Arhitecturi Paralele
Complexitate + Măsurare Timpi

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Elemente preluate din cursul Prof. Ciprian Dobre

FACULTATEA DE
AUTOMATICĂ ȘI
CALCULATOARE

Măsurarea timpului de execuție

```
time ./executabil parametri
```

Măsurare timp – Linia de comandă

```
time sleep 5

real    0m5.001s
user    0m0.000s
sys     0m0.001s
```

```
time sleep 5

sleep 5  0.00s user 0.00s system 0% cpu 5.002 total
```

```
/usr/bin/time sleep 5

0.00user 0.00system 0:05.00elapsed 0%CPU (0avgtext+0avgdata 2076maxresident)k
0inputs+0outputs (0major+73minor)pagefaults 0swaps
```

Măsurare timp – Linia de comandă

time *sleep 5*

```
real    0m5.001s
user     0m0.000s
sys     0m0.001s
```

Wall clock time – Timpul trecut de la pornirea programului – Pe acesta îl folosim

time sleep 5

```
sleep 5  0.00s user 0.00s system 0% cpu 5.002 total
```

/usr/bin/time sleep 5

```
0.00user 0.00system 0:05.00elapsed 0%CPU (0avgtext+0avgdata 2076maxresident)k
0inputs+0outputs (0major+73minor)pagefaults 0swaps
```

Măsurare timp – Linia de comandă

```
time sleep 2

real    0m2.021s
user    0m0.000s
sys     0m0.000s
```

```
time sleep 2

real    0m2.018s
user    0m0.000s
sys     0m0.016s
```

```
time sleep 2

real    0m2.016s
user    0m0.000s
sys     0m0.000s
```

```
time sleep 2

real    0m2.015s
user    0m0.000s
sys     0m0.000s
```

Timpii măsurați nu sunt exacți. Pentru a măsura corect trebuie să facem medie a timpilor după mai multe rulări sau să considerăm doar timpi mari – peste o secundă.

Măsurare timp – Linia de comandă

```
time sleep 5
```

```
real    0m5.001s
user    0m0.000s
sys     0m0.001s
```

Suma timpului petrecut în user space pe fiecare core.

```
time sleep 5
```

```
sleep 5  0.00s user 0.00s system 0% cpu 5.002 total
```

```
/usr/bin/time sleep 5
```

```
0.00user 0.00system 0:05.00elapsed 0%CPU (0avgtext+0avgdata 2076maxresident)k
0inputs+0outputs (0major+73minor)pagefaults 0swaps
```

Măsurare timp – Linia de comandă

Suma timpului petrecut în user space pe fiecare core.

```
time timeout 5 ./useAllCPU 12

real    0m4.075s
user    0m47.797s
sys     0m0.031s
```

Măsurare timp – Linia de comandă

***time** sleep 5*

```
real    0m5.001s
user     0m0.000s
sys      0m0.001s
```

Suma timpului petrecut în kernel pe fiecare core.

time sleep 5

```
sleep 5  0.00s user 0.00s system 0% cpu 5.002 total
```

/usr/bin/time sleep 5

```
0.00user 0.00system 0:05.00elapsed 0%CPU (0avgtext+0avgdata 2076maxresident)k
0inputs+0outputs (0major+73minor)pagefaults 0swaps
```

Măsurare timp – Linia de comandă

Orice I/O este făcut de Kernel

```
time dd if=/dev/zero of=file.txt count=1024 bs=1   048576
1024+0 records in
1024+0 records out
1073741824 bytes (1.1 GB) copied, 9.4847 s, 113 MB/s

real    0m9.490s
user    0m0.000s
sys     0m0.992s
```

Măsurare timp – Din program

```
clock_t t;
t = clock();
WORK();
t = clock() - t;
double time_taken = ((double)t)/CLOCKS_PER_SEC; // in seconds
```

Măsurare timp – Din program

```
clock_t t;
t = clock();
WORK();
t = clock() - t;
double time_taken = ((double)t)/CLOCKS_PER_SEC; // in seconds
```

Măsurare timp – Din program

```
clock_t t;
t = clock();
WORK();
t = clock() - t;
double time_taken = ((double)t)/CLOCKS_PER_SEC; // in seconds
```

From man: the value returned by **clock**() also includes the times of any children.

Măsurare timp – Din program

```
struct timespec start, finish;
double elapsed;
clock_gettime(CLOCK_MONOTONIC, &start);
WORK();
clock_gettime(CLOCK_MONOTONIC, &finish);
elapsed = (finish.tv_sec - start.tv_sec);
elapsed += (finish.tv_nsec - start.tv_nsec) / 1000000000.0;
```

Măsurare timp cu sau fără I/O?

Performanța

- Timp de execuție
- Memorie ocupată
- Număr de procese (thread-uri)
- Scalabilitate
- Toleranță la defecte
- Cost

Măsuri

- T - Timpul total necesar execuției programului paralel
- P - Numărul de procesoare utilizate
- S – Speedup
  - $S = \frac{G}{T}$
- G – Timp execuție cel mai rapid algoritm secvențial

- Costul C = T * P

- Eficienţa $E = \frac{G}{C} = \frac{G}{TP} = \frac{S}{P}$

Complexitate

9	6	9	4	2	7	6	5	6	…	1

* 3

27	18	27	12	6	21	18	15	18	…	3

Complexitate secvențială?

Complexitate

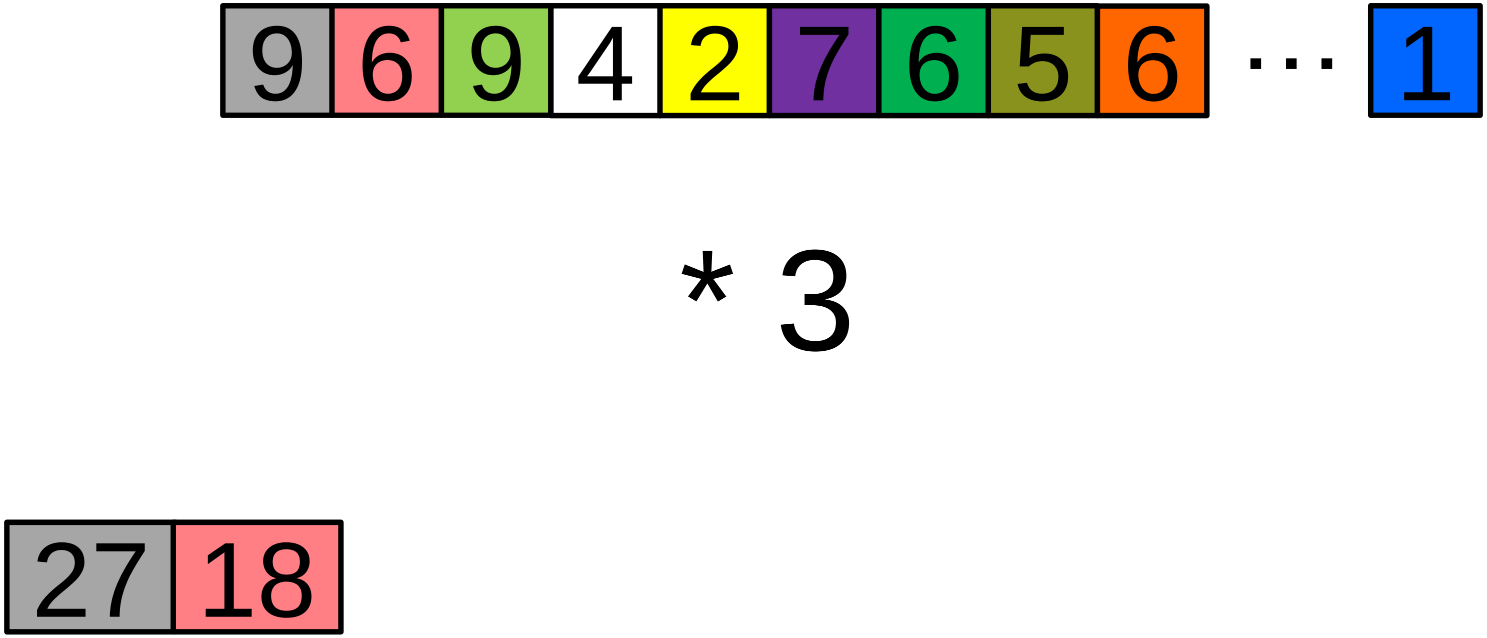
9	6	9	4	2	7	6	5	6	...	1

* 3

27

Complexitate secvențială?

Complexitate



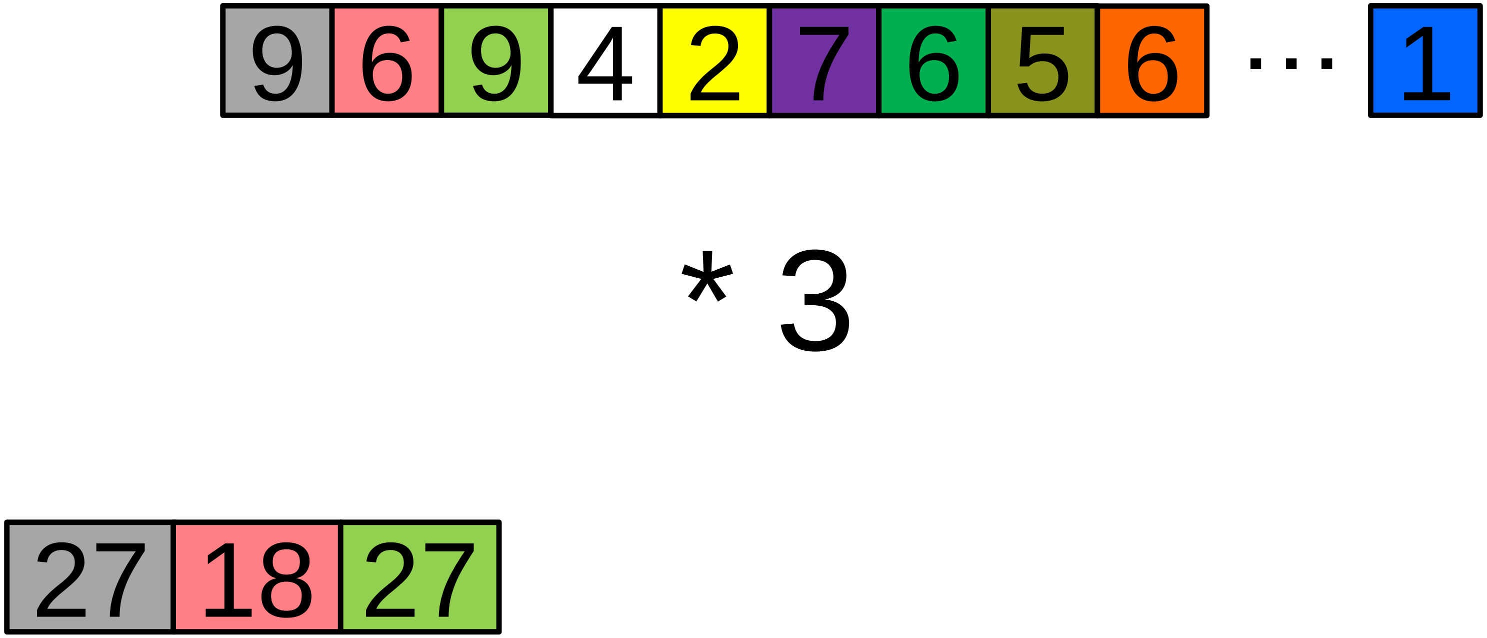
9	6	9	4	2	7	6	5	6	…	1

* 3

27	18

Complexitate secvențială?

Complexitate



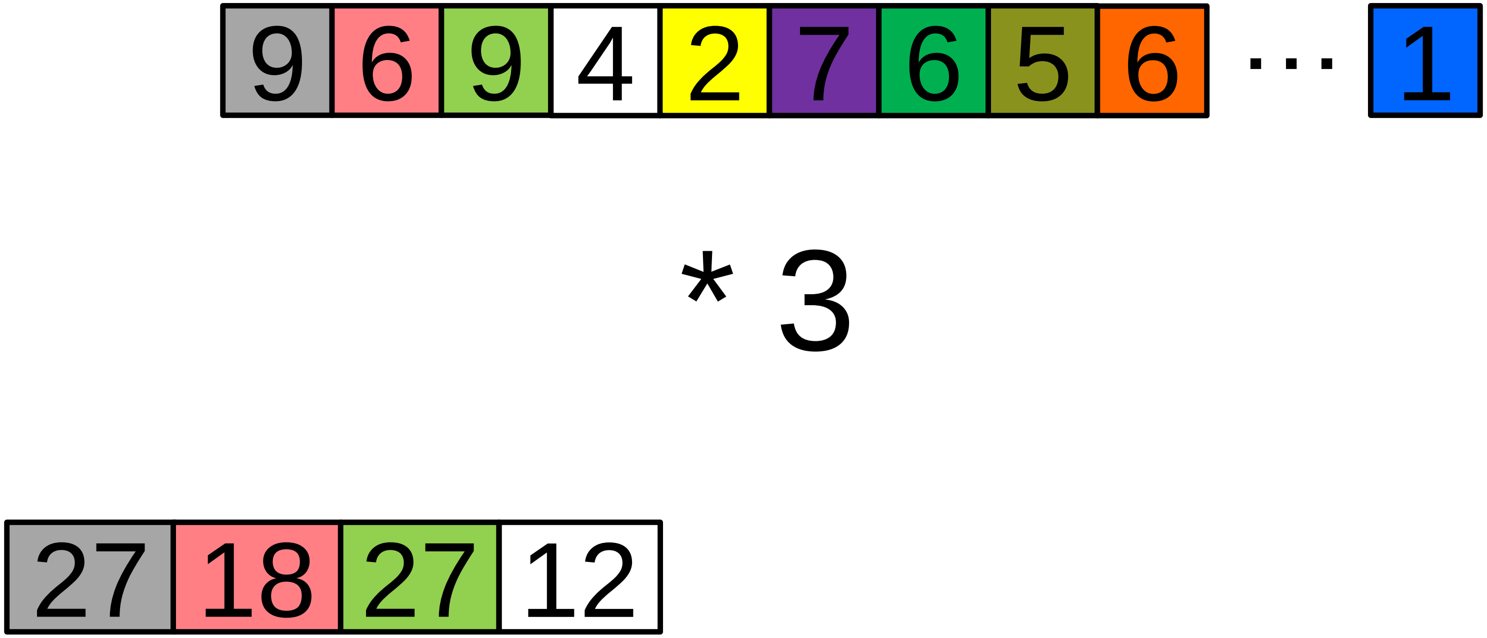
9	6	9	4	2	7	6	5	6	…	1

* 3

27	18	27

Complexitate secvențială?

