Structuri de date și algoritmi
Grafuri – Acoperire și Capacitate

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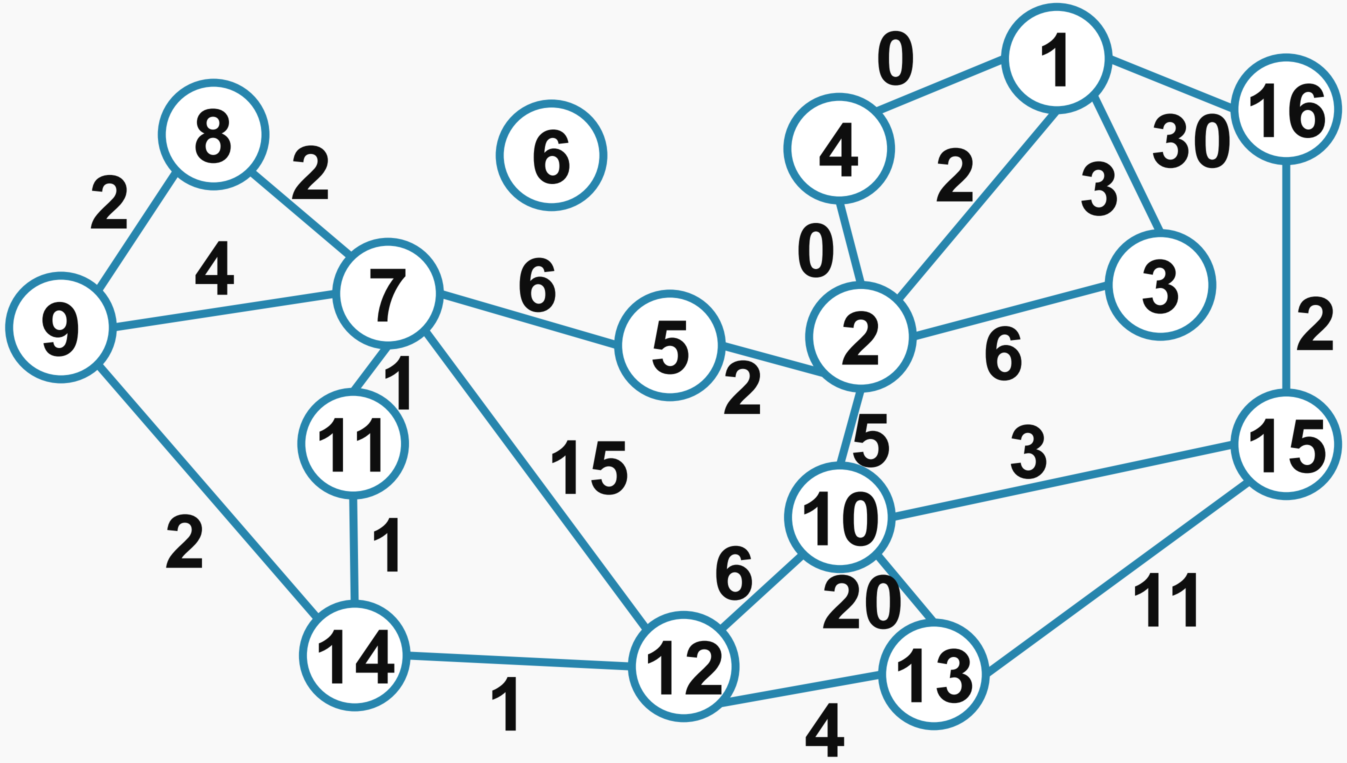
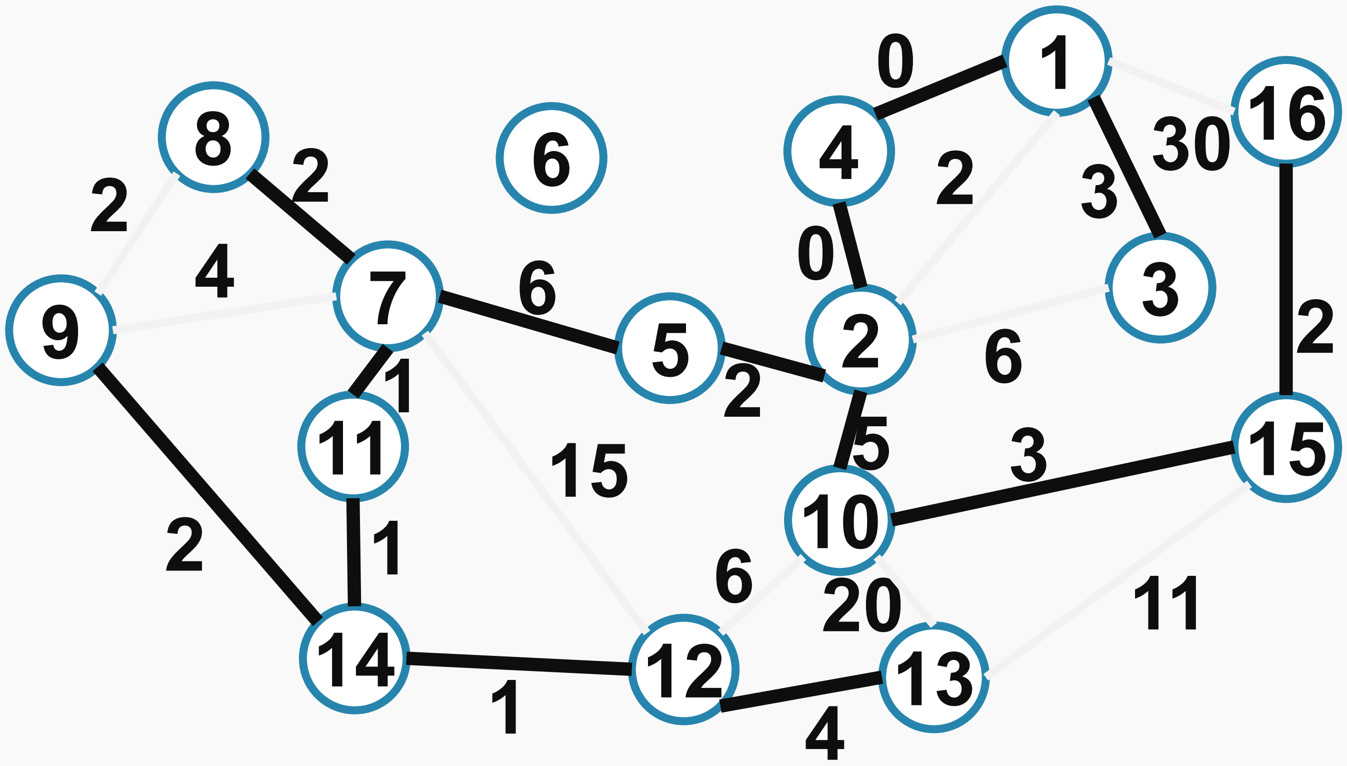
Arbori minimi de acoperire

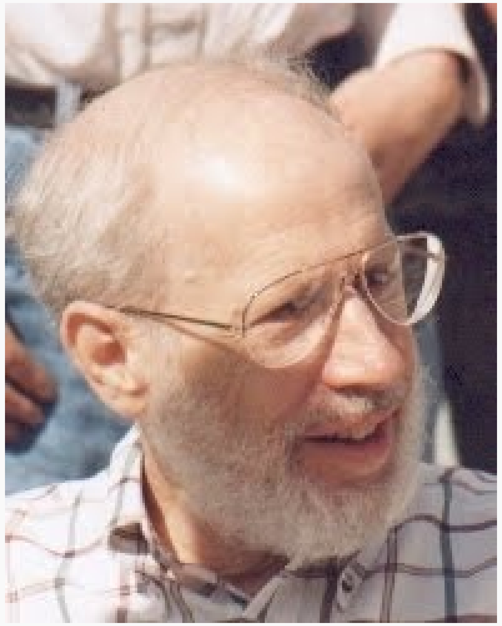


Arbori minimi de acoperire

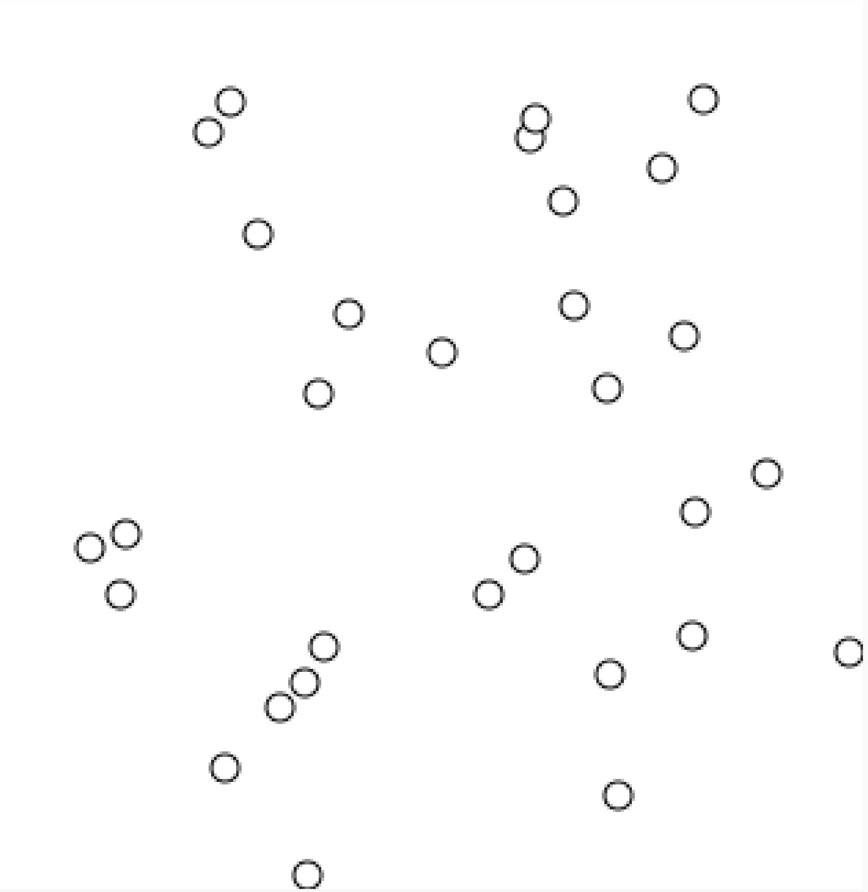


Algoritmul lui Kruskal (1956)

```
tree Kruskal(G) {
    sort(G.E); // sort by weight
    A = {};
    for each (node in G.V)
        Make_set(node);
    for each ((u, v) in G.E) {
        if (Find_set(u) != Find_set(v)) {
            A = A ∪ {(u, v)};
            Union(Find_set(u), Find_set(v));
        }
    }
    return A;
}
```

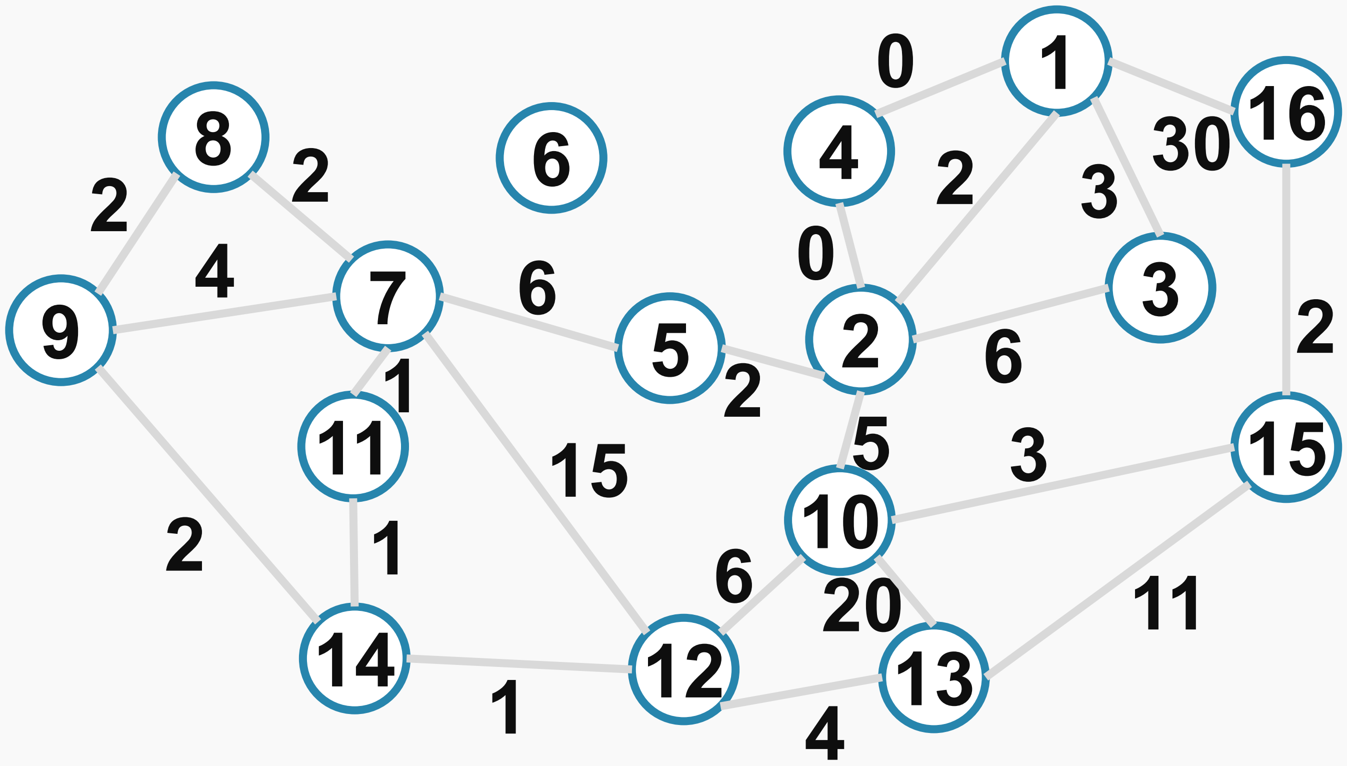


Algoritmul lui Kruskal

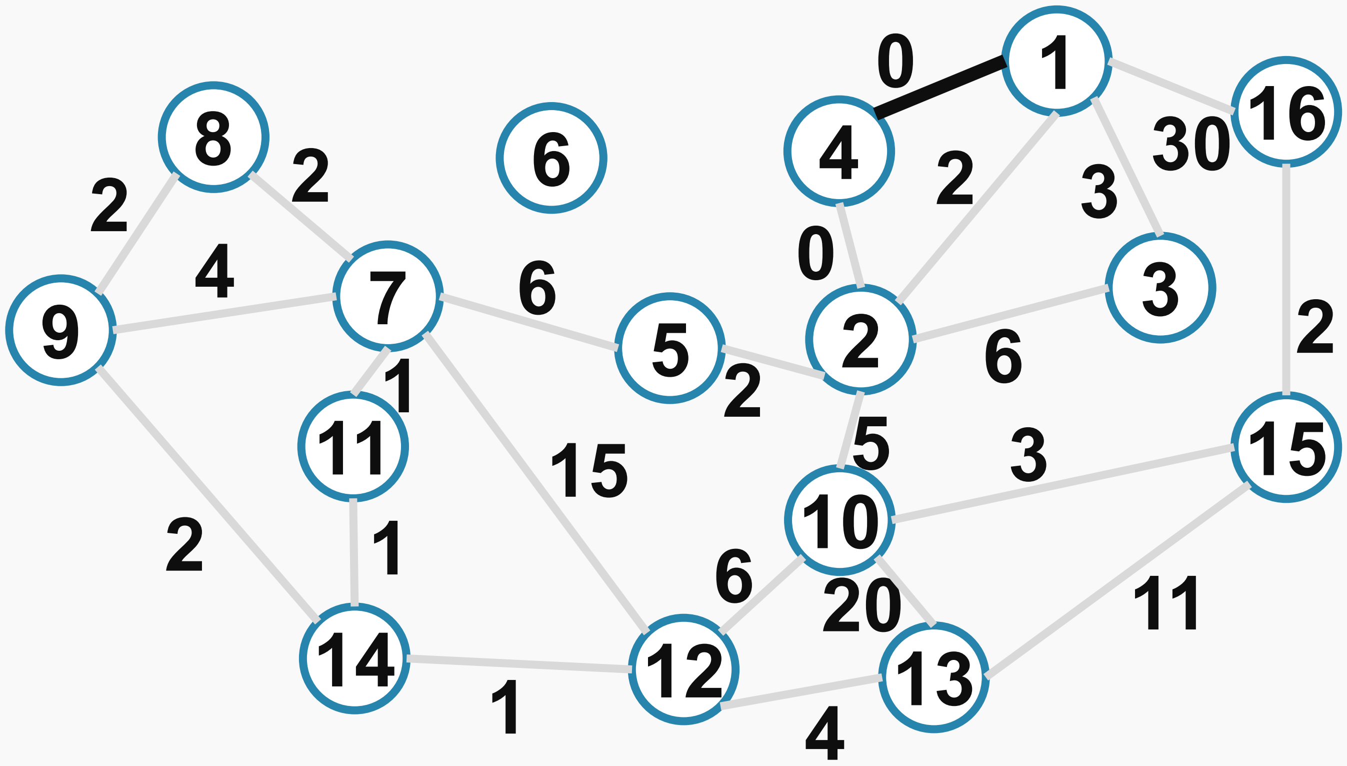


[Kruscal animation]

