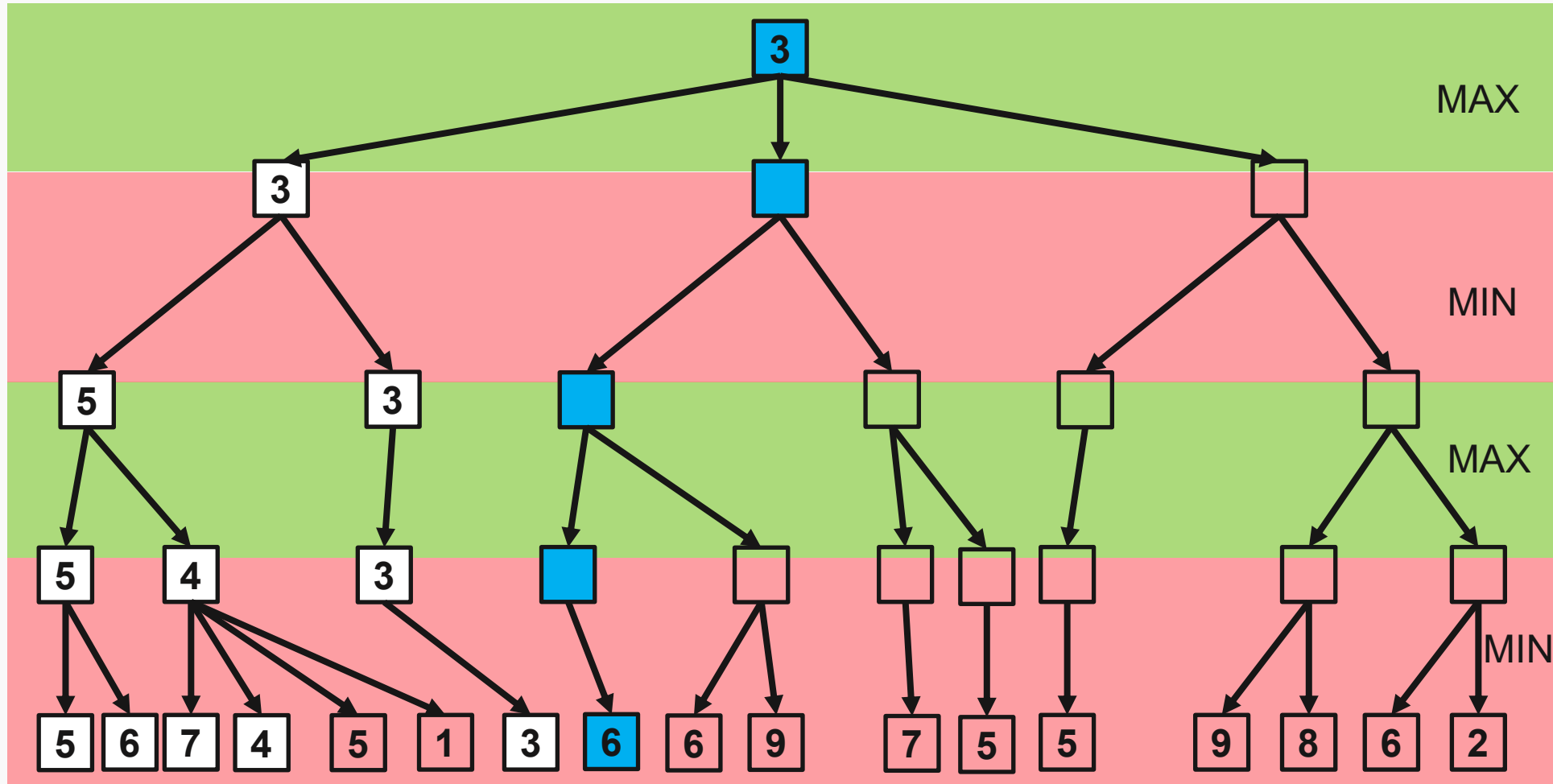
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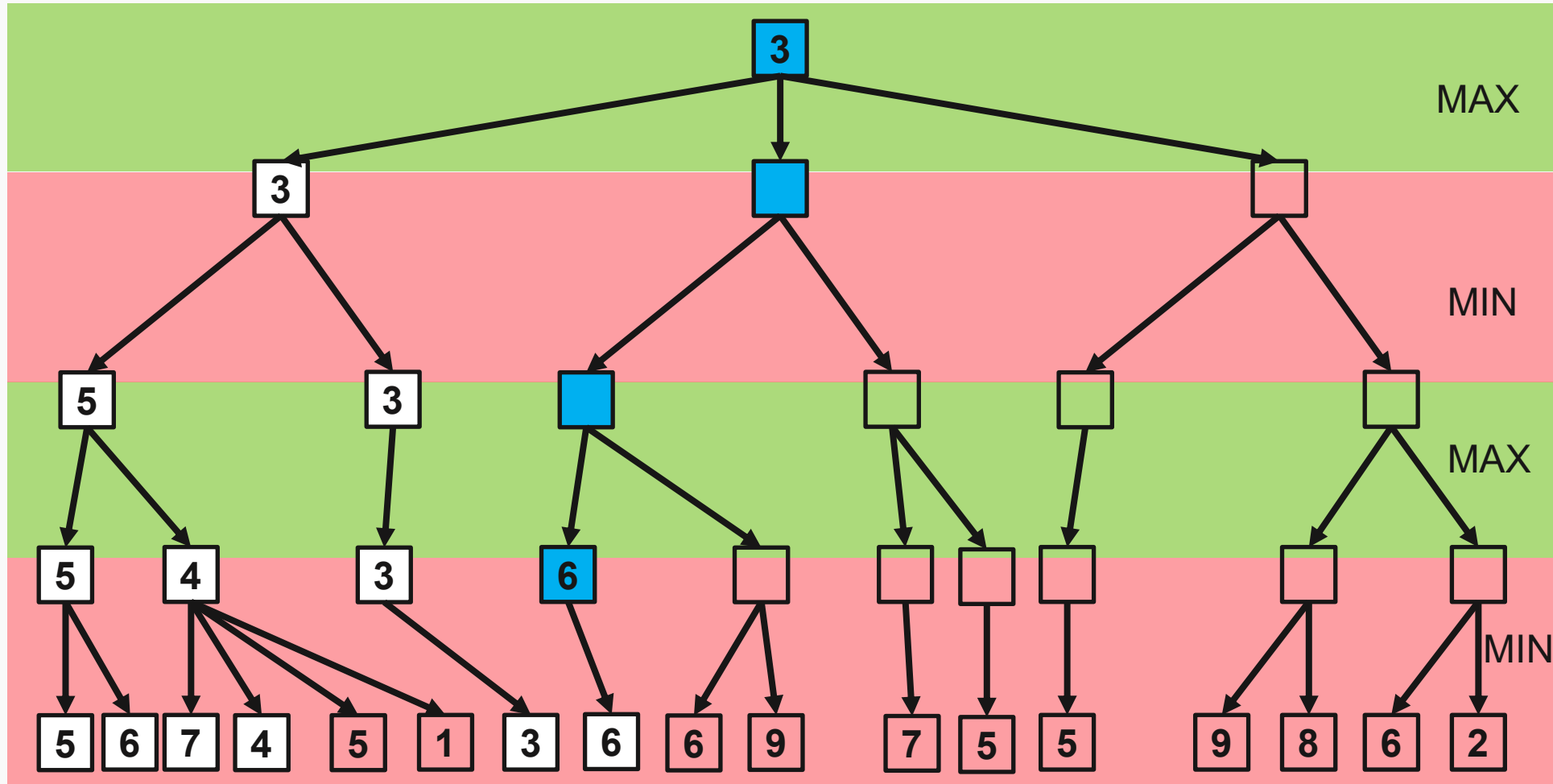


