Business Analytics: Exercises

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Part I: Linear Programming

Exercise 1. Advertising (Cornuejols and Trick) *

MegaMarketing is planning a concentrated one week advertising campaign for their new CutsEverything SuperKnife. The ads have been designed and produced and now they wish to determine how much money to spend in each advertising outlet. In reality, they have hundreds of possible outlets to choose from. We will illustrate their problem with two outlets: Prime-time TV, and newsmagazines. The problem of optimally spending advertising dollars can be formulated in many ways. For instance, given a fixed budget, the goal might be to maximize the number of target customers reached (a target customer is a customer with a reasonable chance of purchasing the product). An alternative approach, which we adopt here, is to define targets for reaching each market segment and to minimize the money spent to reach those targets. For this product, the target segments are Teenage Boys, Affluent Women (ages 40-49), and Retired Men. Each minute of primetime TV and page of newsmagazine advertisement reaches the following number of people (in millions):

Outlet	Boys	Women	Men	Cost
TV	5	1	3	600
Mag	2	6	3	500
Target	24	18	24	

Again, MegaMarketing is interested in straightforward answers like how many units of each outlet to purchase to meet the segment goals. They are also interested in such questions as ``How much will it cost to reach an extra million retired men?'', ``One radio spot reaches 1 million boys, 1 million women, and 1 million men: how much are we willing to pay for such a spot?'', and similar questions.

Model and solve the problem as a linear program.

Exercise 2. Manufacturing (Fox)

A company wants to can two new different drinks for the holiday season. It takes 2 hours to can one gross of Drink A, and it takes 1 hour to label the cans. It takes 3 hours to can one gross of Drink B, and it takes 4 hours to label the cans. The company makes $10 profit on one gross of Drink A and a $20 profit on one gross of Drink B. Given that we have 20 hours to devote for canning the drinks and 15 hours to devote for labeling cans per week, how many cans of each type of drink should the company package to maximize profits?

Exercise 3. Financial planning (Cornuejols and Trick) *

In your finance courses, you will learn a number of techniques for creating « optimal » portfolios. The optimality of a portfolio depends heavily on the model used for defining risk and other aspects of financial instruments. Here is a particularly simple model that is amenable to linear programming techniques. Consider a mortgage team with $100,000,000 to finance various investments. There are five categories of loans, each with an associated return and risk (1-10, 1 best) :

Loan/investment	Return (%)	Risk
First Mortgages	9	3
Second Mortgages	12	6
Personal Loans	15	8
Commercial Loans	8	2
Government Securities	6	1

The goal for the mortgage team is to allocate the money to the categories so as to: (a) Maximize the average return per dollar (b) Have an average risk of no more than 5 (all averages and fractions taken over the invested money (not over the saving account)). (c) Invest at least 20% in commercial loans (d) The amount in second mortgages and personal loans combined should be no higher than the amount in first mortgages.

Exercise 4. Investing over Time (Cornuejols and Trick) *

We are going to manage an investment portfolio over a 6-year time horizon. We begin with $1000, and at various times we can invest in one or more of the following:

Savings account X, annual yield 5%.
Security Y, 2-year maturity, total yield 12% if bought now, 11% thereafter.
Security Z, 3-year maturity, total yield 18%.
Security W, 4-year maturity, total yield 24%.

To keep things simple we will assume that each security can be bought in any denomination. (This assumption can be relaxed if one uses integer or dynamic programming .) We can make savings deposits or withdrawals anytime. We can buy Security Y any year but year 3. We can buy Security Z anytime after the first year. Security W, now available, is a one-time opportunity. We let xt be the amount of money invested in the savings account X at the beginning of year t, and similarly for yt, zt, and wt. We will put any money not tied up in securities into the savings account. The situation is summed up in the figure.

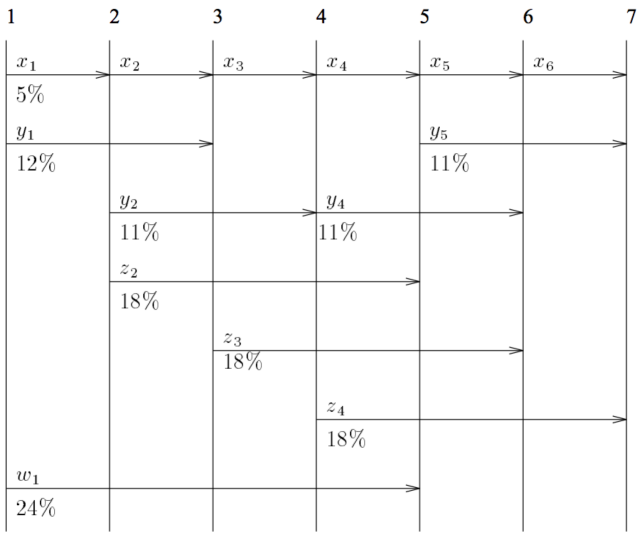


Figure 5.6: Possible investments over a 6-year horizon

The problem is really a kind of inventory problem. In a given year, the amount of money carried forward from the previous year (in savings), plus the yield from securities that mature that year, equals the amount invested in new securities plus the amount of money left over for next year. There is one twist: inventory grows while in storage. (In real inventory problems, stock often decreases over time due to spoilage, etc., and this can be reflected in negative « interest » rates.) Let v be the final yield when all securities are cashed in at the end of the sixth year. The objective is to maximize the final yield. Model and solve as an LP.

Exercise 5. Production planning (Cornuejols and Trick)

A plant produces two types of refrigerators, A and B. There are two production lines,
one dedicated to producing refrigerators of Type A, the other to producing refrigerators of Type B. The capacity of the production line for A is 60 units per day, the capacity of the production line for B is 50 units per day. A requires 20 minutes of labor whereas B requires 40 minutes of labor. Presently, there is a maximum of 40 hours of labor per day which can be assigned to either production line. Profit contributions are $20 per refrigerator of Type A produced and $30 per Type B produced. What should the daily production be?

Exercise 6. McCow Butchers (Fox)

The McCow Butchers is a large-scale distributor of dressed meats for Myrtle Beach restaurants and hotels. Ryan's steak house orders meat for meatloaf (mixed ground beef, pork, and veal) for 1000 lb according to the following specifications:

1. Ground beef must be not less than 400 lb and not more than 600 lb.
2. The ground pork must be between 200 and 300 lb.
3. The ground veal must weigh between 100 and 400 lb.
4. The weight of the ground pork must be not more than one and one half (3/2) times the weight of the veal.

The contract calls for Ryan's to pay $1200 for the meat. The cost per pound for the meat is $0.70 for hamburger,

$0.60 for pork, and $0.80 for the veal. How can this be modeled?

Exercise 7. Bankruptcy (Cornuejols and Trick)

Albert, Bill, Charles, David, and Edward have gotten into a bind. After a series of financial transactions, they have ended up each owing some of the others huge amounts of money. In fact, near as the lawyers can make out, the debts are as follows

Debtor	Creditor	Amount ($millions)
A	E	10
A	C	3
B	A	5
C	B	6
C	D	4
D	A	4
E	C	7
E	D	3

The question is, who is bankrupt? We will say that a person i is bankrupt if there is no possible transfer of funds among the people such that i completely pays off his obligations, and the transfer of funds satisfies the following condition: for every two persons j and k, the amount paid by person j to person k is no greater than the debt of j to k. For instance, Albert is bankrupt since he owes 13, and is only owed 9. Formulate the problem of determining whether Bill is bankrupt as a linear program. Then modify your formulation to determine if each of the others is bankrupt. *This example may look contrived, but it is inspired by a solution to the debts involved in a crash of Kuwait's al-Mankh stock market.*

Exercise 8. Blending (cornuejols and trick) * (just mentioned)

An oil company blends gasoline from three ingredients: butane, heavy naphta and catalytic reformate. The characteristics of the ingredients as well as minimum requirements for regular gasoline are given below:

	Butane	Catalytic Reformate	Heavy Naphta	Gasoline
Octane	120	100	74	≥ 89
Vapor Pressure	60	2.5	4.0	≤ 11
Volatility	105	3	12	≥ 17

The cost (per gallon) of butane is $0.58, it is $1.55 for catalytic reformate and $0.85 for heavy naphta. How many gallons of the three ingredients should be blended in order to produce 12,000 gallons of gasoline at minimum cost?

Exercise 9. Production and Distribution (Wagner) *

A manufacturing firm produces widgets and distributes them to five wholesalers at a fixed delivered price of $2.50 per unit. Sales forecasts indicate that monthly deliveries will be 2700, 2700, 9000, 4500 and 3600 widgets to wholesalers 1-5 respectively. The monthly production capacities are 4500, 9000 and 11,250 at plants 1, 2 and 3, respectively. The direct costs of producing each widget are $2 at plant 1, $1 at plant 2 and $1.80 at plant 3. The transport cost of shipping a widget from a plant to a wholesaler is given below.

Wholesaler	1	2	3	4	5
Plant 1	.05	.07	.11	.15	.16
Plant 2	.08	.06	.10	.12	.15
Plant 3	.10	.09	.09	.10	.16

Formulate an LP model for this production and distribution problem.

Exercise 10. Sensisitity analysis (Cornuejols and trick) *

Tucker Inc. needs to produce 1000 Tucker automobiles. The company has four production plants. Due to differing workforces, technological advances, and so on, the plants differ in the cost of producing each car. They also use a different amount of labor and raw material at each. This is summarized in the following table:

Plant	Cost ('000)	Labor	Material
1	15	2	3
2	10	3	4
3	9	4	5
4	7	5	6

The labor contract signed requires at least 400 cars to be produced at plant 3; there are 3300 hours of labor and 4000 units of material that can be allocated to the four plants. Model as an LP and solve using Excel solver.

Using the sensitivity report analysis, answer the following questions:
1. What are the current production quantities? What is the current cost of production?
2. How much will it cost to produce one more vehicle? How much will we save by producing one less?
3. How would our solution change if it cost only $8,000 to produce at plant 2? For what ranges of costs is our solution (except for the ob jective value) valid for plant 2?
4. How much are we willing to pay for a labor hour?
5. How much is our union contract costing us? What would be the value of reducing the 400 car limit down to 200 cars? To 0 cars? What would be the cost of increasing it by 100 cars? by 200 cars?
6. How much is our raw material worth (to get one more unit)? How many units are we willing to buy at that price? What will happen if we want more?
7. A new plant is being designed that will use only one unit of workers and 4 units of raw material. What is the maximum cost it can have in order for us to consider using it?
8. By how much can the costs at plant 1 increase before we would not produce there?

Exercise 11. Workforce planning (Cornuejols and Trick)

Due to an unexpected glut of orders, Blaster Steel has decided to hire temporary workers for a five day period to clear out the orders. Each temporary worker can work either a two day shift or a three day shift for this period (shifts must be consecutive days). At least 10 workers are needed on days 1, 3, 5, and at least 15 workers are needed on days 2 and 4. A worker on a two day shift gets paid $125/day, while those on a three day shift gets paid $100/day. (a) Formulate the problem of hiring temporary workers to minimize cost while meeting the demand for workers. (b) Due to a limited number of training personnel, no more than 10 workers can start their shift on any day. Update your formulation in (a) to take this into account. (c) Union regulations require that at least half of all money spent on workers go to those who work three day shifts. Update your formulation in (a) to handle this requirement. (d) There are four people who are willing to work a shift consisting of days 1, 2, and 5, for a payment of $110/day. Update your formulation in (a) to handle this possibility.

Exercise 12. Workforce planning (Cornuejols and Trick) *

Consider a restaurant that is open seven days a week. Based on past experience, the number of workers needed on a particular day is given as follows :

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Number	14	13	15	16	19	18	11

Every worker works five consecutive days, and then takes two days off, repeating this pattern indefinitely. How can we minimize the number of workers that staff the restaurant?

