



# Arhitecturi Paralele

## Abordarea algoritmilor în mod paralel

Lect. Dr. Ing. Cristian Chilipirea – [cristian.chilipirea@mta.ro](mailto:cristian.chilipirea@mta.ro)





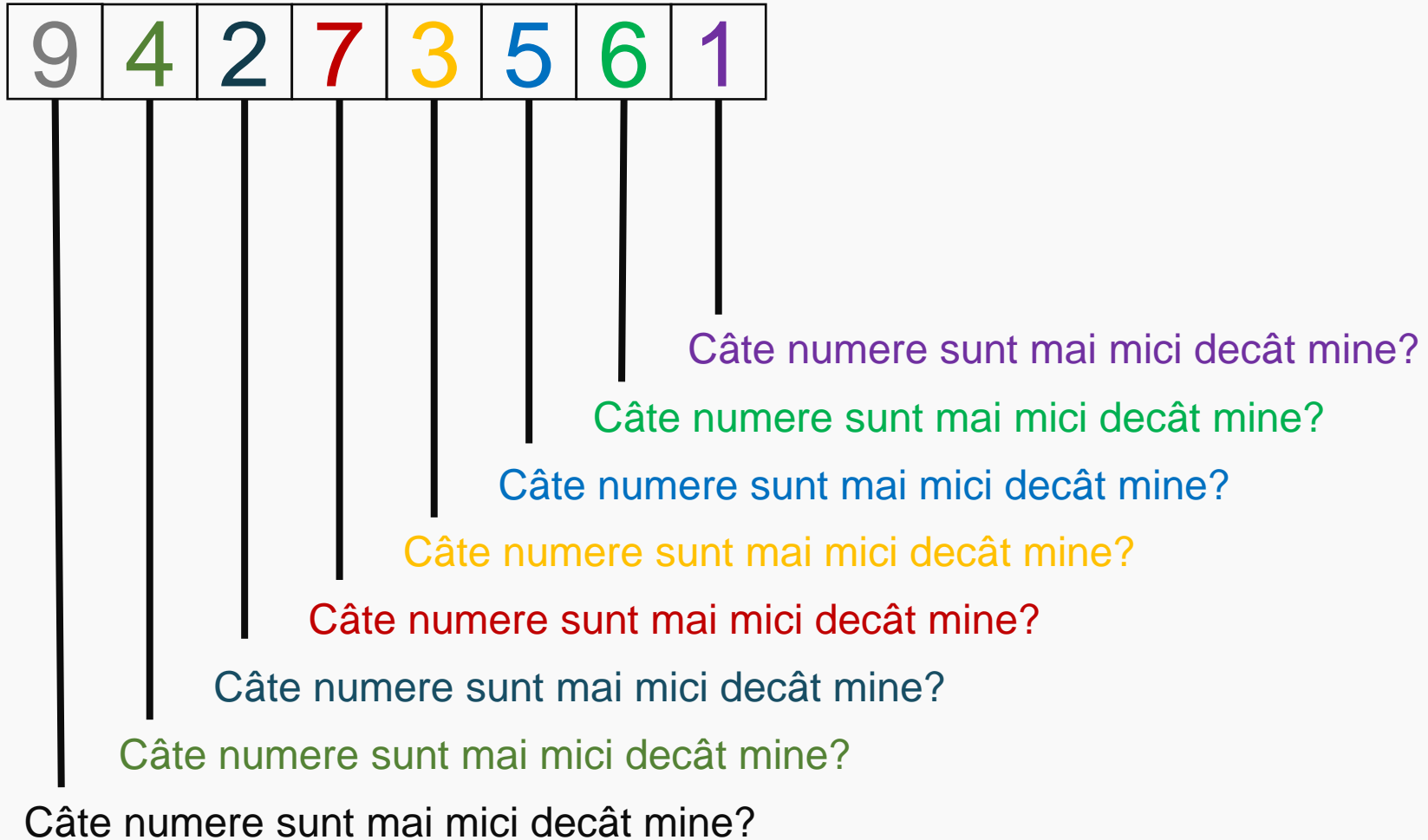
# Rank Sort

9	4	2	7	6	5	6	1
---	---	---	---	---	---	---	---

1	2	4	5	6	6	7	9
---	---	---	---	---	---	---	---



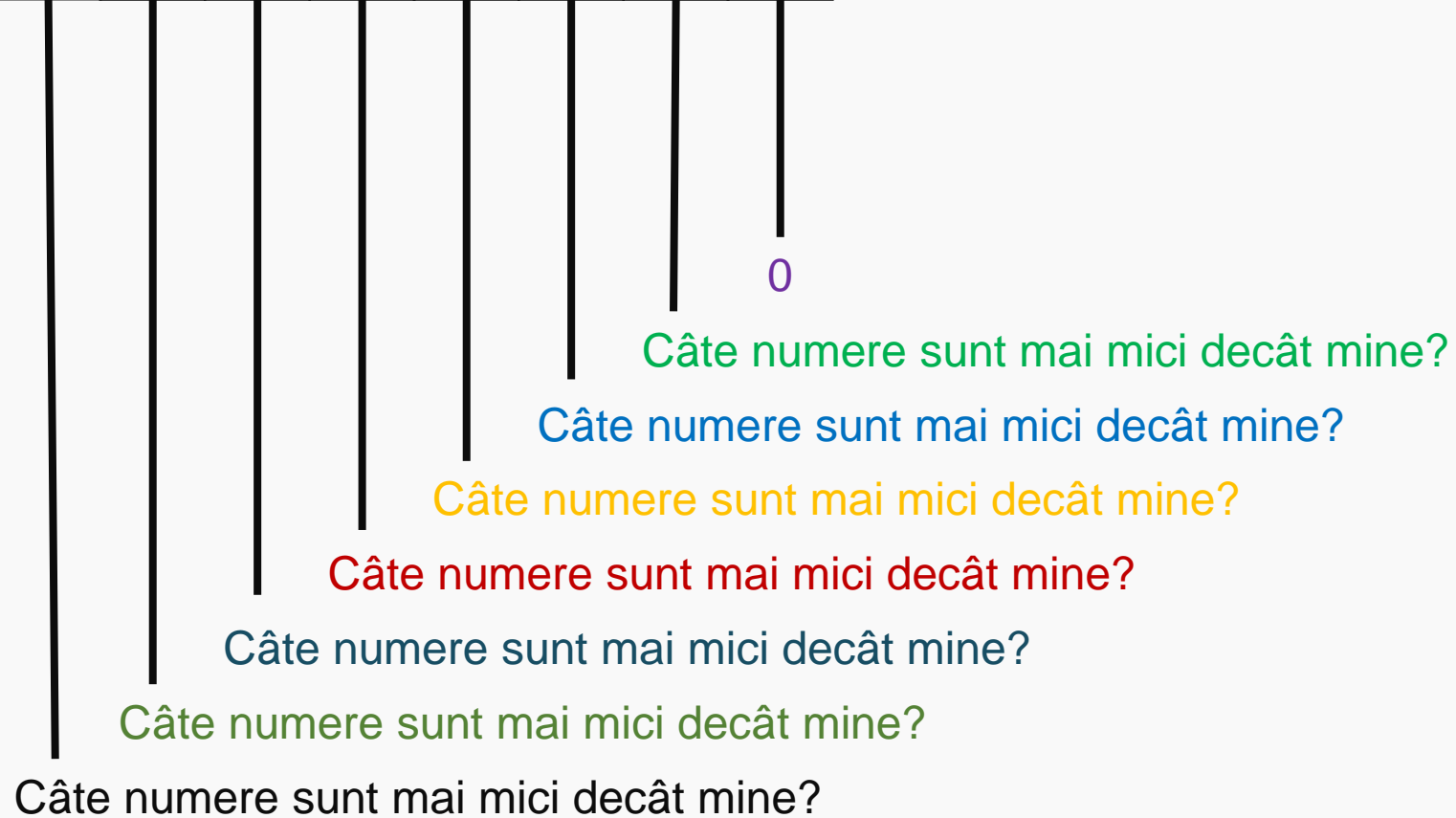
# Rank Sort





# Rank Sort

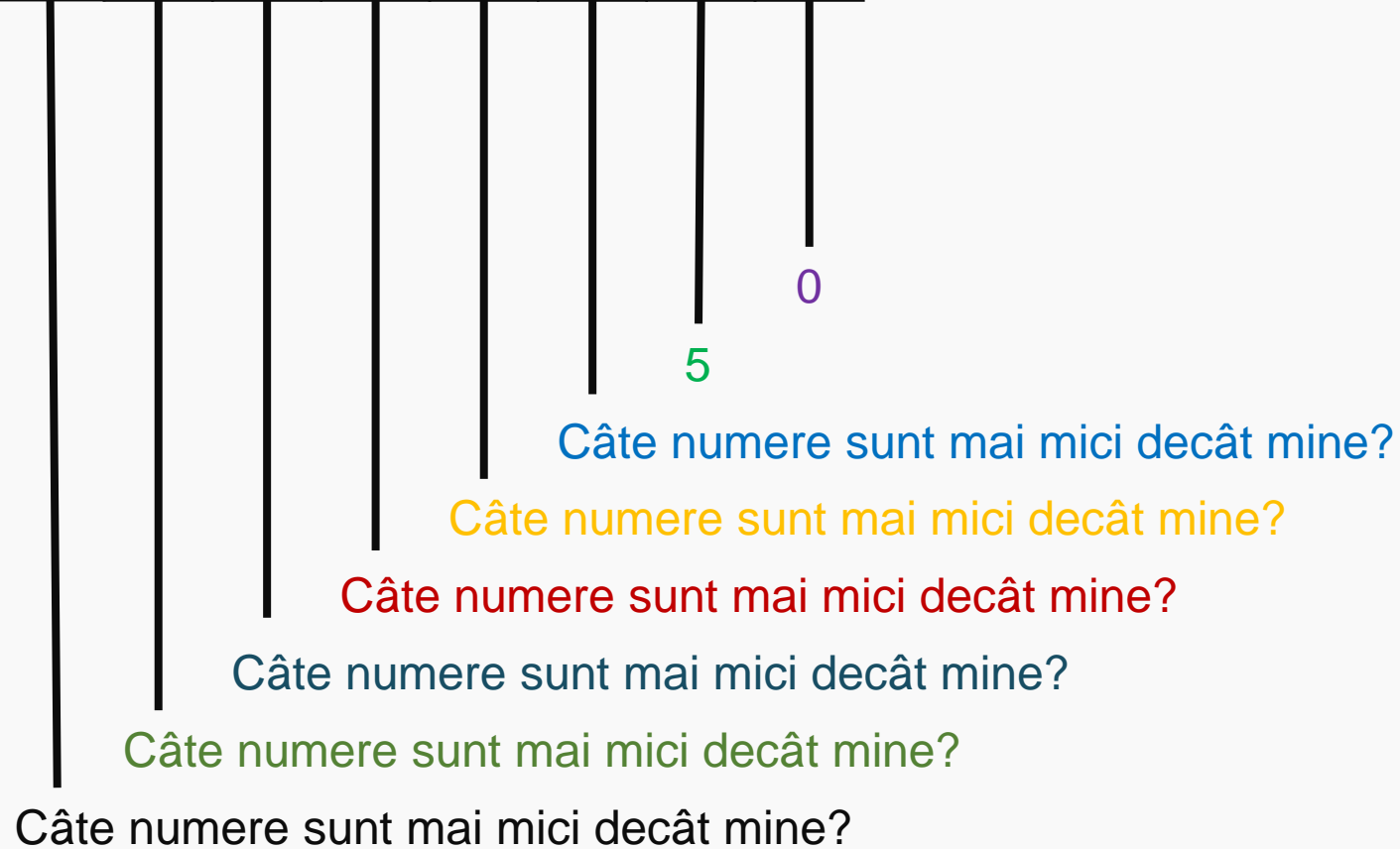
9	4	2	7	3	5	6	1
---	---	---	---	---	---	---	---





# Rank Sort

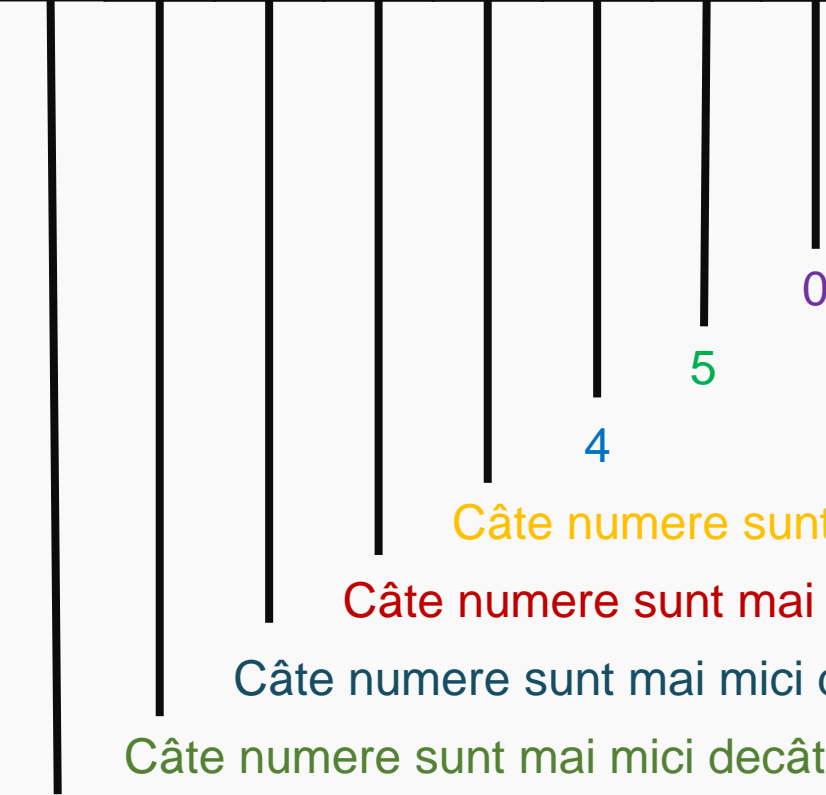
9	4	2	7	3	5	6	1
---	---	---	---	---	---	---	---





# Rank Sort

9	4	2	7	3	5	6	1
---	---	---	---	---	---	---	---



Câte numere sunt mai mici decât mine?

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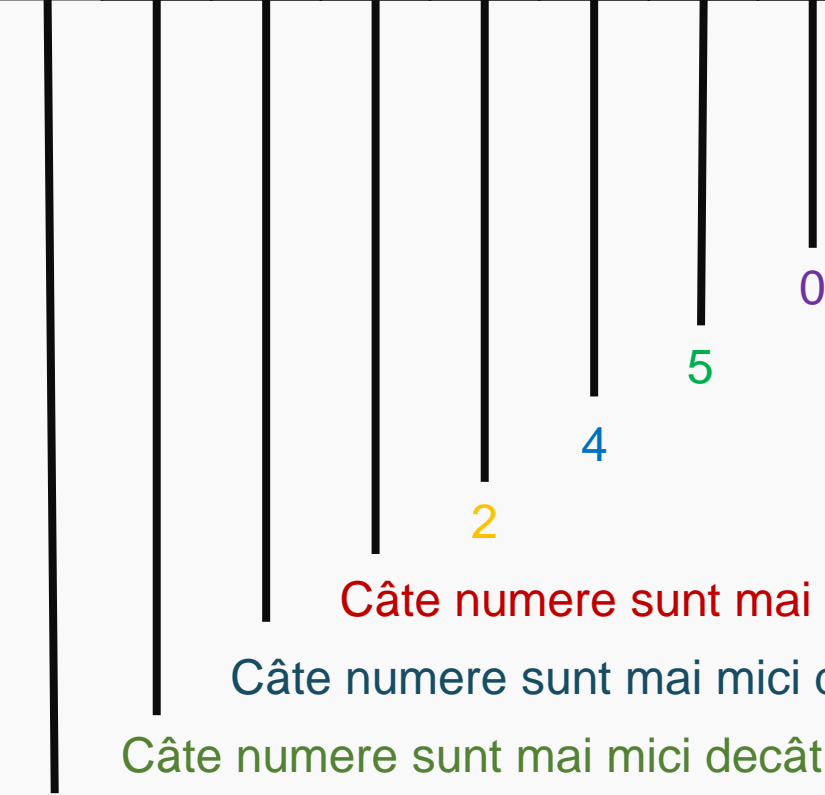
Câte numere sunt mai mici decât mine?

Câte numere sunt mai mici decât mine?



# Rank Sort

9	4	2	7	3	5	6	1
---	---	---	---	---	---	---	---



Câte numere sunt mai mici decât mine?

Câte numere sunt mai mici decât mine?

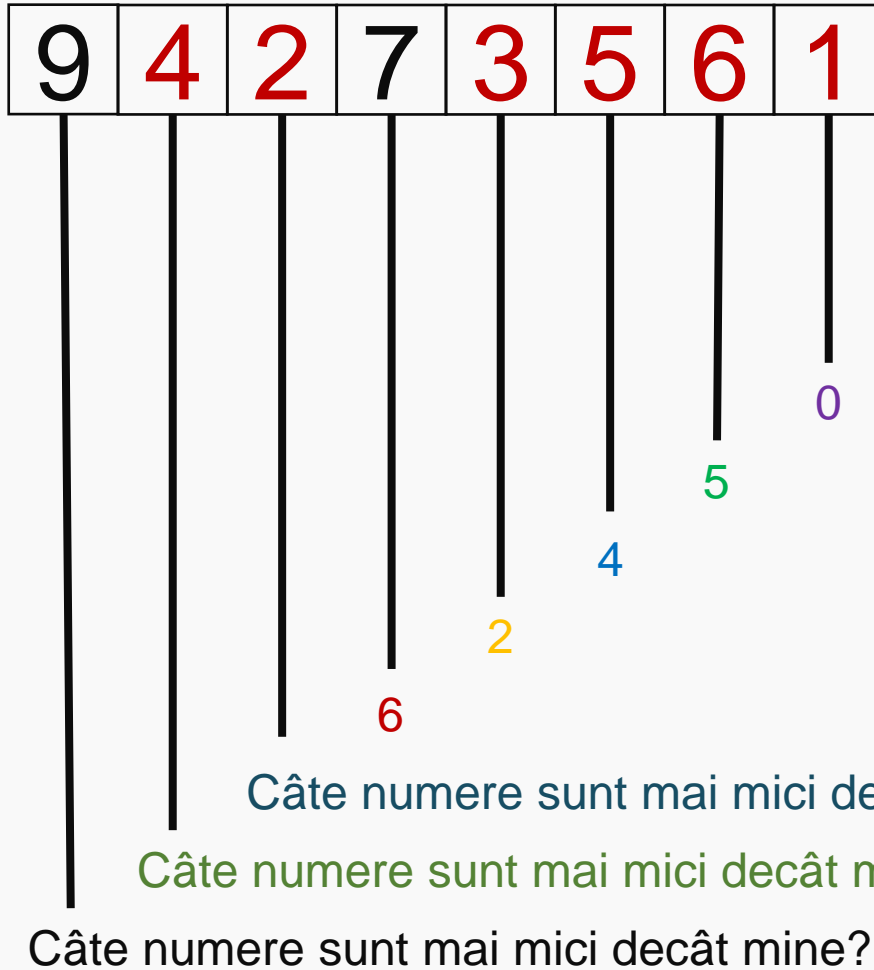
Câte numere sunt mai mici decât mine?