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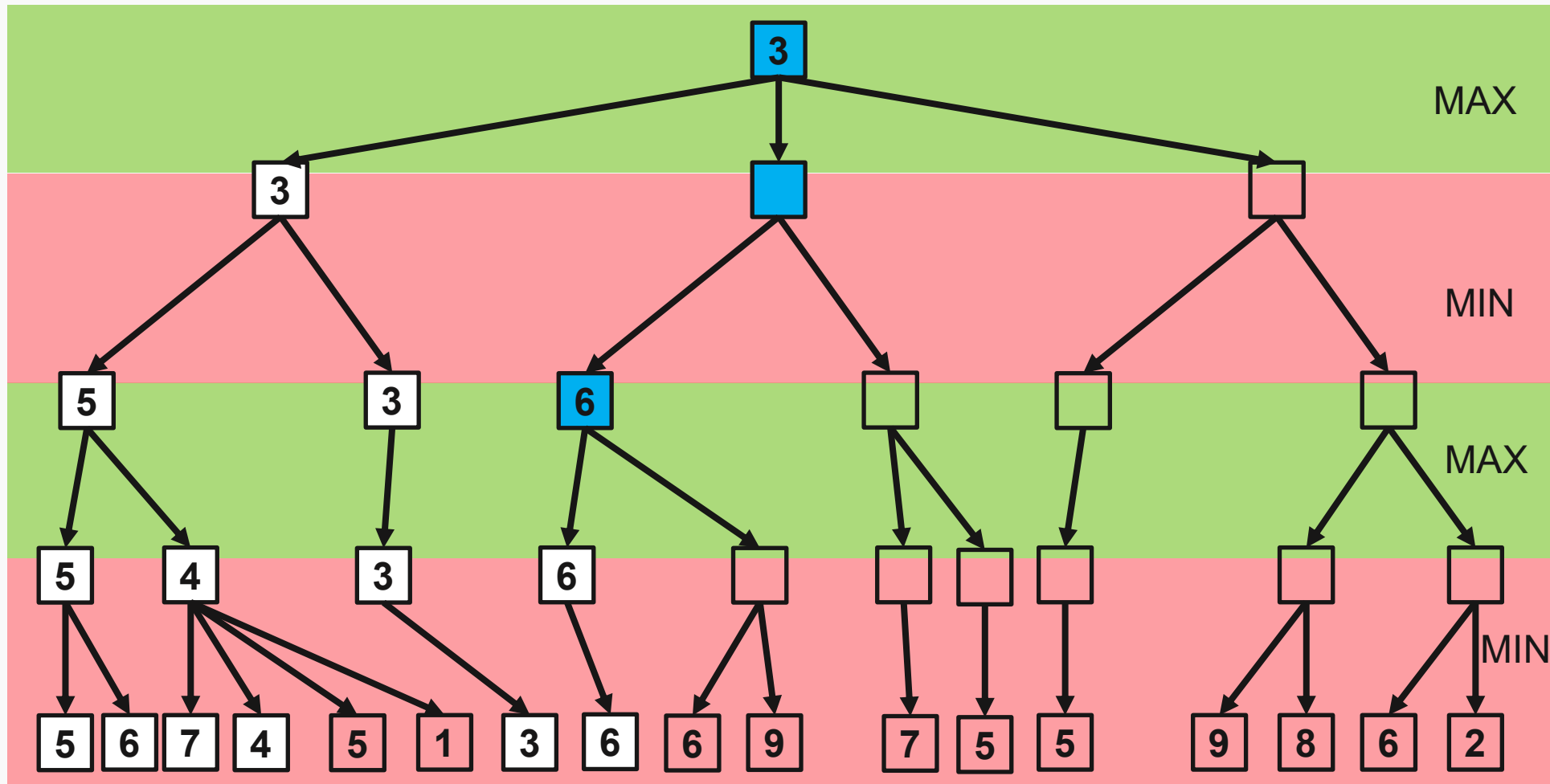


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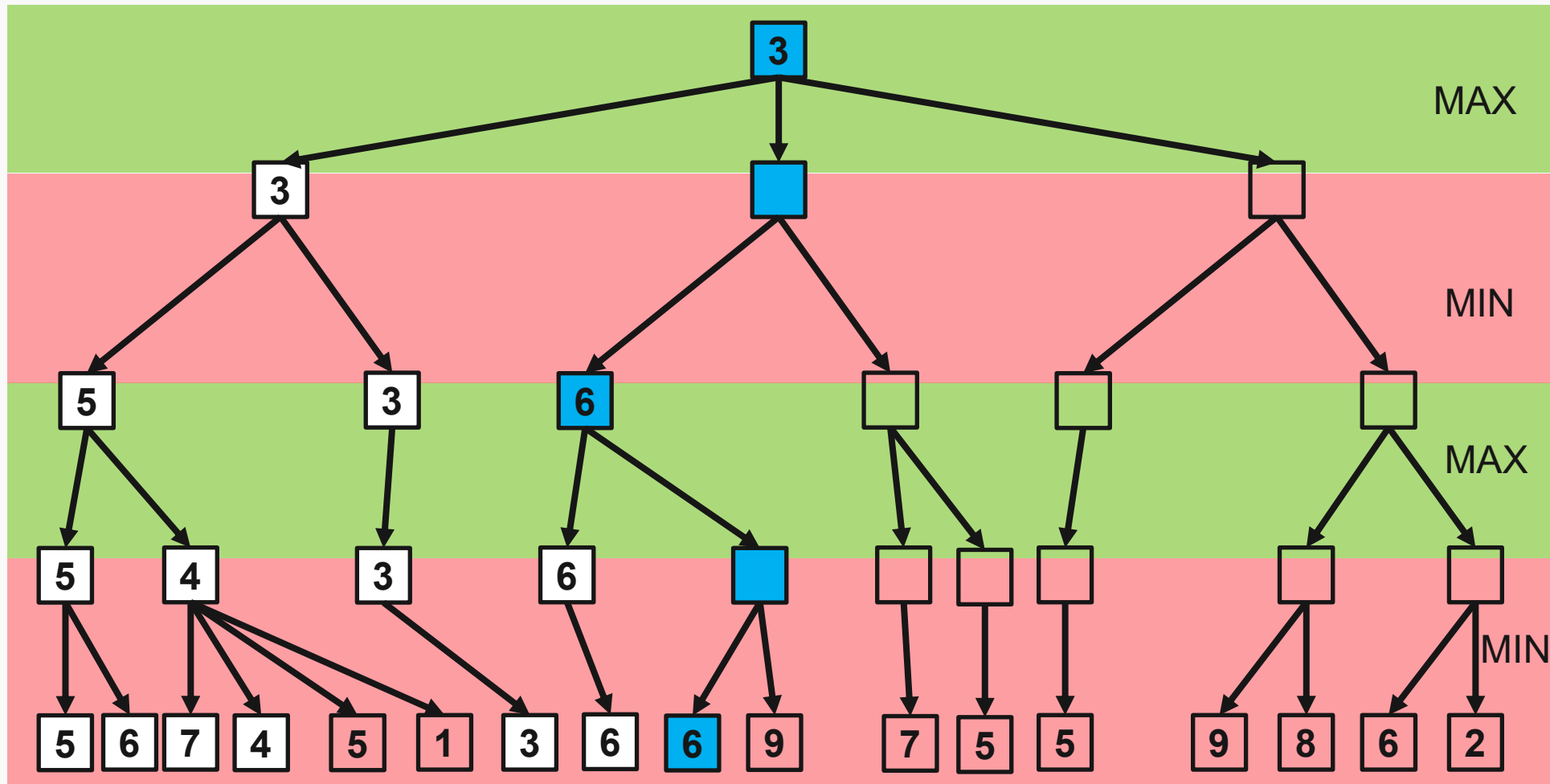
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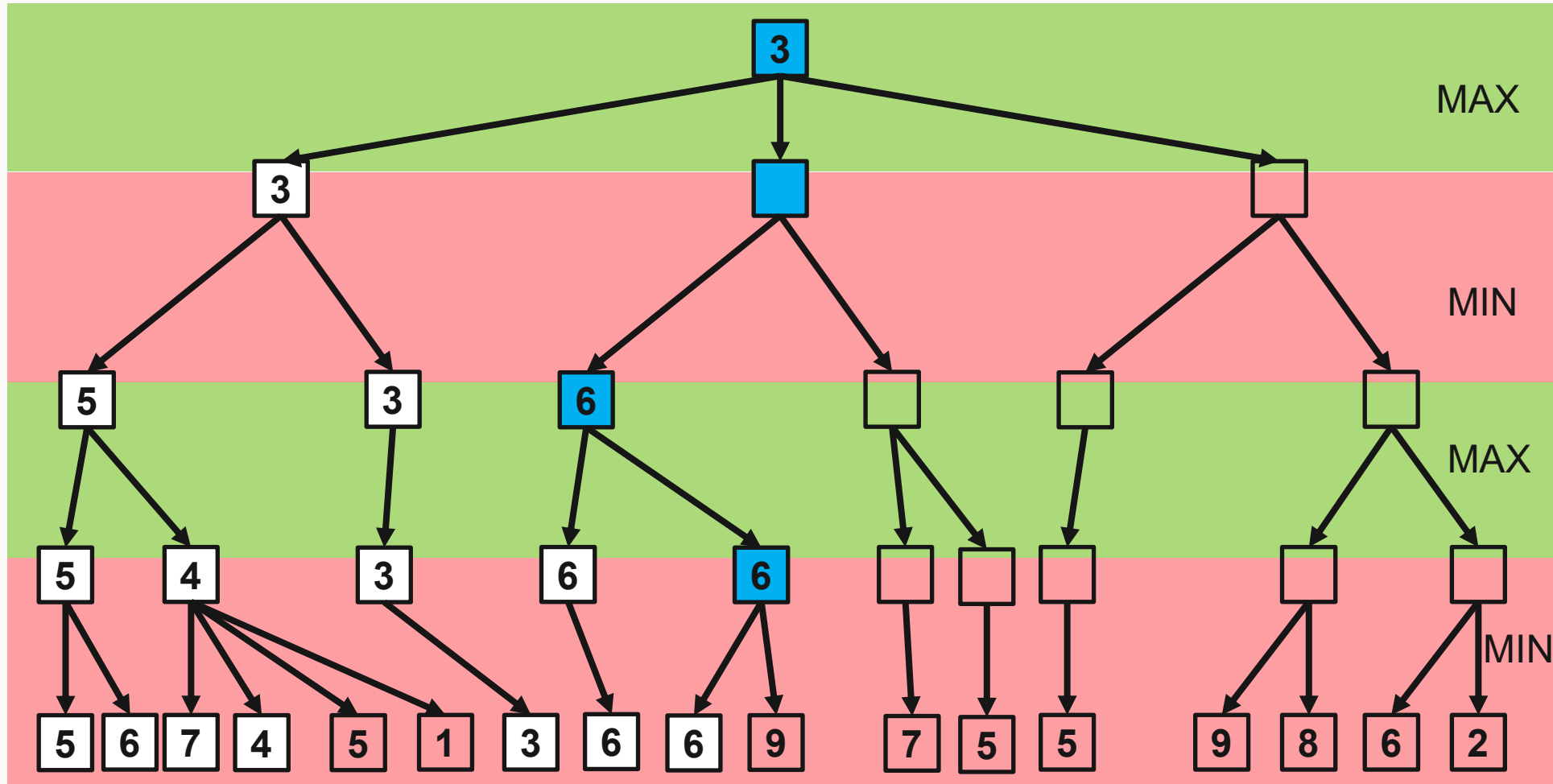