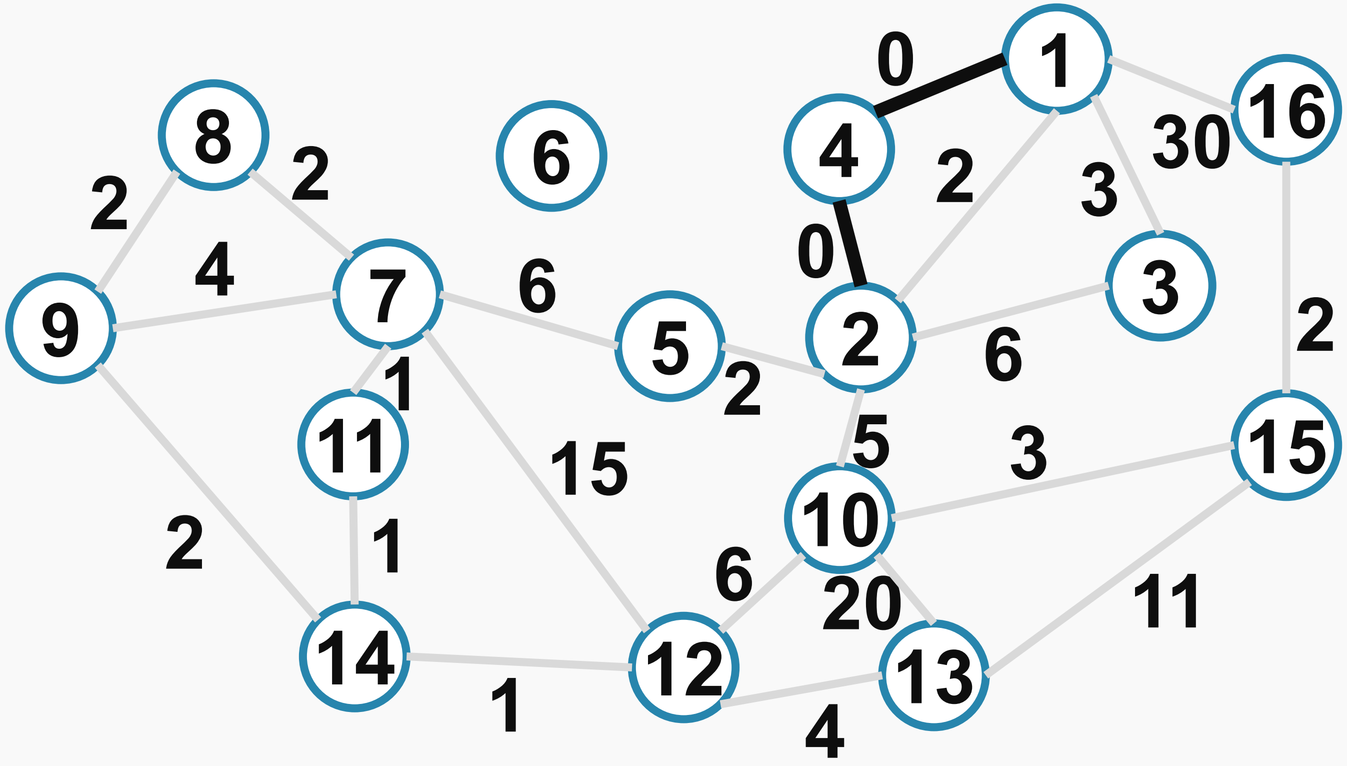
Algoritmul lui Kruskal



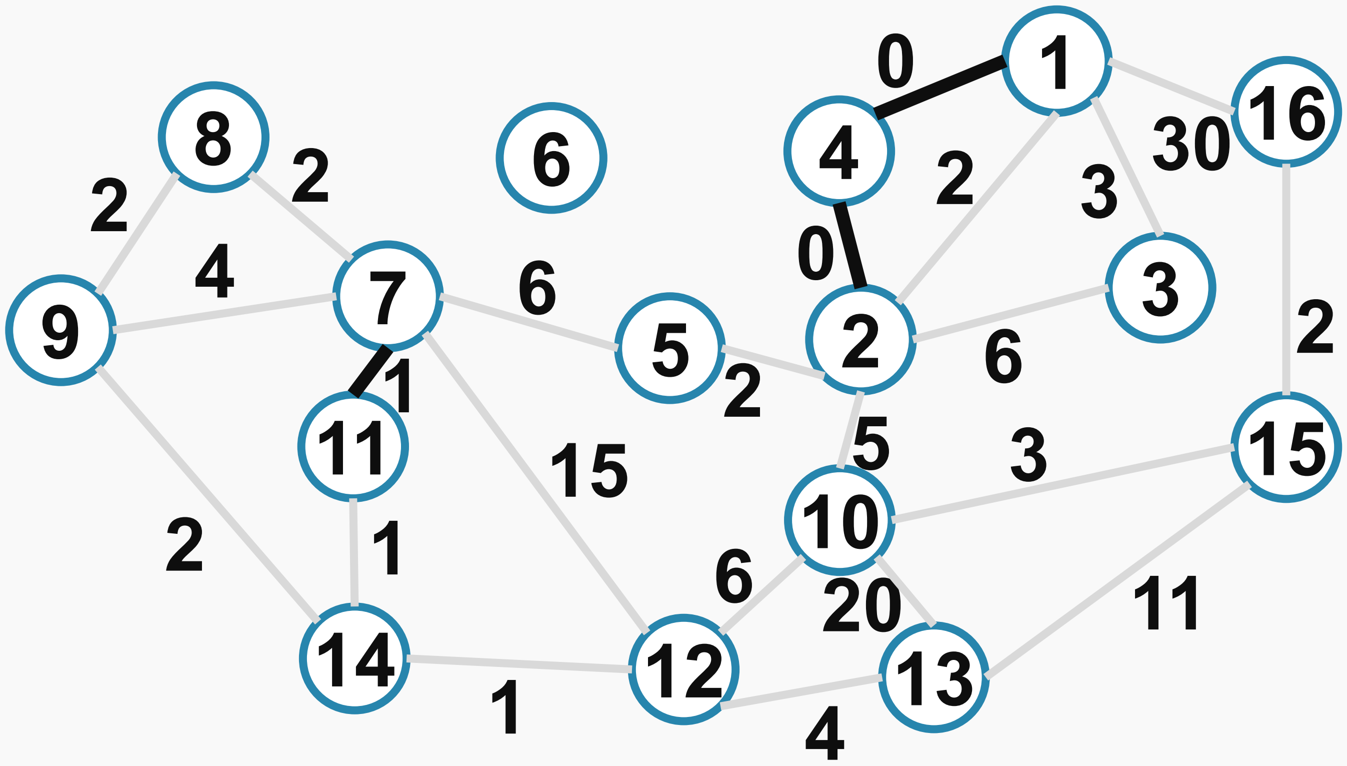
Algoritmul lui Kruskal



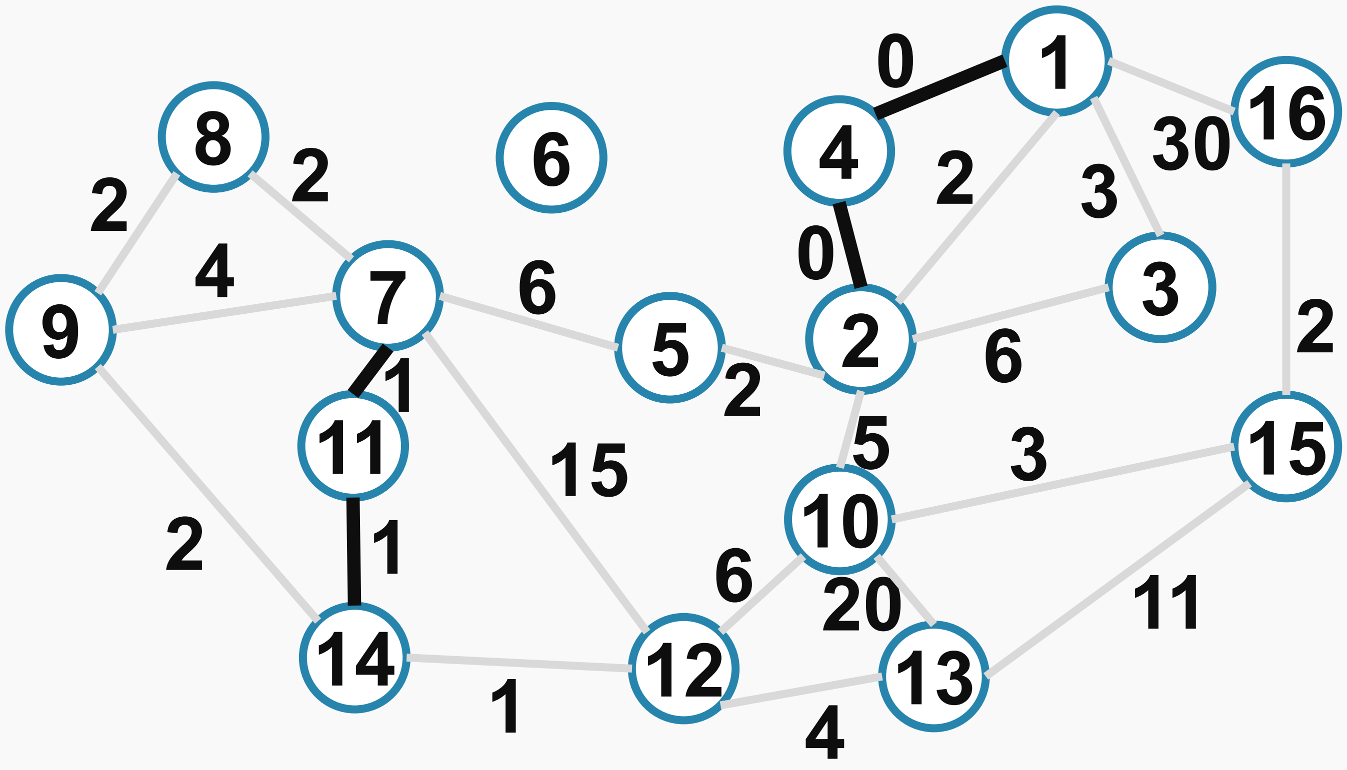
Algoritmul lui Kruskal



Algoritmul lui Kruskal



Algoritmul lui Kruskal



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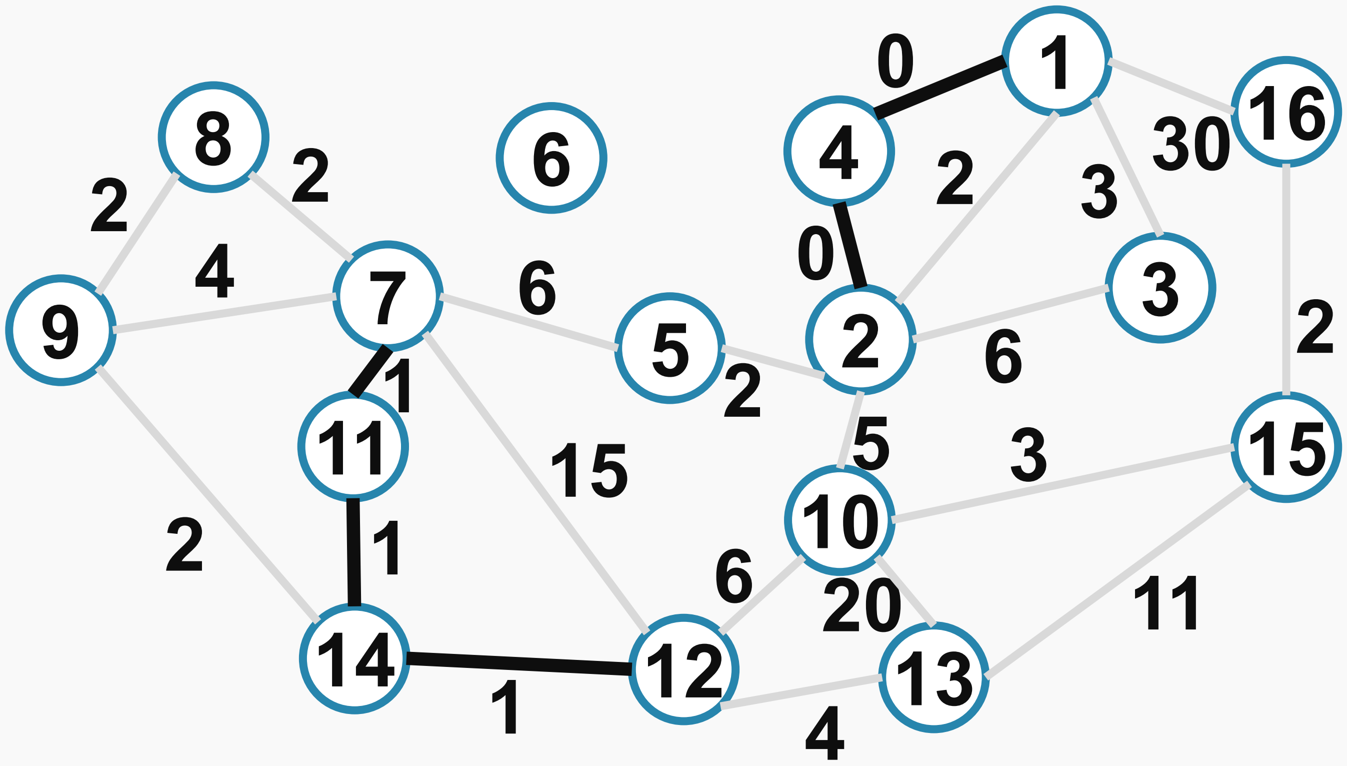
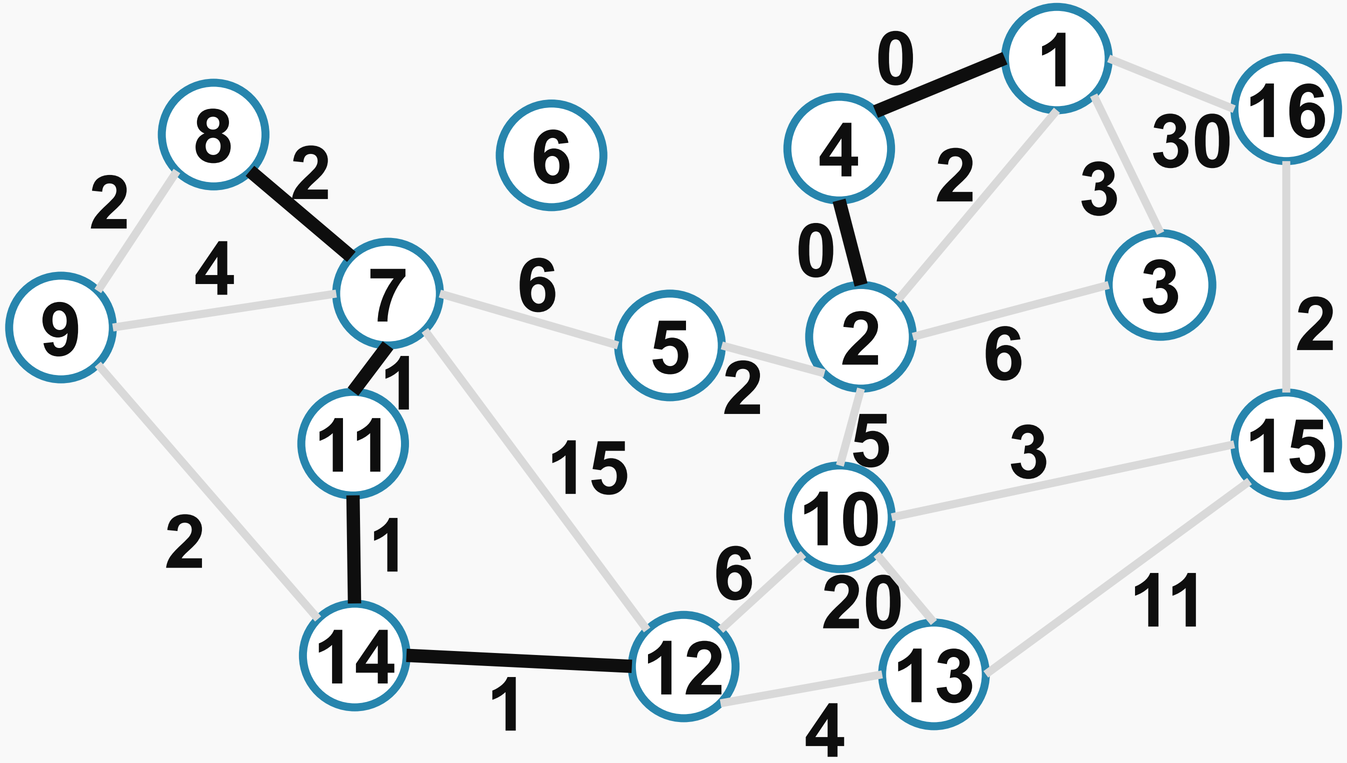


Algoritmul lui Kruskal



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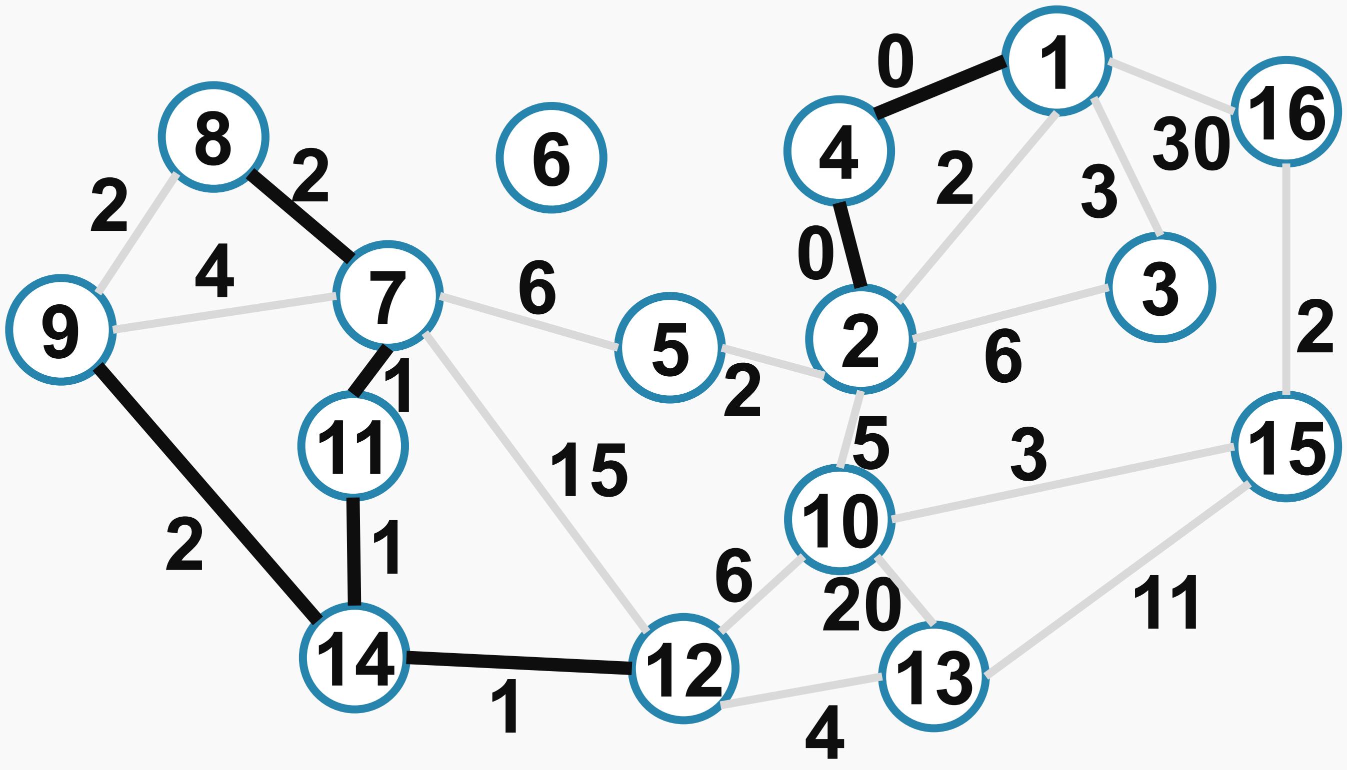

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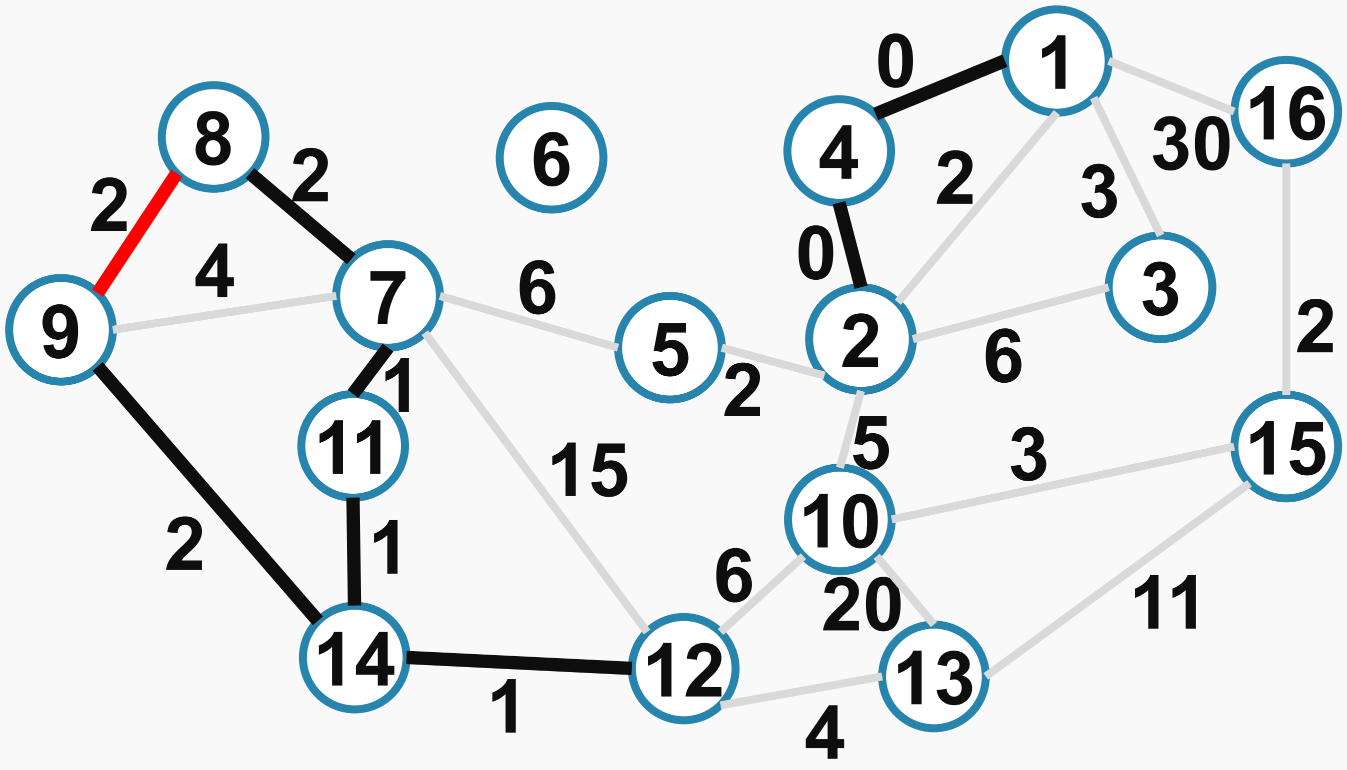

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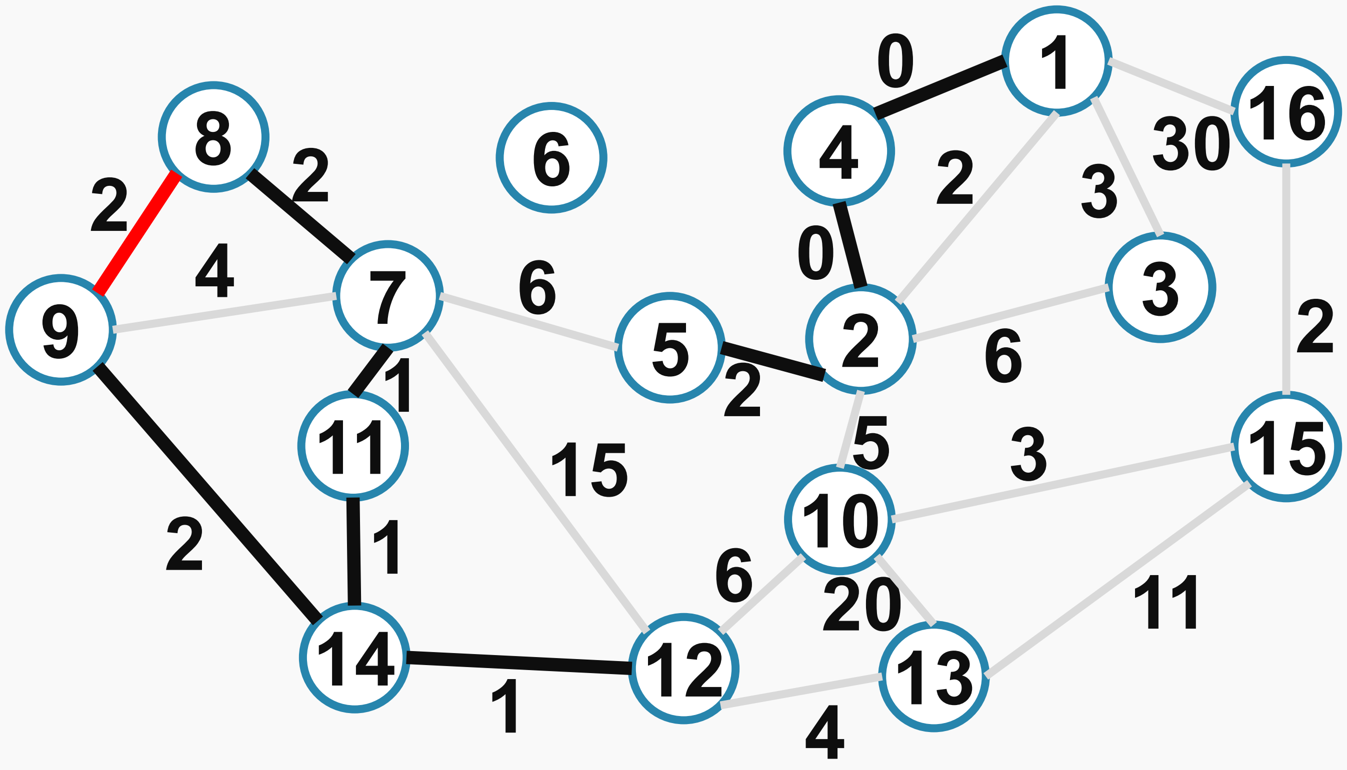
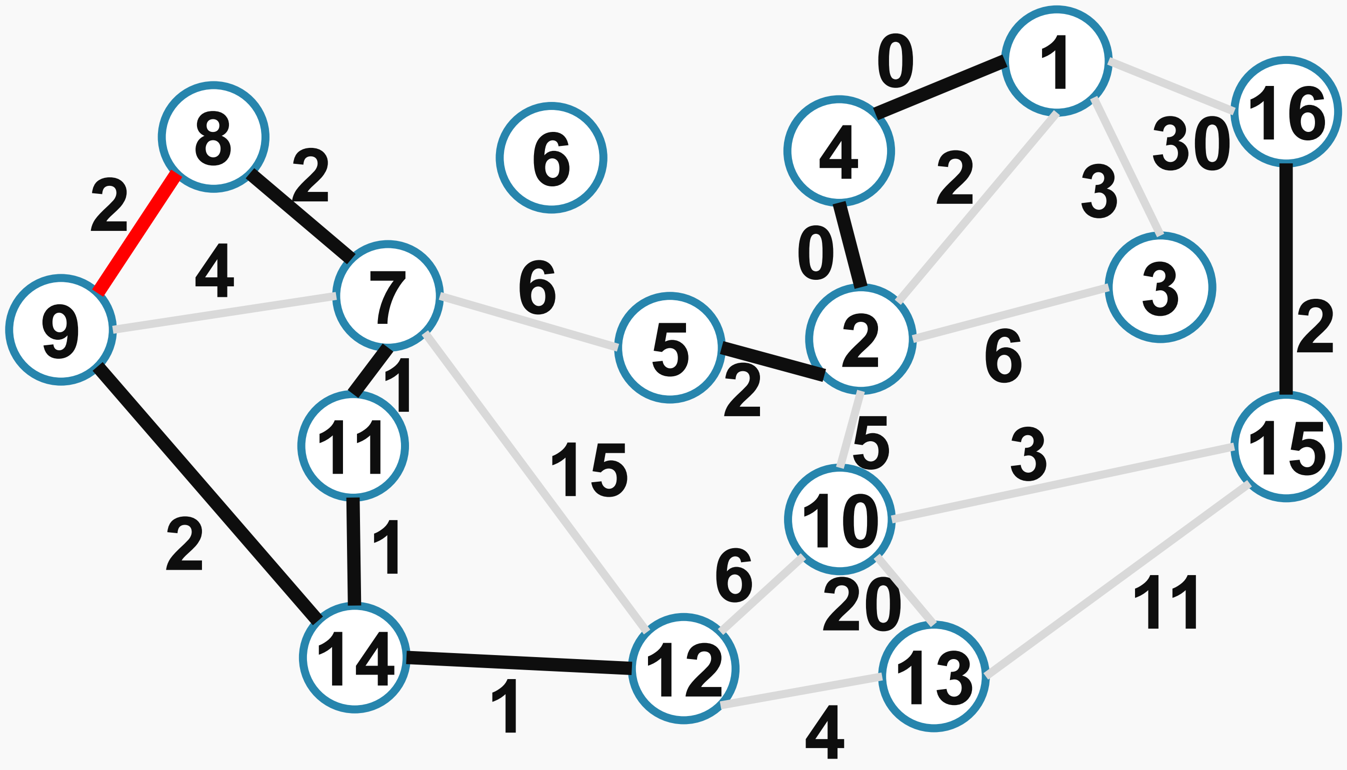


Algoritmul lui Kruskal



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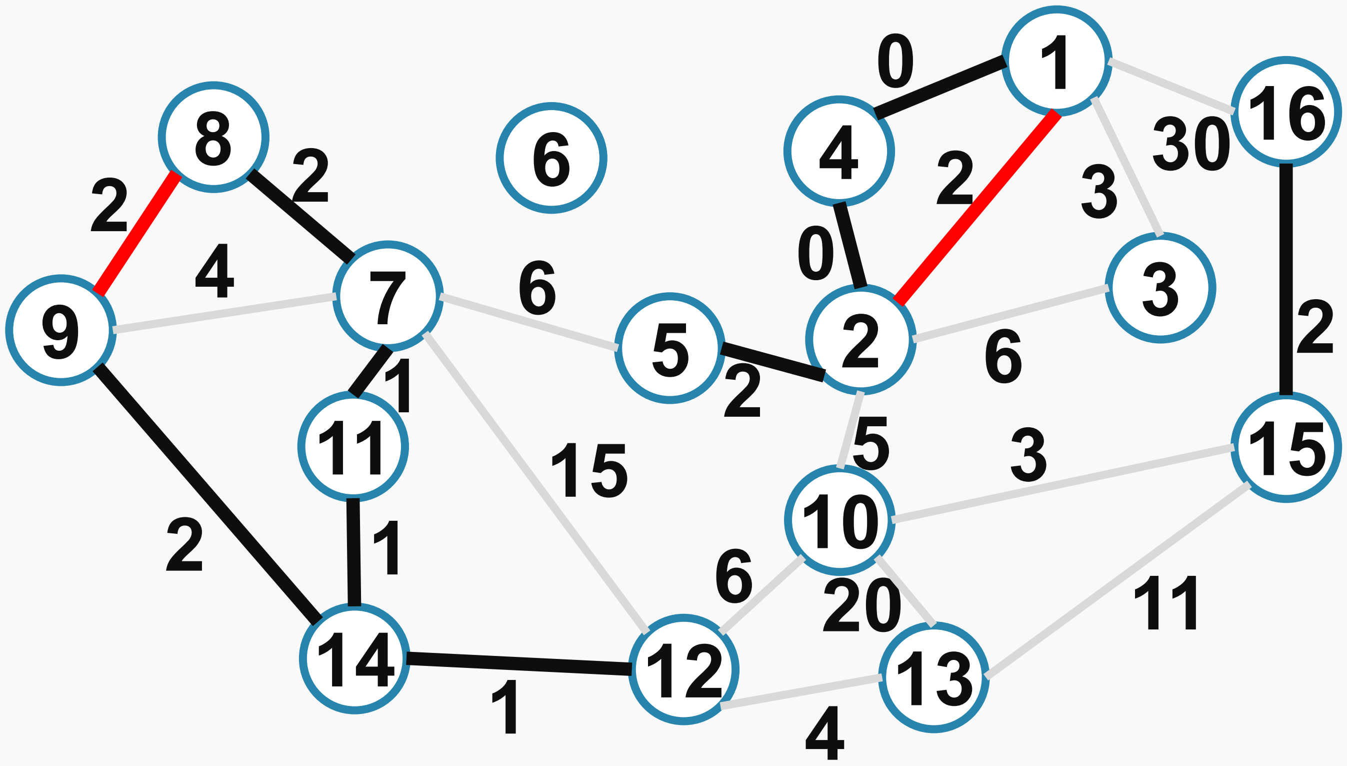