Exercise 13. Sensitivity analysis on workforce planning (Cornuejols and Trick) *

Answer each of the following questions independently of the others.
1. What is the current total number of workers needed to staff the restaurant?
2. Due to a special offer, demand on thurdays increases. As a result, 18 workers are needed instead of 16. What is the effect on the total number of workers needed to staff the restaurant?
3. Assume that demand on mondays decreases: 11 workers are needed instead of 14. What is the effect on the total number of workers needed to staff the restaurant?
4. Currently, 15 workers are needed on wednesdays. In what range can this number vary without changing the optimal basis?
5. Currently, every worker in the restaurant is paid $1000 per month. So the objective function in the formulation can be viewed as total wage expenses (in thousand dollars). Workers have complained that Shift 4 is the least desirable shift. Management is considering increasing the wages of workers on Shift 4 to $1100. Would this change the optimal solution? What would be the effect on total wage expenses?
6. Shift 1, on the other hand, is very desirable (sundays off while on duty fridays and saturdays, which are the best days for tips). Management is considering reducing the wages of workers on Shift 1 to $ 900 per month. Would this change the optimal solution? What would be the effect on total wage expenses?
7. Management is considering introducing a new shift with the days off on tuesdays and sundays. Because these days are not consecutive, the wages will be $ 1200 per month. Will this increase or reduce the total wage expenses?

Exercise 14. Financial Planning (fox)

A bank makes four kinds of loans to its personal customers, and these loans yield the following annual interest rates to the bank:

1. First mortgage—14%

2. Second mortgage—20%

3. Home improvement—20%

4. Personal overdraft—10%

The bank has a maximum foreseeable lending capability of $250 million and is further constrained by the policies:

1. First, mortgages must be at least 55% of all mortgages issued and at least 25% of all loans issued (in $ terms)

2. Second, mortgages cannot exceed 25% of all loans issued (in $ terms)

3. To avoid public displeasure and the introduction of a new windfall tax, the average interest rate on all loans must not exceed 15%

Formulate the bank's loan problem as an LP so as to maximize interest income while satisfying the policy limitations.

Note here that these policy conditions, while potentially the profit that the bank can make, also limit its exposure to risk in a particular area. It is a fundamental principle of risk reduction that risk is reduced by spreading money (appropriately) across different areas.

Exercise 15. Production Planning Problem (fox)

A company manufactures four variants of the same table and in the final part of the manufacturing process, there are assembly, polishing, and packing operations. For each variant, the time required for these operations is shown in the following table (in minutes) as is the profit per unit sold.

	Assembly	Polish	Pack	Profit ($)
Variant 1	2	3	2	1.50
2	4	2	3	2.50
3	3	3	2	3.00
4	7	4	5	4.50

Given the current state of the labor force, the company estimates that, each year, they have 100,000 minutes of assembly time, 50,000 minutes of polishing time, and 60,000 minutes of packing time available. How many of each variant should the company make per year and what is the associated profit?

Exercise 16. Production planning (fox)

In order to produce 1000 tons of nonoxidizing steel for BMW engine valves, at least the following units of manganese, chromium, and molybdenum will be needed weekly: 10 units of manganese, 12 units of chromium, and 14 units of molybdenum (1 unit is 10 lb). These materials are obtained from a dealer who markets these metals in three sizes: small (S), medium (M), and large (L). One S case costs $9 and contains two units of manganese, two units of chromium, and one unit of molybdenum. One M case costs $12 and contains two units of manganese, three units of chromium, and one unit of molybdenum. One L case costs $15 and contains one unit of manganese, one unit of chromium, and ve units of molybdenum. How many cases of each kind (S, M, L) should be purchased weekly so that we have enough manganese, chromium, and molybdenum at the smallest cost?

Exercise 17. Sales (cornuejols and trick)

Sales forecasts for the next four months are (in thousand of units)

October	10
November	16
December	10
January	12

September's production was set at 12,000 units. Varying production rate incurs some cost: production can be increased from one month to the next at a cost of $2 per unit and decreased at a cost of $0.50 per unit. In addition, inventory left at the end of a month can be stored at a cost of $1 per unit per month. Given current demand, there will be no inventory at the end of September. No inventory is desired at the end of January. Formulate a linear program that minimizes the total cost (varying production rate + inventory costs) of meeting the above demand.

Exercise 18. Investment (Cornuejols and Trick)

You have $1000 to invest in Securities 1 and 2; uninvested funds are deposited in a savings account with annual yield of 5%. You will sell the securities when they reach maturity. Security 1 matures after 2 years with a total yield of 12%. Security 2 matures after 3 years with a total yield of 19%. Your planning horizon is 7 years, and you want to maximize total assets at the end of the seventh year. Suppose either security may be purchased in arbitrarily small denominations. a) Let xit be the amount invested in Security i at the beginning of year t; the savings account can be considered Security 0. Write the appropriate LP model. b) Solve the problem by computer and indicate your optimal portfolio in each year 1,...,7.

Exercise 19. Capacity planing (Cornuejols and Trick)

An electric utility has six power plants on the drawing board. The anticipated useful life of these plants is 30 years for the coal-fired plants (plants 1, 2 and 3) and 40 years for the fuel-fired plants (plants 4, 5 and 6). Plants 1, 2 and 4 will be on line in year 5. Plants 3 and 5 in year 15. Plant 6 in year 25. The cost of installing generating capacity at Plant i, discounted to the present, is ci per megawatt, for i = 1,2,…,6. Projected power demand in year t is Dt megawatts, for t = 10; 20; 30; 40: Due to environmental regulations, the fraction of generating capacity at coal-fired plants in year t, relative to total generating capacity in year t, can be at most rt for t = 10; 20; 30; 40. The generating capacity of each plant has yet to be determined. Write a linear program that assigns capacities to the six plants so as to minimize total present cost of installing generating capacity, while meeting demand and satisfying the environmemtal constraints in years t = 10; 20; 30; 40:

Exercise 20. Routing (Wagner)

An air cargo firm has 8 aircraft of type 1, 15 of type 2 and 11 of type 3 available for today's flights. A type 1 craft can carry 45 tons, type 2, 7 tons and type 3, 5 tons. 20 tons of cargo are to be flown to city A and 28 tons to city B. Each plane makes at most one flight a day. The costs of flying a plane from the terminal to each city are as follows.

	Type 1	Type 2	Type 3
City A	23	15	1.4
City B	58	20	3.8

Let xi be the number of type i planes sent to A and yi the number to B. Formulate an LP for this routing problem.

Exercise 21. Distribution (Fox)

There are three warehouses at different cities: Detroit, Pittsburgh, and Buffalo. They have 250, 130, and 235 tons of paper accordingly. There are four publishers in Boston, New York, Chicago, and Indianapolis. They ordered 75, 230, 240, and 70 tons of paper to publish new books. Transportation costs, in dollars, of one ton of paper are listed in the following table:

From\To	Boston (BS)	New York (NY)	Chicago (CH)	Indianapolis (IN)
Detroit (DT)	15	20	16	21
Pittsburgh (PT)	25	13	5	11
Buffalo (BF)	15	15	7	17

Management wants you to minimize the shipping costs while meeting demand. This problem involves the allocation of resources. Model as a linear program and solve.

Exercise 22. Super Bowl Advertising (Fox)

The Super Bowl advertising agency wishes to plan an advertising campaign in three different media: television, radio, and magazines.

The purpose or goal is to reach as many potential customers as possible. Results of a marketing study are given in the following:

	Day Time TV	Prime Time TV	Radio	Magazines
Cost of advertising unit	$40,000	$75,000	$30,000	$15,000
Number of potential customers reached per unit	400,000	900,000	500,000	200,000
Number of woman customers reached per unit	300,000	400,000	200,000	100,000

The company does not want to spend more than $800,000 on advertising. It further requires (1) at least 2 million exposures to take place among woman, (2) TV advertising to be limited to $500,000, (3) at least three advertising units to be bought on day time TV and two units on prime time TV, and (4) the number of radio and magazine advertisement units should each be between 5 and 10 units.

Exercise 23. Portfolio manager

A portfolio manager who is in charge of a bank wants to invest $10 million. The securities available for purchase, as well as their respective quality ratings, maturate, and yields, are shown in the following table:

Bond Name	Bond Type	Moody's Quality Scale	Bank's Quality Scale	Years to Maturity	Yield at Maturity (%)	After-Tax Yield (%)
A	Municipal	Aa	2	9	4.3	4.3
B	Agency	Aa	2	15	5.4	2.7
C	Govt 1	Aaa	1	4	5	2.5
D	Govt 2	Aaa	1	3	4.4	2.2
E	Local	Ba	5	2	4.5	4.5

The bank places certain policy limitations on the portfolio manager's actions:

1. Government and agency bonds must total at least $4 million.

2. The average quality of the portfolios cannot exceed 1.4 on the bank's quality scale. Note that a low number means high quality.

3. The average years to maturity must not exceed 5 years.

Assume that the objective is to maximize after-tax earnings on the investment.

Exercise 24. MarketQuest, Inc.

MarketQuest, Inc specializes in evaluating consumer's reaction to new products and services. A company, introducing new type of washing powder, asked the MarketQuest, Inc. to prepare a campaign with door-to-door personal interviews about households' opinion. Households both with children and without children should be interviewed; both daytime and evening interviews should be conducted. There is a plan to conduct 1000 interviews with the following restrictions:

- ✓ At least 300 households with children should be interviewed.
- ✓ At least 400 households without children should be interviewed.
- ✓ Number of evening interviews ≥ number of daytime interviews
- ✓ At least 35% of the interviews for households with children should be conducted during evening
- ✓ At least 65% of the interviews for households without children should be conducted during evening

It costs 5 (resp. 4) euros to interview a household with (resp. without) children during day time, and 6.25 (resp. 5) euros for a evening interview. Prepare a campaign with door-to-door personal interviews that satisfies the constraints above and minimizes total cost.

Part I (extension): Integer Programming

Exercise 25. Workforce scheduling (cornuejols and Trick)

Gotham City National Bank is open Monday-Friday from 9 am to 5 pm. From past experience, the bank knows that it needs the following number of tellers.

Time Period	Tellers Required
9 – 10	4
10 – 11	3
11 – noon	4
noon – 1	6
1 – 2	5
2 – 3	6
3 – 4	8
4 – 5	8

The bank hires two types of tellers. Full-time tellers work 9-5 five days a week, except for 1 hour off for lunch, either between noon and 1 pm or between 1 pm and 2 pm. Full-time tellers are paid (including fringe benefits) $25/hour (this includes payment for lunch hour). The bank may also hire up to 3 part-time tellers. Each part-time teller must work exactly 4 consecutive hours each day. A part-time teller is paid $20/hour and receives no fringe benefits. Formulate a linear program to meet the teller requirements at minimum cost.

Exercise 26. Facility location (Fox)

The Emergency Service Coordinator (ESC) for a county is interested in locating the county's three ambulances to maximize the residents that can be reached within 8 minutes in emergency situations. The county is divided into six zones, and the average time required to travel from one region to the next under semiperfect conditions is summarized in Table 1.1.

The population in zones 1, 2, 3, 4, 5, and 6 is given in Table 1.2.

We want better coverage to improve the ability to take care of patients who require an ambulance to go to a hospital. Determine the location for placement of the ambulances to maximize coverage within the pre-determined allotted time.

Assumptions: We initially assume that time travel between zones is negligible. We further assume that the times in the data are averages under ideal circumstances.

TABLE 1.1

Average Travel Times from Zone *i* to Zone *j* in Perfect Conditions

	1	2	3	4	5	6
1	1	8	12	14	10	16
2	8	1	6	18	16	16
3	12	18	1.5	12	6	4
4	16	14	4	1	16	12
5	18	16	10	4	2	2
6	16	18	4	12	2	2

TABLE 1.2

Populations in Each Zone

1	50,000
2	80,000
3	30,000
4	55,000
5	35,000
6	20,000
Total	270,000

Exercise 27. Knapsack (Fox)

Your company is considering for investments. Investment 1 yields a net present value (NPV) of $17,000; investment 2 yields a NPV of $23,000; investment 3 yields a NPV of $13,000; and investment 4 yields a NPV of $9,000. Each investment requires a current cash ow of investment 1, $6000; investment 2, $8000; investment 3, $5000; and investment 4, $4000. At present, $21,000 is available for investment. Formulate and solve as an integer programming problem assuming that you can only invest at most one time in each investment.

Part I (application): Data Envelopment Analysis

Exercise 28. Bank (Cornuejols and Trick)

To illustrate how DEA works, let's take an example of three banks. Each bank has exactly 10 tellers (the only input), and we measure a bank based on two outputs: Checks cashed and Loan applications. The data for these banks is as follows:

Bank A: 10 tellers, 1000 checks, 20 loan applications
Bank B: 10 tellers, 400 checks, 50 loan applications
Bank C: 10 tellers, 200 checks, 150 loan applications

Which bank is efficient ? What is the efficiency rate of the non efficient ones ?