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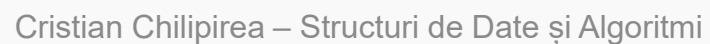


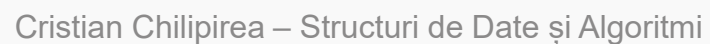


# AlphaBeta Prunning Ideea



Suntem deja siguri că la acest nivel competitorul va alege 6 sau mai mic.  
Dar dacă el alege 6 sau mai mic, noi știm siguri că vom alege 6.



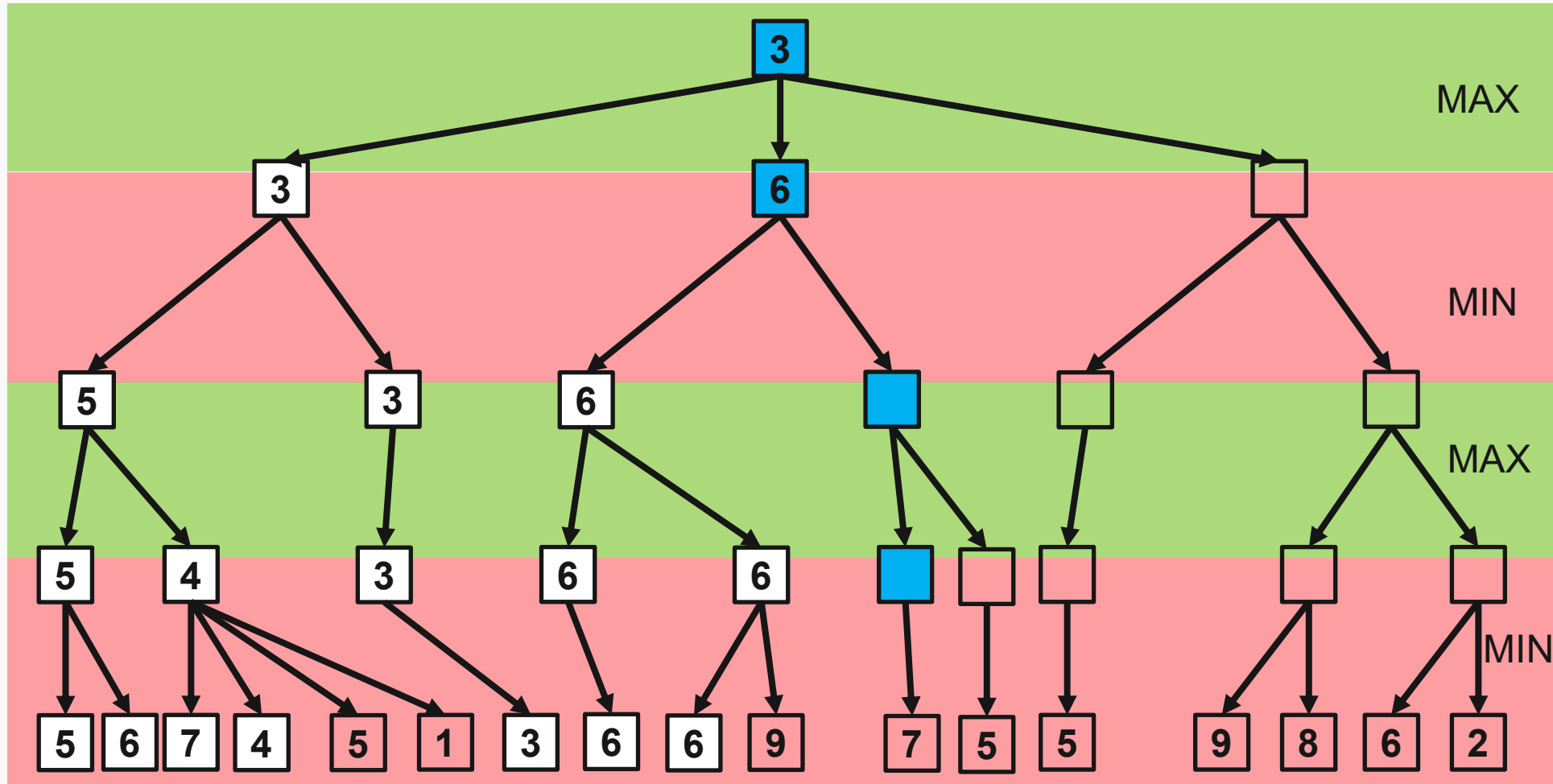