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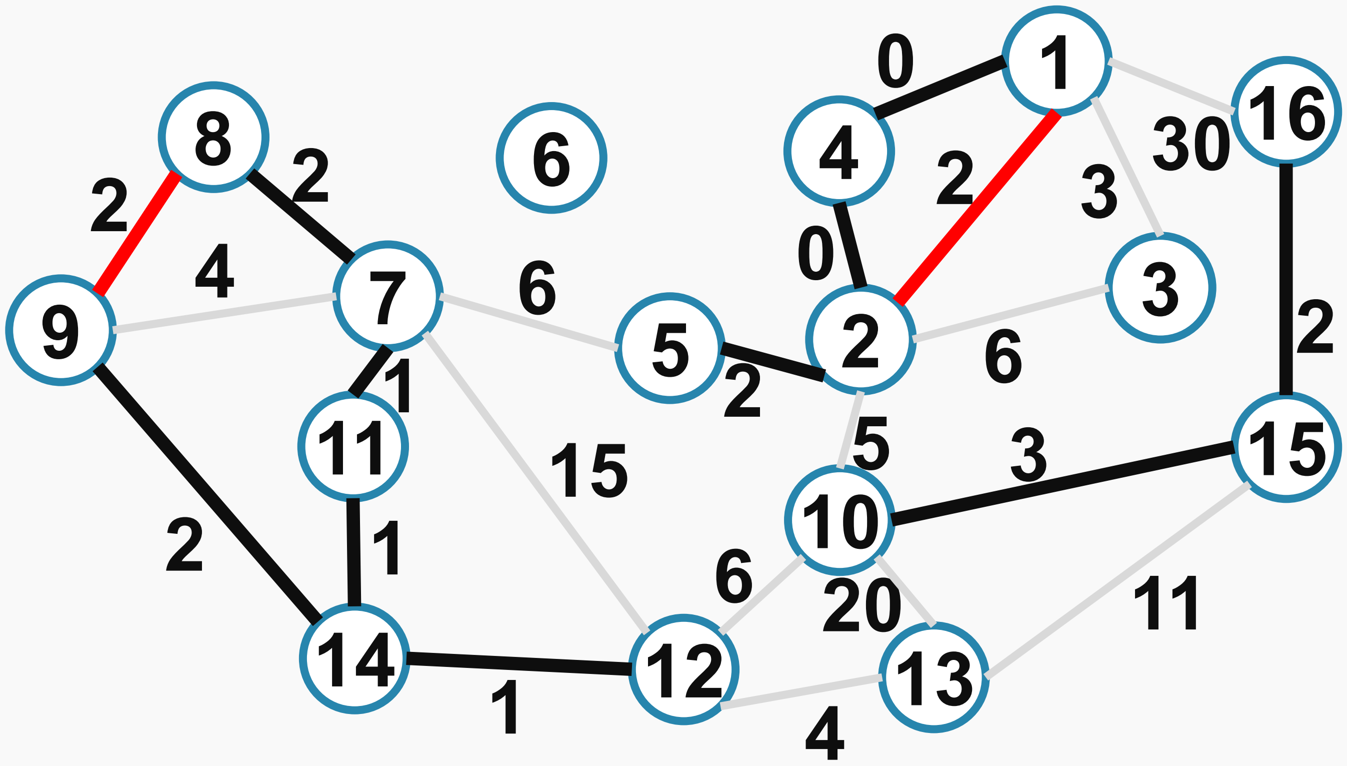
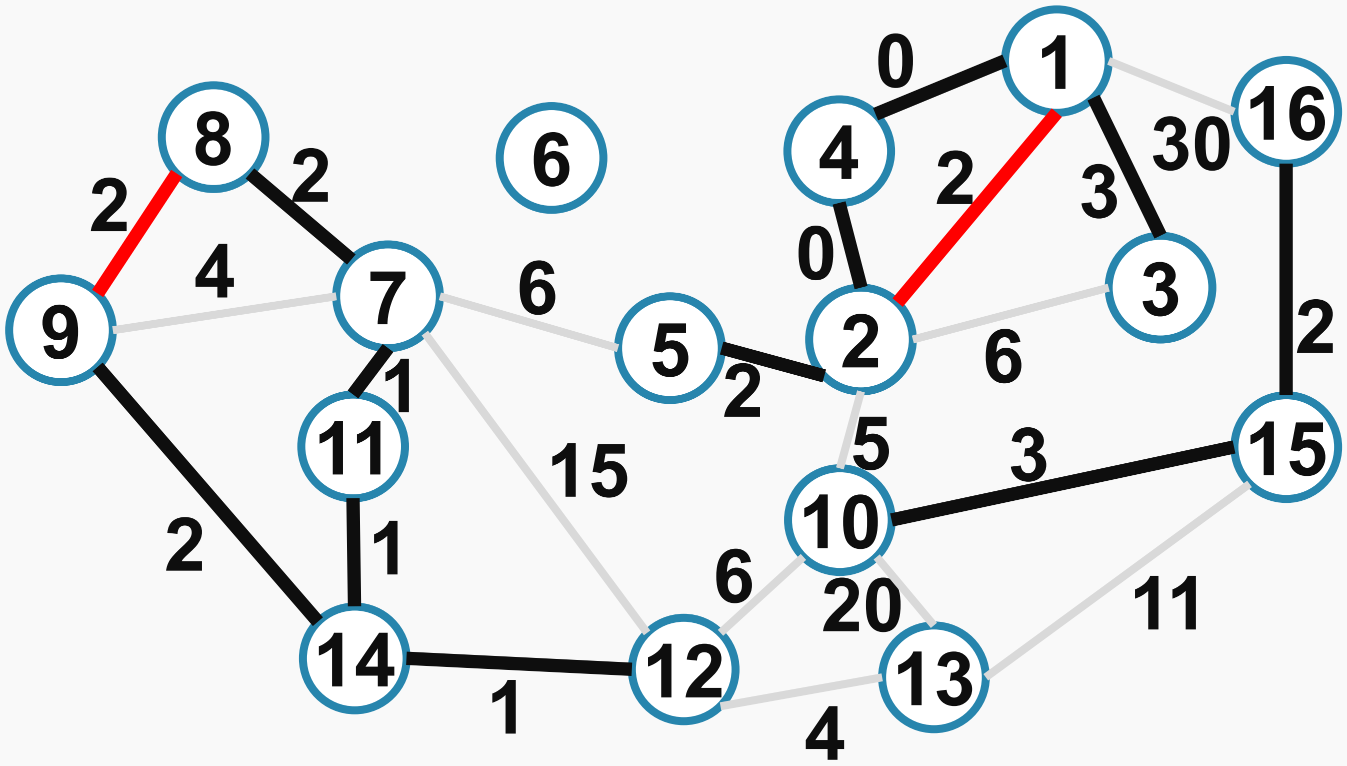
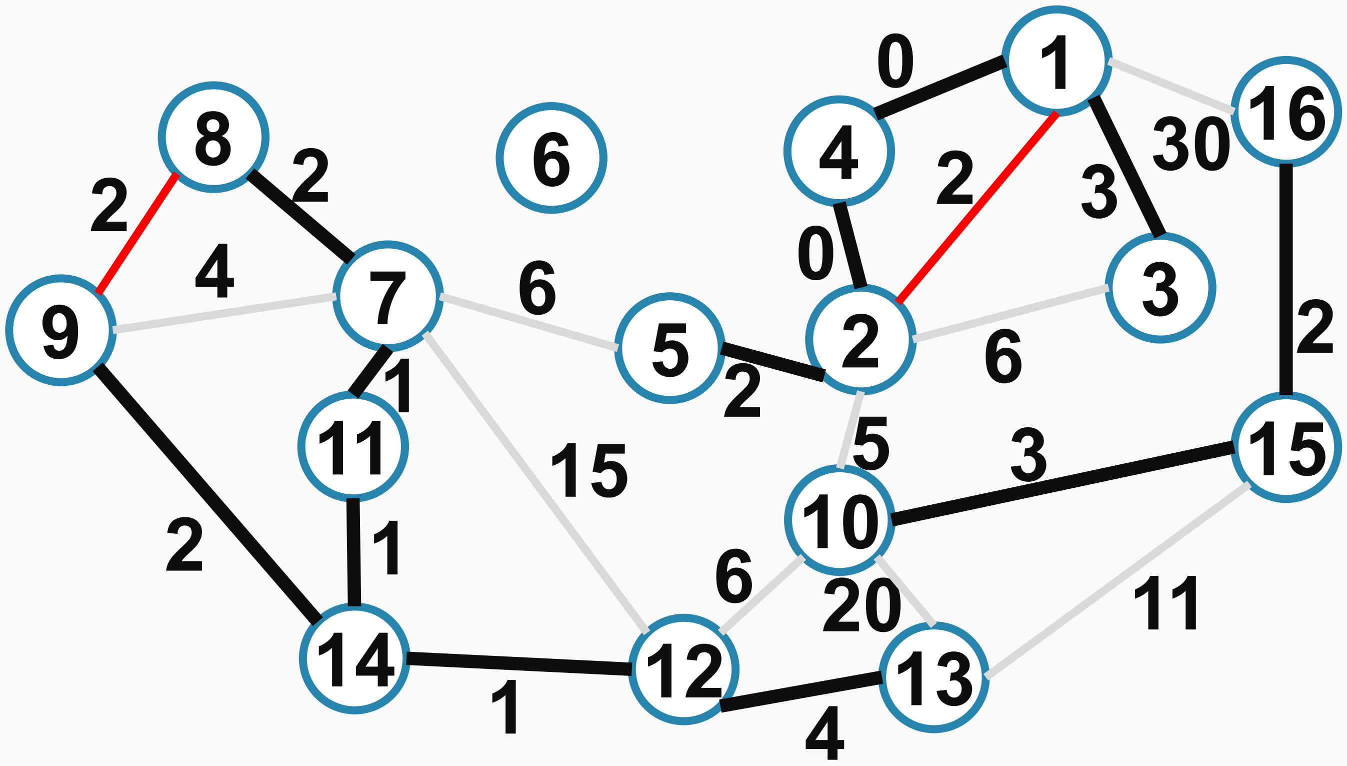


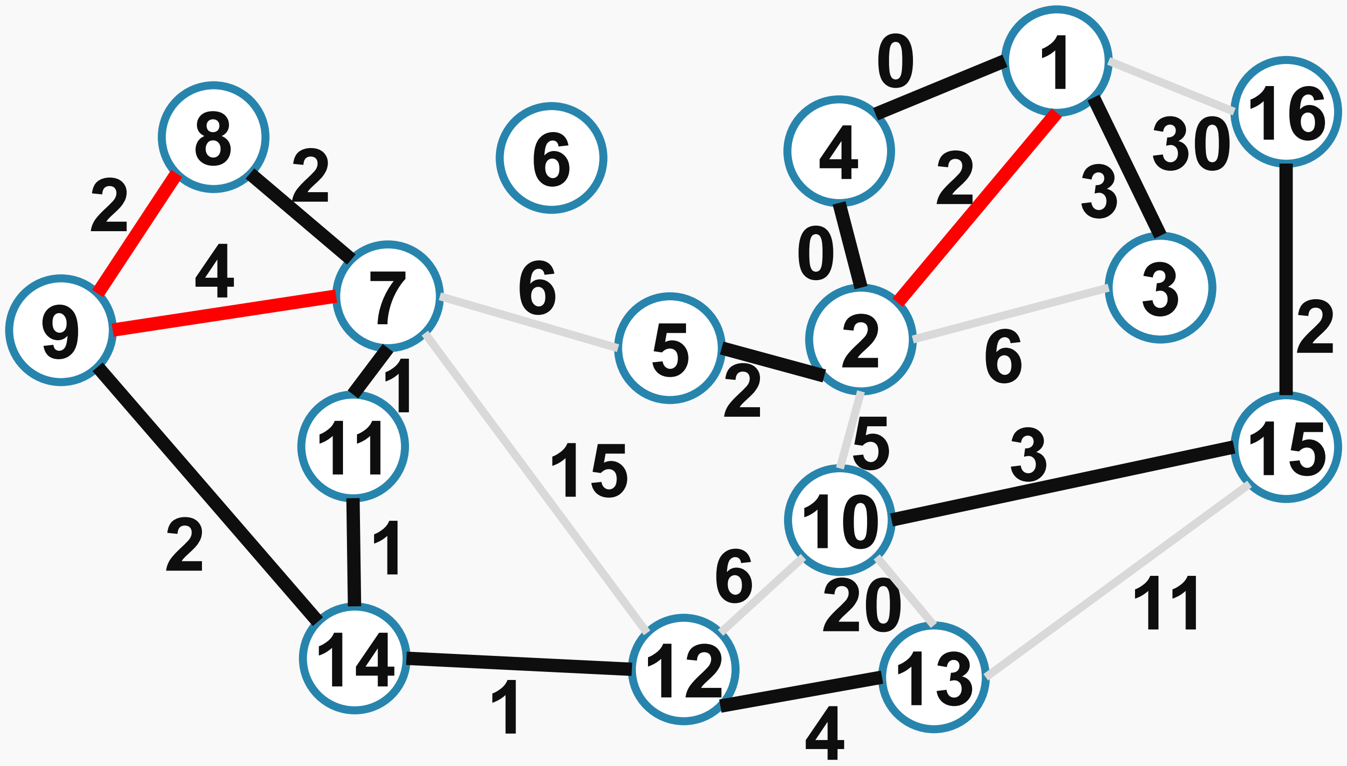
Algoritmul lui Kruskal



Algoritmul lui Kruskal



Algoritmul lui Kruskal



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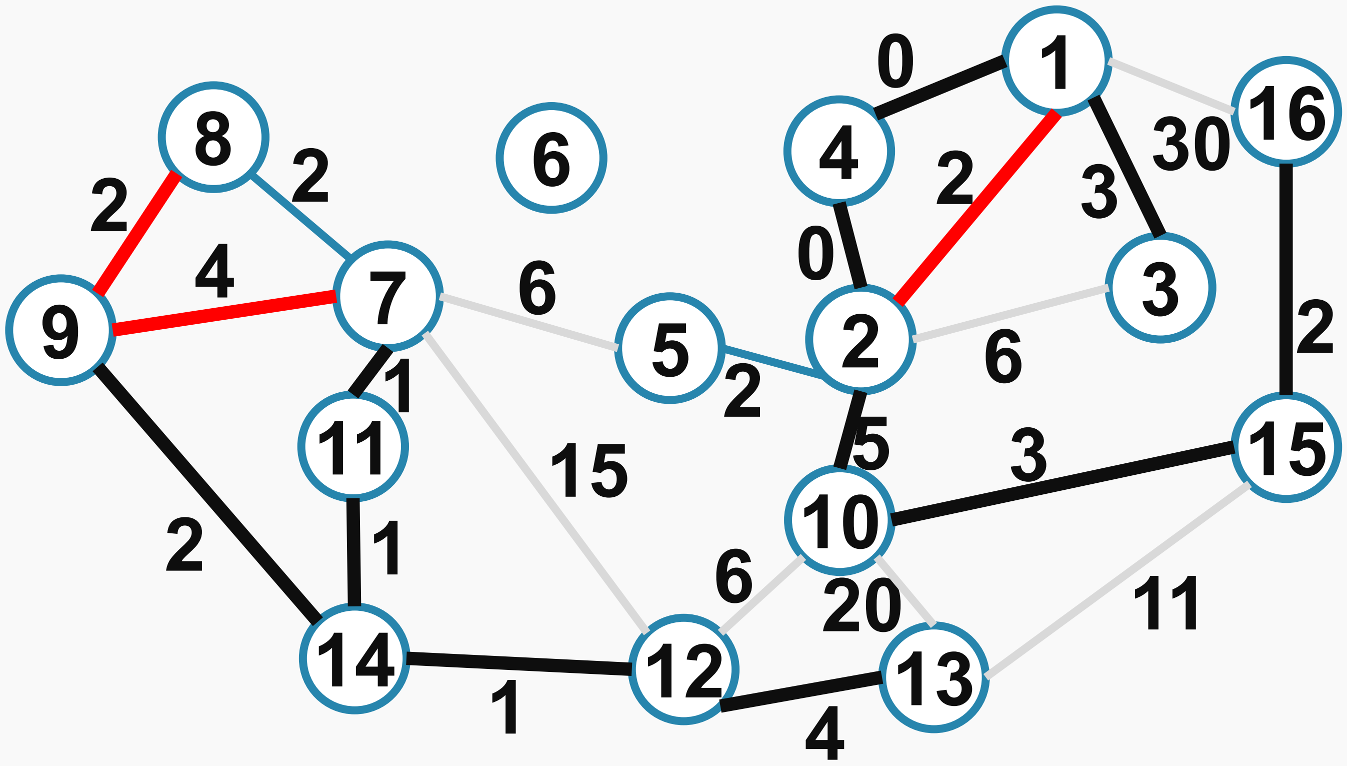

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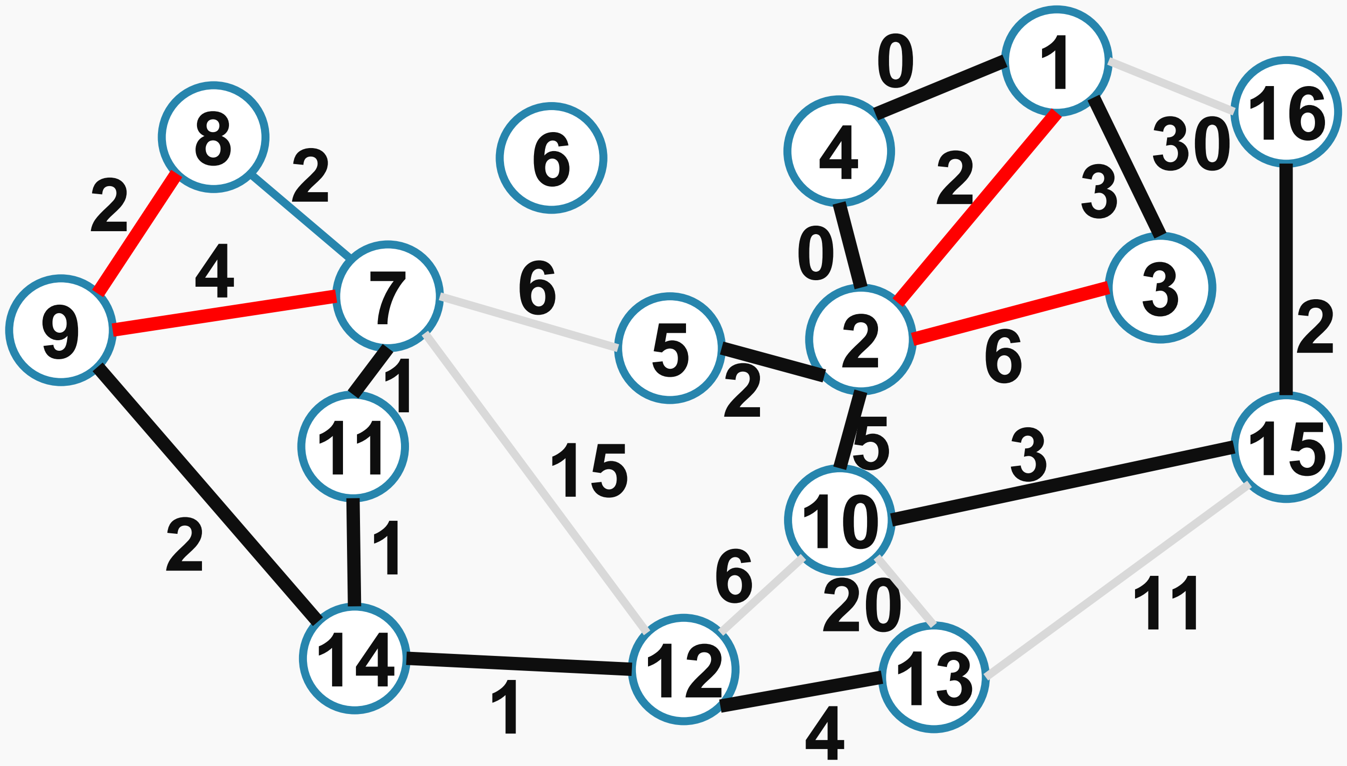
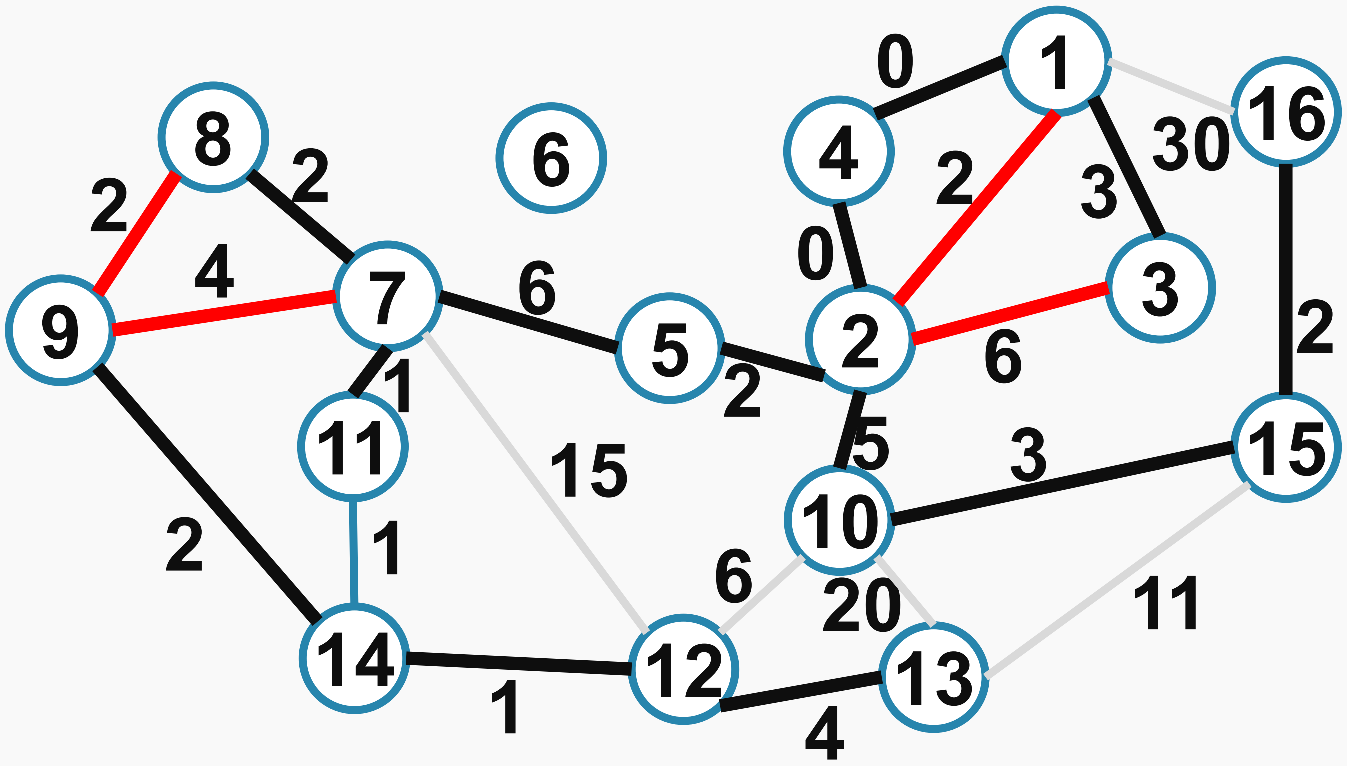
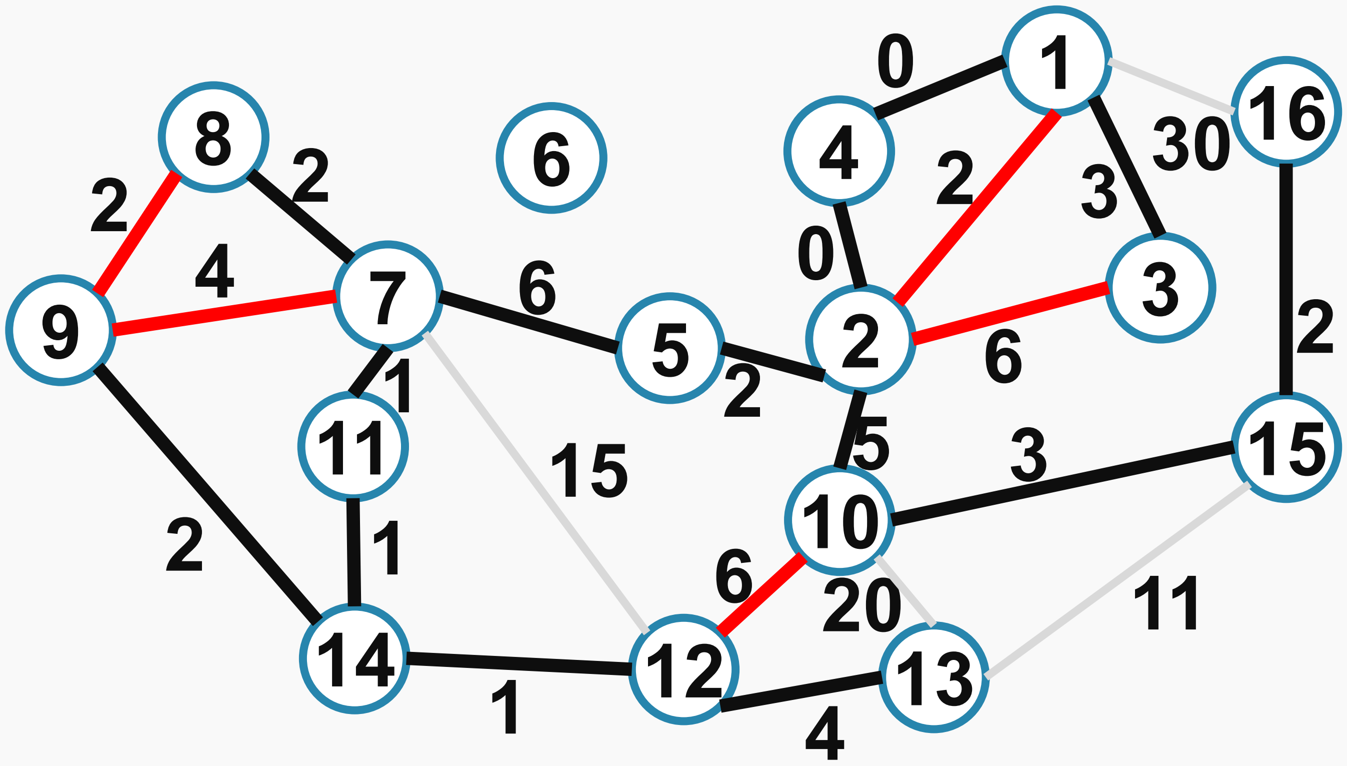


