

# Structuri de date și algoritmi

## Liste, Stive, Cozi (1)

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PER ASPERA AD ASTRA



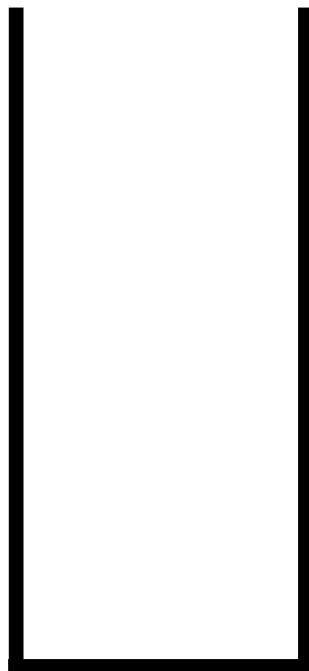


# Inversarea unui şir



# Inversarea unui șir

ABCDEF

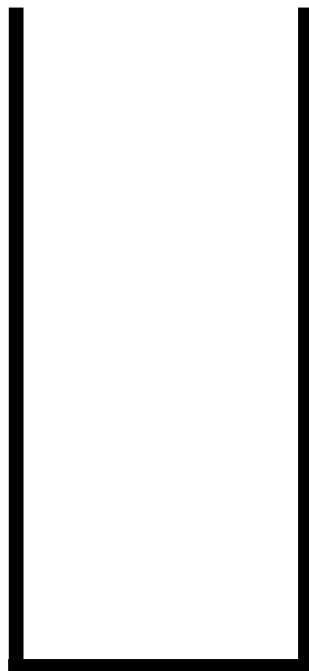




# Inversarea unui șir

BCDEF

push( A )

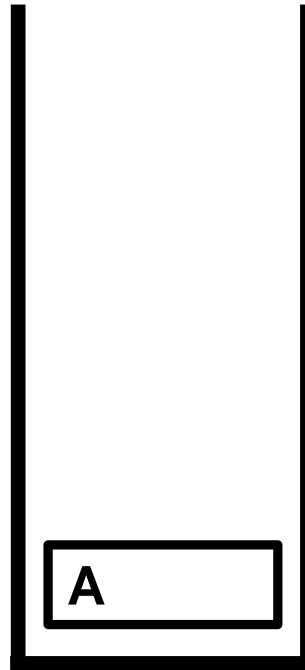




# Inversarea unui șir

CDEF

push( B )

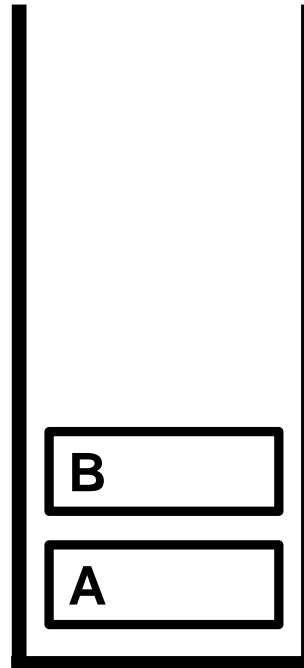




# Inversarea unui șir

DEF

push( c )

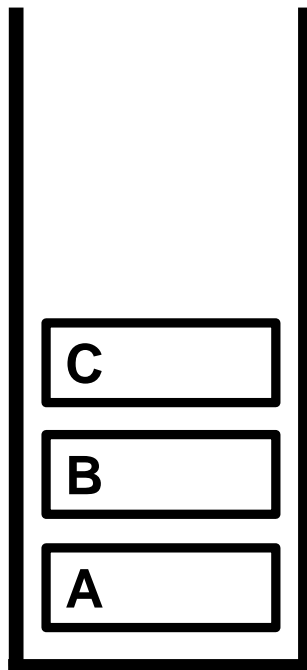




# Inversarea unui şir

EF

push( D )

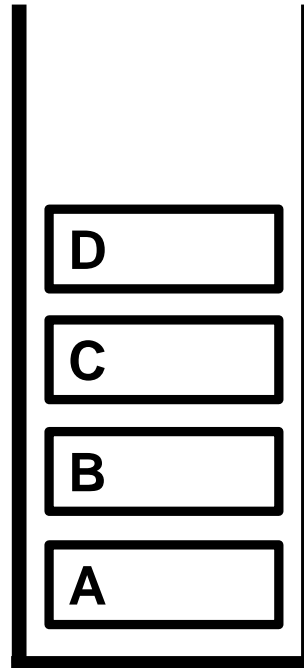




# Inversarea unui șir

F

push( E )

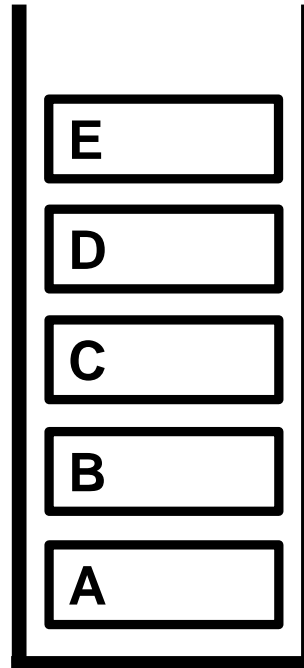






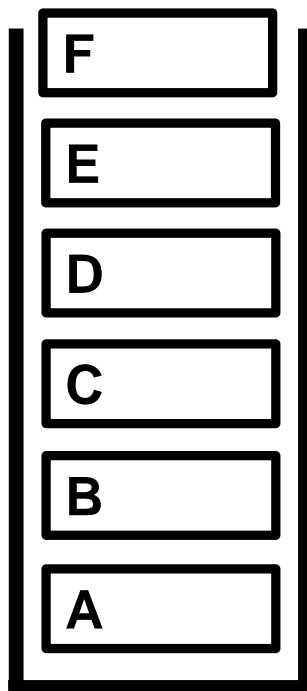
# Inversarea unui șir

push( **F** )





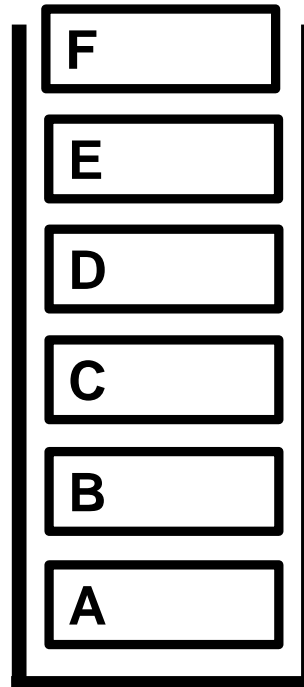
# Inversarea unui șir





# Inversarea unui șir

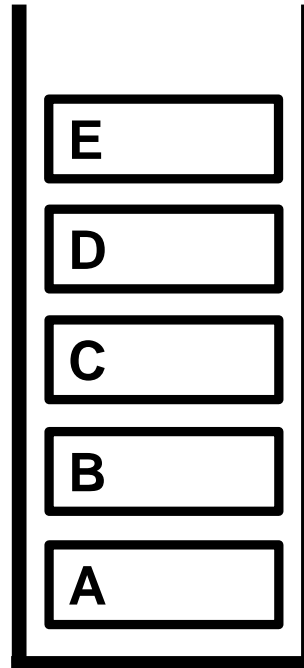
pop()





# Inversarea unui șir

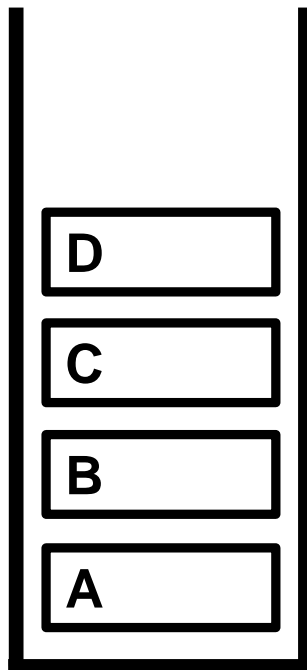
F pop()





# Inversarea unui șir

E pop()

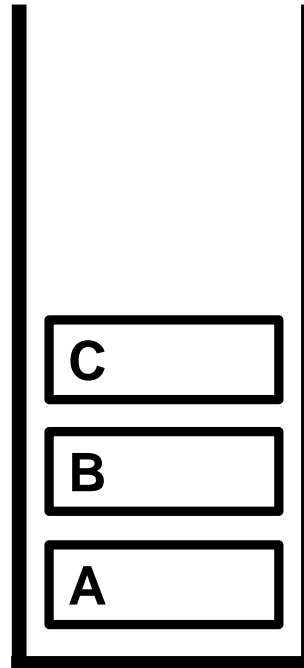


Afișare: F



# Inversarea unui șir

D pop()

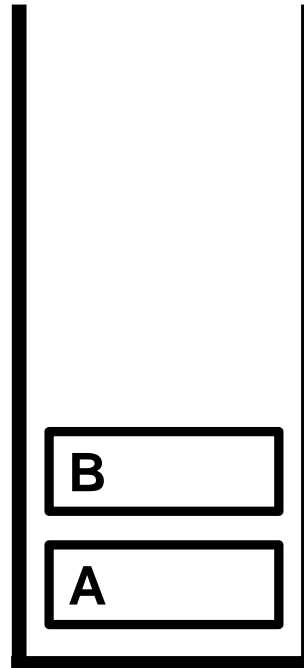


Afișare: FE



# Inversarea unui șir

C pop()

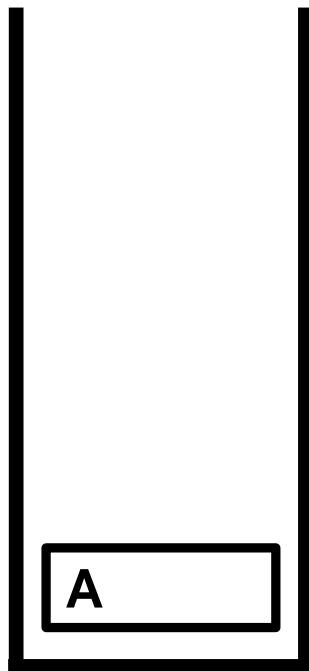


Afișare: FED



# Inversarea unui șir

**B** pop()



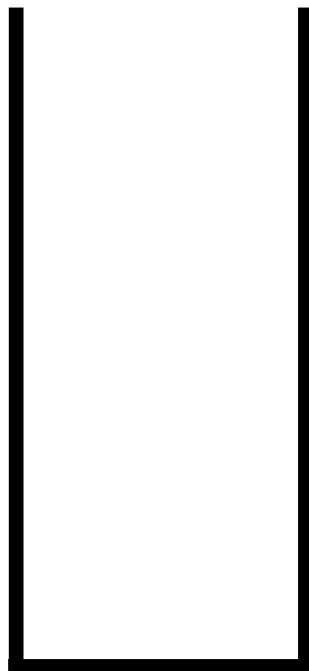
Afișare: FEDC





# Inversarea unui șir

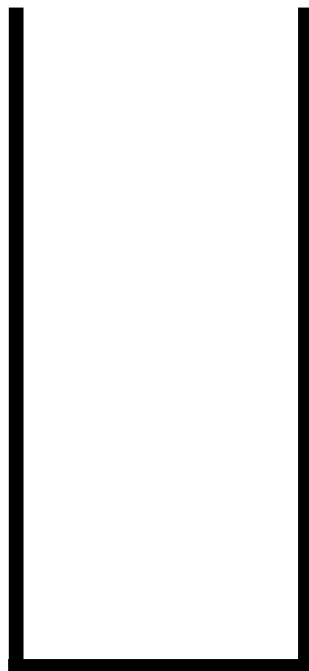
A



Afișare: FEDCB



# Inversarea unui şir



Afişare: FEDCBA



# Corectitudinea parantezelor unui expresii

<http://chilipirea.ro/sda>



## Corectitudinea parantezelor unui expresii

1.  $[(1+2)-3*(2+1)]$
2.  $\{[(())]\}$
3.  $[(()]$
4.  $[]()$
5.  $[(1+)*2(3)]$
6.  $\{1+2+3+[2*3]*(1+2)\}$
7.  $\{1+2+3+[2*3*(1+2)]\}$
8.  $\{1+2+3+[2*3]*(1+2)))$

■ Care expresii sunt corecte?



# Corectitudinea parantezelor unui expresii

Parcurgem șirul de caractere

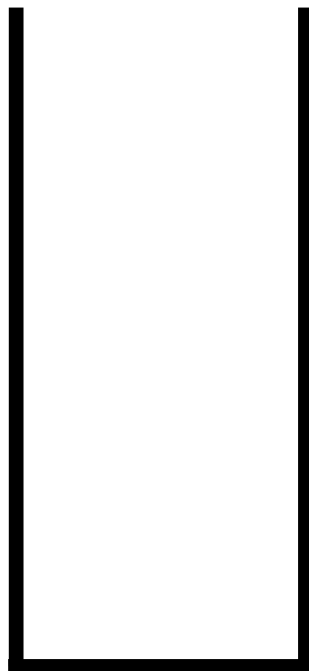
- Ignorăm orice caracter cu excepția [ ] ( ) { }
- Pentru caracterele ( [ { adăugăm pe acestea pe stivă
- Pentru caracterele ) ] } scoatem capul stivei și verificăm dacă se potrivește
  - Dacă nu avem ce scoate expresia este greșită
  - Dacă nu expresia este greșită

Când terminăm șirul știm că expresia este corectă



## Corectitudinea parantezelor unui expresii

$[(1+2)-3*(2+1)]$

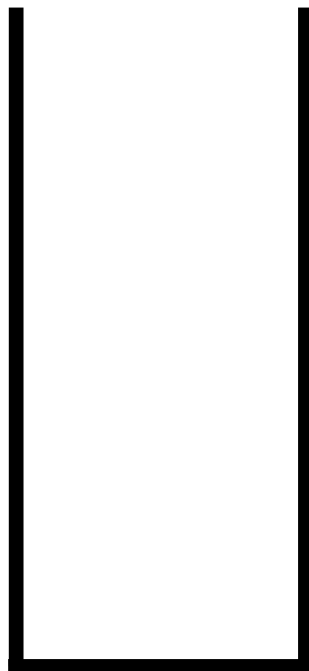




## Corectitudinea parantezelor unui expresii

$(1+2)-3*(2+1)]$

push( [ ] )

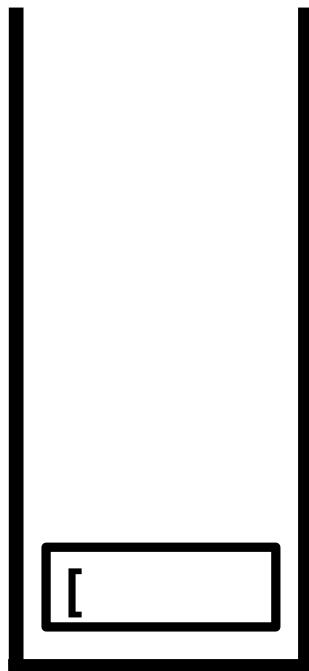




## Corectitudinea parantezelor unui expresii

$1+2)-3*(2+1)]$

push( ( ) )

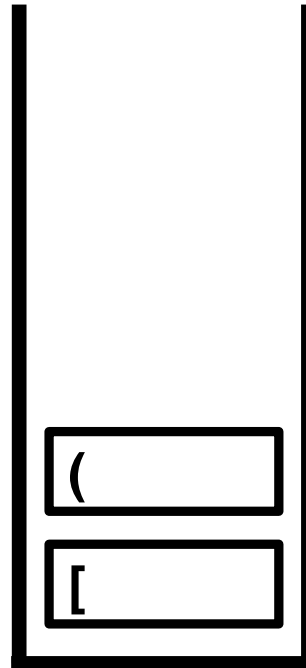






# Corectitudinea parantezelor unui expresii

$1+2)-3*(2+1)]$





## Corectitudinea parantezelor unui expresii

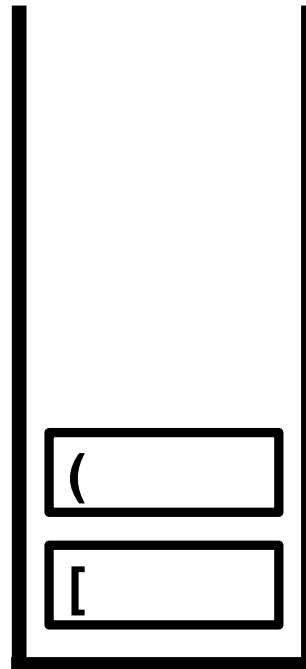
$+2)-3*(2+1)]$

A diagram illustrating a stack data structure. It consists of a large vertical rectangle representing the stack container. Inside this container, at the bottom, are two smaller horizontal rectangles representing the elements in the stack. The top element contains the opening parenthesis '(', and the bottom element contains the opening square bracket '['. This represents the state of the stack after processing the expression up to the second closing parenthesis in the given string.