Câte numere sunt mai mici decât mine?



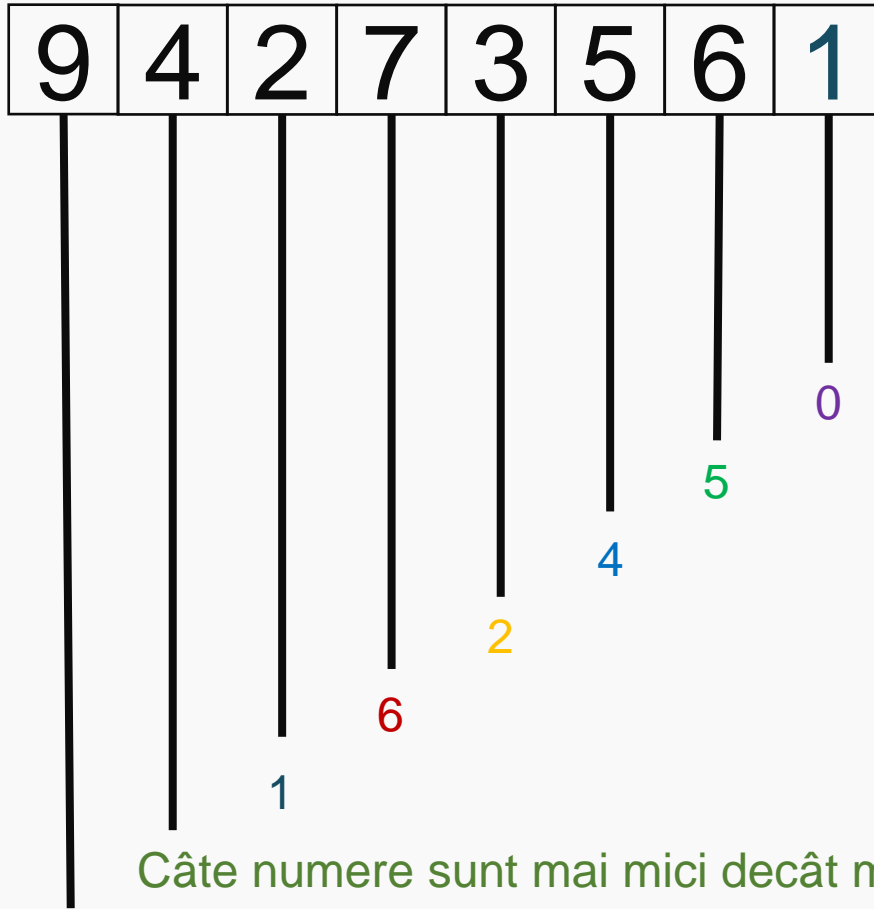


# Rank Sort





# Rank Sort

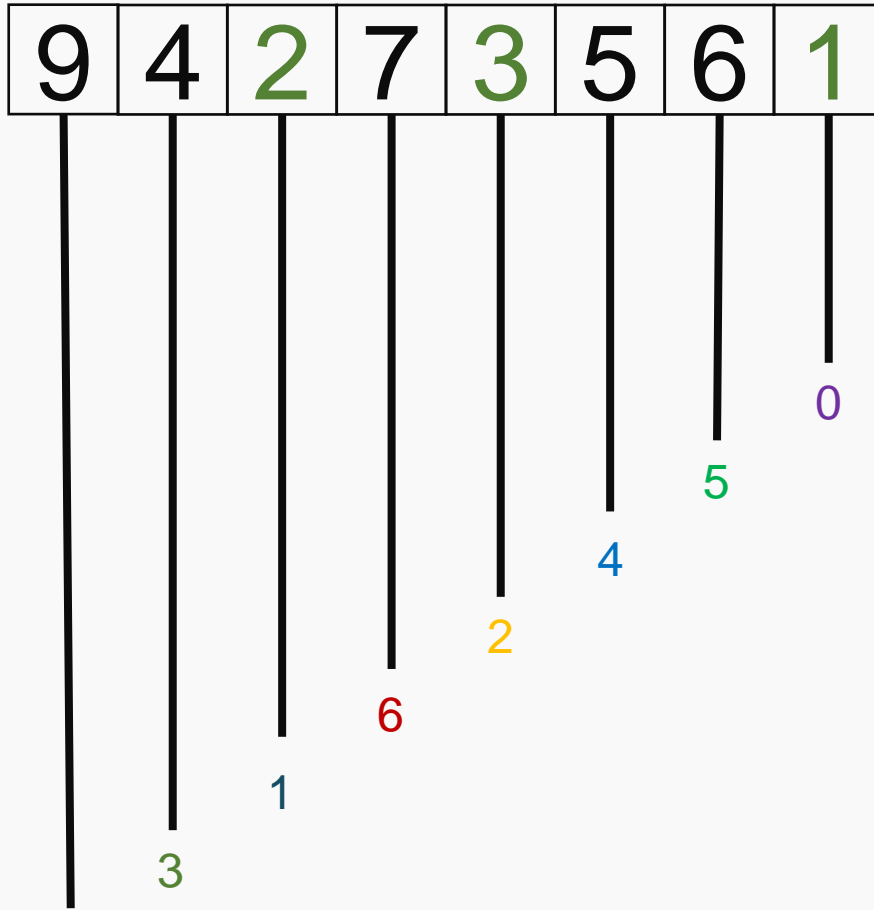


Câte numere sunt mai mici decât mine?

Câte numere sunt mai mici decât mine?



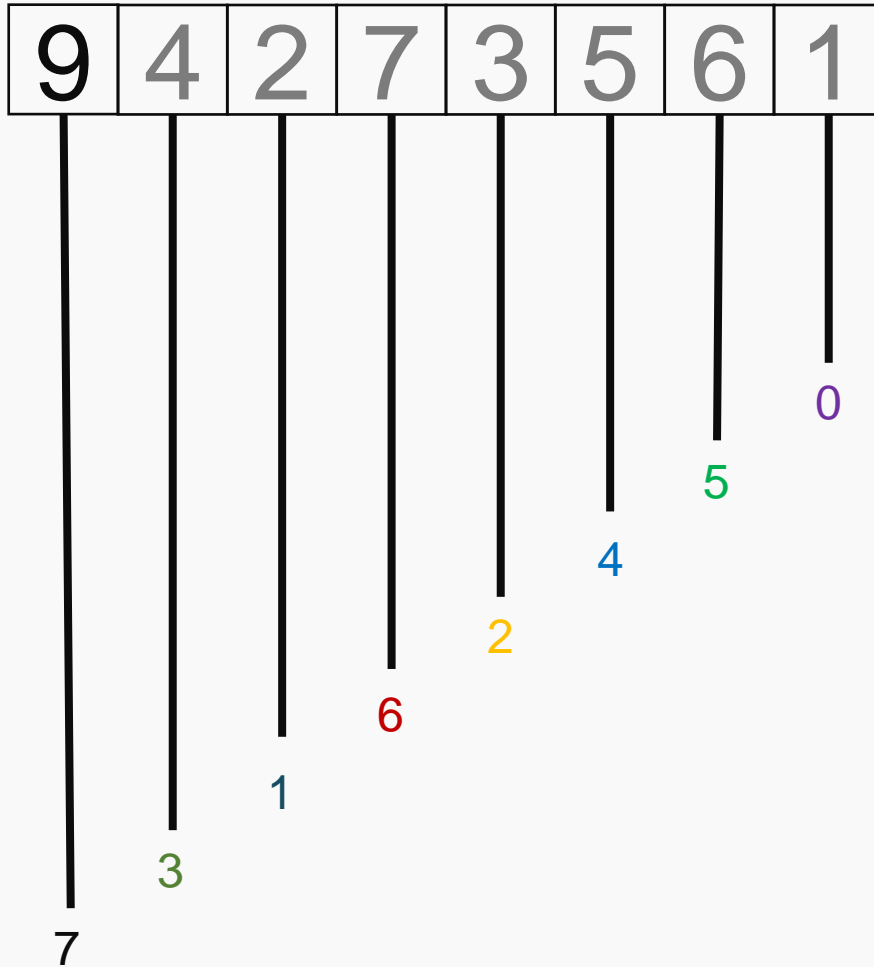
# Rank Sort



Câte numere sunt mai mici decât mine?