Exercise 29. Hospital (Fox)

Given the following input–output table for three hospitals where inputs are number of beds and labor hours in thousands per month and outputs, all measured in hundreds, are patient days for patients under 14, patient days for patients between 14 and 65, and patient days for patients more than 65.

Determine the efficiency of the three hospitals.

	Inputs		Outputs		
Hospital	1	2	1	2	3
1	5	14	9	4	16
2	8	15	5	7	10
3	7	12	4	9	13

Exercise 30. Banks branches (Fox)

Consider ranking four bank branches in a particular city. The inputs are as follows:

- Input 1 = labor hours in hundreds per month
- Input 2 = space used for tellers in hundreds of square feet
- Input 3 = supplies used in dollars per month
- Output 1 = loan applications per month
- Output 2 = deposits made in thousands of dollars per month
- Output 3 = checks that processed thousands of dollars per month

The following data table is for the bank branches:

Branches	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3
1	15	20	50	200	15	35
2	14	23	51	220	18	45
3	16	19	51	210	17	20
4	13	18	49	199	21	35

What *best practices* might you suggest to the branches that are less efficient ?

Exercise 31. Recruiting brigage (Fox)

We want to measure the efficiency of 42 recruiting companies that are part of a recruiting brigade in the United States. The model uses six input measures and two output measures created from data that are obtained directly from the sixth brigade in 2014. The outputs are the percent fill-to-demand ratio for the unit and the percent language capability of the unit. The inputs are the number of recruiters and the percent of populations from which to recruit in a region. The main question was to determine if a larger percentage of recruiters' ability to speak languages other than English improved their units' ability to attract recruits. The goal is to identify those units that are not operating at the highest level so that improvement can be made to improve their ef ciency. The data envelopment will calculate which of the companies, in this case DMU_1 , DMU_2 , ... , DMU_{42}, are more efficient when compared to the others.

Input and Output Coefficients for the DEA Approach (Array Named Matrix A)

6th REC BDE	Input Coefficients						Output Coefficients	
Company	%PopAPI	%PopAA	%PopH	%PopW	%PopNative	#Recruiters	Fill-to-Demand Ratio	Language-to-Recruiter Ratio
6F2—San Gabri El Vl	0.263722	0.031573	0.517156	0.185787	0.001763	38	0.872152	0.2105
6F3—Long Beach	0.087205	0.105755	0.672485	0.133068	0.001486	41	0.959648	0.2683
6F5—Sn Fernando Vl	0.080011	0.035218	0.533119	0.350004	0.001648	41	0.763623	0.0976
6F7—Coastal	0.157075	0.199102	0.295339	0.347086	0.001397	27	0.840470	0.2963
6f8—Los Angeles	0.136177	0.055508	0.546700	0.260140	0.001474	51	0.896214	0.3000
6h1—Eugene	0.036067	0.007294	0.093672	0.846818	0.016149	28	0.798541	0.0909
6h2—Vancouver	0.074656	0.038769	0.133544	0.745682	0.007349	49	0.888734	0.2083
6H3—Wi Lsonvi Lle	0.035723	0.010366	0.173345	0.769427	0.011140	25	0.763134	0.1000
6H5—Honolulu	0.600513	0.014254	0.122297	0.260485	0.002450	24	1.014687	0.0800
6H7—Guam	0.926119	0.010042	0.000000	0.063839	0.000000	24	1.016260	0.0000
6I 0—Si Erra Nevada	0.047907	0.019145	0.250423	0.664284	0.018241	29	0.910788	0.0000
6I 1—Reddi Ng	0.040783	0.010726	0.151065	0.772217	0.025210	42	0.808599	0.0870
6I 3—Sacramento Vl	0.081499	0.036714	0.216618	0.657542	0.007626	39	0.920515	0.0000
6I 4—San Joaqui N	0.113513	0.053919	0.445580	0.381530	0.005457	36	0.931590	0.0000
6I 5—Capi Tol	0.202628	0.107138	0.257041	0.427997	0.005197	52	0.888965	0.0000
6I 6—North Bay	0.086219	0.066036	0.319470	0.519875	0.008401	31	0.656557	0.0000
6J1—Ogden	0.018162	0.008748	0.129949	0.833434	0.009707	27	0.797701	0.1000
6J2—Salt Lake	0.043967	0.010922	0.149390	0.784530	0.011190	29	0.752606	0.0000
6J3—Butte	0.010439	0.002670	0.039882	0.885437	0.061573	25	0.714416	0.0000
6J4—Boi Se	0.017945	0.006657	0.165861	0.800868	0.008668	28	0.860633	0.0000
6J6—Las Vegas	0.102264	0.107663	0.309920	0.474534	0.005620	109	0.890533	0.2174

(*Continued*)

Input and Output Coefficients for the DEA Approach (Array Named Matrix A)

6th REC BDE	Input Coefficients						Output Coefficients	
Company	%PopAPI	%PopAA	%PopH	%PopW	%PopNative	#Recruiters	Fill-to-Demand Ratio	Language-to-Recruiter Ratio
6J9—Bi G Horn	0.006928	0.003742	0.067161	0.844465	0.077704	17	0.754591	0.0435
6K1—Redlands	0.046855	0.080681	0.566912	0.300264	0.005287	43	0.969034	0.1111
6K2—Fullerton	0.201510	0.019564	0.492608	0.284349	0.001969	40	0.857143	0.1364
6K4—La Mesa	0.117789	0.053590	0.506028	0.317863	0.004730	39	0.715259	0.0690
6K5—Newport Beach	0.139674	0.013223	0.270650	0.574473	0.001979	32	0.743743	0.0800
6K6—San Marcos	0.044696	0.032839	0.508044	0.408251	0.006170	50	0.876591	0.0000
6K7—Ri Versi De	0.086515	0.090924	0.563245	0.256525	0.002791	55	0.855292	0.1163
6K8—San Di Ego	0.173149	0.050862	0.249331	0.523374	0.003284	35	0.757979	0.3500
6L1—Everett	0.086824	0.018779	0.109900	0.76868	0.015817	25	0.837500	0.3846
6L2—Seattle	0.201467	0.073000	0.102603	0.616732	0.006198	32	0.766444	0.1364
6L3—Spokane	0.023348	0.012660	0.055685	0.891007	0.017300	23	0.796624	0.1200
6L4—Tacoma	0.092312	0.068854	0.121083	0.704523	0.013227	27	0.990457	0.1818
6L5—Yaki Ma	0.016830	0.008463	0.338310	0.614801	0.021596	22	0.775581	0.3143
6L6—Alaska	0.074101	0.028043	0.061478	0.655457	0.180921	24	0.855114	0.2000
6L7—Olympi A	0.048443	0.018887	0.094673	0.814933	0.023064	29	0.901454	0.1034
6N1—Fresno	0.081141	0.043657	0.577334	0.292158	0.00571	54	0.792119	0.0238
6N2—Bakersfi Eld	0.036745	0.072447	0.546160	0.338127	0.006521	40	0.836003	0.0769
6N6—Gold Coast	0.055439	0.015520	0.471056	0.454545	0.003440	34	0.721532	0.1111
6N7—South Bay	0.321473	0.038378	0.254535	0.383649	0.001965	37	0.638575	0.1220
6N8—East Bay	0.225293	0.125870	0.299244	0.346959	0.002633	55	0.692921	0.1290
6N9—Monterey Bay	0.21374	0.023038	0.451766	0.309182	0.002274	29	0.726957	0.0909

Part II: Decision Trees

Exercise 32. Life Insurance (fox) *

A term life insurance policy will pay a beneficiary a certain sum of money upon the death of the policy holder. These policies have premiums that must be paid annually. The company can sell either a $250,000 or a $300,000 policy. Suppose a life insurance company sells a $250,000 1-year-term life insurance policy to a 50-year-old female for $750. According to the National Vital Statistics Report (1997), Vol. 47, No. 28 (p. 74), the probability that the 50-year-old female will die is 2.88 per thousand or 0.00288; thus, the probability that the female will survive the year is $1 - 0.00288 = 0.99712$. Compute the expected value of this policy to the insurance company. If successful, should the company sell the $300,000 policy?

Exercise 33. Bonds or Stocks (Fox) *

We have a choice of two investment strategies: stocks and bonds. The returns for each under two possible economic conditions are as follows:

States of Nature	
$p_1 = 0.68$	$p_2 = 0.32$

Alternative	Condition 1	Condition 2
Stocks	$11,500	–$4,300
Bonds	$7,700	$1,400

Compute the expected value and select the best alternative.

What probabilities for conditions 1 and 2 would have to exist to be indifferent toward stocks and bonds?

Exercise 34. Concessions at Sporting Events (fox)

Consider a firm that handles concessions for a sporting event. The firm's manager needs to know whether to stock up with coffee or coke-cola products. A local agreement restricts you to only one beverage. You estimate a $1500 profit of selling coke-cola products if it is cold and a $5000 profit of selling cola if it is warm. You also estimate a $4000 profit of selling coffee if it is cold and a $1000 profit of selling coffee if it is warm. The forecast says that there is a 30% of a cold front, otherwise the weather will be warm. What do you do?

Exercise 35. Ski Resort Decisions (fox)

The financial success of a ski resort in Squaw Valley is dependent on the amount of early snow fall in the fall and winter months. If the snow fall is greater than 40 in., the resort always has a successful ski season. If the snow is between 30 and 40 in., the resort has a moderate season, and if the snow fall is less than 30 in., the season is poor and the resort will lose money. The seasonal snow probabilities from the weather service are displayed in the following table with the expected revenue that was historically calculated for the previous 10 seasons. A hotel chain has offered to lease the resort during the winter for $100,000. You must decide whether to operate yourself or lease the resort. What decision should you make?

	States of Nature		
	Snow > 40″ $p_1 = 0.40$	30″ < Snow < 40″ $p_2 = 0.20$	Snow < 30″ $p_3 = 0.40$
Financial return if we operate	$280,000	$100,000	−$40,000
Lease	$100,000	$100,000	$100,000

Exercise 36. A Paving Company (Fox) *

Consider a company that needs to decide whether or not to bid and build large or small parking lots. The three alternatives under consideration with the respective demand revenues and losses with the estimated demand probabilities are as follows:

	Outcomes		
	High Demand	Moderate Demand	Low Demand
Alternatives	(p_{hd} = 0.3)	(p_{md} = 0.5)	(p_{ld} = 0.2)
Large parking lot	$200,000	$120,000	−$125,000
Small parking lot	$100,000	$55,000	−$25,000
No parking lots	$0	$0	$0

Exercise 37. Strawberry (fox)

Assume the following probability distribution of daily demand for strawberries:

Daily demand	0	1	2	3	4
Probability	0.2	0.3	0.25	0.2	0.05

Also assume that unit cost = $2.50, selling price = $4.99 (i.e., profit on sold unit = $2.49), and salvage value on unsold units = $2 (i.e., loss on unsold unit = $1). We can stock either 0, 1, 2, 3, or 4 units. How many units should be stocked each day? Assume that units from one day cannot be sold the next day.

Exercise 38. A Paving Company revisited (Fox) *

Let us assume that before making a decision, the company has an option to hire a market research company for $4000. This company will survey the markets that are serviced by this company as to the attractiveness of the new outdoor play sets. The company knows that the market research does not provide perfect information but does provide updated information based on their sample survey. The company has additionally to decide whether or not to hire the market research team. If the research is conducted, the assumed probabilities of a success survey and an unsuccessful survey are 0.57 and 0.43, respectively. Further, as we have gained more information, our probabilities for the demand will change. Given a successful survey outcome, the probability of high demand is 0.509, for moderate demand is 0.468, and/or for low demand is 0.023. Given an unsuccessful survey outcome, the probability of high demand is 0.023, for moderate demand is 0.543, and/or for low demand is 0.434. Build the decision tree and solve.

Exercise 39. Oil Company (Fox) *

An oil company is considering making a bid on a shale oil development contract to be awarded by the government. The company has decided to bid $210 million. They estimate that they have a 70% chance of winning the contract bid. If the company wins the contract, the management has three alternatives for processing the shale. It can develop a new method for processing the oil, can use the existing method, or can ship the shale overseas for processing. The development cost of the new process is estimated at $30 million. The outcomes and probabilities associated in developing the new method are given as follows:

Event	Probability	Financial Outcome (in Millions)
Extremely successful	0.6	$350
Moderately successful	0.3	$250
Failure	0.1	$90

The existing methods cost $6.5 million to execute, and the outcomes and probabilities are given as follows:

Event	Probability	Financial Outcome (in Millions)
Extremely successful	0.5	$300
Moderately successful	0.25	$200
Failure	0.25	$40

The cost to ship overseas is $5 million. If it is shipped overseas, the contract guarantee is $230 million. Construct a decision tree and determine the best strategy.