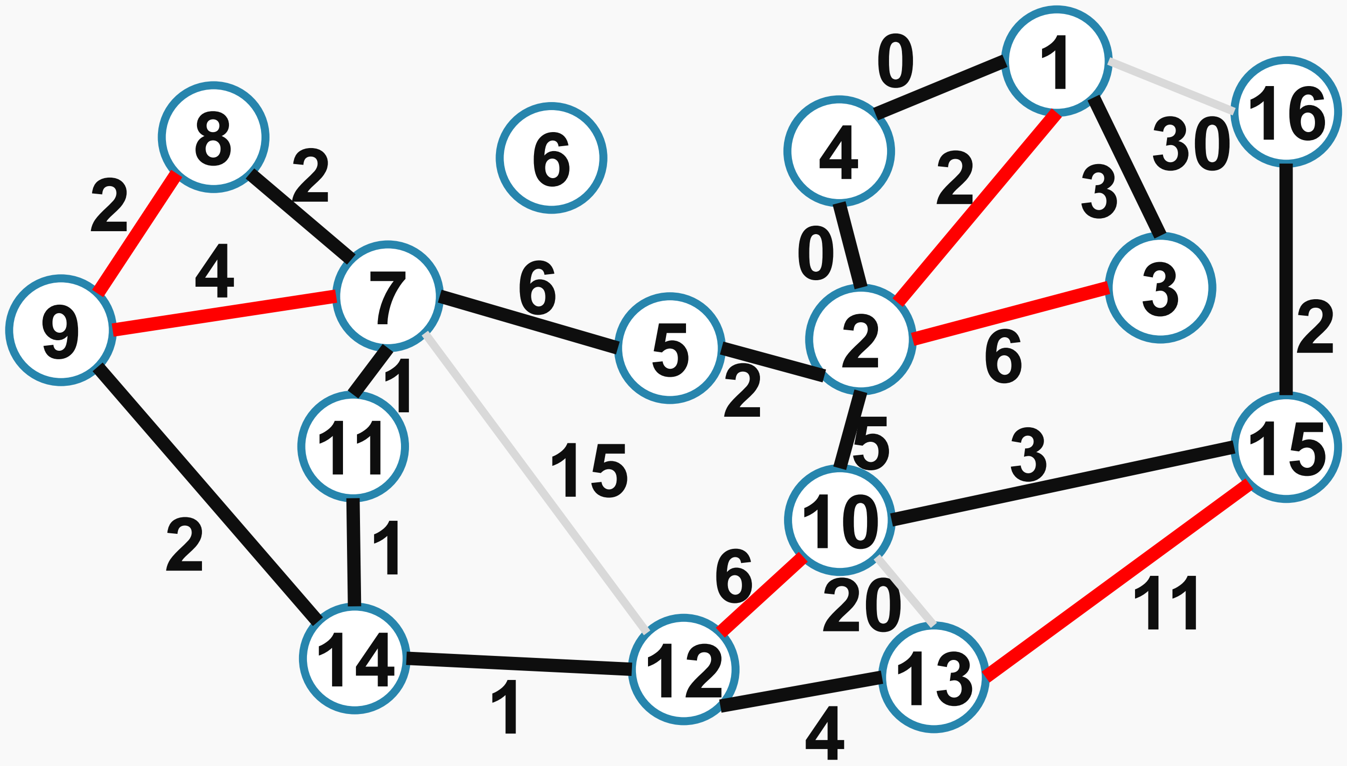
Algoritmul lui Kruskal



Algoritmul lui Kruskal



Algoritmul lui Kruskal



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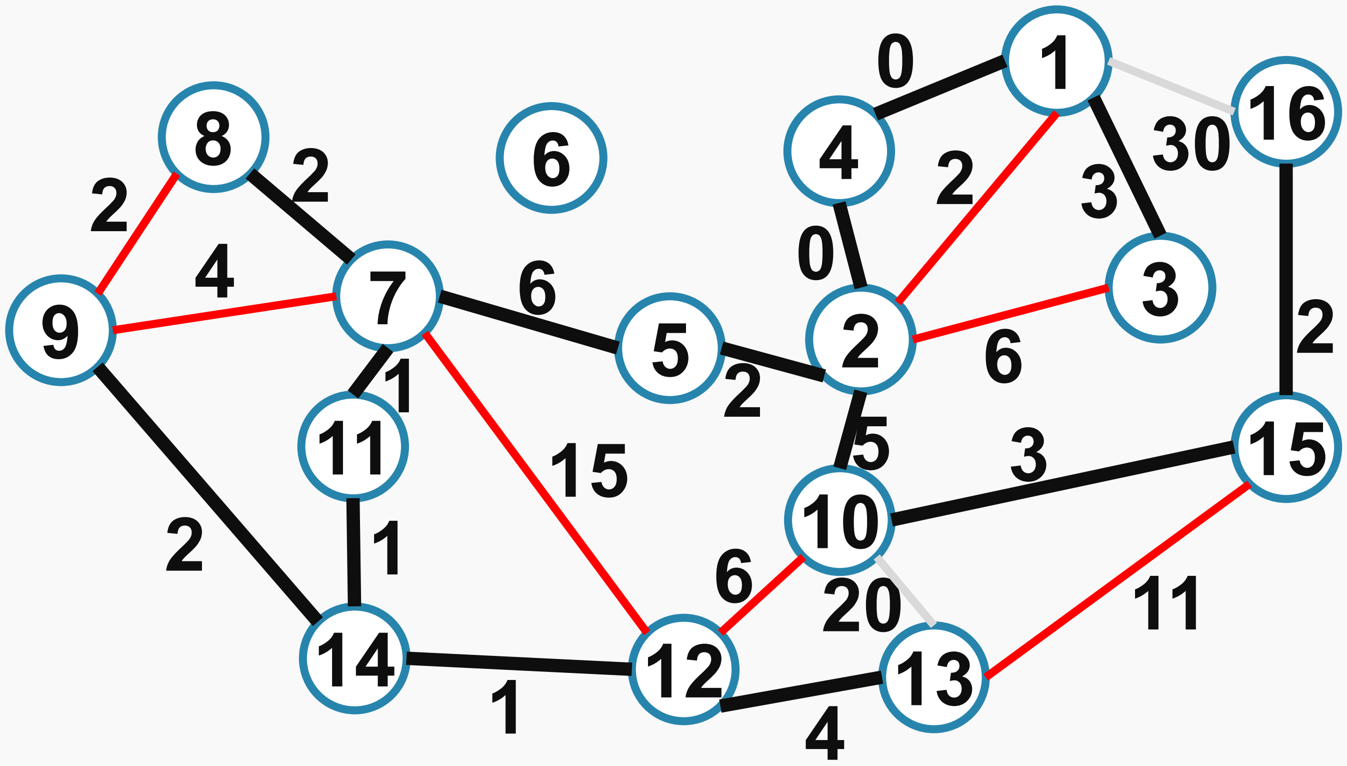

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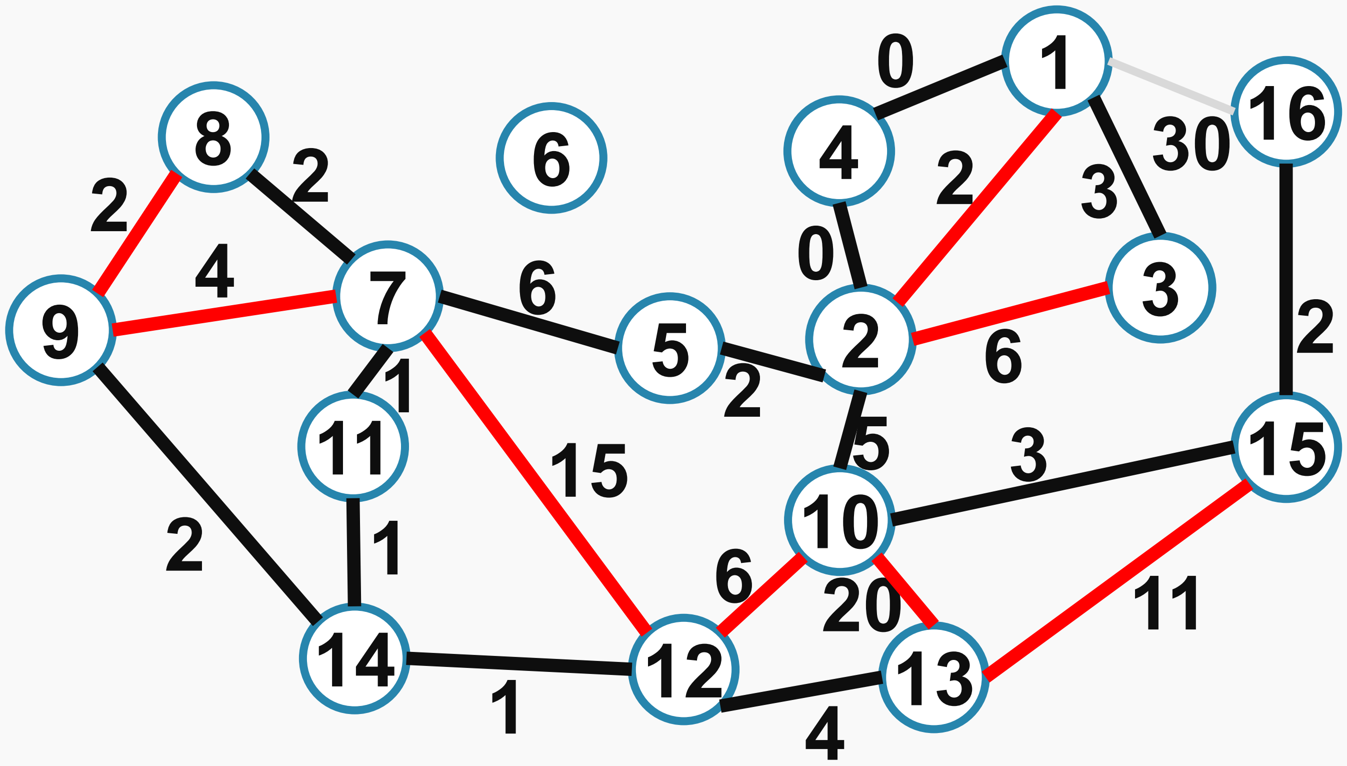

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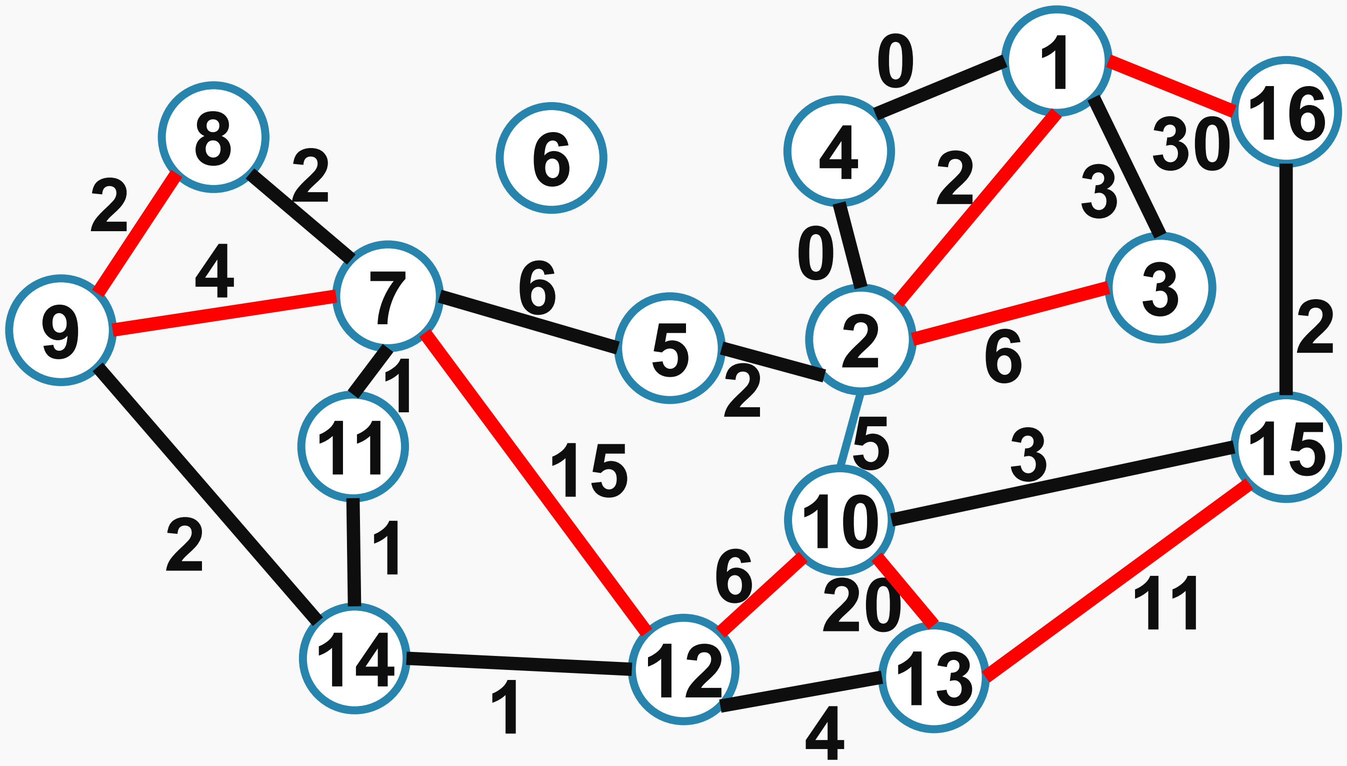
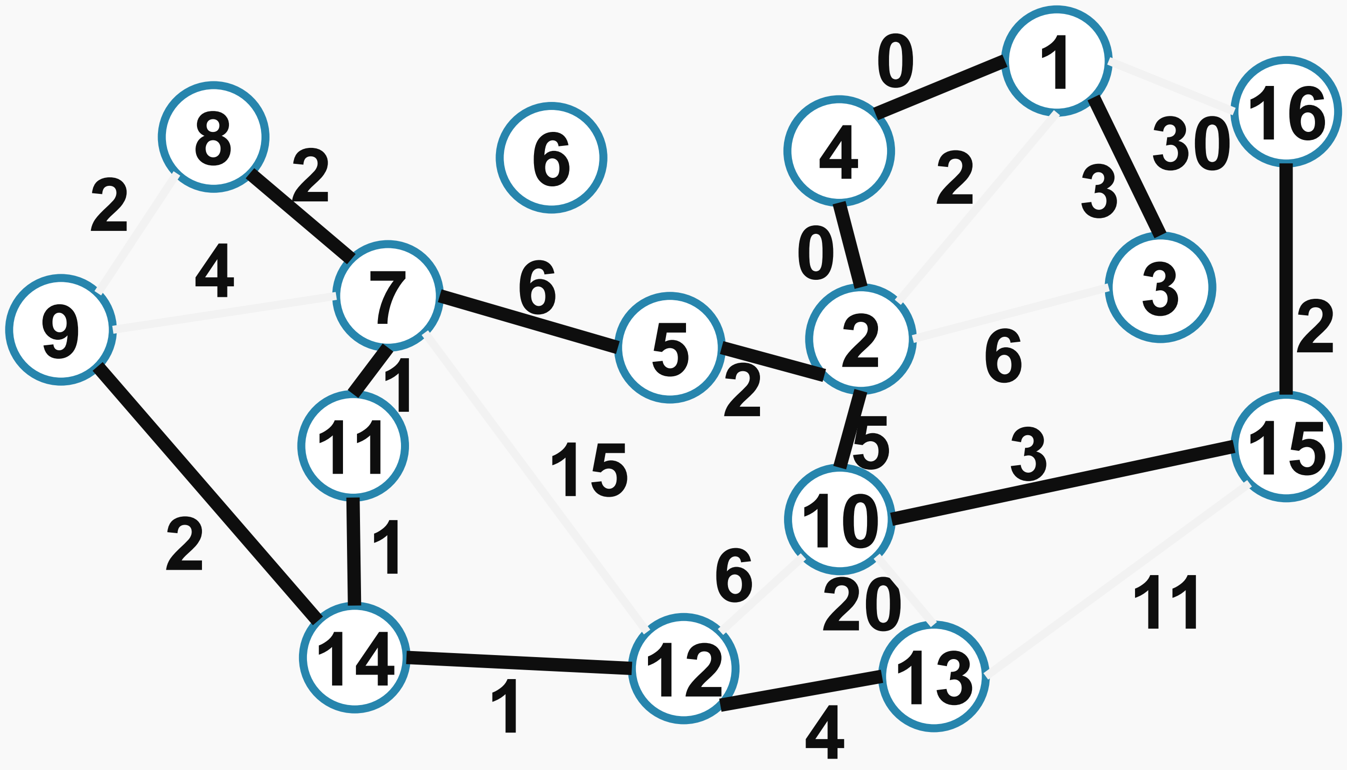


Algoritmul lui Kruskal



Complexitate?

```
tree Kruskal(G) {
    sort(G.E); // sort by weight
    A = {};
    for each (node in G.V)
        Make_set(node);
    for each ((u, v) in G.E) {
        if (Find_set(u) != Find_set(v)) {
            A = A U {(u, v)};
            Union(Find_set(u), Find_set(v));
        }
    }
    return A;
}
```

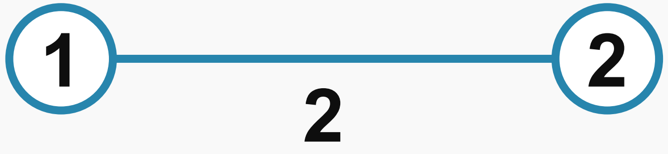
Complexitate?

$$O(E \log(E))$$

Flux maxim

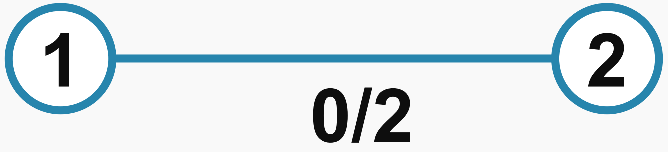
Graf capacitate

În general valoarea de pe muchie reprezintă o distanță.



- O distanță mai mare face muchia mai greu de parcurs.

Valoarea muchiei poate reprezenta o capacitate.



- Similar apei/curentului, cu cât capacitatea e mai mare cu atât e mai ușor de parcurs.

Șoselele au și distanță și capacitate (număr benzi/viteză max)

Flux maxim Algoritmul Ford-Fulkerson

c capacitate muchie

f flow muchie. Capacitate folosită

$c_f(u, v) = c(u, v) - f(u, v)$ diferența de capacitate

G_f graful cu muchii c_f

FORD-FULKERSON (G, s, t)
for each edge $(u, v) \in G.E$
    $(u, v).f = 0$
while there exists a path p from s to t in the residual network G_f
    $c_f(p) = \min\{c_f(u, v) : (u, v)\ is\ in\ p\}$
    for each edge (u, v) in p
        if $(u, v) \in G.E$
            $(u, v).f = (u, v).f + c_f(p)$
        else
            $(v, u).f = (v, u).f - c_f(p)$

Flux maxim Algoritmul Ford-Fulkerson (1956)

c capacitate muchie

f flow muchie. Capacitate folosită

$c_f(u,v) = c(u,v) - f(u,v)$ diferența de capacitate

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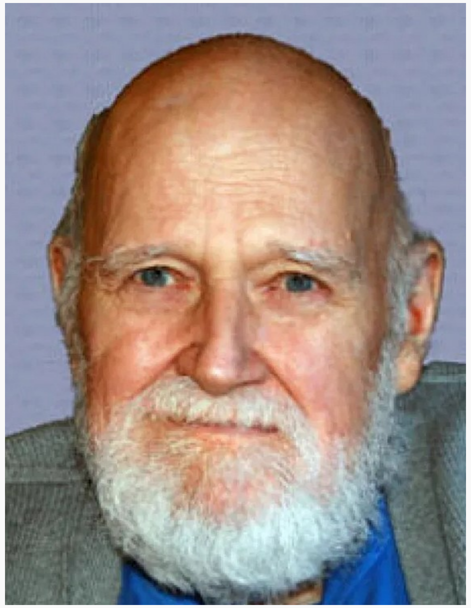
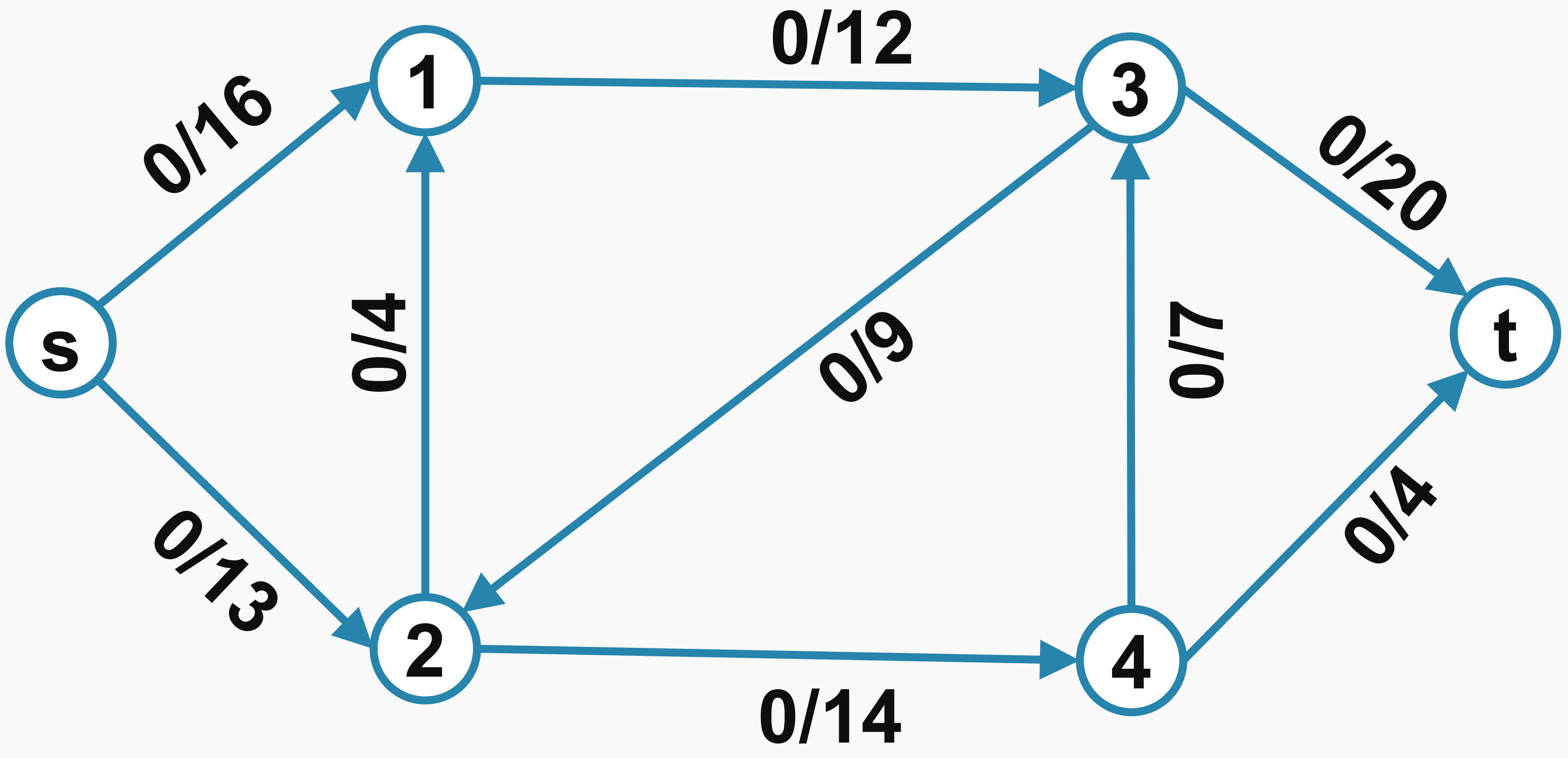
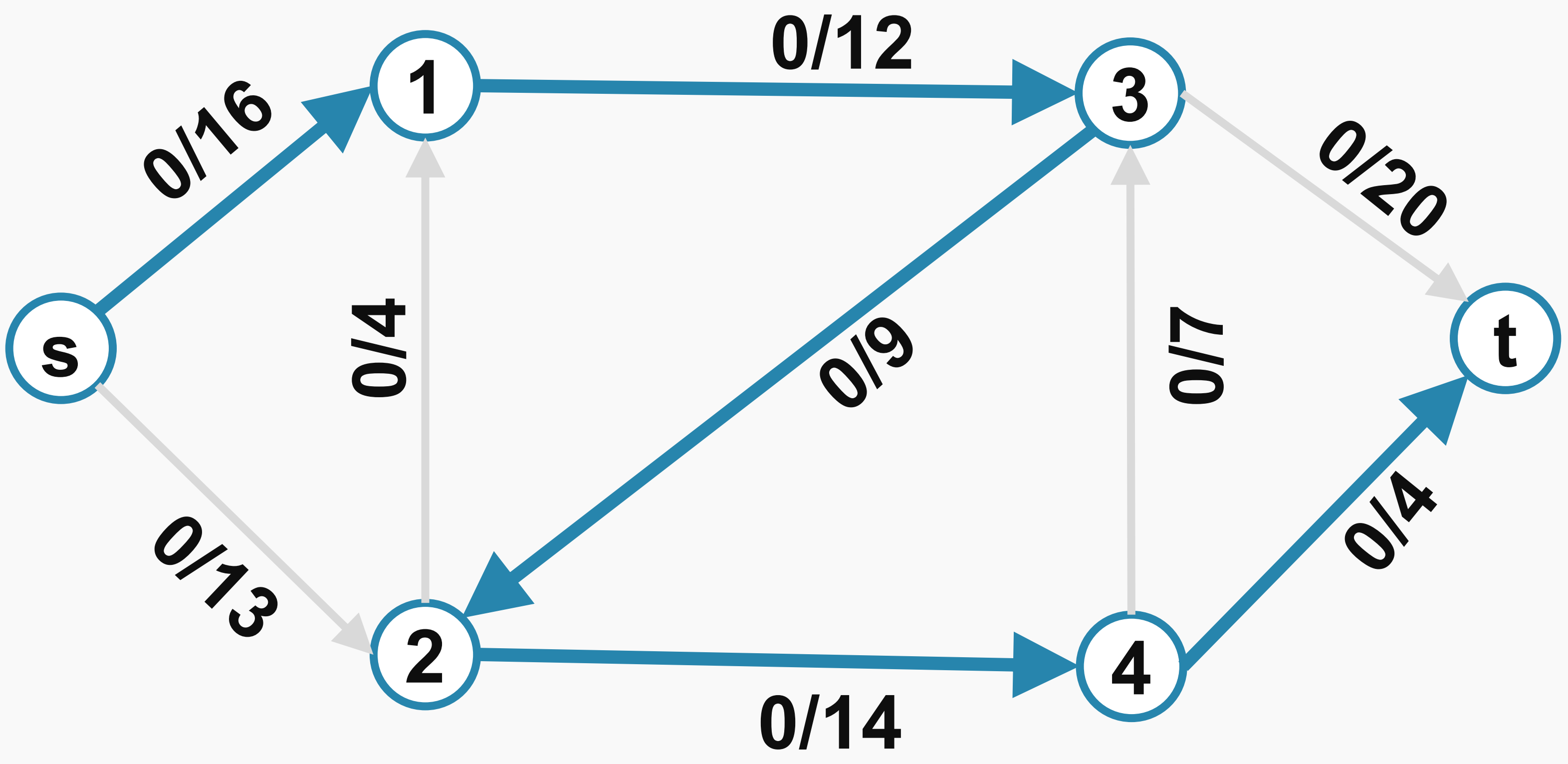