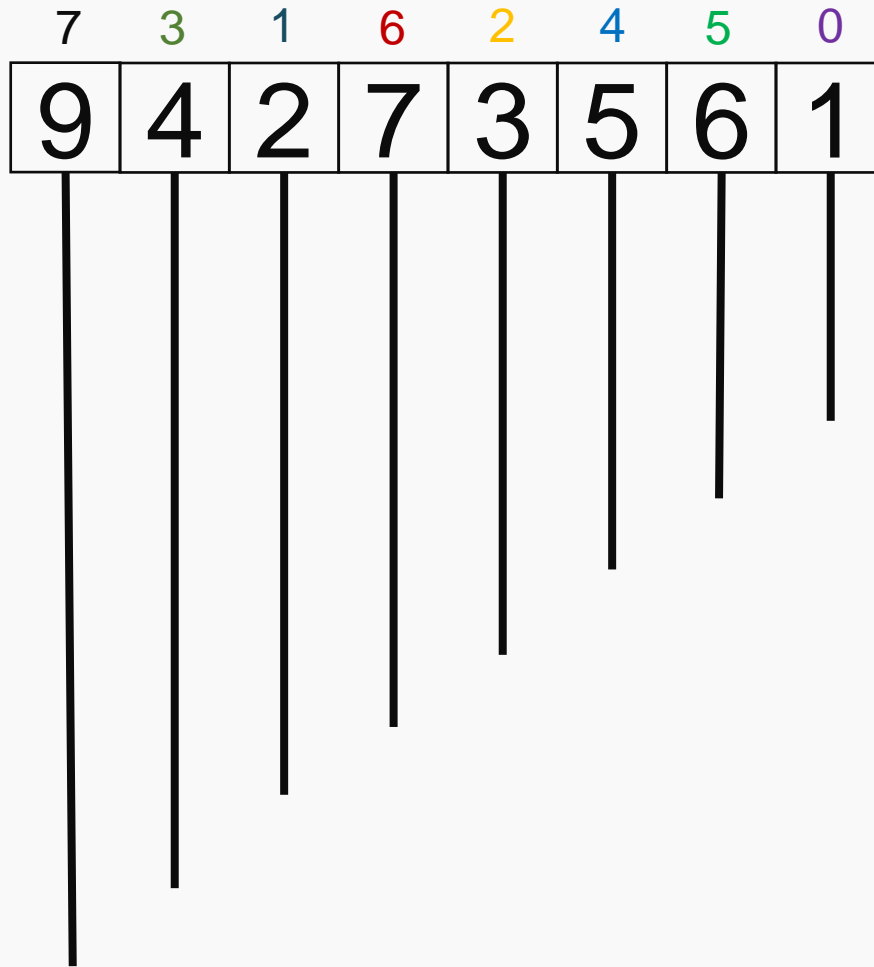


# Rank Sort





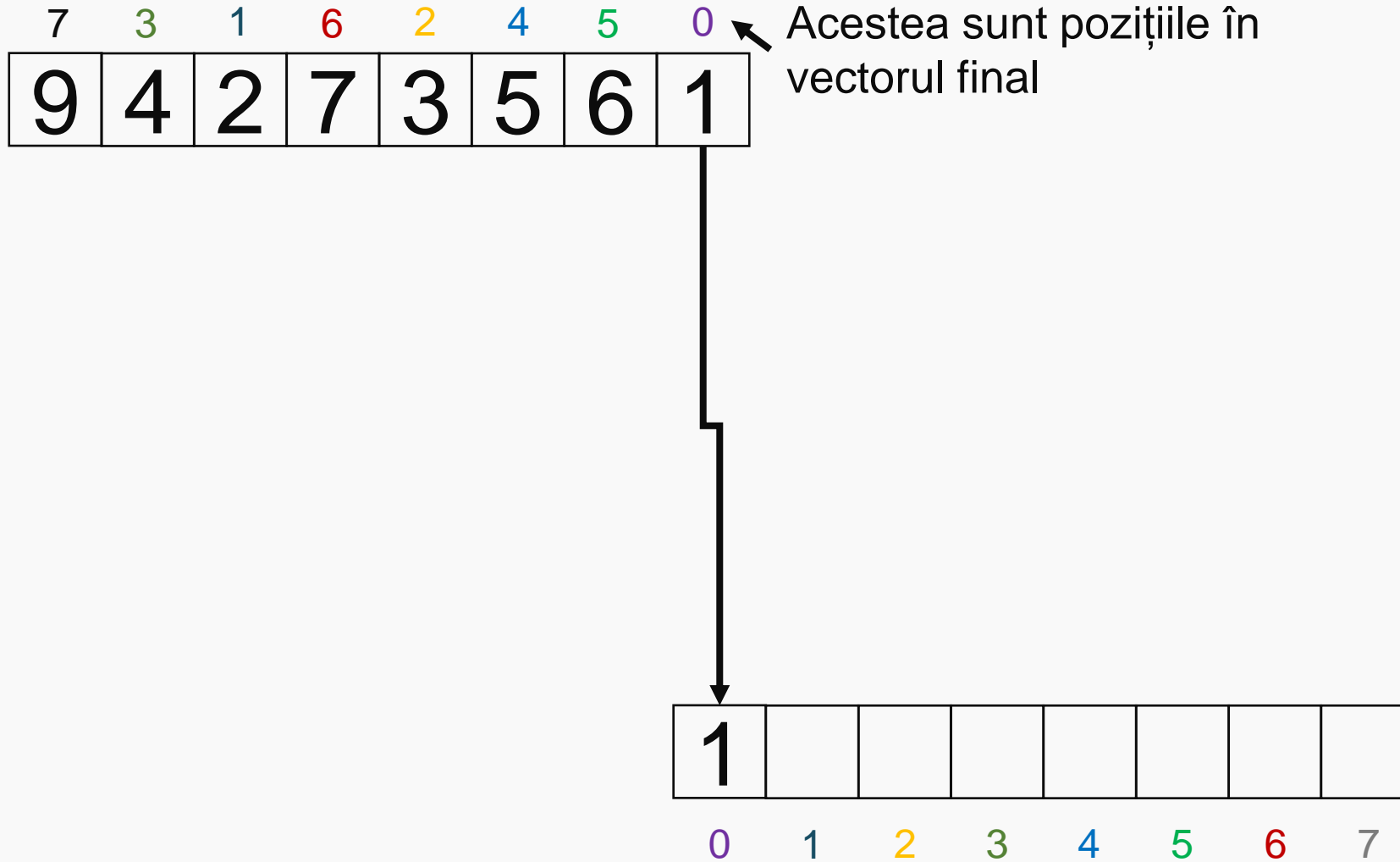
# Rank Sort



Acestea sunt pozițiile în vectorul final

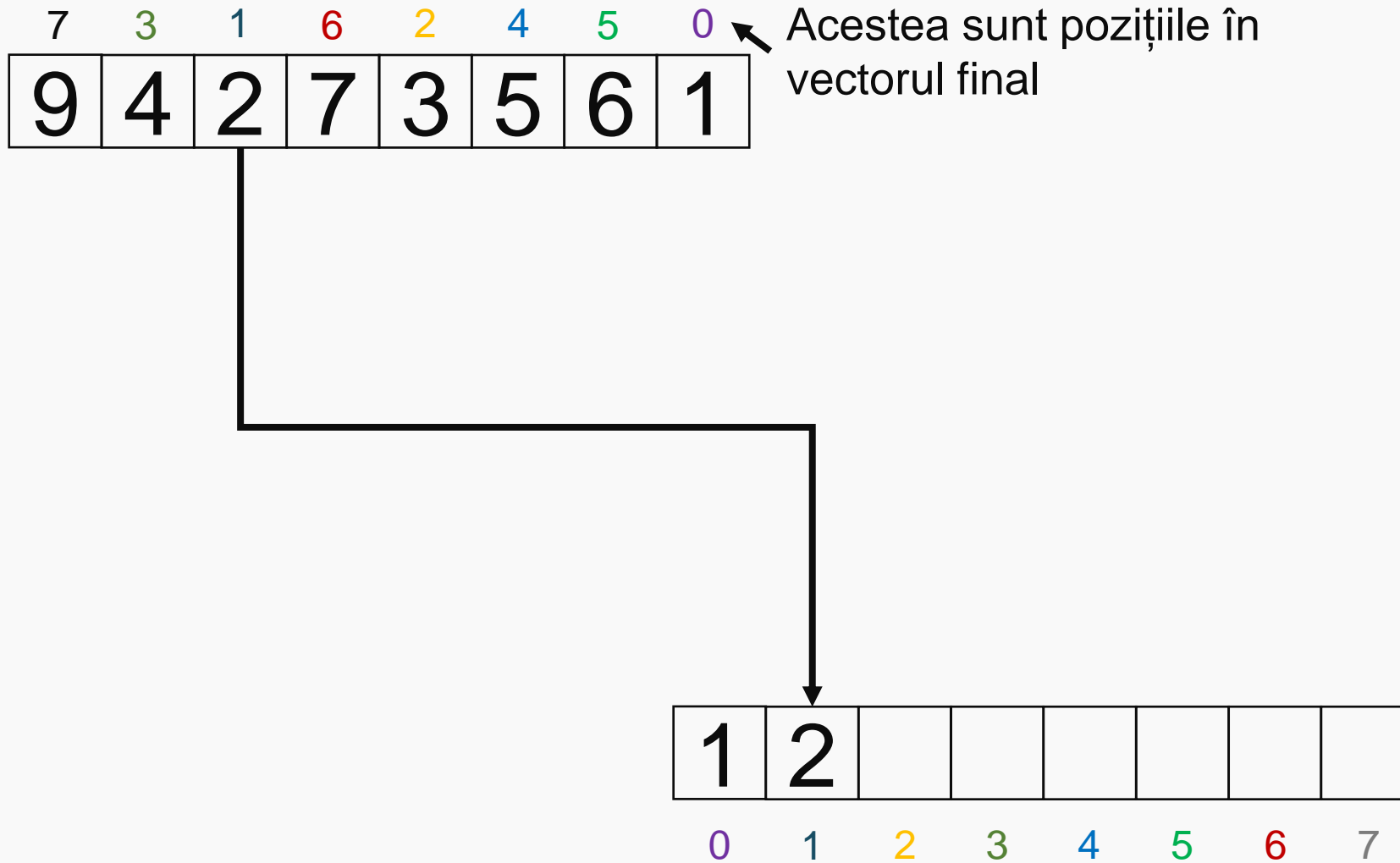


# Rank Sort



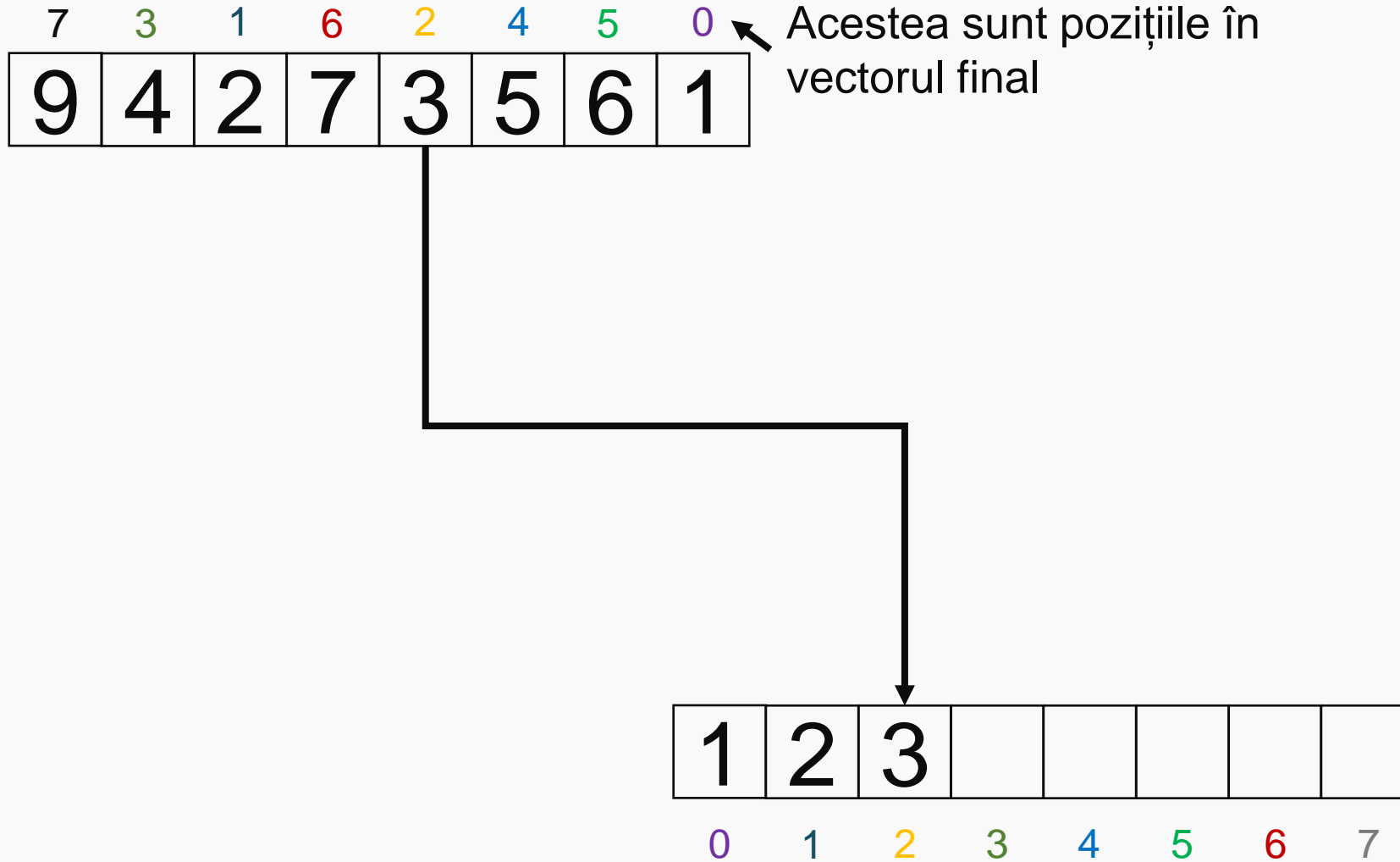


# Rank Sort





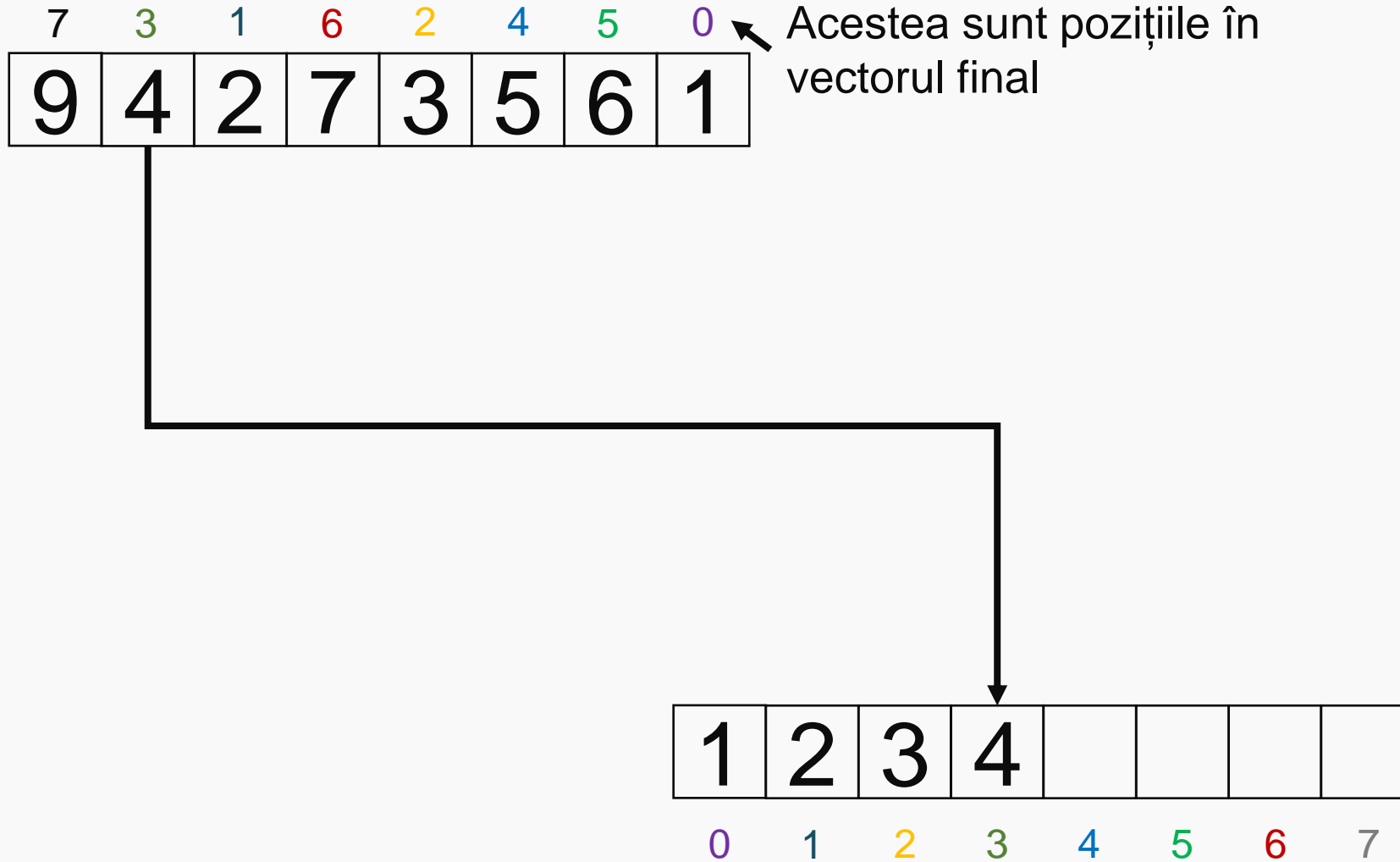
# Rank Sort





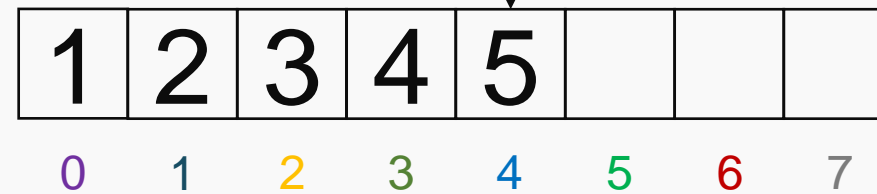
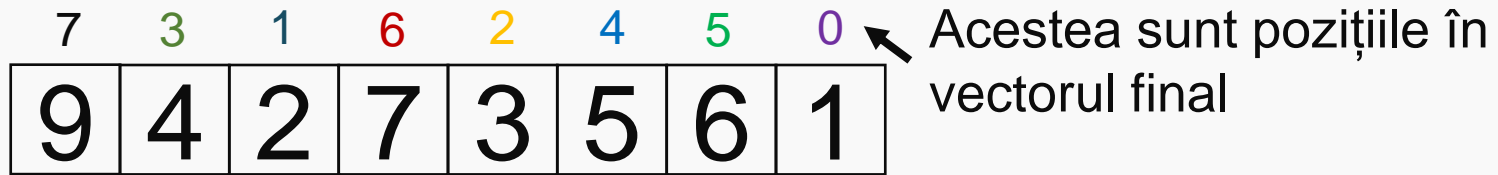


# Rank Sort



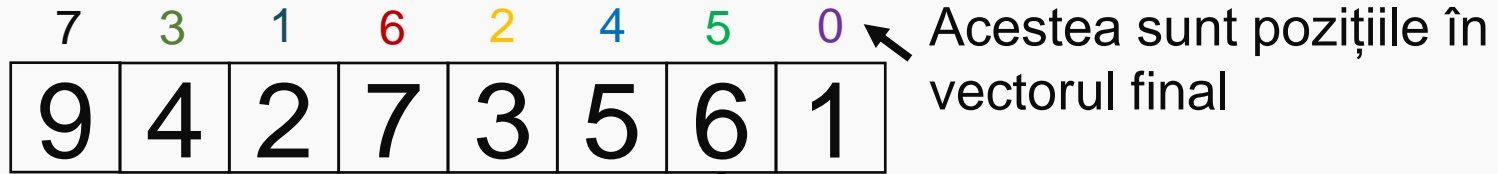


# Rank Sort



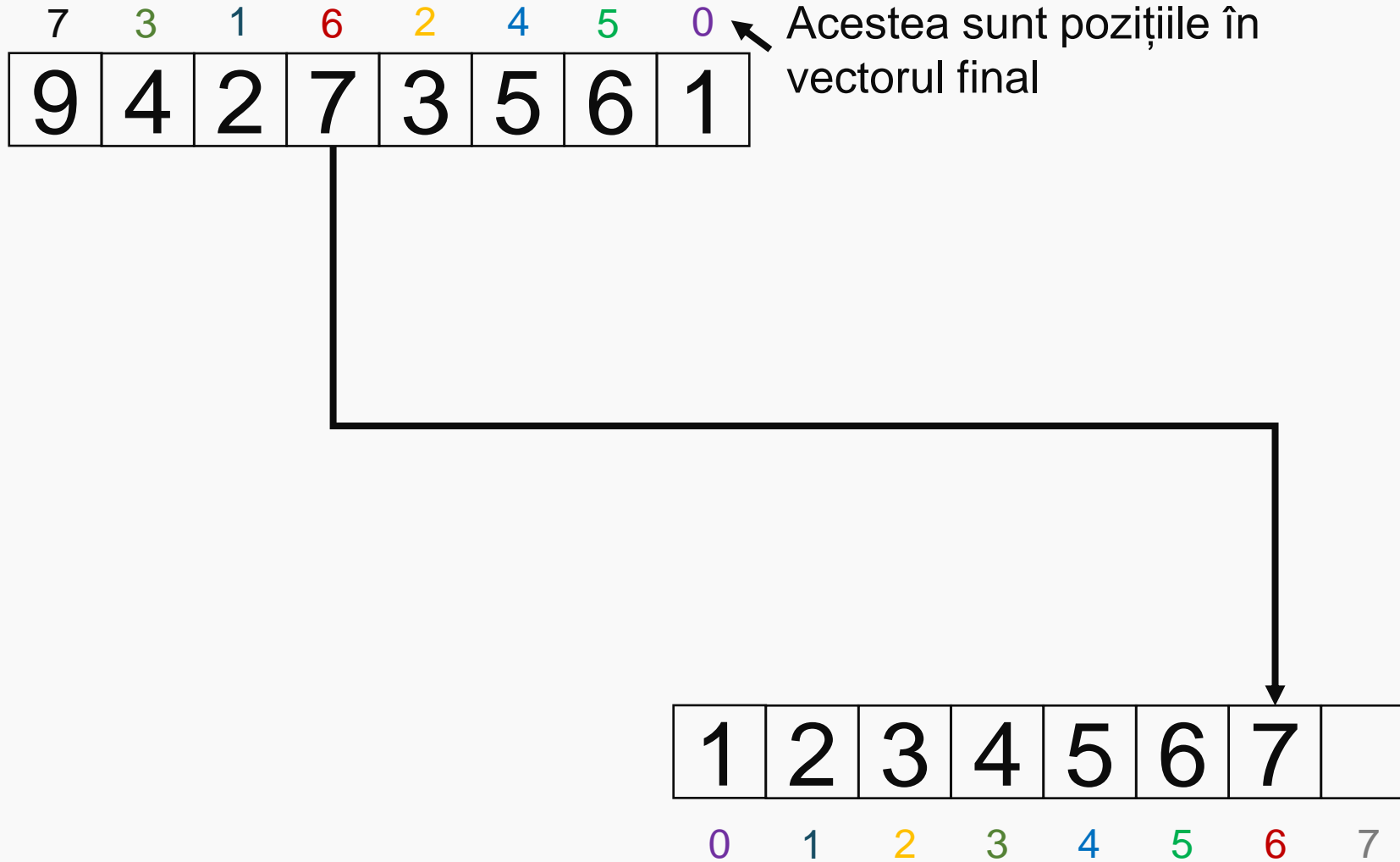


# Rank Sort





# Rank Sort





# Rank Sort

7	3	1	6	2	4	5	0
9	4	2	7	3	5	6	1

Acestea sunt pozițiile în  
vectorul final

1	2	3	4	5	6	7	9
0	1	2	3	4	5	6	7



# Rank Sort Paralel

9	4	2	7	6	5	6	1
---	---	---	---	---	---	---	---

1	2	4	5	6	6	7	9
---	---	---	---	---	---	---	---



# Rank Sort Paralel

9	4	2	7	3	5	6	1
---	---	---	---	---	---	---	---

**Răspunsul la toate  
întrebările poate fi  
determinat în paralel**

Câte numere sunt mai mici decât mine?

Câte numere sunt mai mici decât mine?

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Câte numere sunt mai mici decât mine?

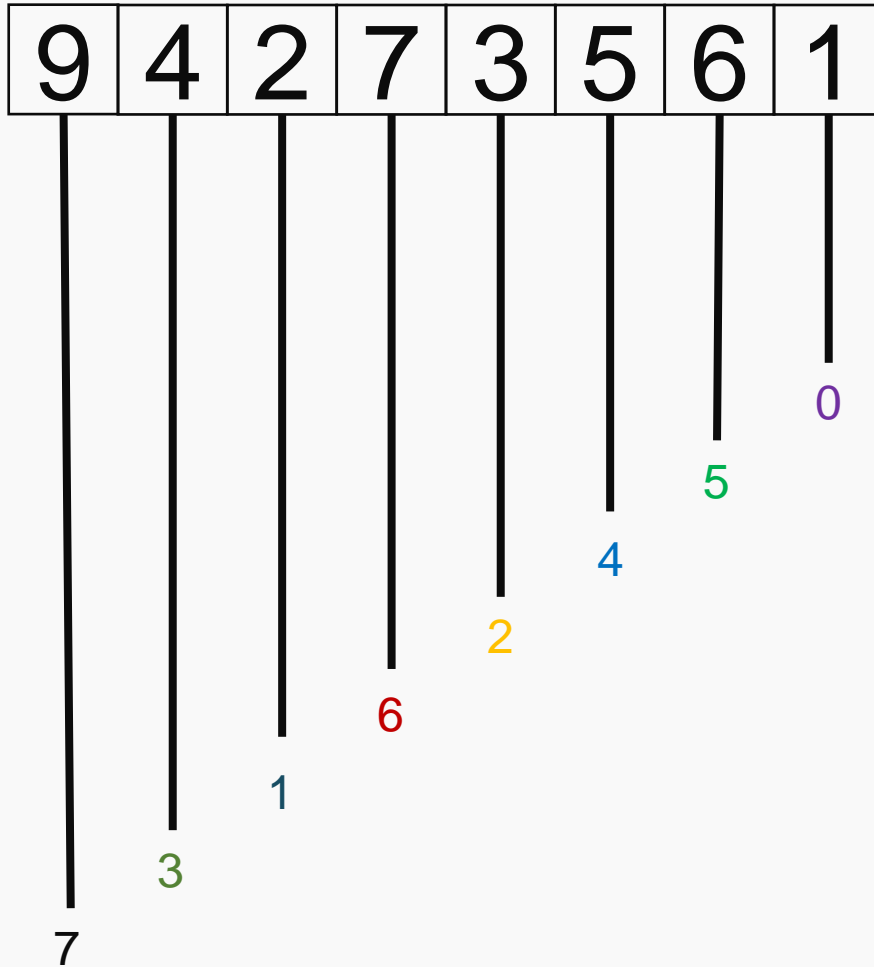
Câte numere sunt mai mici decât mine?

Câte numere sunt mai mici decât mine?

Câte numere sunt mai mici decât mine?