Complexitate



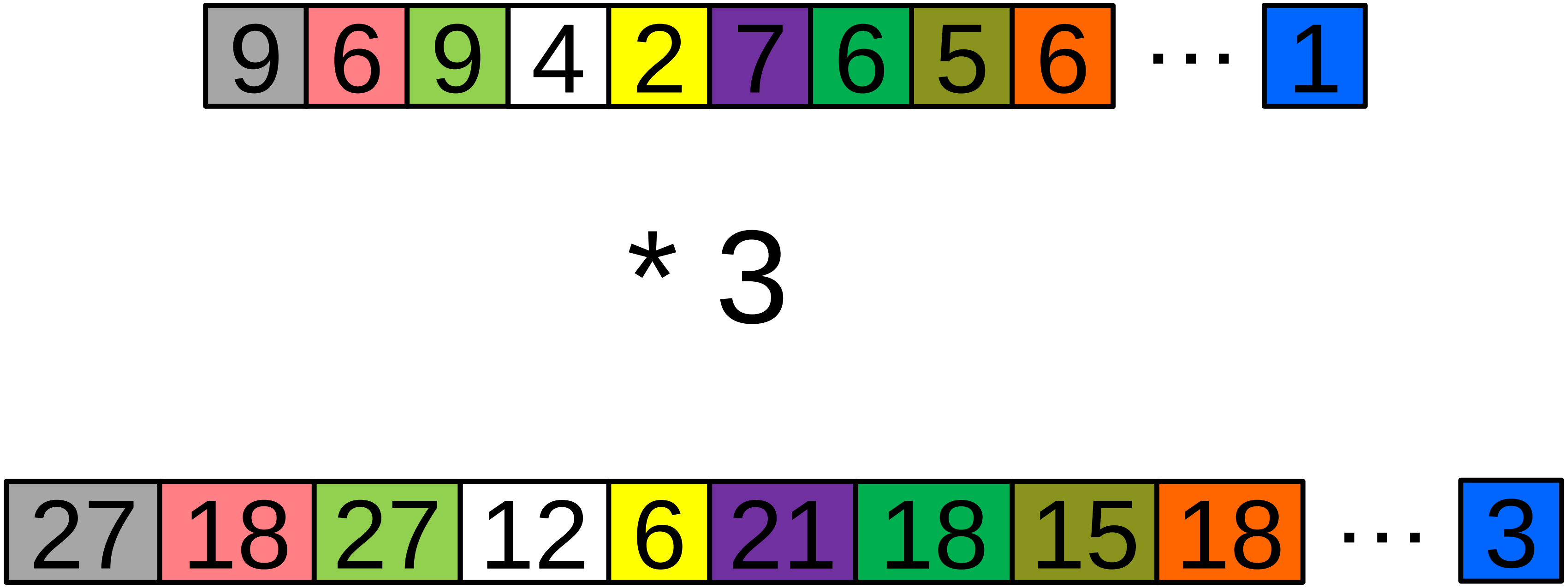
9	6	9	4	2	7	6	5	6	⋯	1

$* \ 3$

27	18	27	12

Complexitate secvențială?

Complexitate



9 6 9 4 2 7 6 5 6 ... 1

* 3

27 18 27 12 6 21 18 15 18 ... 3

Complexitate secvențială? O(**N**)

Complexitate

9	6	9	4	2	7	6	5	6	· · ·	1

* 3

Complexitate paralelă?

Complexitate

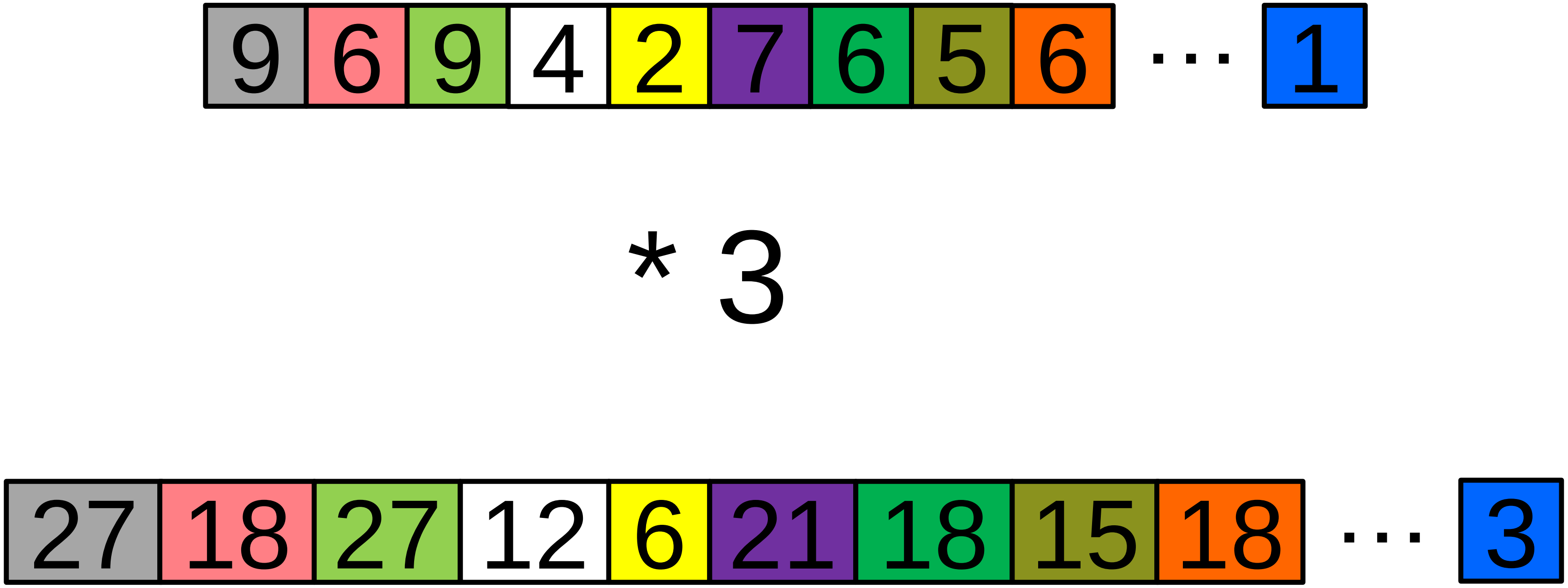
9	6	9	4	2	7	6	5	6	…	1

* 3

27	18	27	12	6	21	18	15	18	…	3

Complexitate paralelă? **O(1)** ?

Complexitate



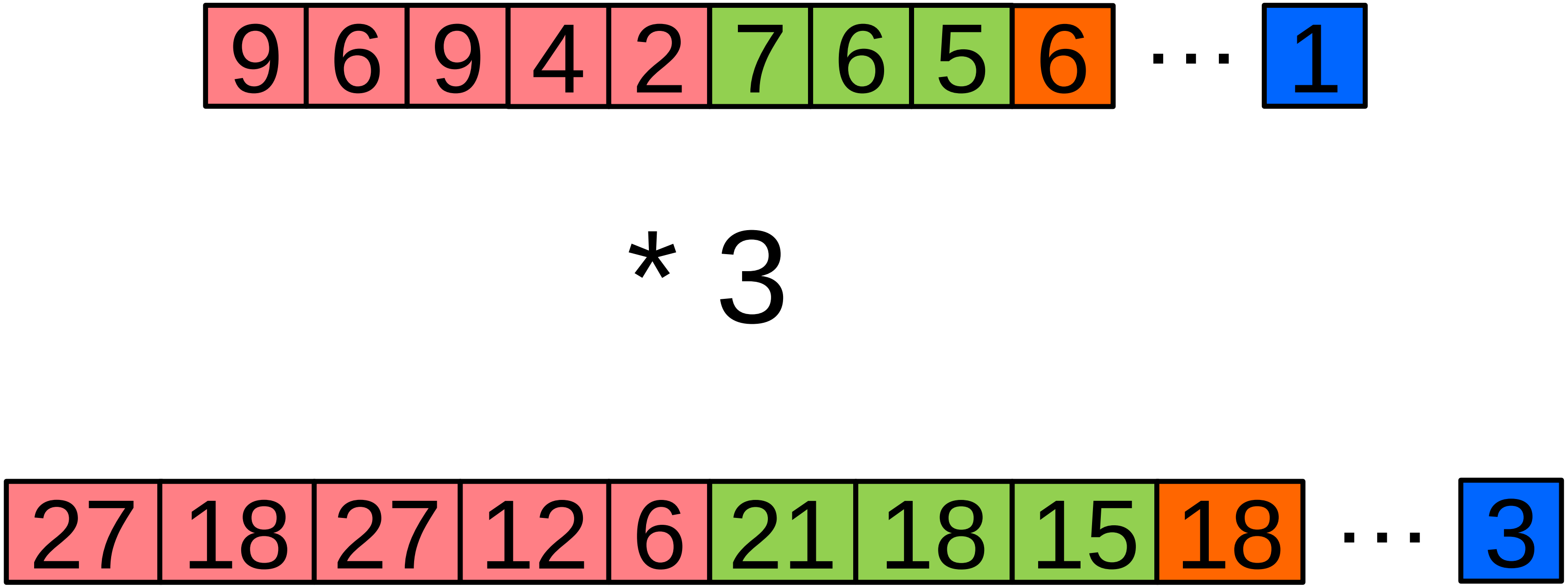
9	6	9	4	2	7	6	5	6	...	1

* 3

27	18	27	12	6	21	18	15	18	...	3

Complexitate paralelă? **O(1)** ? **P** = **N**

Complexitate



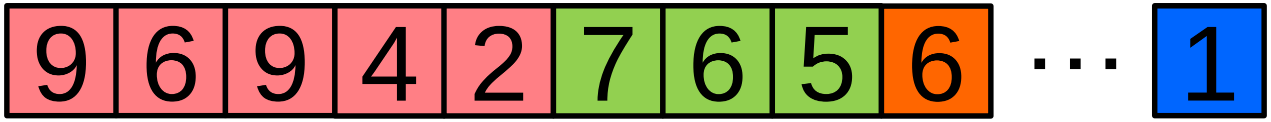
9	6	9	4	2	7	6	5	6	…	1

$* \ 3$

27	18	27	12	6	21	18	15	18	…	3

Complexitate paralelă? $\mathbf{O}(\frac{N}{P})$

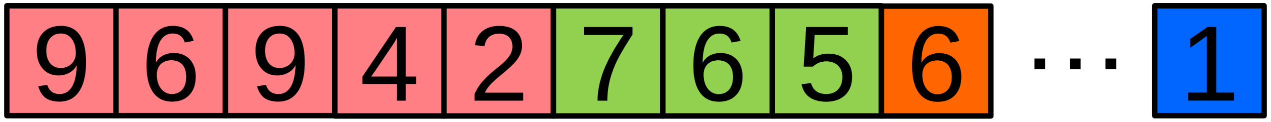
Complexitate



9	6	9	4	2	7	6	5	6	...	1

Speedup?

Complexitate

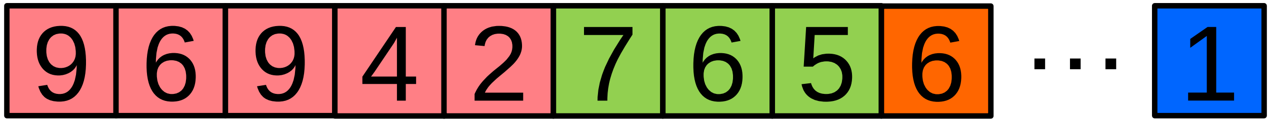


9 6 9 4 2 7 6 5 6 … 1

Speedup?

$$T = O(\frac{N}{P})$$

Complexitate

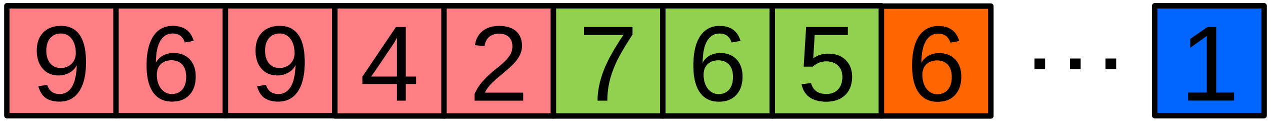


Speedup?

T = O($\frac{N}{P}$)

G = O(N)

Complexitate



9	6	9	4	2	7	6	5	6	…	1

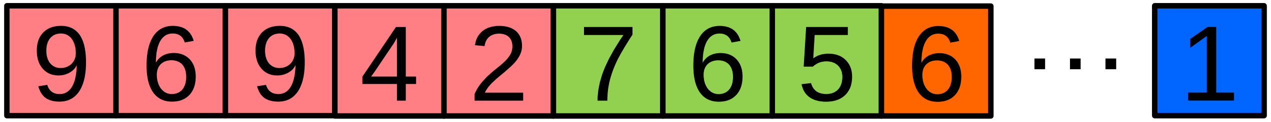
Speedup?

$\mathbf{T = O(\frac{N}{P})}$

$\mathbf{G = O(N)}$

$$S = \frac{O(N)}{O(\frac{N}{P})}$$

Complexitate


9 6 9 4 2 7 6 5 6 ⋯ 1

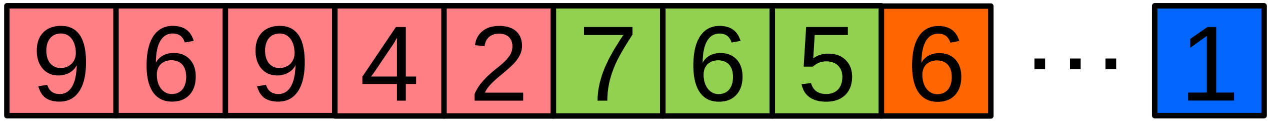
Speedup?

T = $O(\frac{N}{P})$

G = O(N)

$$S = \frac{O(N)}{O(\frac{N}{P})} = O(P)$$

Complexitate



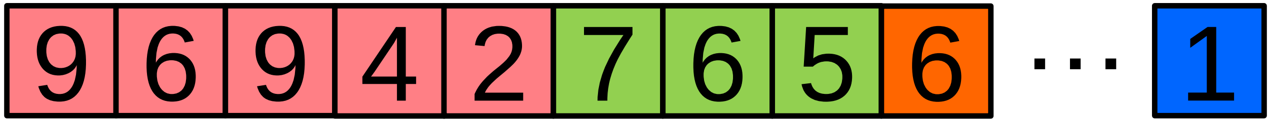
$$S = \frac{O(N)}{O(\frac{N}{P})} = O(P)$$

Eficiența?

T = O($\frac{N}{P}$)

G = O(N)

Complexitate



$$S = \frac{O(N)}{O(\frac{N}{P})} = O(P)$$

Eficiența?

T = O($\frac{N}{P}$)

G = O(N)

$$E = \frac{S}{P} = \frac{O(P)}{P} = 1$$

Timpi adunare a doi vectori C = A + B

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s

Timpi adunare a doi vectori C = A + B

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1			

$$S = \frac{6.14}{6.14}$$

Timpi adunare a doi vectori C = A + B

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1	S = 0.8		

$$S = \frac{6.14}{7.76}$$

Timpi adunare a doi vectori C = A + B

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1	S = 0.8	S = 0.78	

$$S = \frac{6.14}{7.78}$$

Timpi adunare a doi vectori C = A + B

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1	S = 0.8	S = 0.78	S = 0.78

$$S = \frac{6.14}{7.78}$$

Timpi adunare a doi vectori C = A + B

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1	S = 0.8	S = 0.78	S = 0.78

WRONG

$$S = \frac{6.14}{7.78}$$

Timpi adunare a doi vectori C = A + B

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1			

$$S = \frac{6.15}{6.15}$$

Timpi adunare a doi vectori C = A + B

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1	S = 0.8		

$$S = \frac{6.15}{7.77}$$

Timpi adunare a doi vectori C = A + B

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1	S = 0.8	S = 1.55	

$$S = \frac{6.15}{3.95}$$

Timpi adunare a doi vectori C = A + B

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1	S = 0.8	S = 1.55	S = 3.05

$$S = \frac{6.15}{2.01}$$

Timpi adunare a doi vectori C = A + B

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1	S = 0.8	S = 1.55	S = 3.05

De ce nu **S = P** ?

Timpi adunare a doi vectori C = A + B

Nu se ține cont de timpul de citire/scriere RAM.

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1	S = 0.8	S = 1.55	S = 3.05

De ce nu **S = P** ? S = O(P) este **ideal**.