Exercise 40. Local TV (Fox)

The local TV station has $150,000 available for research and wants to decide whether to market a new advertising strategy for their station. The station is located in the city, but its viewers are statewide. They have three alternatives:

Alternative 1: Test locally with a small test group, then utilize the results of the local study to determine if a statewide study is needed.

Alternative 2: Immediately market with no studies.

Alternative 3: Immediately decide not to use the new strategy and keep everything *status quo*.

In the absence of a study, they believe that the new strategy has a 55% chance of success and a 45% chance of failure at the state level. If successful, the new strategy will bring $300,000 additional assets and if a failure, we will lose $100,000 in assets. If they do the study (which costs $30,000), there is a 60% chance of favorable outcome and a 40% chance of an unfavorable outcome. If the study shows that it is a local success, then there is an 85% chance that it will be a state success. If the study shows that it was a local failure, then there is only a 10% chance that it will be a state success. What should the local TV station do?

Exercise 41. Marketing and production strategy (Cornuejols and Trick)

Company ABC has developed a new line of products. Top management is attempting to decide on the appropriate marketing and production strategy. Three strategies are being considered, which we will simply refer to as A (aggressive), B (basic) and C (cautious). The market conditions under study are denoted by S (strong) or W (weak). Management's best estimate of the net profits (in millions of dollars) in each case are given in the following payoff table.

decision	*state of nature*	
	S	W
A	30	-8
B	20	7
C	5	15

Management's best estimates of the probabilities of a strong or a weak market are 0.45 and 0.55 respectively. Which strategy should be chosen ?

Although the basic strategy B is appealing, ABC's management has the option of asking the marketing research group to perform a market research study. Within a month, this group can report on whether the study was encouraging (E) or discouraging (D). In the past, such studies have tended to be in the right direction: When market ended up being strong, such studies were encouraging 60% of the time and they were discouraging 40% of the time. Whereas, when market ended up being weak, these studies were discouraging 70% of the time and encouraging 30% of the time. Such a study would cost $500,000. Should management request the market research study or not ?

Exercise 42. Art Dealer (Cornuejols and Trick)

An art dealer has a client who will buy the masterpiece Rain Delay for $50,000. The dealer can buy the painting now for $40,000 (making a profit of $10,000). Alternatively, he can wait one day, when the price will go down

to $30,000. The dealer can also wait another day when the price will be $25,000. If the dealer does not buy by that day, then the painting will no longer be available. On each day, there is a 2/3 chance that the painting will be sold elsewhere and will no longer be available.
(a) Draw a decision tree representing the dealers decision making process.
(b) Solve the tree. What is the dealers expected profit? When should he buy the painting?
(c) What is the Expected Value of Perfect Information (value the dealer would place on knowing when the item will be sold)?

Exercise 43. Walter's Dog and Pony show (Cornuejols and Trick)*

Walter's Dog and Pony show is scheduled to appear in Cedar Rapids on July 4. The profits obtained are heavily dependent on the weather. In particular, if the weather is rainy, the show loses $28,000 and if sunny, the show makes a profit of $12,000. (We assume that all days are either rainy or sunny.) Walter can decide to cancel the show, but if he does, he forfeits a $1,000 deposit he put down when he accepted the date. The historical record shows that on July 4, it has rained 1/4 of the time for the last 100 years.
(a) What decision should Walter make to maximize his expected net dollar return?
(b) What is the expected value of perfect information?

Walter has the option to purchase a forecast from Victor's Weather Wonder. Victor's accuracy varies. On those occasions when it has rained, Victor has been correct (i.e. predicted rain) 90% of the time. On the other hand, when it has been sunny, he has been right (i.e. he predicted sun) only 80% of the time.
(c) If Walter had the forecast, what strategy should he follow to maximize his expected net dollar return?
(d) How much should Walter be willing to pay to have the forecast?

Exercise 44. Drilling (Cornuejols and Trick)

Wildcat Oil is considering spending $100,000 to drill at a particular spot. The result of such a drilling is either a « Dry Well » (of no value), a « Wet Well » (providing $150,000 in revenues) or a « Gusher » (providing $250,000 in revenues). The probabilities for these three possibilities are .5, .3 and .2 respectively.
(a) Draw a decision tree for the problem of deciding whether to drill or not.
(b) Solve the decision tree assuming the goal is to maximize the expected net revenue. Should the company drill? The following two problems should be done independently.
(c) SureFire Consultants is able to determine for certain the type of well before drilling. They offer to tell Wildcat the type for $50,000. Should Wildcat accept their offer?
(d) Close-enough Consultants offer to use their specialized seismic hammer. This hammer returns either encouraging or discouraging results. In the past, when applied to a Gusher, the hammer always returned encouraging results. When applied to a Wet Well, it was encouraging 75% of the time and discouraging 25% of the time. When applied to a Dry Well, it was encouraging one-third of the time and discouraging two-thirds of the time. What is the maximum Wildcat should pay Close-enough Consultants for use of their hammer?

Exercise 45. Stygian Chemical Industries (hbr.com)

The management of Stygian Chemical Industries, Ltd., must decide whether to build a small plant or a large one to manufacture a new product with an expected market life of ten years. The decision hinges on what size the market for the product will be.

Possibly demand will be high during the initial two years but, if many initial users find the product unsatisfactory, will fall to a low level thereafter. Or high initial demand might indicate the possibility of a sustained high-volume market. If demand is high and the company does not expand within the first two years, competitive products will surely be introduced.

If the company builds a big plant, it must live with it whatever the size of market demand. If it builds a small plant, management has the option of expanding the plant in two years in the event that demand is high during the introductory period; while in the event that demand is low during the introductory period, the company will maintain operations in the small plant and make a tidy profit on the low volume.

Management is uncertain what to do. The company grew rapidly, it kept pace with the chemical industry generally. The new product, if the market turns out to be large, offers the present management a chance to push the company into a new period of profitable growth. The development department, particularly the development project engineer, is pushing to build the large-scale plant to exploit the first major product development the department has produced in some years.

The chairman, a principal stockholder, is wary of the possibility of large unneeded plant capacity. He favors a smaller plant commitment, but recognizes that later expansion to meet high-volume demand would require more investment and be less efficient to operate. The chairman also recognizes that unless the company moves promptly to fill the demand which develops, competitors will be tempted to move in with equivalent products. Marketing estimates indicate a 60% chance of a large market in the long run and a 40% chance of a low demand, developing initially as follows:

Initially high demand, sustained high:	60%	
Initially high demand, long-term low:	10%	Low = 40%
Initially low and continuing low:	30%	
Initially low and subsequently high:	0%	

Therefore, the chance that demand initially will be high is 70% (60 + 10). If demand is high initially, the company estimates that the chance it will continue at a high level is 86%(60 ÷ 70). Comparing 86% to 60%, it is apparent that a high initial level of sales changes the estimated chance of high sales in the subsequent periods. Similarly, if sales in the initial period are low, the chances are 100% (30 ÷ 30) that sales in the subsequent periods will be low. Thus the level of sales in the initial period is expected to be a rather accurate indicator of the level of sales in the subsequent periods.
Estimates of annual income are made under the assumption of each alternative outcome:
1. A large plant with high volume would yield $1,000,000 annually in cash flow.
2. A large plant with low volume would yield only $100,000 because of high fixed costs and inefficiencies.
3. A small plant with low demand would be economical and would yield annual cash income of $400,000.
4. A small plant, during an initial period of high demand, would yield $450,000 per year, but this would drop to $300,000 yearly in the long run because of competition. (The market would be larger than under Alternative 3, but would be divided up among more competitors.)
5. If the small plant were expanded to meet sustained high demand, it would yield $700,000 cash flow annually, and so would be less efficient than a large plant built initially.
6. If the small plant were expanded but high demand were not sustained, estimated annual cash flow would be $50,000.
It is estimated further that a large plant would cost $3 million to put into operation, a small plant would cost $1.3 million, and the expansion of the small plant would cost an additional $2.2 million.

Draw the decision tree and solve.

Exercise 46. Xanadu Traders (decision tree primer)

Xanadu Traders, a privately held U.S. metals broker, has acquired an option to purchase one million kilograms of partially refined molyzirconium ore from the Zeldavian government for $5.00 per kilogram. Molyzirconium can be processed into several different products which are used in semiconductor manufacturing, and George Xanadu, the owner of Xanadu Traders, estimates that he would be able to sell the ore for $8.00 per kilogram after importing it. However, the U.S. government is currently negotiating with Zeldavia over alleged dumping of certain manufactured goods which that country exports to the United States. As part of these negotiations, the U.S. government has threatened to ban the import from Zeldavia of a class of materials that includes molyzirconium. If the U.S. government refuses to issue an import license for the molyzirconium after Xanadu has purchased it, then Xanadu will have to pay a penalty of $1.00 per kilogram to the Zeldavian government to annul the purchase of the molyzirconium. Xanadu has used the services of Daniel A. Analyst, a decision analyst, to help in making decisions of this type in the past, and George Xanadu calls on him to assist with this analysis. From prior analyses, George Xanadu is well-versed in decision analysis terminology, and he is able to use decision analysis terms in his discussion with Analyst.

Analyst: As I understand it, you can buy the one million kilograms of molyzirconium ore for $5.00 a kilogram and sell it for $8.00, which gives a profit of ($8.00 -$5.00) x 1 000 000 = $3 000 000. However, there is some chance that you cannot obtain an import license, in which case you will have to pay $1.00 per kilogram to annul the purchase contract. In that case, you will not have to actually take the molyzirconium and pay Zeldavia for it, but you will lose $1.00 x1 000 000 = $1 000 000 due to the cost of annulling the contract.

Xanadu: Actually, some chance may be an understatement. The internal politics of Zeldavia make it hard for their government to agree to stop selling their manufactured goods at very low prices here in the United States. The chances are only fifty-fifty that I will be able to obtain the import license. As you know, Xanadu Traders is not a very large company. The $1,000,000 loss would be serious, although certainly not fatal. On the other hand, making $3,000,000 would help the balance sheet…

Build a decision tree and solve the problem.

We now consider an expanded version of the decision that includes dependent uncertainties and extend the analysis procedure to handle this new issue. We continue to follow the discussion between Daniel Analyst and George Xanadu.

Analyst: Maybe there is a way to reduce the risk. As I understand it, the reason you need to make a quick decision is that Zeldavia has also offered this deal to other brokers, and one of them may take it before you do. Is that really very likely? Perhaps you can apply for the import license and wait until you know whether it is approved before closing the deal with Zeldavia.

Xanadu: That s not very likely. Some of those brokers are pretty big operators, and dropping $1,000,000 would not make them lose any sleep. I'd say there is a 0.70 probability that someone else will take Zeldavia's offer if I wait until the import license comes through. Of course, it does not cost anything to apply for an import license, so maybe it is worth waiting to see what happens.

Should Xanadu Traders wait to see if an import license is issued before purchasing the molyzirconium?

Exercise 47. ABC Computer Company (decision tree primer).

ABC Computer Company is considering submission of a bid for a government contract to provide 10,000 specialized computers for use in computer-aided design. There is only one other potential bidder for this contract, Complex Computers, Inc., and the low bidder will receive the contract. ABC s bidding decision is complicated by the fact that ABC is currently working on a new process to manufacture the computers. If this process works as hoped, then it may substantially lower the cost of making the computers. However, there is some chance that the new process will actually be more expensive than the current manufacturing process. Unfortunately, ABC will not be able to determine the cost of the new process without actually using it to

manufacture the computers. If ABC decides to bid, it will make one of three bids: $9,500 per computer, $8,500 per computer, or $7,500 per computer. Complex Computers is certain to bid, and it is equally likely that Complex will bid $10,000, $9,000, or $8,000 per computer. If ABC decides to bid, then it will cost $1,000,000 to prepare the bid due to the requirement that a prototype computer be included with the bid. This $1,000,000 will be totally lost regardless of whether ABC wins or loses the bidding competition. With ABC's current manufacturing process, it is certain to cost $8,000 per computer to make each computer. With the proposed new manufacturing process, there is a 0.25 probability that the manufacturing cost will be $5,000 per computer and a 0.50 probability that the cost will be $7,500 per computer. Unfortunately, there is also a 0.25 probability that the cost will be $8,500 per computer.