# Corectitudinea parantezelor unui expresii

2)-3\*(2+1)]





## Corectitudinea parantezelor unui expresii

$) - 3 * (2 + 1) ]$

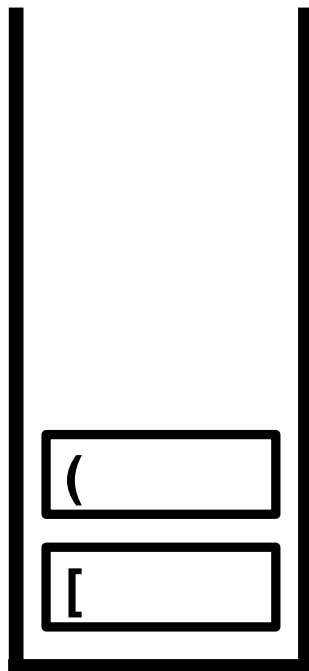
(
[



# Corectitudinea parantezelor unui expresii

$-3*(2+1)]$

pop()  
)

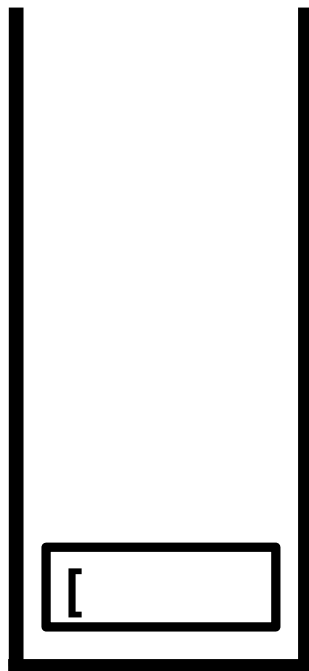




## Corectitudinea parantezelor unui expresii

$-3*(2+1)]$

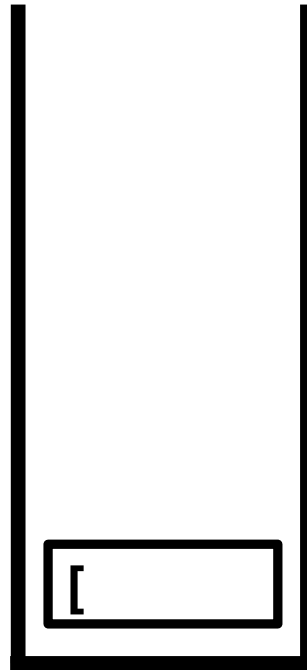
(  pop()  
)





# Corectitudinea parantezelor unui expresii

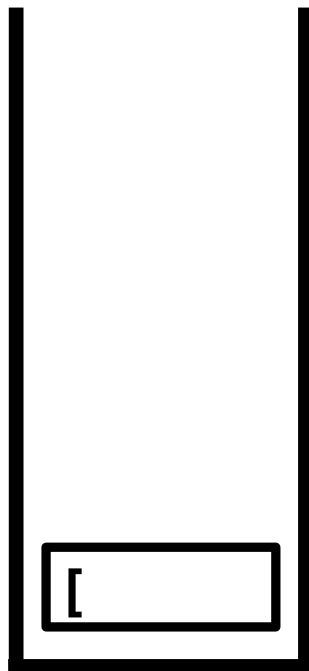
$3^*(2+1)]$





# Corectitudinea parantezelor unui expresii

\*(2+1)]

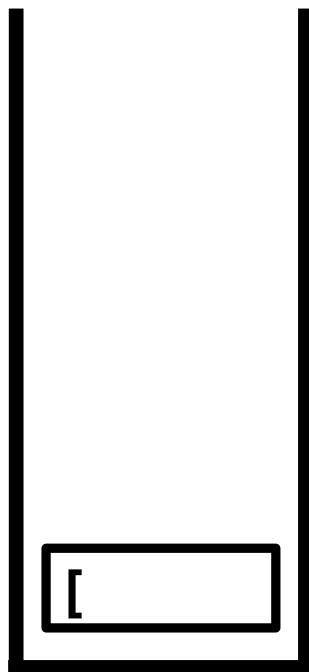






# Corectitudinea parantezelor unui expresii

$(2+1)]$

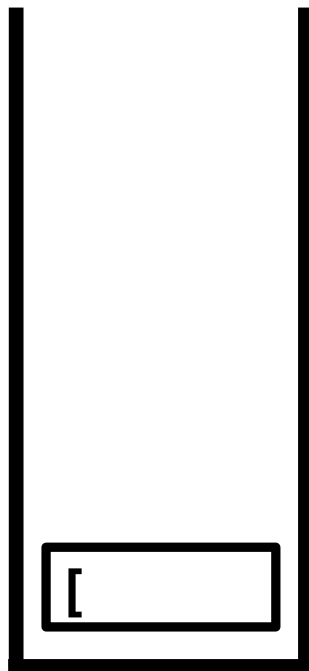




# Corectitudinea parantezelor unui expresii

2+1)]

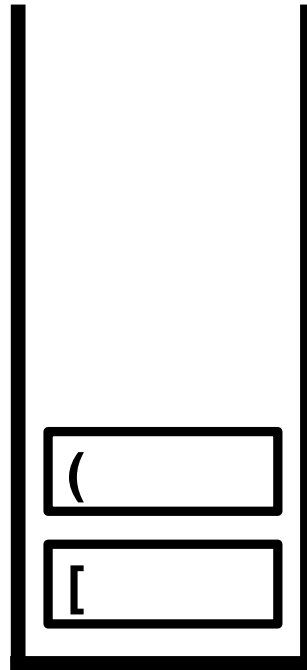
push( ( ) )





# Corectitudinea parantezelor unui expresii

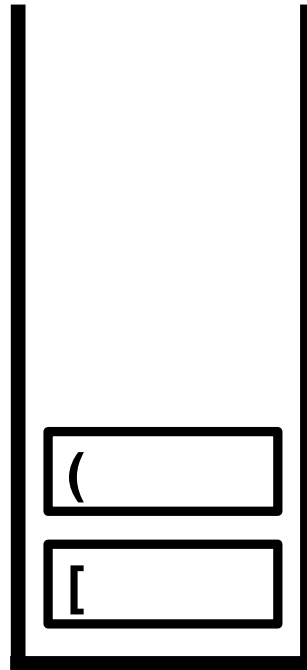
2+1)]





# Corectitudinea parantezelor unui expresii

+1)]





# Corectitudinea parantezelor unui expresii

1)]



# Corectitudinea parantezelor unui expresii

)]

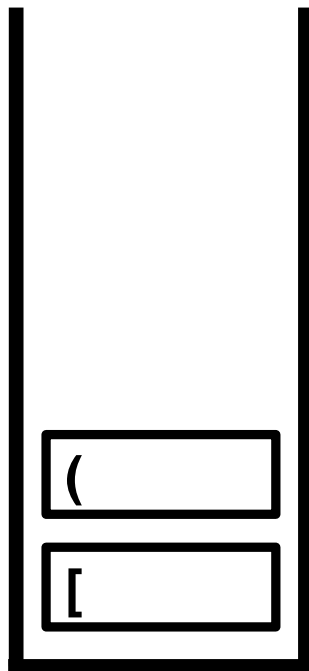
(
[



# Corectitudinea parantezelor unui expresii

]

pop()  
)

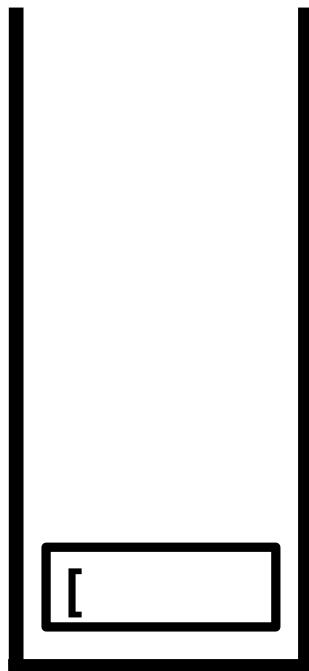




# Corectitudinea parantezelor unui expresii

]

(  pop()  
)

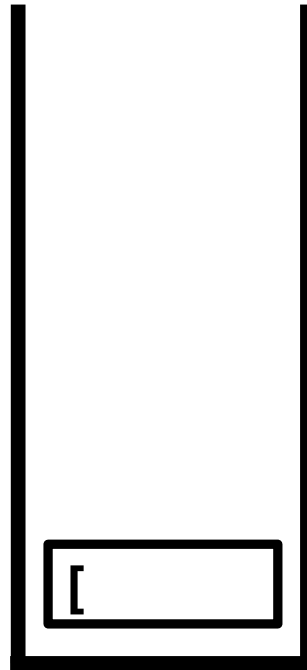






# Corectitudinea parantezelor unui expresii

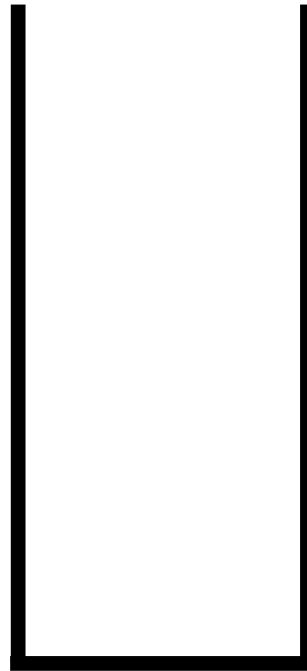
pop()  
]





# Corectitudinea parantezelor unui expresii

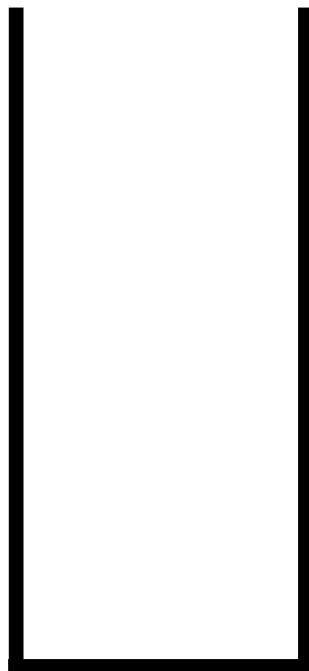
[  pop()  
]





## Corectitudinea parantezelor unui expresii

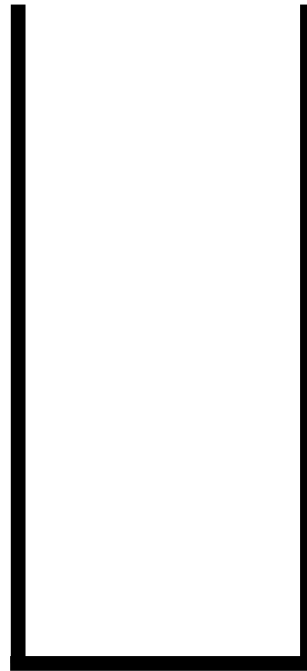
$[(1+2)-3*(2+1)]$     Corectă





# Corectitudinea parantezelor unui expresii

[)([

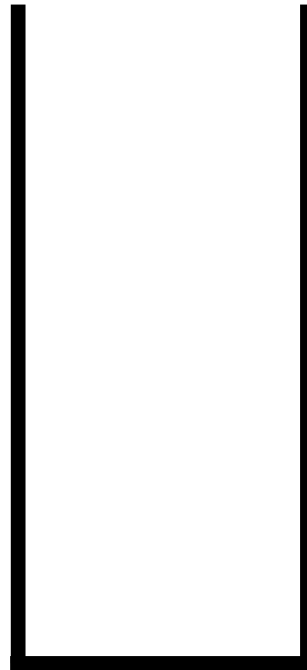




# Corectitudinea parantezelor unui expresii

)([

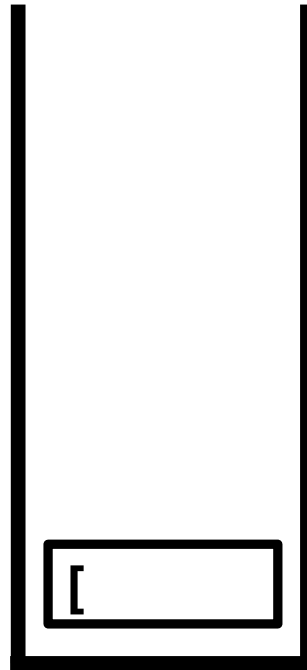
push( [ ] )





# Corectitudinea parantezelor unui expresii

)([

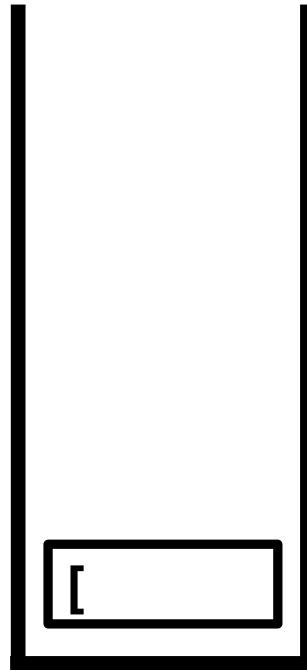




# Corectitudinea parantezelor unui expresii

( ]

pop()  
)

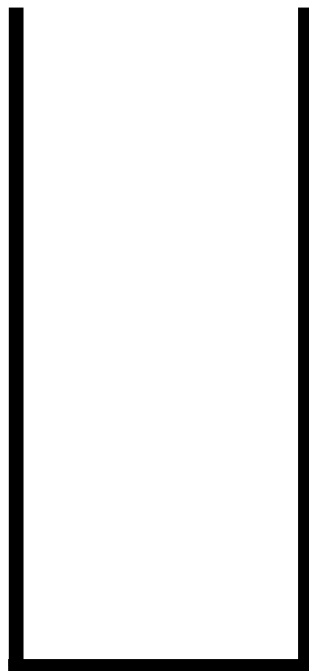




# Corectitudinea parantezelor unui expresii

(] **Incorect**

[  pop()  
)

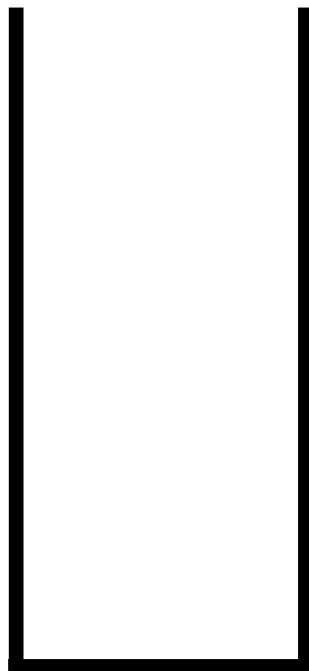






# Corectitudinea parantezelor unui expresii

[)]( **Incorect**







# Stiva programului

<http://pythontutor.com/c.html#mode=display>

Ce problemă observați?

```
#include<stdlib.h>
```

```
int fun1() {  
    return 1;  
}
```

```
void fun2() {  
    int d = 0;  
    fun1();  
    return;  
}
```

```
int main() {  
    int a = 5;  
    int v[4] = { 0,1,2,3 };  
    int* vv = (int*)malloc(5);  
    vv[3] = 5;  
    fun2();  
    a = fun1();  
    return 0;  
}
```



# Stiva programului

<http://pythontutor.com/c.html#mode=display>

```
#include<stdlib.h>
```

```
int fun1() {  
    return 1;  
}
```

```
void fun2() {  
    int d = 0;  
    fun1();  
    return;  
}
```

```
int main() {  
    int a = 5;  
    int v[4] = { 0,1,2,3 };  
    int* vv = (int*)malloc(5 * sizeof(int));  
    vv[3] = 5;  
    fun2();  
    a = fun1();  
    return 0;  
}
```



# Stiva programului

```
#include<stdio.h>
```

```
#define N 5
```

```
void fun1() {  
    int v[N];  
    int i;  
    for (i = 0; i < N; i++) {  
        v[i] = i;  
        printf("fun1: %p %i\n", v + i, v[i]);  
    }  
}
```

```
void fun2() {  
    int z[N];  
    int i;  
    for (i = 0; i < N; i++) {  
        printf("fun2: %p %i\n", z + i, z[i]);  
        z[i] = 0;  
    }  
}
```

```
int main() {  
    fun1();  
    fun2();  
    return 0;  
}
```

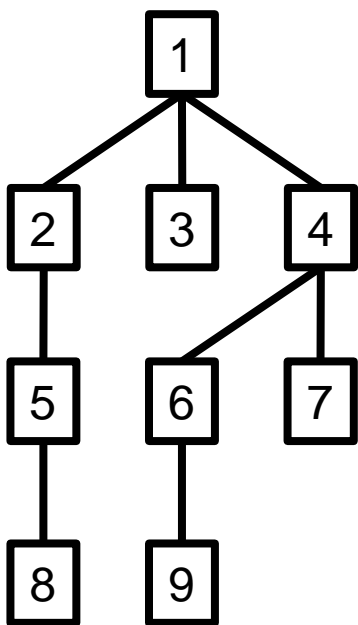
Ce valori vor fi afișate de fun2?