Timpi adunare a doi vectori C = A + B

$$E = \frac{1}{1}$$

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1	S = 0.8	S = 1.55	S = 3.05
E = 1			

Timpi adunare a doi vectori C = A + B

$$E = \frac{0.8}{1}$$

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1	S = 0.8	S = 1.55	S = 3.05
E = 1	E = 0.8		

Timpi adunare a doi vectori C = A + B

$$E = \frac{1.55}{2}$$

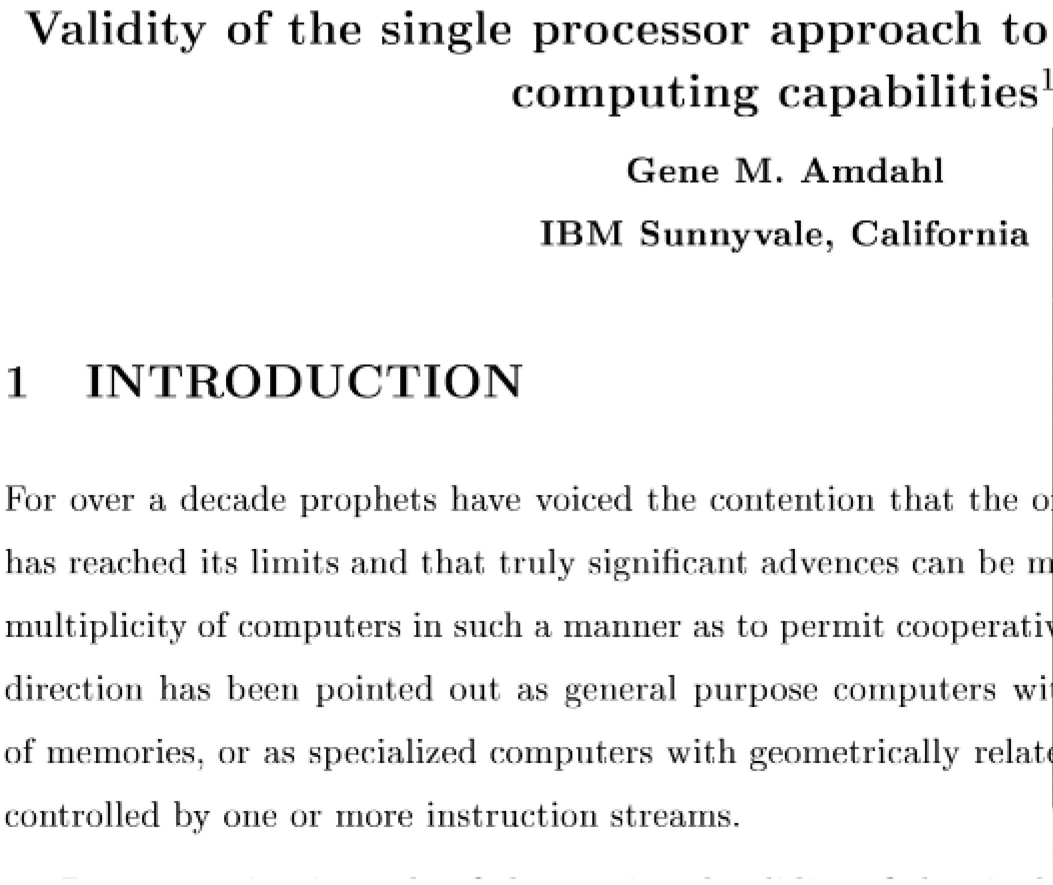
Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1	S = 0.8	S = 1.55	S = 3.05
E = 1	E = 0.8	E = 0.77	

Timpi adunare a doi vectori C = A + B

$$E = \frac{3.05}{4}$$

Sequential	Pthread (1 Thread)	Pthread(2 Thread)	Pthread(4 Thread)
real 0m6.151s user 0m6.141s sys 0m0.000s	real 0m7.777s user 0m7.766s sys 0m0.000s	real 0m3.954s user 0m7.828s sys 0m0.000s	real 0m2.011s user 0m7.875s sys 0m0.031s
S = 1	S = 0.8	S = 1.55	S = 3.05
E = 1	E = 0.8	E = 0.77	E = 0.76

Amdahl's law



Validity of the single processor approach to achieving large scale computing capabilities[1]

Gene M. Amdahl

IBM Sunnyvale, California

1 INTRODUCTION

For over a decade prophets have voiced the contention that the o
has reached its limits and that truly significant advences can be m
multiplicity of computers in such a manner as to permit cooperativ
direction has been pointed out as general purpose computers wi
of memories, or as specialized computers with geometrically relate
controlled by one or more instruction streams.

Amdahl's law - prerequisits

f - Procent de operații din algoritmul secvențial care se pot executa paralel.

$$T = fG + (1 - f)\frac{G}{P}$$

$$S = \frac{G}{T}$$

Amdahl's law - prerequisits

f - Procent de operații din algoritmul secvențial care se pot executa paralel.

$$T = fG + (1 - f)\frac{G}{P}$$

$$S = \frac{G}{T} = \frac{G}{fG + \frac{(1-f)G}{P}}$$

Amdahl's law

$$S = \frac{1}{f + \frac{(1-f)}{P}}$$

Amdahl's law

$$S = \frac{1}{f + \frac{(1-f)}{P}}$$

Ce se întâmplă dacă P e foarte mare (chiar mai mare decât N)?

Amdahl's law

$$S = \frac{1}{f + \frac{(1-f)}{P}}$$

Ce se întâmplă dacă P e foarte mare (chiar mai mare decât N)?

$$\boldsymbol{S \leq \frac{1}{f}}$$

Amdahl's law - prerequisits

f - Procent de operații din algoritmul secvențial care se pot executa paralel.

$$T = fG + (1 - f)\frac{G}{P}$$

Ce se întâmplă dacă P e foarte mare (chiar mai mare decât N)?

Amdahl's law - prerequisits

f - Procent de operații din algoritmul secvențial care se pot executa paralel.

$$T = fG + (1 - f)\frac{G}{P}$$

Ce se întâmplă dacă P e foarte mare (chiar mai mare decât N)?

$$\boldsymbol{T \geq fG}$$

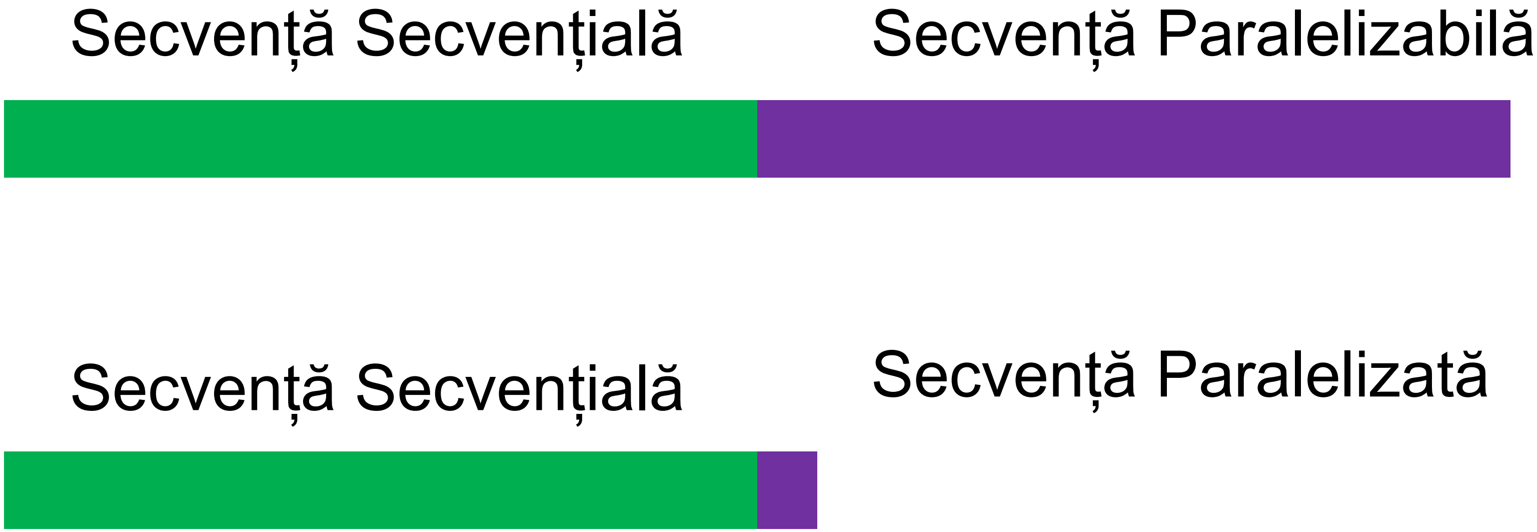
Pauză

```
int a = 0
co [Tid=1 to 2]
{
        for(int i = 0; i < 10000; i++)
                a = a + 2
}

print(a)
```

Care este valoarea minimă ce poate fi printată?

Amdahl's law



Secvență Secvențială

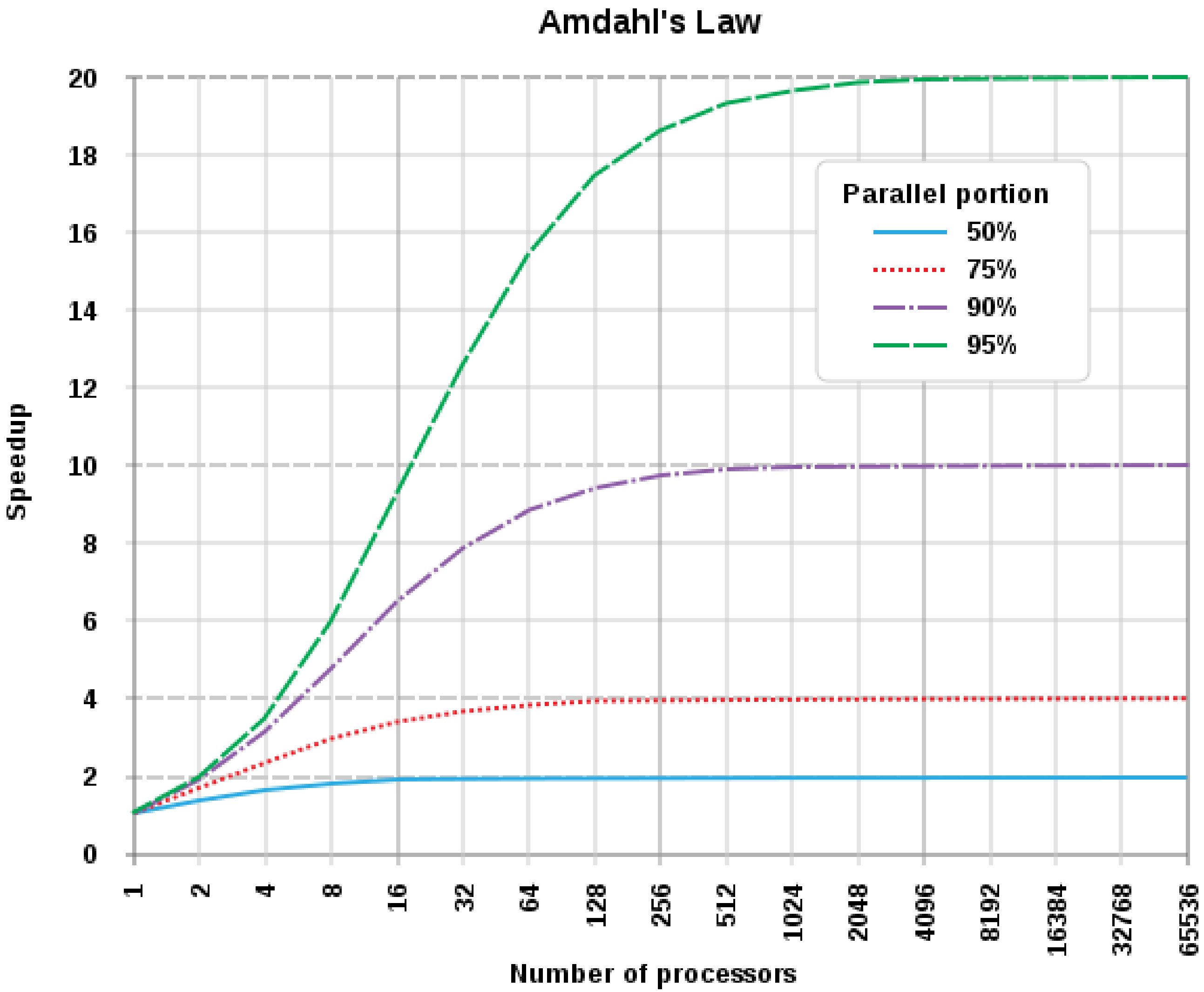
Secvență Paralelizabilă

Secvență Secvențială

Secvență Paralelizată

Măsurare timp cu sau fără I/O?

Amdahl's law

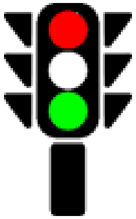

Amdahl's Law
Speedup
20
18
16
14
12
10
8
6
4
2
0
1
2
4
8
16
32
64
128
256
512
1024
2048
4096
8192
16384
32768
65536
Number of processors
Parallel portion
50%
75%
90%
95%

Amdahl's law

$$S = \frac{1}{f + \frac{(1-f)}{P}}$$

Reminder

- Ce este un semafor?



- Ce este un mutex?

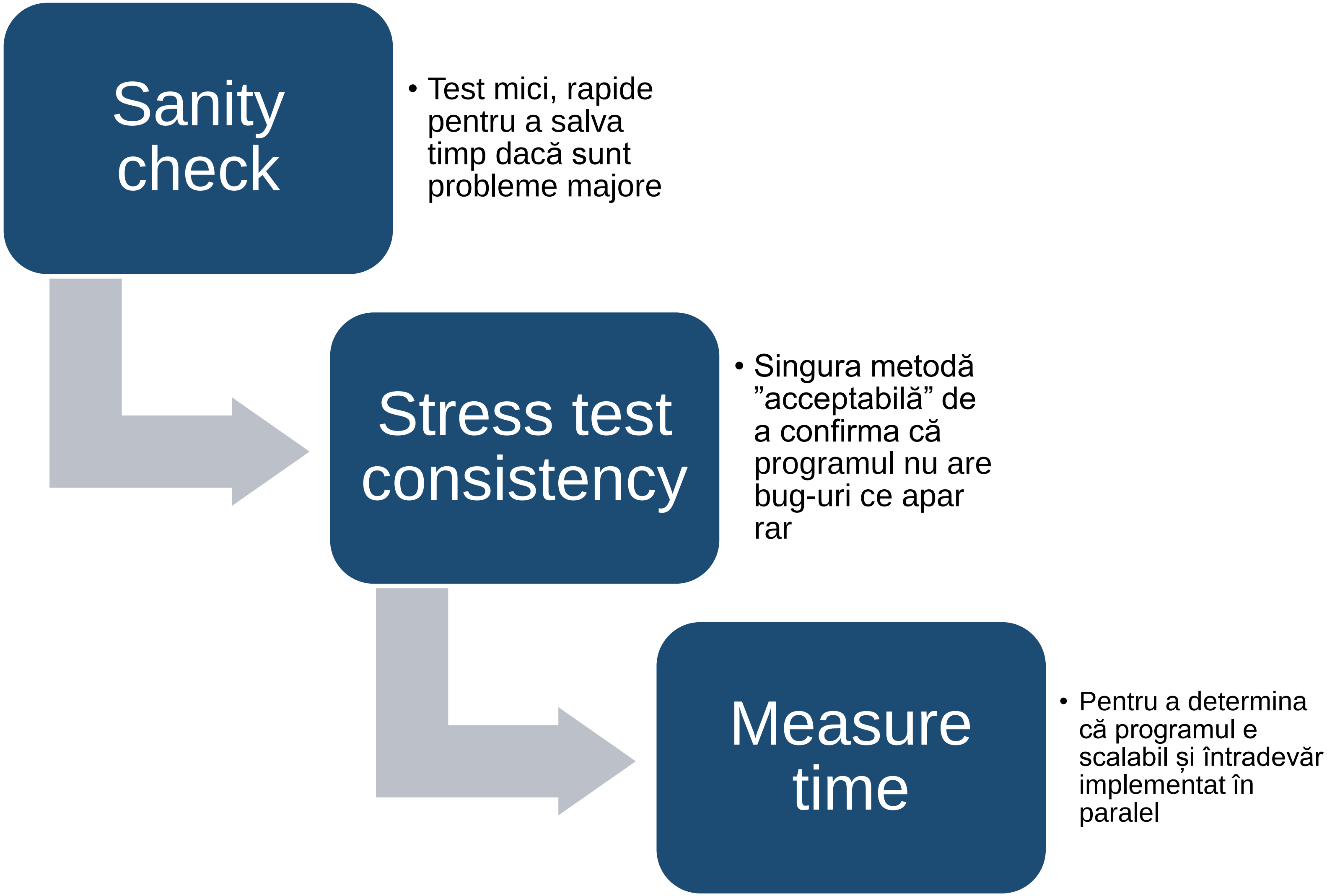
- Ce este o barieră?