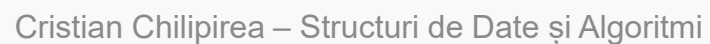






# AlphaBeta Prunning Idea



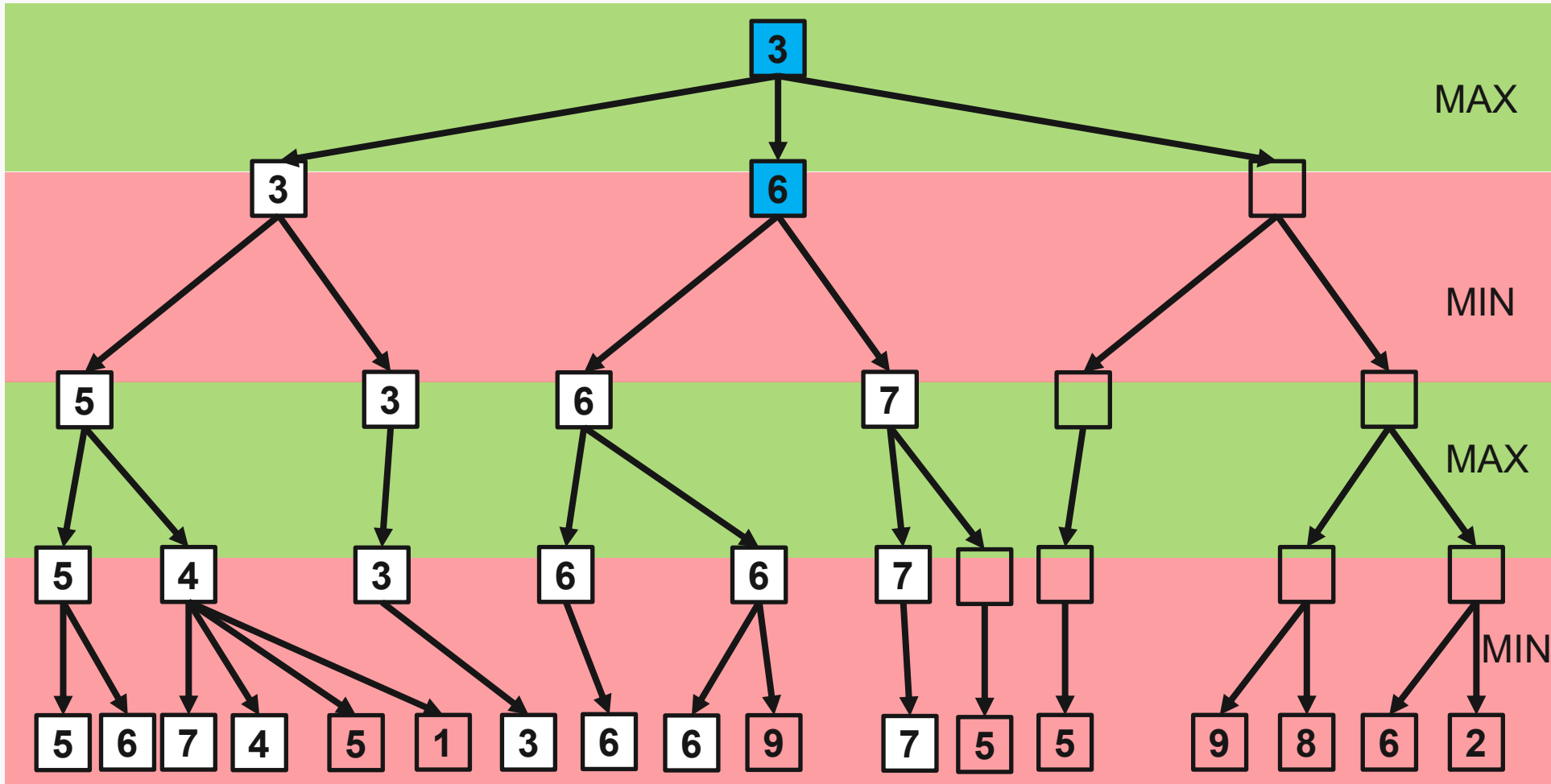


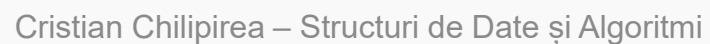






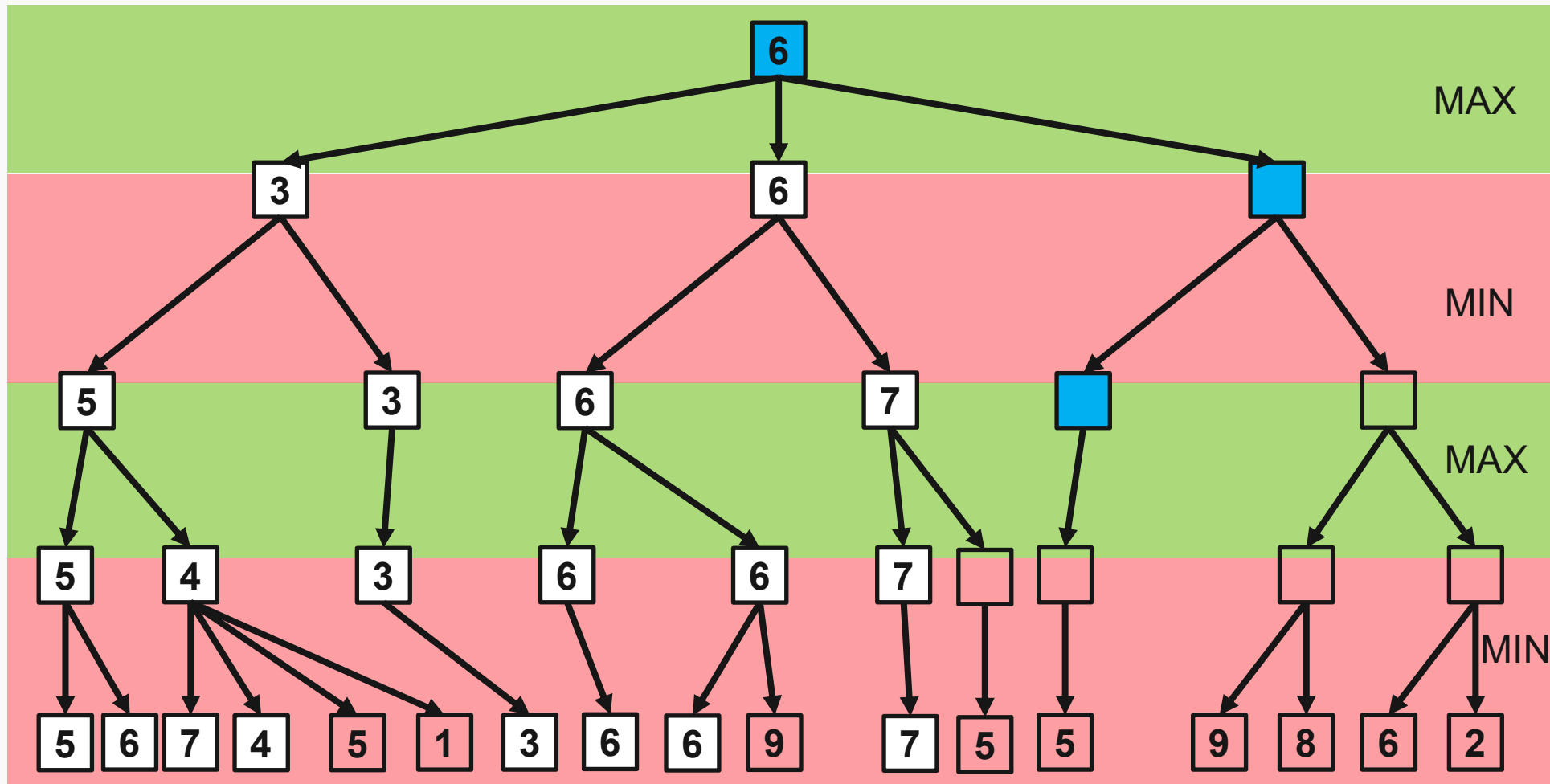
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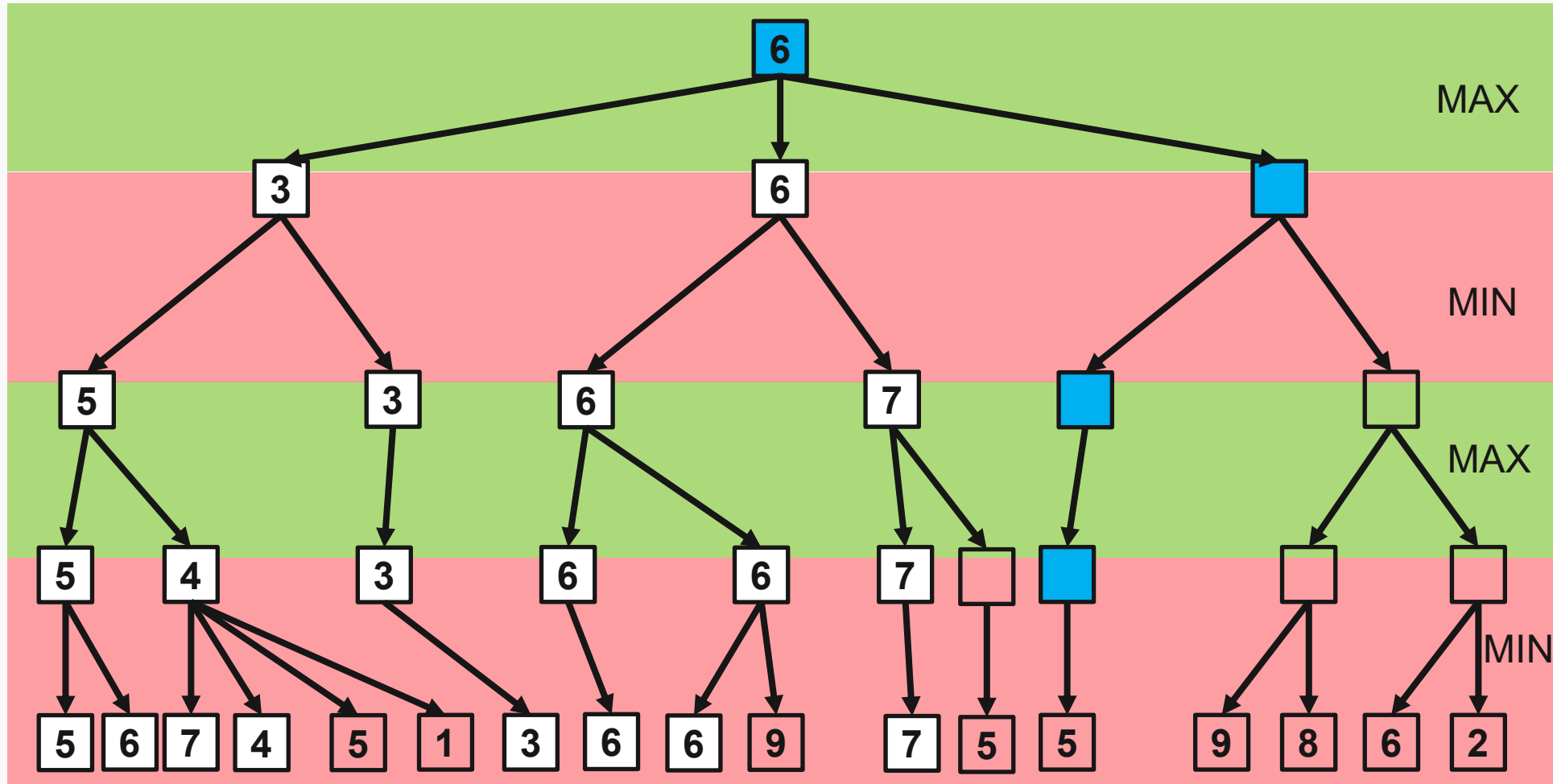
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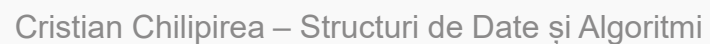