# Rank Sort Paralel



**Răspunsul la toate  
întrebările poate fi  
determinat în paralel**



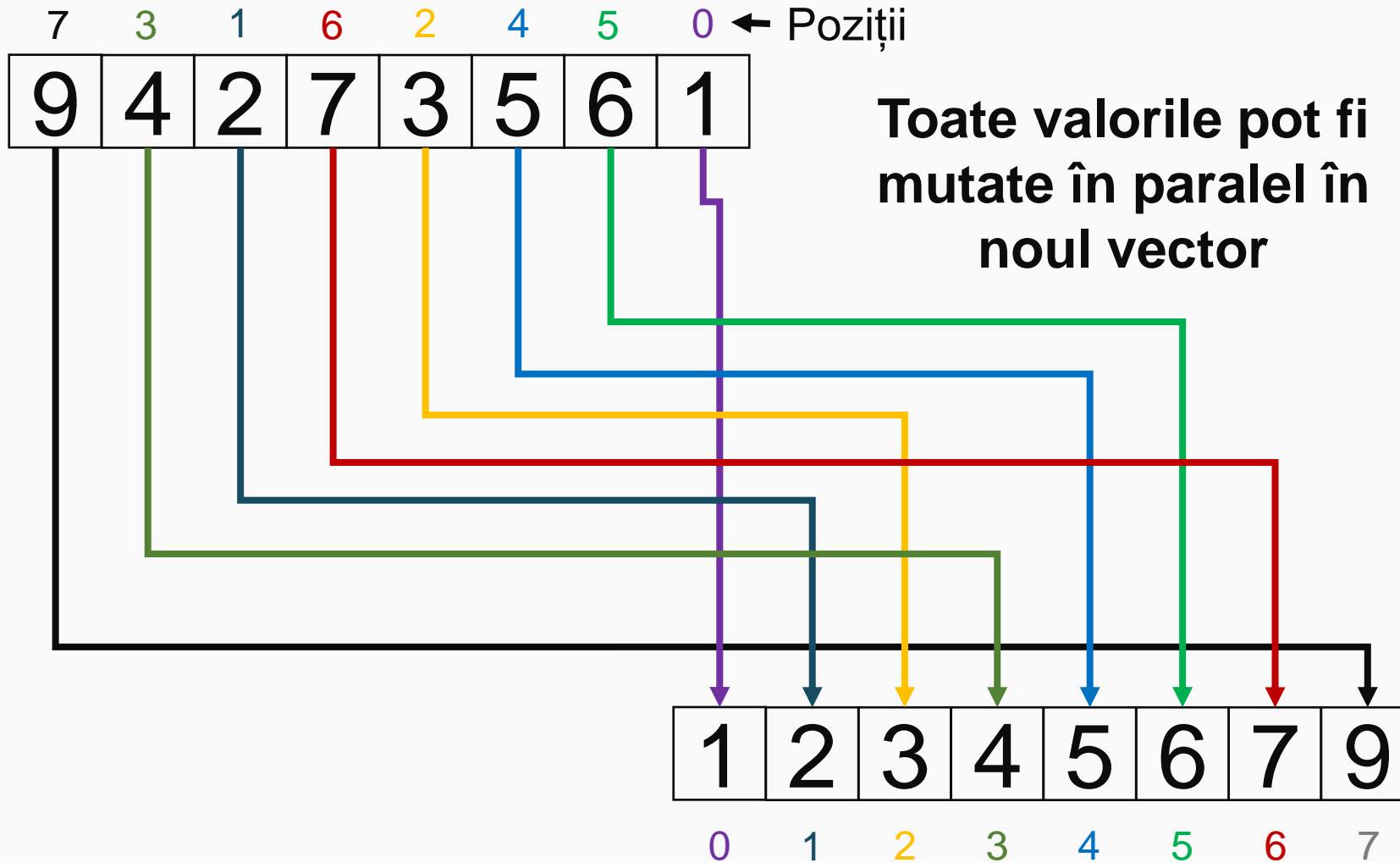


# Rank Sort Paralel





# Rank Sort Paralel







# Algoritm merge

2	3	4	5	7
---	---	---	---	---

1	2	4	4	6
---	---	---	---	---

Avem ca intrare două  
liste **sortate** dorim să  
le unim într-o listă  
**sortată**

1	2	2	3	4	4	4	5	6	7
---	---	---	---	---	---	---	---	---	---



# Algoritm merge

2	3	4	5	7
---	---	---	---	---

1	2	4	4	6
---	---	---	---	---

Soluție:

Se extrage mereu cel mai mic element  
(Garantat să fie pe prima poziție în  
una din cele două liste)