Consistența?

- Demonstrație formală

- Stres test
  - mereu comparați cu rezultatul algoritmului secvențial sau care sunteți siguri că oferă rezultat corect

Workflow - Testarea programelor



Sanity check

- Test mici, rapide pentru a salva timp dacă sunt probleme majore

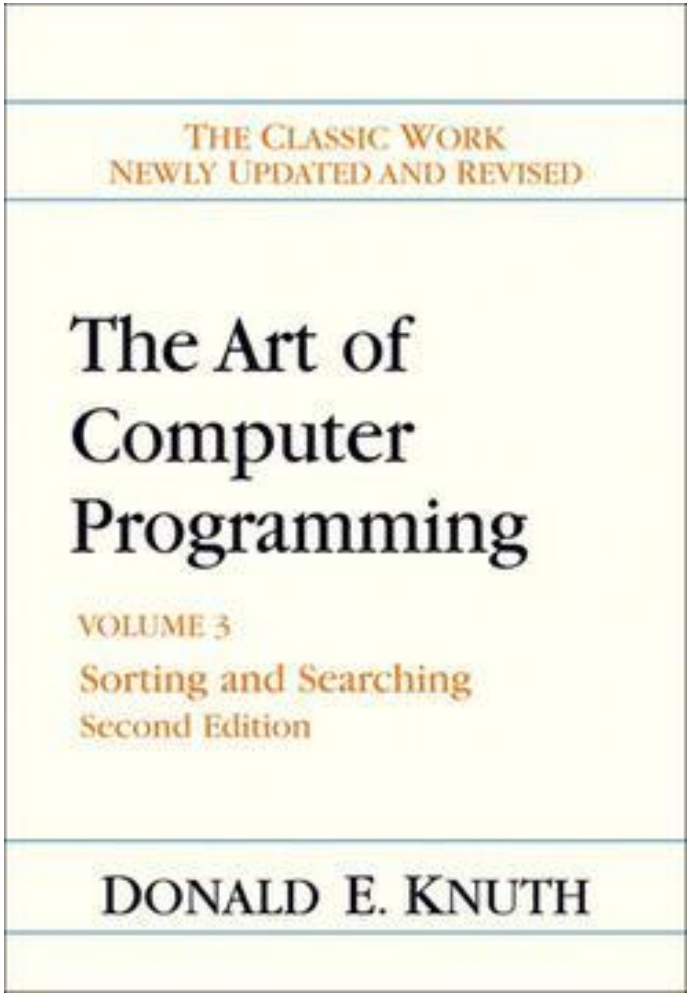
Stress test consistency

- Singura metodă ”acceptabilă” de a confirma că programul nu are bug-uri ce apar rar

Measure time

- Pentru a determina că programul e scalabil și întradevăr implementat în paralel

