

Fundamentos de Machine Learning para Geometalurgia **Regression Models**

24 de abril al 4 de mayo 2023

Agenda

Machine Learning basis



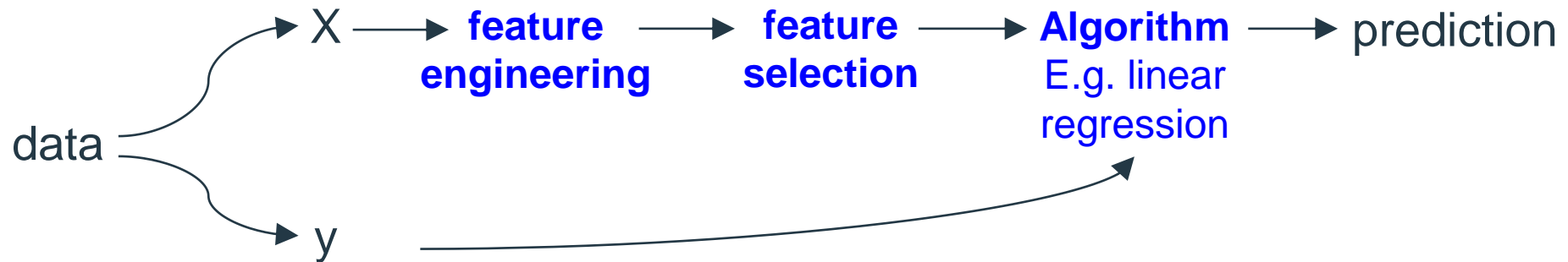
Case study

**Univariate
Exploratory Data Analysis (EDA)**

Data Preparation

**Regression model (proxy) for
geometallurgical parameter A_i**

Machine Learning Process



X
(predictor matrix)

information such as
mineralogy,
minzone, etc.

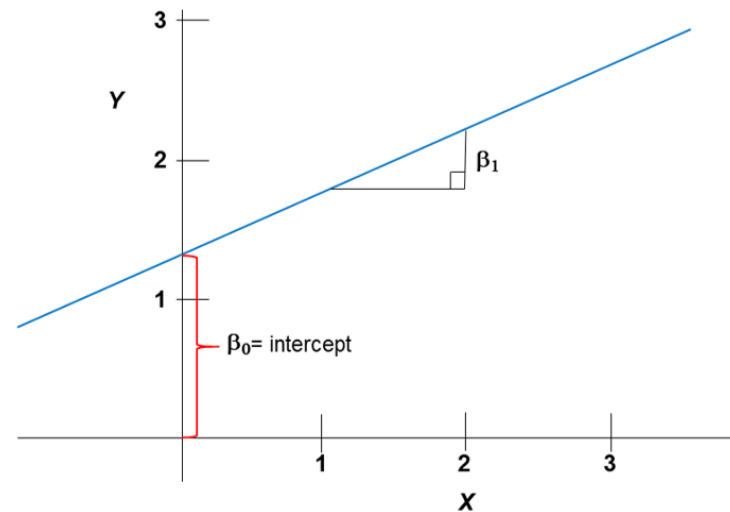
y
(target variable)

A_i values

Algorithm: Linear regression

Simple linear regression can be expressed as a function of slope (β_1) and the intercept (β_0) of a straight line:

$$Y = \beta_0 + \beta_1 X$$



Algorithm: Linear regression

Multiple linear regression model with response Y and terms X_1, \dots, X_p can be expressed as:

$$y = \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n + \beta_0$$

where

y : response variable

n : number of features

x_n : n -th feature

β_n : regression coefficient (weight) of the n -th feature











β_0 : y-intercept

Animation:

https://aegis4048.github.io/mutiple_linear_regression_and_visualization_in_python