Complexitate:  $O(N)$

1	2	2	3	4	4	4	5	6	7
---	---	---	---	---	---	---	---	---	---



# Algorithm merge

2	3	4	5	7
1	2	4	4	6

---



# Algorithm merge

2	3	4	5	7
	2	4	4	6

---

1
---



# Algorithm merge

3	4	5	7
---	---	---	---

2	4	4	6
---	---	---	---



1	2
---	---





# Algoritm merge

3	4	5	7
4	4	6	

---

1	2	2
---	---	---



# Algoritm merge

4	5	7
4	4	6

---

1	2	2	3
---	---	---	---



# Algoritm merge

	5	7
4	4	6

1	2	2	3	4
---	---	---	---	---



# Algoritm merge

5	7
---	---

4	6
---	---



1	2	2	3	4	4
---	---	---	---	---	---



# Algoritm merge

5	7
6	

---

1	2	2	3	4	4	4
---	---	---	---	---	---	---



# Algorithm merge

7

6



1	2	2	3	4	4	4	5
---	---	---	---	---	---	---	---



# Algorithm merge

7

---

1	2	2	3	4	4	4	5	6
---	---	---	---	---	---	---	---	---



# Algorithm merge

---

1	2	2	3	4	4	4	5	6	7
---	---	---	---	---	---	---	---	---	---







# Algoritm merge

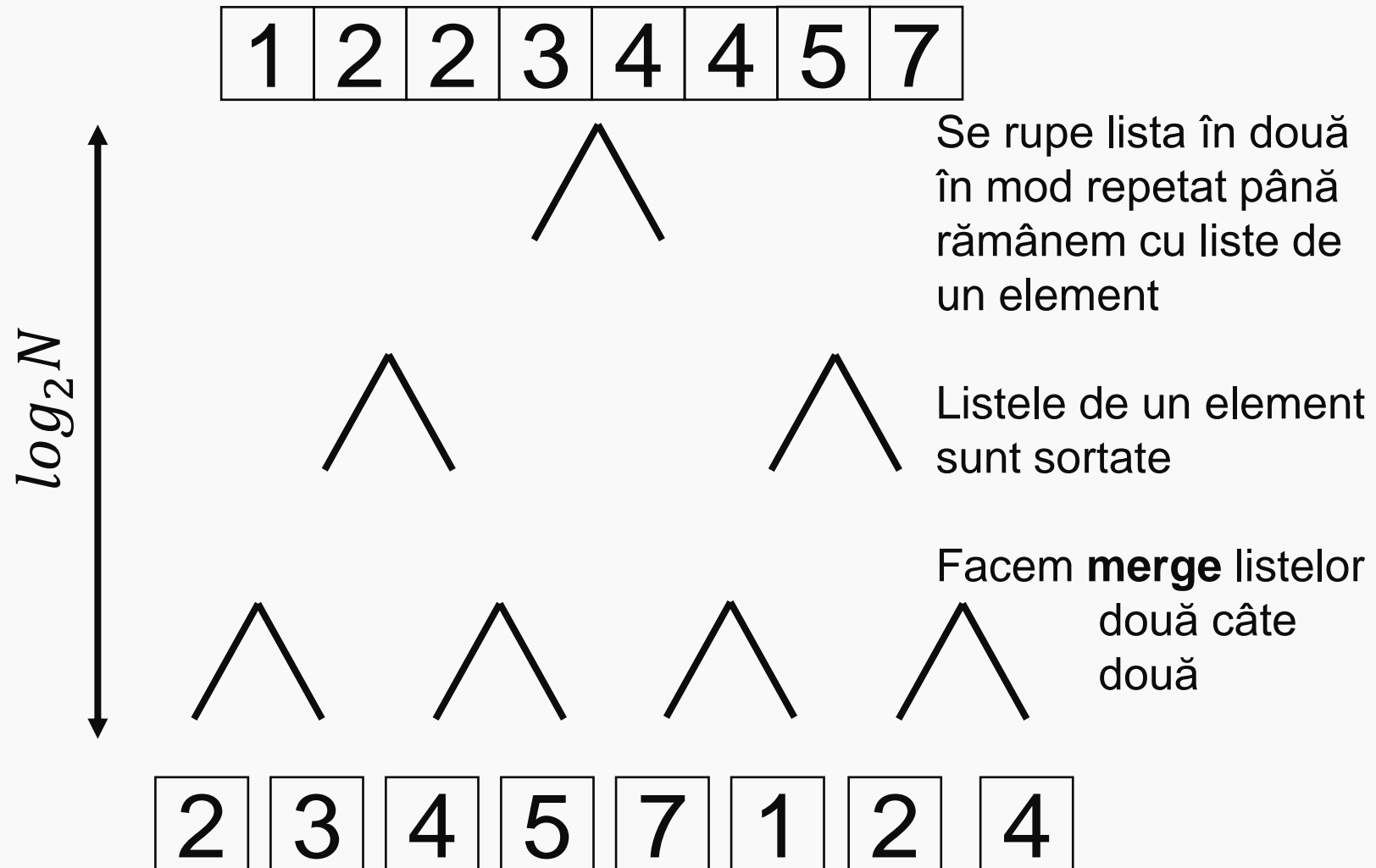


Folosim acest semn pentru a  
reprezenta operația **MERGE**





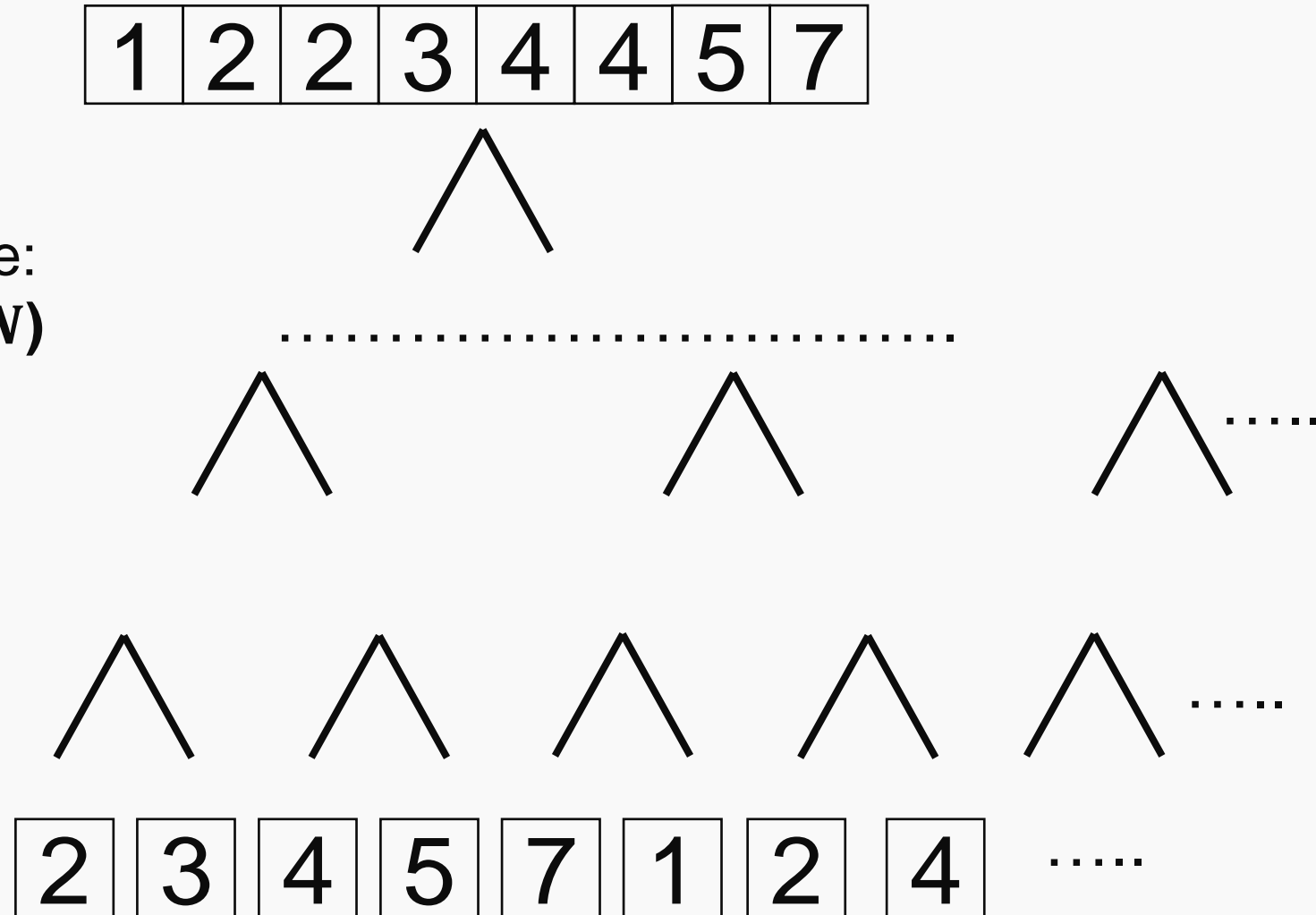
# Merge sort





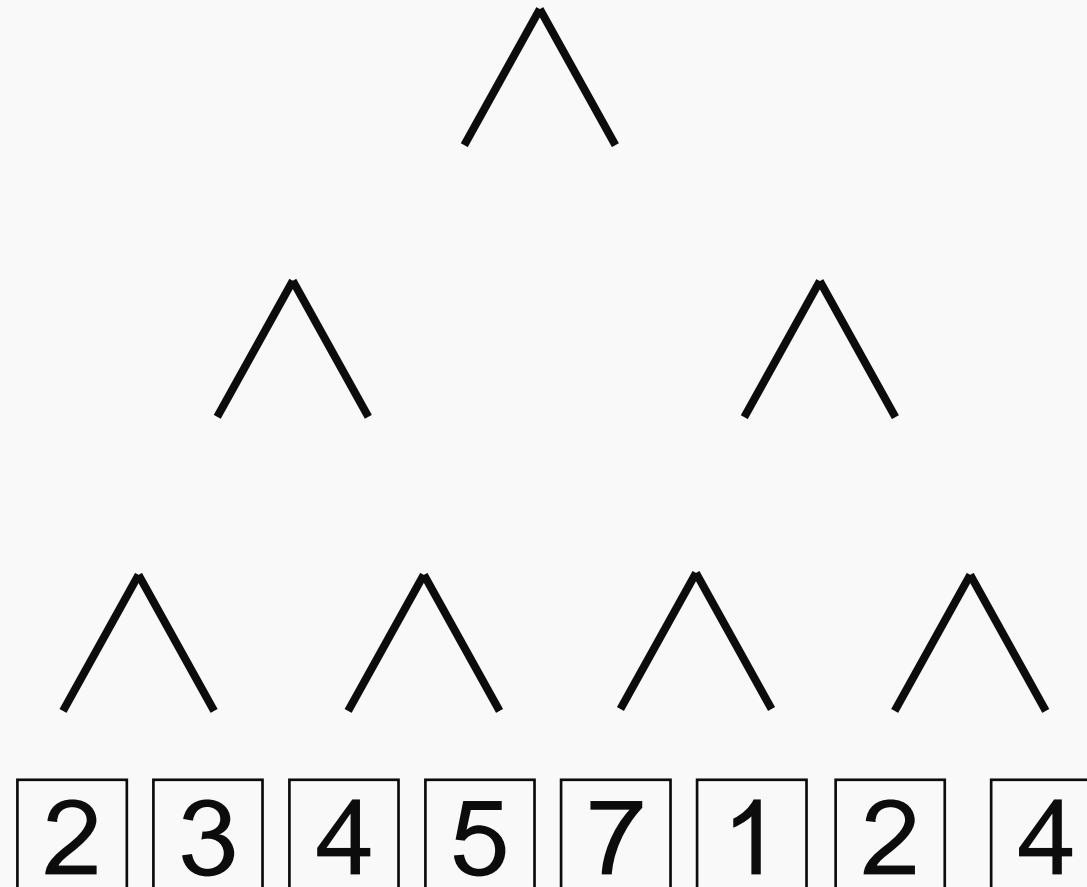
# Merge sort

Complexitate:  
 $O(N * \log_2 N)$



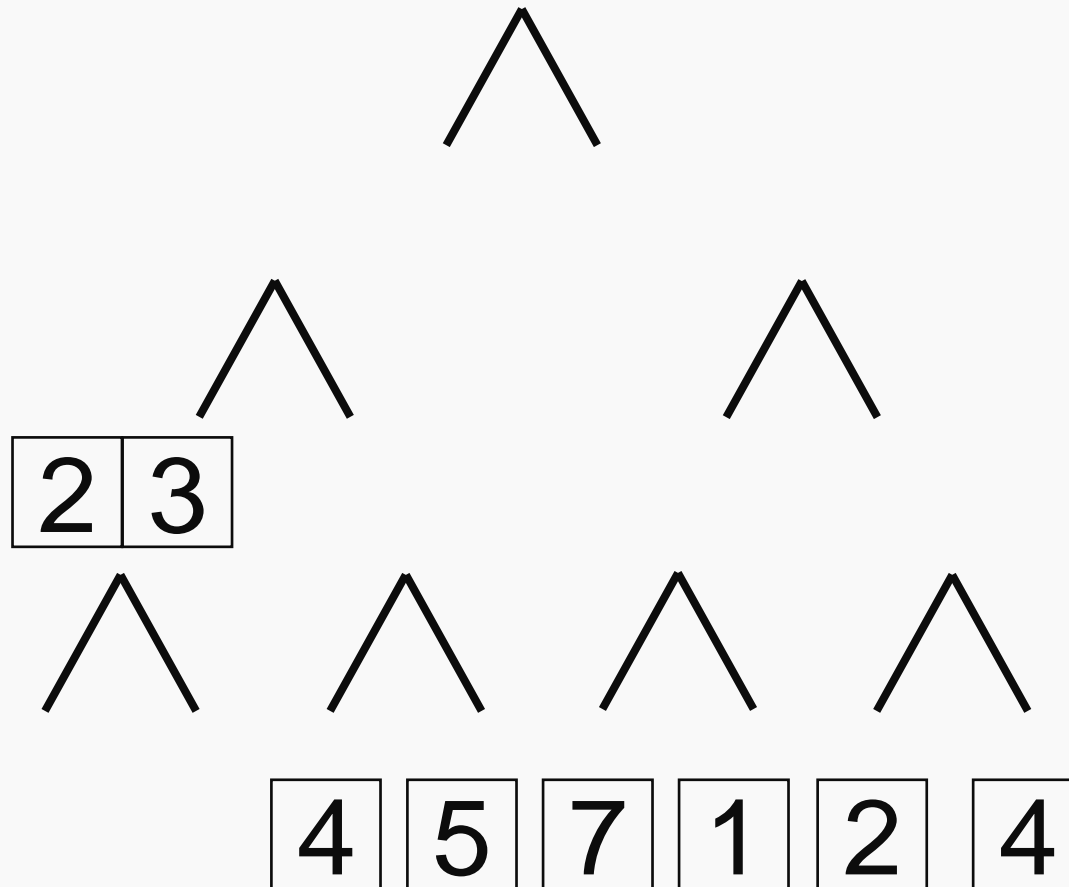


# Merge sort



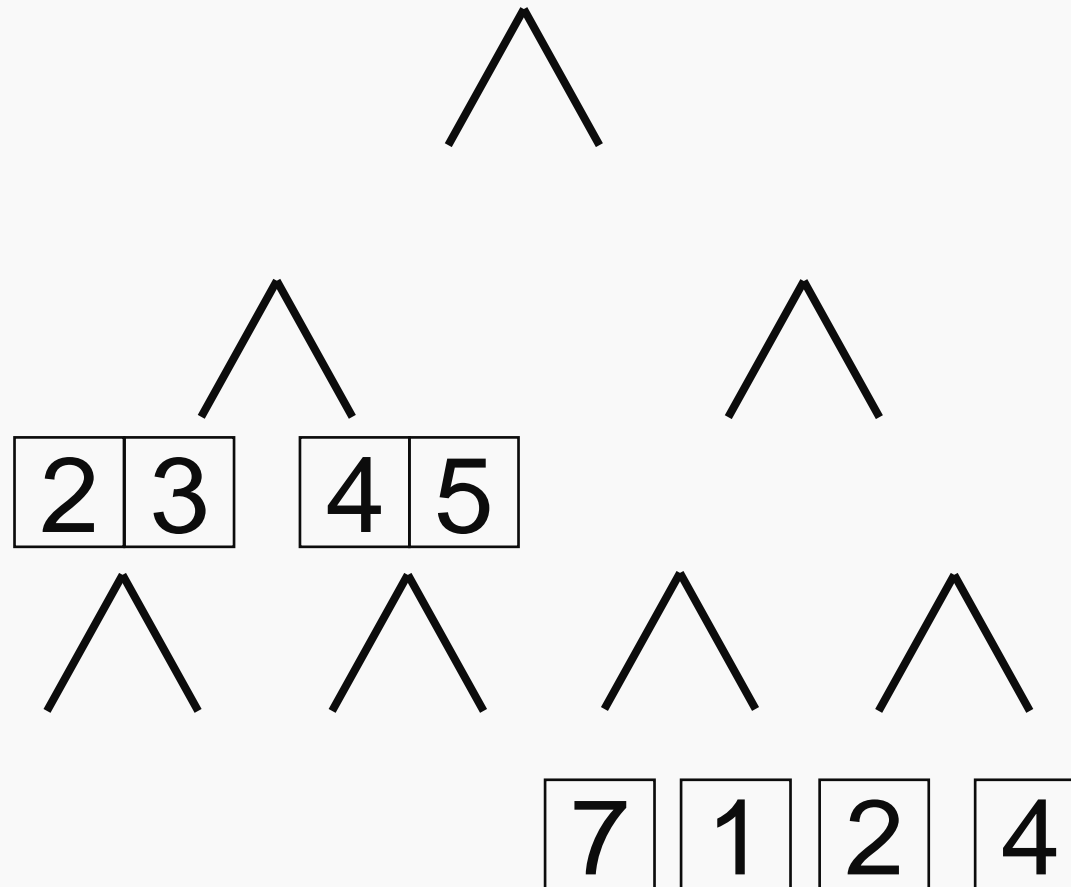


# Merge sort



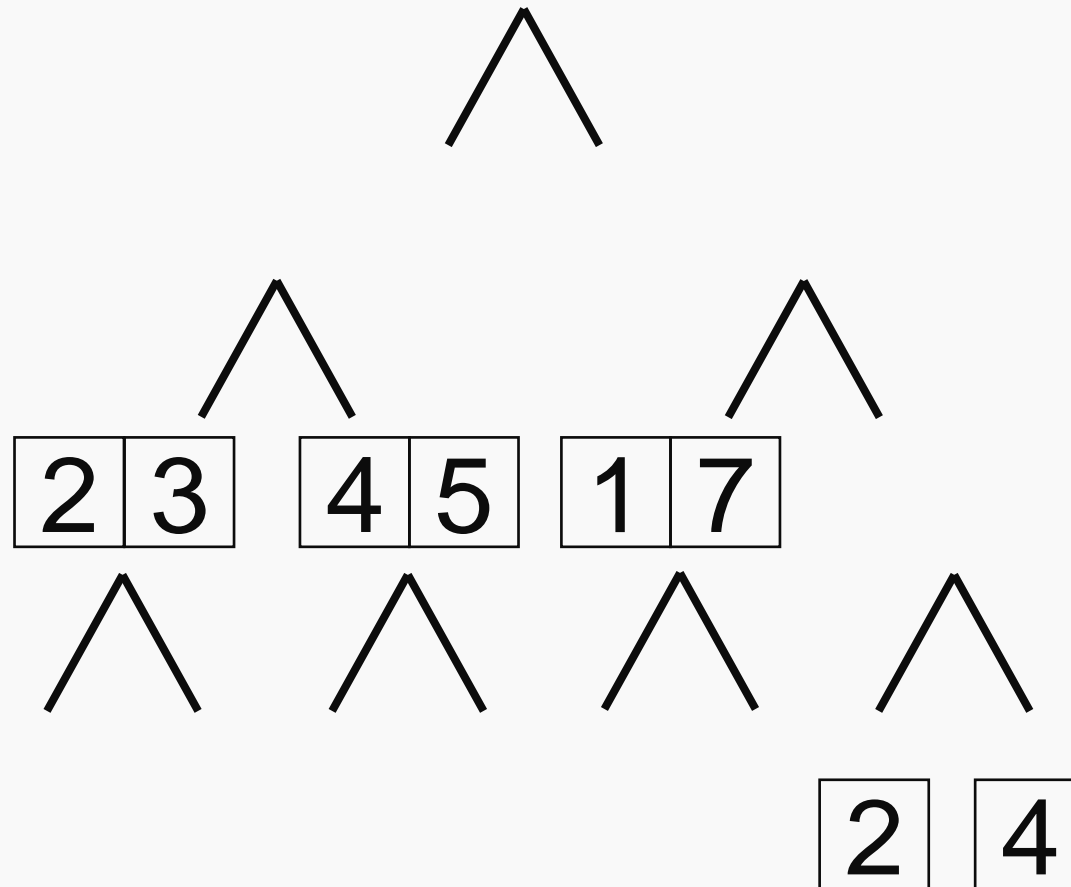


# Merge sort





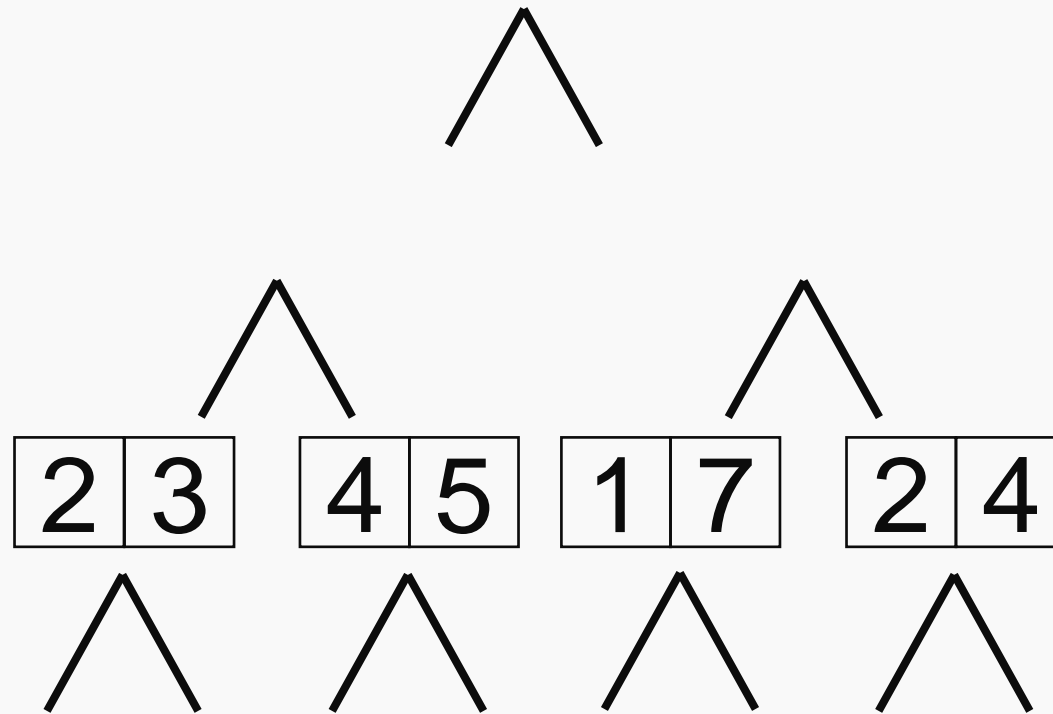
# Merge sort





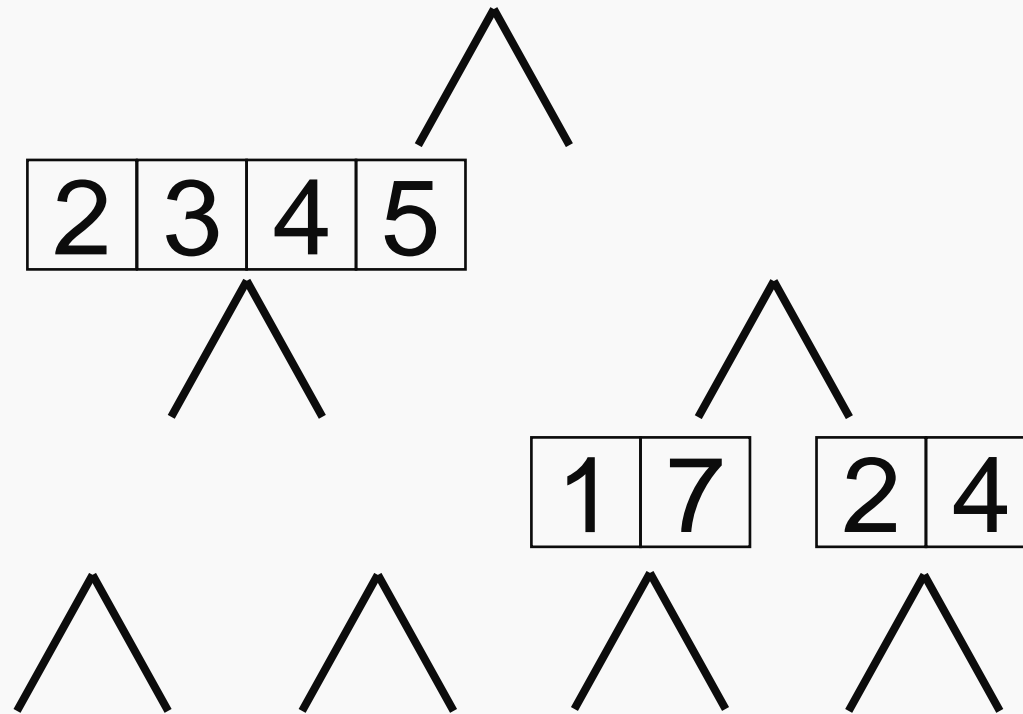


# Merge sort



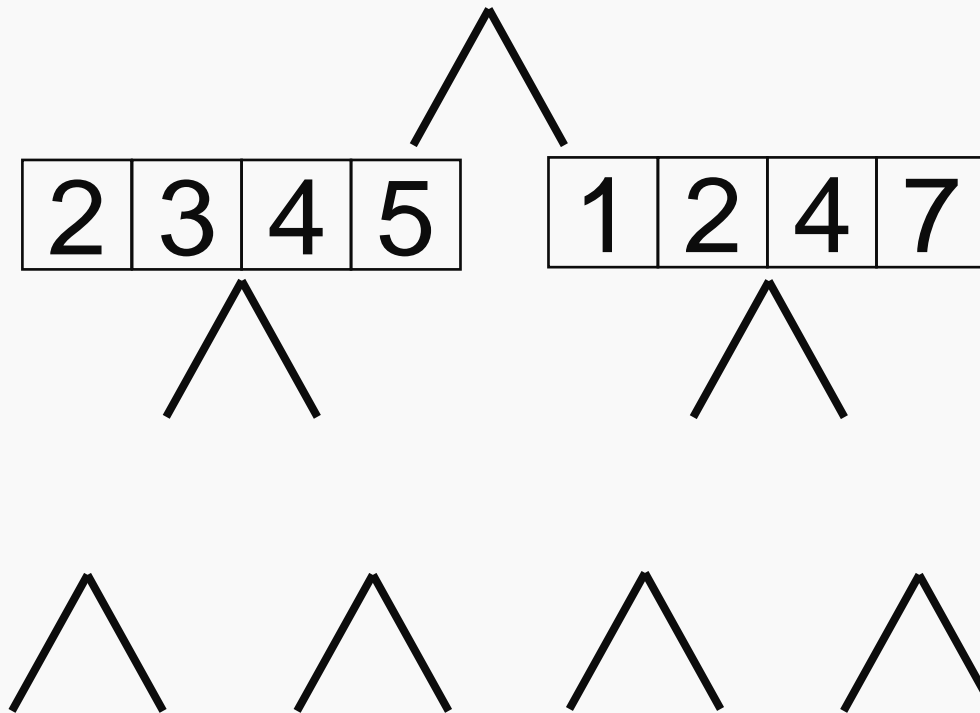


# Merge sort





# Merge sort





# Merge sort

1	2	2	3	4	4	5	7
---	---	---	---	---	---	---	---

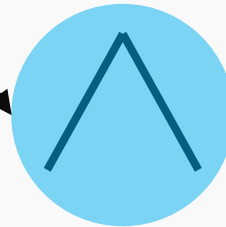




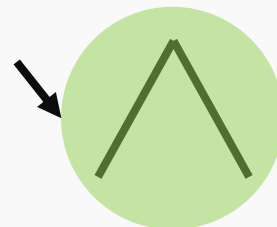


# Merge sort paralel

Operațiile  
**albastre**  
pot fi  
executate în  
paralel



Operațiile  
**verzi** pot fi  
executate în  
paralel





# Merge sort paralel

O operație  
**verde** cu una  
**albastră** nu  
poate fi  
executată în  
paralel

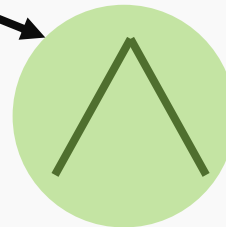
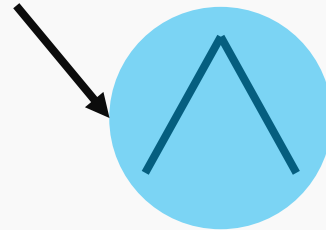
Operația



depinde de  
rezultatul  
operațiilor



1	2	2	3	4	4	5	7
---	---	---	---	---	---	---	---



2	3	4	5	7	1	2	4
---	---	---	---	---	---	---	---

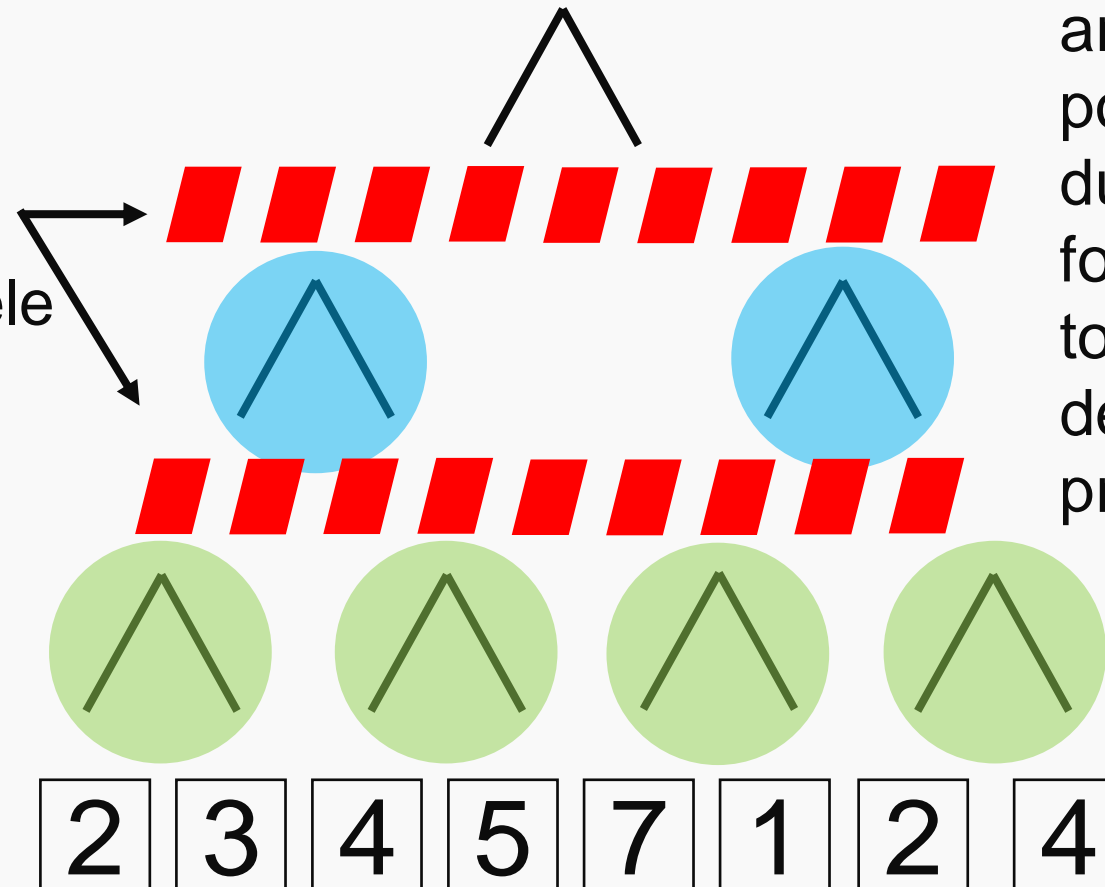


# Merge sort paralel

Soluție:

**Barrier**

Între nivelele  
arborelui

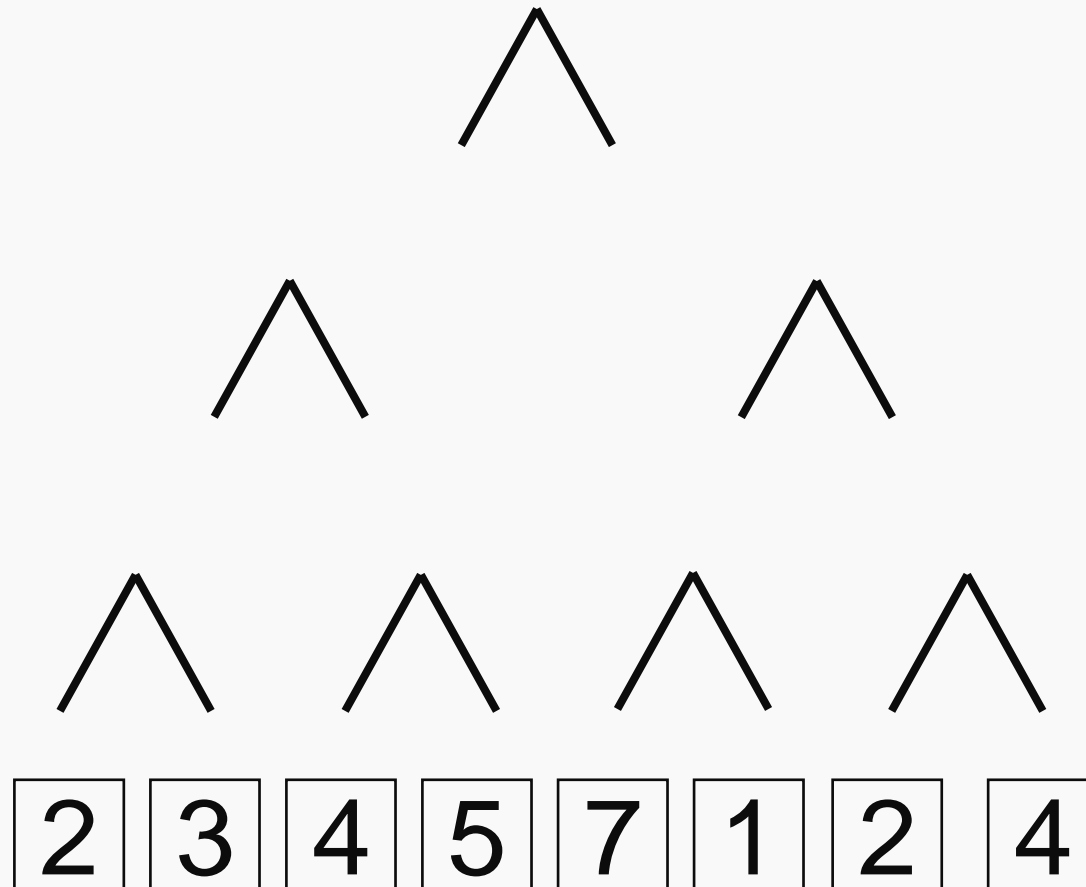


Operațiile după  
un nivel al  
arborelui pot  
porni doar  
după ce au  
fost terminate  
toate operațiile  
de pe nivelul  
precedent.