Let’s parallelize some algorithms



THE CLASSIC WORK
NEWLY UPDATED AND REVISED

The Art of Computer Programming

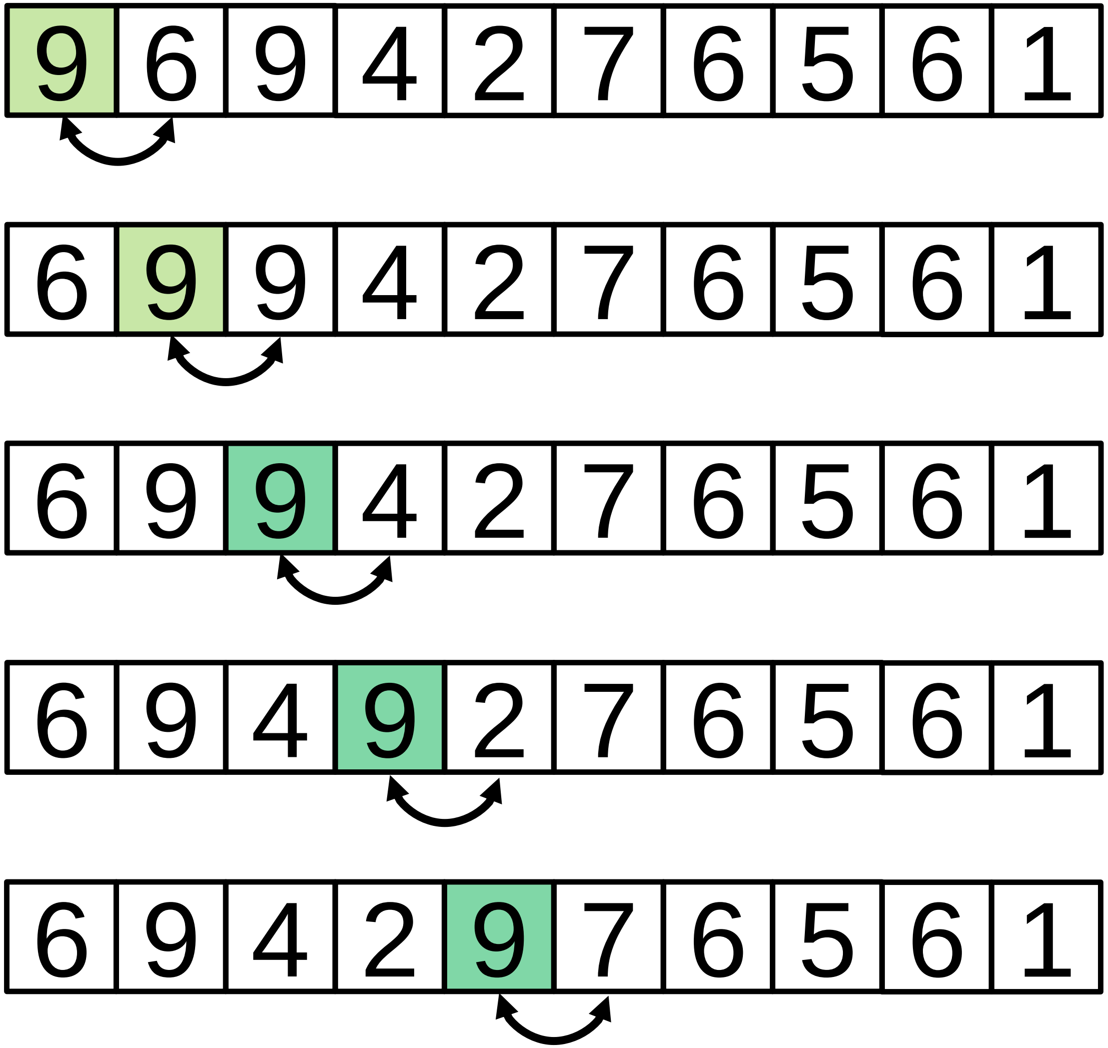
VOLUME 3

Sorting and Searching

Second Edition

DONALD E. KNUTH

Bubble sort



9 6 9 4 2 7 6 5 6 1

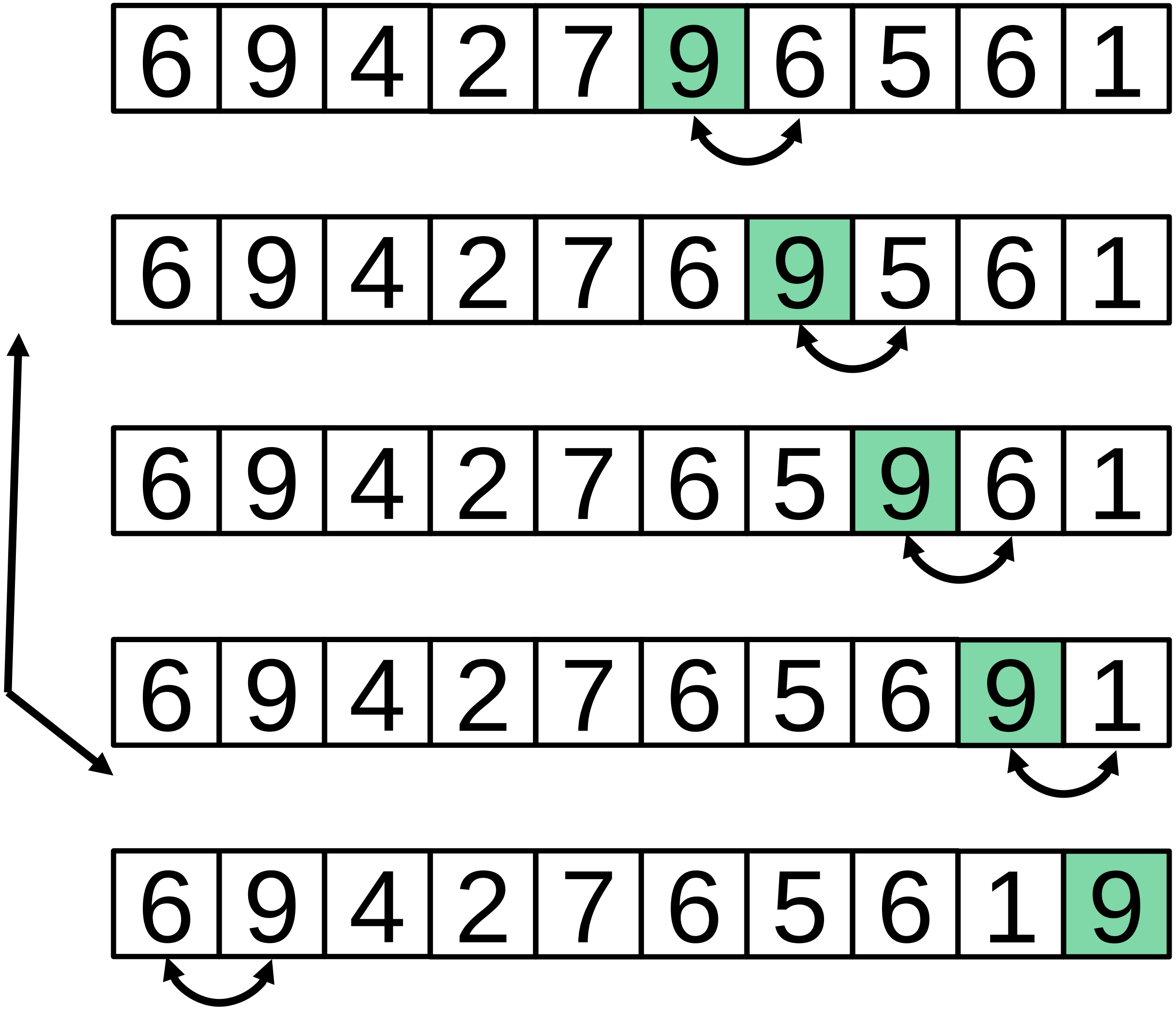
6 9 9 4 2 7 6 5 6 1

6 9 9 4 2 7 6 5 6 1

6 9 4 9 2 7 6 5 6 1

6 9 4 2 9 7 6 5 6 1

Bubble sort



6 9 4 2 7 9 6 5 6 1

6 9 4 2 7 6 9 5 6 1

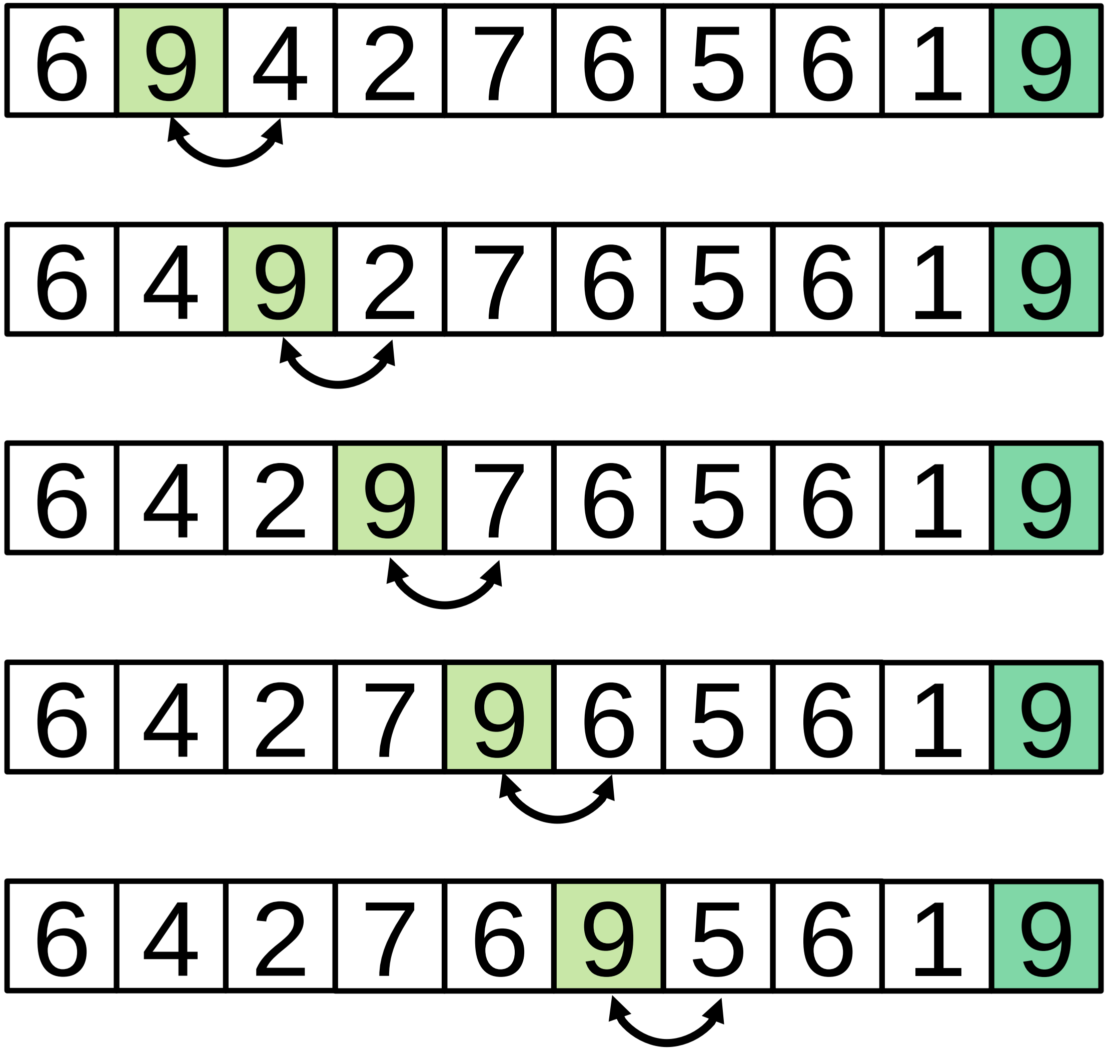
6 9 4 2 7 6 5 9 6 1

6 9 4 2 7 6 5 6 9 1

6 9 4 2 7 6 5 6 1 9

Repeat until sorted

Bubble sort



6 9 4 2 7 6 5 6 1 9

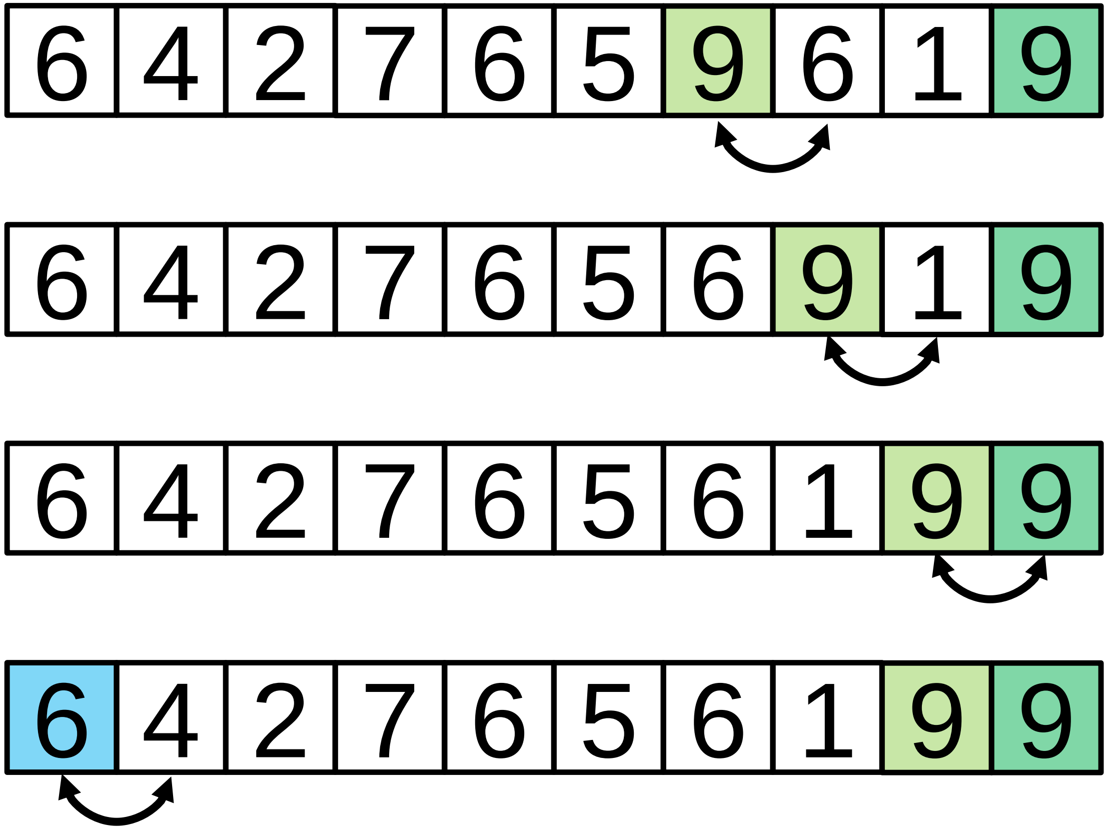
6 4 9 2 7 6 5 6 1 9

6 4 2 9 7 6 5 6 1 9

6 4 2 7 9 6 5 6 1 9

6 4 2 7 6 9 5 6 1 9

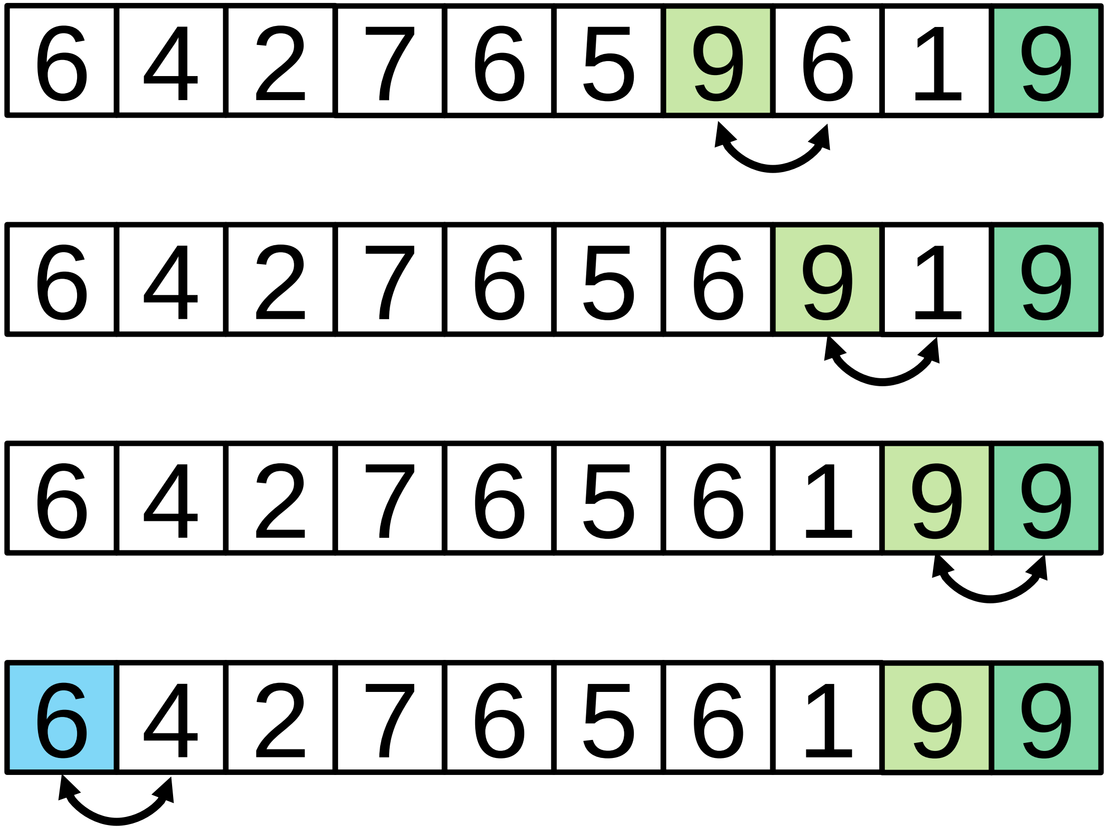
Bubble sort



6	4	2	7	6	5	9	6	1	9
6	4	2	7	6	5	6	9	1	9
6	4	2	7	6	5	6	1	9	9
6	4	2	7	6	5	6	1	9	9

....

Bubble sort

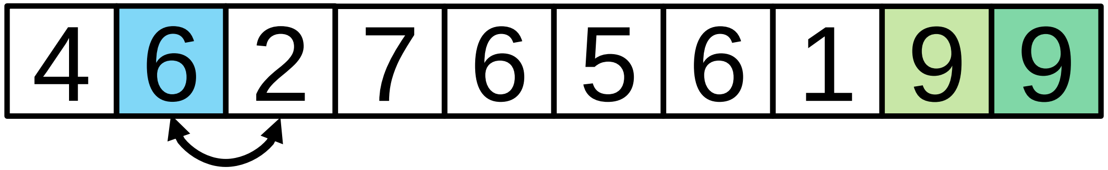


6	4	2	7	6	5	9	6	1	9
6	4	2	7	6	5	6	9	1	9
6	4	2	7	6	5	6	1	9	9
6	4	2	7	6	5	6	1	9	9

....

Complexitate?

Bubble sort



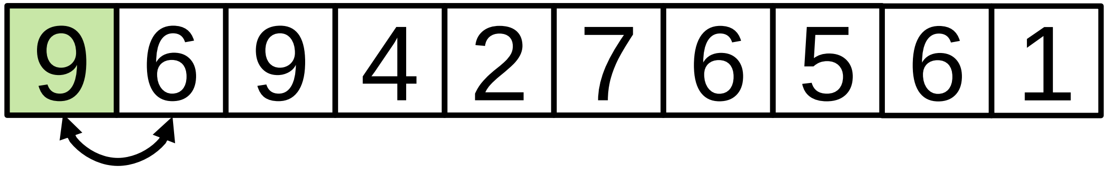
4	6	2	7	6	5	6	1	9	9

....

Complexitate: **O(n^2)**
Un pas trece prin toate elementele
Garantat să termine după n repetiții

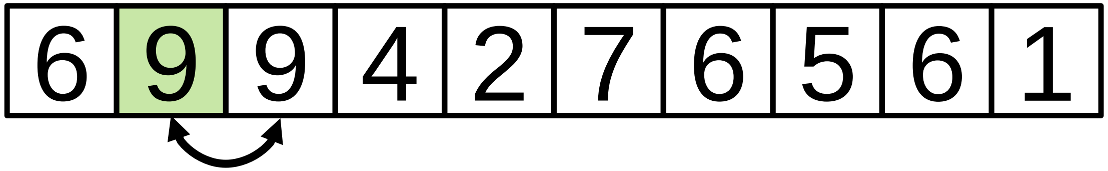
1	2	4	5	6	6	6	7	9	9

Parallel bubble sort



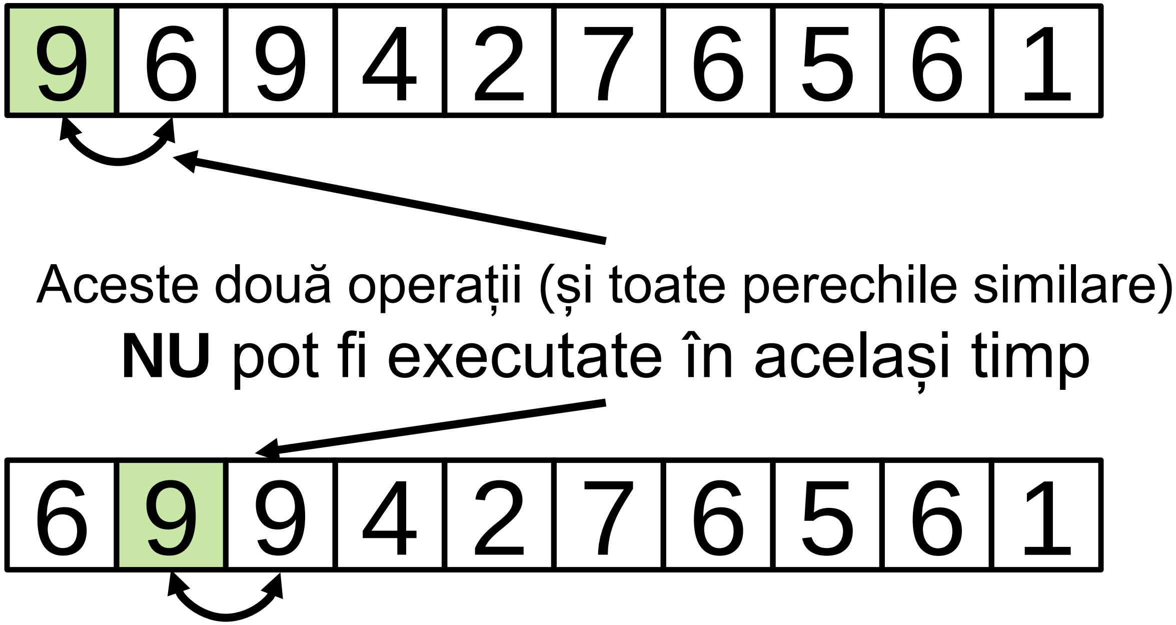
9	6	9	4	2	7	6	5	6	1

Cum paralelizăm?



6	9	9	4	2	7	6	5	6	1

Parallel bubble sort

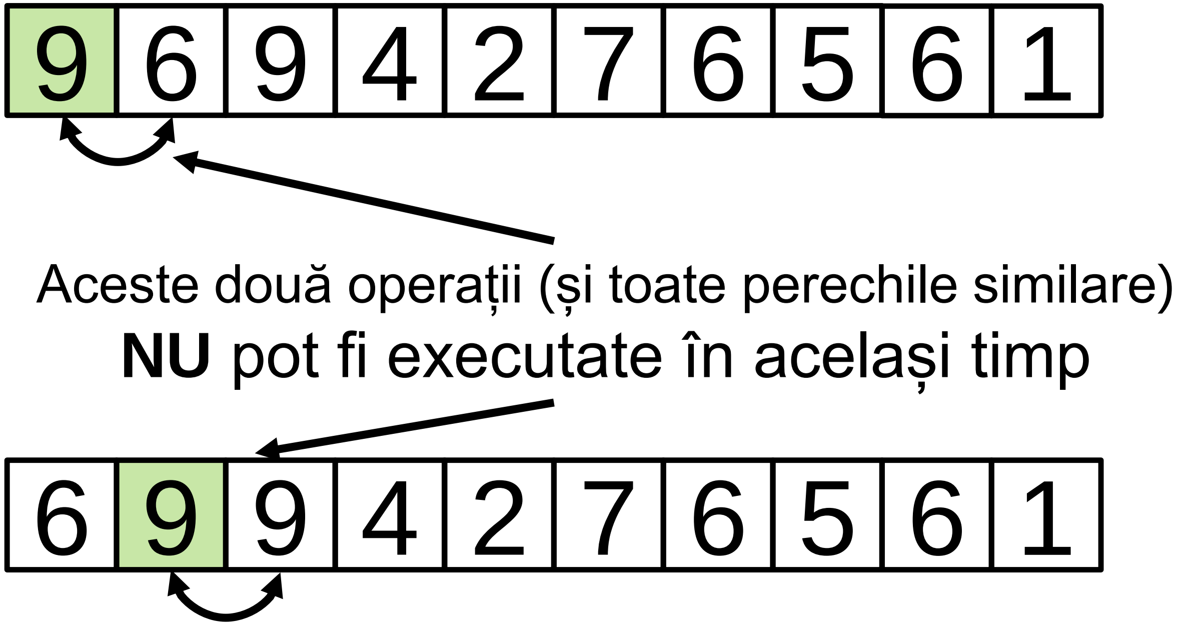


9	6	9	4	2	7	6	5	6	1

Aceste două operații (și toate perechile similare)
NU pot fi executate în același timp

6	9	9	4	2	7	6	5	6	1

Parallel bubble sort



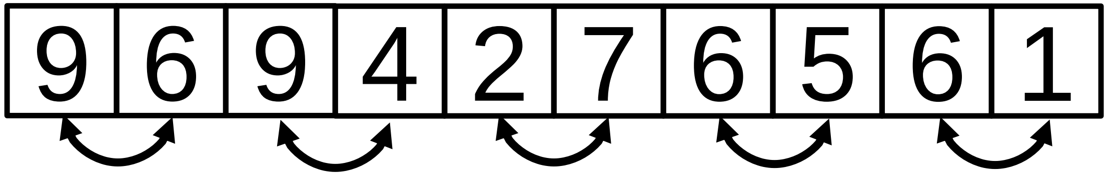
🟩 9	6	9	4	2	7	6	5	6	1

Aceste două operații (și toate perechile similare)
NU pot fi executate în același timp

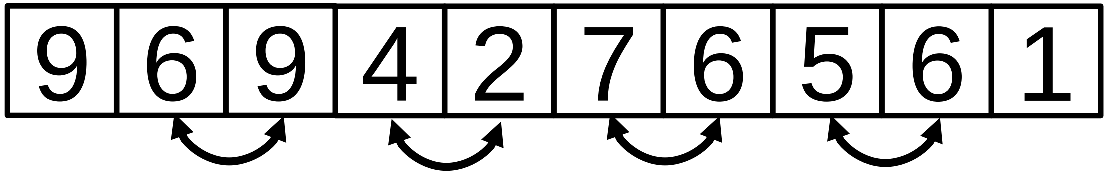
6	🟩 9	9	4	2	7	6	5	6	1

Solution hint: Operațiile nu trebuie
executate în această ordine

OETS: Odd-Even Transposition Sort



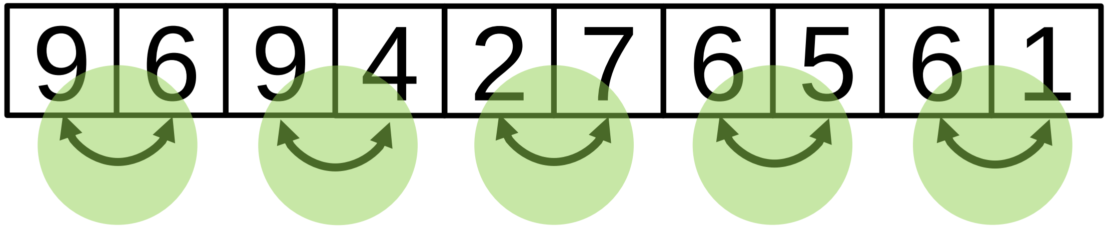
9	6	9	4	2	7	6	5	6	1



9	6	9	4	2	7	6	5	6	1

OETS: Odd-Even Transposition Sort

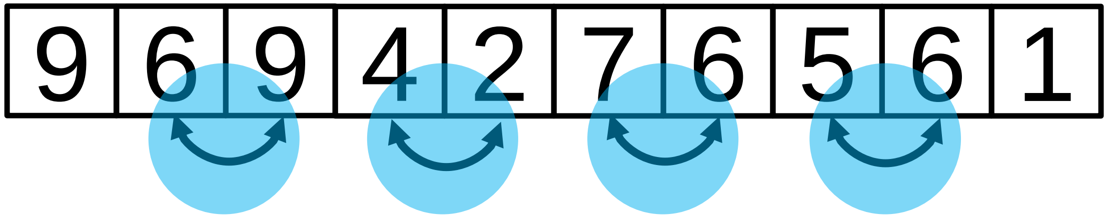
Pot fi
executate →
în paralel



9	6	9	4	2	7	6	5	6	1

………..