## Implementare reală linked lists

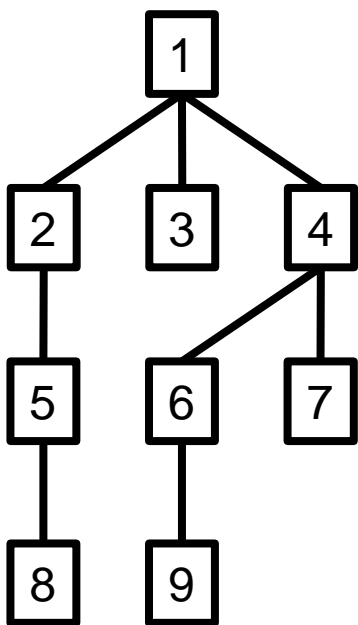
[https://gcc.gnu.org/onlinedocs/gcc-4.6.2/libstdc++/api/a00922\\_source.html](https://gcc.gnu.org/onlinedocs/gcc-4.6.2/libstdc++/api/a00922_source.html)





Care este rădăcina arborelui?

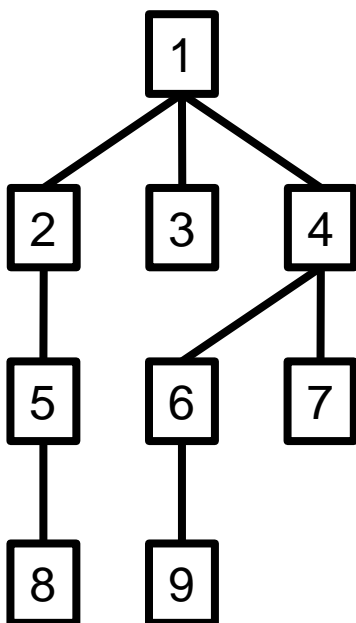




Care sunt frunzele arborelui?

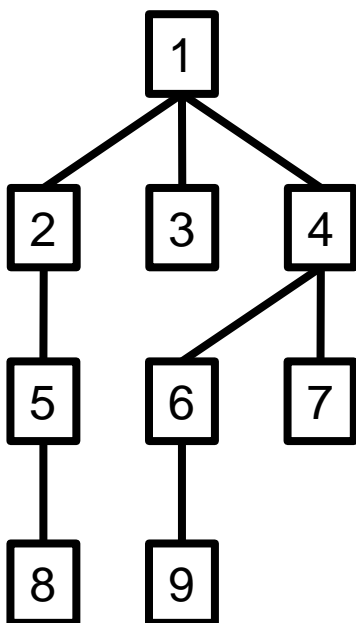


# Afișarea unui arbore fără recursivitate





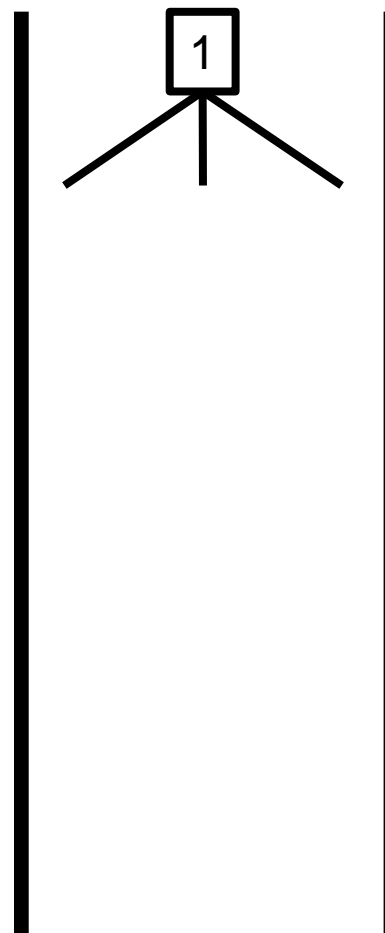
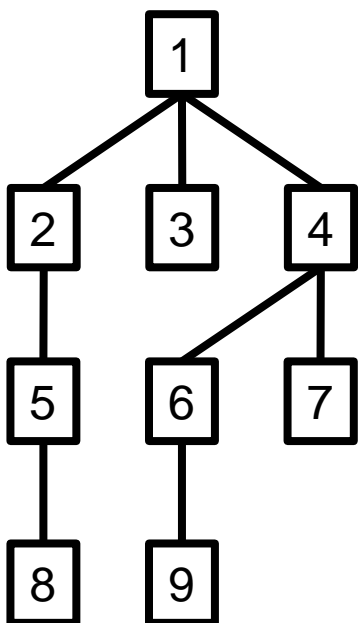
# Afișarea unui arbore folosind o coadă



- Adăugăm rădăcina în coadă
- Până ce coada este goală
  - Scoatem un nod din coadă
  - Adăugăm toți copiii în coadă

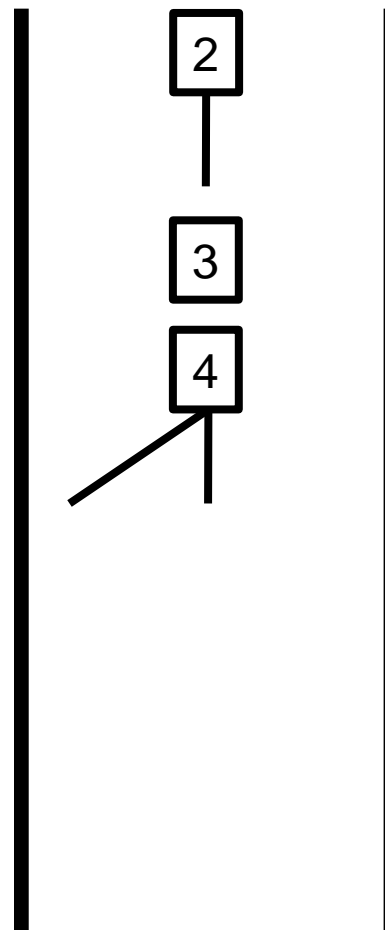
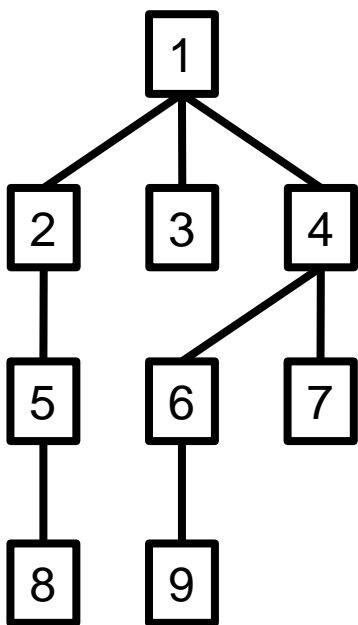


# Afișarea unui arbore folosind o coadă





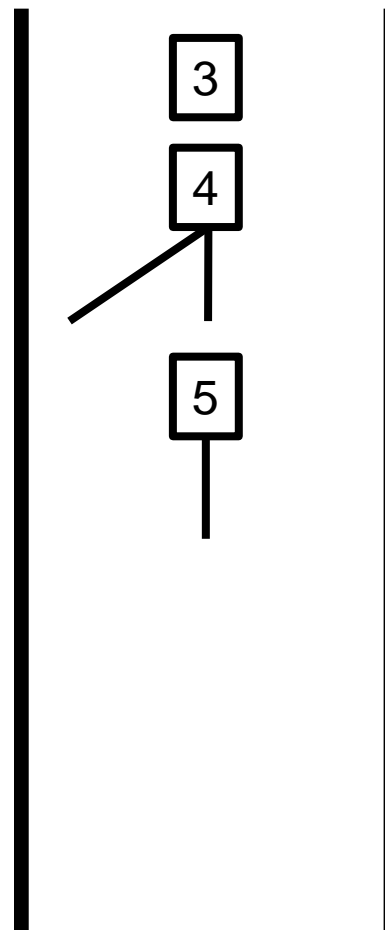
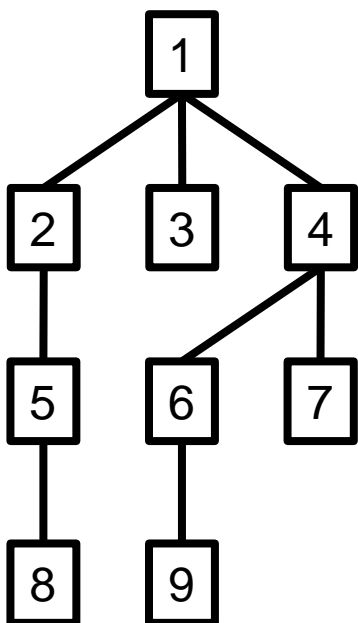
# Afișarea unui arbore folosind o coadă



Afișare: 1



# Afișarea unui arbore folosind o coadă

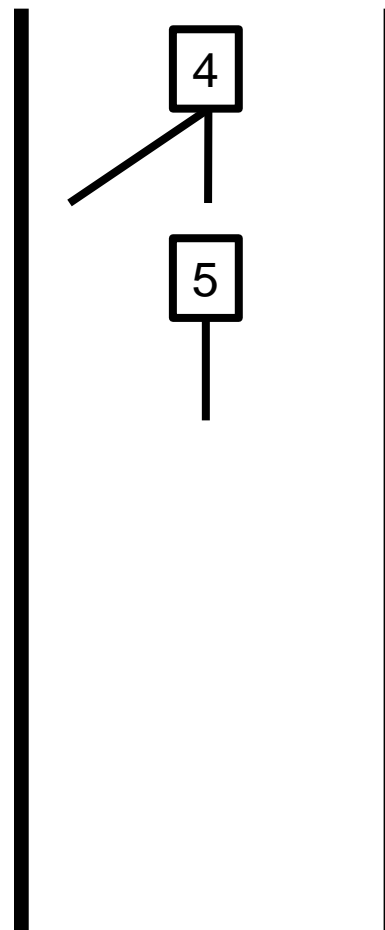
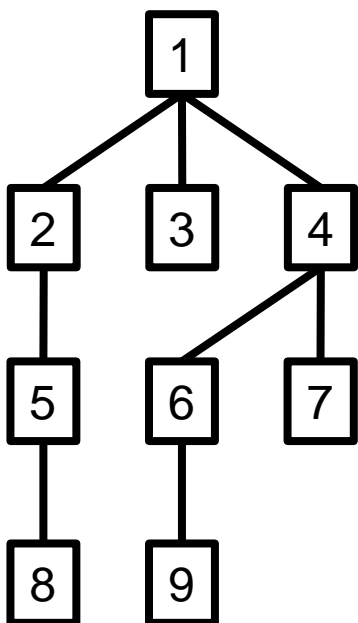


Afișare: 

1	2
---	---



# Afișarea unui arbore folosind o coadă

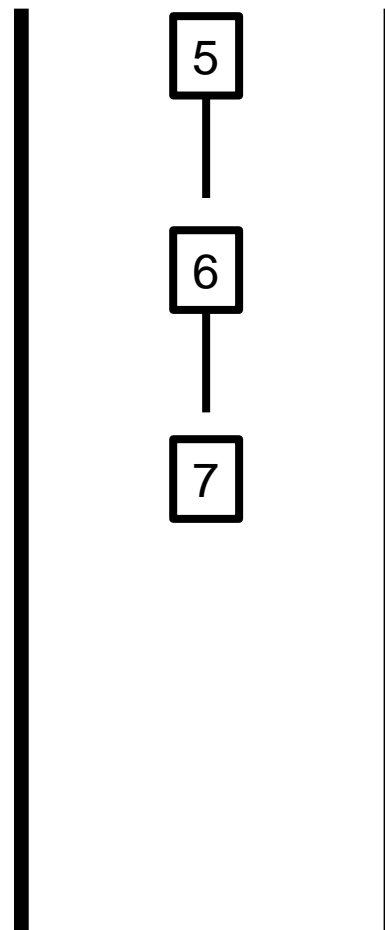
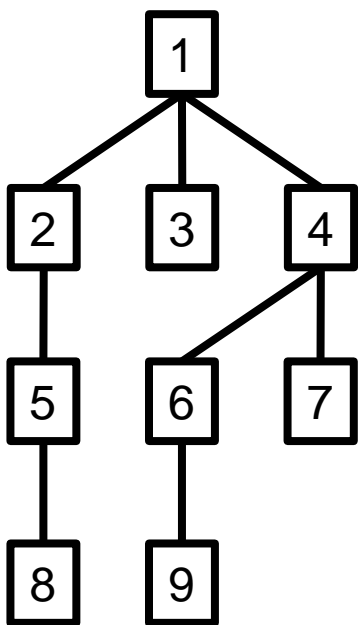


Afișare: 

1	2	3
---	---	---



# Afișarea unui arbore folosind o coadă



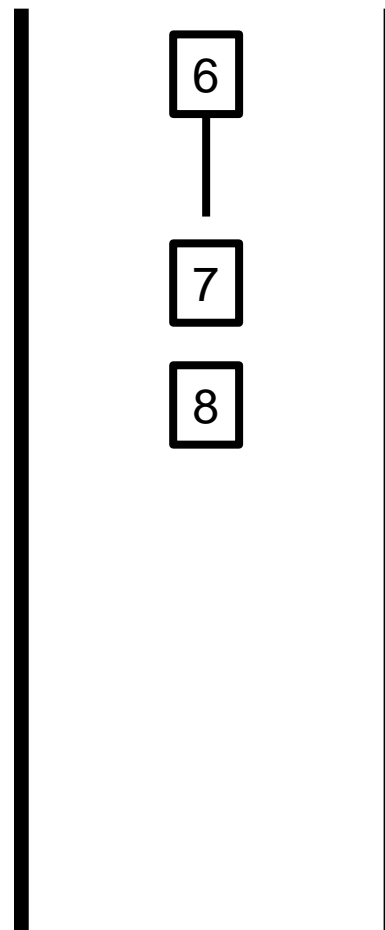
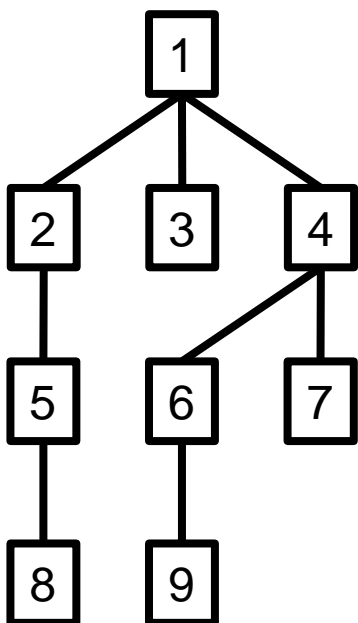
Afișare: 

1	2	3	4
---	---	---	---





# Afișarea unui arbore folosind o coadă

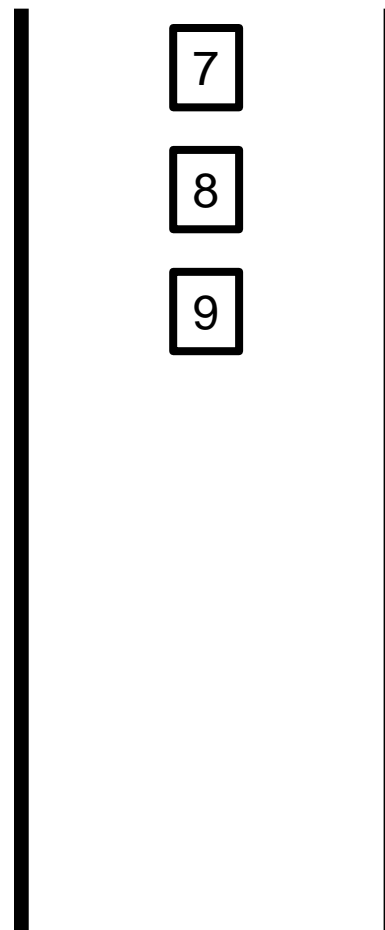
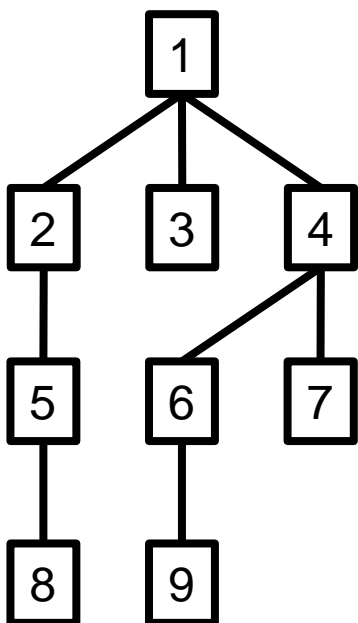


Afișare: 