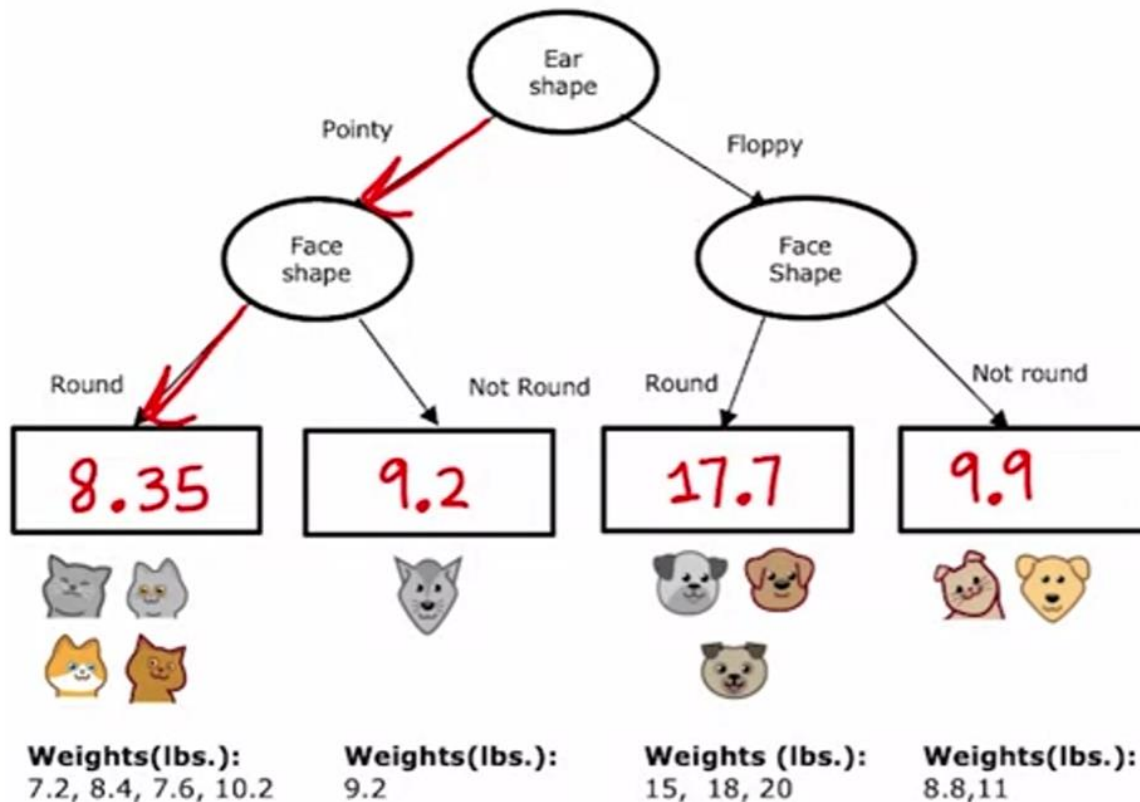
Algorithm: regression tree

Regression with Decision Trees: Predicting a number

| | Ear shape | Face shape | Whiskers | Weight (lbs.) |
|---|-----------|------------|----------|---------------|
|  | Pointy | Round | Present | 7.2 |
|  | Floppy | Not round | Present | 8.8 |
|  | Floppy | Round | Absent | 15 |
|  | Pointy | Not round | Present | 9.2 |
|  | Pointy | Round | Present | 8.4 |
|  | Pointy | Round | Absent | 7.6 |
|  | Floppy | Not round | Absent | 11 |
|  | Pointy | Round | Absent | 10.2 |
|  | Floppy | Round | Absent | 18 |
|  | Floppy | Round | Absent | 20 |

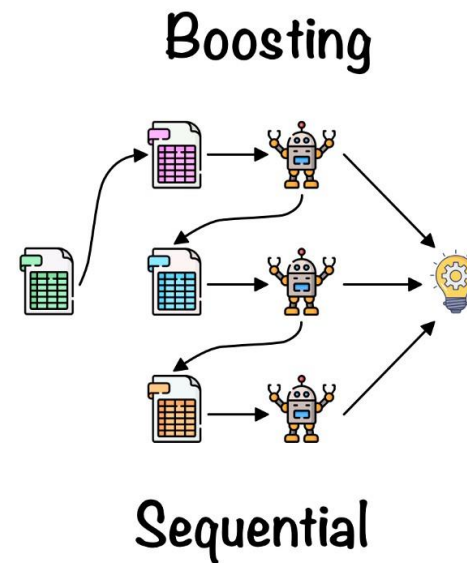
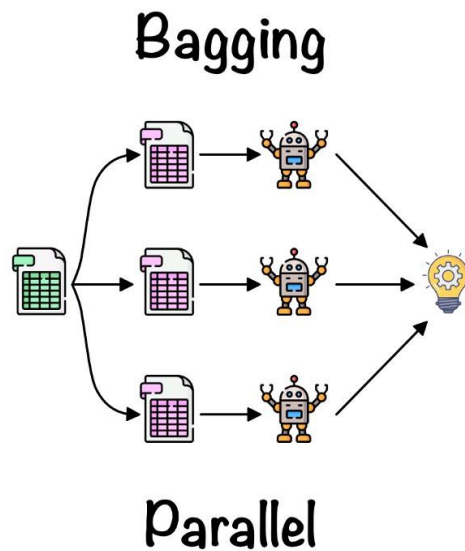
Algorithm: regression tree

Built through binary recursive partitioning, and then continues splitting each partition into smaller groups.