Pot fi
executate →
în paralel



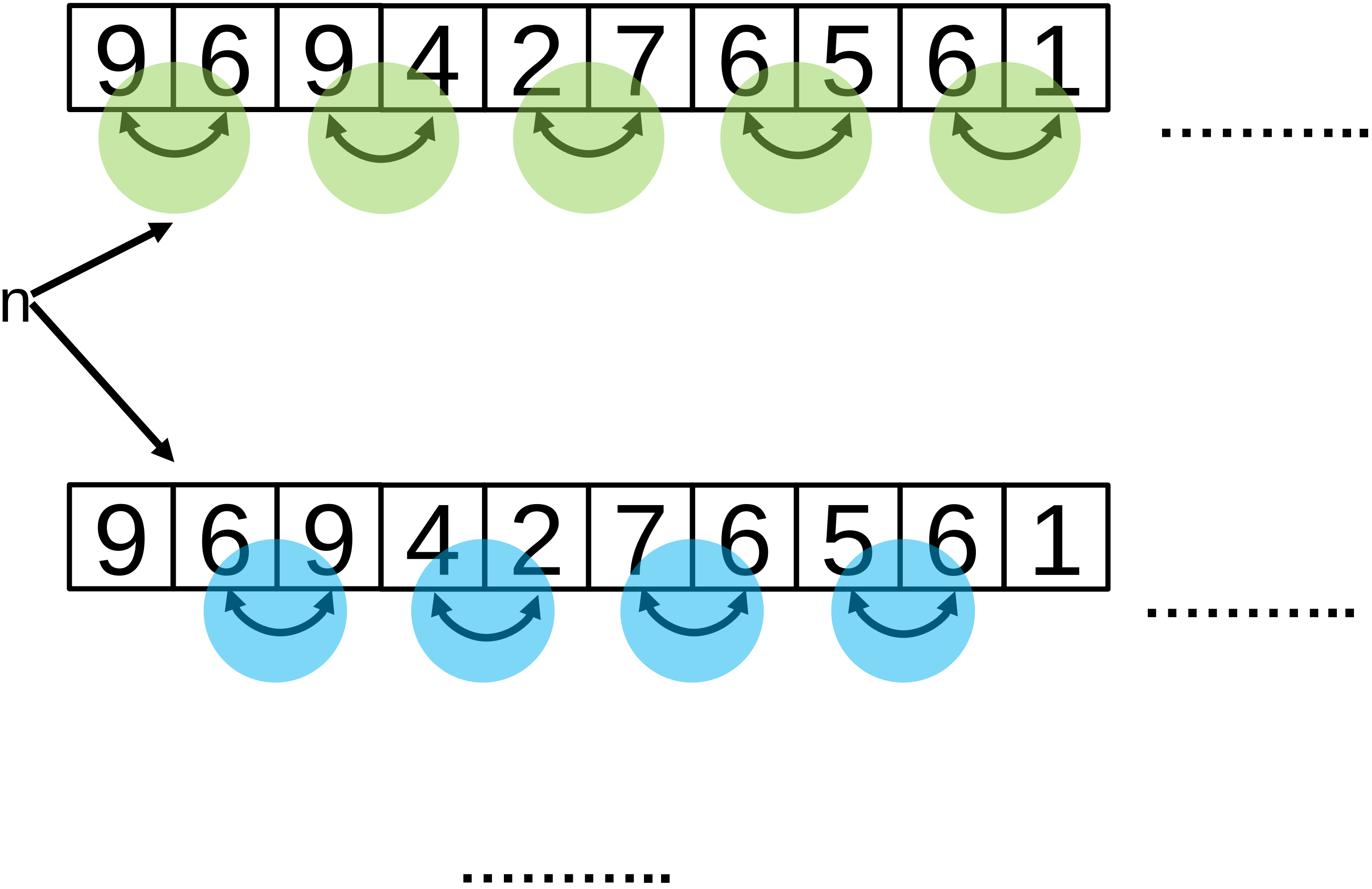
9	6	9	4	2	7	6	5	6	1

………..

………..

OETS: Odd-Even Transposition Sort

NU pot fi executate în paralel



9	6	9	4	2	7	6	5	6	1

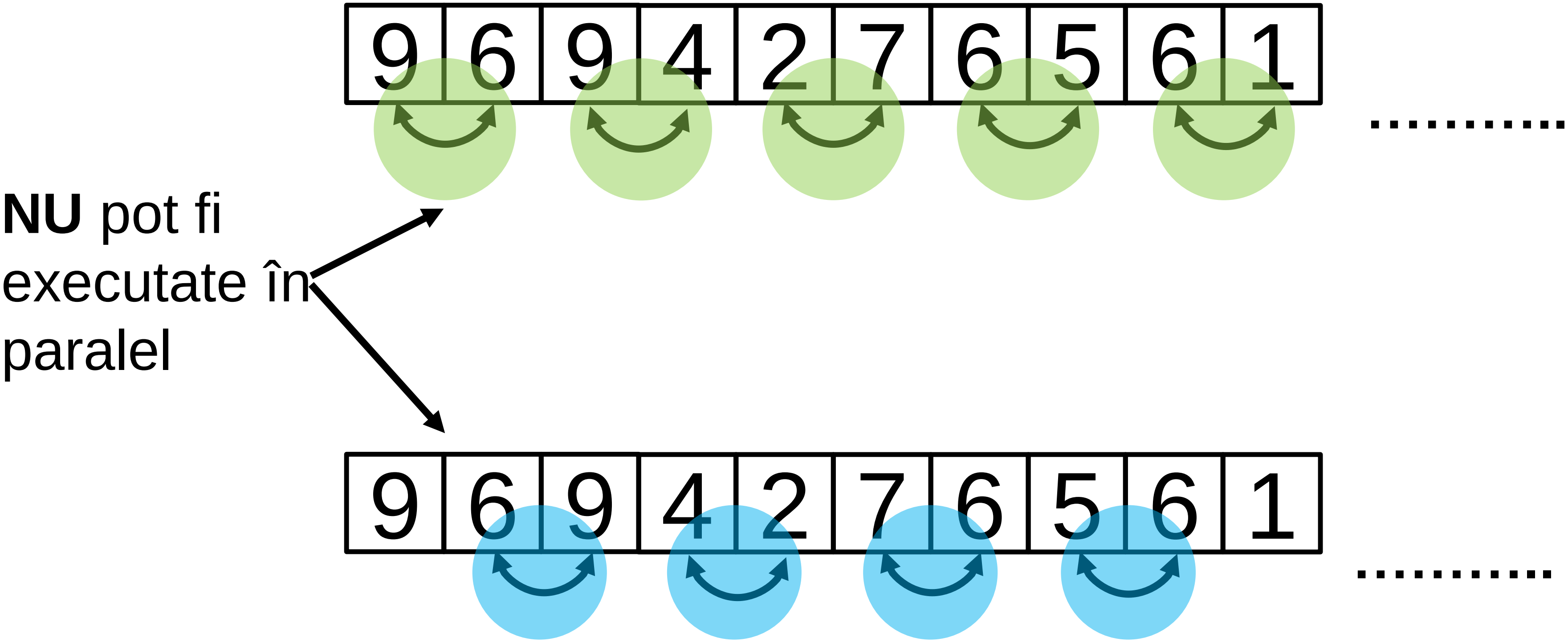
………..

9	6	9	4	2	7	6	5	6	1

………..

………..

OETS: Odd-Even Transposition Sort



9	6	9	4	2	7	6	5	6	1

....

NU pot fi executate în paralel

9	6	9	4	2	7	6	5	6	1

....

Ce facem? Ce folosim?

....

OETS: Odd-Even Transposition Sort

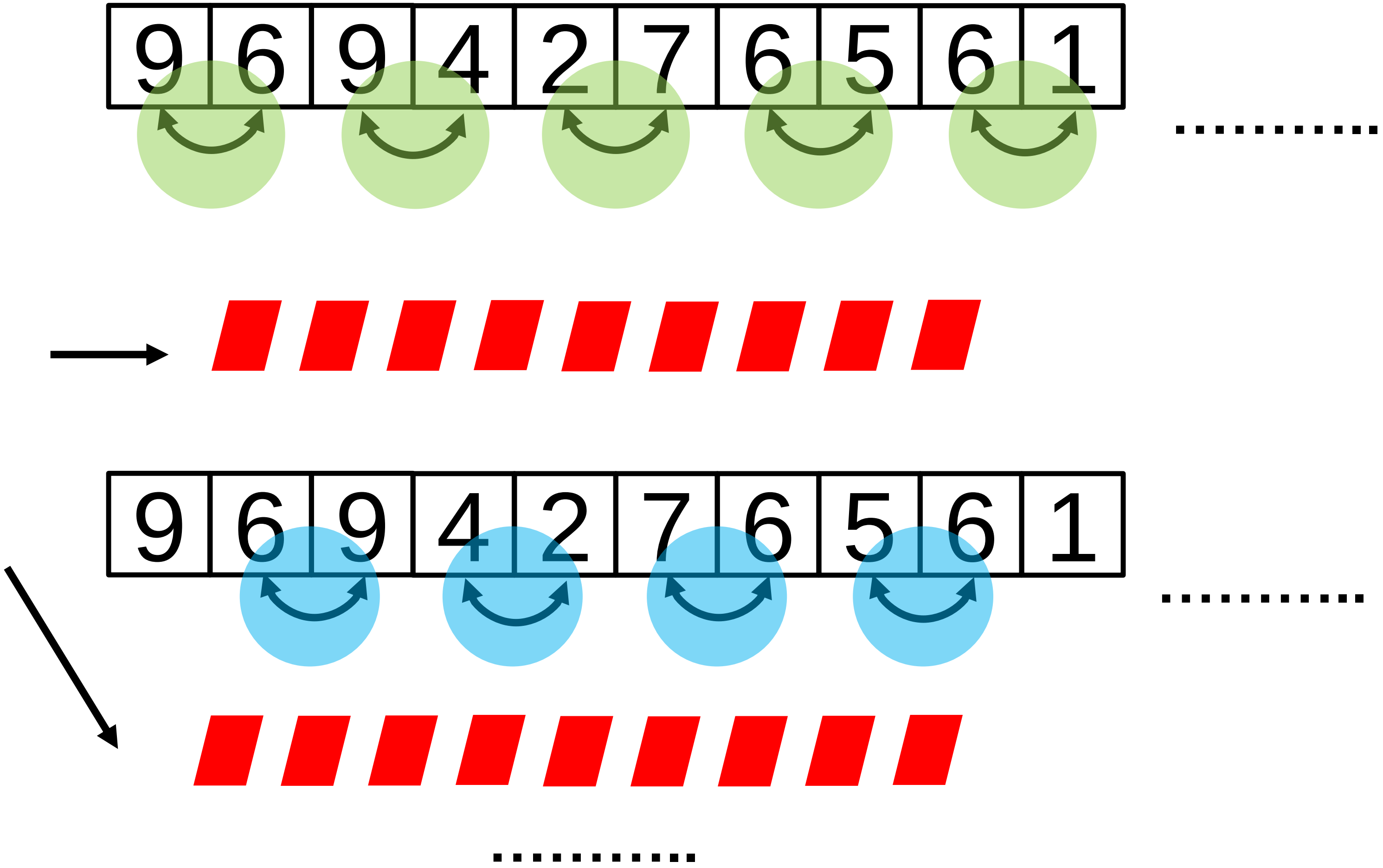
Soluția:

Barrier

între

fiecare

repetiție



9	6	9	4	2	7	6	5	6	1

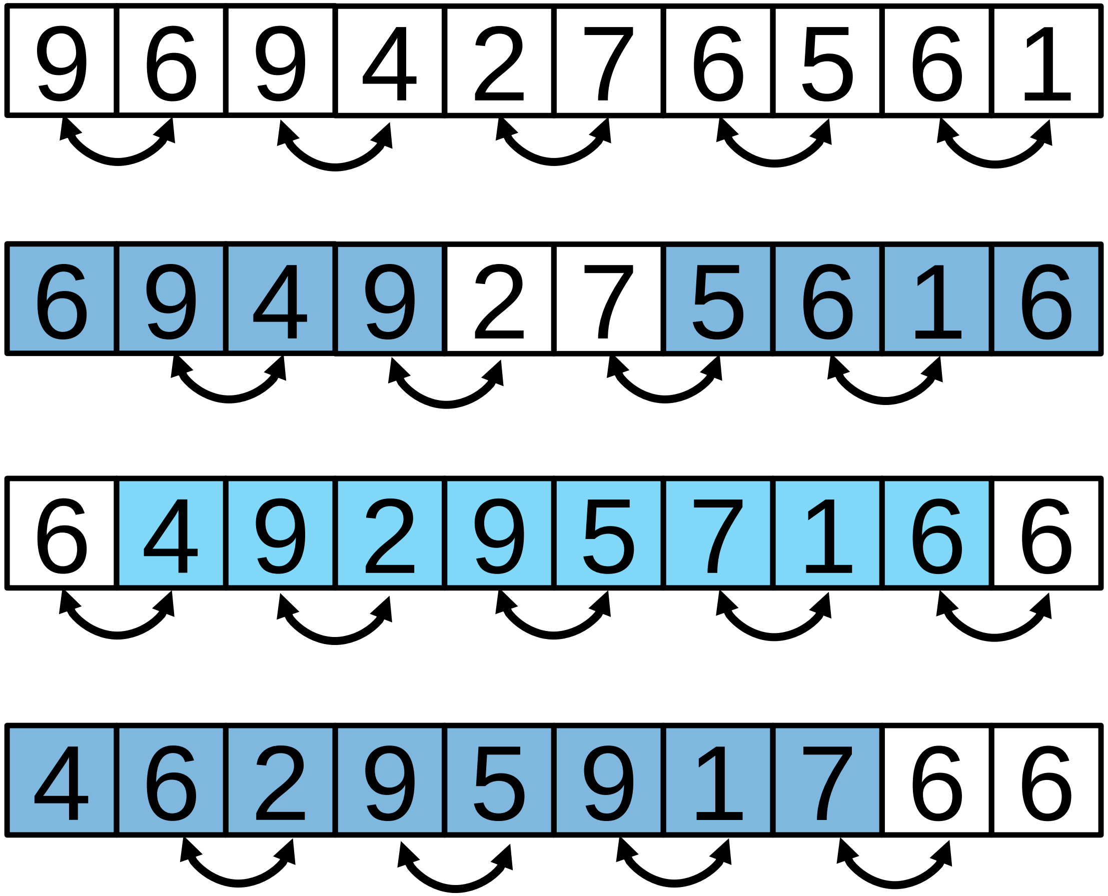
....

9	6	9	4	2	7	6	5	6	1

....