1	2	3	4	5
---	---	---	---	---



# Afișarea unui arbore folosind o coadă

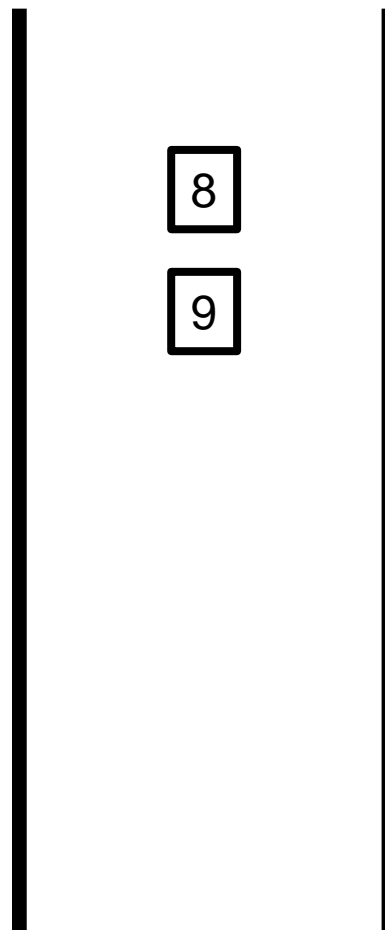
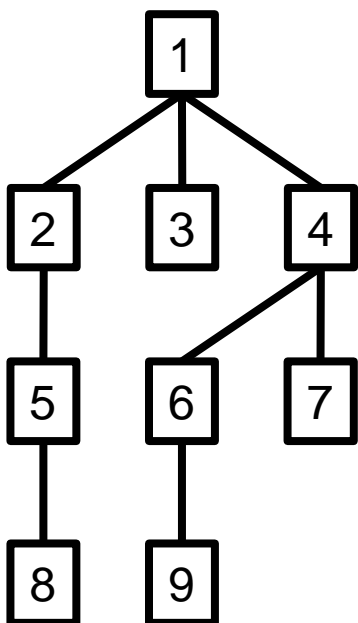


Afișare: 

1	2	3	4	5	6
---	---	---	---	---	---



# Afișarea unui arbore folosind o coadă

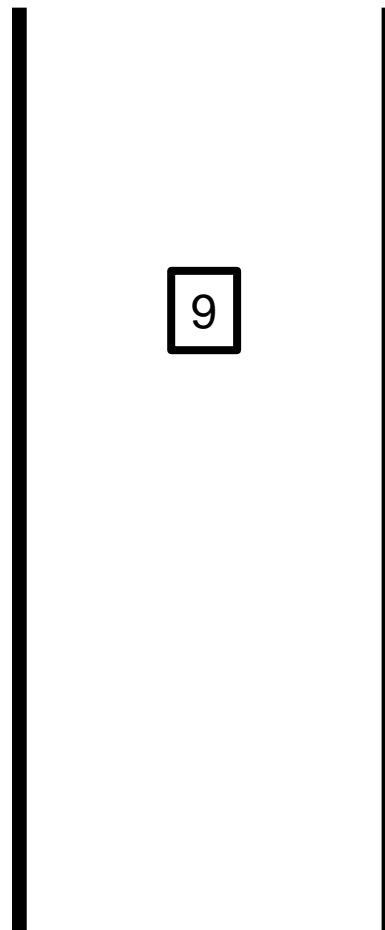
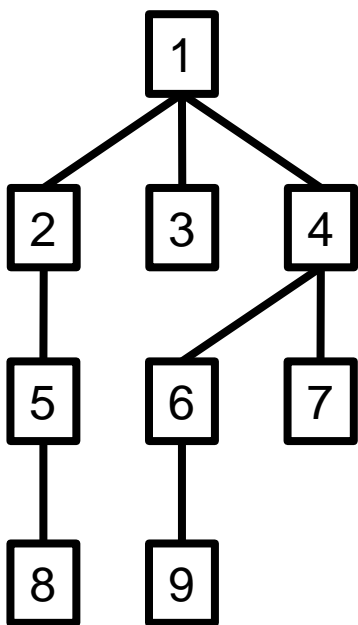


Afișare: 

1	2	3	4	5	6	7
---	---	---	---	---	---	---



# Afișarea unui arbore folosind o coadă

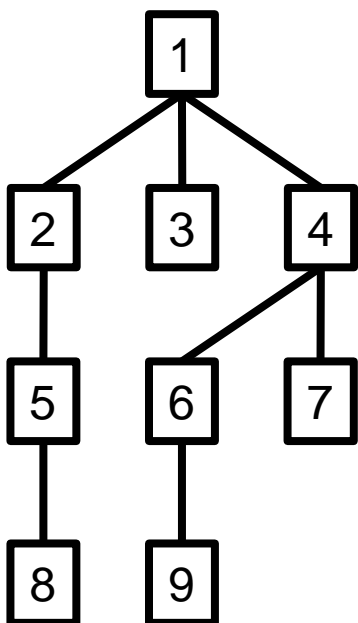


Afișare: 

1	2	3	4	5	6	7	8
---	---	---	---	---	---	---	---



# Afișarea unui arbore folosind o coadă

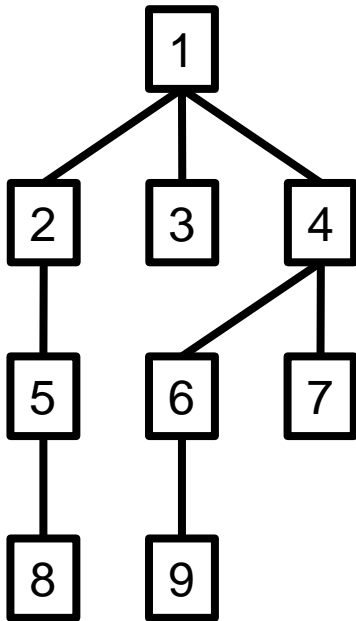


Afișare: 

1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---

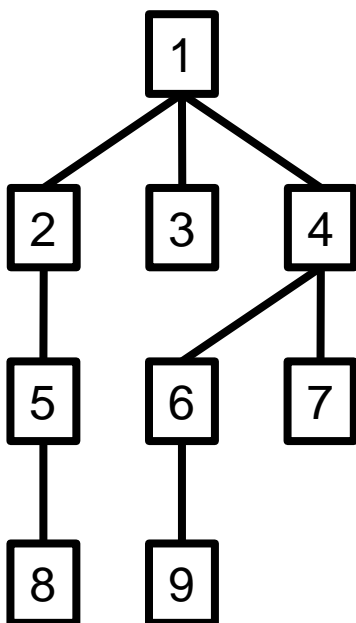


# Afișarea unui arbore folosind o stivă





# Afișarea unui arbore folosind o stivă

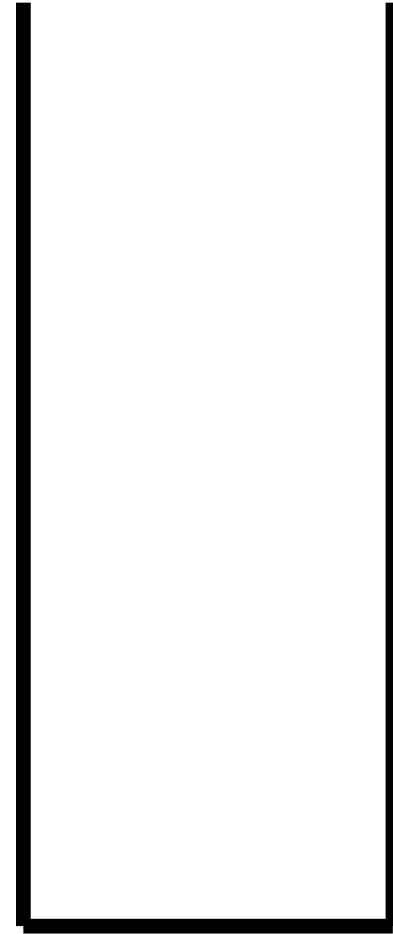
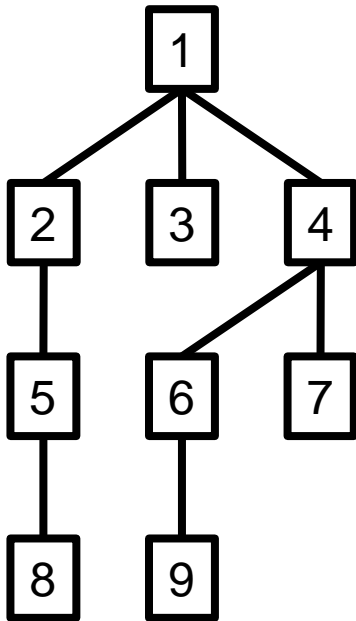


- Adăugăm rădăcina în stivă
- Până ce stiva este goală
  - Scoatem un nod din stivă
  - Adăugăm toți copiii în stivă

Cum va fi afișat arborele?



# Afișarea unui arbore folosind o stivă

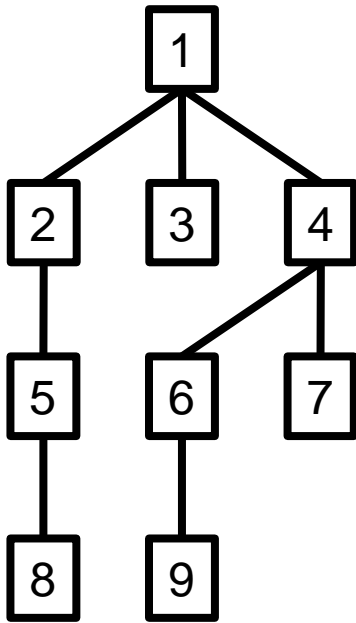


Afișare:

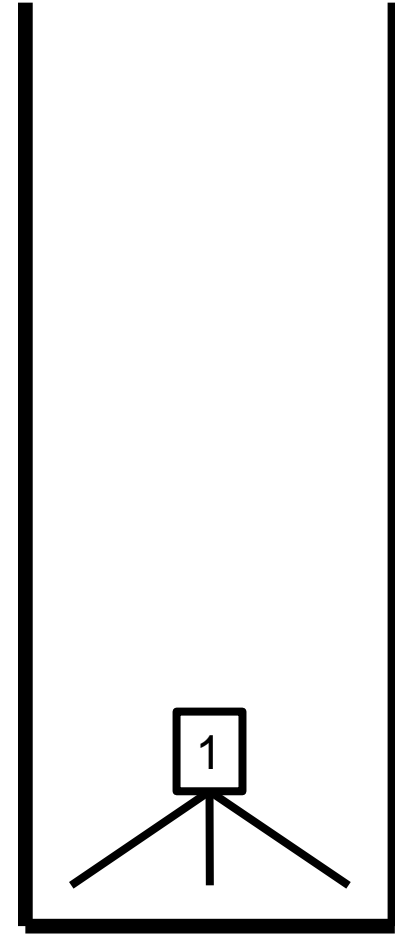




# Afișarea unui arbore folosind o stivă

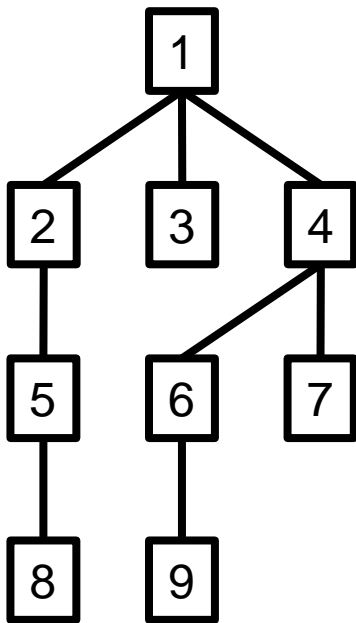


Afișare:

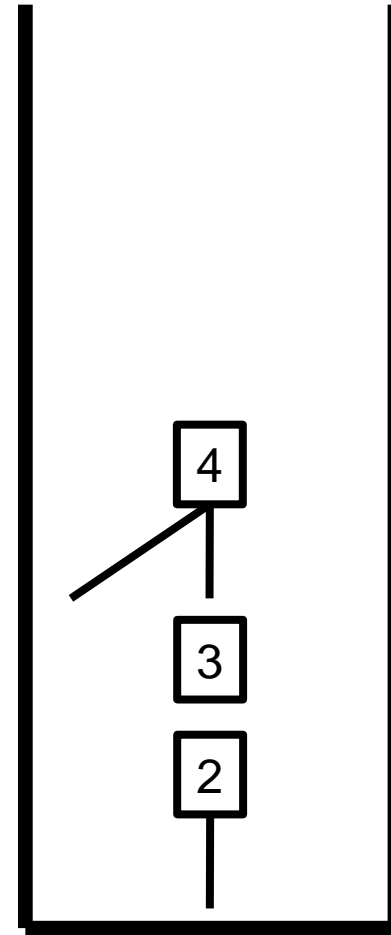




# Afișarea unui arbore folosind o stivă

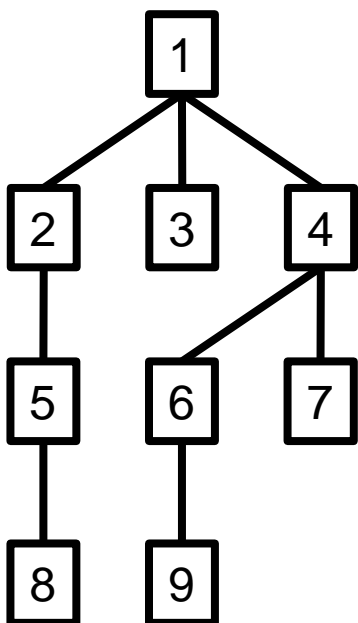


Afișare: 1



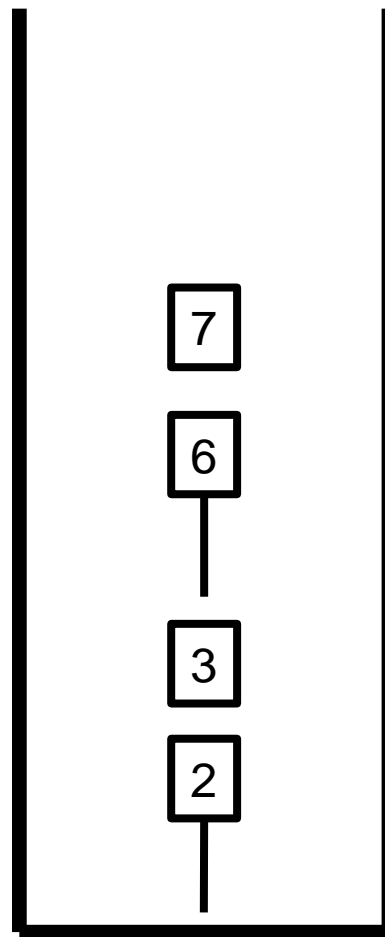


# Afișarea unui arbore folosind o stivă



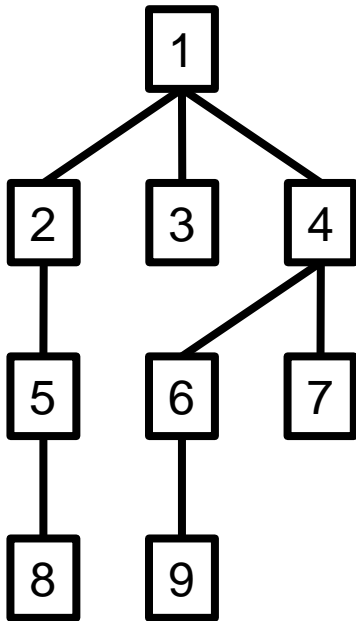
Afișare: 