Should ABC Computer Company submit a bid, and if so, what should they bid per computer?

Exercise 48. Arthrodax Company (decision tree primer).

Arthrodax Company has been approached by Ranger Sound with a rush order offer to purchase 100 units of a customized version of Arthrodax's SoundScreamer audio mixer at $5,000 per unit, and Arthrodax needs to decide how to respond. The electronic modifications of the standard SoundScreamer needed for this customized version are straightforward, but there will be a fixed cost of $100,000 to design the modifications and set up for assembly of the customized SoundScreamers, regardless of the number of units produced. It will cost $2,000 per unit to manufacture the circuit boards for the units. Since Arthrodax has some short term spare manufacturing capacity, the Ranger offer is potentially attractive. However, the circuit boards for the customized units will not fit into the standard SoundScreamer case, and Arthrodax must decide what to do about acquiring cases for the customized units as it decides whether to accept Ranger s purchase offer. An appropriate case can be purchased at $500 per case, but Arthrodax could instead purchase an injection molder to make the cases. It will cost $20,000 to purchase the molder, and there is a 0.6 probability that it will be possible to successfully make the cases using the molder. If the molder does not work, then the purchase price for the molder will be totally lost and Arthrodax must still purchase the cases at $500 per case. If the molder works, then it will cost $60 per case to make the cases using the molder. Regardless of which case is used, the cost of assembling the SoundScreamer circuit boards into the case is $20 per unit. Unfortunately, there is no way to test the molder without purchasing it. Assume that there is no other use for the molder except to make the cases for the Ranger order.
(i) Draw a decision tree for Arthrodax's decision about whether to accept the Ranger offer and how to acquire the cases for the customized SoundScreamers.
(ii) Using expected net profit as the decision criterion, determine the preferred course of action for Arthrodax.

Assume that all information above is still valid, except as discussed in this paragraph. Ranger now tells Arthrodax that there is uncertainty about the number of customized SoundScreamers that will be needed. Specifically, there is a 0.35 probability that it will need 100 units, and a 0.65 probability that it will need 50 units. If Arthrodax will agree now to produce either number of units, then Ranger will pay $6,000 per unit if it ultimately orders 50 units, and will pay $5,000 per unit if it ultimately orders 100 units. The timing is such on this rush order that Arthrodax will have to make a decision about purchasing the injection molder before it knows how many units Ranger will take. However, Arthrodax will only need to purchase or manufacture the number of circuit boards and cases needed for the final order of either 50 or 100 units.

(i) Draw a decision tree for Arthrodax's decision about whether to accept the Ranger offer and how to acquire the cases for the customized SoundScreamers.
(ii) Using expected net profit as the decision criterion, determine the preferred course of action for Arthrodax

Assume now that Arthrodax could delay the decision about purchasing the injection molder until after it knows how many units Ranger will take.
(i) Draw a decision tree for Arthrodax s decision about whether to accept the Ranger offer and how to acquire the cases for the customized SoundScreamers.
(ii) Using expected net profit as the decision criterion, determine the preferred course of action for Arthrodax.

Exercise 49. Aba Manufacturing (decision tree primer).

Aba Manufacturing has contracted to provide Zyz Electronics with printed circuit (PC) boards under the following terms: (1) 100,000 PC boards will be delivered to Zyz in one month, and (2) Zyz has an option to take delivery of an additional 100,000 boards in three months by giving Aba 30 days notice. Zyz will pay $5.00 for each board that it purchases. Aba manufactures the PC boards using a batch process, and manufacturing costs are as follows: (1) there is a fixed setup cost of $250,000 for any manufacturing batch run, regardless of the size of the run, and (2) there is a marginal manufacturing cost of $2.00 per board regardless of the size of the batch run. Aba must decide whether to manufacture all 200,000 PC boards now or whether to only manufacture 100,000 now and manufacture the other 100,000 boards only if Zyz exercises its option to buy those boards. If Aba manufactures 200,000 now and Zyz does not exercise its option, then the manufacturing cost of the extra 100,000 boards will be totally lost. Aba believes there is a 50% chance Zyz will exercise its option to buy the additional 100,000 PC boards.
(i) Explain why it might potentially be more profitable to manufacture all 200,000 boards now.
(ii) Draw a decision tree for the decision that Aba faces.
(iii) Determine the preferred course of action for Aba assuming it uses expected profit as its decision criterion.

Aba Manufacturing has created a new option: It can conduct some research and development in an attempt to lower the fixed setup cost associated with manufacturing a batch of the PC boards. This research and development would not be completed in time to influence the setup cost for the initial batch that Zyz has ordered, but would be completed before the second batch would have to be manufactured. The research and development will cost $25,000, and there is a 0.4 probability that it will be successful. If it is successful, then the fixed setup cost per batch will be reduced by $200,000 to $50,000. If the research and development is not successful, then there will be no reduction in the setup cost. There will be no other benefits from the research and development besides the potential reduction in setup cost for the Zyz reorder. (i) Using expected profit as the decision criterion, determine whether Aba should undertake the research and development. (ii) Using expected profit as the decision criteria, determine the value of learning for certain whether the research and development will be successful before a decision has to be made about whether to initially manufacture 100,000 or 200,000 PC boards.

Exercise 50. Intermodular Semiconductor Systems case (decision tree primer).

The Special Products Division of Intermodular Semiconductor Systems has received a Request for Quotation from Allied Intercontinental Corporation for 100 deep sea semiconductor electrotransponders, a specialized instrument used in testing undersea engineered structures. While Intermodular Semiconductor Systems has never produced deep sea electrotransponders, they have manufactured subsurface towed transponders, and it is clear that they could make an electrotransponder that meets Allied s specifications. However, the production cost is uncertain due to their lack of experience with this particular type of transponder. Furthermore, Allied has also requested a quotation from the Undersea Systems Division of General Electrodevices. Intermodular Semiconductor Systems and General Electrodevices are the only companies capable of producing the electrotransponders within the time frame required to meet the construction schedule for Allied s new undersea habitat project. Mack Reynolds, the Manager of the Special Products Division, must decide whether to bid or not, and if Intermodular Semiconductor Systems does submit a bid, what the quoted price should be. He has assembled a project team consisting of Elizabeth Iron from manufacturing and John Traveler from marketing to assist with the analysis. Daniel A. Analyst, a consulting decision analyst, has also been called in to assist with the analysis.

Analyst: For this preliminary analysis, we have agreed to consider only a small number of different possible bids, production costs, and General Electrodevices bids.

Reynolds: That's correct. We will look at possible per-unit bids of $3,000, $5,000, and $7,000. We will look at possible production costs of $2,000, $4,000, and $6,000 per unit, and possible per-unit bids by General Electrodevices of $4,000, $6,000, and $8,000. Iron: There is quite a bit of uncertainty about the cost of producing the electrotransponders. I'd say there is a 50% chance we can produce them in a volume of 100 units at $4,000 per unit. However, that still leaves a 50% chance that they will either be $2,000 or $6,000 per unit.

Analyst: Is one of these more likely than the other?

Iron: No. It's equally likely to be either $2,000 or $6,000. We don't have much experience with deep sea transponders. Our experience with subsurface towed transponders is relevant, but it may take some effort to make units that hold up to the pressure down deep. I'm sure we can do it, but it may be expensive.

Analyst: Could you do some type of cost-plus contract?

Reynolds: No way! This isn't the defense business. Once we commit, we have to produce at a fixed price. Allied would take us to court otherwise. They're tough cookies, but they pay their bills on time.

Iron: I want to emphasize that there is no problem making the electrotransponders and meeting Allied's schedule. The real issue is what type of material we have to use to take the pressure. We may be able to use molyaluminum like we do in the subsurface towed units in which case the cost will be lower. If we have to go to molyzirconium, then it will be more expensive. Most likely, we will end up using some of each, which will put the price in the middle.

Analyst: What is General Electrodevices likely to bid?

Traveler: They have more experience than we do with this sort of product. They have never made deep sea electrotransponders, but they have done a variety of other deep sea products. I spent some time with Elizabeth discussing their experience, and also reviewed what they did on a couple of recent bids. I'd say there is a 50% chance they will bid $6,000 per unit. If not, they are more likely to bid low than high there is about a 35% percent chance they will bid $4,000 per unit.

Analyst: So that means there is 15% chance they will bid $8,000.

Traveler: Yes.

Reynolds: Suppose we had a better handle on our production costs. Would that give us more of an idea what General Electrodevices would bid?

Iron: No. They use graphite-based materials to reinforce their transponders. The cost structure for that type of production doesn t have any relationship to our system using moly alloys.

(i) Draw a decision tree for the decision that Reynolds must make.
(ii) Determine the expected values for each of the alternatives, and specify which alternative Reynolds should select if he uses expected value as a decision criterion.

Assume that all information presented in that exercise still holds.

Analyst: Would it be possible to get a better handle on production costs before making the bid?

Iron: As I said earlier, the main issue is what it will cost to reinforce the electrotransponders to take the pressure. We could make up some material samples and borrow the high pressure chamber over in the Submersible Systems Division to do some tests. We'd get some information out of that, but there would still be a lot of uncertainty. Also, it would be expensive I would have to put people on overtime to meet the bid schedule. The main problem is that we don t have time to do very extensive testing before the bid is due. We could make up a rack of samples from materials we have in stock and take some measurements under pressure, but these materials aren t exactly the same as what we would use in the actual electrotransponders. Because of this, we would still not know for sure what we will have to do to make the electrotransponders work. [This option was discussed at some length. Following this discussion Analyst summarizes as follows.]

Analyst: As I understand it, the result of doing material tests would be an indication that the production will either be expensive or inexpensive. If molyaluminum is going to work, it is more likely that you will get an inexpensive result while if you have to use molyzirconium you are more likely to get an expensive result.

Iron: Yes. In previous cases when we have done tests like this and molyaluminum ultimately worked, then 80% of the time we had gotten an inexpensive indication. On the other hand, when it has worked out that we needed molyzirconium, then 90% of the time we had gotten an expensive indication.

Analyst: What about if a mixture worked?

Iron: We haven't gotten very much useful information in those cases. In cases where a mixture has worked, 60% of the time we had gotten an inexpensive indication and 40% of the time it came out expensive.

Analyst: Based on our earlier discussion, I understand that if molyaluminum works the production costs will be $2,000 per unit, if molyzirconium is needed the costs will be $6,000 per unit, and if a mixture works the costs will be $4,000.

Iron: That's correct for the 100-unit quantity we are discussing here.

Reynolds: How much would the material tests cost?

Iron: There will be a lot of hand labor. I ll go talk with my people and get a figure back to you in a couple of hours. [Iron leaves the meeting and later reports that it would cost $7,000 to conduct the material tests.]

(i) Determine the expected value of perfect information about what material must be used.
(ii) Determine whether it is worth doing the experiment that is outlined above.

Exercise 51. Sue Reynolds (utility) *

Sue Reynolds has to decide if she should get information (at a cost of $17,000) to invest in a retail store. If she gets the information, there is a 0.6 probability that the information will be favorable and a 0.4 probability that the information will not be favorable. If the information is favorable, there is a 0.9 probability that the store will be a success. If the information is not favorable, the probability of a successful store is only 0.2. Without any information, Sue estimates that the probability of a successful store will be 0.6. A successful store will give a return of $100,000. If the store is built but is not successful, Sue will see a loss of $80,000. Of course, she could always decide not to build the retail store.

(a) Do you think that Sue's estimate of 0.6 for the probability of a successful store is correct. Adjust if necessary. What do you recommend?
(b) What impact would a 0.7 probability of obtaining favorable information have on Sue's decision? The probability of obtaining unfavorable information would be 0.3.
(c) Sue believes that the probabilities of a successful and an unsuccessful retail store given favorable information might be 0.8 and 0.2, respectively, instead of 0.9 and 0.1, respectively. What impact, if any, would this have on Sue's decision and the best EMV?
(d) Sue had to pay $17,000 to get information. Would her decision change if the cost of the information increased to $20,000?
(e) Using the original data in this problem (and a cost of information of $20,000) and the following utility table, compute the maximum expected utility. Is this the curve of a risk seeker or a risk avoider?

MONETARY VALUE	UTILITY
$100,000	1
$80,000	0.4
$0	0.2
–$20,000	0.1
–$80,000	0.05
–$100,000	0

(f) Compute the maximum expected utility given the following utility table. Does this utility table represent a risk seeker or a risk avoider?