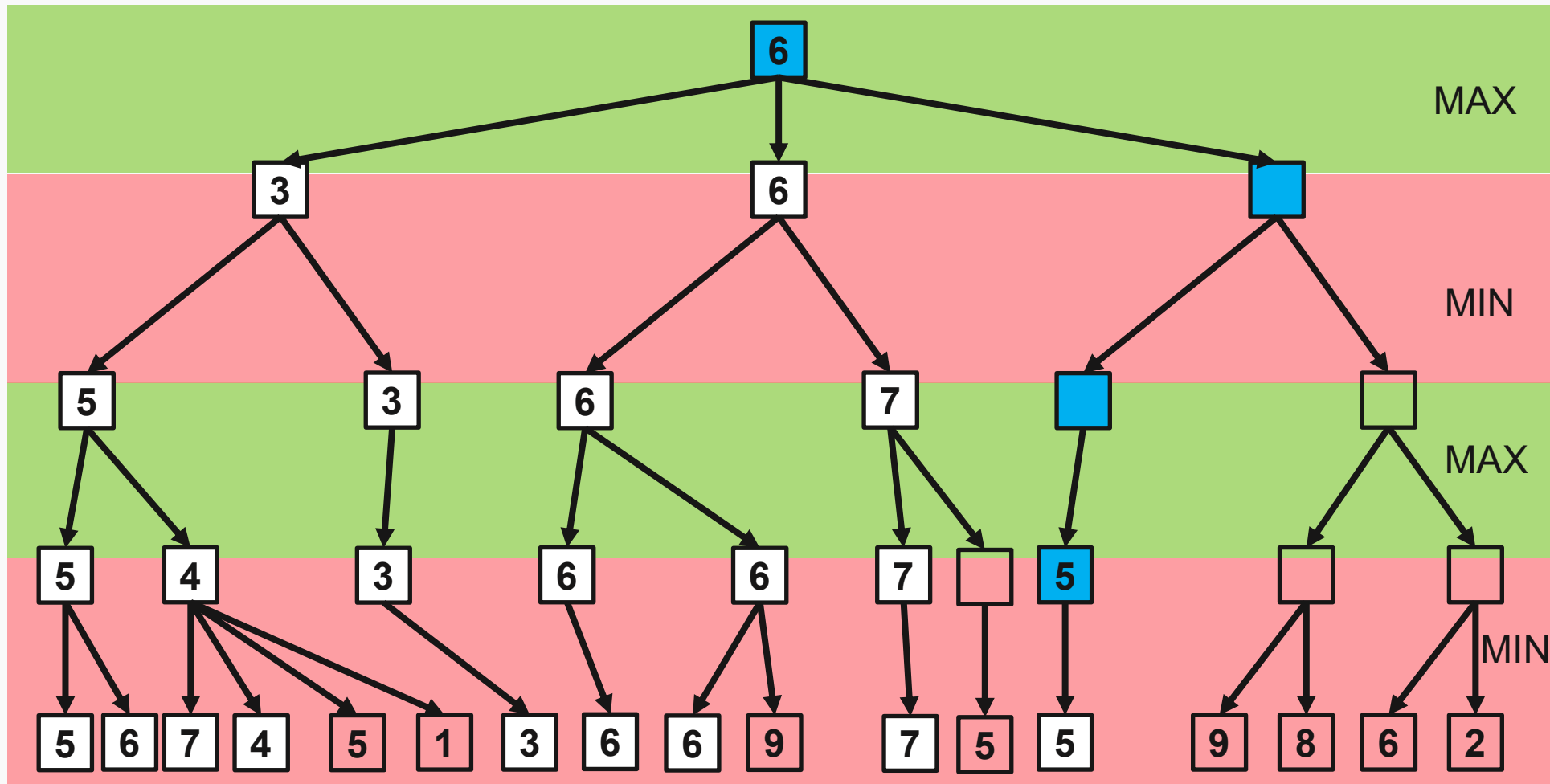
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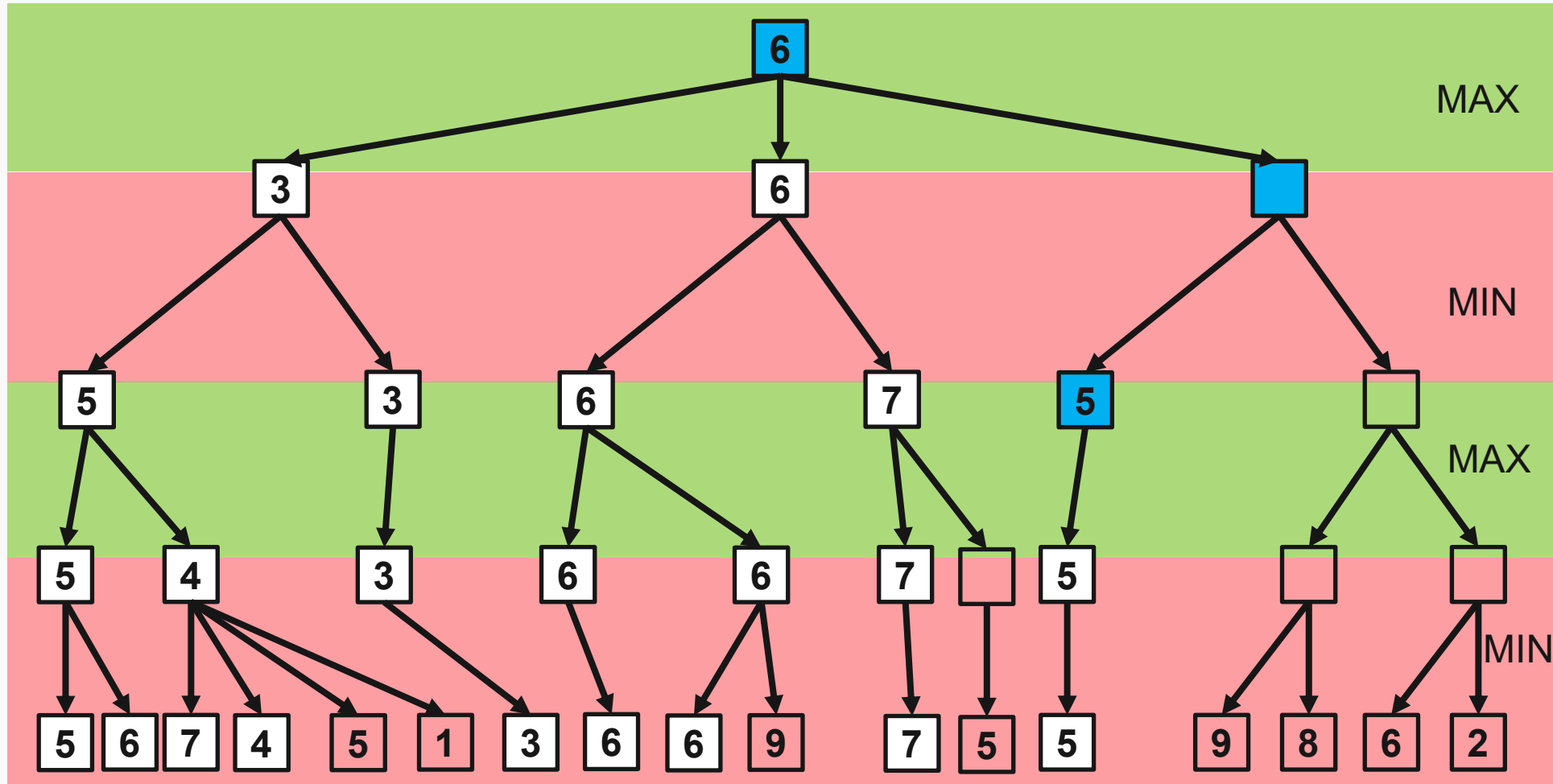