1	4
---	---



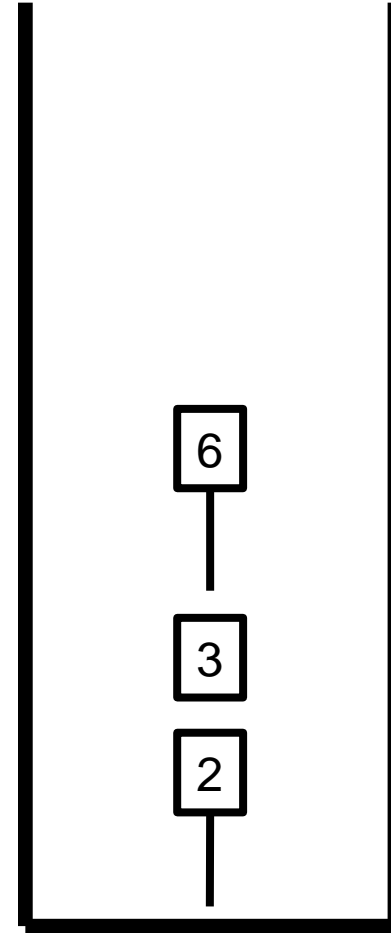


# Afișarea unui arbore folosind o stivă



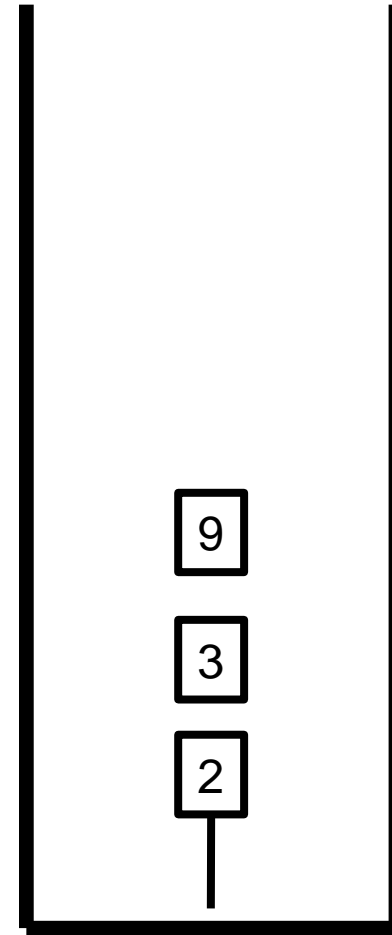
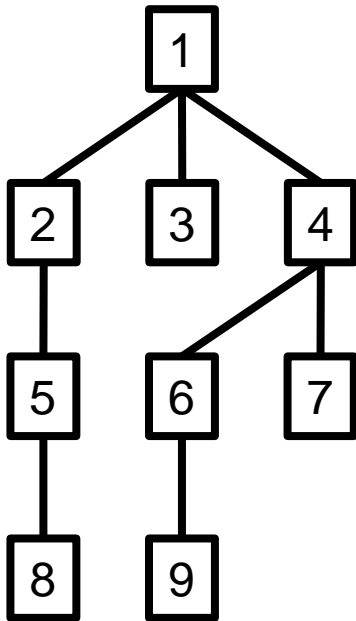
Afișare: 

1	4	7
---	---	---





# Afișarea unui arbore folosind o stivă

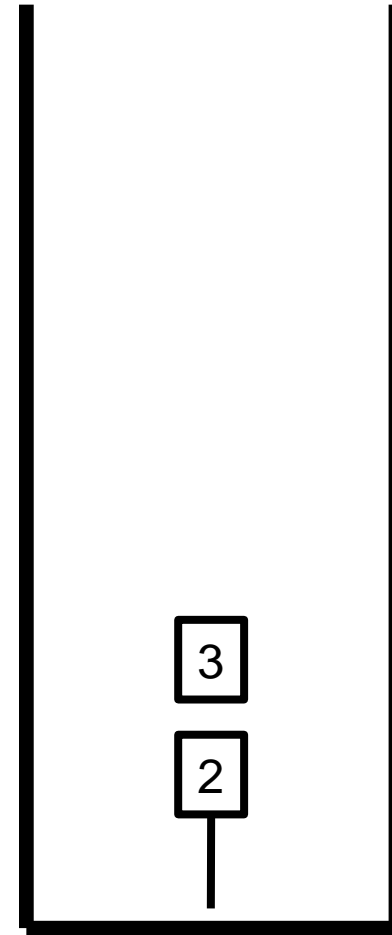
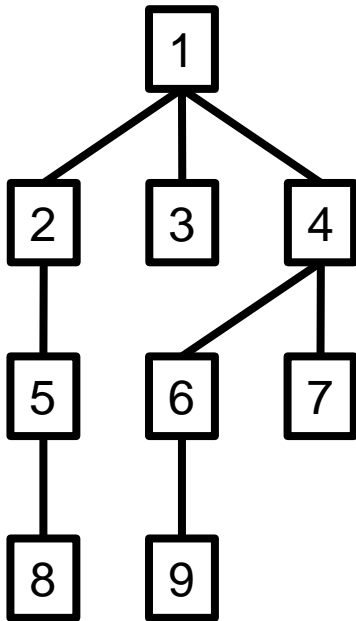


Afișare: 

1	4	7	6
---	---	---	---



# Afișarea unui arbore folosind o stivă

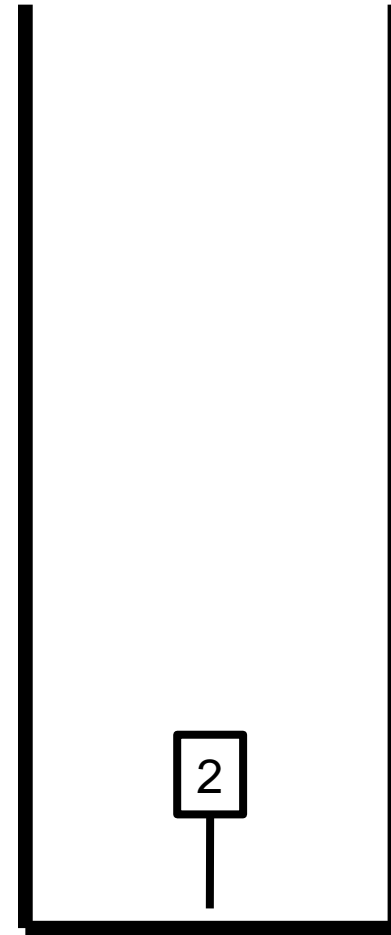
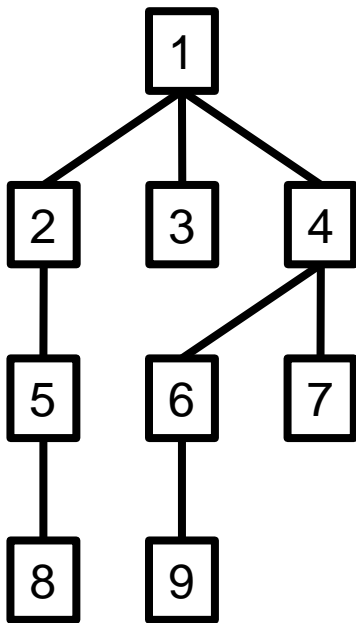


Afișare: 

1	4	7	6	9
---	---	---	---	---



# Afișarea unui arbore folosind o stivă

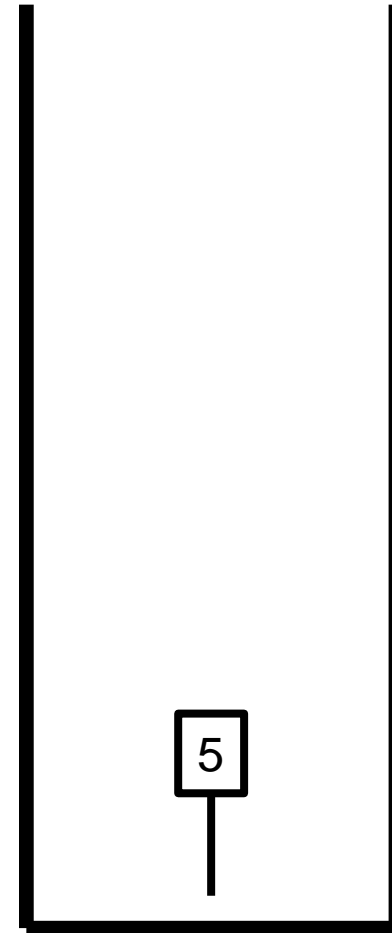
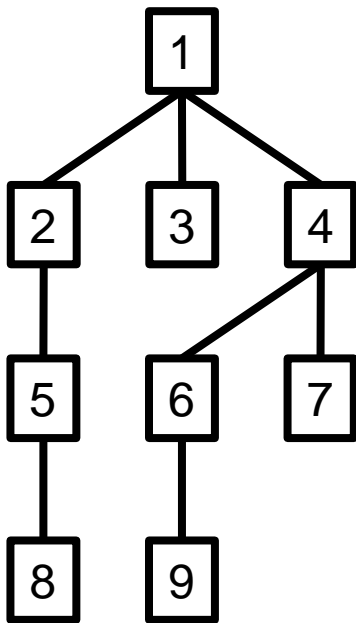


Afișare: 

1	4	7	6	9	3
---	---	---	---	---	---



# Afișarea unui arbore folosind o stivă



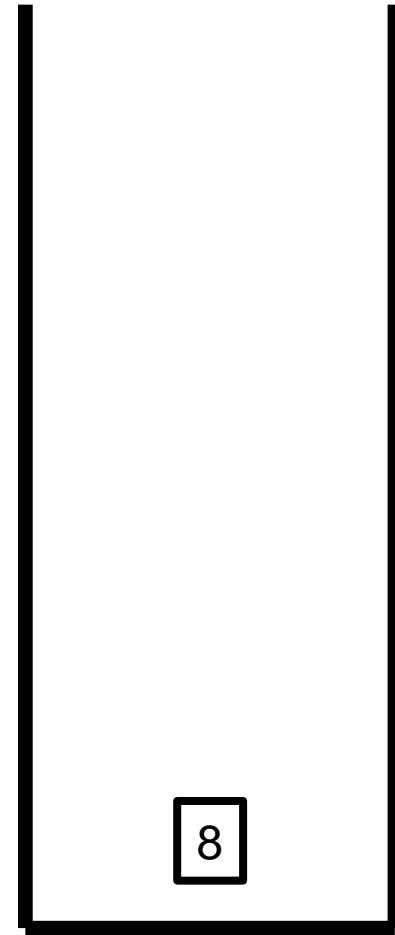
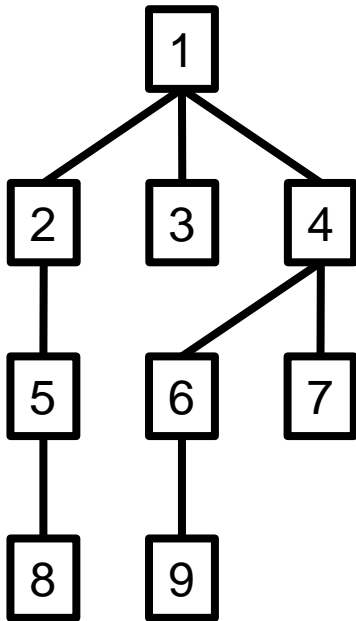
Afișare: 

1	4	7	6	9	3	2
---	---	---	---	---	---	---





# Afișarea unui arbore folosind o stivă

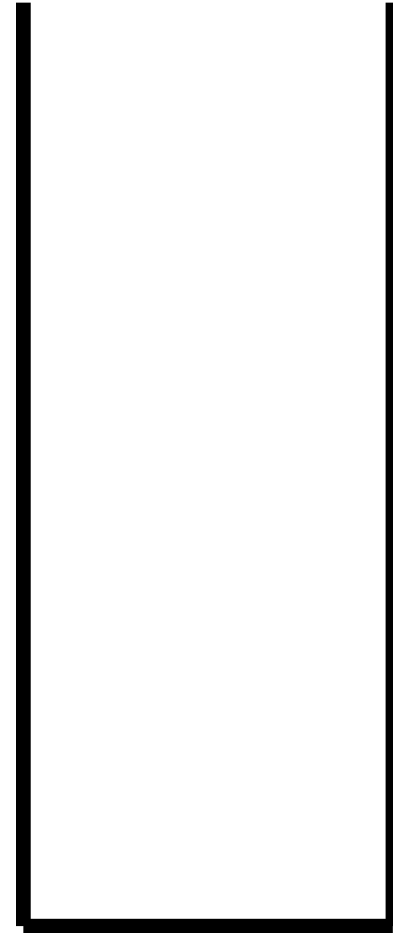
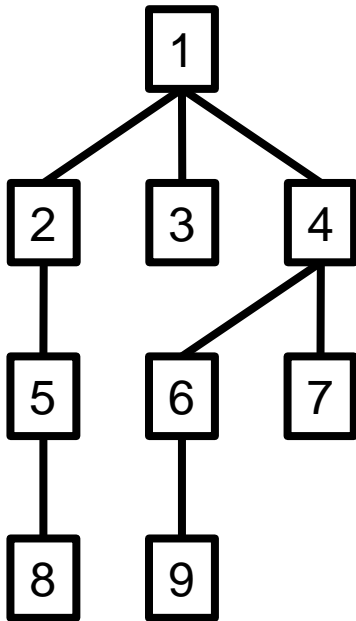


Afișare: 

1	4	7	6	9	3	2	5
---	---	---	---	---	---	---	---



# Afișarea unui arbore folosind o stivă



Afișare: 

1	4	7	6	9	3	2	5	8
---	---	---	---	---	---	---	---	---



# Skip list



# Skip Lists

Algorithms and  
Data Structures

Jeffrey Vitter  
Editor

## Skip Lists: A Probabilistic Alternative to Balanced Trees

*Skip lists are data structures that use probabilistic balancing rather than strictly enforced balancing. As a result, the algorithms for insertion and deletion in skip lists are much simpler and significantly faster than equivalent algorithms for balanced trees.*

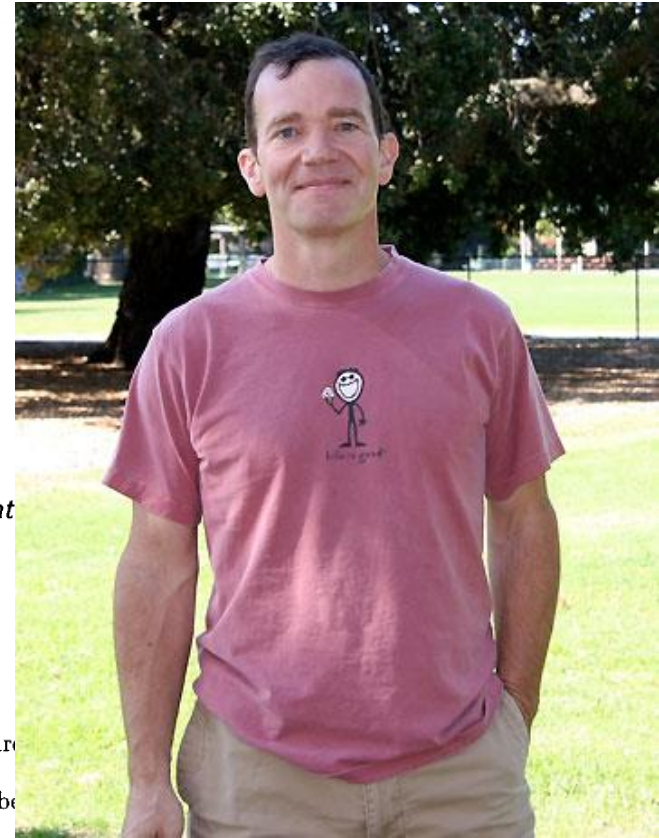
### William Pugh

Binary trees can be used for representing abstract data types such as dictionaries and ordered lists. They work well when the elements are inserted in a random order. Some sequences of operations, such as inserting the elements in order, produce degenerate data structures that perform very poorly. If it were possible to randomly permute the list of items to be inserted, trees would work well with high probability for any input

self-adjusting tree algorithms. Skip lists are also very space efficient. They can easily be configured to require an average of  $1\frac{1}{2}$  pointers per element (or even less) and do not require balance or priority information to be stored with each node.