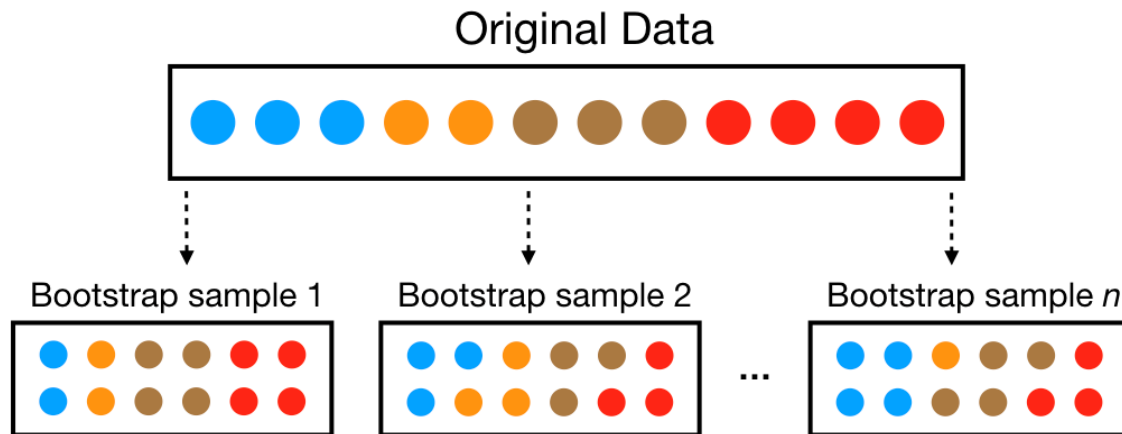
Algorithm: random forest

Ensemble learning is the process of using multiple models, trained over the same data, averaging the results to find a better predictive result. Combining weak learners to build a stronger learner usually increase the model performance. Random forest is a bagging ensemble.



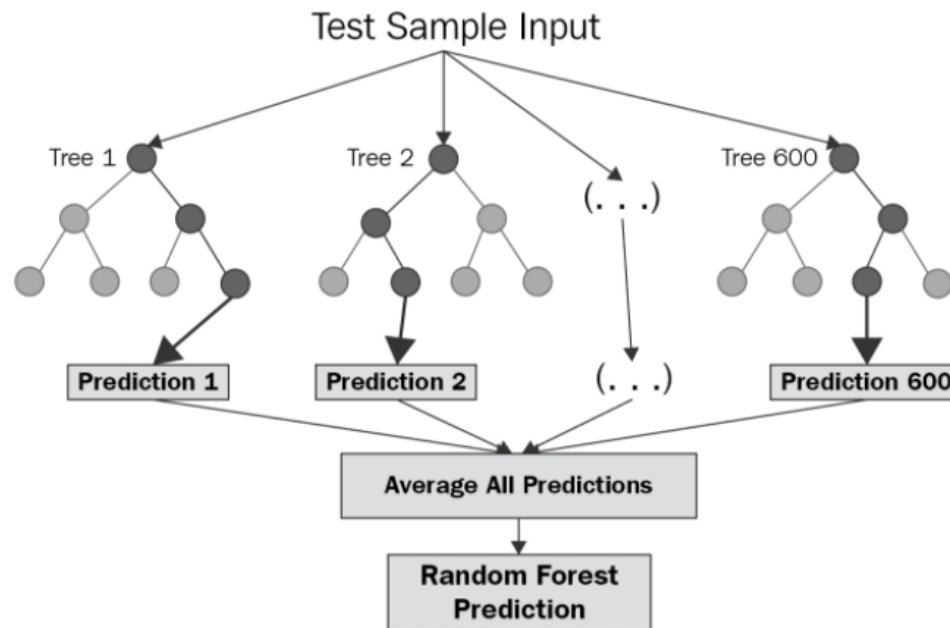
Algorithm: random forest

Bootstrapping is the process of randomly sampling subsets of a dataset over a given number of iterations and a given number of variables. Since samples are drawn with replacement, each bootstrap sample is likely to contain duplicate values.

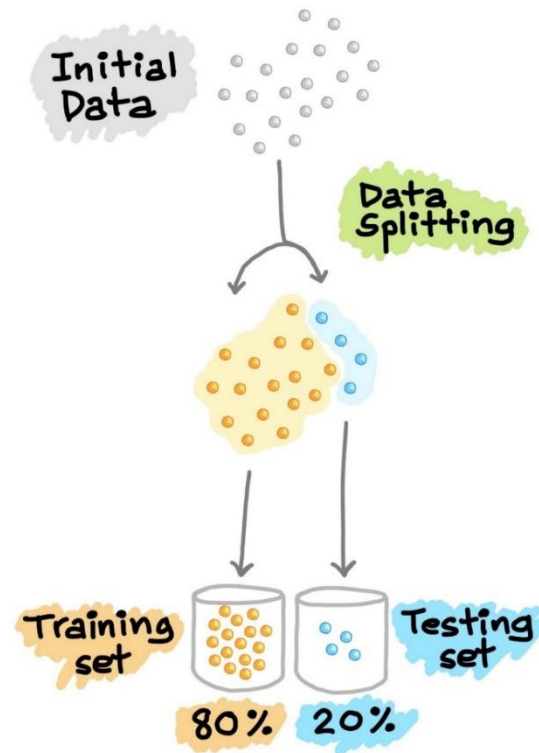


Algorithm: random forest

Bootstrapping algorithm that ensemble multiple randomly drawn decision trees from the data, averaging the results to obtain the prediction. In addition, a subset of the features is randomly selected at each node.