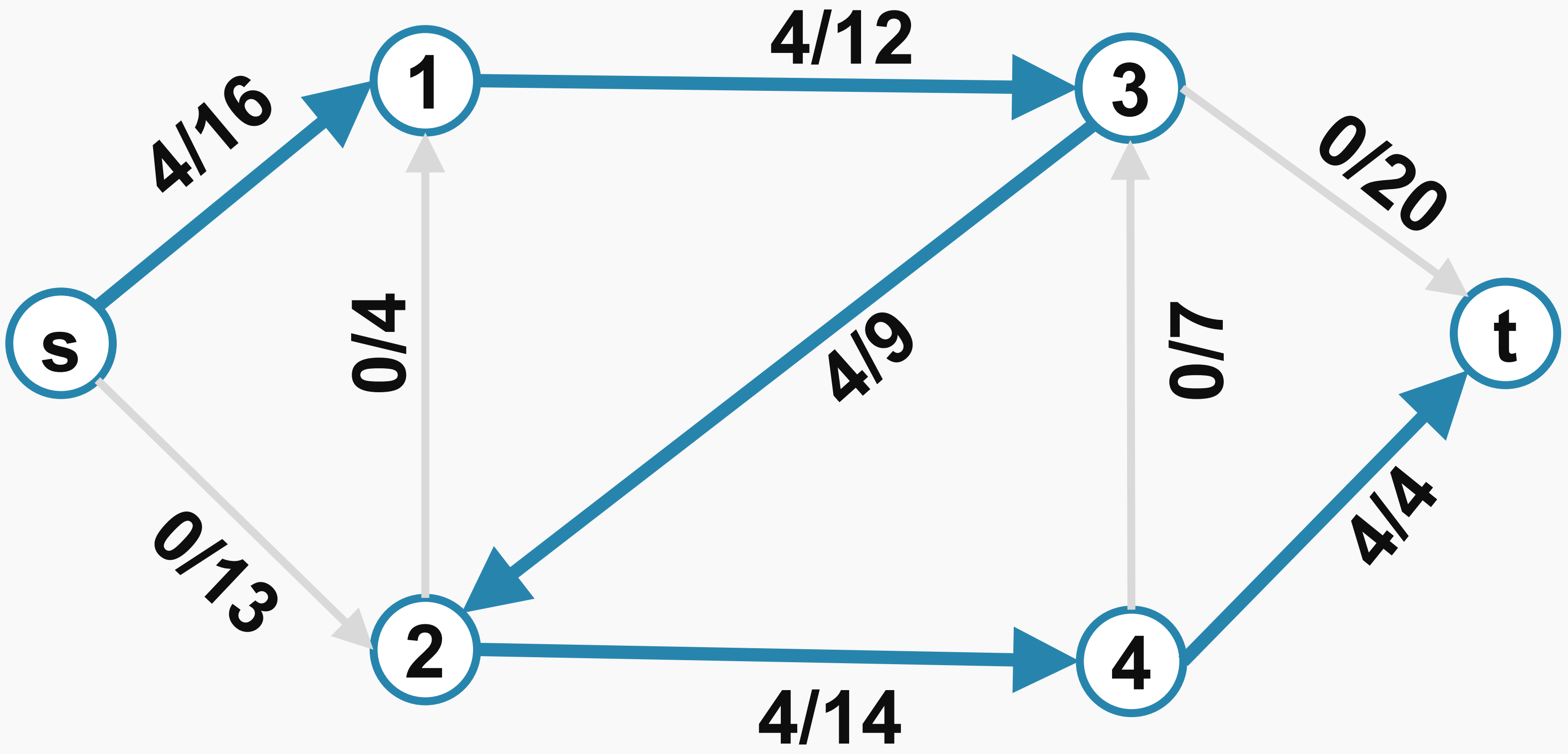
Algoritmul Ford-Fulkerson



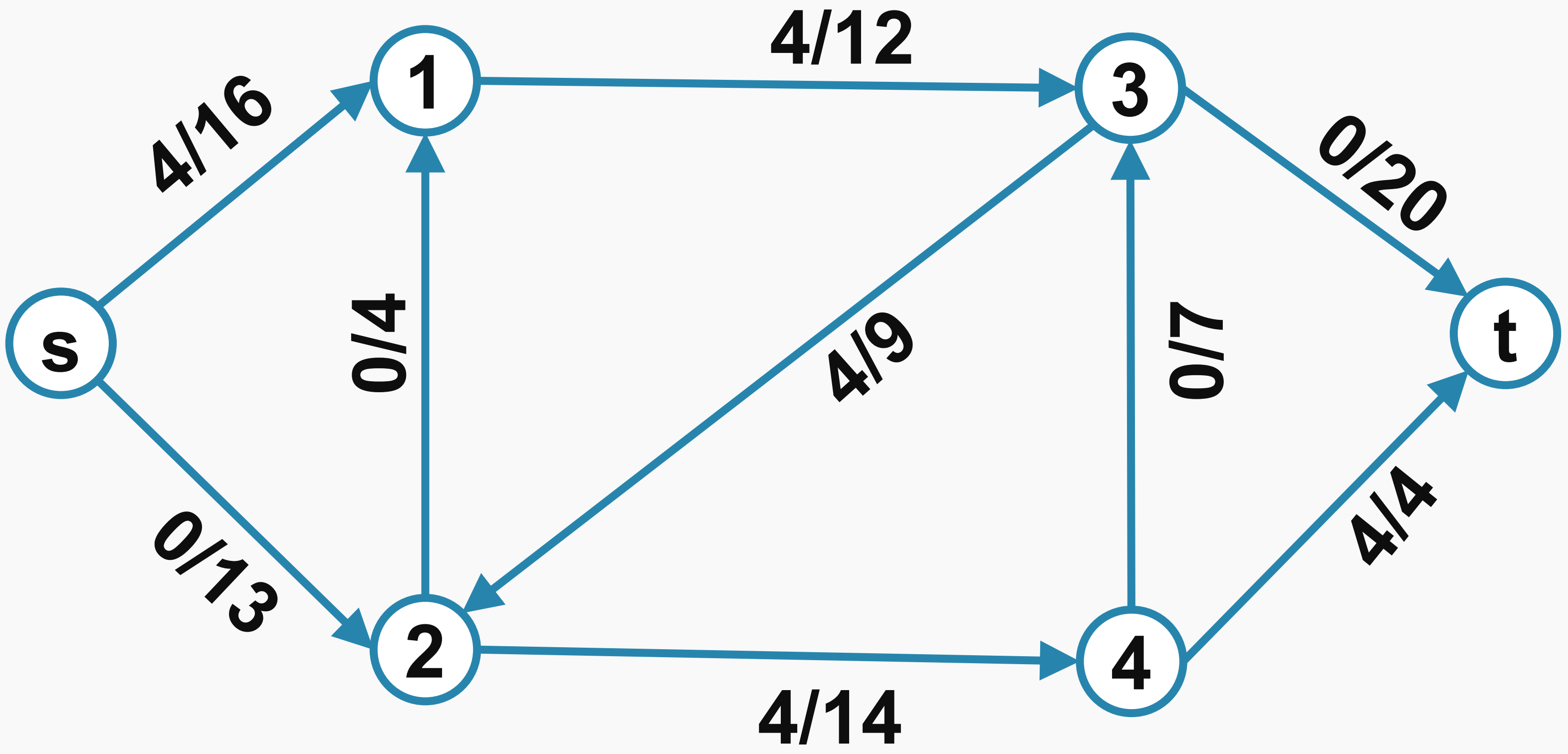
Algoritmul Ford-Fulkerson



Algoritmul Ford-Fulkerson



Algoritmul Ford-Fulkerson



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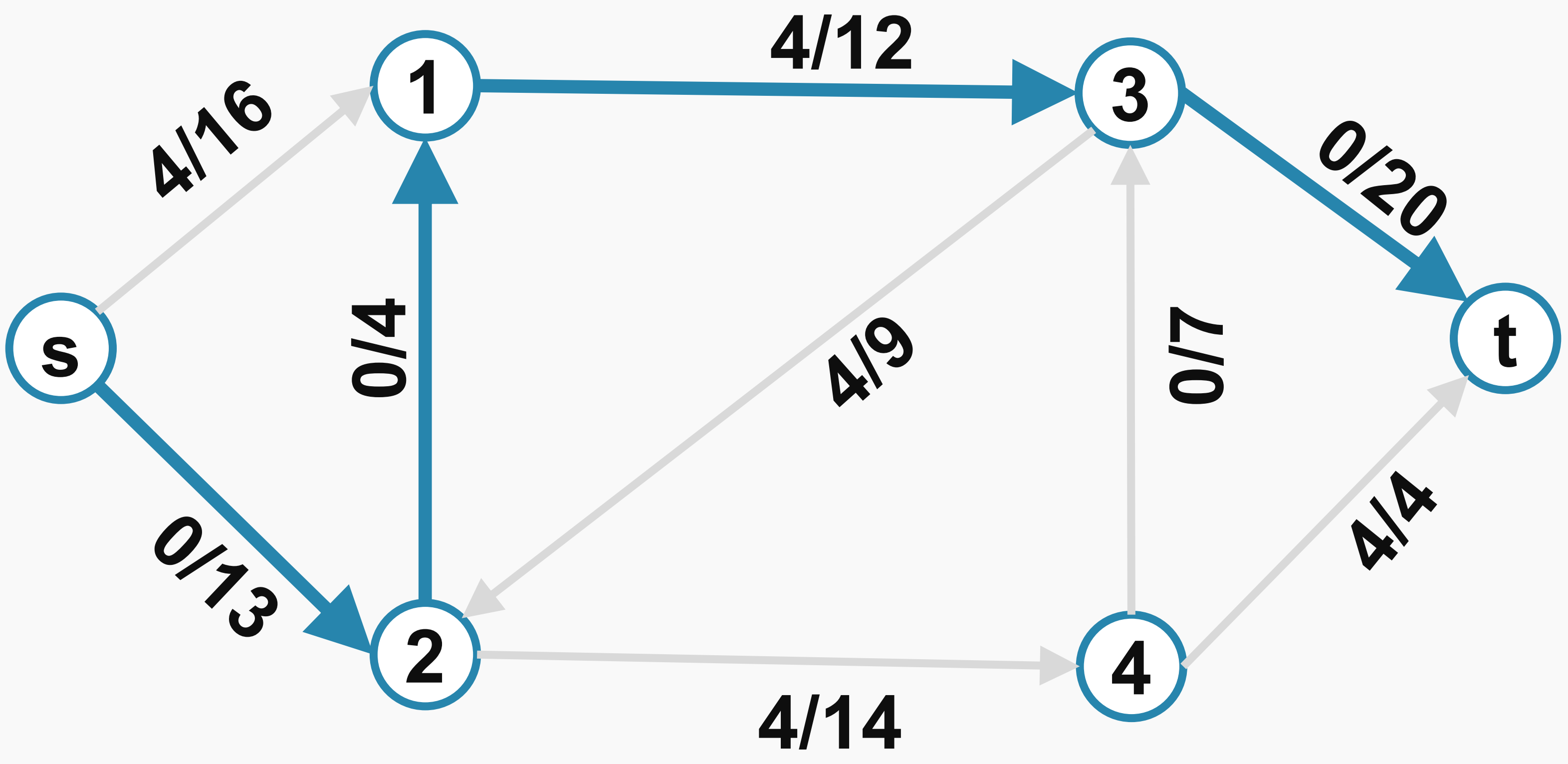

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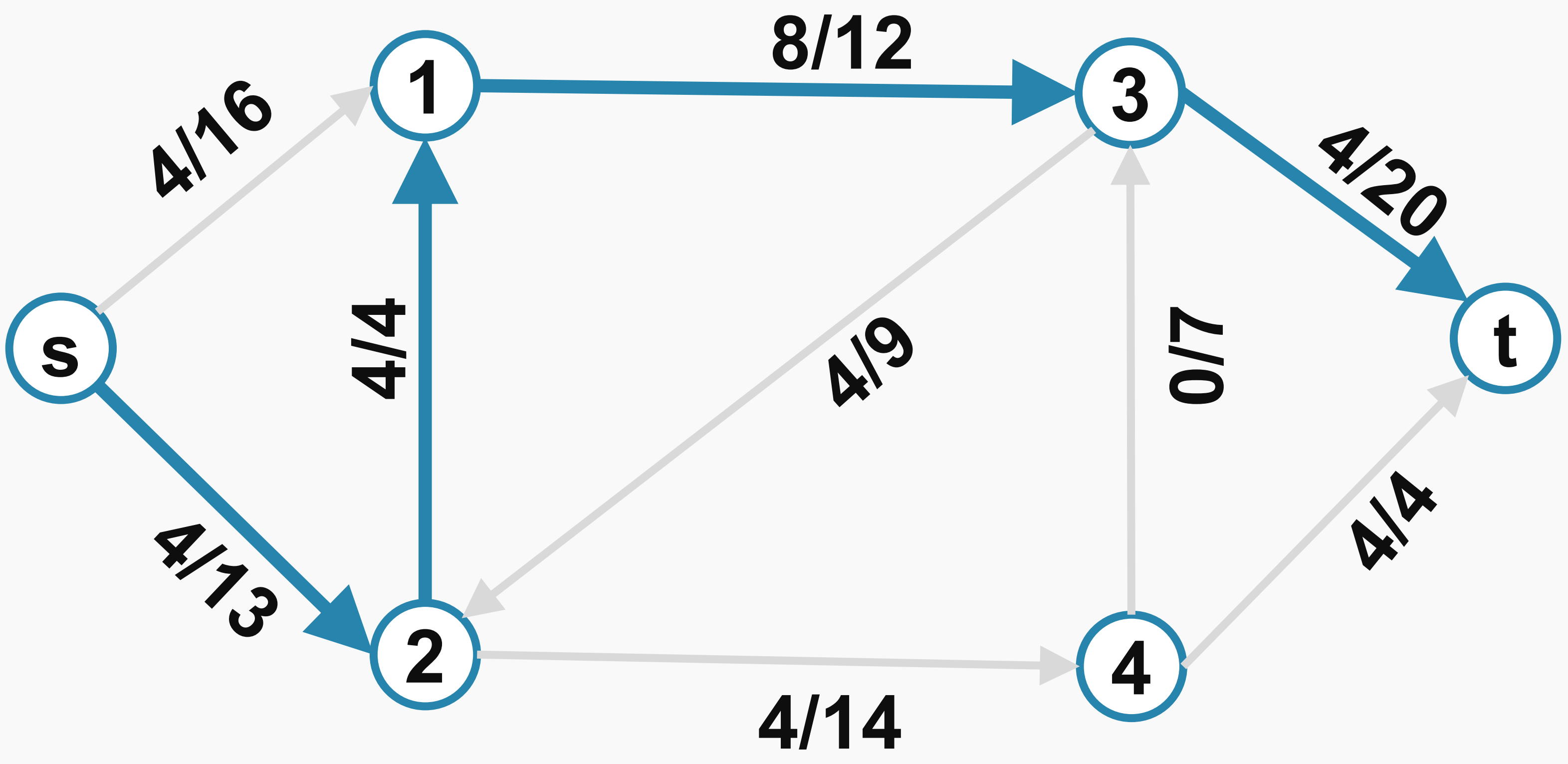

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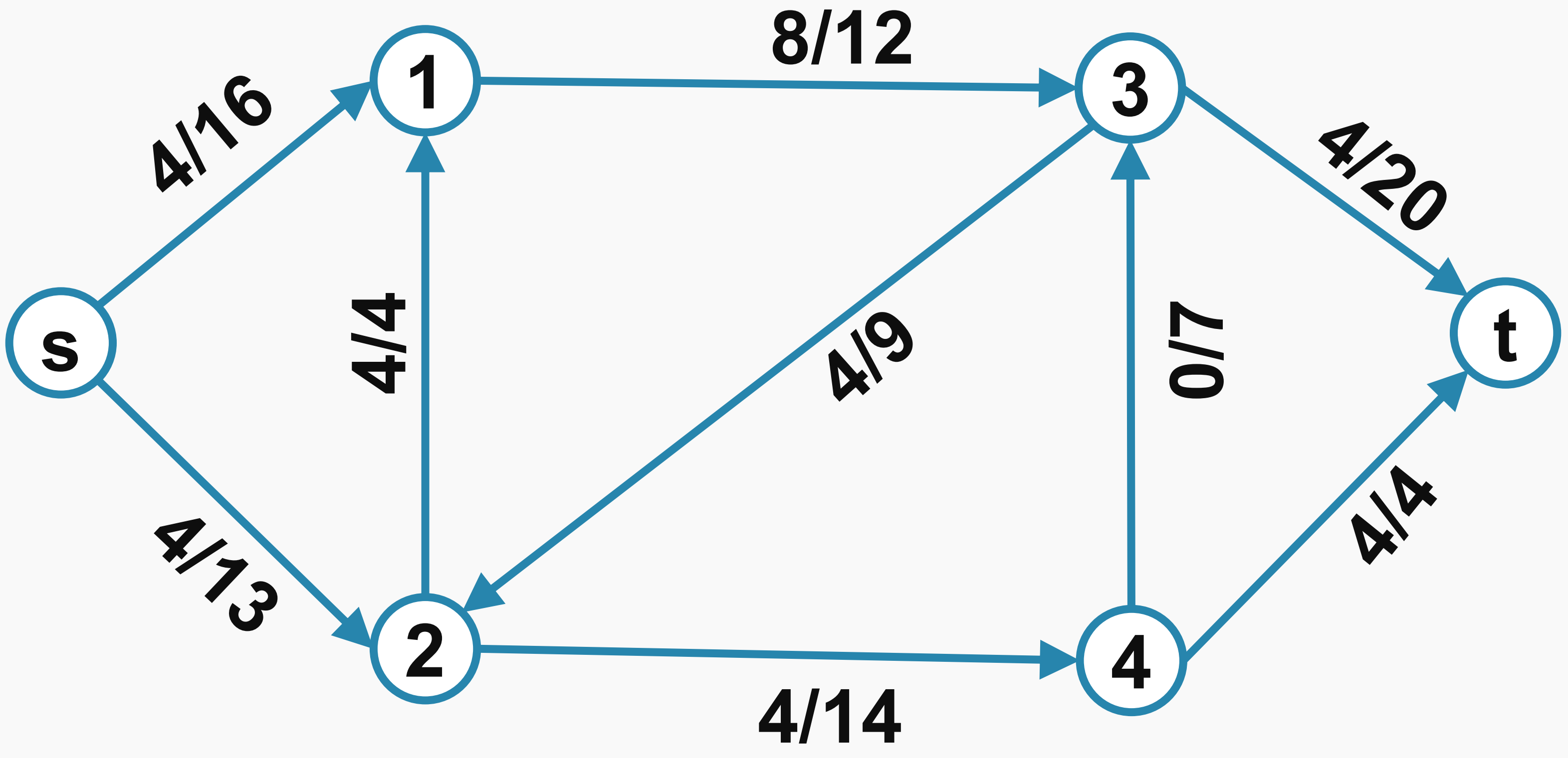

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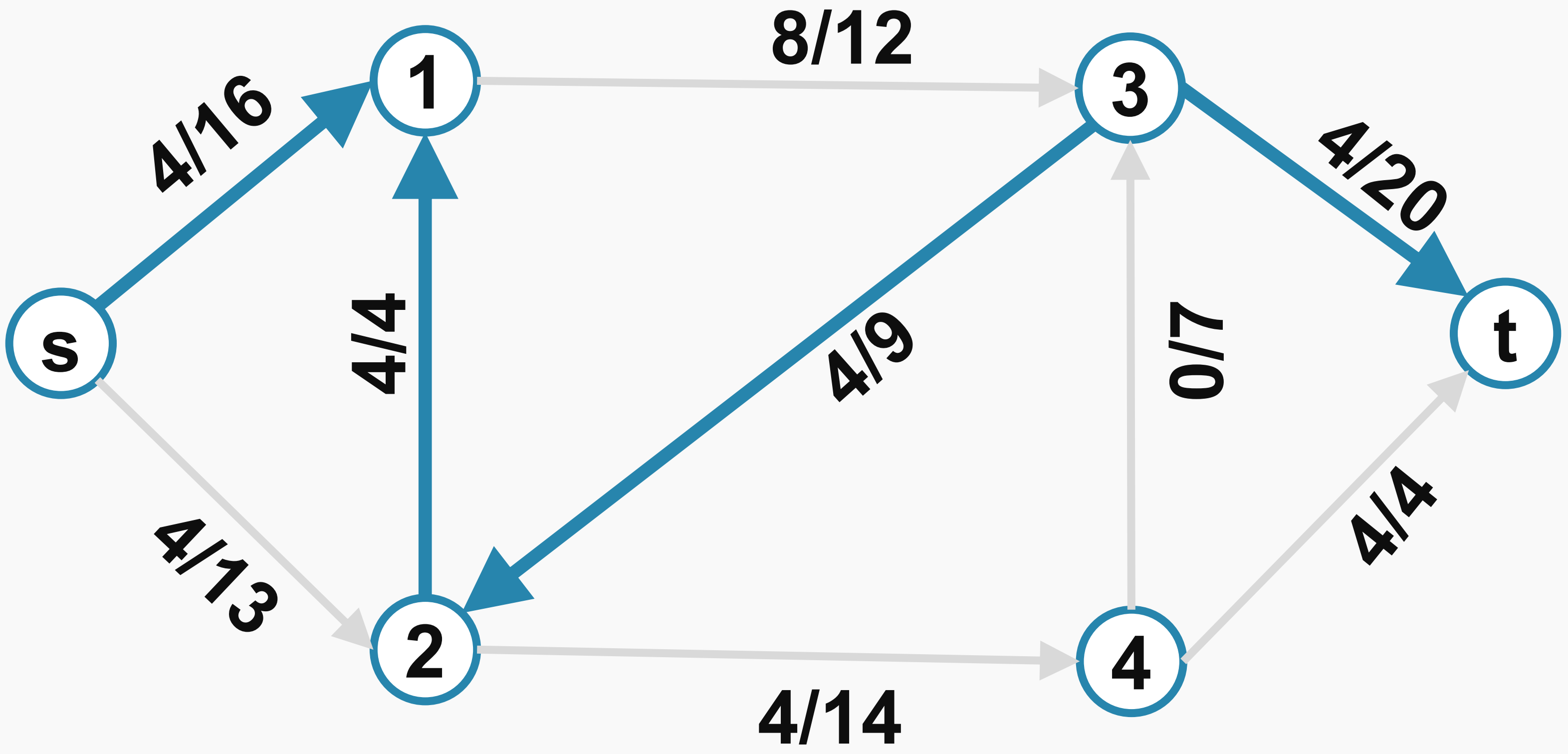