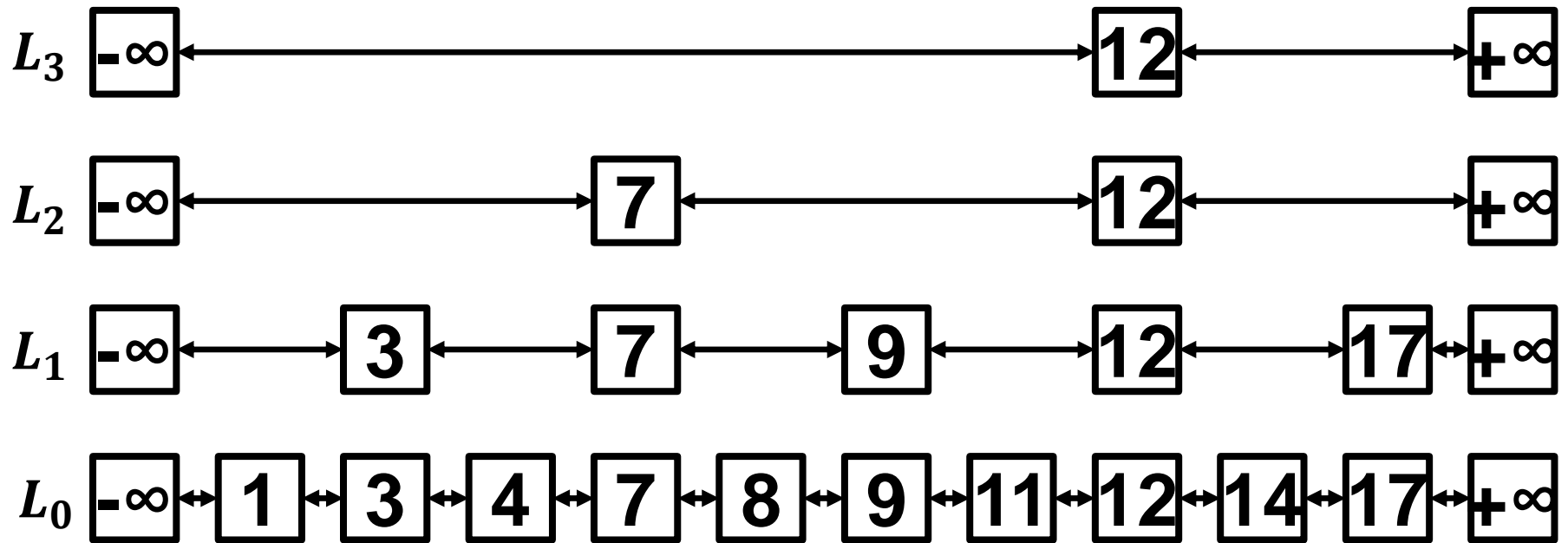
### SKIP LISTS

We might need to examine every node of the list when





# Skip list





## Skip list - search

```
for (i = top; i >= 0; i--) { //Li
    while (searchValue > iterator->next->value) {
        iterator = iterator->next;
    }
    if (searchValue == iterator->value)
        return iterator;
    iterator = iterator->down;
}
```



## Skip list - remove

```
search(node);  
removeFromAllLevels(node);
```



## Skip list - insert

```
prevNode = search(node);  
insert(prevNode->next, node, L0);  
i = 1;  
while (random % 2 != 0) {  
    insert(node, Li)  
    i++;  
}
```





## Skip list complexities

	Vector	Listă	Skip list
Complexitate acces	$O(1)$	$O(N)$	$O(\log_2(N))$
Complexitate inserție	$O(N)$	$O(N)$	$O(\log_2(N))$
Complexitate inserție capete	$O(N)/O(1)$	$O(1)$	$O(\log_2(N))$
Complexitate ștergere	$O(N)$	$O(N)$	$O(\log_2(N))$
Complexitate ștergere capete	$O(N)/O(1)$	$O(1)$	$O(\log_2(N))$



# Bloom Filters



# Bloom Filters

## Space/Time Trade-offs in Hash Coding with Allowable Errors

BURTON H. BLOOM

*Computer Usage Company, Newton Upper Falls, Mass.*

In this paper trade-offs among certain computational factors in hash coding are analyzed. The paradigm problem considered is that of testing a series of messages one-by-one for membership in a given set of messages. Two new hash-coding methods are examined and compared with a particular conventional hash-coding method. The computational factors considered are the size of the hash area (space), the time required to identify a message as a nonmember of the given set (reject time), and an allowable error frequency.

The new methods are intended to reduce the amount of space required to contain the hash-coded information from that associated with conventional methods. The reduction in space is accomplished by exploiting the possibility that a

iteratively generating hash addresses of cells. The contents of these cells are then compared with the test messages. A match indicates the test message is a member of the set; an empty cell indicates the opposite. The reader is assumed to be familiar with this and similar conventional hash-coding methods [1, 2, 3].

The new hash-coding methods to be introduced are suggested for applications in which the great majority of messages to be tested will not belong to the given set. For these applications, it is appropriate to consider as a unit of time (called *reject time*) the average time required to classify a test message as a nonmember of the given set. Furthermore, since the contents of a cell can, in general, be recognized as not matching a test message by examining only part of the message, an appropriate assumption will be introduced concerning the time required to access individual bits of the hash area.

In addition to the two computational factors, reject time and space (i.e. hash area size), this paper considers a third computational factor, allowable fraction of errors. It will be shown that allowing a small number of test messages to be falsely identified as members of the given set will permit a much smaller hash area to be used without increasing the reject time. In some practical applications,



# Silly Bloom Filters

5	0000_0101	OR
1	0000_0001	OR
3	0000_0011	OR
32	0010_0000	OR
33	0010_0001	
		=
	????_????	



# Silly Bloom Filters

**5      0000\_0101   OR**

**1      0000\_0001   OR**

**3      0000\_0011   OR**

**32     0010\_0000   OR**

**33     0010\_0001**

**=**

**Rez     0010\_0111**



# Silly Bloom Filters

**Rez**      **0010\_0111**   **OR**  
**Număr**                      **inițial**  
  
   **=**  
  
     **??**      **????\_????**



# Silly Bloom Filters

**Rez**      **0010\_0111 OR**  
**1**      **0000\_0001**  
**=**  
**1**      **0000\_0001**



# Silly Bloom Filters

$$\begin{array}{rcl} \text{Rez} & 0010\_0111 & \text{OR} \\ 5 & 0000\_0101 & \\ & & = \\ 5 & 0000\_0101 & \end{array}$$





# Silly Bloom Filters

**Rez**      **0010\_0111 OR**  
**33**      **0010\_0001**  
**=**  
**33**      **0010\_0001**



# Silly Bloom Filters

$$\begin{array}{rcl} \text{Rez} & 0010\_0111 & \text{OR} \\ x & \text{????\_????} & \\ & & = \\ x & \text{????\_????} & \end{array}$$

Găsiți x diferit de numerele inițiale scris în zecimal și binar:



# Confusion matrix

	<b>Pozitive reale</b>	<b>Negative reale</b>
<b>Pozitive prezise</b>	<b>True pozitive</b>	<b>False positives</b>
<b>Negative prezise</b>	<b>False negatives</b>	<b>True negatives</b>



## Confusion matrix

		Fac parte din calcul inițial	NU fac parte din calcul inițial
		Pozitive reale	Negative reale
$x \mid \text{BF} == x$	<b>Pozitive prezise</b>	<b>True positives</b>	<b>False positives</b>
$x \mid \text{BF} != x$	<b>Negative prezise</b>	<b>False negatives</b>	<b>True negatives</b>

Ce poate întoarce un (silly) bloom filter testat pe multiple numere?



## Confusion matrix – bloom filters

		Fac parte din calcul inițial	NU fac parte din calcul inițial
		Pozitive reale	Negative reale
$x \mid \text{BF} == x$	Pozitive prezise	True positives	False positives
$x \mid \text{BF} != x$	Negative prezise	False negatives	True negatives



## Bloom filter - creare

- Un set de funcții hash ( $h_1()$ ;  $h_2()$ ; ...  $h_k()$ )
  - Funcțiile returnează valori între 1 și  $N$
- Un șir de  $N$  biți initializați cu 0
- Un input  $x$  pe care dorim să îl adăugăm filtrului, setăm cu 1 pozițiile  $h_i()$ , oricare ar fi  $i$ .



## Bloom filter - Verificare apartenență

Pentru un **y** pentru care dorim să știm dacă este în filtru.

- Aplicăm toate funcțiile **hi()**
- Dacă pe toate pozițiile returnate găsim **1** în vectorul de biți putem considera că **y** aparține setului de numere cu care a fost creat filtrul. (Putem avea **false positives**)
- Dacă pe oricare poziție din cele returnate găsim **0** în vectorul de biți putem fi siguri că **y** nu a fost introdus în Bloom filter



# Bloom filter – inițializare

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---





# Bloom filter – inițializare

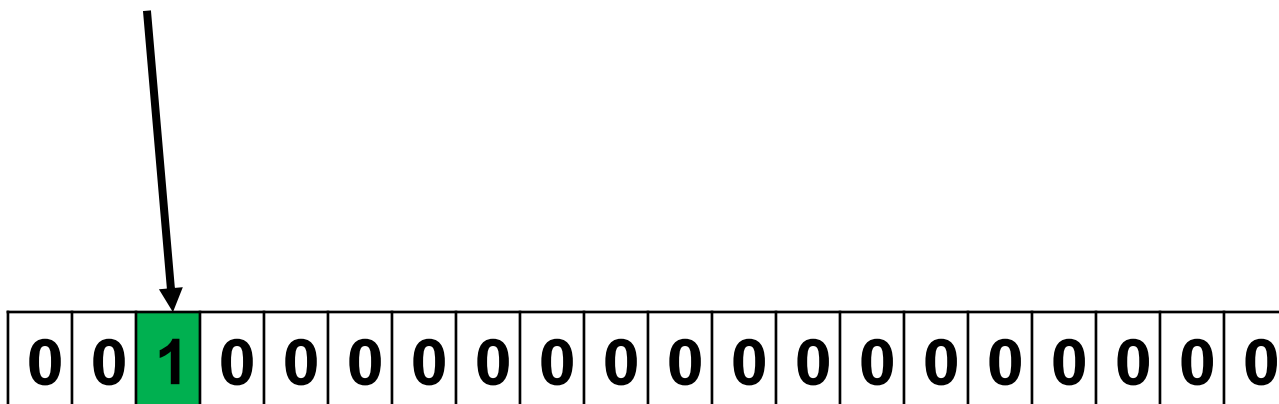
$X_1$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---