Data splitting



| | Features | Labels |
|--------------|----------|---------|
| Training Set | X_train | y_train |
| Test Set | X_test | y_test |

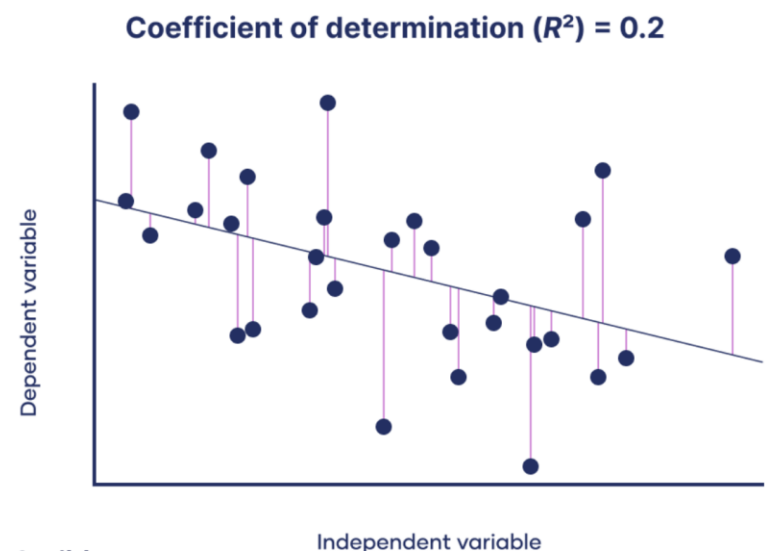
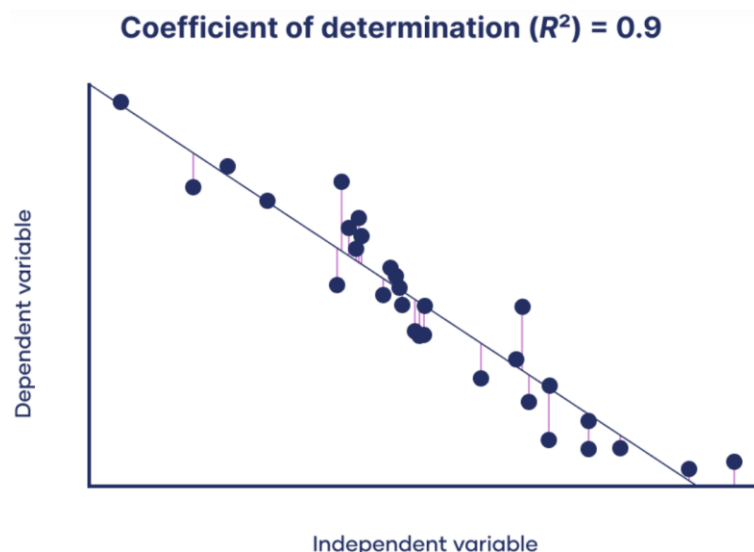
Trained model must perform well on new, unseen data. In order to simulate the new, unseen data, the available data is subjected to data splitting whereby it is split into 2 portions. 80% of the original data is used as the training set and the remaining 20% is used as the testing set

Error Metrics

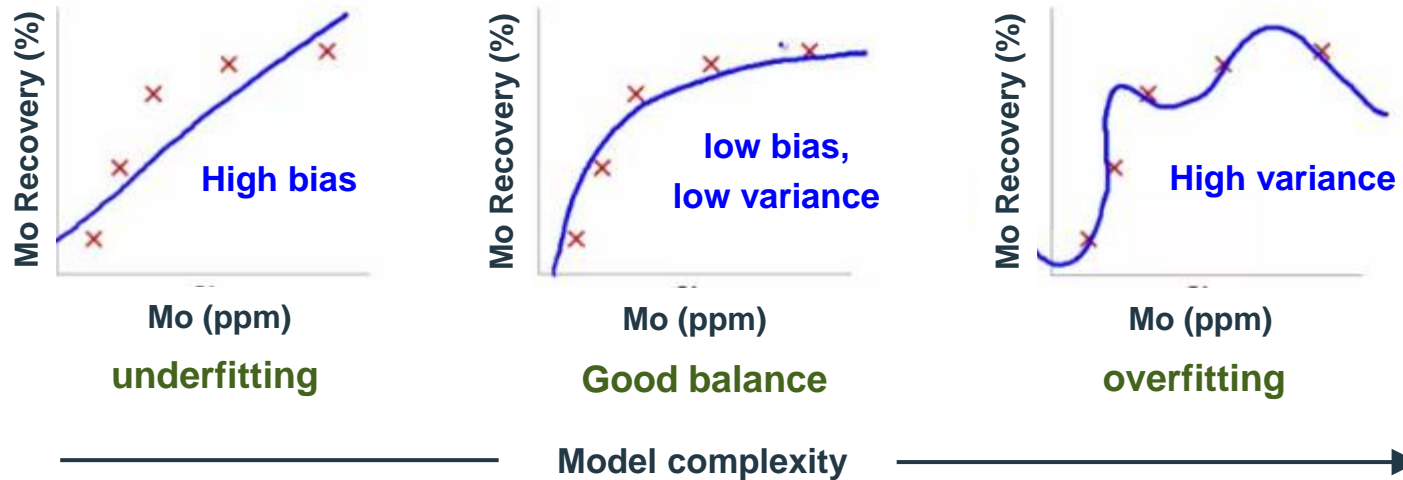
RMSE is the most used metric in regression.

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (Predicted_i - Actual_i)^2}{N}}$$

Coefficient of Determination (r^2) determines the proportion of variance in the dependent variable that can be explained by the independent variable.