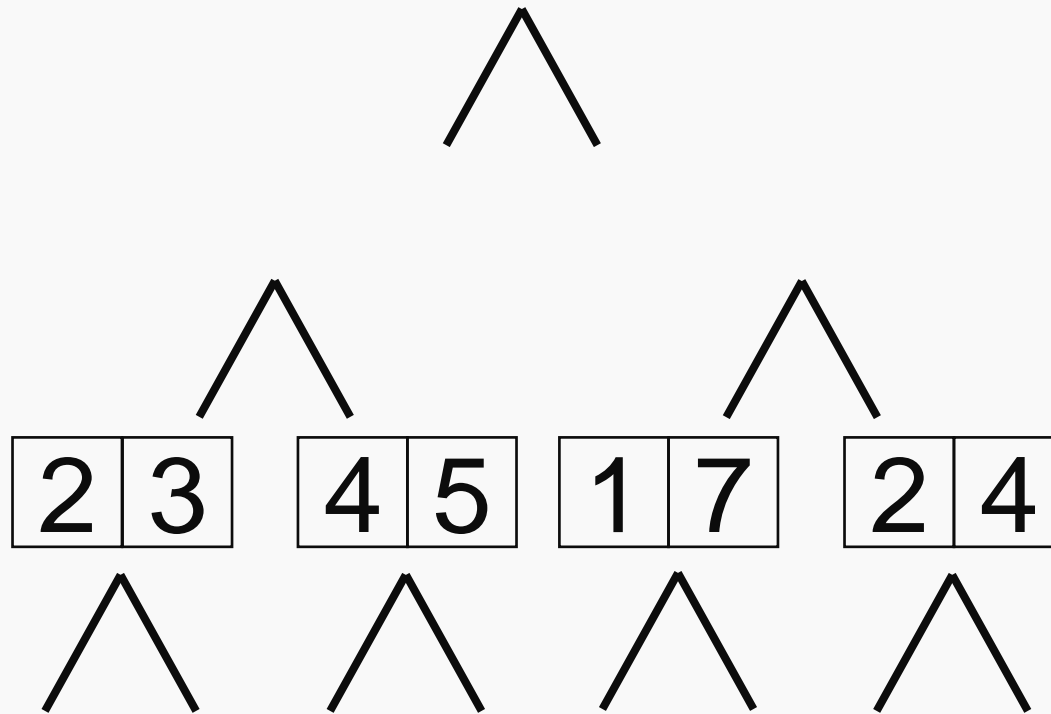


# Merge sort paralel



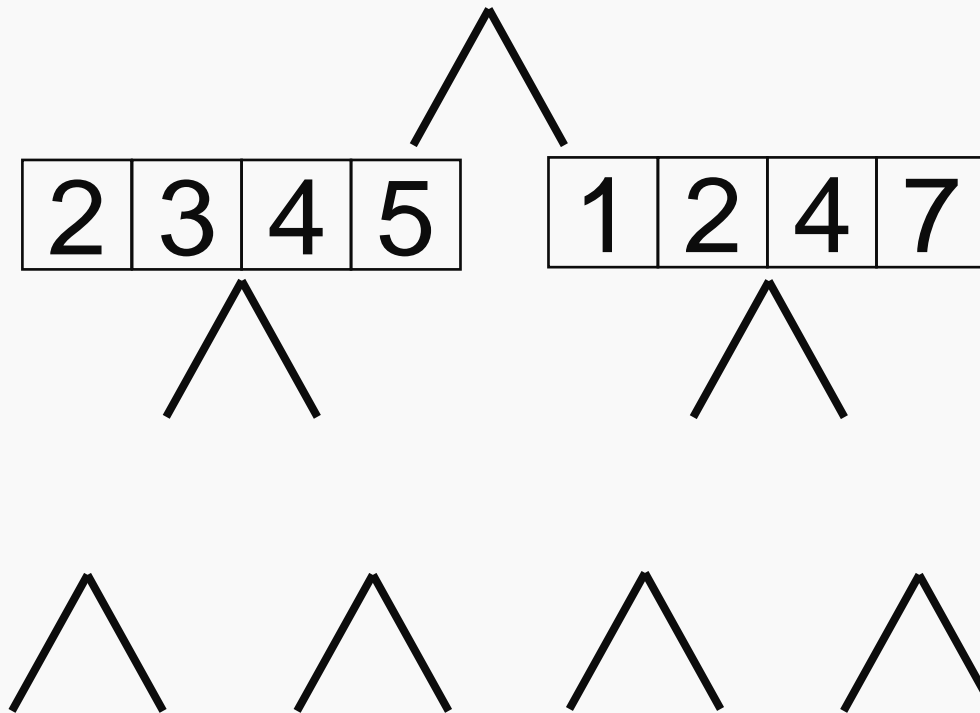


# Merge sort paralel





# Merge sort paralel





# Merge sort paralel

1	2	2	3	4	4	5	7
---	---	---	---	---	---	---	---



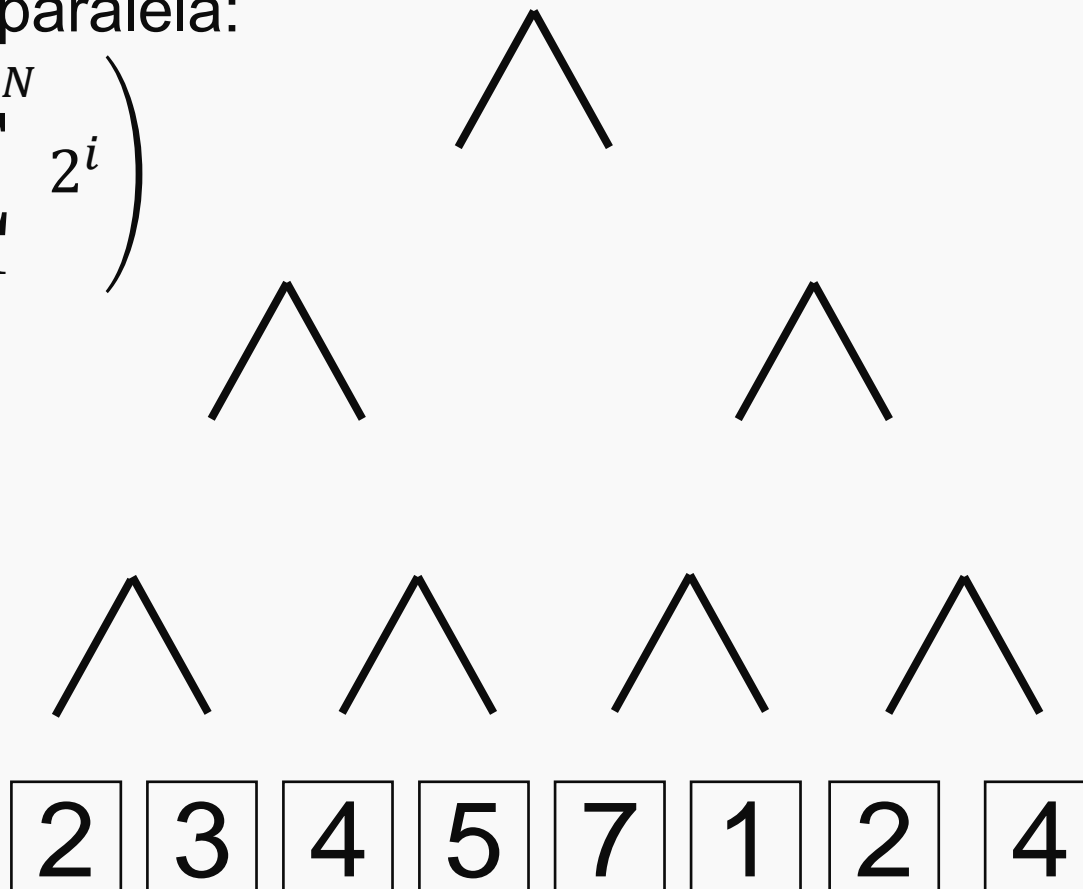


# Merge sort paralel - complexitate

Complexitate paralelă:

$$O\left(\sum_{i=1}^{\log_2 N} 2^i\right)$$

Dacă  $P = N$





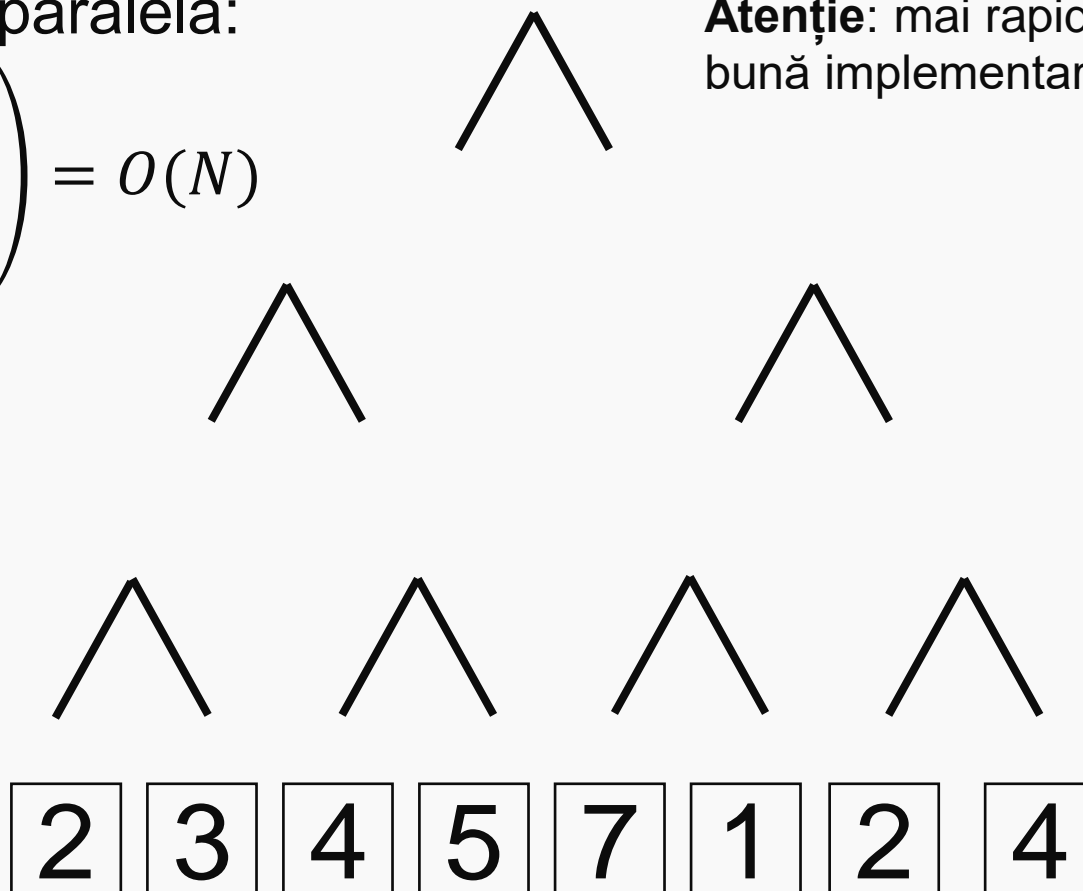
# Merge sort paralel - complexitate

Complexitate paralelă:

$$O\left(\sum_{i=1}^{\log_2 N} 2^i\right) = O(N)$$

Dacă  $P = N$

**Atenție:** mai rapid decât cea mai bună implementare secvențială





# Merge sort paralel - complexitate

Cea mai bună soluție paralelă: paralelizează și operația merge

Articol

[Parallel Merge Sort – Richard Cole](#)



## Parallel Merge Sort

*Richard Cole*

New York University



**Abstract.** We give a parallel implementation of merge sort on a CREW PRAM that uses  $n$  processors and  $O(\log n)$  time; the constant in the running time is small. We also give a more complex version of the algorithm for the EREW PRAM; it also uses  $n$  processors and  $O(\log n)$  time. The constant in the running time is still moderate, though not as small.

### 1. Introduction

1975]; this procedure merges two sorted arrays, each of length at most  $n$ , in time  $O(\log \log n)$  using a linear number of processors. When used in the obvious way, Valiant's procedure leads to an implementation of merge sort on  $n$  processors using  $O(\log n \log \log n)$  time. More recently, Kruskal [K, 1983] improved this sorting algorithm to obtain a sorting algorithm that runs in time  $O(\log n \log \log n / \log \log \log n)$  on  $n$  processors. (The