## Bloom filter – inițializare

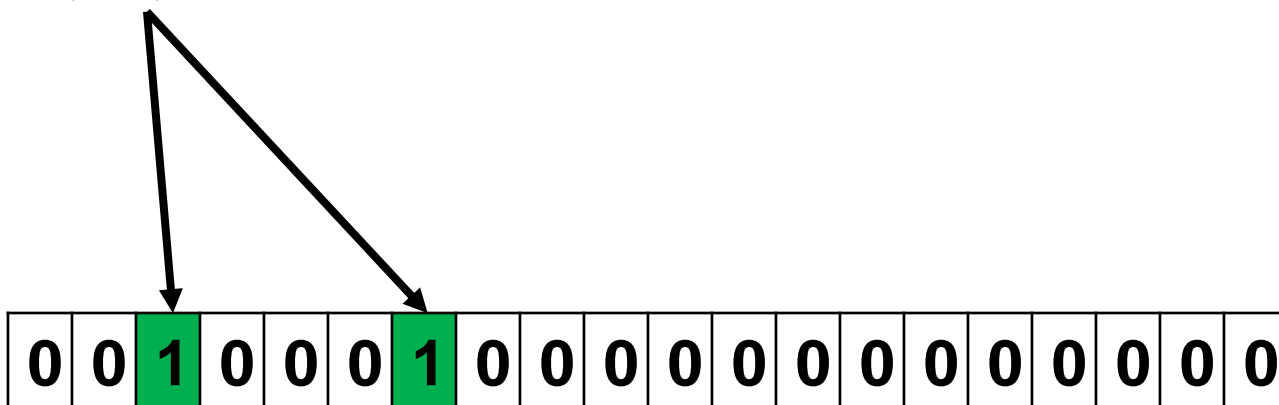
$$h_1(X_1) = 3$$





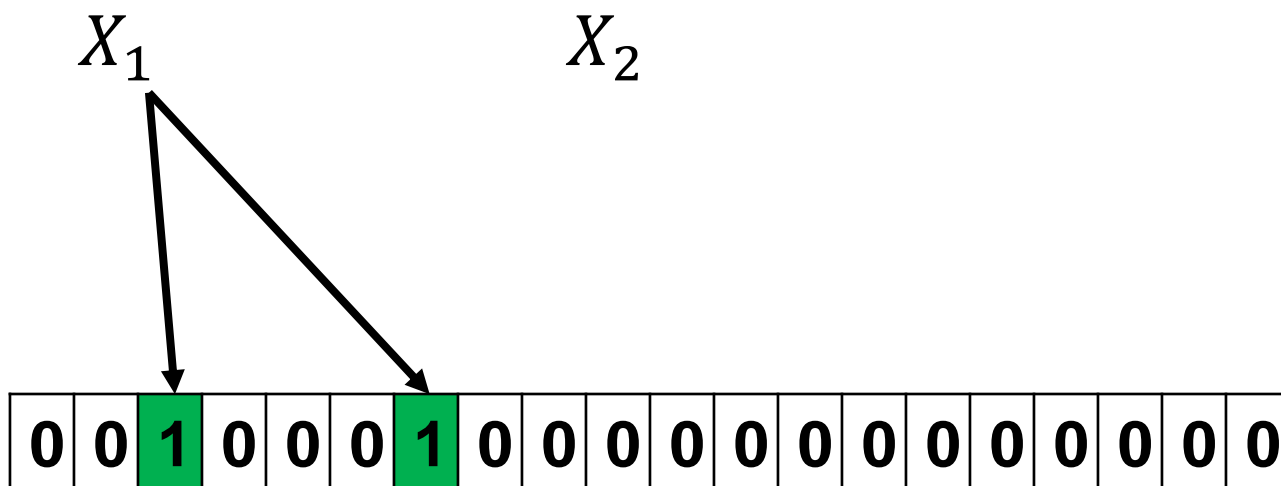
## Bloom filter – inițializare

$$h_2(X_1) = 7$$



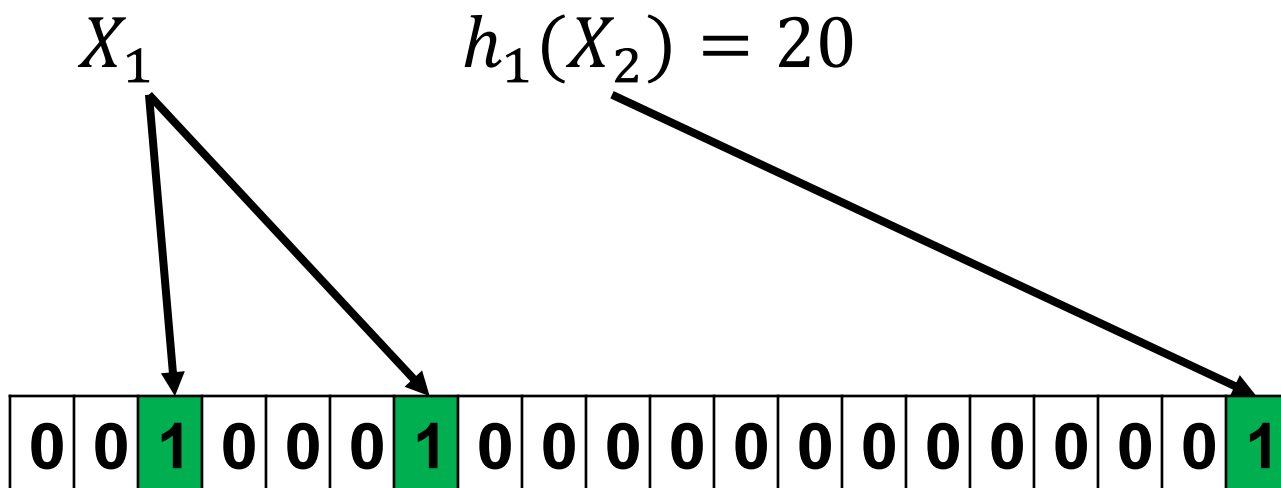


## Bloom filter – inițializare



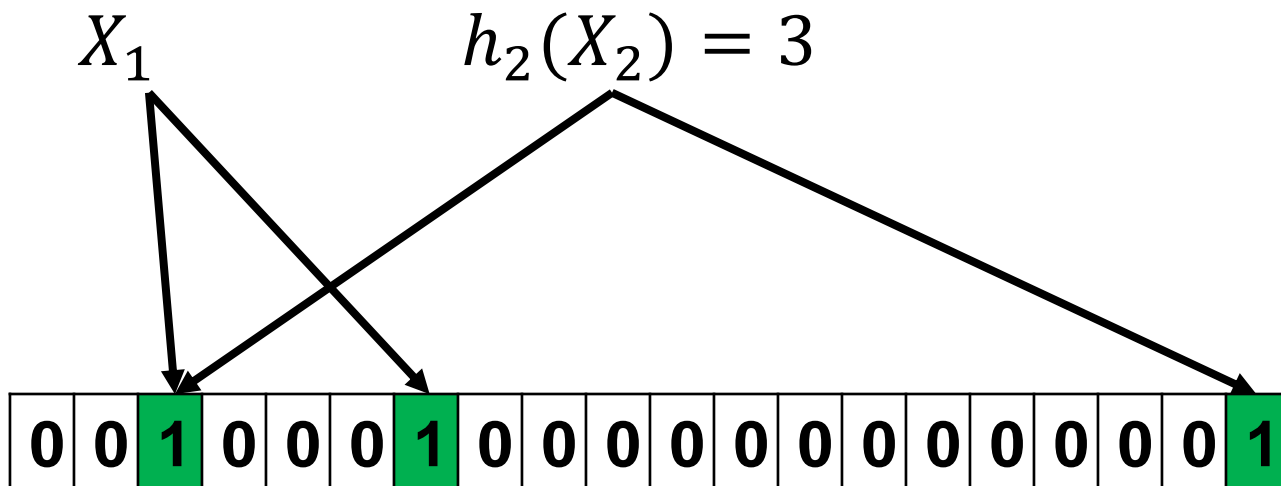


## Bloom filter – inițializare



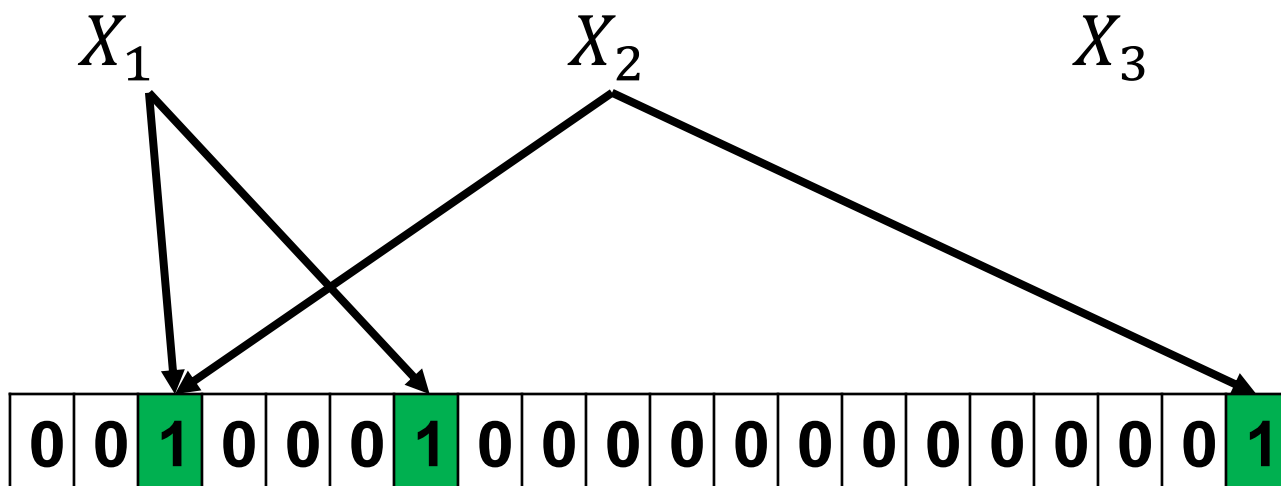


## Bloom filter – inițializare



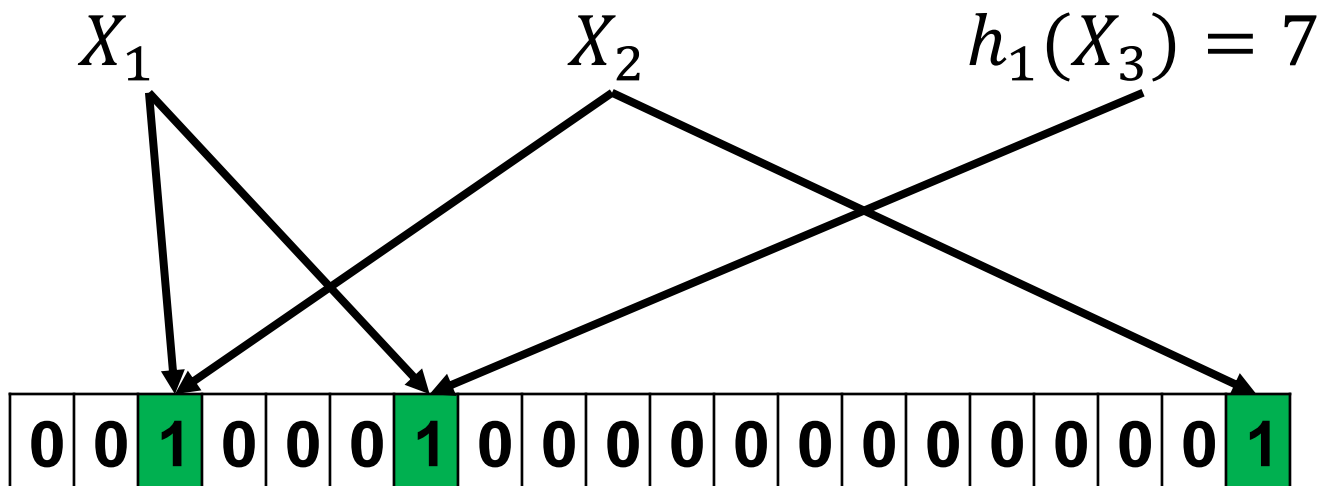


## Bloom filter – inițializare





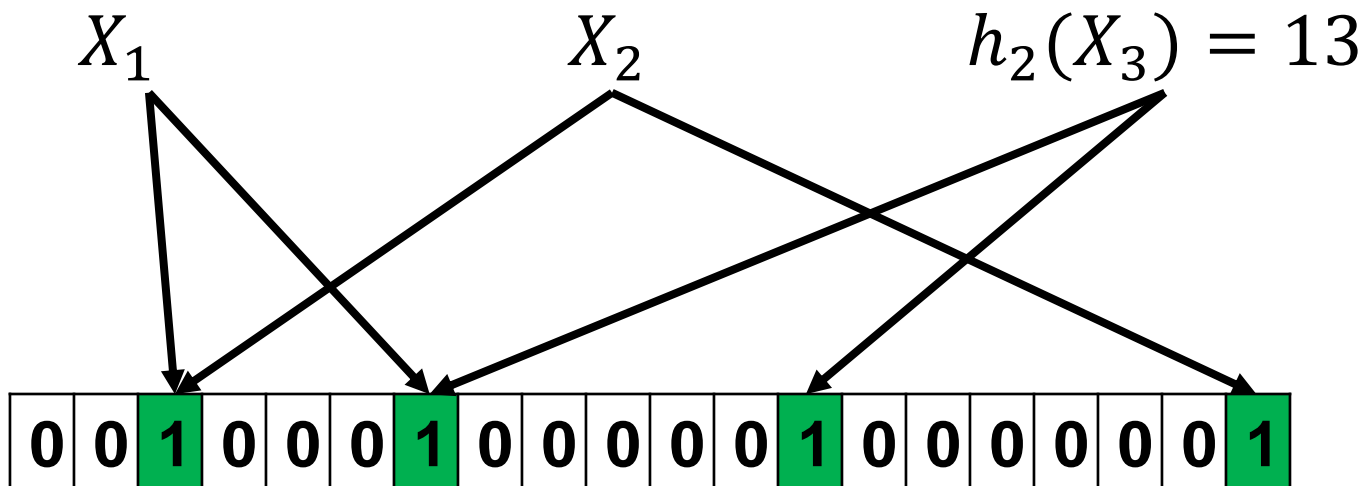
## Bloom filter – inițializare





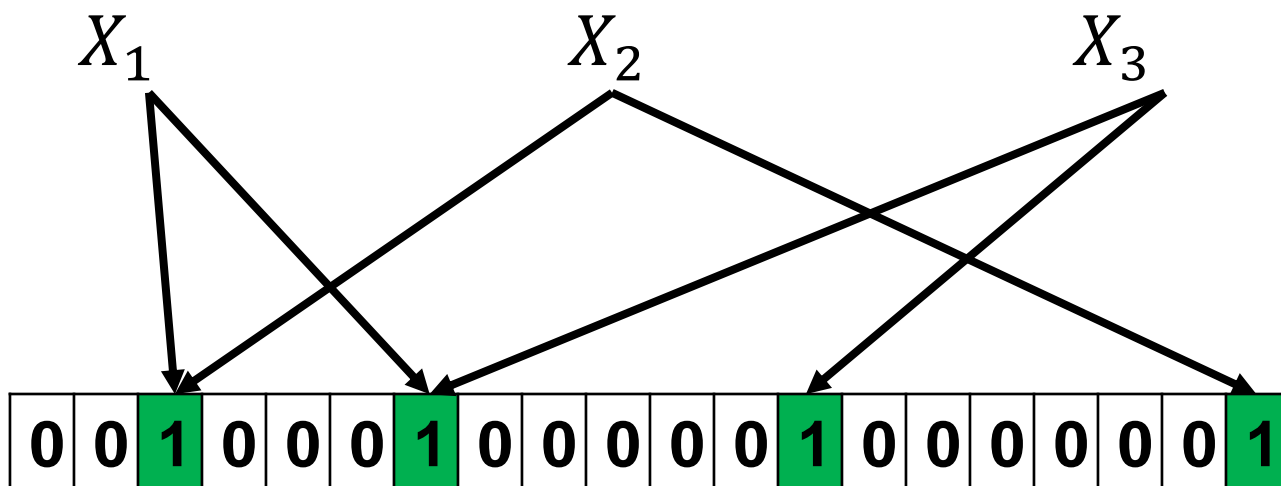


## Bloom filter – inițializare



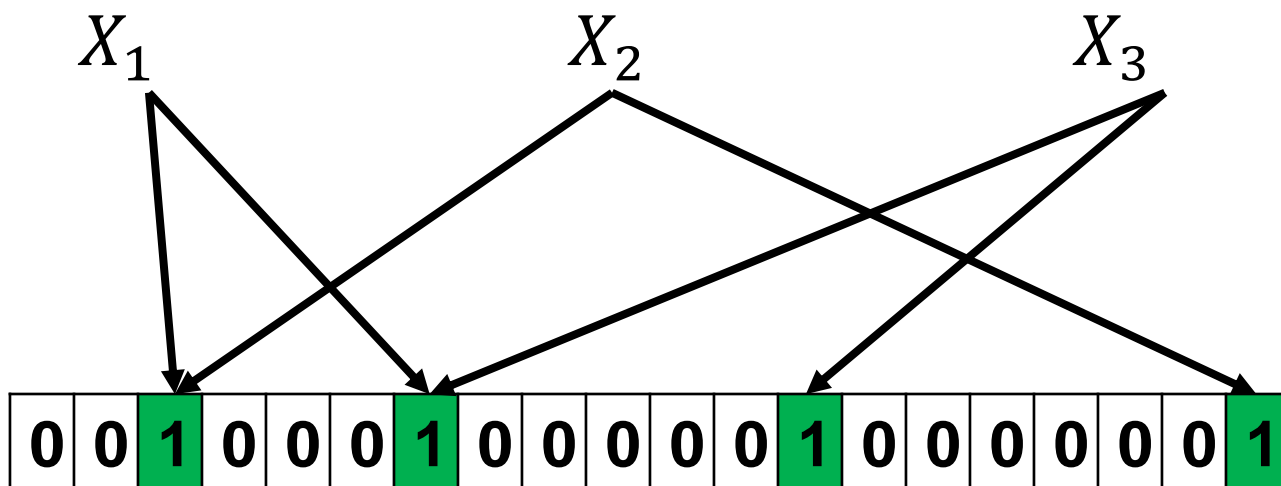


## Bloom filter – inițializare





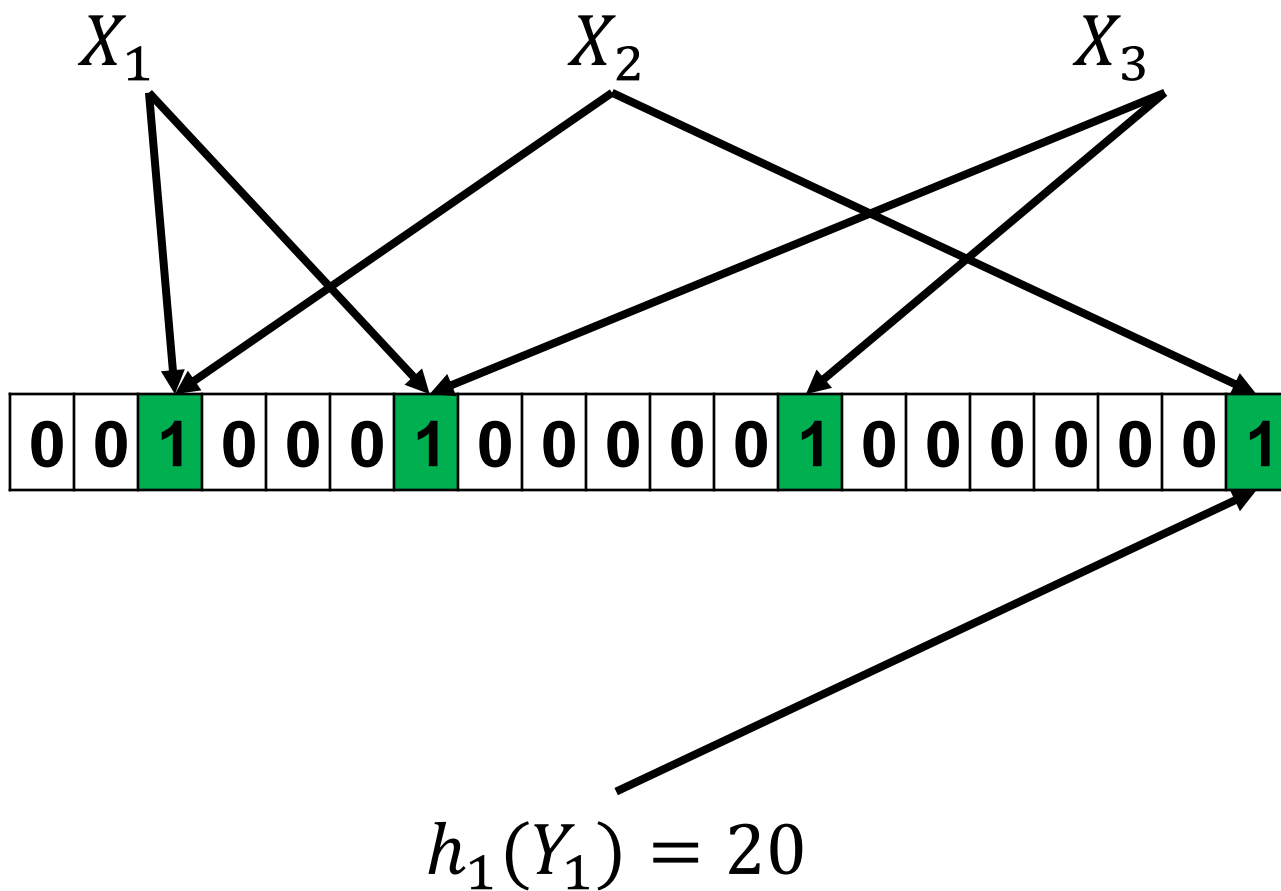
## Bloom filter – folosire



$$Y_1 = X_2$$

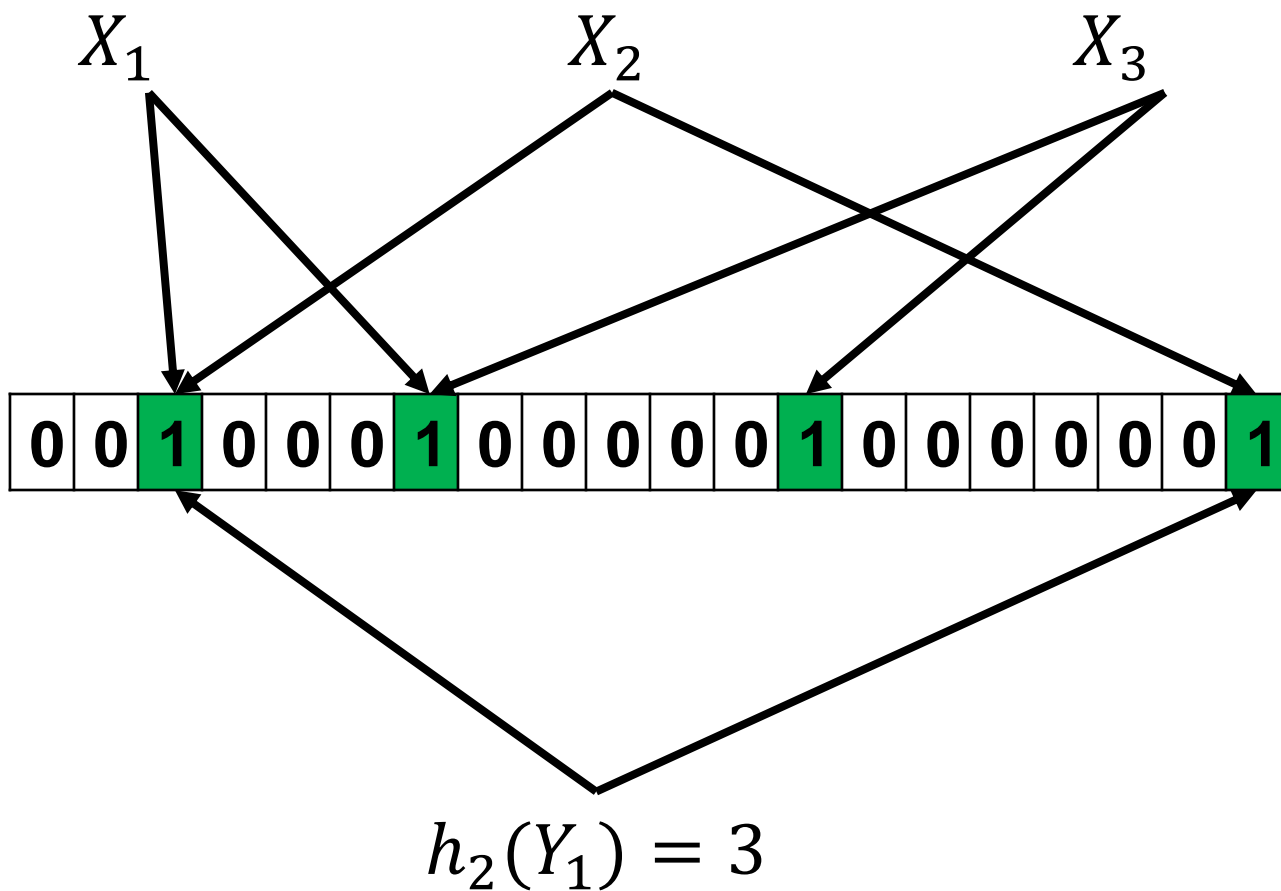


## Bloom filter – folosire





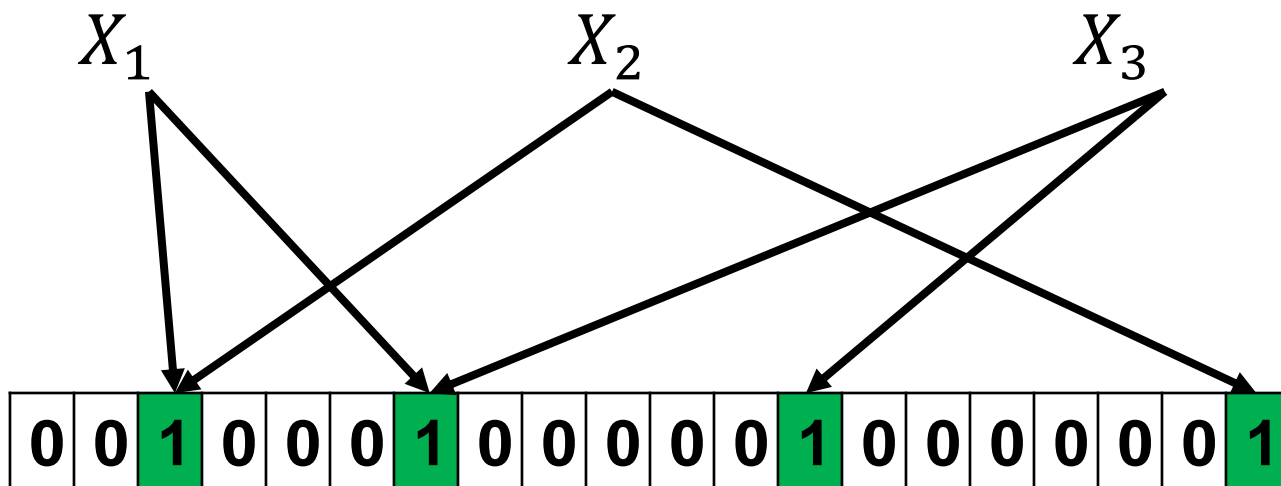
## Bloom filter – folosire



OK



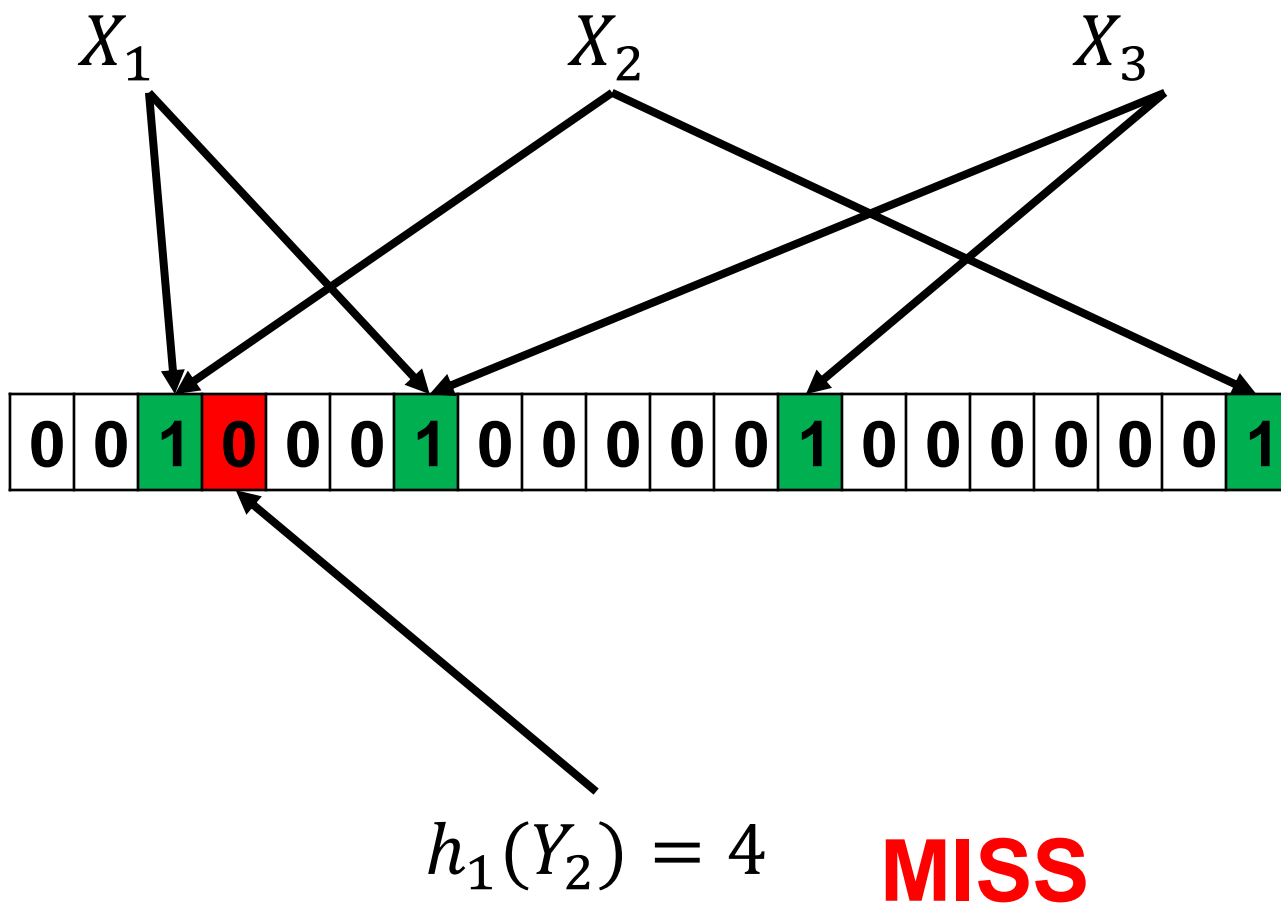
## Bloom filter – folosire



$Y_2$

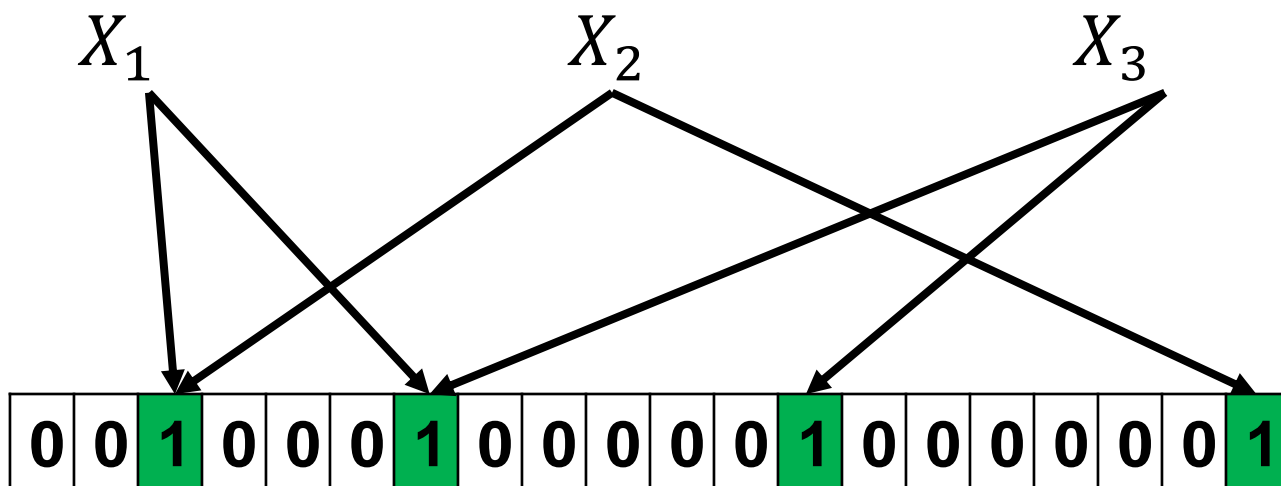


## Bloom filter – folosire





## Bloom filter – folosire

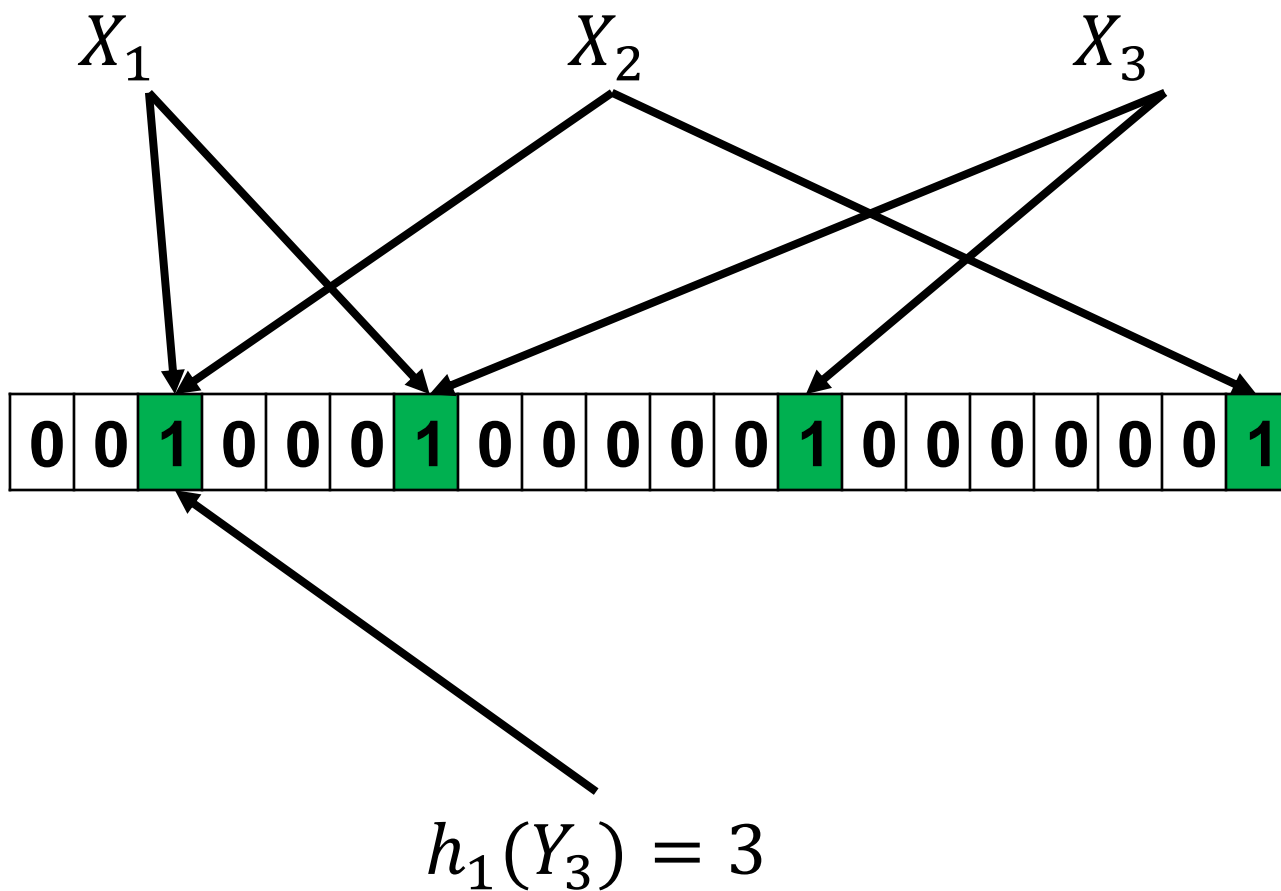