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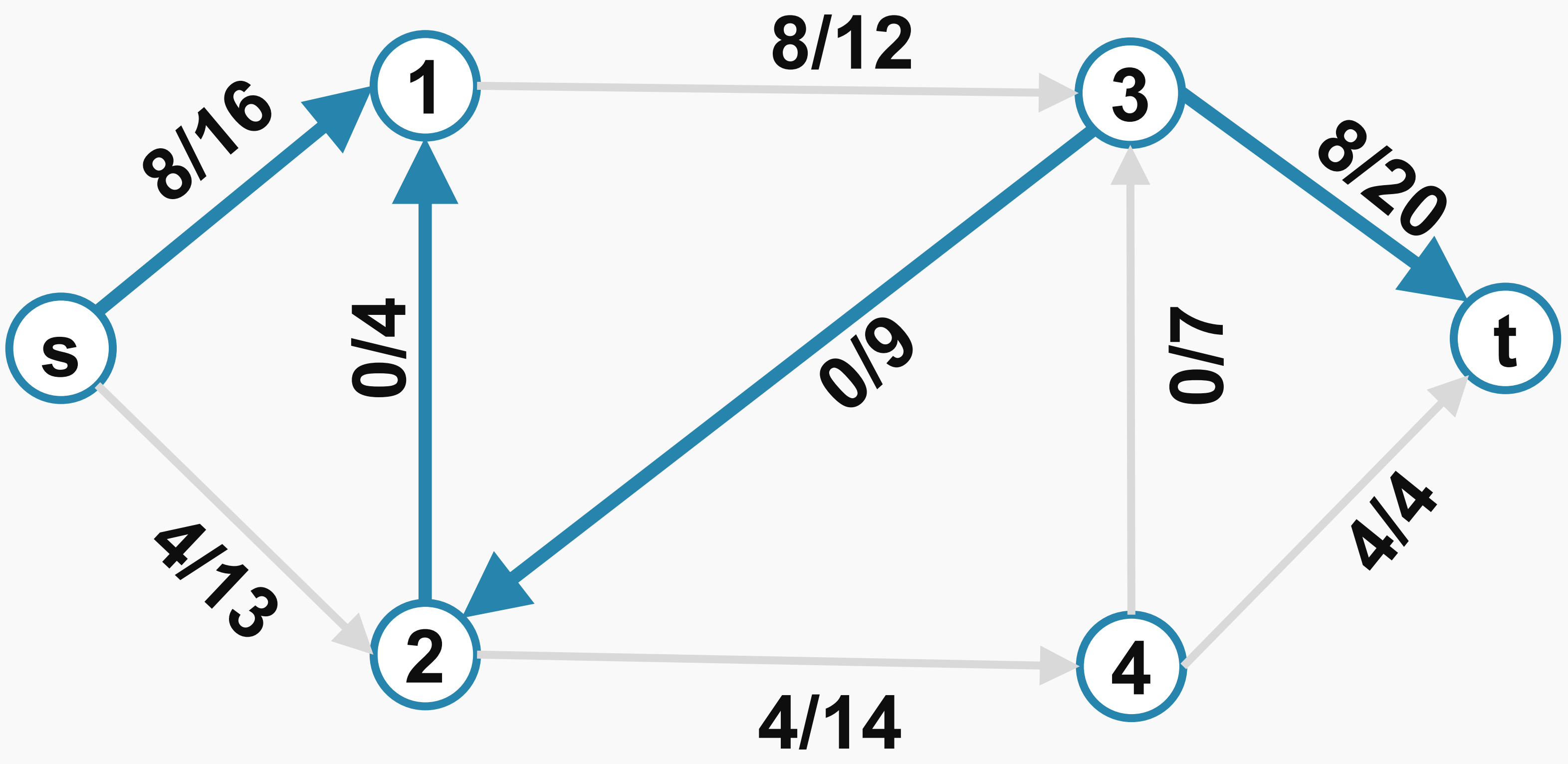

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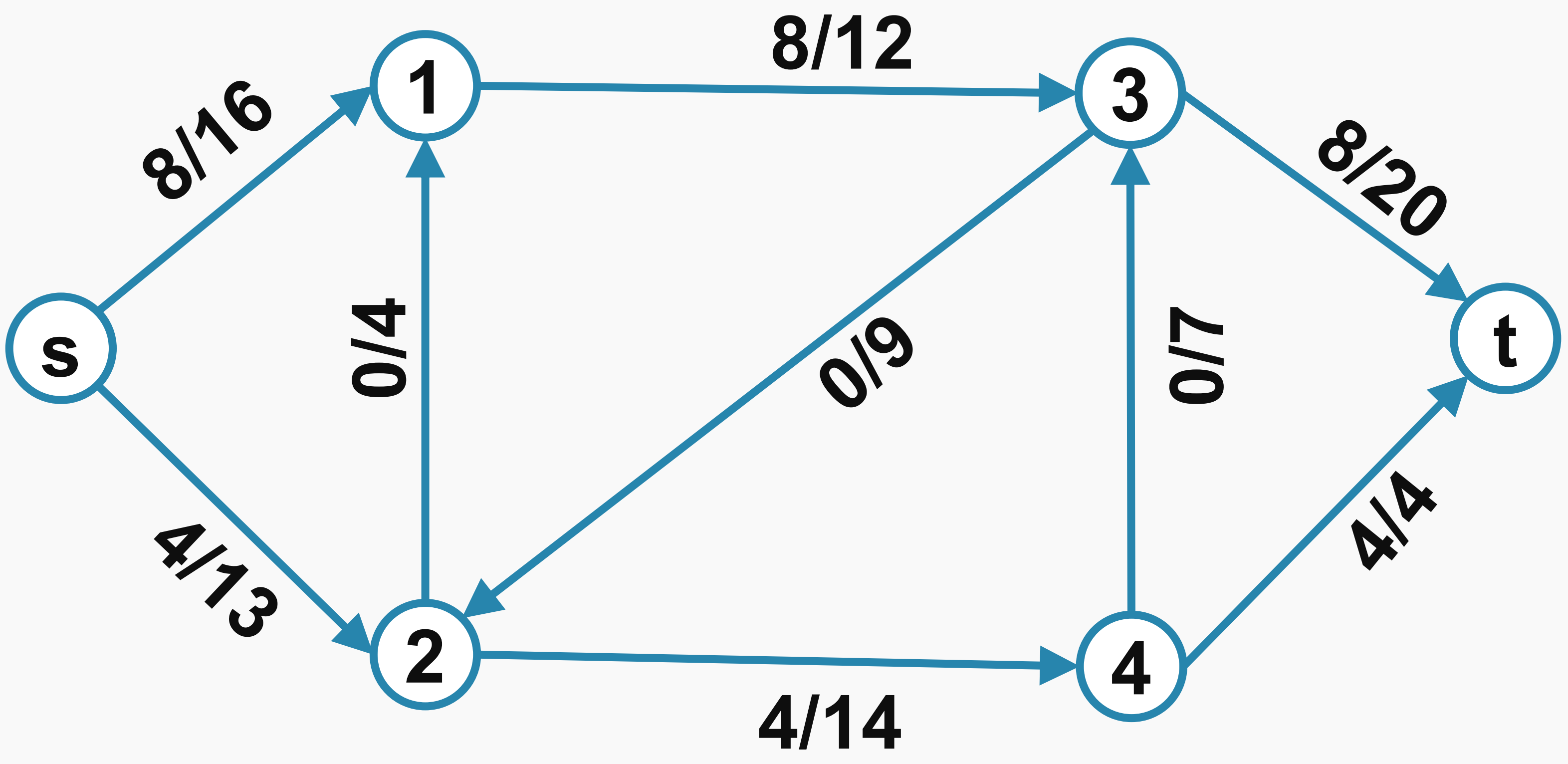

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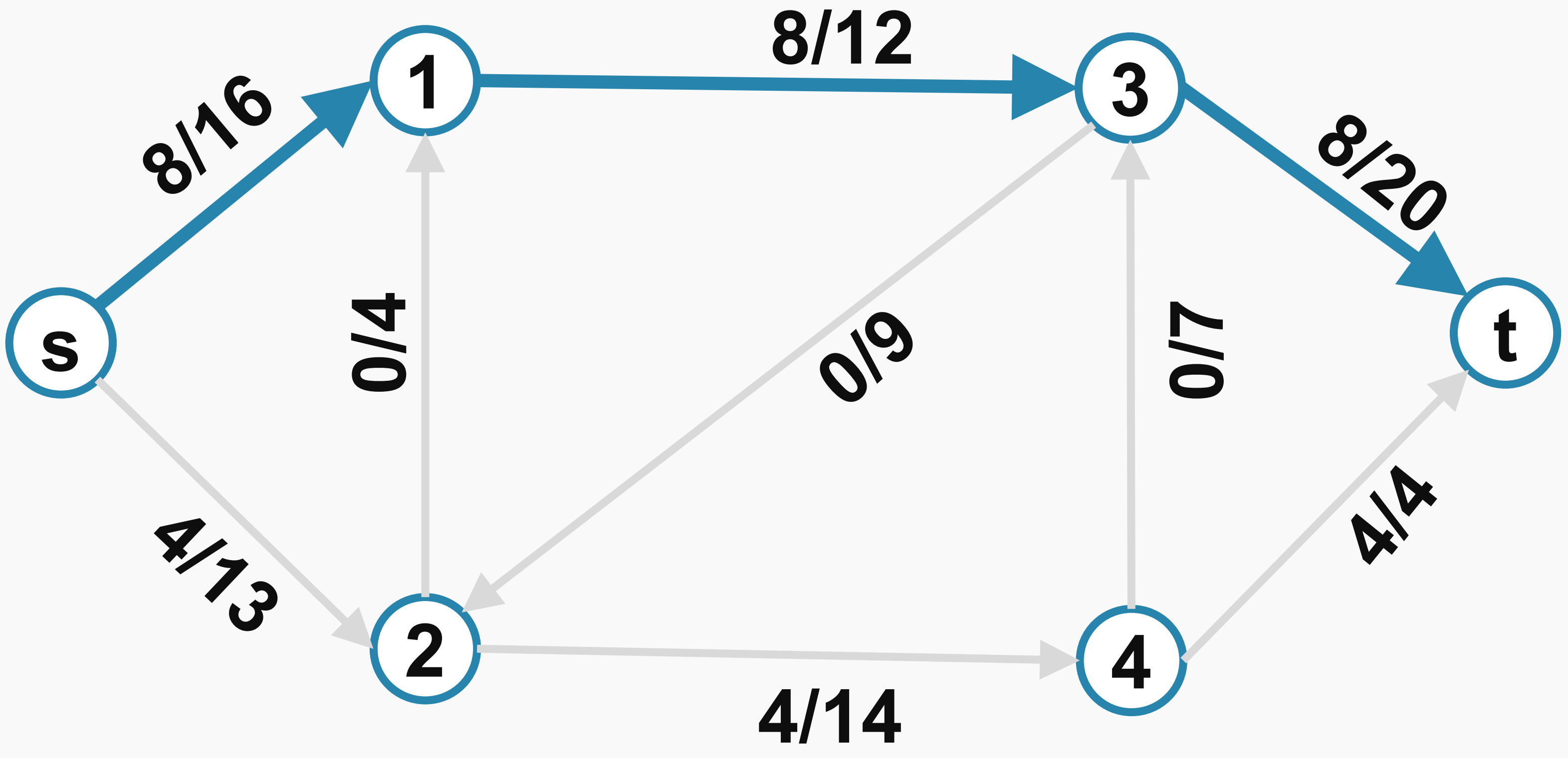

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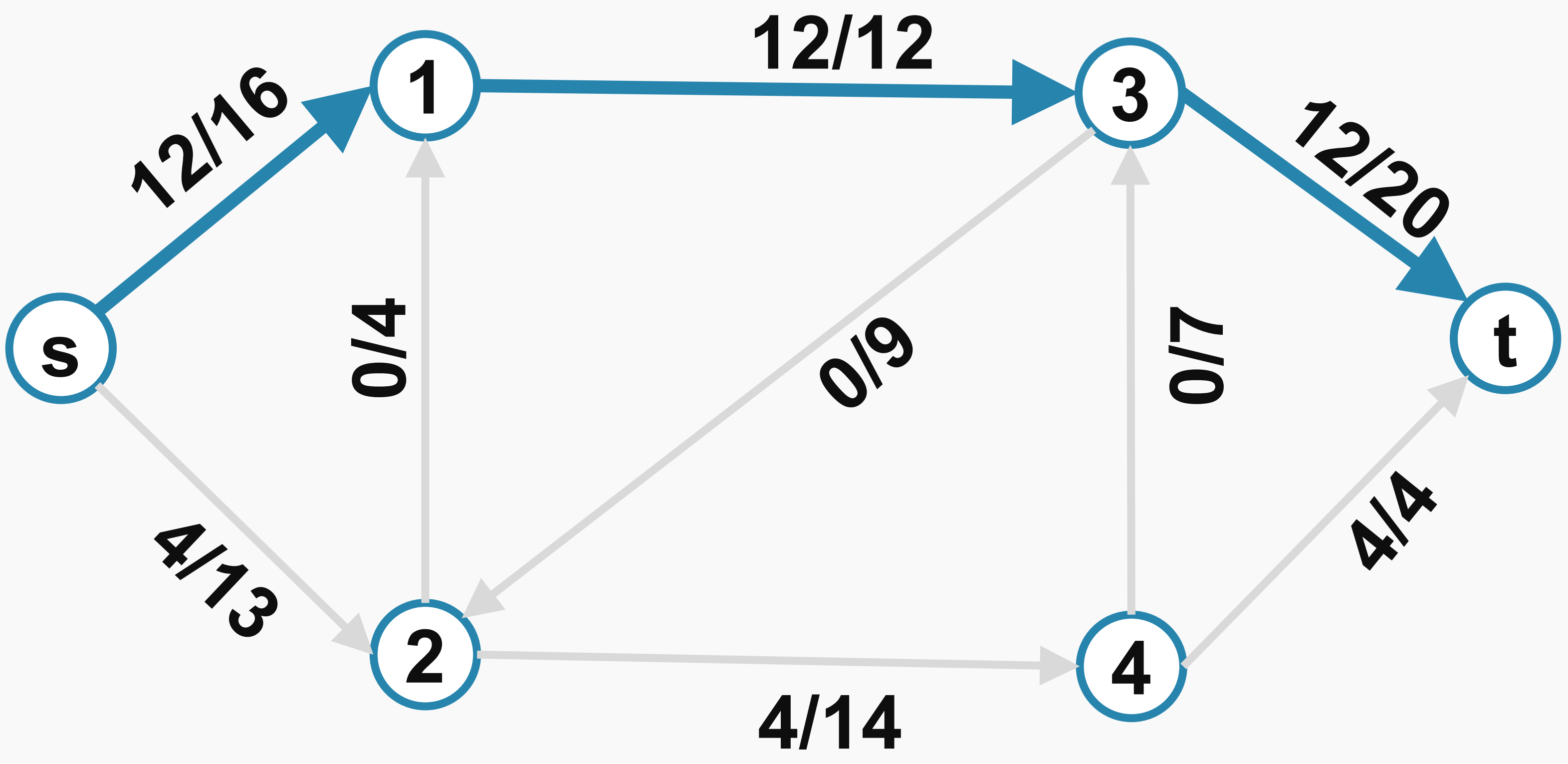

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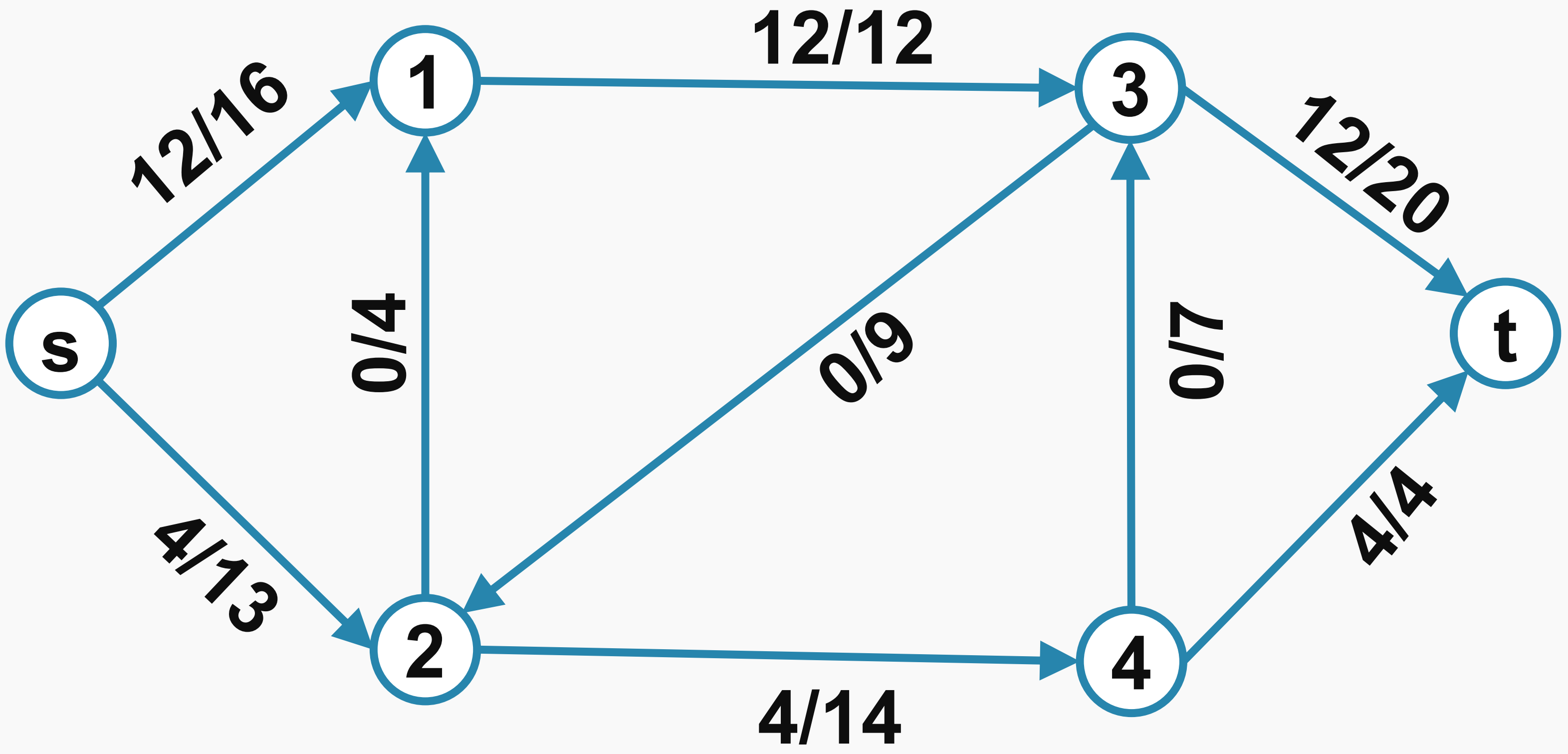

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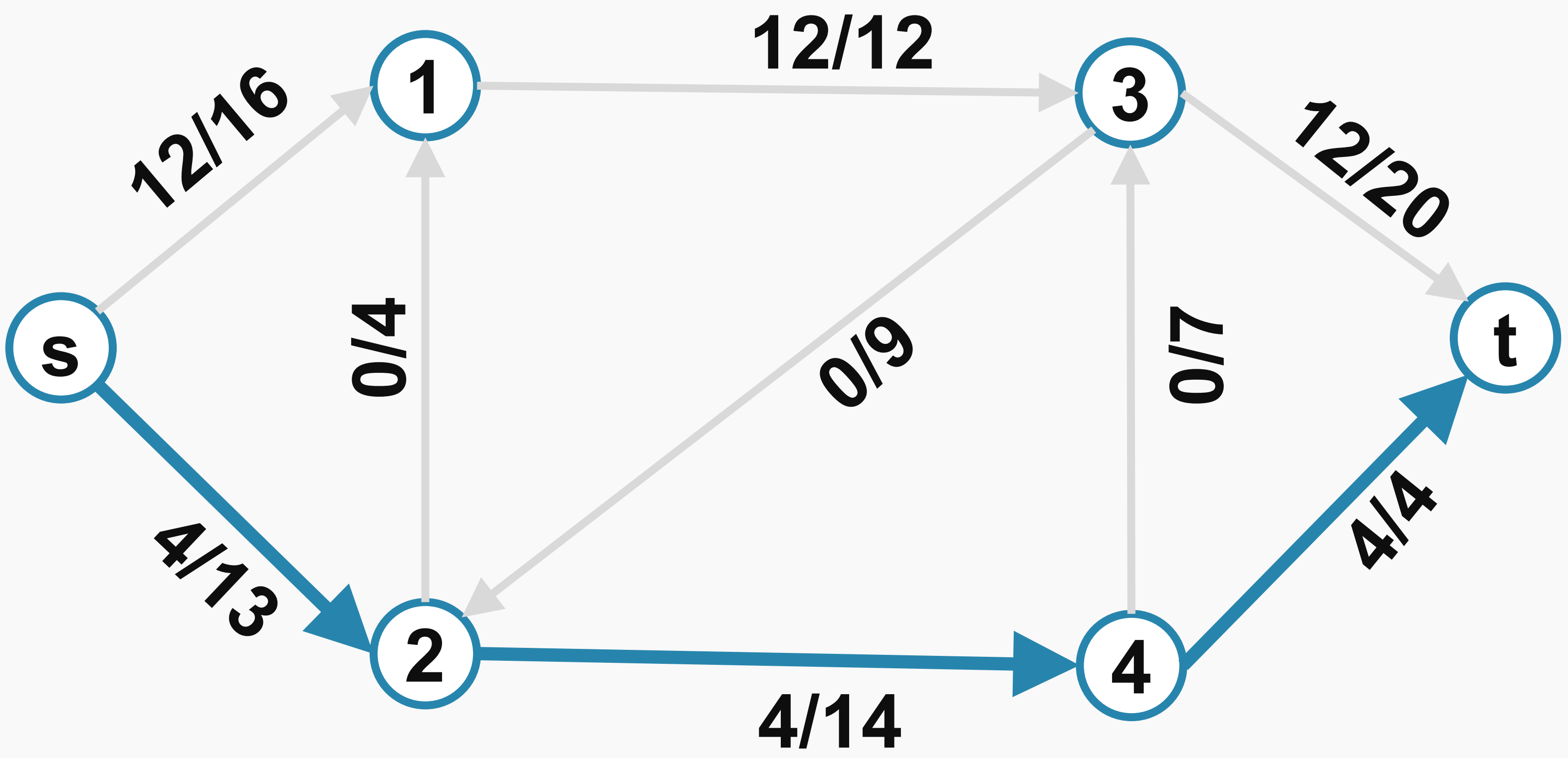

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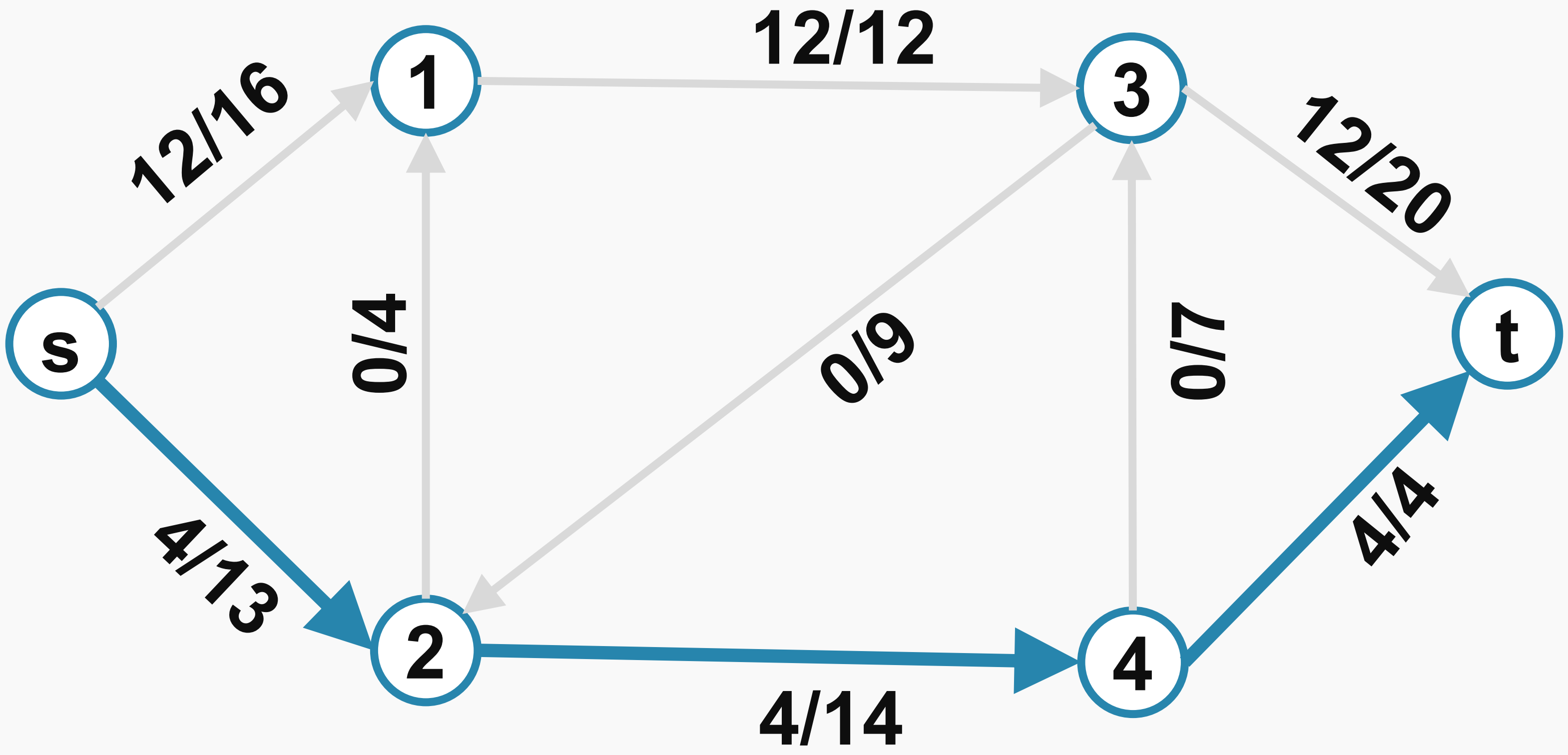