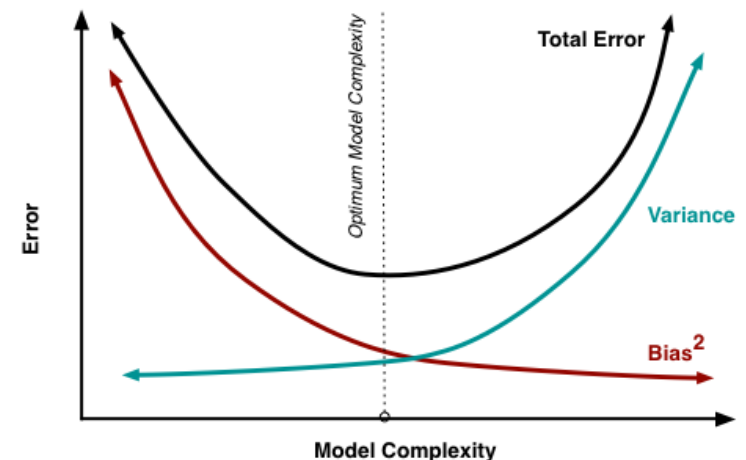


Bias-variance Tradeoff

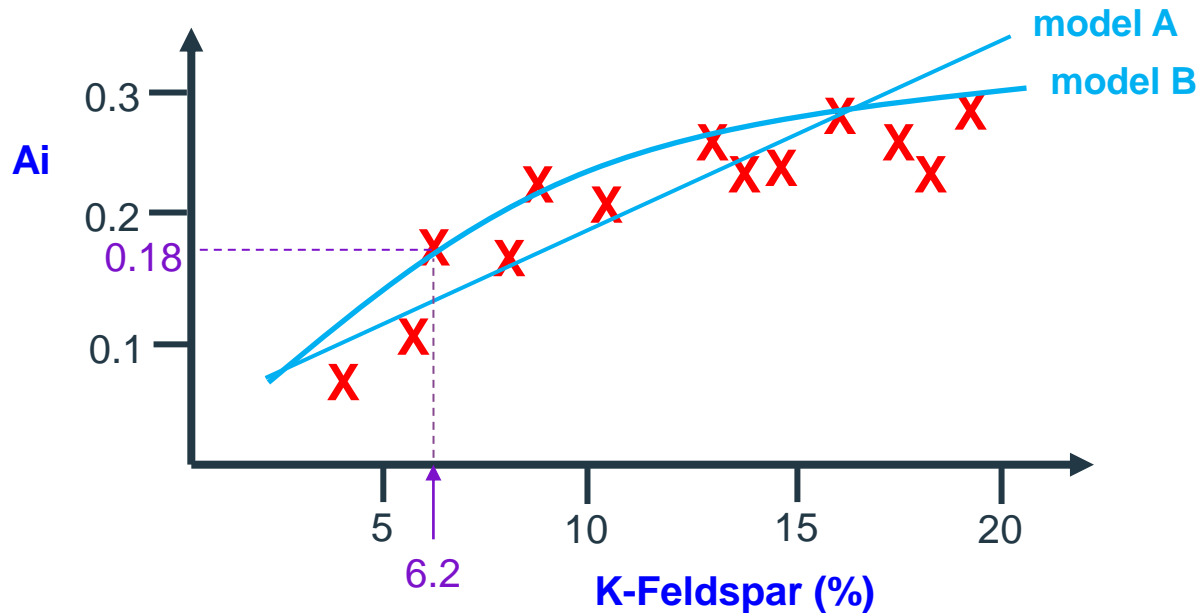


bias: difference between the model prediction and the actual value. Leads to high error on training and test data.

variance: variability of model prediction. Perform well on training data but has high error on test data.



How does a regression algorithm learn?



| predictor K-Feldspar (%) | target Ai |
|-----------------------------|--------------|
| 10.4 | 0.21 |
| 6.2 | 0.18 |
| 15.9 | 0.28 |
| ... | ... |

Regression: predict an infinite number of possible outputs. Model:

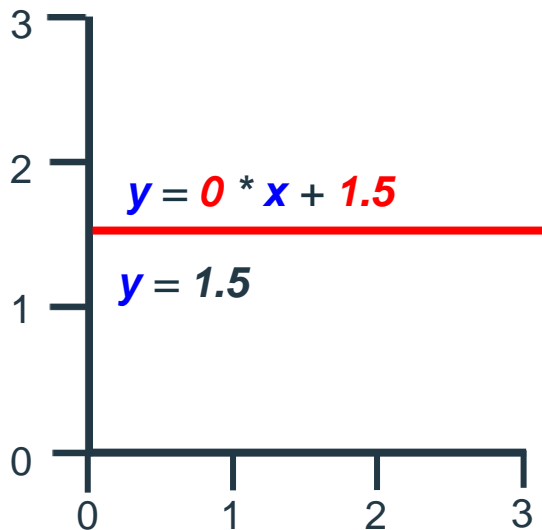
$$y = wx + b \rightarrow Ai = w * [K-Feldspar (%)] + b$$

Our goal is to find an algorithm that selects the most appropriate line/curve to fit the data. Which model is better, **model A** or **model B**? How can we choose the best?