$$Y_3! = X_1 \quad Y_3! = X_2 \quad Y_3! = X_3$$



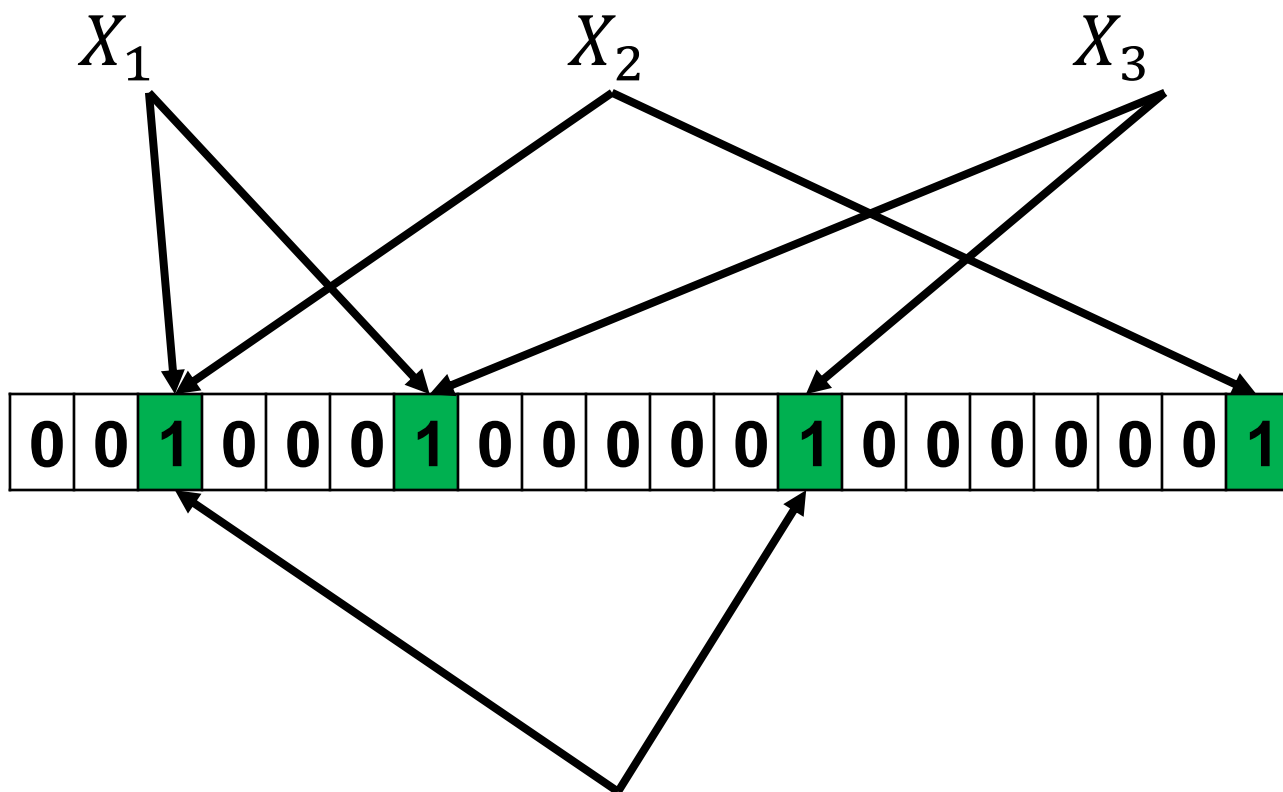


## Bloom filter – folosire





## Bloom filter – folosire



$h_2(Y_3) = 13$  **FALSE OK**



## Bloom filter – Performanță

- Probabilitate de false positive =  $\left(\frac{(\#1)}{N}\right)^k$
- Procent de biți egali cu 0:

$$\left(1 - \frac{1}{N}\right)^{d*k} = \left(1 - \frac{1}{N}\right)^{\frac{1}{N}N*d*k} = e^{-\frac{d*k}{N}}$$

- Probabilitate de false positive:

$$\left(1 - e^{-\frac{d*k}{N}}\right)^k$$

d numărul de valori din filtru  
k numărul de funcții de hash  
N numărul de biți



# Bloom filter

## ■ Avantaje

- Verificare extrem de rapidă (chiar mai rapid decât la hash map)
  - La hash map am avea  $O(1)$  dar constanta poate fi mare, unele cazuri  $O(N)$
- Memorie extrem de puțină (foarte bun pentru sisteme distribuite)

## ■ Dezavantaj

- Nu poate fi folosit mereu deoarece este aproximativ (**false positives**)

## ■ Folosire:

- Baze de date distribuite (se filtrează cererile)
- Distribuirea liste imense (gen URL-uri nesigure)
- Web Crawling – listă cu URL-uri deja crawluite
- Sistem recomandări – bloom filter pentru fiecare utilizator verifică dacă a primit deja o recomandare