MONETARY VALUE	UTILITY
$100,000	1
$80,000	0.9
$0	0.8
–$20,000	0.6
–$80,000	0.4
–$100,000	0

Exercise 52. Xanadu Traders (continuation)

This is a continuation of the Xanadu Traders decision problem. We continue to follow the conversation between Daniel Analyst and George Xanadu.
Analyst: I understand from my previous work with you that financial risks of the size involved in this deal would be uncomfortable but would not sink Xanadu Traders. If you could, you would buy some insurance against the potential loss, but you are not going to avoid the deal just because of the possible loss.
Xanadu: That's correct.
Analyst: I recall that you told me in the past that you would be just willing to accept a deal with a fifty-fifty chance of making $2,000,000 or losing $1,000,000. However, if the upside were $2,100,000 and the downside were $1,050,000, you would not take the deal.
Xanadu: That's correct.
Question: Taking into account Xanadu s attitude toward risk taking, what is the preferred alternative.

Remark: *To implement the expected utility approach discussed in the class, it is necessary to first determine a utility function. Both theory and practical experience have shown that it is often appropriate to use a particular form of utility function called the exponential. For risk averse decision makers, in decisions involving profits (more of the evaluation measure is better), this function has the form* $u(x) = 1 - \exp(-x/R)$, *for some* $R > 0$ *where* $u(x)$ *represents the utility function,* x *is the evaluation measure,* R *is a constant called the risk tolerance, and exp represents the exponential function. The following procedure can be used to determine the approximate value of* R *for a particular decision maker: Ask the decision maker to consider a hypothetical alternative that has equal chances of yielding a profit of* R_0 *or a loss of* $R_0 / 2$ *(Calculus shows that the true comparison should be between* R_0 *or a loss of* $R_0 * \ln(2-1/e)$, *and* $\ln(2-1/e) \approx 0.49 \approx 1/2$*). Then ask the decision maker to specify the value of* R_0 *for which he or she would be indifferent between receiving or not receiving the alternative. (Or, put another way, ask the decision maker to adjust* R_0 *until the certainty equivalent for this hypothetical alternative is just equal to zero.) When the decision maker has adjusted* R_0 *in this way, then R is approximately equal to* R_0 *(Note that the expected value for this hypothetical alternative is* $EV = 0.5 * R_0 - 0.5 * (R_0 /2) = 0.25 * R_0$, *and therefore as long as* R_0 *is greater than zero the decision maker is specifying a risk averse utility function.*

Suppose a source of perfect information existed that would let Xanadu know if the import license would be issued.

Question: How much money would it be worth to obtain perfect information about issuance of the import license?

Now consider a potential source of imperfect information

Analyst: Is there any way of obtaining additional information about the chances of obtaining a license other than waiting and seeing what happens? Perhaps there is something that doesn't take as long as waiting for the import approval.
Xanadu: Well, there's always John S. Lofton. He is a Washington-based business consultant with good connections in the import licensing bureaucracy. For a fee, he will consult his contacts and see if they think the license will be granted. Of course, his assessment that the license will come through is no guarantee. If somebody in Congress starts screaming, they might shut down imports from Zeldavia. They are really upset about this in the Industrial Belt, and Congress is starting to take some heat. On the other hand, even if Lofton thinks the license won t come through, he might be wrong. He has a pretty good record on calling these things, but not perfect. And he charges a lot for making a few telephone calls.

Analyst: How good has he been?
Xanadu: He's done some assessments for me, as well as other people I know. I'd say in cases where the import license was ultimately granted, he called it right 90% of the time. However, he hasn't been so good on the license requests that were turned down. In those cases, he only called it right 60% of the time.
Analyst: You commented earlier that he was expensive. How much would he charge?
Xanadu: This is a pretty standard job for him. His fee for this type of service is $10,000.

Should Xanadu hire Lofton, and if so, what is the maximum amount that he should pay Lofton for his services?

Exercise 53. Order Skis for Rentals (fox)

A large skiing group in Lake Tahoe runs a very profitable business renting skis and ski wear. The management is planning on next year's skiing season and wants to know how many skis to order? They can order a small amount, a medium amount, and a large amount of skis for next season. The account staff members have estimated the demand and the profit:

	Outcomes		
Alternatives	**Light Snow ($p = 0.3$)**	**Moderate Snow ($p = 0.6$)**	**Heavy Snow ($p = 0.1$)**
Small order	$4,000	$5,700	$7,200
Medium order	$2,400	$8,000	$10,000
Large order	–$1,600	$4,200	$12,000

What would you advise ?

Exercise 54. Company ABC (Fox)

Company ABC has developed a new line of products. Top management is attempting to decide on both marketing and production strategies. Three strategies are considered and will be referred as *A* (aggressive), *B* (basic), and *C* (cautious). The conditions under which the study will be conducted are *S* (strong) and *W* (weak) conditions. Management's best estimates for net profits (in millions of dollars) are given in the following table. Build a decision tree to assist the company to determine the best strategy.

Decision	Strong (with probability 48%)	Weak (with probability 52%)
A	33	–5
B	22	8
C	15	13

Exercise 55. Newsboy (Cornuejols and Trick)

Let us consider the example of the newsboy problem: a newsboy buys papers from the delivery truck at the beginning of the day. During the day, he sells papers. Leftover papers at the end of the day are worthless. Assume that each paper costs 15 cents and sells for 50 cents and that the following probability distribution is known.

$$\begin{aligned} p_0 &= Prob\{\ demand = 0\ \} &= 2/10 \\ p_1 &= Prob\{\ demand = 1\ \} &= 4/10 \\ p_2 &= Prob\{\ demand = 2\ \} &= 3/10 \\ p_3 &= Prob\{\ demand = 3\ \} &= 1/10 \end{aligned}$$

How many papers should the newsboy buy from the delivery truck?

Exercise 56. Cleaning service (Cornuejols and Trick)

The Scrub Professional Cleaning Service receives preliminary sales contracts from two sources: its own agent and building managers. Historically, 3/8 of the contracts have come from the Scrub agent and 5/8 from building managers. Unfortunately, not all preliminary contracts result in actual sales contracts. Actually, only 1/2 of those preliminary contracts received from building managers result in a sale, whereas 3/4 of those received from the Scrub agent result in a sale. The net return to Scrub from a sale is $6400. The cost of processing and following up on a preliminary contract that does not result in a sale is $320. What is the expected return associated with a preliminary sales contract?

Exercise 57. Product decision (decision tree primer)

To absorb some short-term excess production capacity at its Arizona plant, Special Instrument Products is considering a short manufacturing run for either of two new products, a temperature sensor or a pressure sensor. The market for each product is known if the products can be successfully developed. However, there is some chance that it will not be possible to successfully develop them. Revenue of $1,000,000 would be realized from selling the temperature sensor and revenue of $400,000 would be realized from selling the pressure sensor. Both of these amounts are net of production cost but do not include development cost. If development is unsuccessful for a product, then there will be no sales, and the development cost will be totally lost. Development cost would be $100,000 for the temperature sensor and $10,000 for the pressure sensor. The probability of development success has been estimated at 0.5 for the temperature sensor and 0.8 for the pressure sensor.

Part II (extension): Decision under ignorance

Exercise 58. Investments by Expected Value (Fox) *

Values for a 5-year investment of alternative investment strategies as a function of the nature of the economy:

		Nature of the Economy Investment			
		Fast, *F*	Moderate, *M*	Normal, *N*	Slow, *S*
You	Fortune 500 stocks, *A*	2	3	1	1
	Bonds, *B*	2	1	2	1
	Options, *C*	0.5	4	1	1
	Mutual funds, *D*	1	3	0.5	1

Case 1: One-time decisions and probabilities are known, and we want to maximize our expected outcome.

Maximize expected value criterion: Compute the expected value for each option. Choose the largest expected value. Let us assume that an accomplished econometrician has estimated a *subjective* probability. A subjective probability differs from the relative frequency definition in that it is not the ratio of favorable outcomes to total outcomes because experimental data are not available, but rather it is the best estimate of a qualified expert. Let us assume that the probabilities for the states of the economy *F*, *M*, *N*, and *S* are 0.2, 0.4, 0.3, and 0.1, respectively.

Case 2: One-time decisions, probabilities unknown.

Laplace criterion: This decision criterion assumes that the unknown probabilities are equal. Therefore, we can simply average the payoffs (expected value) for each investment, or, equivalently, can choose the investment strategy with the highest sum because the weights are equal. The Laplace method is simply the same as maximizing the expected value assuming the states of nature are equally likely.

Maximin criterion: Here, we want to compute the worst that can happen if we choose each strategy. We look at each row and find the minimum value. We now examine the columns of the row minimums and select the maximum of these minimum values. Hence the term *maximum* (*minima*) or *maximin*.

The maximin strategy is *pessimistic* in that it chooses the strategy based only on the worst case for each strategy under consideration, completely neglecting the better cases that could result. When considering an investment that you wish to be conservative, such as investing funds for the college education of your children, and a minimax strategy gets the job done, it may be the decision criterion for which you are looking.

Maximax criterion: There may be instances where we want to obtain the best result possible. We examine each row and obtain the maximum in each row. Then, we examine the columns of max's and choose the largest value.

Obviously, this strategy is *optimistic* because it considers only the best case while neglecting the risk of each strategy. If all your investment needs were taken care of, if you want to invest some *extra money*, and if you are willing to suffer the risk involved for the possibility of a big gain, the optimistic maximax criterion may be of

interest to you.

Minimax regret criterion: The minimax regret strategy is the one that minimises the maximum regret. Essentially, this is the technique for a decision maker who does not wish to make the wrong decision. 'Regret' in this context is defined as the opportunity loss through having made the wrong decision.

Since the minimax criterion applied here is to the regret (difference or ratio of the payoffs) rather than to the payoff itself, it is not as pessimistic as the ordinary minimax approach.

Test the different methods and outcomes on the example.

Exercise 59. Investment Strategy (fox)

Assume that we have 100,000 to invest (one time). We have three alternatives: stocks, bonds, or savings. We place the estimated returns in a payoff matrix.

Alternatives	Conditions		
Investments	**Fast Growth (Risk)**	**Normal Growth**	**Slow Growth**
Stocks	$10,000	$6,500	−$4,000
Bonds	8,000	$6,000	1,000
Savings	5,000	$5,000	5,000

Evalute the best strategy according to the different criterion defined in the previous exercise.

Exercise 60. Hotel, Restaurant or Convenience store ? (fox)

A local investor is considering three alternative real estate investments, a hotel, a restaurant, and a convenience store. The hotel and the convenience store will be adversely or favorably affected depending on their closeness to gasoline stations, whereas the restaurant will be assumed to be relatively stable. The payoff is given as follows:

	Conditions		
Alternative	**#1 Gas Close Distance**	**#2 Gas Medium Distance**	**#3 Gas Far Away Distance**
Hotel	$25,000	10,000	−8,000
Convenience store	4,000	8,000	−12,000
Restaurant	5,000	6,000	6,000

Determine the best plan by each of the following criteria and show work:

1. Laplace
2. Maximin
3. Maximax
4. Minmax regret

Part III: Game Theory

Exercise 61. Sharing a market (introduction to Operations Research) *

Two companies share the bulk of the market for a particular kind of product. Each is now planning its new marketing plans for the next year in an attempt to wrest some sales away from the other company. (The total sales for the product are relatively fixed, so one company can increase its sales only by winning them away from the other.) Each company is considering three possibilities: (1) better packaging of the product, (2) increased advertising, and (3) a slight reduction in price. The costs of the three alternatives are quite comparable and sufficiently large that each company will select just one. The estimated effect of each combination of alternatives on the increased percentage of the sales for company 1 is as follows:

		Player 2		
Strategy		**1**	**2**	**3**
Player 1	1	2	5	1
	2	1	6	0
	3	3	−2	−1

Each company must make its selection before learning the decision of the other company.
(a) Without eliminating dominated strategies, use the minimax (or maximin) criterion to determine the best strategy for each company.
b) Now identify and eliminate dominated strategies as far as possible. Make a list of the dominated strategies, showing the order in which you were able to eliminate them. Then show the resulting reduced payoff table with no remaining dominated strategies.

Exercise 61 bis. Sharing a market*

The estimated effect of each combination of alternatives on the increased percentage of the sales for company 1 is now as follows:

		Player 2		
Strategy		**1**	**2**	**3**
Player 1	1	2	5	4
	2	1	6	0
	3	3	−2	−1

What is the best strategy now ?