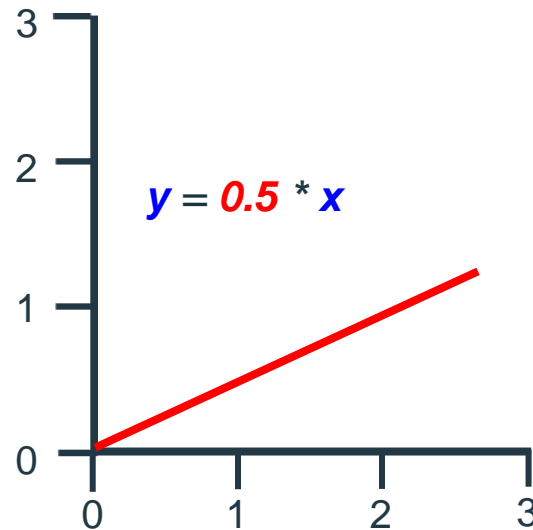
How does a regression algorithm learn?

Model: $y = wx + b$ | w, b = parameters (coefficients)

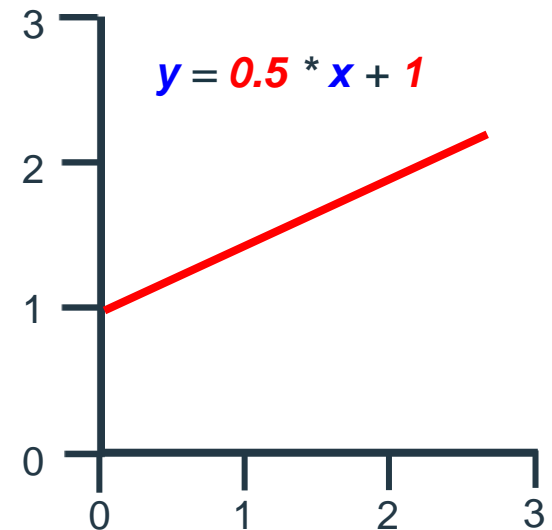
What w and b do?



$$w = 0$$
$$b = 1.5$$



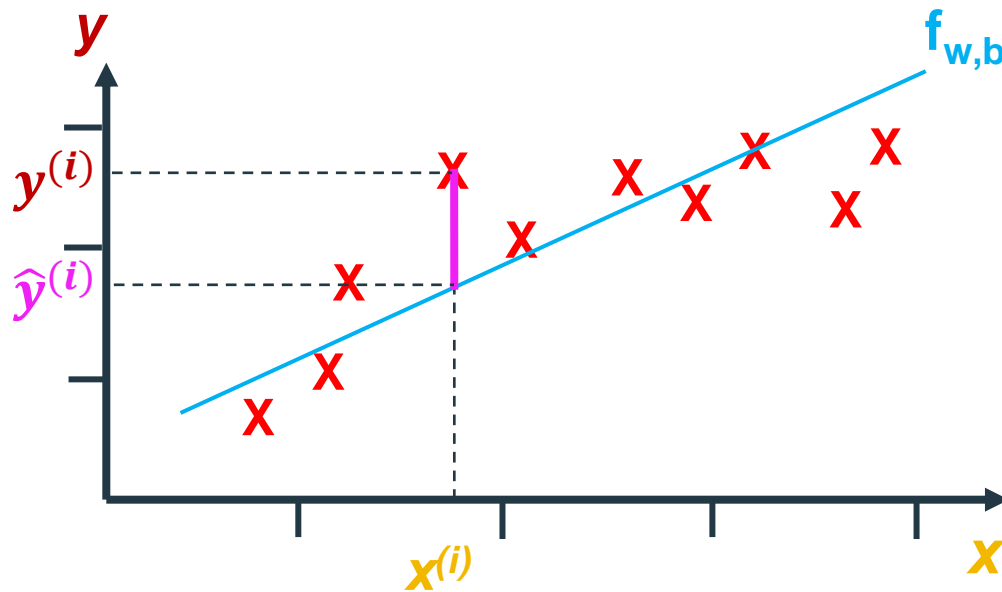
$$w = 0.5$$
$$b = 0$$



$$w = 0.5$$
$$b = 1$$

Modified from Andrew Ng, "machine learning specialization" (Coursera)

How does a regression algorithm learn?



prediction:

$$\hat{y}^{(i)} = f_{w,b}(x^{(i)})$$

cost function:

$$J(w,b) = \frac{1}{2m} \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)})^2$$

Find w, b :

$\hat{y}^{(i)}$ is close to $y^{(i)}$ for all $(x^{(i)}, y^{(i)})$

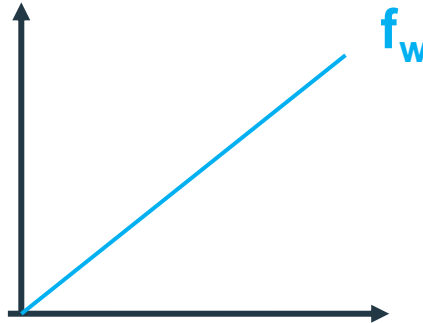
How does a regression algorithm learn?

