Stochastic Processes 2WB08 problem set 2

- Problem 6.8, 6.9.
- Problem: Let $(X_n)_{n\in\mathbb{N}_0}$ be a Markov chain with finite state space S and transition matrix p. Let $g:S\to\mathbb{R}$ be a function with the following property: there exists $\lambda>0$ such that for all $i\in S$ the inequality

$$\sum_{j \in S} p(i,j)g(j) \leq \lambda g(i)$$

holds. Prove that $(\lambda^{-n}g(X_n))_{n\geq 0}$ is a supermartingale.

- Read Section 6.3 in Ross.
- Problems 6.15, 6.16, 6.17, 6.18, 6.19, 6.20, 6.21.