Exercise 62. Trader Joe's versus Whole Foods *

Suppose that grocery chain stores want to enter into the market in a new area. They have choices to locate in

more densely populated area or less densely populated town surrounded by other towns. Let us assume a smaller grocery store such as Trader Joe's will locate a franchise in either densely populated area or a less densely populated area. Further, a mega grocery store franchise such as Whole Foods is making the same decision—they will locate either in denser populated area or the less dense area. Analysts have estimated the market shares and we place both sets of payoffs in a single game matrix. Listing the row player's payoffs first, we have the payoff as shown below. The payoff matrix represents a constant sum total conflict game.

		Trader Joe's	
		Densely populated	Less densely populated
Whole Foods	Densely populated	65	67
	Less densely populated	55	60

What is the best strategy for both firms ?

Exercise 63. Battle of the network

Two television networks are battling for viewer shares. Viewer share is important because, the higher it is, the more money the network can make from selling advertising time during that program. Consider the following situation: the networks make their programming decisions independently and simultaneously. Each network can show either sports or a sitcom. Network 1(NBC) has a programming advantage in sitcoms and Network 2 (CBS) has one in sports: If both networks show sitcoms, then Network 1 gets a 56% viewer share. If both networks show sports, then Network 2 gets a 54% viewer share. If Network 1 shows a sitcom and Network 2 shows sports, then Network 1 gets a 51% viewer share and Network 2 gets 49%. Finally, if Network 1 shows sports and Network 2 shows a sitcom, then each gets a 50% viewer share.

The possible outcomes are best represented in a table.

		Network 2	
		Sitcom	Sports
Network 1	Sitcom	(56%, 44%)	(51%, 49%)
	Sports	(50%, 50%)	(46%, 54%)

What is the best strategy for each network ?

Exercise 64. Competitive Advantage *

In a technologically advanced economy like that of the United states, firms constantly encounter the following situation. A new technological advance becomes available. If one firm adopts the new technology, it gains an advantage over its competitors, a competitive advantage. If all firms adopt the new technology, then the advantage vanishes. This is represented in the next table, where a measures the size of the competitive advantage

		Firm 2	
		Adopt	*Stay put*
Firm 1	*Adopt*	$(0,0)$	$(a,-a)$
	Stay put	$(-a,a)$	$(0,0)$

Each firm has two strategies, either Stay put, or Adopt the new technology.

Take, for example, the hospital industry. Magnetic Resonance Imaging (MRI) was a new technology that enhances conventional X rays. It allows doctors to see body damage in ways that were not previously possible. From a public policy standpoint, it may not make much sense for every hospital to have its own MRI unit. These units are expensive to buy and to operate. They can eat up millions of dollars. Often, one MRI unit could handle the traffic of several hospitals. But for an individual hospital does it make sens to not offer MRI ?

Exercise 65. Competitive Advantage with Three Firms (cornuejols and Trick)

As before, each firm has the choice between staying put or adopting the new technology. If no firm adopts the new technology, then there is no competitive advantage and the payoff vector is (0; 0; 0). If exactly one firm adopts the new technology, then that firm gets the competitive advantage a, while each firm at a competitive disadvantage looses a/2. Thus if only Firm 1 adopts the new technology,then the payoff vector is (a, -a/2, -a/2). You can think of this as Firm 1 taking market share from both Firm 2 and Firm 3. If exactly two firms adopt the new technology, then these two firms split the competitive advantage, each gaining a/2, and the firm at a disadvantage looses a. Finally, if all firms adopt the new technology, there is no competitive advantage and the payoff
vector is (0; 0; 0). We can represent the payoffs in a table as follows

Firm 3 Adopt

		Firm 2	
		Adopt	Stay put
Firm 1	Adopt	$(0,0,0)$	$(\frac{a}{2},-a,\frac{a}{2})$
	Stay put	$(-a,\frac{a}{2},\frac{a}{2})$	$(\frac{-a}{2},\frac{-a}{2},a)$

Firm 3 Stay put

		Firm 2	
		Adopt	Stay put
Firm 1	Adopt	$(\frac{a}{2},\frac{a}{2},-a)$	$(a,\frac{-a}{2},\frac{-a}{2})$
	Stay put	$(\frac{-a}{2},a,\frac{-a}{2})$	$(0,0,0)$

Exercise 66. Manufacturers (introduction to Operations Research) *

Two manufacturers currently are competing for sales in two different but equally profitable product lines. In both cases the sales volume for manufacturer 2 is three times as large as that for manufacturer 1. Because of a recent technological breakthrough, both manufacturers will be making a major improvement in both products.

However, they are uncertain as to what development and marketing strategy to follow. If both product improvements are developed simultaneously, either manufacturer can have them ready for sale in 12 months. Another alternative is to have a "crash program" to develop only one product first to try to get it marketed ahead of the competition. By doing this, manufacturer 2 could have one product ready for sale in 9 months, whereas manufacturer 1 would require 10 months (because of previous commitments for its production facilities). For either manufacturer, the second product could then be ready for sale in an additional 9 months. For either product line, if both manufacturers market their improved models simultaneously, it is estimated that manufacturer 1 would increase its share of the total future sales of this product by 8 percent of the total (from 25 to 33 percent). Similarly, manufacturer 1 would increase its share by 20, 30, and 40 percent of the total if it marketed the product sooner than manufacturer 2 by 2, 6, and 8 months, respectively. On the other hand, manufacturer 1 would lose 4, 10, 12, and 14 percent of the total if manufacturer 2 marketed it sooner by 1, 3, 7, and 10 months, respectively.

Formulate this problem as a two-person, zero-sum game, and then determine which strategy the respective manufacturers should use according to the minimax criterion.

Exercise 67. Labor union (introduction to Operations Research)

The labor union and management of a particular company have been negotiating a new labor contract. However, negotiations have now come to an impasse, with management making a "final" offer of a wage increase of $1.10 per hour and the union making a "final" demand of a $1.60 per hour increase. Therefore, both sides have agreed to let an impartial arbitrator set the wage increase somewhere between $1.10 and $1.60 per hour (inclusively). The arbitrator has asked each side to submit to her a confidential proposal for a fair and economically reasonable wage increase (rounded to the nearest dime). From past experience, both sides know that this arbitrator normally accepts the proposal of the side that gives the most from its final figure. If neither side changes its final figure, or if they both give in the same amount, then the arbitrator normally compromises halfway between ($1.35 in this case). Each side now needs to determine what wage increase to propose for its own maximum advantage. Formulate this problem as a two-person, zero-sum game.

Exercise 68. Cigarette Advertising on Television (cornuejols and Trick)

On January 1, 1971, cigarette advertizing was banned on american television. It came as something of a surprise to the industry, and a welcome one at that, that profits rose by $91 million in 1971. Why did this happen? Consider the strategic interaction in the industry before and after the agreement went into effect. Although there were four large tobacco companies involved, let us consider the strategic interaction between only two of them, for simplicity. In 1970, the payoff matrix was :

		Company 2	
		No TV ads	*TV Ads*
Company 1	*No TV Ads*	$(50, 50)$	$(20, 60)$
	TV Ads	$(60, 20)$	$(27, 27)$

Exercise 69. Market Niche (cornuejols and Trick) *

Two firms are competing for a single market niche. If one firm occupies the market niche, it gets a return of 100. If both firms occupy the market niche, each loses 50. If a firm stays out of the market, it breaks even. The payoff table is (fixed cost to invest 200, market 300):

		Firm B	
		Enter	*Stay out*
Firm A	*Enter*	$(-50, -50)$	$(100, 0)$
	Stay out	$(0, 100)$	$(0, 0)$

This game has two pure strategy equilibria, namely one of the two firms enters the market niche and the other stays out. But, unlike the games we have encountered thus far, neither player has a dominant strategy. When a player has no dominant strategy, she should consider playing a mixed strategy. In a mixed strategy, each of the various pure strategies is played with some probability, say p1 for Strategy 1, p2 for Strategy 2, etc with p1 + p2 + … = 1. What would be the (optimal) mixed strategies for Firms A and B?

Exercise 70. Fishing in Jamaica (Davenport) *

In the fifties, Davenport studied a village of 200 people on the south shore of Jamaica, whose inhabitants made their living by fishing :

« Twenty-six fishing crews in sailing, dugout canoes fish this area [fishing grounds extend outward from shore about 22 miles] by seting fish pots, which are drawn and reset, weather and sea permiting, on three regular fishing days each week ... The fishing grounds are divided into inside and outside banks. The inside banks lie from 5-15 miles offshore, while the outside banks all lie beyond ... Because of special underwater contours and the location of one prominent headland, very strong currents set across the outside banks at frequent intervals ... These currents are not related in any apparent way to weather and sea conditions of the local region. The inside banks are almost fully protected from the currents. » [Davenport 1960]

There were 26 wooden canoes. The captains of the canoes might adopt 3 fishing strategies:

– IN – put all pots on the inside banks
– OUT – put all pots on the outside banks
– IN-OUT – put some pots on the inside banks, some pots on the outside

It takes time to reach outside banks, so those who use OUT or IN-OUT can set fewer pots. When the current is running, it is harmful to outside pots: marks are dragged away, pots may be smashed while moving, changes in temperature may kill fish inside the pots, etc. The outside banks produce higher quality fish, both in variaties and in size. If many outside fish are available, they may drive the inside fish off the market. The OUT and IN-OUT strategies require befer canoes. Their captains dominate the sport of canoe racing, which is prestigious and offers large rewards.

Davenport collected the data concerning the fishermen average monthly profit depending on the fishing strategies they used to adopt.

Fishermen\Current	FLOW	NO FLOW
IN	17,3	11,5
OUT	-4,4	20,6
IN-OUT	5,2	17,0

What is the optimal strategy for an `adversary current' ?

Collecting data from past years, it was observed that the probability of flow is 25%. How does it affect the strategy ? Explain why it could still make sense to use the previous strategy ? (hint : evaluate the expected profit with respect to perturbation around the mean).

Exercise 71. Resource economics (Cornuejols and Trick)

Here is a game that arises in resource economics. (a) There are two fishing spots, one good and the other excellent. The good spot has 6 fish whereas the excellent spot has 10 fish. Two fishermen know how good each of the spots is. Each fisherman must choose one of the two spots to go fishing. If they end up going to the same spot, they divide the catch equally. If they go to different spots, the fisherman at the good spot gets 6 fish, the one at the excellent spot gets 10. Find three equilibria for this game. (b) In such fishing games, conflicts often break over the fishing grounds. How might an assymetrical equilibrium, such as the one provided by the 200-mile fishing limit among nations, lead to a more efficient outcome?

Exercise 72. Best Buy vs Amazon (ignitionframework.com)

In the early years of the 2010s, the fate of Best Buy has been in question. Business pundits have long predicted their demise thanks to the triumph of the online juggernaut – Amazon. Part of the issue for Best Buy, pundits said, was something called "showrooming" where customers would stop in to Best Buy only to look at a product in person, check out how it looks and feels in its physical form, then purchase it online at Amazon. So basically Best Buy was Amazon's brick and mortar "showroom." Not a good situation for Best Buy to be in.

The issue usually came down to price. The prices of the same or similar electronics at Best Buy were typically higher than the prices available online at Amazon. A major reason for higher prices at Best Buy could be that their cost structure is saddled with brick and mortar stores all across the country and the sales staff required to support those stores. Given this scenario, what would you do if you were the CEO of Best Buy?

To figure this out, let's suppose all you could do was change Best Buy's pricing and then analyze the following scenarios from a game theory perspective:

1. **Hold Price –** Status quo
2. **Best Price –** Always offer the lowest price versus competitors
3. **Price Match –** Always offer to match competitor's prices even when they are lower than yours

Put yourself in the shoes of the consumer as someone who wants to buy a 15″ flat screen TV. Assume for a moment that the TV is sold at both retailers (it is the exact same TV) and (marginal) costs $60 from the manufacturer for both Best Buy and Amazon.

When you go into Best Buy to look at this 15″ TV, they have it listed for $75 out the door – a price that would give them $15 in profit for every unit sold. You see the price and wonder what the price is on Amazon. So you use your Amazon price check app and find out that Amazon has the exact same TV listed for $74 delivered to your home.

