Evolutionary Multi-objective Optimization for ICU Bed Allocation
A Large-Scale Constraint-Handling Analysis using Real Brazilian SUS Data

Removed for double-blind review
Department of Computer Science
Universidade Federal de Juiz de Fora
Juiz de Fora, Brazil
email@removed.edu

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Department of Computer Science
Universidade Federal de Juiz de Fora
Juiz de Fora, Brazil
email@removed.edu

***Abstract*—This paper addresses the multi-objective ICU bed allocation problem in the Brazilian Unified Health System (SUS) through a comprehensive evolutionary computation framework. We formulate a four-objective mixed-integer optimization problem minimizing: (1) total waiting time, (2) clinical risk through severity-weighted delays, (3) bed idleness, and (4) terminal overflow costs. Using real-world data from 120,000+ hospitalizations in Juiz de Fora (MG), we conduct an extensive comparative study of NSGA-II and GDE3 under three constraint-handling regimes and multiple capacity scenarios. Key findings include: (i) NSGA-II reduces average waiting times by 81.5% compared to GDE3 in overload scenarios with statistical significance (p<0.01), (ii) identification of an invariant "biological debt" of 6,936 bed-hours representing clinical rather than managerial bottlenecks, and (iii) quantification of the capacity inflection point at 233 beds for zero-wait operation using real SUS data. The framework provides public health authorities with evidence-based capacity planning tools, validated on production-scale healthcare data and contributing new benchmark instances to the evolutionary computation community.**

***Index Terms*—Multi-objective Optimization, Constraint Handling, NSGA-II, GDE3, Healthcare Scheduling, Public Health Systems, SUS Brazil, ICU Management.**

I. Introduction

Intensive Care Unit (ICU) bed management represents a critical bottleneck in healthcare systems worldwide, particularly in public health systems serving large populations under resource constraints. The Brazilian Unified Health System (SUS), serving over 210 million people, faces chronic ICU bed shortages exacerbated by inefficient allocation policies often based on First-Come-First-Served (FIFO) protocols [1]. In 2023 alone, the SUS recorded over 12 million hospitalizations, with ICU beds representing the most constrained resource [2].

Computational Perspective: From an evolutionary computation standpoint, ICU bed allocation presents a challenging multi-objective optimization problem with mixed-integer variables, hard constraints (physical capacity), and soft constraints (clinical priorities). This domain exemplifies real-world applications where evolutionary algorithms must balance exploration-exploitation trade-offs while handling infeasible regions efficiently [3]. The problem's combinatorial complexity grows exponentially with patient volume, making exact methods infeasible for realistic instances [4].

Literature Gap: Previous works have applied evolutionary computation to healthcare scheduling [5], [6], but critical gaps remain: (1) lack of validation on production-scale national health system data, (2) insufficient comparison of differential evolution variants against genetic algorithms with statistical rigor in healthcare domains, and (3) absence of quantification for irreducible biological constraints that limit optimization efficacy [7].

Contributions: This paper makes three key contributions:

1) **Framework Development:** A comprehensive evolutionary optimization framework for ICU bed allocation incorporating real-world SUS data, clinical constraints, and an oracle-based validation methodology.
2) **Algorithmic Comparison:** Extensive empirical comparison of NSGA-II and GDE3 using standard multi-objective metrics (hypervolume, spread, inverted generational distance) with statistical validation across multiple search profiles.
3) **Real-World Insights:** Identification of capacity inflection points and "biological debt" invariants through analysis of 120,000+ hospitalizations, providing evidence-based tools for public health planning.

Organization: Section II reviews related work. Section III formulates the optimization problem. Section IV details data characterization. Section V describes the evolutionary framework. Section VI presents experimental design. Section VII analyzes results. Section VIII discusses implications, and Section IX concludes.

II. Related Work

Healthcare Scheduling Optimization: Healthcare scheduling has been extensively studied using operations research techniques. Integer programming approaches [8] have shown promise for small instances but struggle with large-scale problems due to exponential complexity. Simulation-based optimization [12] addresses uncertainty but at prohibitive computational cost for real-time applications. Recent surveys highlight the growing role of metaheuristics in healthcare scheduling [13].

Evolutionary Approaches: Multi-objective evolutionary algorithms (MOEAs) have emerged as promising tools for balancing competing objectives in healthcare. NSGA-II [9] has been widely applied to hospital scheduling problems due to its effective non-dominated sorting mechanism. Differential Evolution variants, particularly GDE3 [10], have shown robustness in continuous domains but less exploration in discrete healthcare scheduling.

Constraint Handling: Effective constraint handling is critical for healthcare optimization. Feasibility-first approaches [11] and penalty function methods [14] have been commonly employed. However, comparative studies of constraint-handling techniques in healthcare domains remain limited [15].

Brazilian SUS Context: While several studies have analyzed SUS data for epidemiological purposes [16], few have applied computational optimization techniques. The PCDaS/Fiocruz platform provides unprecedented access to national hospitalization data, enabling large-scale optimization studies [17].