Simplified Model:

$$f_{w,b}(x) = wx + b$$

if $b = 0$,

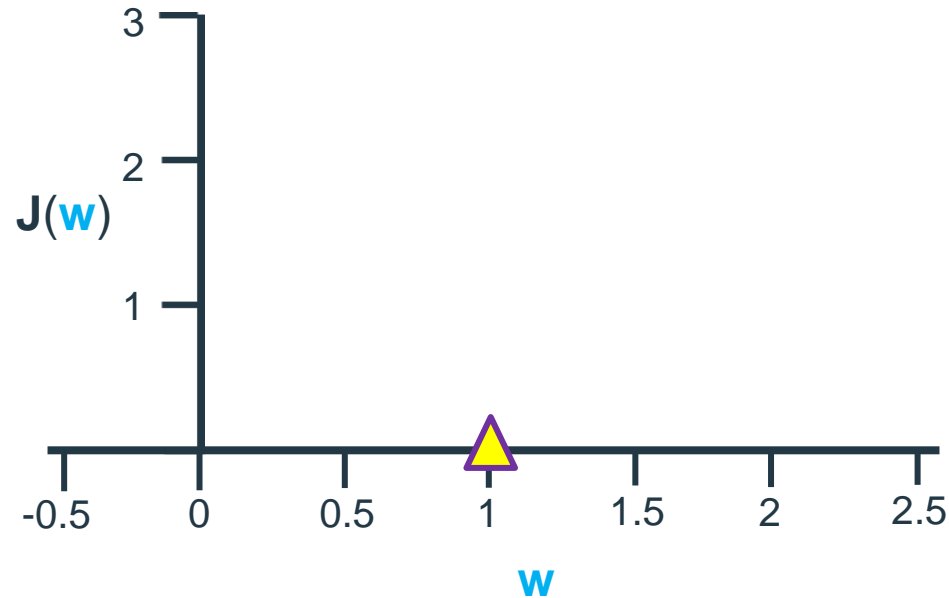
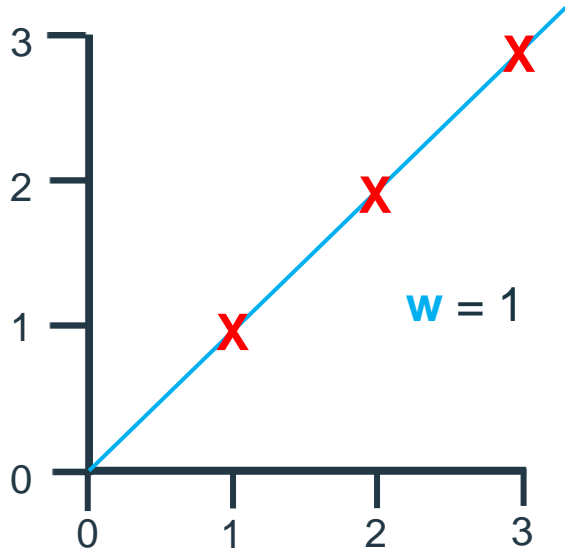
$$f_w(x) = wx$$



cost function:
$$J(w) = \frac{1}{2m} \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)})^2$$

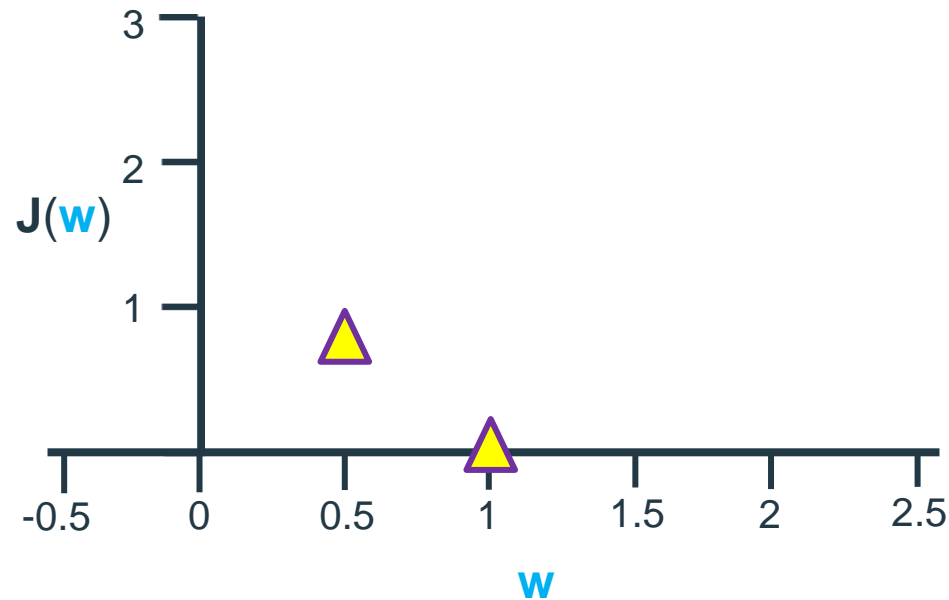
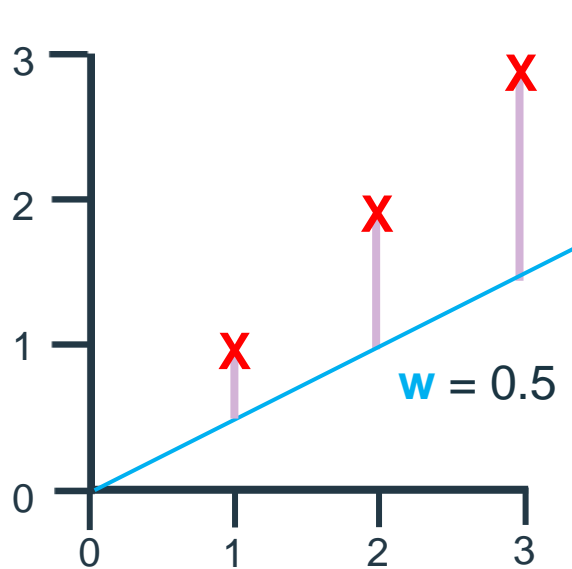
goal: minimize $J(w)$