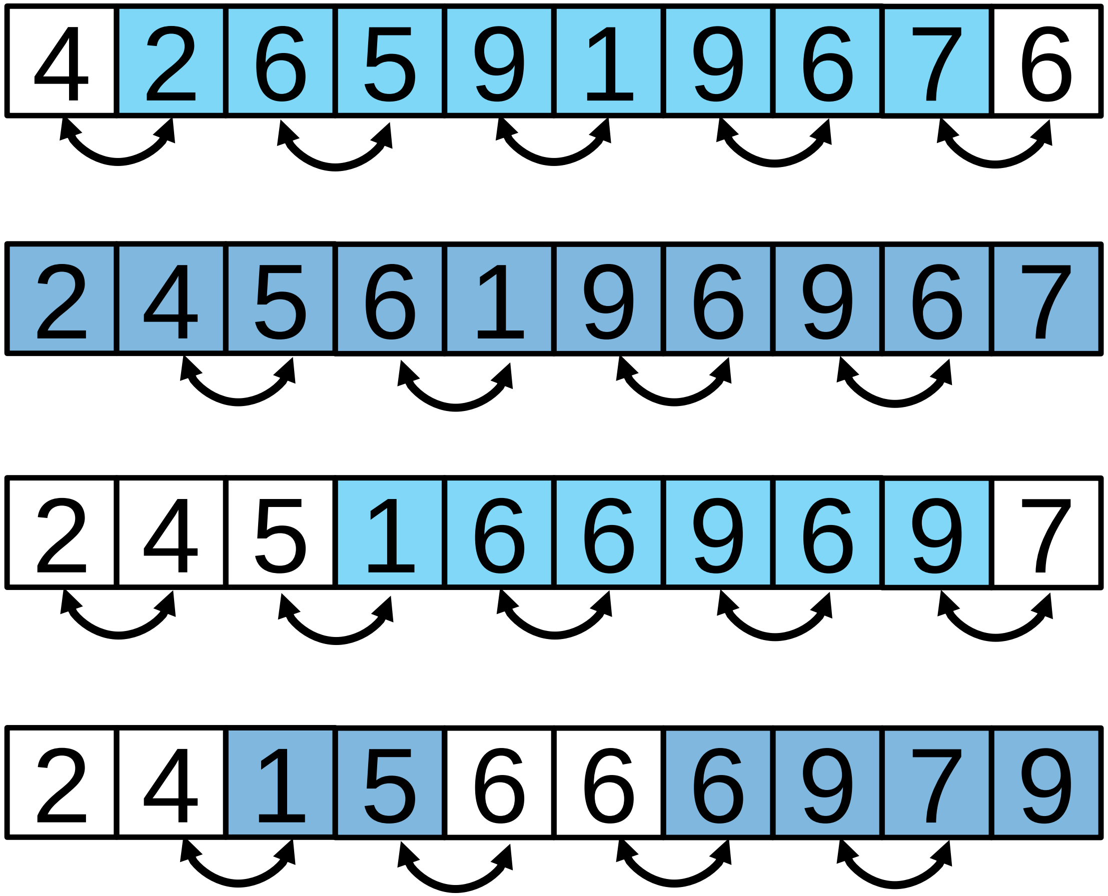
....

OETS: Odd-Even Transposition Sort



9	6	9	4	2	7	6	5	6	1
6	9	4	9	2	7	5	6	1	6
6	4	9	2	9	5	7	1	6	6
4	6	2	9	5	9	1	7	6	6

OETS: Odd-Even Transposition Sort



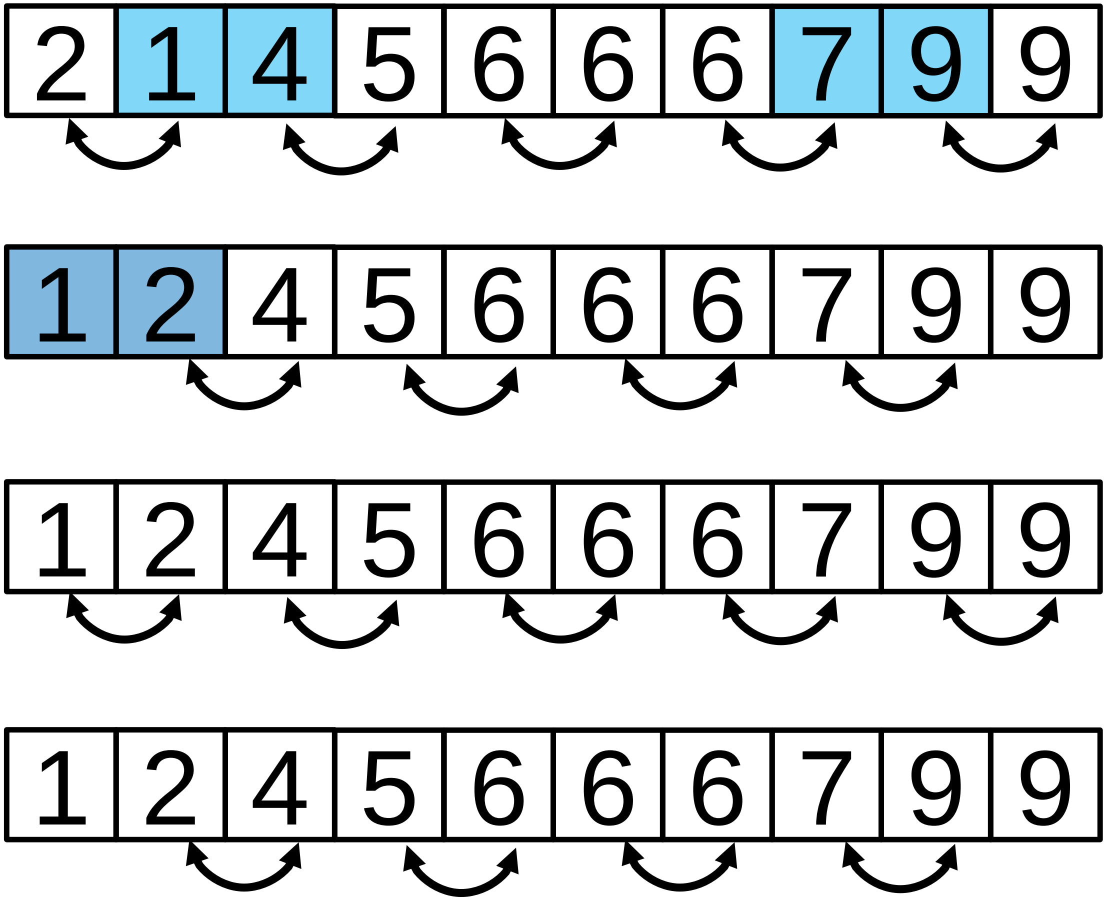
4 2 6 5 9 1 9 6 7 6

2 4 5 6 1 9 6 9 6 7

2 4 5 1 6 6 9 6 9 7

2 4 1 5 6 6 6 9 7 9

OETS: Odd-Even Transposition Sort



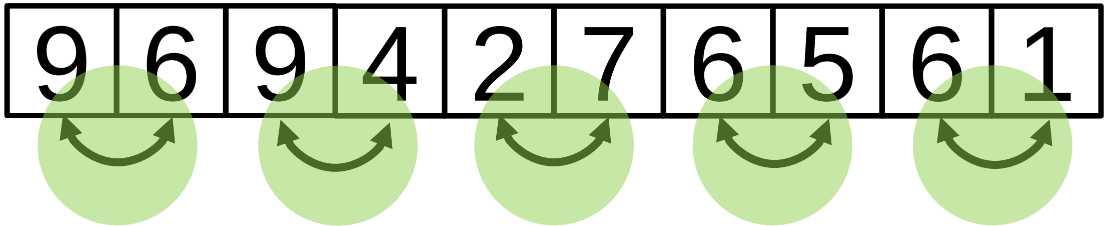
2 1 4 5 6 6 6 7 9 9

1 2 4 5 6 6 6 7 9 9

1 2 4 5 6 6 6 7 9 9

1 2 4 5 6 6 6 7 9 9

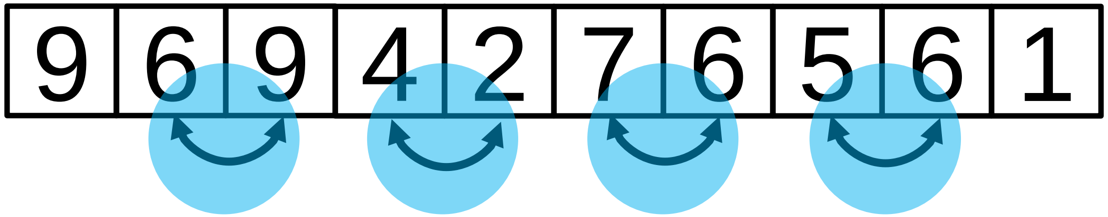
OETS: Odd-Even Transposition Sort



9	6	9	4	2	7	6	5	6	1

....

Complexitate a soluției paralele?

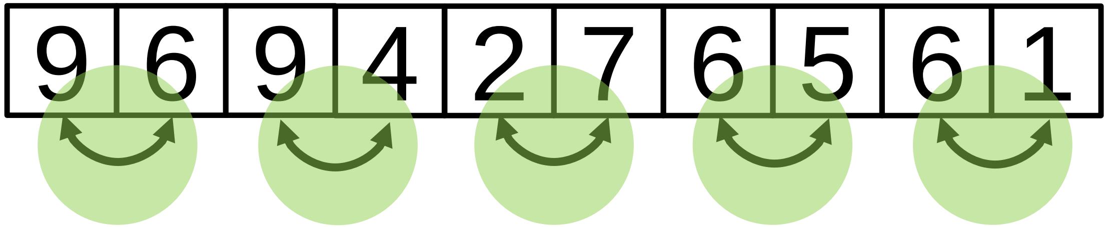


9	6	9	4	2	7	6	5	6	1

....

....

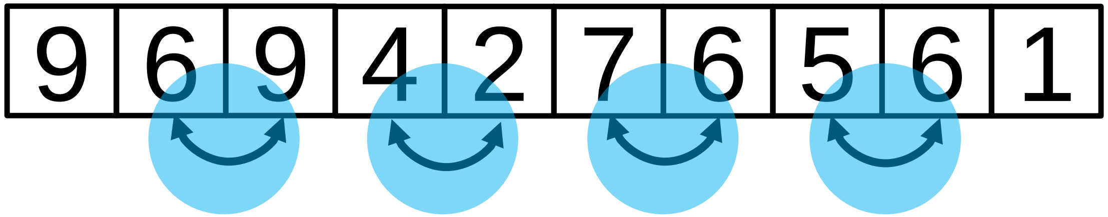
OETS: Odd-Even Transposition Sort



9	6	9	4	2	7	6	5	6	1

....

Complexitate a soluției paralele?



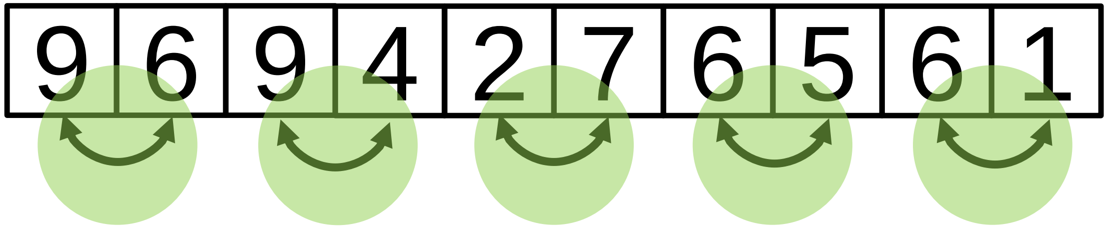
9	6	9	4	2	7	6	5	6	1

....

$T = O\left(\frac{N}{P} * N\right)(= O(N)$ pentru $P = N)$

....

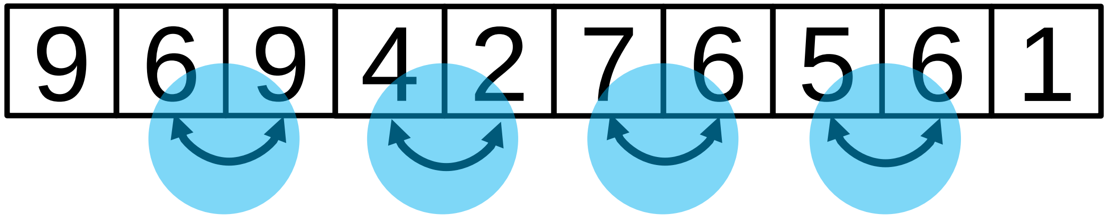
OETS: Odd-Even Transposition Sort



9 6 9 4 2 7 6 5 6 1

....

Speedup?



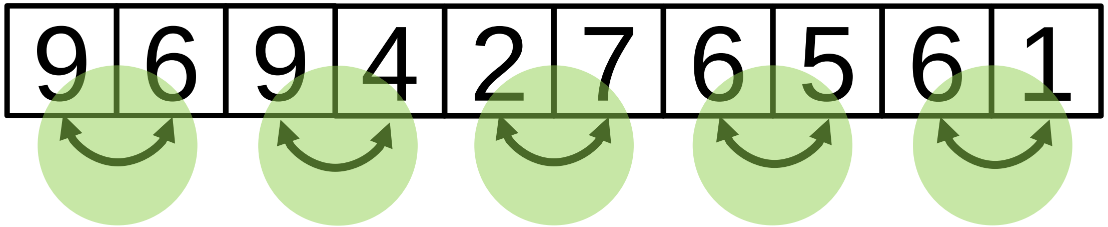
9 6 9 4 2 7 6 5 6 1

....

$T = O\left(\frac{N}{P} * N\right)(= O(N)$ pentru $P = N)$

....

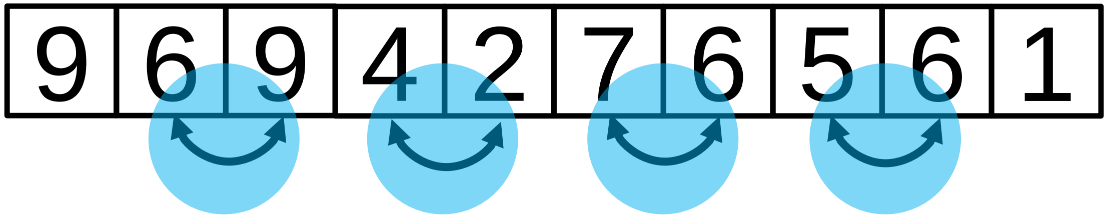
OETS: Odd-Even Transposition Sort



9	6	9	4	2	7	6	5	6	1

....

Speedup?



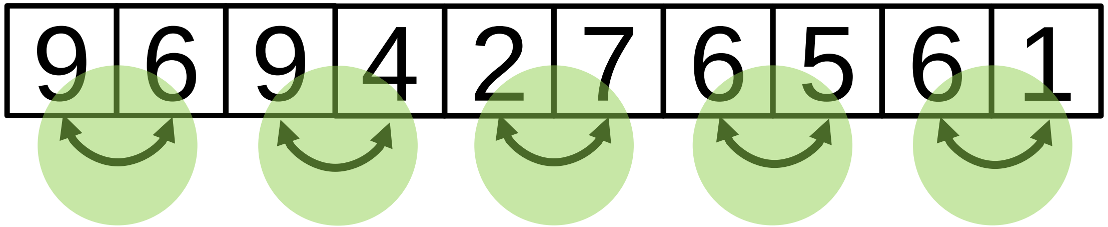
9	6	9	4	2	7	6	5	6	1

....

$$S = \frac{N^2}{\frac{N^2}{P}} = P$$

....

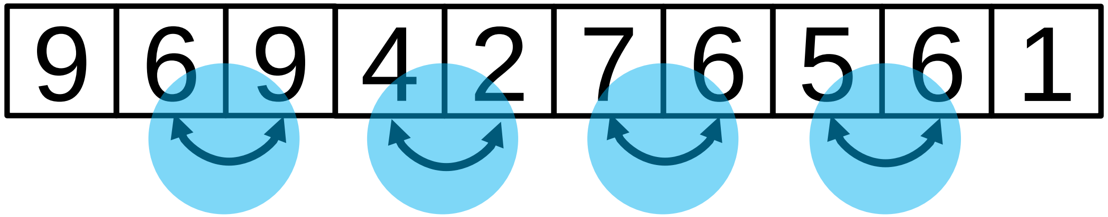
OETS: Odd-Even Transposition Sort



9	6	9	4	2	7	6	5	6	1

....

Speedup? **But is it really**?