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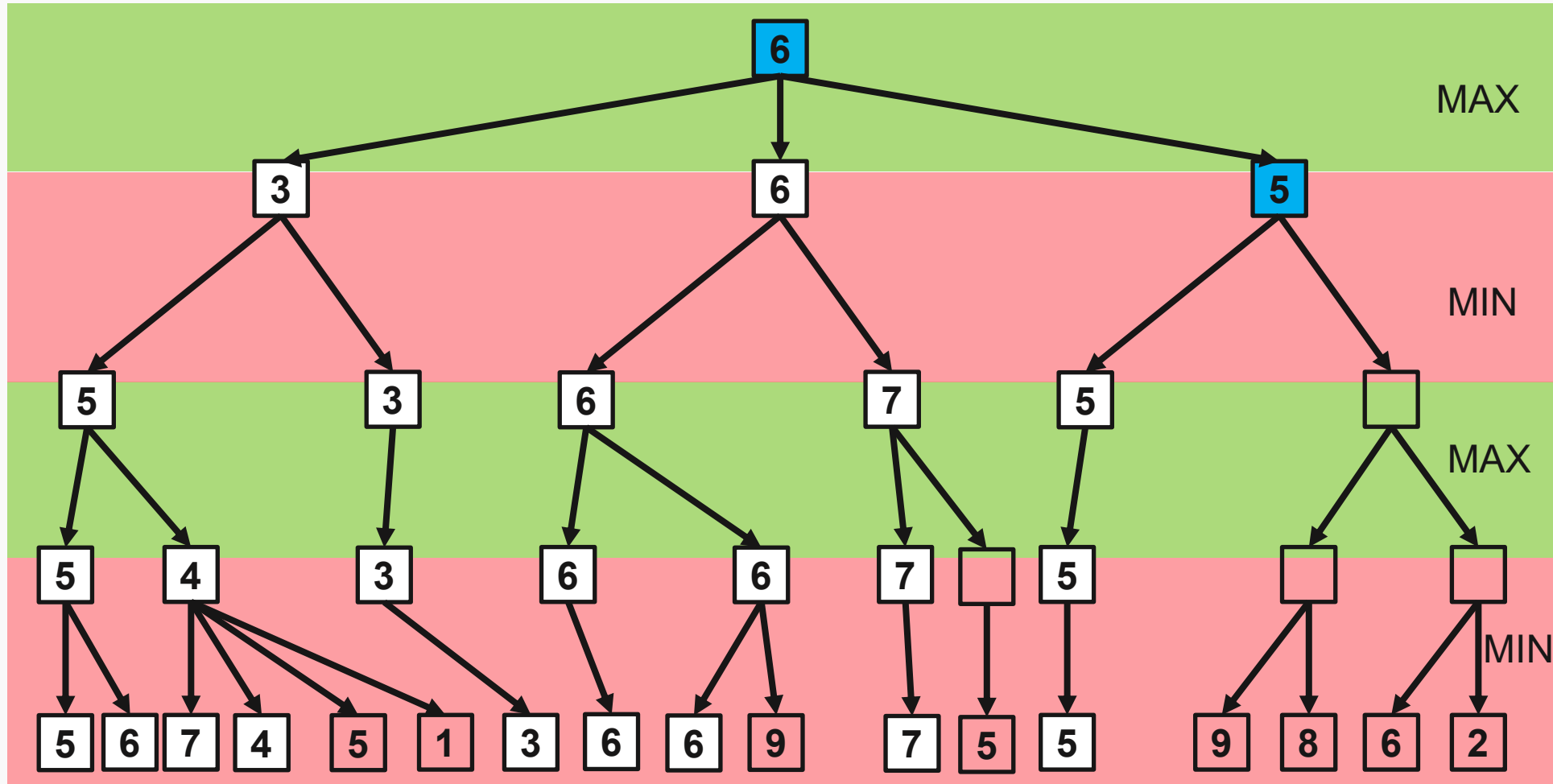