How does a regression algorithm learn?



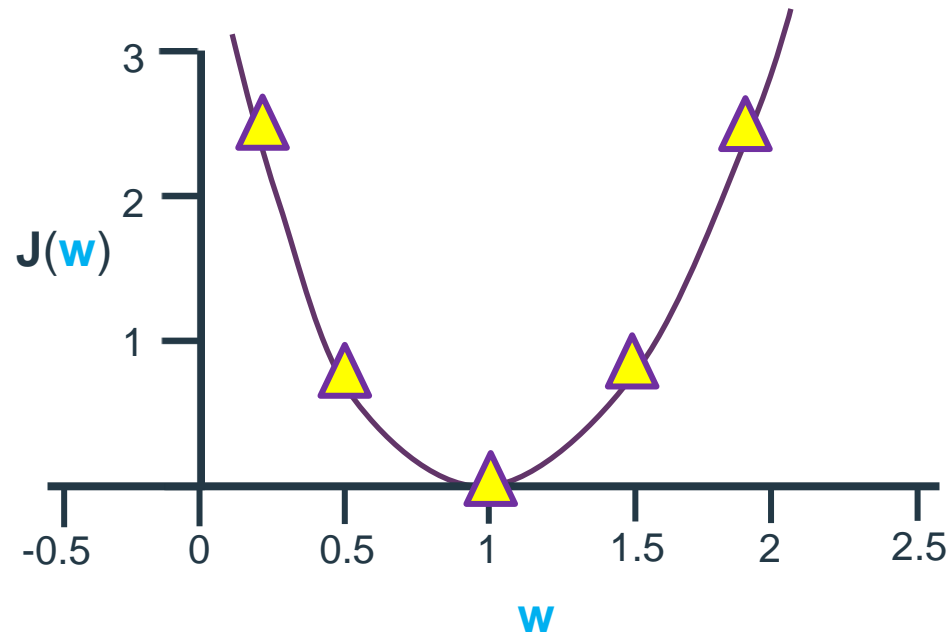
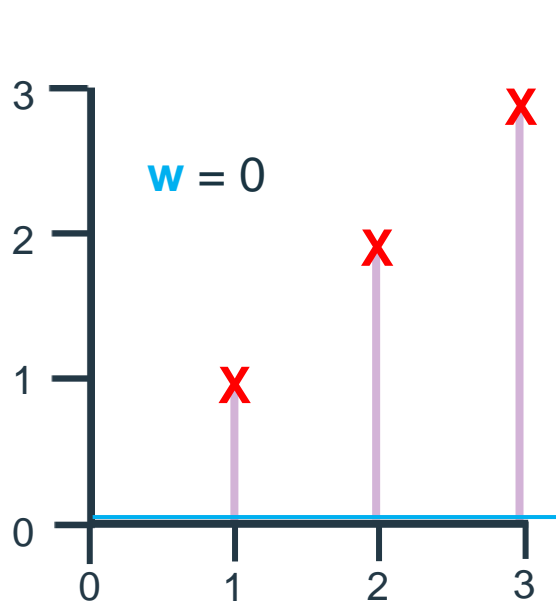
$$J(w = 1) = \frac{1}{2m} \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)})^2 = \frac{1}{2m} (0^2 + 0^2 + 0^2) = 0$$

How does a regression algorithm learn?



$$J(w = 0.5) = \frac{1}{2m} \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)})^2 = \frac{1}{2m} [(0.5-1)^2 + (2-1)^2 + (1.5-3)^2] = \frac{1}{2 \cdot 3} [3.5] = 0.58$$

How does a regression algorithm learn?



$$J(w = 0) = \frac{1}{2m} \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)})^2 = \frac{1}{2m} (1^2 + 2^2 + 3^2) = \frac{1}{2 \cdot 3} [14] = 2.3$$

goal: minimize $J(w)$

How does a regression algorithm learn?

Any point represents a particular choice of w and b . The high in that point is the value of $J(w,b)$.

(GPT-3 parameters)