# Căutare binară

Căutăm

3

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9



# Căutare binară

Căutăm 3

Între pozițiile 0 15

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9



**$3 < 7$**




# Căutare binară

Căutăm 3

Între pozițiile 0 7

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9

  
 $3 < 5$




# Căutare binară

Căutăm 3

Între pozițiile 0 3

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9

  
**3 > 2**



# Căutare binară

Căutăm 3

Între pozițiile 3 3

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9

↑  
**3 = 3**  
**end**

**Complexitate  $O(\log_2(N))$**



# Căutare binară

Căutăm 3

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9





# Căutare paralelă – implementare naivă

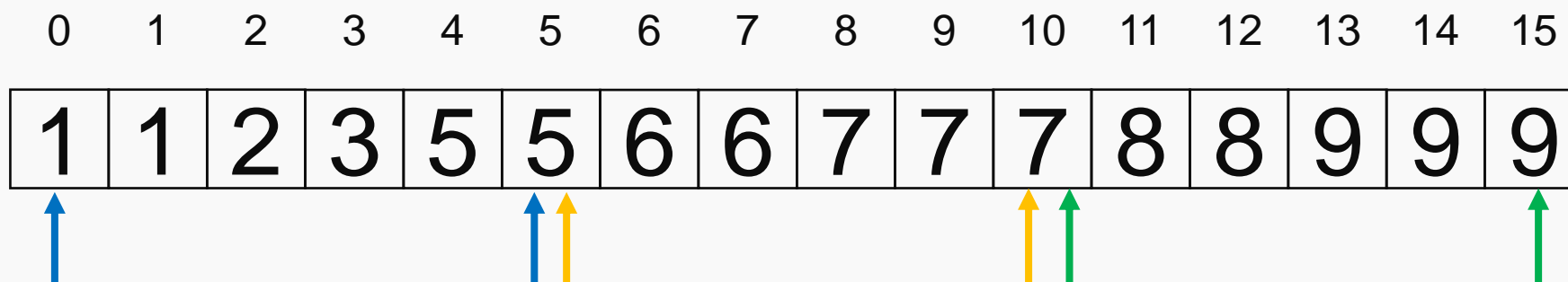
Căutăm

3

Între pozițiile

0

15



**Fiecare thread este responsabil de o zonă**





# Căutare paralelă – implementare naivă

Căutăm 3

Între pozițiile 0 15

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9



Elementul căutat este în bucata mea





# Căutare paralelă – implementare naivă

Căutăm 3

Între pozițiile 0 15



Elementul nu este la mine, mă opresc



# Căutare paralelă – implementare naivă

Căutăm 3

Între pozițiile 0 15



Elementul nu este la mine, mă opresc




# Căutare paralelă – implementare naivă

Căutăm 3

Între pozițiile 1 4

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9

  
Caut binar



# Căutare paralelă – implementare naivă

Căutăm 3

Între pozițiile 1 4

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9

↑  
Caut binar

**Doar un singur pas s-a executat în paralel.  
Am pornit thread-uri degeaba.**

**Complexitate:  $O(\log_2(N))$  la fel ca secvențial**

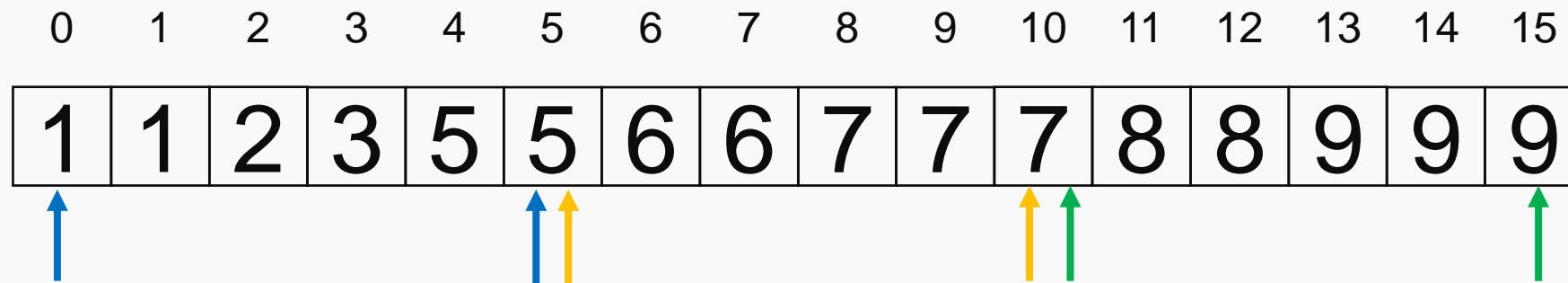




# Căutare paralelă

Căutăm 3

Între pozițiile 0 15



**Fiecare thread este responsabil de o zonă.**

**Când trecem la pasul următor toate thread-urile se mută în noua zonă**



# Căutare paralelă

Căutăm 3

Între pozițiile 0 15

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9



Elementul căutat este în bucata mea





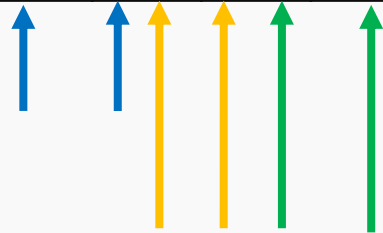


# Căutare paralelă

Căutăm 3

Între pozițiile 1 4

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9



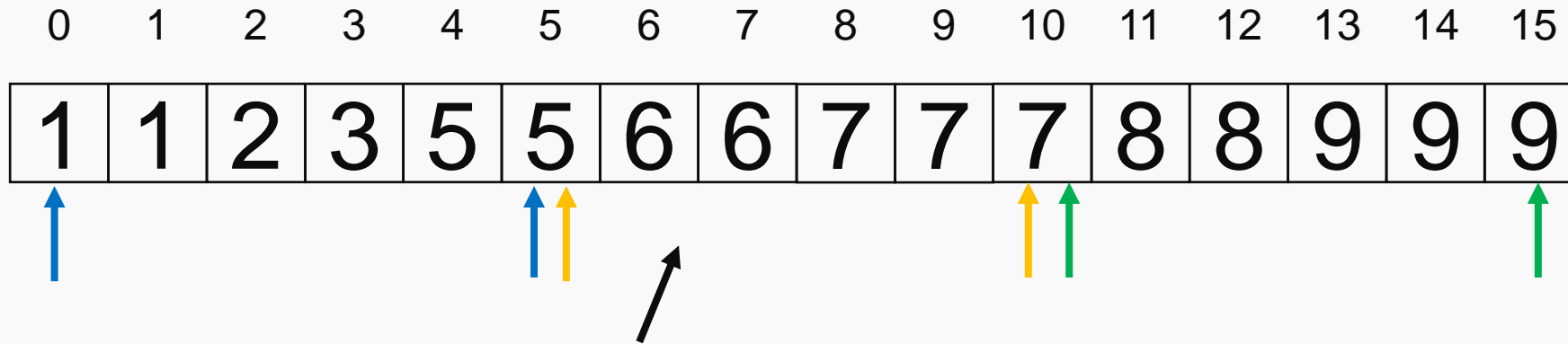
Am găsit elementul

Am găsit elementul

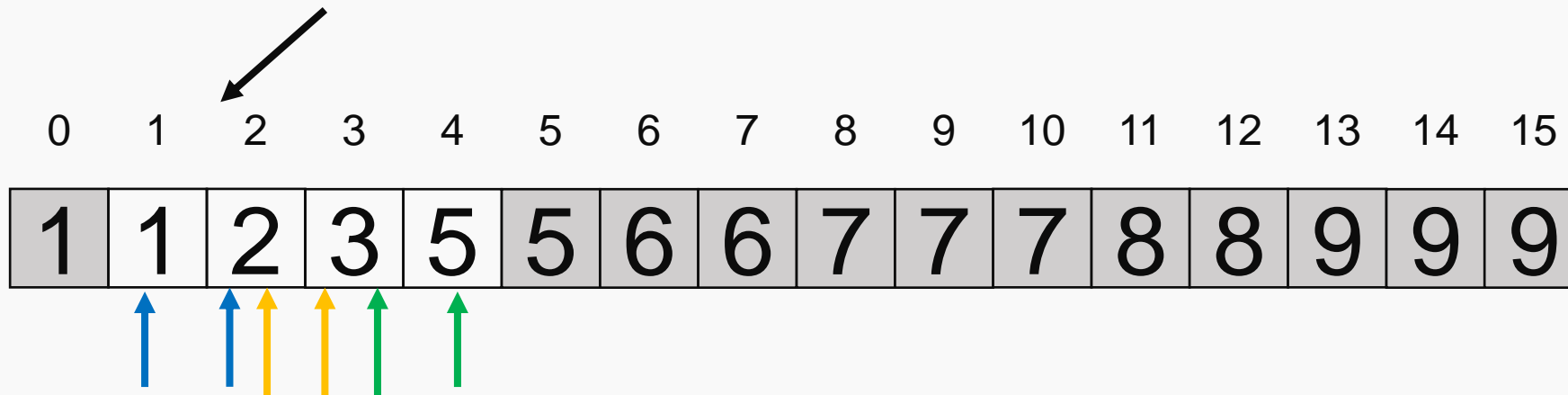
**Toate** thread-urile caută în noua zonă



# Căutare paralelă

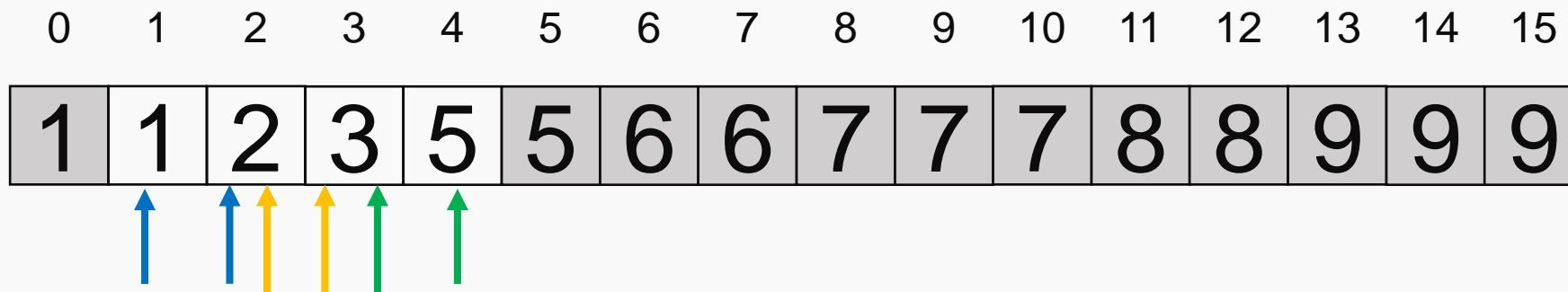


Operațiile aceste **NU** pot executa paralel





# Căutare paralelă





## Căutare paralelă – soluția 2

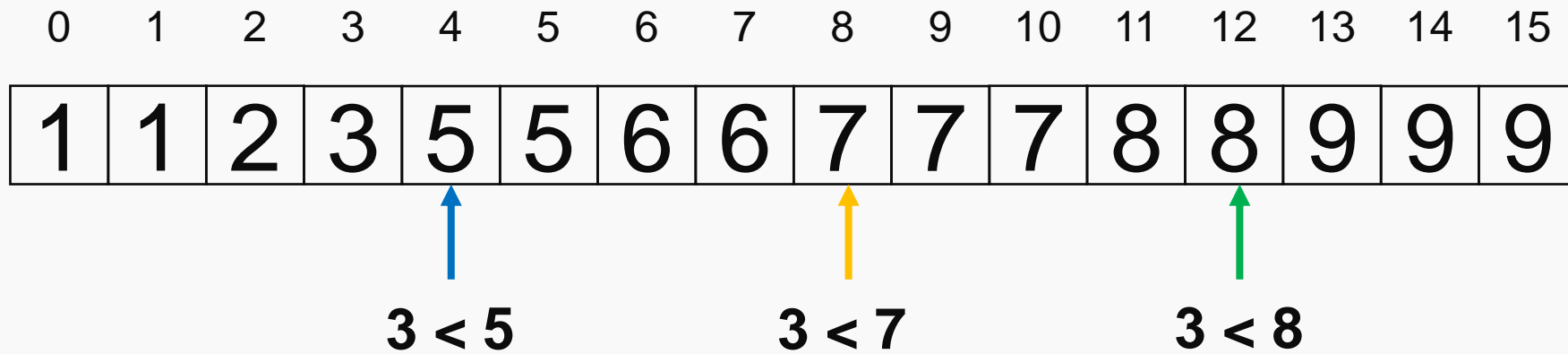
Mai greu de implementat  
Mai puține thread-uri



## Căutare paralelă – soluția 2

Căutăm 3

Între pozițiile 0 15





## Căutare paralelă – soluția 2

Căutăm 3

Între pozițiile 0 3

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9

$3 > 1$   
 $3 > 2$   
 $3 = 3$  end

Complexitate:  $O(\log_p(n))$



## Căutare paralelă – soluția 2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9

$3 < 5$

$3 < 7$

$3 < 8$

Operațiile aceste **NU** pot executa paralel

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9

$3 > 1$

$3 = 3$  end

$3 > 2$



## Căutare paralelă – soluția 2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9

$3 < 5$

$3 < 7$

$3 < 8$



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	2	3	5	5	6	6	7	7	7	8	8	9	9	9

$3 > 1$

$3 = 3$

$3 > 2$





# Căutare paralelă - Complexitate

$$O(\log_p(N))$$

Speedup?



# Căutare paralelă - Complexitate

$$O(\log_p(N))$$

Speedup?

$$S = \frac{\log_2(N)}{\log_p(N)}$$



# Căutare paralelă - Complexitate

$$O(\log_p(N))$$

Speedup?

$$S = \frac{\log_2(N)}{\log_p(N)} = \frac{\log(P)}{\log(2)} = \log_2(P)$$





# Merge sort paralel - idee

Operația de merge poate și ea fi paralelizată.  
Pentru a o paraleliza ne bazăm pe căutare binară  
(sau chiar paralelă) și pe rank sort.





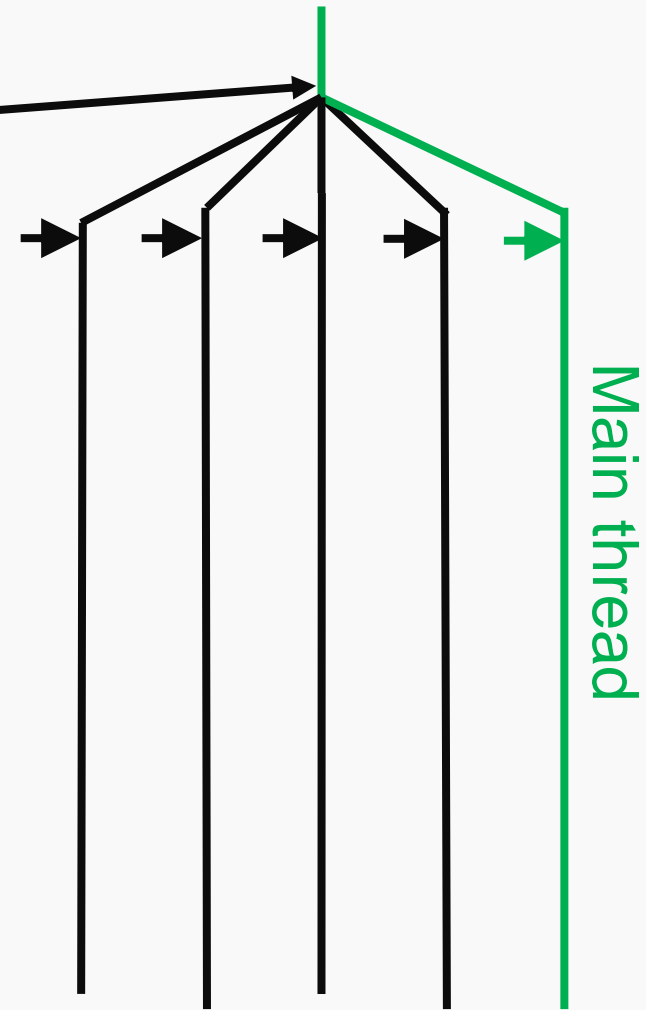
# **Executor Service sau Replicated Workers sau Thread Pool**

## **Abordare de probleme recursive în paralel**



# Replicated Workers

`startWorkers();`



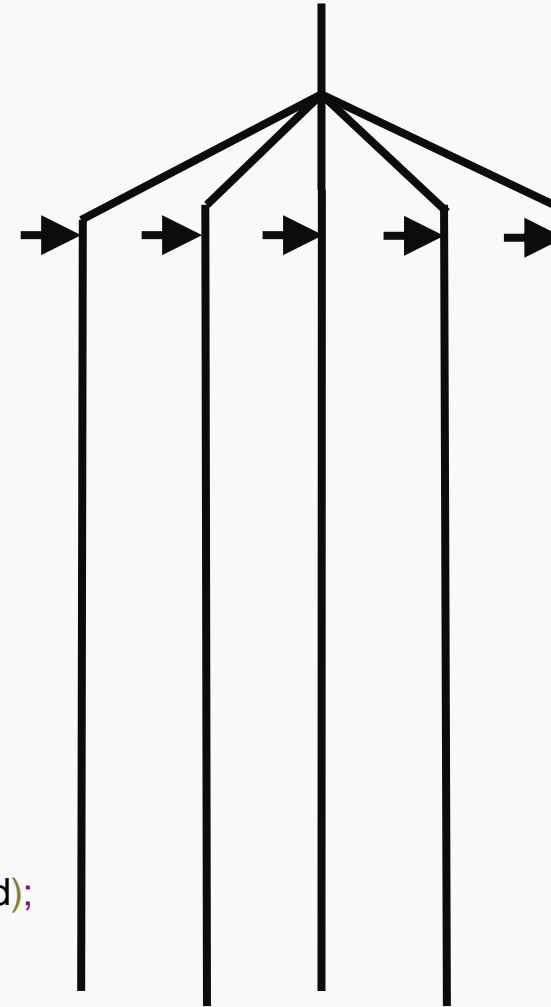




# Replicated Workers

```
Task makeTask(int i)
{
    Task task;
    int * newData = (int*)malloc(sizeof(int));
    newData[0]=i;
    task.data=newData;
    task.runTask = printSomething;
    return task;
}

void printSomething (void * data, int thread_id)
{
    int task_id = *(int*)data;
    if(task_id>N) {
        forceShutDownWorkers();
        return;
    }
    printf("Something %i from thread %i\n", task_id, thread_id);
    putTask(makeTask(task_id+1));
}
```





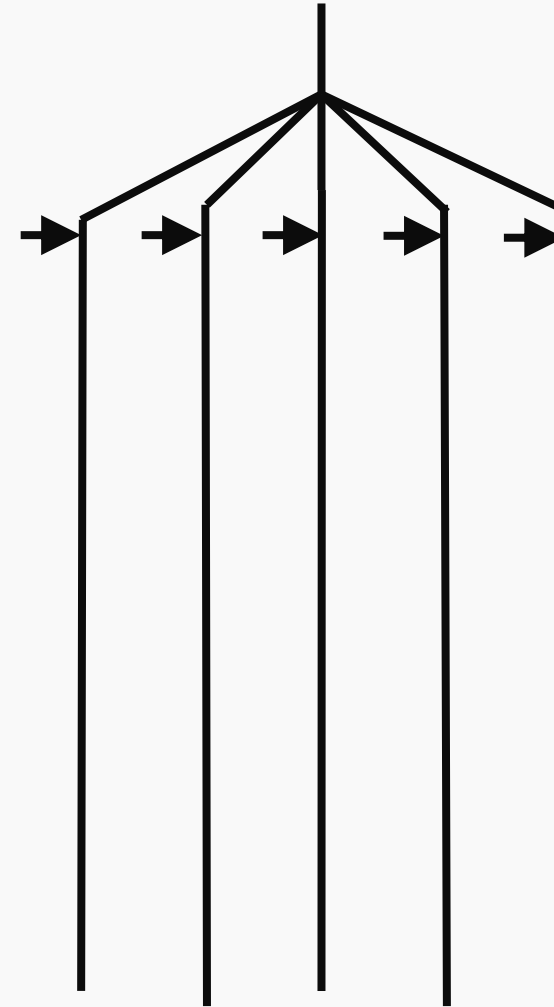
# Replicated Workers

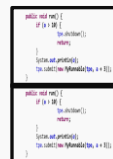
```
putTask(Task1);  
putTask(Task2);  
putTask(Task3);  
.....
```

.....



```
public void run() {  
    if (isDone()) {  
        return;  
    }  
    try {  
        putTask(Task1);  
    } catch (InterruptedException e) {  
        e.printStackTrace();  
    }  
}  
  
public void run() {  
    if (isDone()) {  
        return;  
    }  
    try {  
        putTask(Task2);  
    } catch (InterruptedException e) {  
        e.printStackTrace();  
    }  
}  
  
public void run() {  
    if (isDone()) {  
        return;  
    }  
    try {  
        putTask(Task3);  
    } catch (InterruptedException e) {  
        e.printStackTrace();  
    }  
}  
  
public void run() {  
    if (isDone()) {  
        return;  
    }  
    try {  
        putTask(Task4);  
    } catch (InterruptedException e) {  
        e.printStackTrace();  
    }  
}  
  
public void run() {  
    if (isDone()) {  
        return;  
    }  
    try {  
        putTask(Task5);  
    } catch (InterruptedException e) {  
        e.printStackTrace();  
    }  
}
```