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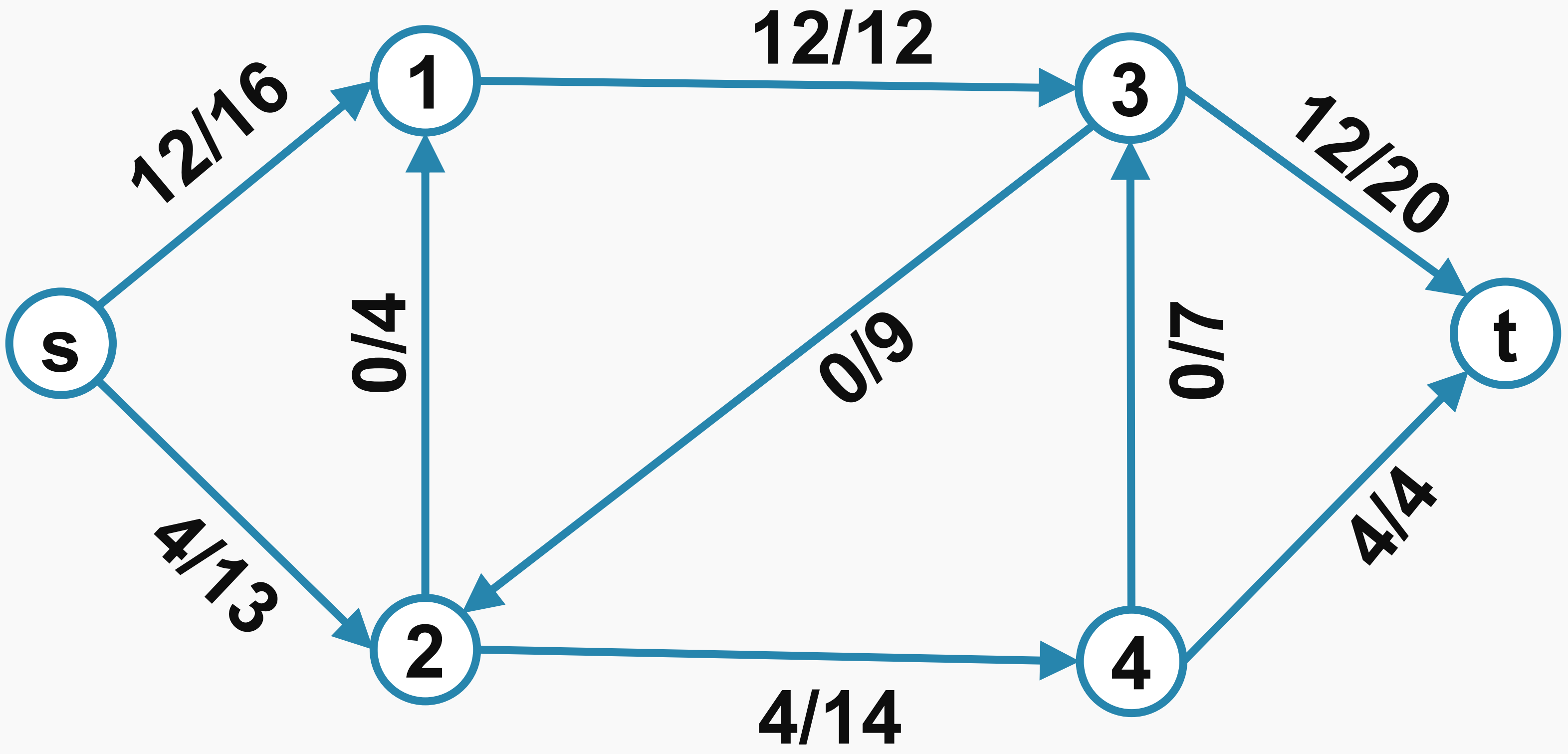

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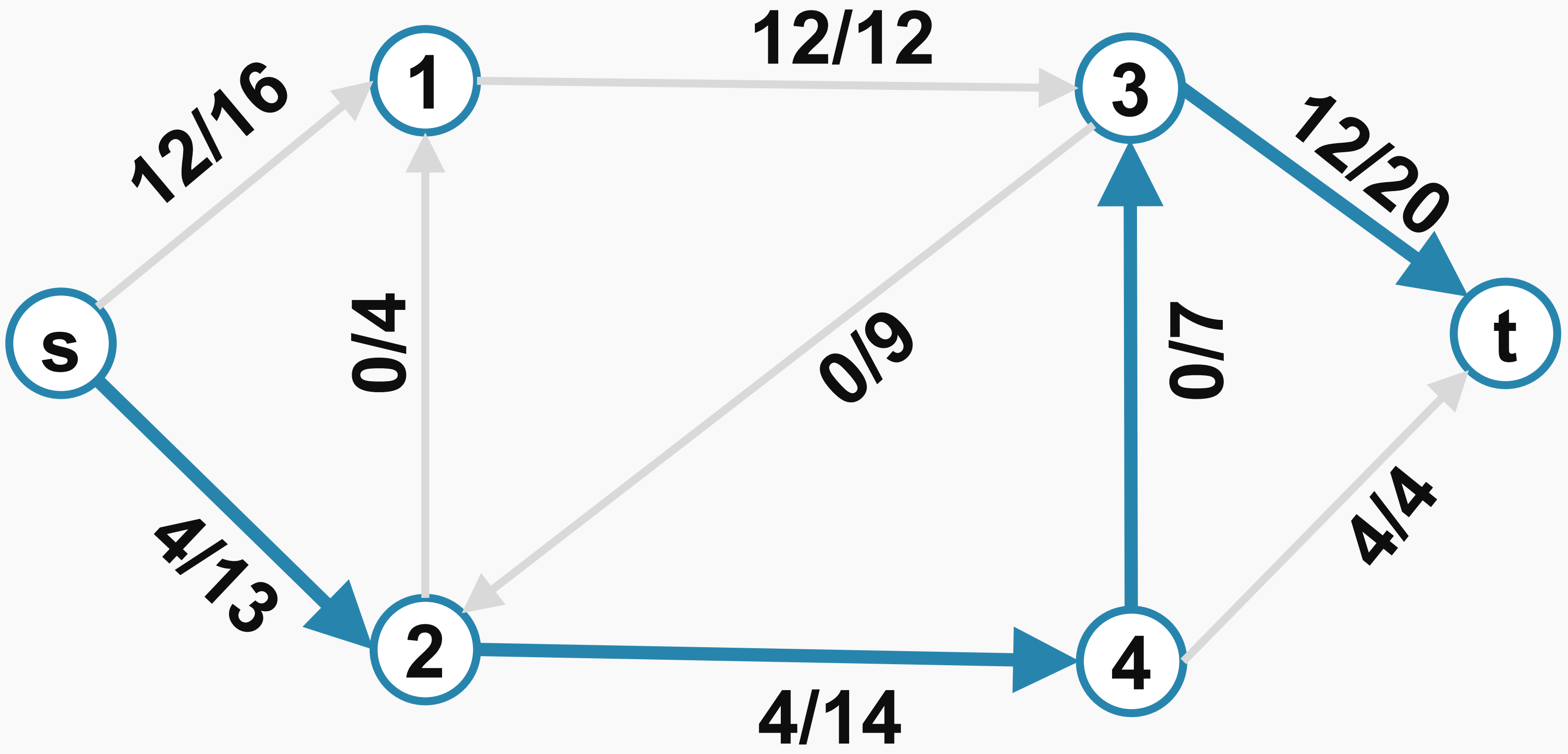

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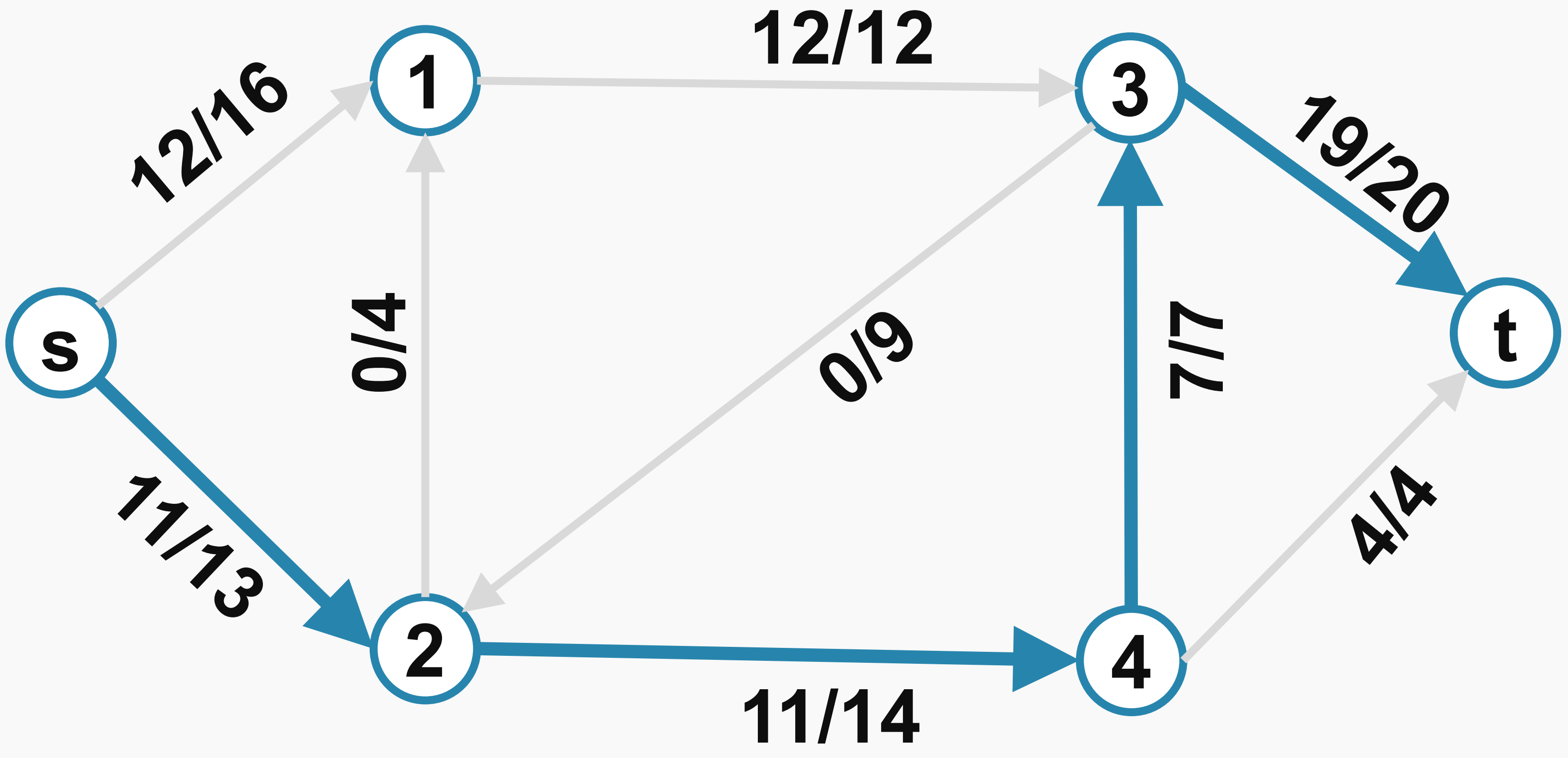

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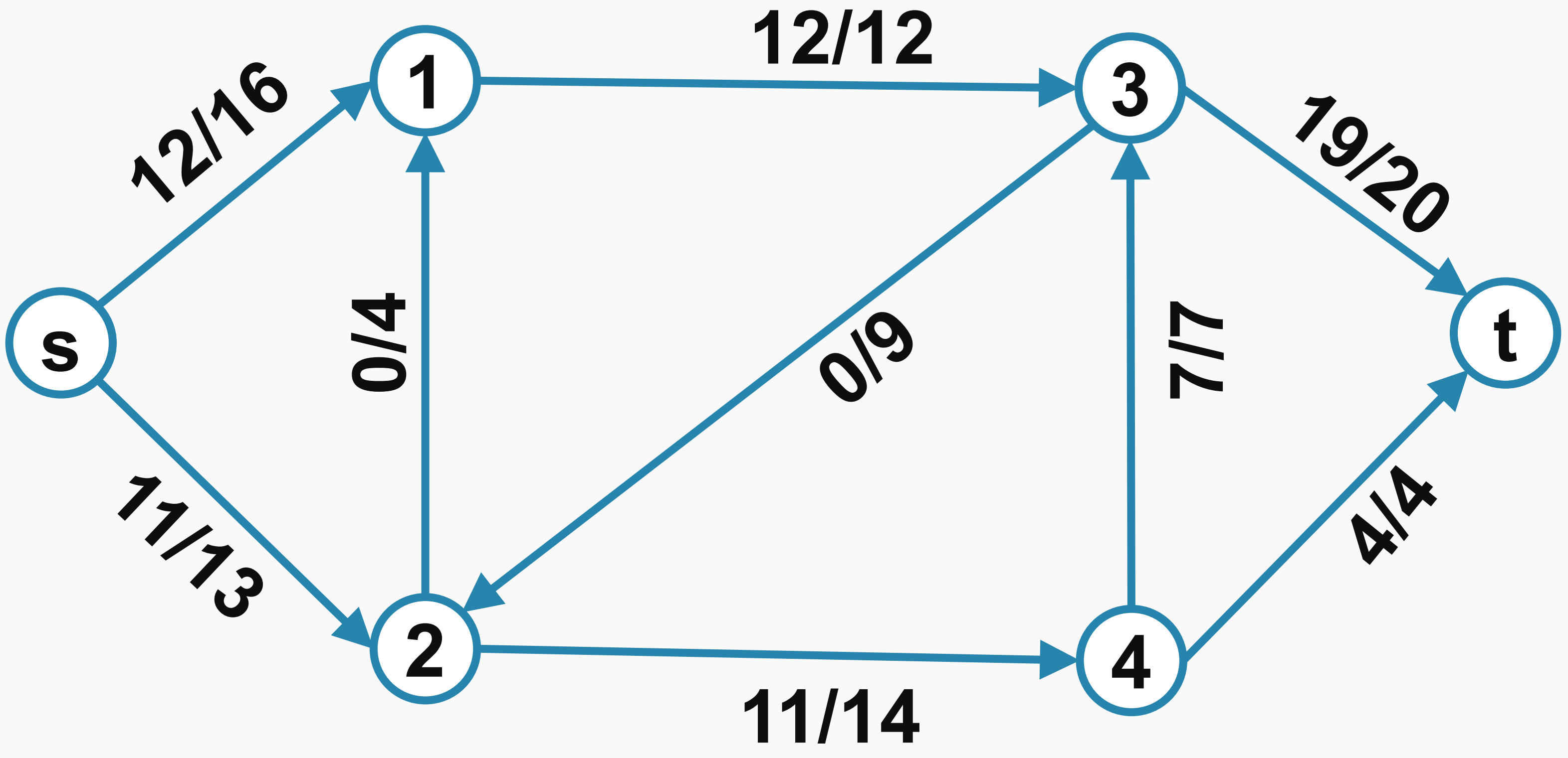

Complexitate?

FORD-FULKERSON (G, s, t)
for each edge $(u, v) \in G.E$
$\quad (u, v).f = 0$
while there exists a path p from s to t in the residual network G_f
$\quad c_f(p) = \min\{c_f(u, v): (u, v) \text{ is in } p\}$
$\quad$ **for** each edge (u, v) in p
$\quad\quad$ **if** $(u, v) \in G.E$
$\quad\quad\quad (u, v).f = (u, v).f + c_f(p)$
$\quad\quad$ **else**
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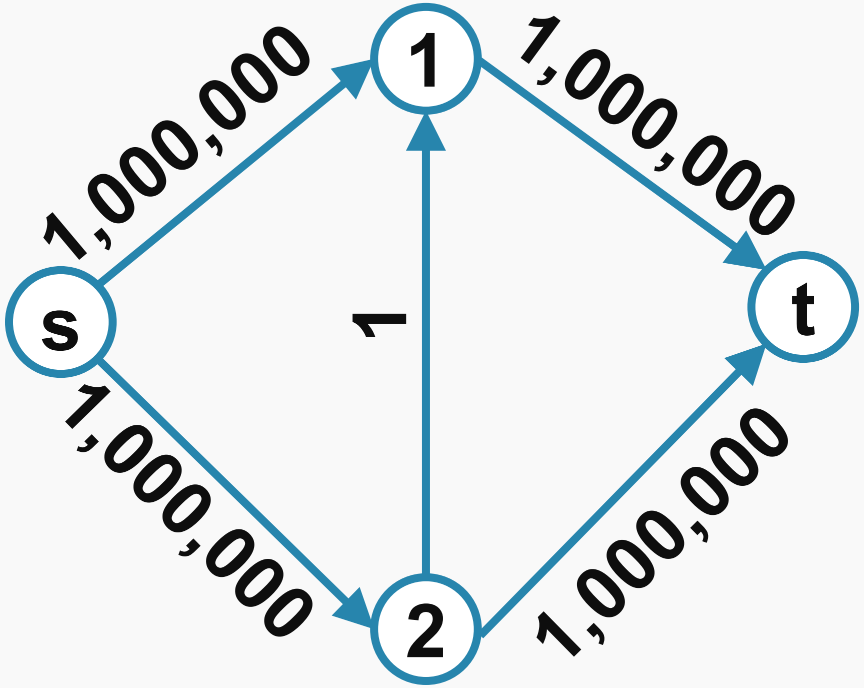

Complexitate?

Dacă alegem p random

$$O(E|f^*|)$$

FORD-FULKERSON (G, s, t)
for each edge $(u, v) \in G.E$
    $(u, v).f = 0$
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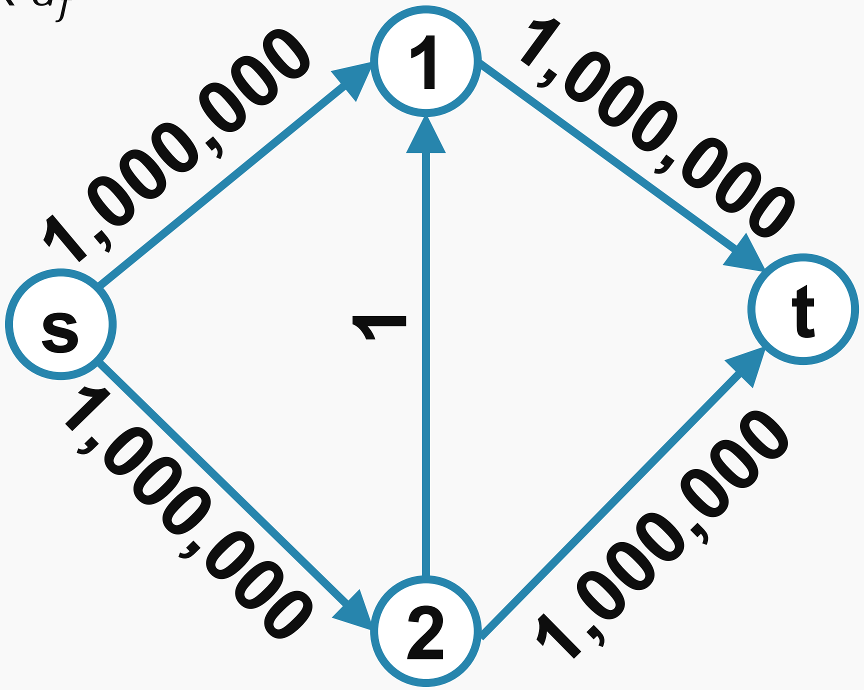
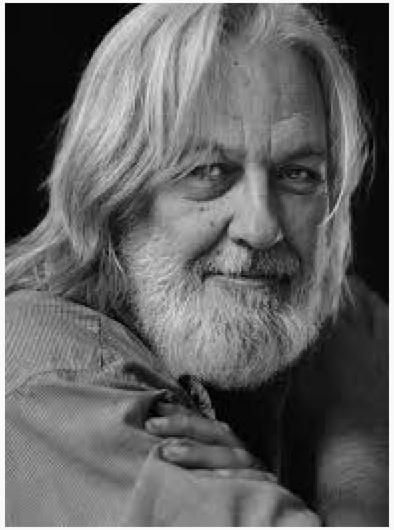


Algoritmul Edmonds–Karp (1972)

Dacă alegem p folosind BFS



$$O(VE^2)$$

Există și algoritmi cu complexitate mai bună

Table 1: Polynomial algorithms for the max flow problem

#	Due to	Year	Running Time
1	Ford & Fulkerson [11]	1956	$O(nmU)$
2	Edmonds and Karp [10]	1972	$O(nm^2)$
3	Dinic [9]	1970	$O(n^2m)$
4	Karzanov [19]	1974	$O(n^3)$
5	Cherkasky [7]	1977	$O(n^2\sqrt{m})$
6	Malhotra, Kumar & Maheshwari [22]	1977	$O(n^3)$
7	Galil [14]	1980	$O(n^{5/3}m^{2/3})$
8	Galil & Naaman [15]	1980	$O(nm\log^2 n)$
9	Sleator & Tarjan [23]	1983	$O(nm\log n)$
10	Gabow [13]	1985	$O(nm\log U)$
11	Goldberg & Tarjan [17]	1988	$O(nm\log(n^2/m))$
12	Ahuja & Orlin [2]	1989	$O(nm + n^2\log U)$
13	Ahuja, Orlin & Tarjan [3]	1989	$O(nm\log(n\sqrt{U}/(m+2))$
14	King, Rao & Tarjan [20]	1992	$O(nm + n^{2+\epsilon})$
15	King, Rao & Tarjan [21]	1994	$O(nm\log_{m/nlogn} n)$
16	Cheriyan, Hagerup & Mehlhorn [6]	1996	$O(n^3/\log n)$
17	Goldberg & Rao [16]	1998	$O(\min\{n^{2/3}, m^{1/2}\}m\log(n^2/m)\log U)$
18	Orlin [this paper]	2012	$O(nm)$
19	Orlin [this paper]	2012	$O(n^2/\log n)$ if $m = O(n)$

"Max flows in O(nm) time, or better" – James Orlin

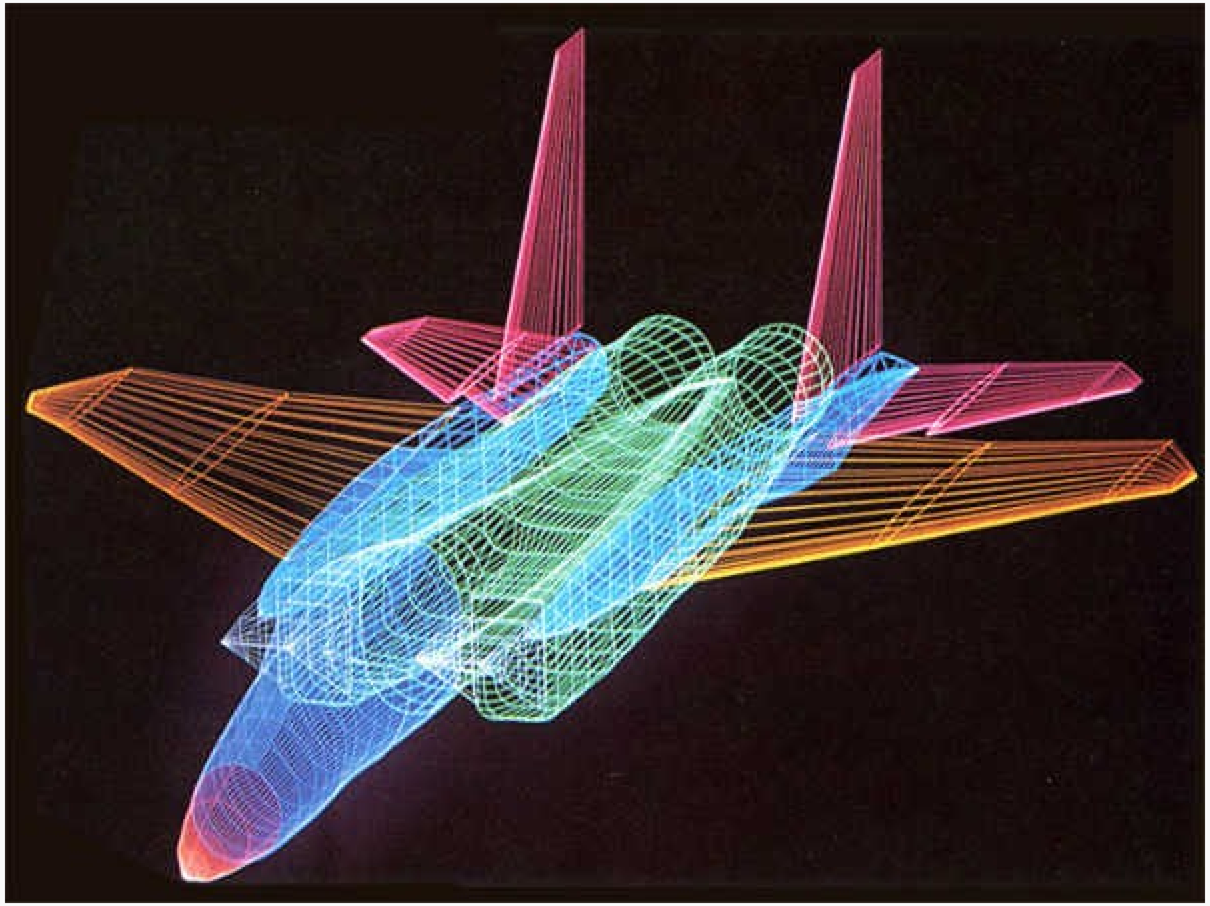
Alte considerente grafuri

Se pot schimba în timp.

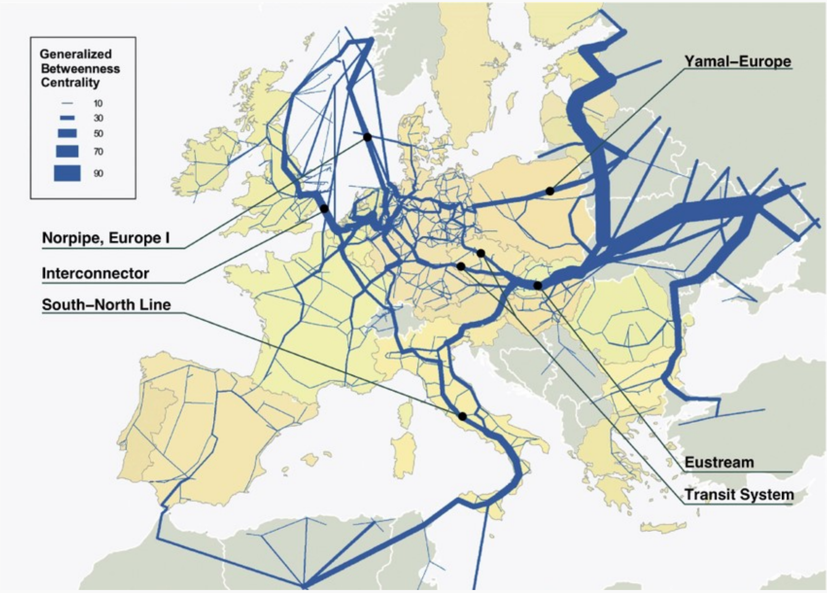
LineGraph – Pentru un graf non-direcțional muchiile devin noduri și nodurile muchii.