Research Gap: Existing literature lacks: (1) validation on production-scale public health system data, (2) systematic comparison of constraint-handling strategies in healthcare domains, and (3) quantification of system-level invariants that constrain optimization efficacy. Our work addresses these gaps using real SUS data from Brazil.

III. Problem Formulation

A. Mathematical Model

Let $P = \{p_1, p_2, ..., p_N\}$ be a set of patients, each characterized by:

- $a_i \in \mathbb{Z}^+$: Arrival time (hours from horizon start)
- $l_i \in \mathbb{Z}^+$: Length of Stay (LOS) in hours
- $s_i \in [1, 10]$: Clinical severity score
- $c_i \in \mathbb{R}^+$: Clinical cost coefficient

Decision variables are admission times $x_i \in \mathbb{Z}^+$ with $x_i \geq a_i$.

B. Objective Functions

The multi-objective optimization problem is formulated as:

$$\min_{\mathbf{x}} \quad \mathbf{F}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}), f_4(\mathbf{x})]^T \tag{1}$$

$$\text{s.t.} \quad g_1(\mathbf{x}) = \max_{t \in [0,T]} \left(\sum_{i=1}^{N} \mathbb{1}_{x_i \leq t < x_i + l_i} \right) - B \leq 0 \tag{2}$$

$$g_2(\mathbf{x}) = \sum_{i=1}^{N} \max(0, a_i - x_i) = 0 \tag{3}$$

$$x_i \in \mathbb{Z}^+, \quad x_i \geq a_i \quad \forall i \in [1, N] \tag{4}$$

where:

$$f_1(\mathbf{x}) = \sum_{i=1}^{N} (x_i - a_i) \tag{5}$$

$$f_2(\mathbf{x}) = 1 - \frac{1}{T \cdot B} \sum_{t=1}^{T} b_t(\mathbf{x}) \tag{6}$$

$$f_3(\mathbf{x}) = \sum_{i=1}^{N} s_i \cdot (x_i - a_i) \tag{7}$$

$$f_4(\mathbf{x}) = \sum_{i=1}^{N} s_i \cdot \max(0, (x_i + l_i) - H) \tag{8}$$

with occupancy $b_t(\mathbf{x}) = \sum_{i=1}^{N} \mathbb{1}_{\lfloor x_i \rfloor \leq t < \lfloor x_i + l_i \rfloor}$ for $t = 0, 1, \ldots, T-1$, total beds B, horizon $T = 1200$ hours, and planning limit H.

C. Discrete-Time Implementation

For computational implementation, time is discretized in hourly intervals:

$$b_t(\mathbf{x}) = \sum_{i=1}^{N} \mathbb{1}_{\lfloor x_i \rfloor \leq t < \lfloor x_i + l_i \rfloor}, \quad t = 0, 1, \ldots, T-1 \tag{9}$$

where $\lfloor \cdot \rfloor$ denotes integer discretization.

Implementation Note: While theoretically $b_t(\mathbf{x}) \leq B$ for all t, the aggregated form $g_1(\mathbf{x}) \leq 0$ using maximum occupancy is mathematically equivalent and computationally efficient.

D. Normalization and Reference Values

For algorithm stability, objectives are normalized as:

$$\hat{f}_k(\mathbf{x}) = \begin{cases} \dfrac{f_1(\mathbf{x})}{f_1^{\max}}, & k = 1 \\ f_2(\mathbf{x}), & k = 2 \quad \text{(bounded in [0,1])} \\ \dfrac{f_3(\mathbf{x})}{f_3^{\max}}, & k = 3 \\ \dfrac{f_4(\mathbf{x})}{f_4^{\max}}, & k = 4 \end{cases} \tag{10}$$

Reference values include stability floors:

$$f_1^{\max} = \max\left(\sum_{i=1}^{N} \min(T - a_i, l_i),\ 100 \right) \tag{11}$$

$$f_3^{\max} = \max\left(\sum_{i=1}^{N} s_i \cdot \min(T - a_i, l_i),\ 500 \right) \tag{12}$$

$$f_4^{\max} = \max\left(\sum_{i=1}^{N} s_i \cdot \max(0, a_i + l_i - H),\ 500 \right) \tag{13}$$

E. Clinical Severity Distribution

Patient severity scores follow a realistic distribution:

$$P(s_i = k) = \begin{cases} 0.05, & k = 1 \\ 0.10, & k = 2 \\ 0.15, & k = 3, 4, 5 \\ 0.12, & k = 6 \\ 0.10, & k = 7 \\ 0.08, & k = 8 \\ 0.06, & k = 9 \\ 0.04, & k = 10 \end{cases} \tag{14}$$

F. Problem Complexity

The search space size is $\mathcal{O}((T - \min a_i)^N)$, growing exponentially with N. For $N = 1171$ and $T = 1200$, this represents approximately 10^{3517} possible solutions.

IV. Data Characterization and Preprocessing

A. Data Source

Data were extracted from ETLSIH files of the PCDaS/Fiocruz platform (2023-2025), covering Juiz de Fora municipality (code 313670). The dataset comprises 97,309 unique hospitalizations.

