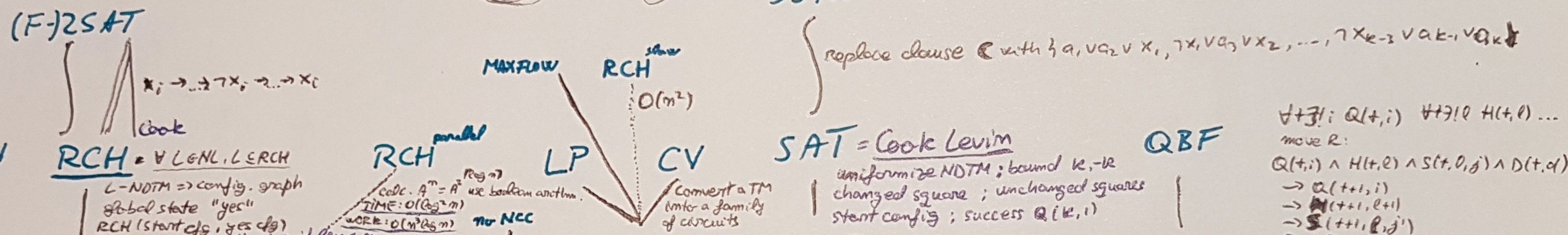
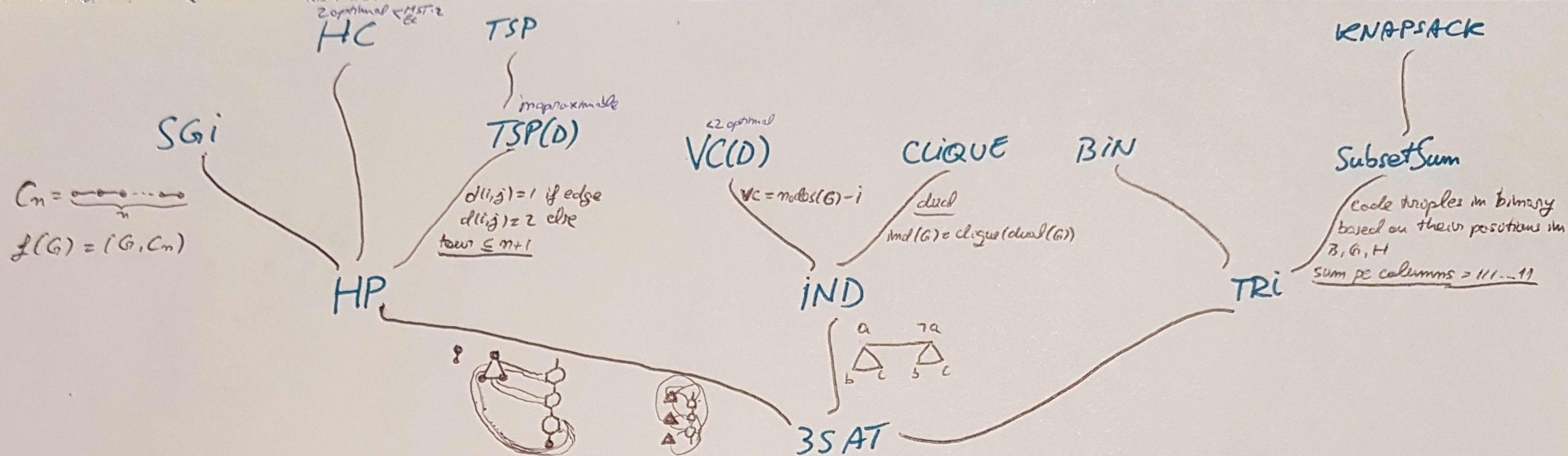


Complete problems



$$NC_1 \subseteq L \subseteq NL \subseteq NC_2 \dots NC \subseteq P \subseteq NP \subseteq PSPACE = NPSpace$$

\parallel \parallel \parallel \parallel
 co-L co-NL co-P co-NP co-PSPACE

Szelepcenyi / Immerman $(N(x))$
 calc. $|S(k+1)|$ based on $|S(k)|$ and $RCH(x, z, k)$

Bottom
 calculate $C_n(x)$ of depth $O(\log n)$
 ← build without storing while evaluating

$NSPACE(f(n)) \subseteq TIME(k \log(n) + f(n))$ if $f(n) \geq \log n$
 $M - k$ -tape NDTM operates in space $f(n)$
 States: Q - constant
 Input tape: n positions for head
Work tapes: $|\Sigma|$ symbols, $f(n)$ squares, $f(n)$ pos for head
 Output tapes: 2 (yes/no) - constant
 total: $Q \cdot n \cdot (|\Sigma|^{f(n)} \cdot f(n))^{\frac{k-2}{2}} \in O^{log(n) + f(n)}$
RCH - from initial configuration, to a yes configuration
 time $O(n^2) \Rightarrow O(c^{(\log(n) + f(n))^2}) = O(c^{log(n) + f(n)})$
config graph (like above) has size $O(n^k)$, $f(n) = \log n$

$$\underline{RCH} \in NC_2 \Rightarrow NL \subseteq NC_2$$

NDTM depth $f(n)$, degree D

f -proper \Rightarrow can generate $f(n)$ zeros in space $O(f(n))$

Count up to $D^{f(n)} \Rightarrow f(n)$ memory used
 $\Rightarrow O(f(n))$ space

Savitch: RCH is space $O(\log^2 n)$

$R(x, y, 0) \text{ iff } x = y \vee (x, y) \in G$

$R(x, y, i+1) \text{ iff } \exists z R(x, z, i) \wedge R(z, y, i)$

use stack $\leq \log n$

Corollary:

$NSPACE(f(n)) \subseteq SPACE(f^2(n))$

if $f(n) \geq \log(n)$