# AlphaBeta Pruning Idea





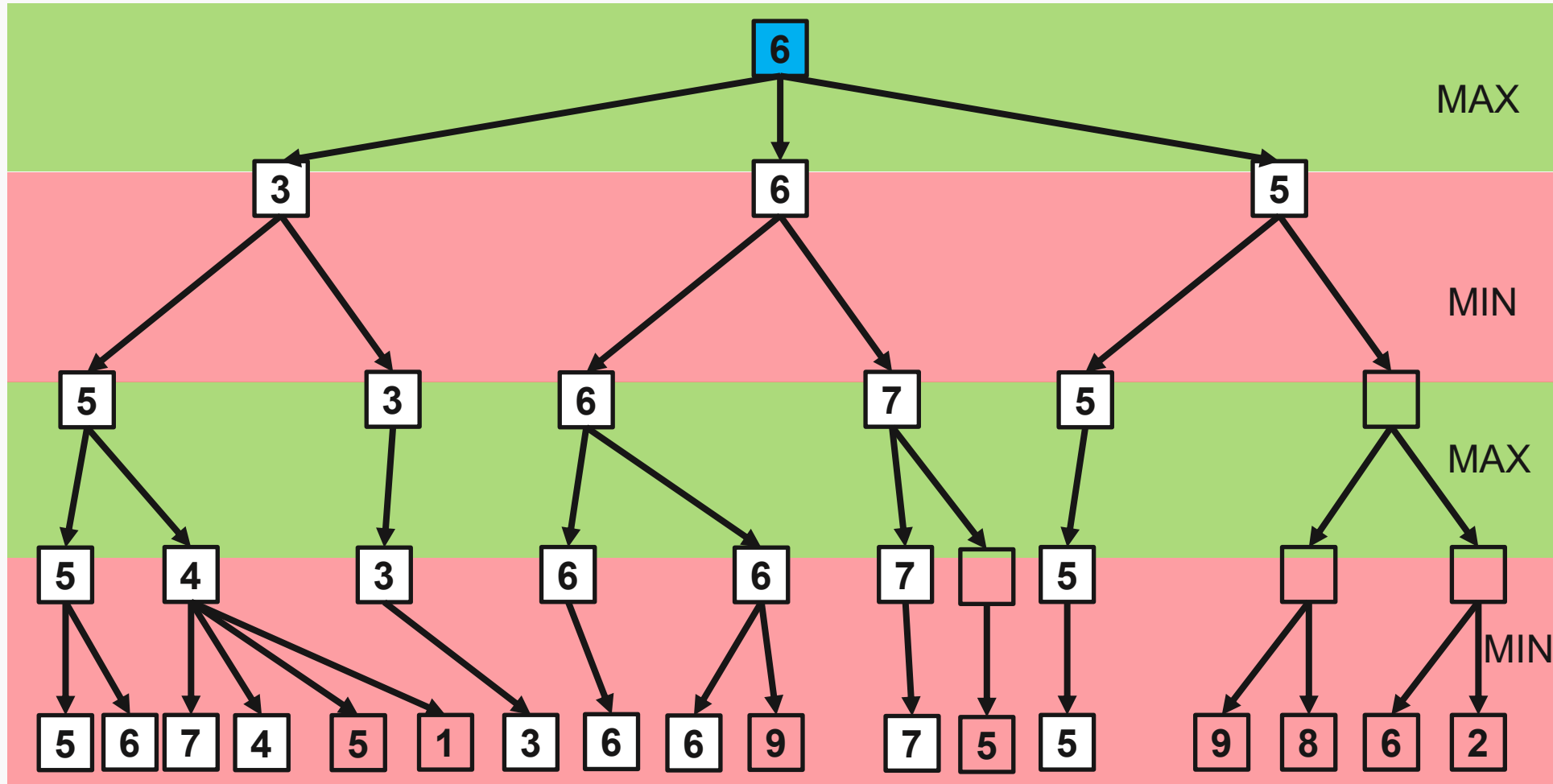
# AlphaBeta Pruning Ideea

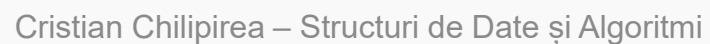


Suntem deja siguri că la acest nivel competitorul va alege 5 sau mai mic.  
Dar dacă el alege 5 sau mai mic, noi știm siguri că vom alege 6.



# AlphaBeta Pruning Idea







# MinMax + AlphaBeta Pruning

```
int alphaBeta(gameNode node, int level, int alpha, int beta) {  
    if (level == maxDepth || isEndGame(node))  
        return heuristic(node);  
    if (level % 2 == maximizingPlayer) {  
        int value = -∞;  
        for each (child of node) {  
            value = max(value, alphaBeta(child, level + 1, alpha, beta));  
            alpha = max(alpha, value);  
            if (alpha >= beta)  
                break;  
        }  
        return value;  
    } else {  
        int value = +∞;  
        for each (child of node) {  
            value = min(value, alphaBeta(child, level + 1, alpha, beta));  
            beta = min(beta, value);  
            if (alpha >= beta)  
                break;  
        }  
        return value;  
    }  
}
```





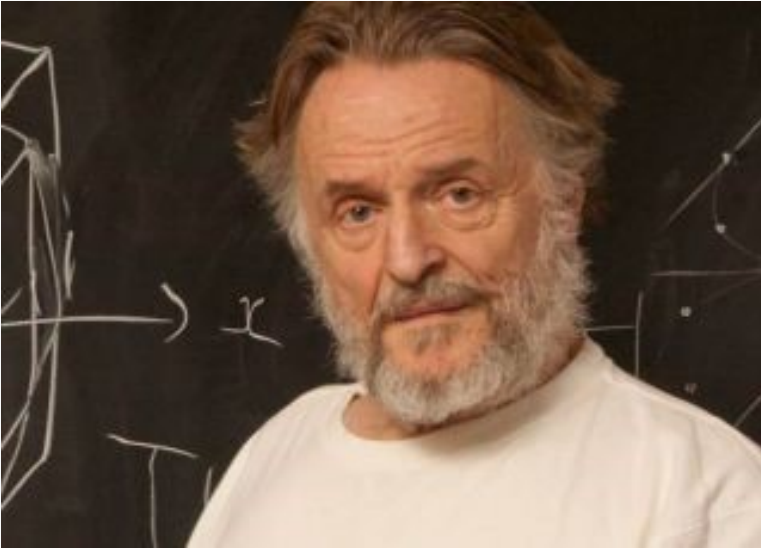
# Apelare AlphaBeta

```
for each (child of node)
    score[child] = alphaBeta(child, 0,  $-\infty$ ,  $+\infty$ );
```

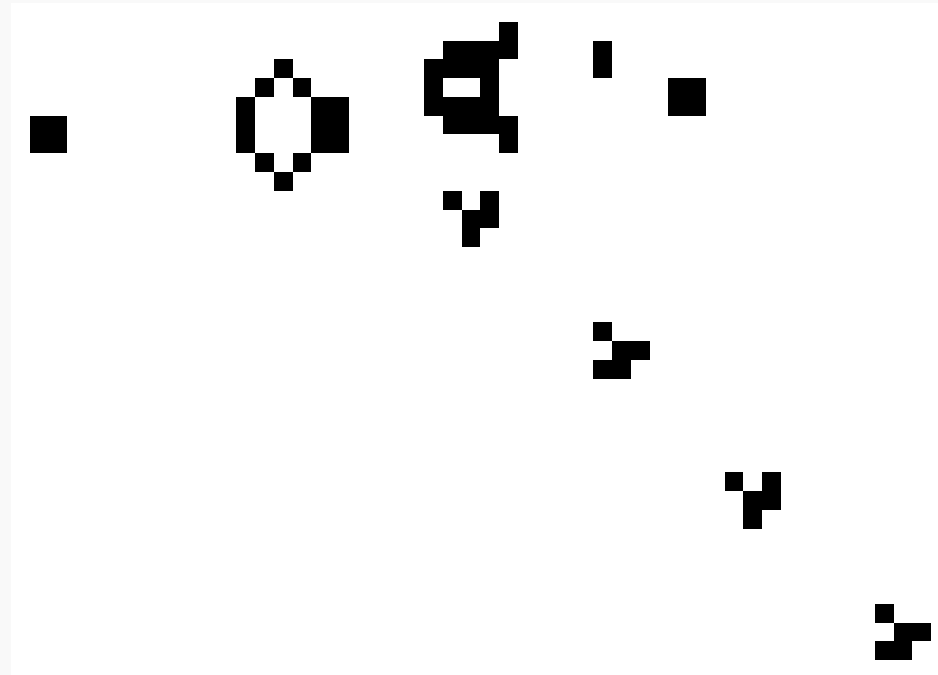
Mutarea este acel child pentru care score e maxim