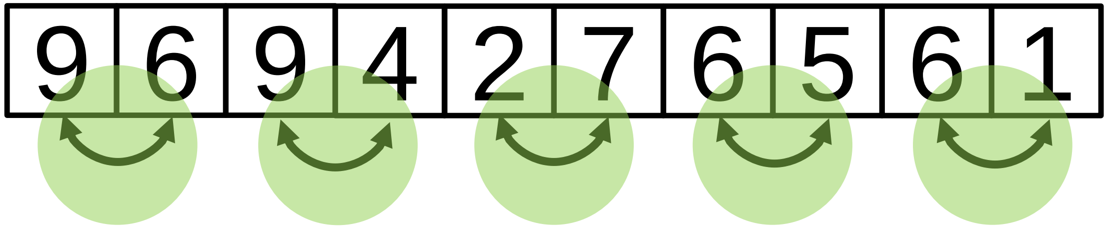
9	6	9	4	2	7	6	5	6	1

....

$$S = \frac{N^2}{\frac{N^2}{P}} = P$$

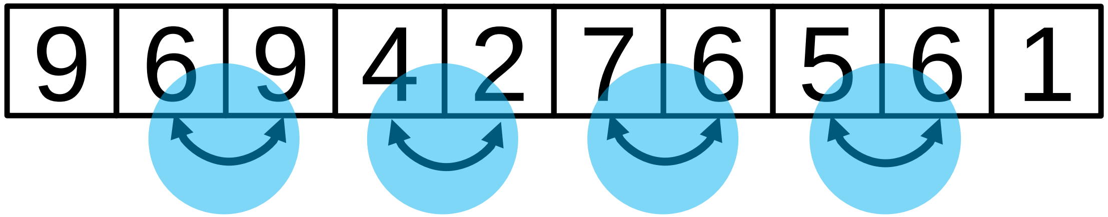
....

OETS: Odd-Even Transposition Sort



....

Speedup?

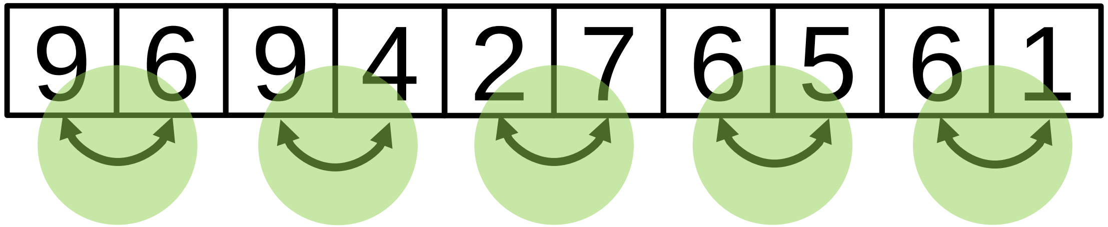


....

$$S = \frac{N log_2 N}{\frac{N^2}{P}}$$

....

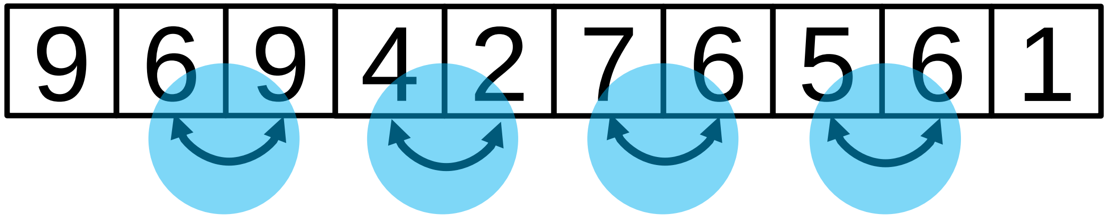
OETS: Odd-Even Transposition Sort



9	6	9	4	2	7	6	5	6	1

....

Speedup?



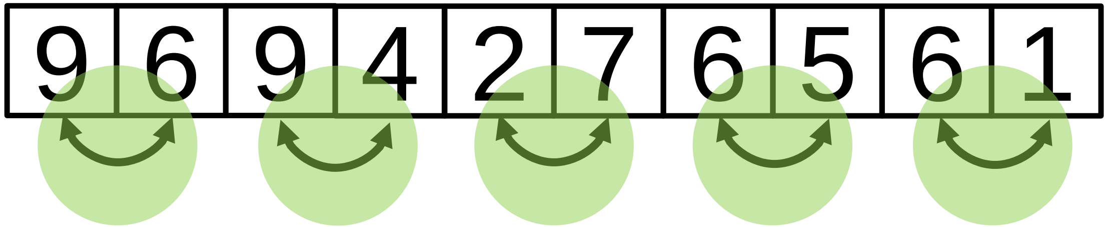
9	6	9	4	2	7	6	5	6	1

....

$$S = \frac{Plog_2N}{N}$$

....

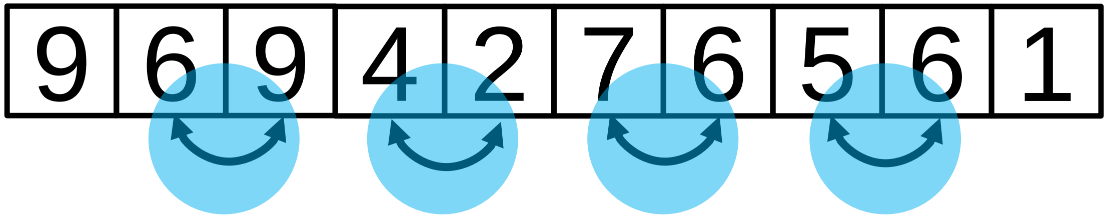
OETS: Odd-Even Transposition Sort



9	6	9	4	2	7	6	5	6	1

....

Nu uitați așteptatul la barieră poate introduce timpi mari!



9	6	9	4	2	7	6	5	6	1

....

$$S = \frac{Plog_2N}{N}$$

....

Shear sort
(Row-column sort)
(Snake sort)

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<

Sortează fiecare linie **pară** în mod **ascendent**
Sortează fiecare linie **impară** în mod **descendent**

Shear sort

9	6	9	4
2	7	6	5
9	3	6	2
5	4	1	5

> > > >

Sortează crescător coloanele

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Repetă tot de $log_2 n$ ori

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

De ce liniile pare crescător și celelalte descrescător?

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

De ce liniile pare crescător și celelalte descrescător?
Dorim compararea celui mai mare element de pe linia i cu cel mai mic de pe linia i+1

Shear sort

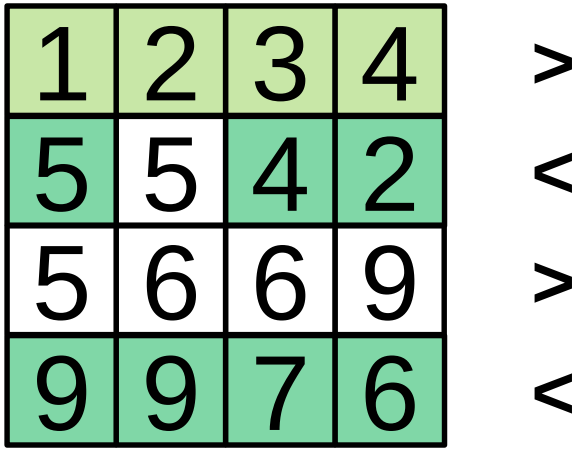
4	6	9	9	>
7	6	5	2	<
2	3	6	9	>
5	5	4	1	<

Shear sort

2	3	4	1
4	5	5	2
5	6	6	9
7	6	9	9

> > > >

Shear sort



1	2	3	4	>
5	5	4	2	<
5	6	6	9	>
9	9	7	6	<

Shear sort

1	2	3	2
5	5	4	4
5	6	6	6
9	9	7	9
>	>	>	>

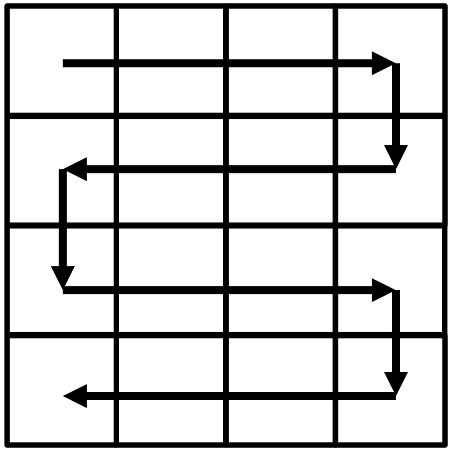
Shear sort

1	2	2	3	>
5	5	4	4	<
5	6	6	6	>
9	9	9	7	<

Shear sort

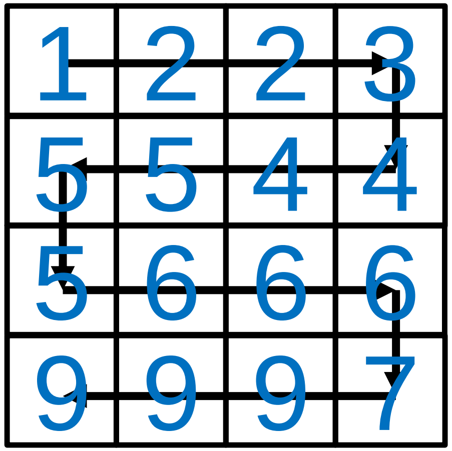
1	2	2	3
5	5	4	4
5	6	6	6
9	9	9	7
>	>	>	>

Shear sort



Lista finală se obține citind în formă de șerpuită (snake sort)

Shear sort



1	2	2	3
5	5	4	4
5	6	6	6
9	9	9	7

1	2	2	3	4	4	5	5	5	6	6	6	7	9	9	9

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Complexitate? $G =$

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Complexitate? $G = log_2N \ * \ log_2N$ repetiții

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Complexitate? $G = log_2 N \; * \; 2 \; * \sqrt{N}$

$\sqrt{N}$ linii/coloane

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Complexitate? $G = log_2 N \ * \ 2 * \sqrt{N} \ * \ \sqrt{N} * \ log_2 \sqrt{N}$

$\sqrt{N} * \ log_2 \sqrt{N}$ cel mai bun algoritm secvențial de sortare

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Complexitate? $G = log_2 N \ * N \ * \ log_2\sqrt{N}$

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Complexitate? $G = Nlog_2N \ * log_2\sqrt{N}$

Cel mai bun algoritm secvențial rămâne $Nlog_2N$

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

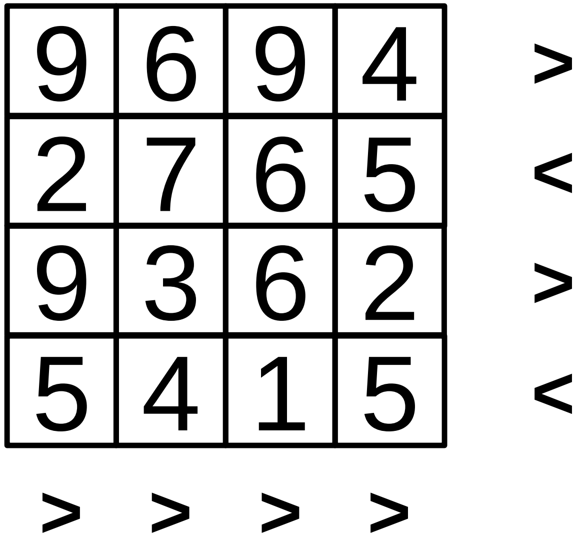
Cum paralelizăm?

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Cum paralelizăm?
Toate liniile în paralel, barieră, apoi toate coloanele, apoi barieră, și repetăm.

Shear sort



9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

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Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Complexitate versiune paralelă?

$$G = log_2 N \; * \; 2 \; * \; \sqrt{N} \; * \; \sqrt{N} * \; log_2\sqrt{N}$$

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Complexitate versiune paralelă?   N = P

$$T = log_2 N \ * \ 2 \ * \ \sqrt{N} * log_2 \sqrt{N}$$

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Complexitate versiune paralelă? N = P

$$T = \sqrt{N}\ log_2\sqrt{N} * log_2N$$

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Complexitate versiune paralelă?

$$T = log_2 N \ * \ \ 2 \ * \frac{\sqrt{N}}{P} * \ \sqrt{N} * \ log_2 \sqrt{N}$$

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Complexitate versiune paralelă?

$$T = \frac{N}{P} log_2 N \ * \ log_2 \sqrt{N}$$

Shear sort

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Speedup?

$$T = \frac{N}{P} log_2 N \ * \ log_2 \sqrt{N}$$

$$G = \ N log_2 N$$

Shear sort

$$S = \frac{Nlog_2N}{\frac{N}{P}log_2N \; * \; log_2\sqrt{N}}$$

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Speedup?

$$T = \frac{N}{P}log_2N \; * \; log_2\sqrt{N}$$

$$G = \; Nlog_2N$$

Shear sort

$$S = \frac{P}{log_2\sqrt{N}}$$

9	6	9	4	>
2	7	6	5	<
9	3	6	2	>
5	4	1	5	<
>	>	>	>	

Speedup?

$$T = \frac{N}{P} log_2 N \; * \; log_2\sqrt{N}$$

$$G = \; N log_2 N$$

Matrix multiply

```
for(i=0; i<N; i++)
        for(j=0; j<N; j++)
                for(k=0; k<N; k++)
                        c[i][j] += a[i][k] * b[k][j]
```

Matrix multiply

```
for(i=0; i<N; i++)
        for(j=0; j<N; j++)
                for(k=0; k<N; k++)
                        c[i][j] += a[i][k] * b[k][j]
```

Complexitate?

Matrix multiply

```
for(i=0; i<N; i++)
        for(j=0; j<N; j++)
                for(k=0; k<N; k++)
                        c[i][j] += a[i][k] * b[k][j]
```

Complexitate?

$$G = N^3$$

Matrix multiply

Atenție avem N^2 elemente

```
for(i=0; i<N; i++)
        for(j=0; j<N; j++)
                for(k=0; k<N; k++)
                        c[i][j] += a[i][k] * b[k][j]
```

Complexitate?

$$G = N^3$$

Matrix multiply

```
co[Tid = 1 to P]
{
        start = Tid * ceil(N/P)
        end = min((Tid+1) * ceil(N/P),N)
        for(i=start; i<end; i++)
                for(j=0; j<N; j++)
                        for(k=0; k<N; k++)
                                c[i][j] += a[i][k] * b[k][j]
}
```

Matrix multiply

Complexitate?

```
co[Tid = 1 to P]
{
    start = Tid * ceil(N/P)
    end = min((Tid+1) * ceil(N/P),N)
    for(i=start; i<end; i++)
        for(j=0; j<N; j++)
            for(k=0; k<N; k++)
                c[i][j] += a[i][k] * b[k][j]
}
```

Matrix multiply

Complexitate?

$$T = \frac{N^3}{P}$$

```
co[Tid = 1 to P]
{
        start = Tid * ceil(N/P)
        end = min((Tid+1) * ceil(N/P),N)
        for(i=start; i<end; i++)
                for(j=0; j<N; j++)
                        for(k=0; k<N; k++)
                                c[i][j] += a[i][k] * b[k][j]
}
```

Matrix multiply

Speedup?

$$T = \frac{N^3}{P}$$

```
co[Tid = 1 to P]
{
        start = Tid * ceil(N/P)
        end = min((Tid+1) * ceil(N/P),N)
        for(i=start; i<end; i++)
                for(j=0; j<N; j++)
                        for(k=0; k<N; k++)
                                c[i][j] += a[i][k] * b[k][j]
}
```

Matrix multiply

Speedup?

```
co[Tid = 1 to P]
{
        start = Tid * ceil(N/P)
        end = min((Tid+1) * ceil(N/P),N)
        for(i=start; i<end; i++)
                for(j=0; j<N; j++)
                        for(k=0; k<N; k++)
                                c[i][j] += a[i][k] * b[k][j]
}
```

$$S = \frac{N^3}{\frac{N^3}{P}}$$

Matrix multiply

Speedup?

$$S = P$$

```
co[Tid = 1 to P]
{
        start = Tid * ceil(N/P)
        end = min((Tid+1) * ceil(N/P),N)
        for(i=start; i<end; i++)
                for(j=0; j<N; j++)
                        for(k=0; k<N; k++)
                                c[i][j] += a[i][k] * b[k][j]
}
```

Floyd Warshall

```
for(k=0; k<N; k++)
        for(i=0; i<N; i++)
                for(j=0; j<N; j++)
                        c[i][j] = min(c[i][k] + c[k][j], c[i][j])
```

Super-linear speedup?

$$S > P?$$