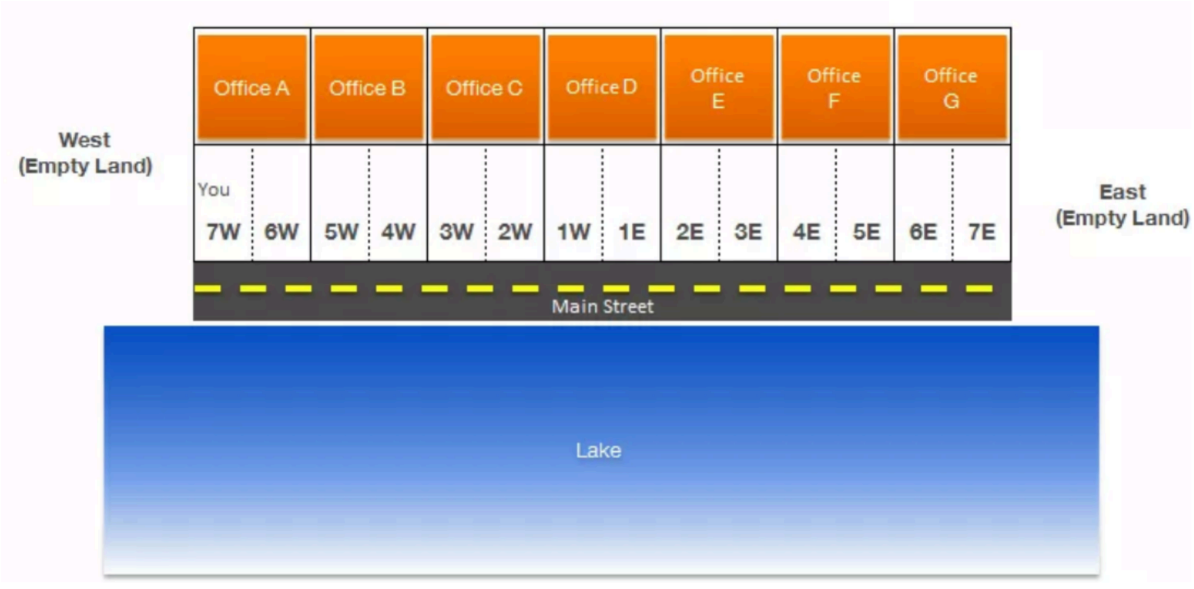
Best price means that you offer always a lower price than the best offer found by your potential client. Match price means that you simply align with the best offer.

What should the CEO of Best Buy decide upon as a strategy ?

Exercise 73. Hot Dog placement (ignitionframework.com)

This example involves the decision of where to locate your hot dog stand. We'll use a hot dog stand for this example but the reality is that it could be any business – a fast food restaurant, retail store, a gas station, etc. Let's suppose you were considering placing your new hot dog stand on Main Street in a city called Anywhere USA. Let's also suppose that you heard from a friend about another person who is planning to open up a hot dog stand on Main Street as well but you don't know exactly where yet. Let's also suppose that the citizens of Anywhere USA like both your hot dogs and your competition's hot dogs equally well – they have no preference between the two.

Now because you are a shrewd business person, you know that where on Main Street you decide to locate your hot dog stand is a big decision that will have a major impact on your success. You also know that where the other person decides to locate their hot dog stand will have a big impact on your business as well because they will be going after the exact same customers as you. So you decide to do some serious analysis of Main Street to determine where the best place would be to locate your new stand. And since you have a competitor who's actions will significantly impact your business, you decide to use game theory to try and predict where they will place their stand and consequentially where you should place yours.



Upon surveying the Main Street area, you notice a few important things:

1. It runs East to West with office buildings evenly spread on the North side and each office building has the same number of employees who need to eat lunch every day
2. On the south side of Main Street is a giant lake

3. To your surprise, there are no other restaurants along Main Street and the sidewalk is completely open!

Let's say your best friend is the mayor of Anywhere USA and since he likes you and your hot dogs so much, he convinced the City Council to pass a City law that states that your hot dog stand is the only lunch food location allowed in town. You jump for joy because this basically means you won't have any competition because the person who was going to bring in another hot dog stand is suddenly out of luck. Now the question is, where would you choose to locate your hot dog stand?

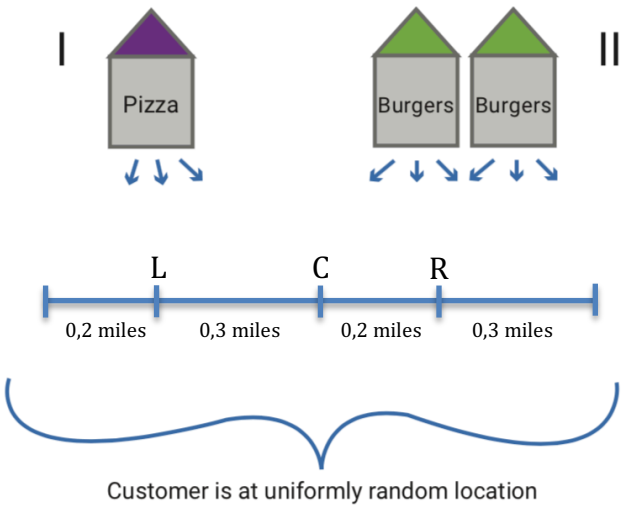
One day your friend the Mayor calls with bad news: the law that stated you were the only hot dog stand allowed has been repealed. And as a result, you find out that a competitor is going to open a hot dog stand on Main Street as well. Your monopoly is over.

Instead of sit back and sulk about your situation, you decide you want to anticipate their moves before they setup their stand and so you start to run a few scenarios. For example, if you're still located at 7W, where would your competition decide to locate? Where would you locate if you were in his shoes? What position on Main Street would provide your competition with the most customers and profits?

Set up the problem as a game and compute the payoff matrix. Analyse the game and decide where to put your hot-dog stand.

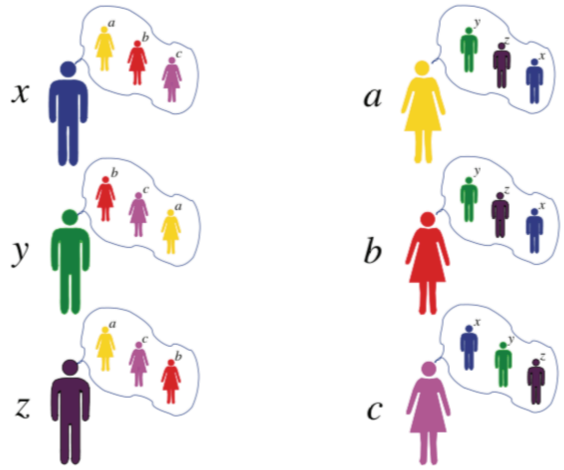
Exercise 74. Another restaurant placement problem

Consider the one-mile stretch of road shown in the next Figure. There are three locations (squares on the street) at which restaurants can be opened: Left, Central, and Right. Company I opens a restaurant at one of these locations and company II opens two restaurants (both restaurants can be at the same location). Customers are located uniformly at random along the one-mile stretch. A customer walks to the closest location at which there is a restaurant and then into one of the restaurants there, chosen uniformly at random. The payoff to company I (respectively company II) is the expected market share for company I (respectively company II) restaurant(s). Determine the value of the game, and find some optimal mixed strategies for the companies.



Part III (extension): Stable matching

Exercise 75. Stable marriage *



How should you organize the matching so that each marriage is `stable' ?

Exercise 76. Headhunter *

A headhunter was assigned by a large company to find qualified young business graduate to fill six positions in six different departments: Sales, Marketing, Purchasing, Supply Chain, Finance and Business Development. The headhunter has identified 6 strong potentiel candidates from Kedge Business School. The headhunter wants to create trust with the company so that he wants to recommend a matching between the young graduates and the jobs that will ensure that both the departments and the graduates will be satisfied by the solution (no candidate will envy a position where they would be more qualified that the hired person) so that no conflict arise between the different departments and future employees. Propose a solution.

	Adam	Beatrice	Camille	David	Emilien	Françoise
Sales	1	2	4	5	3	6
Marketing	5	4	2	3	1	6
Finance	4	1	2	5	6	3
Supply Chain	3	1	6	5	2	4
Purchasing	1	6	4	5	3	2
Business Developper	2	3	4	6	5	1

	Sales	Marketing	Finance	Supply Chain	Purchasing	Business Developper
Adam	3	5	2	1	6	4
Beatrice	4	5	3	2	5	1
Camille	1	2	3	5	6	4
David	5	2	3	4	1	6
Emilien	2	5	3	6	4	1
Françoise	6	1	4	5	3	2

Exercise 77. Stable matching as an integer program

Propose an integer program to solve the stable matching problem.

Exercise 78. Peripatetic (Kleinberg & Tardos)

Peripatetic Shipping Lines, inc., is a shipping company that owns n ships and provides service to n ports. Each of its ships has a schedule that says, for each day of the month, which of the ports it's currently visiting, or whether it's out at sea. (You can assume the "month" here has m days, for some m > n.) Each ship visits each port for exactly one day during the month. For safety reasons, PSL Inc. has the following strict requirement:

(t) No two ships can be in the same port on the same day.

The company wants to perform maintenance on all the ships this month, via the following scheme. They want to truncate each ship's schedule: for each ship Si, there will be some day when it arrives in its scheduled port and simply remains there for the rest of the month (for maintenance). This means that Si will not visit the remaining ports on its schedule (if any) that month, but this is okay. So the truncation of Si's schedule will simply consist of its original schedule up to a certain specified day on which it is in a port P; the remainder of the truncated schedule simply has it remain in port P. Now the company's question to you is the following: Given the schedule for each ship, find a truncation of each so that condition (t) continues to hold: no two ships are ever in the same port on the same day. Show that such a set of truncations can always be found, and give an algorithm to find them.

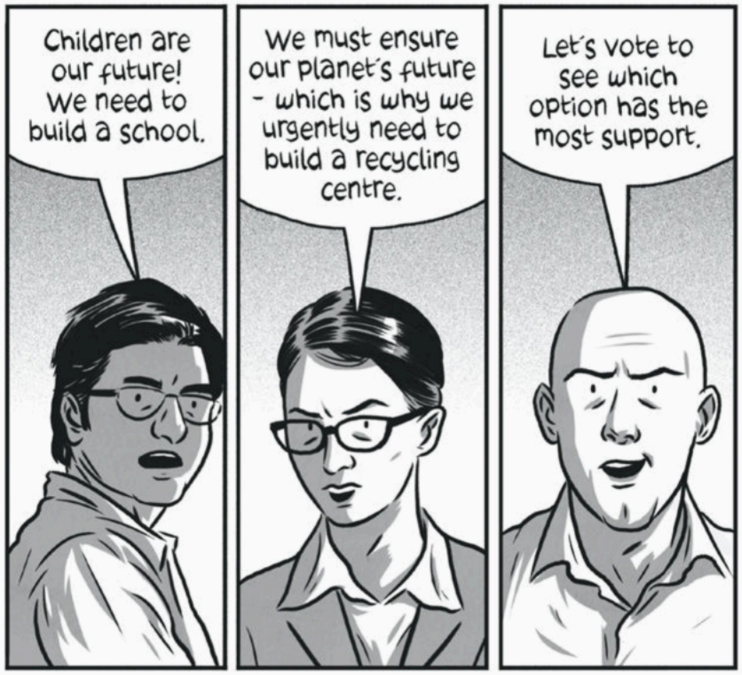
Example: Suppose we have two ships and two ports, and the "month" has four days. Suppose the first ship's schedule is port P1; at sea; port P2 ; at sea and the second ship's schedule is at sea; port P1 ; at sea; port P2 - Then the (only) way to choose truncations would be to have the first ship remain in port P2 starting on day 3, and have the second ship remain in port P1 starting on day 2.

Part IV: Group Decisions

Exercise 79. Voting Theory : a simple example (Introducing Game Theory: A Graphic Guide)

A city owns a vacant lot. There are three proposals for what to do with the land. It could be used as a park, as a recycling centre or as a new school.
The city council has to decide which of these options to choose. There are three people on the council. Each council member individually prefers a different alternative as their top choice.

	Mr Peters	Ms Reynolds	Mr Singh
First choice	Park	Recycling	School
Second choice	Recycling	School	Park
Third choice	School	Park	Recycling



In a series of votes, the committee compares two options at a time. Suppose that each council member votes for the option that he or she actually prefers, which is referred to as sincere voting.

(i) Compute the preferences of the group.
(ii) What would happen if we would decide upon the winner by running two consecutive election say 1-2 and then best(1,2) – 3 ? (Condorcet paradox).
(iii) Tactical voting: analyze what happens when Mr Peters votes for his second best.