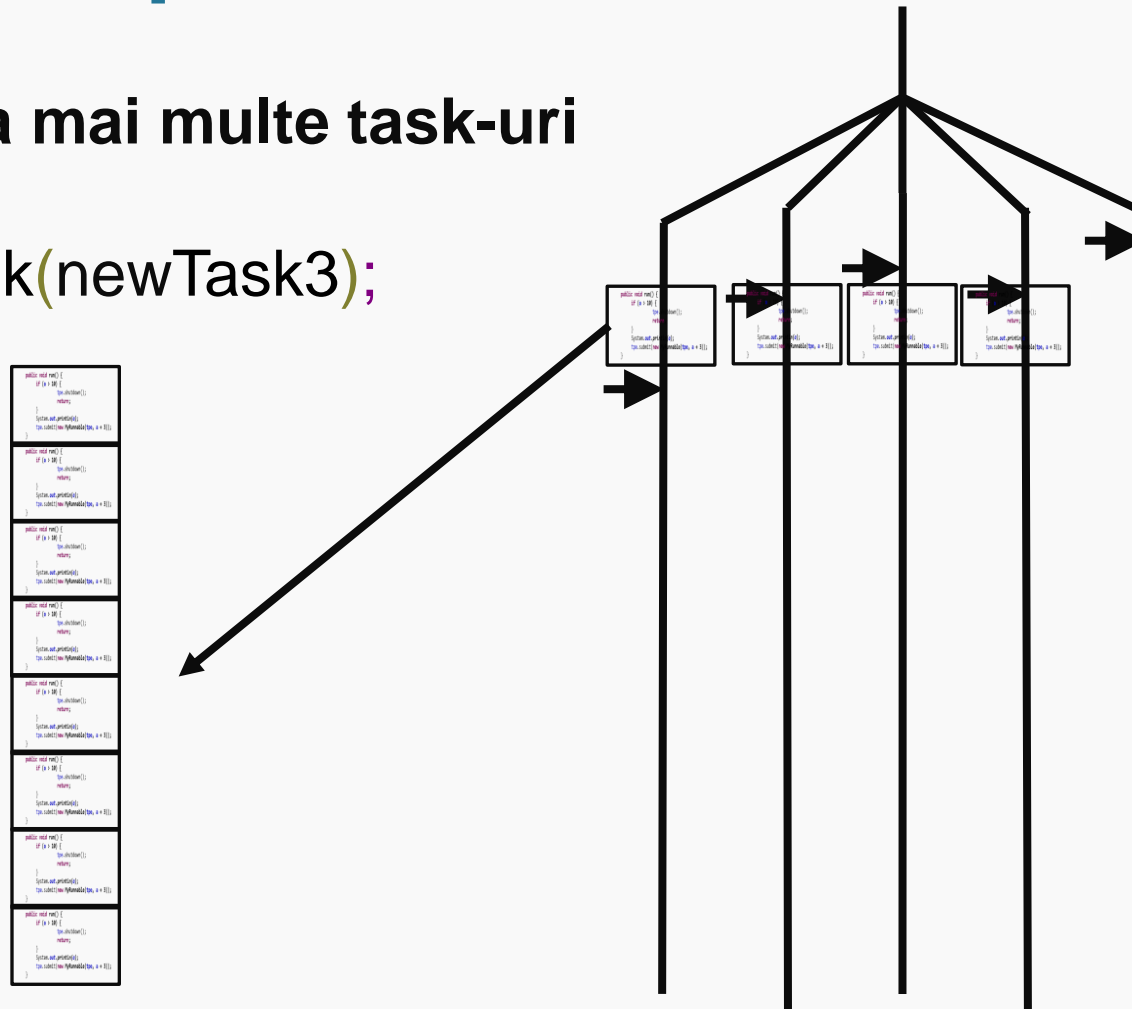




# Replicated Workers

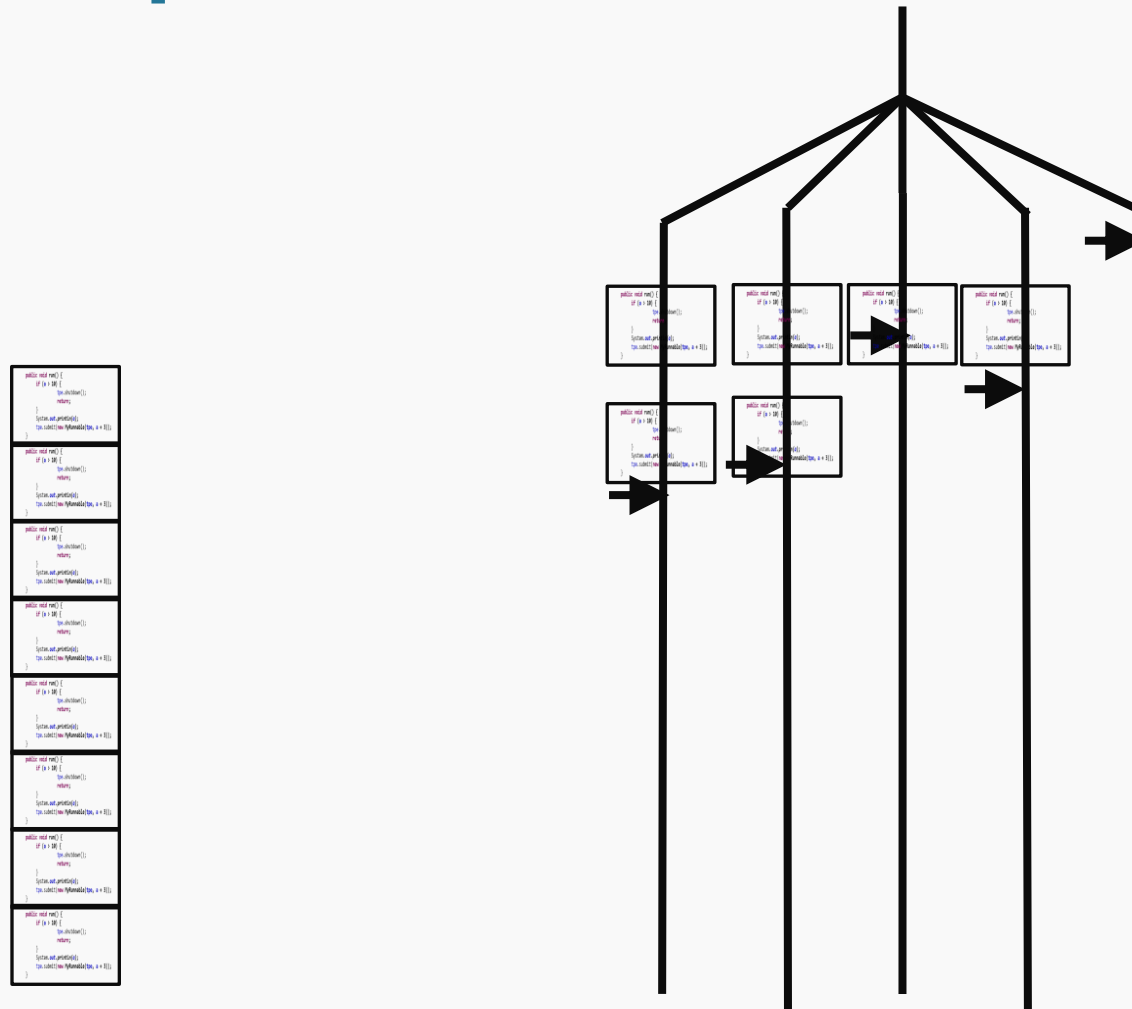
Task-urile pot crea mai multe task-uri

`putTask(newTask3);`





# Replicated Workers





# Replicated Workers

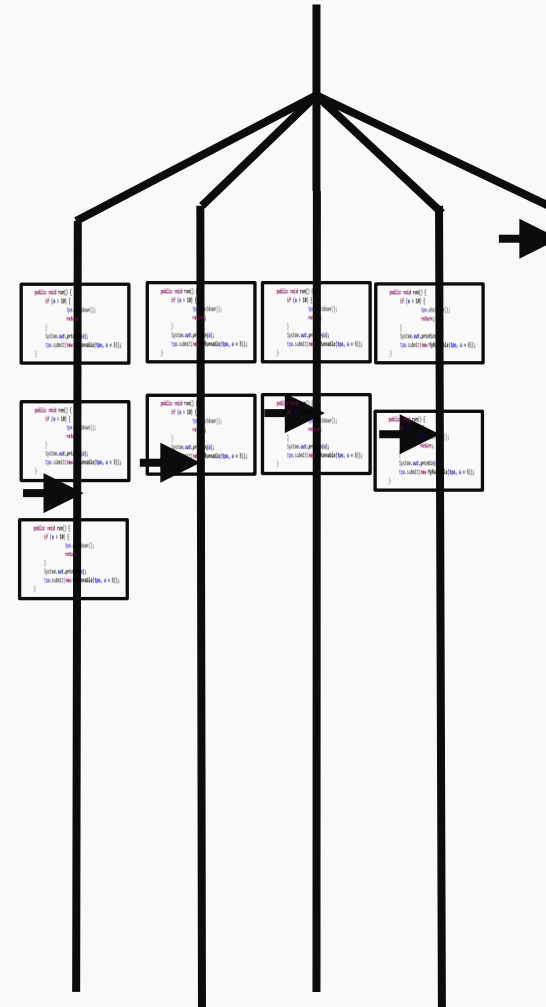
```
public void run() {  
    if (isAlive()) {  
        doAction();  
    }  
    System.out.println();  
    try { Thread.sleep(1000); } catch (InterruptedException e) {}  
}
```

```
public void run() {  
    if (isAlive()) {  
        doAction();  
    }  
    System.out.println();  
    try { Thread.sleep(1000); } catch (InterruptedException e) {}  
}
```

```
public void run() {  
    if (isAlive()) {  
        doAction();  
    }  
    System.out.println();  
    try { Thread.sleep(1000); } catch (InterruptedException e) {}  
}
```

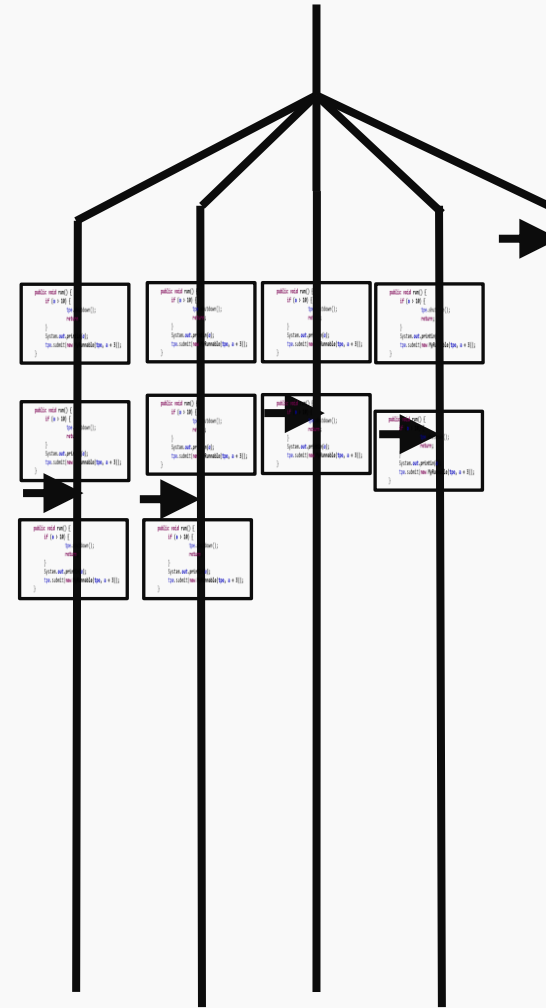
```
public void run() {  
    if (isAlive()) {  
        doAction();  
    }  
    System.out.println();  
    try { Thread.sleep(1000); } catch (InterruptedException e) {}  
}
```

```
public void run() {  
    if (isAlive()) {  
        doAction();  
    }  
    System.out.println();  
    try { Thread.sleep(1000); } catch (InterruptedException e) {}  
}
```





# Replicated Workers



```
public void run() {
    if (isPrime()) {
        System.out.println("Prime");
    }
    System.out.println("Not Prime");
}

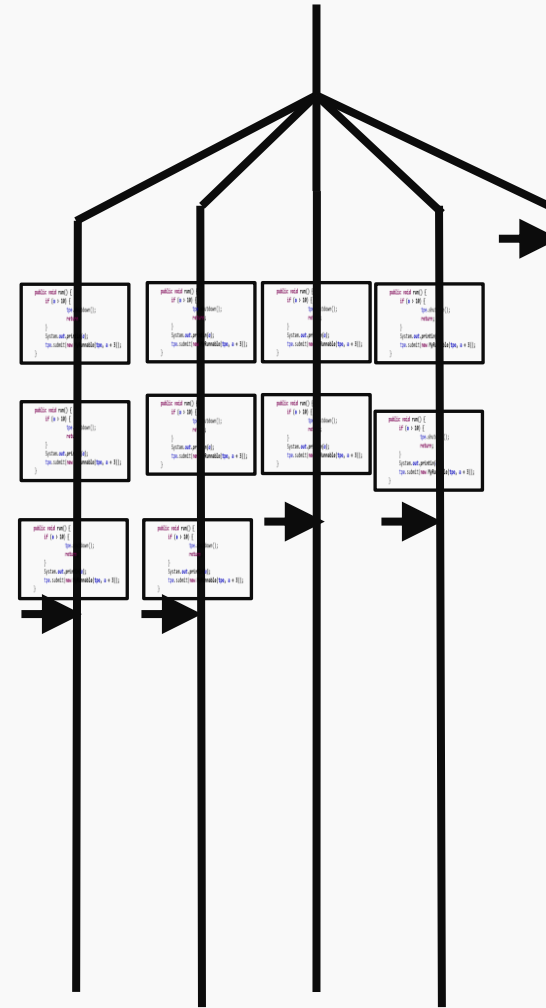
public void run() {
    if (isPrime()) {
        System.out.println("Prime");
    }
    System.out.println("Not Prime");
}

public void run() {
    if (isPrime()) {
        System.out.println("Prime");
    }
    System.out.println("Not Prime");
}

public void run() {
    if (isPrime()) {
        System.out.println("Prime");
    }
    System.out.println("Not Prime");
}
```



# Replicated Workers



```
public void run() {  
    if (is + BE) {  
        // ...  
    }  
    // ...  
    // ...  
}
```

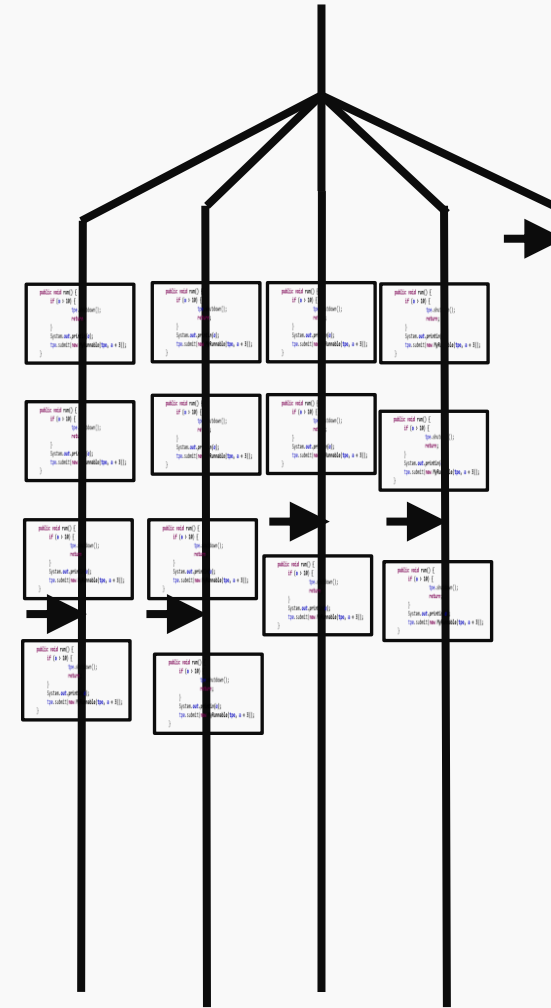




# Replicated Workers

Când oprim thread-urile?  
Depinde de problemă.  
Dacă e suficient putem să oprim  
imediat după găsirea unei soluții.  
Trebuie ținut cont că unele probleme  
nu au soluție.

`joinWorkerThreads();`







# N Queens Problem

		Q	
Q			
			Q
	Q		

**Se cere aranjare a  $N$  regine pe o tablă  $N \times N$  în așa fel încât să nu se atace**



# N Queens Problem

**Nu avem voie mai mult de o regină pe linie**

		Q	
Q			
			Q
	Q		



# N Queens Problem

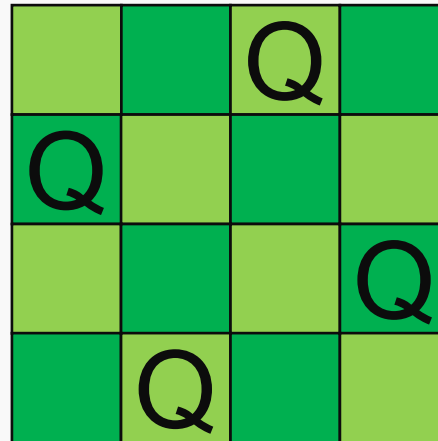
**Nu avem voie mai mult de o regină pe coloană**

		Q	
Q			
			Q
	Q		



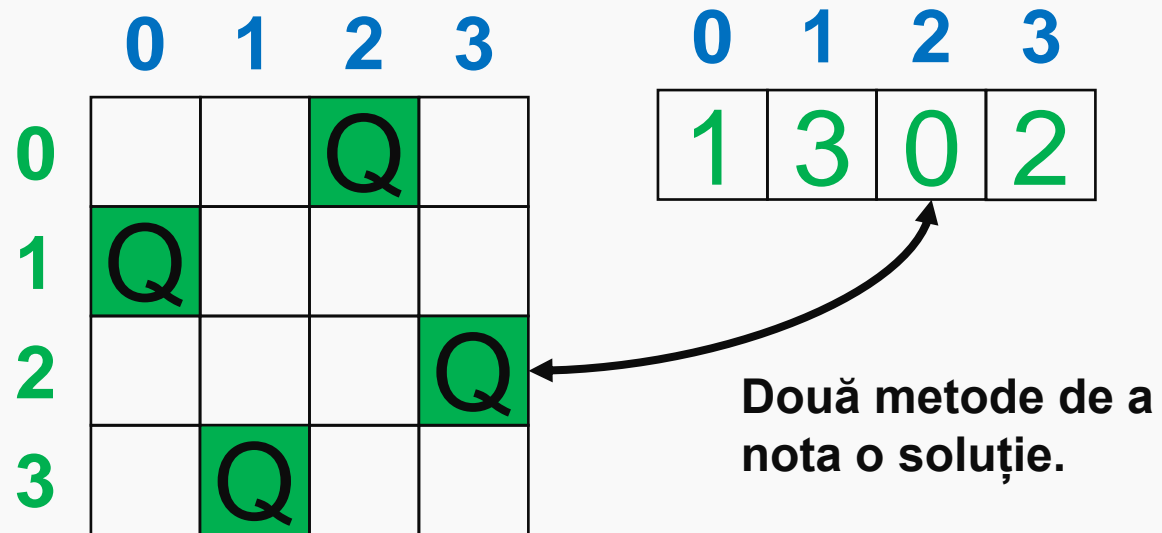
# N Queens Problem

**Nu avem voie mai mult de o regină pe diagonală**





# N Queens Problem



**Poziția din vector reprezintă coloana pe care este așezată regina.**

**Valoarea din vector reprezintă linia pe care este așezată regina.**



# N Queens Problem – Soluție

	0	1	2	3
0	Q			
1				
2				
3				

0	1	2	3
0			





# N Queens Problem – Soluție

	0	1	2	3
0	Q	Q		
1				
2				
3				

0	1	2	3
0	0		

**Conflict linie**



# N Queens Problem – Soluție

	0	1	2	3
0	Q			
1		Q		
2				
3				

0	1	2	3
0	1		

**Conflict diagonală**



# N Queens Problem – Soluție

	0	1	2	3
0	Q			
1				
2		Q		
3				

0	1	2	3
0	2		

**OK.**

**Și tot așa până  
punem toate  
reginele**



# N Queens Problem – Soluție paralelă

0 1 2 3

0			
---	--	--	--

1			
---	--	--	--

2			
---	--	--	--

3			
---	--	--	--



# N Queens Problem – Soluție paralelă

0	1	2	3	
0	0			x
0	1			x
0	2			
0	3			

0	1	2	3	
2	0			
2	1			x
2	2			x
2	3			x

Și tot așa...

0	1	2	3	
1	0			x
1	1			x
1	2			x
1	3			

0	1	2	3	
3	0			
3	1			
3	2			x
3	3			x