# John Conway – game of life

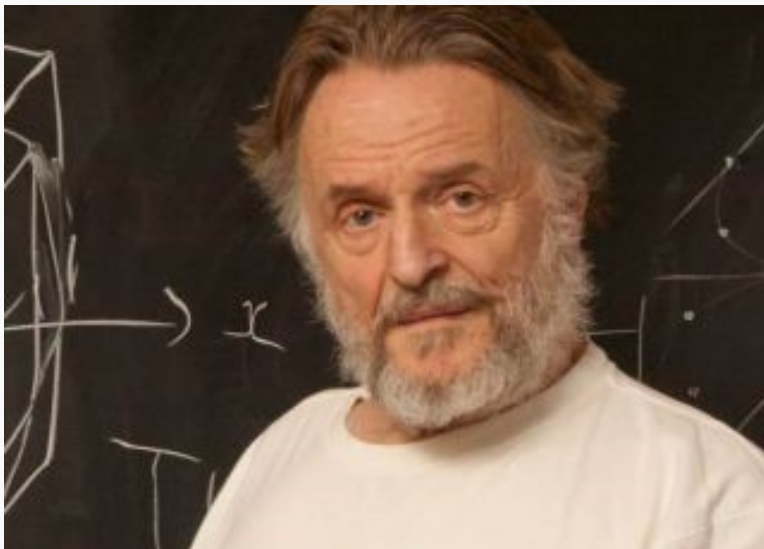


26-Decembrie-1937 -> 11-Aprilie-2020





# John Conway – game of life



Celulele unei matrici pot avea valori **0** sau **1**

Orice celulă **1** fără vecini devine **0**

Orice celulă **1** cu 1 vecin devine **0**

Orice celulă **1** cu 2 vecini rămâne **1**

Orice celulă **1** cu 3 vecini rămâne **1**

Orice celulă **1** cu 4+ vecini devine **0**

Orice celulă **0** cu 3 vecini devine **1**

26-Decembrie-1937 -> 11-Aprilie-2020

Cause of death: Coronavirus

<https://bitstorm.org/gameoflife/>

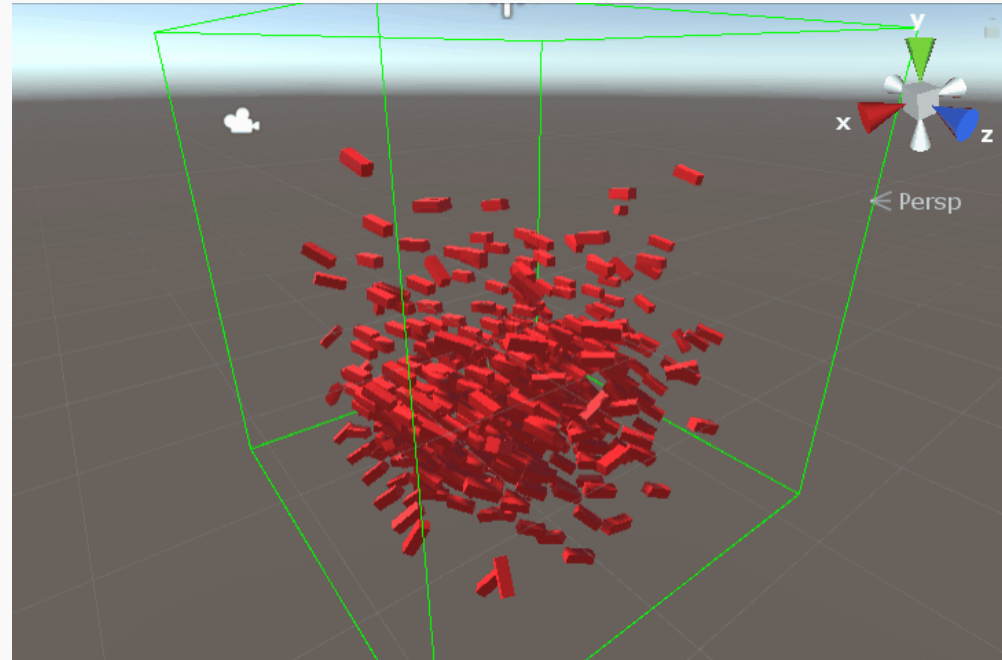
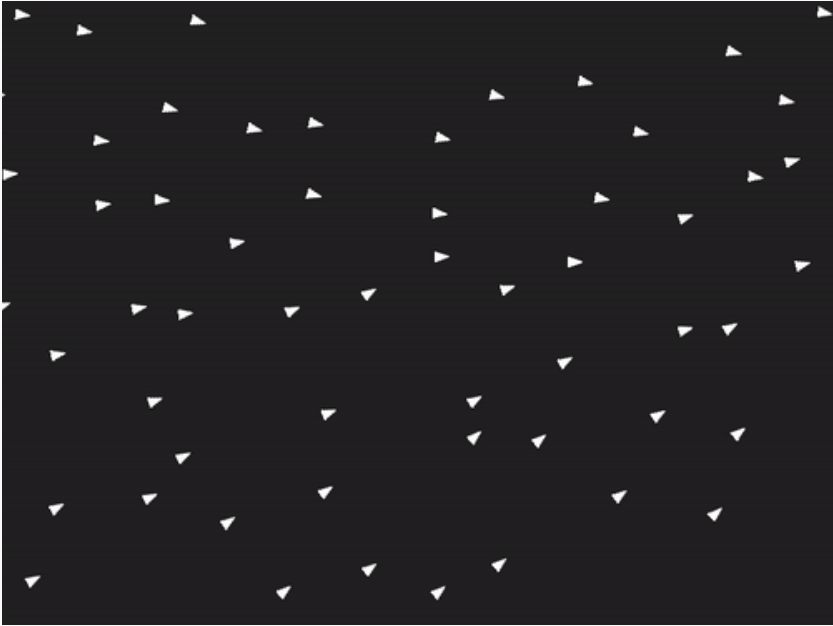


## Pentru curioși – John Conway – surreal numbers

- [https://en.wikipedia.org/wiki/Surreal\\_number](https://en.wikipedia.org/wiki/Surreal_number)
- <https://www.youtube.com/watch?v=E8kUJL04ELA>
- <https://www.youtube.com/watch?v=1eAmxgINXrE>
- <https://www.youtube.com/watch?v=R9Plq-D1gEk>



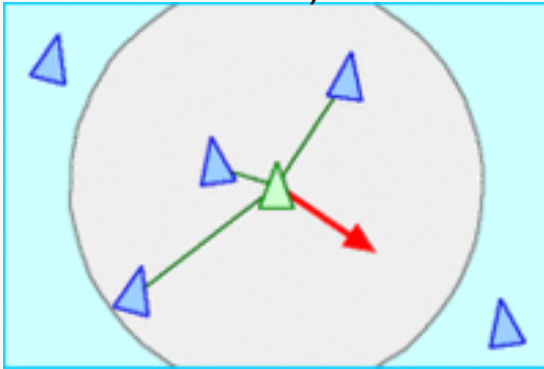
# Boids - Craig Reynolds



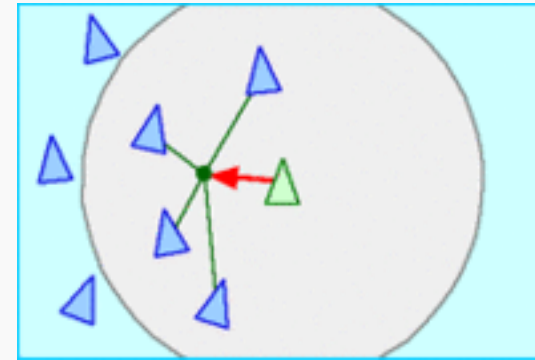


# Boids – Reguli simple comportament complex

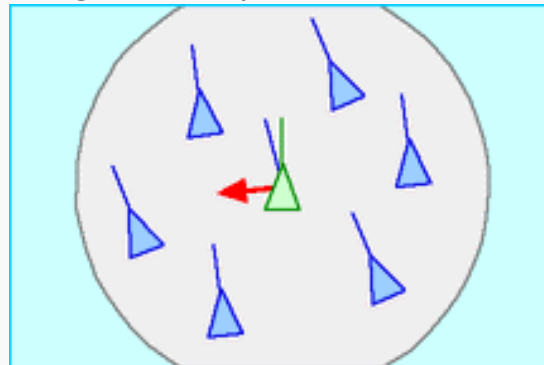
Separare (incearca sa evite coliziuni)



Coeziune (sta aproape de centrul grupului)



Aliniere (zboara in aceeaasi directie cu directia generala a grupului)





# Further discussion

- Nash equilibrium + Pareto Optimal
  - ▣ Prisoner's Dilema
  
- Monte Carlo + Las Vegas (random algorithms)