Use case grafuri – Granițe

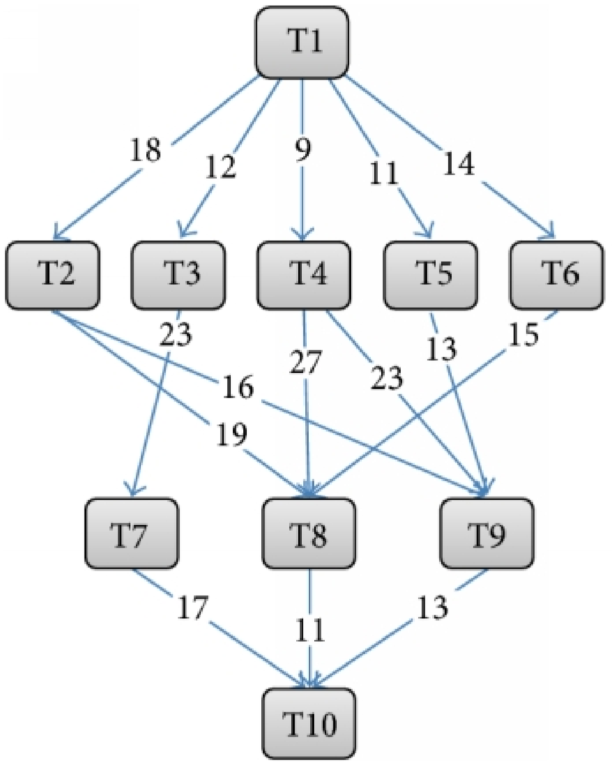
Use case grafuri – Grafică calculator



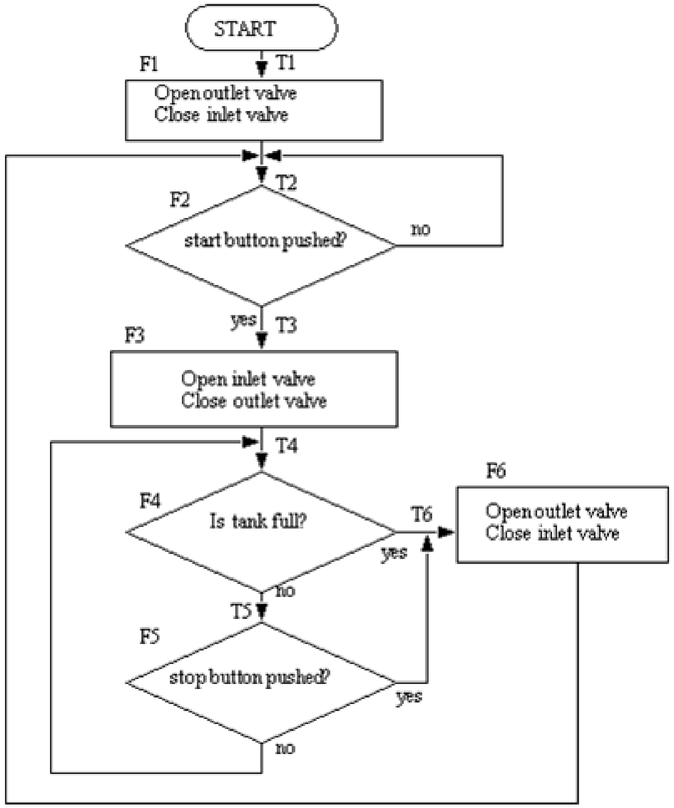
Use caser grafuri – utilități



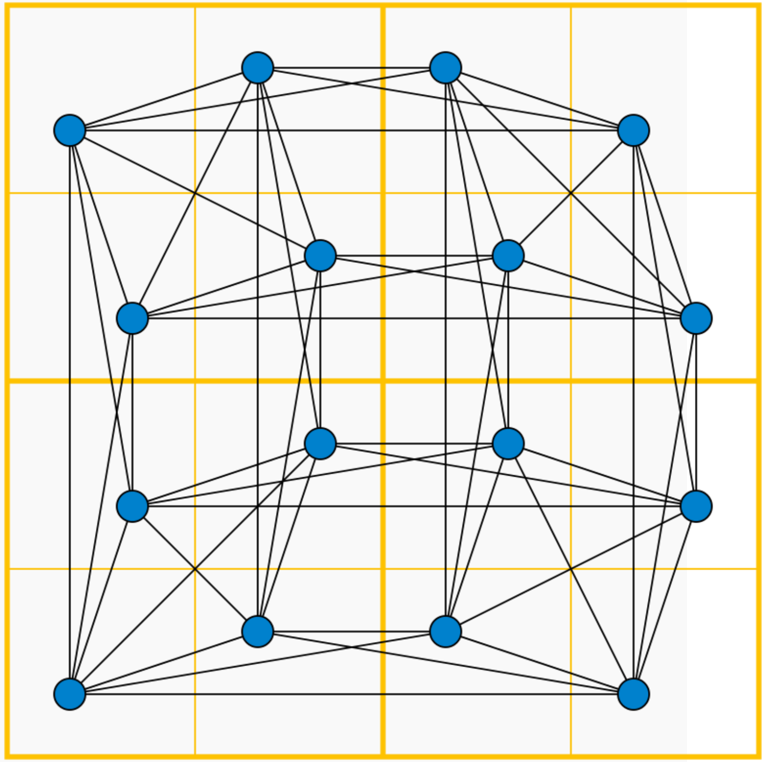
Use case grafuri – Code

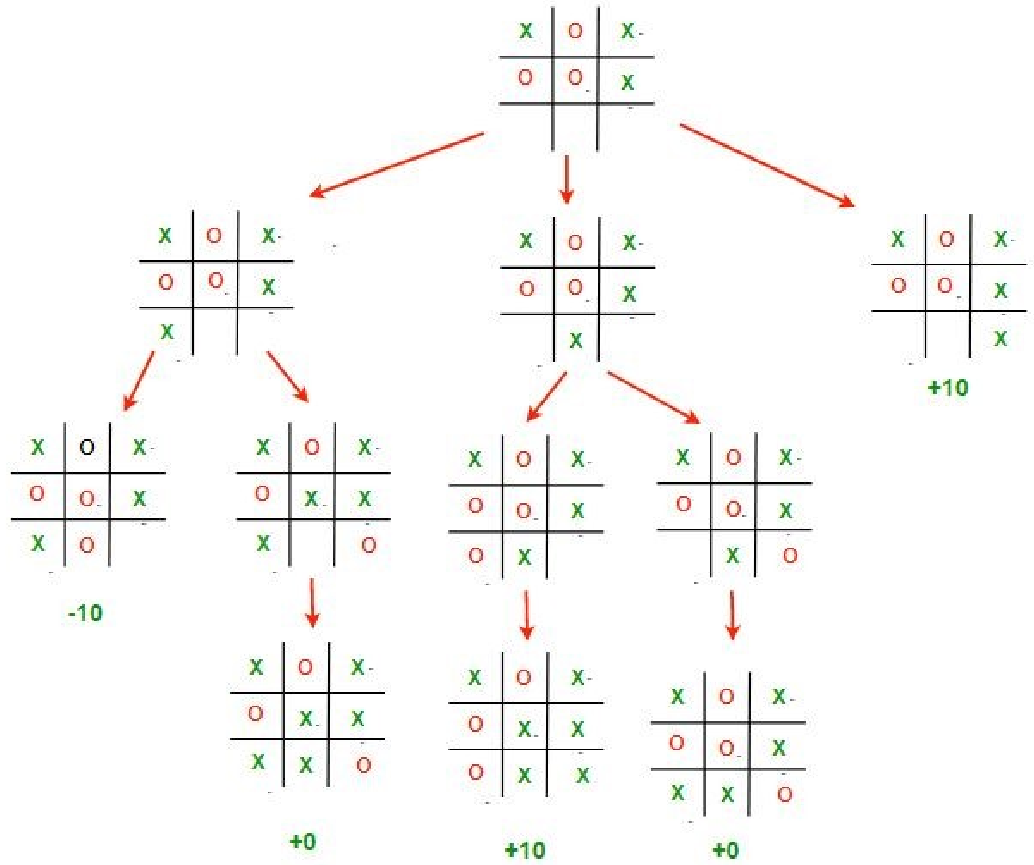


Task	P_1	P_2	P_3
T1	14	16	9
T2	13	19	18
T3	11	13	19
T4	13	8	7
T5	12	13	10
T6	13	16	9
T7	7	15	11
T8	5	11	14
T9	18	12	20
T10	21	7	16

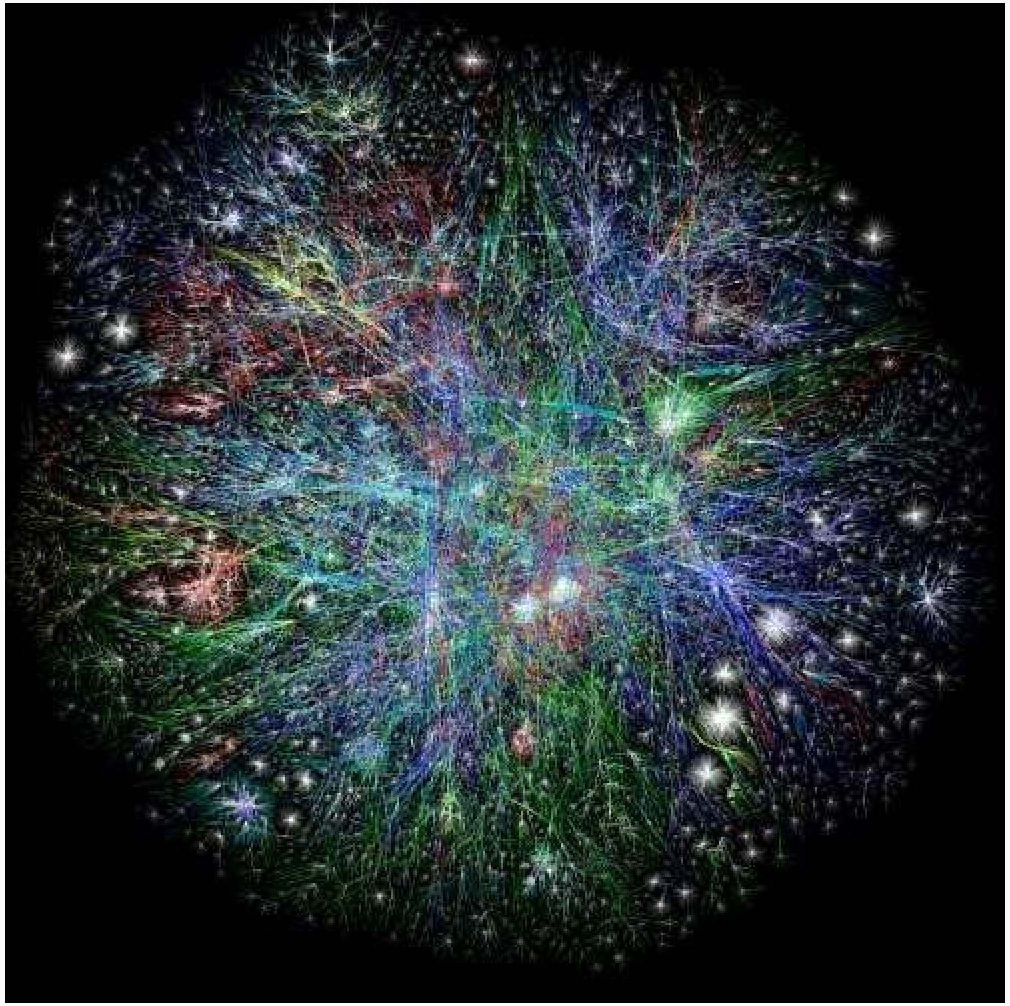


Use case grafuri – Reprezentare Jocuri

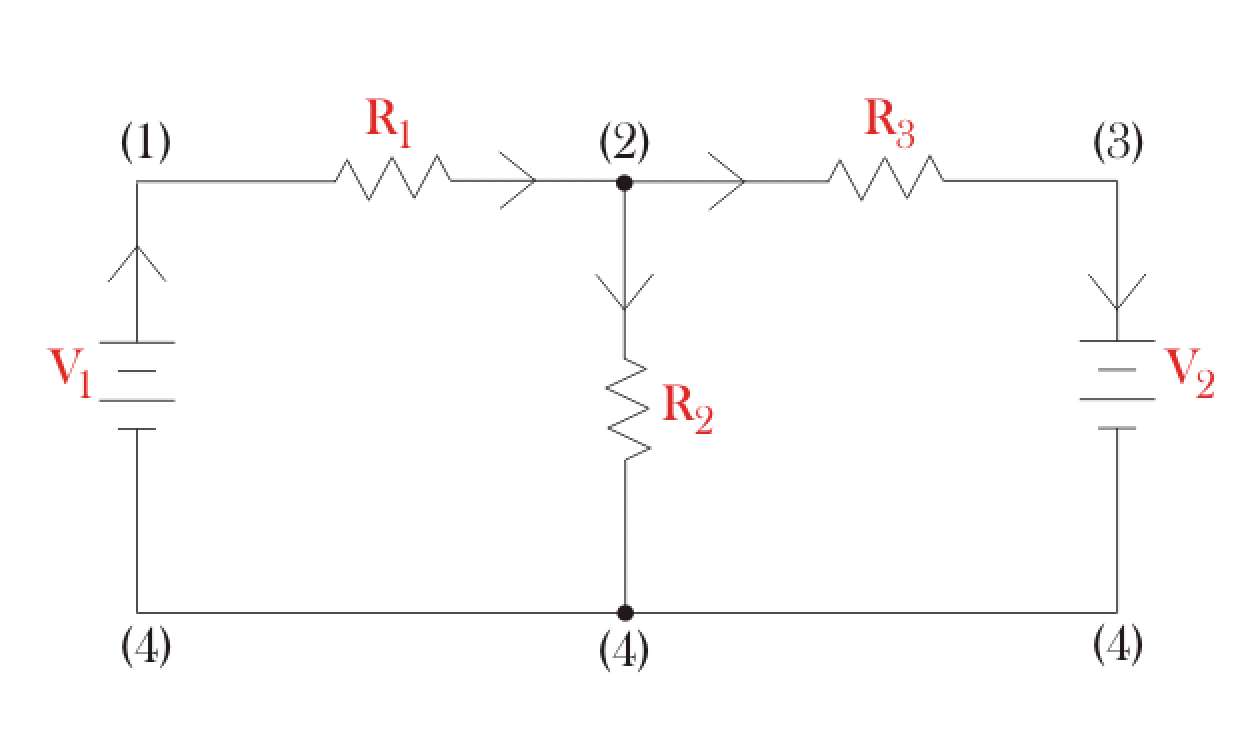




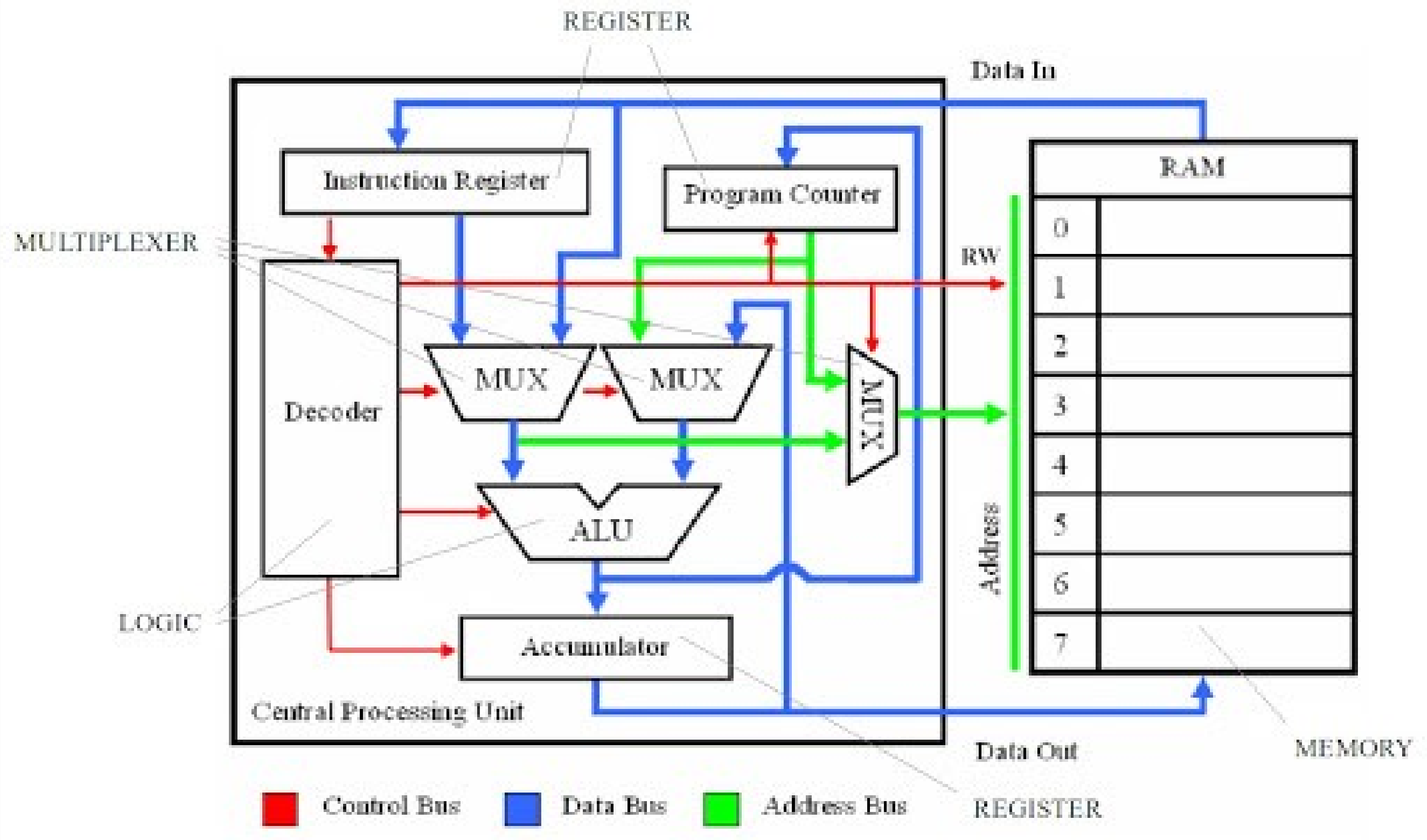
Use case grafuri - Internet



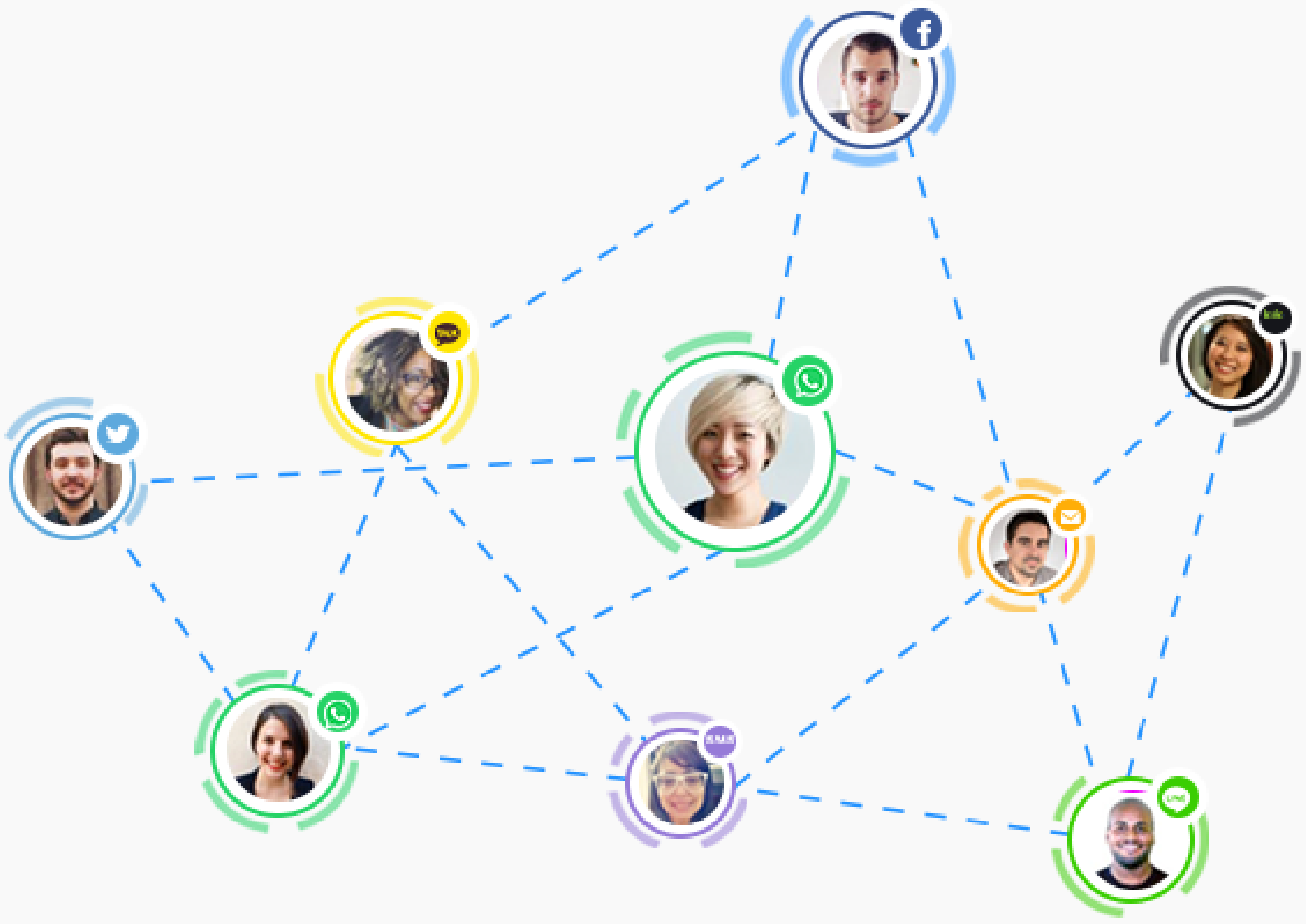
Use case grafuri – Circuite electrice



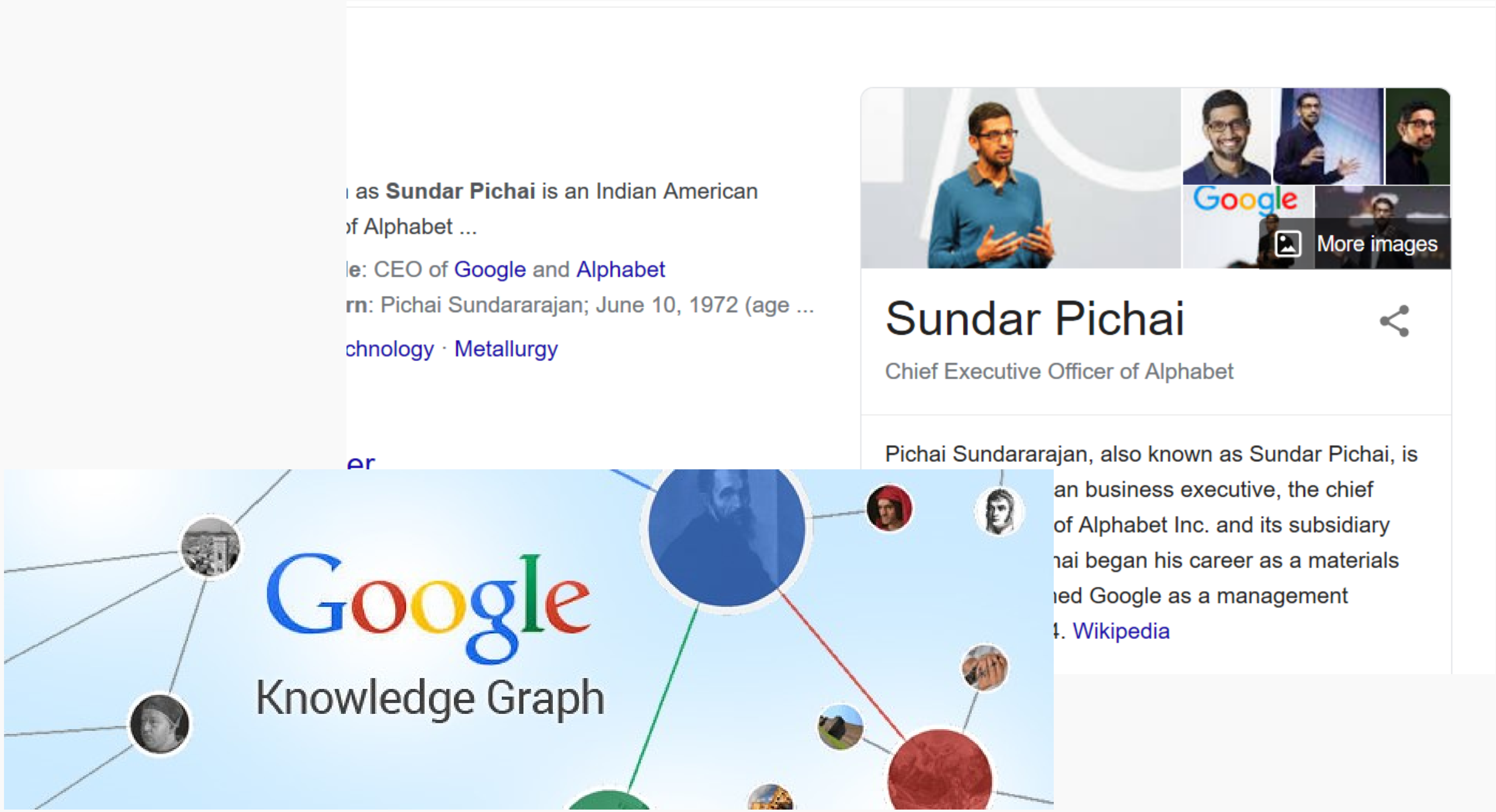
Use case grafuri – Circuite logice



Use case grafuri – Grafuri sociale



Use case-uri – Knowledge graph



Use case-uri – Organigrame