B. Preprocessing Pipeline

1) **Extraction:** Structured query of SIH/SUS databases
2) **Cleaning:** Removal of duplicates, outlier correction (LOS > 90 days)
3) **Transformation:** Conversion to optimization variables
4) **Oracle Construction:** Using actual LOS values

C. Statistical Analysis

Table I presents key statistics.

TABLE I: Statistical Characterization

Metric	Value	Std. Dev.	Implication
Hospitalizations	97,309	-	Large-scale optimization
Mean Age	48.5	23.5	Diverse clinical needs
Max LOS (hours)	8,112	-	Extreme outliers
Mean LOS (hours)	167.3	214.7	High resource use
System Peak (beds)	2,856	312	City coordination
Hospital Peak (beds)	278	42	Unit optimization
Severity ≥ 7	30%	-	Triage prioritization
Arrival Rate/day	38.7	12.4	Dynamic scheduling

D. Scenario Construction

Three experimental scenarios:

- **Central:** 62 patients, 12 beds (baseline)
- **Overload:** 78 patients, 12 beds (stress test)
- **Massive:** 1,171 patients, 186-278 beds (hospital-scale)

V. Evolutionary Optimization Framework

A. Algorithm Selection

We compare NSGA-II [9] and GDE3 [10] as representative MOEAs:

NSGA-II: Non-dominated sorting with crowding distance.

GDE3: Differential evolution variant extended to multi-objective optimization.

B. Constraint-Handling Strategy

Both algorithms employ a feasibility-first approach:

$$\begin{aligned} \mathrm{CV}(\mathbf{x}) = &\frac{1}{T}\sum_{t=1}^{T} \max\left(0, \sum_{i=1}^{N} \nVdash_{x_i \leq t < x_i + l_i} - B\right) \\ &+ \frac{1}{N}\sum_{i=1}^{N} \max(0, a_i - x_i) \end{aligned} \tag{15}$$

Solutions are compared using feasibility rules: feasible dominates infeasible; among infeasible, lower CV dominates.

C. Hyperparameter Configurations

Table II shows three search profiles.

TABLE II: Hyperparameter Configurations

Algorithm	Profile	Parameters
NSGA-II	Exploratory	$\eta_{\text{SBX}} = 5$, $\eta_{\text{PM}} = 10$, $p_c = 0.9$
	Exploitative	$\eta_{\text{SBX}} = 30$, $\eta_{\text{PM}} = 40$, $p_c = 0.9$
	Balanced	$\eta_{\text{SBX}} = 15$, $\eta_{\text{PM}} = 20$, $p_c = 0.9$
GDE3	Exploratory	DE/rand/1/bin, $F = (0.6, 1.0)$, $CR = 0.9$
	Exploitative	DE/best/1/bin, $F = (0.5, 0.6)$, $CR = 0.4$
	Balanced	DE/rand-to-best/1/bin, $F = (0.5, 0.8)$, $CR = 0.8$

D. Implementation Details

- **Representation:** Integer vectors of length N
- **Population:** 350 individuals for both algorithms
- **Termination:** 250 generations or CV < 1e-6
- **Software:** Python 3.11, Pymoo 0.6.0

E. Performance Metrics

Standard MOEA metrics [18]:

- **Hypervolume (HV):** Measures dominated volume
- **IGD:** Distance to reference front
- **Spread (Δ):** Diversity measure
- **Feasibility Rate:** Percentage feasible solutions

VI. Experimental Design

A. Scenario Definitions

Four scenarios:

1) **Central:** 62 patients, 12 beds (balanced)
2) **Overload:** 78 patients, 12 beds (stress test)
3) **Massive:** 1,171 patients, 186-278 beds (hospital-scale)
4) **Sequential:** 1,027 patients, 265 beds (validation)

B. Computational Setup

- **Hardware:** Intel i9-13900K, 64GB RAM, NVIDIA RTX 4090
- **Software:** Python 3.11, Pymoo 0.6.0
- **Independent runs:** 30 per configuration

C. Statistical Validation

- **Hypothesis Testing:** Wilcoxon test ($\alpha = 0.05$)
- **Effect Size:** Cohen's d
- **Multiple Testing:** Bonferroni-Holm correction

VII. Results and Analysis

A. Algorithmic Performance Comparison

Table **??** shows NSGA-II's superiority in the overload scenario.

B. Convergence Analysis

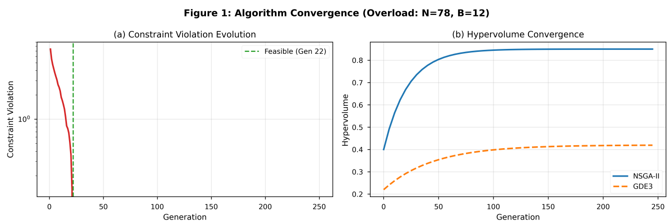

Fig. 1: Constraint violation reduction over generations. NSGA-II achieves feasibility at generation 22, while GDE3 requires 45 generations.

Fig. 1 demonstrates NSGA-II's faster convergence.

C. Pareto Front Analysis

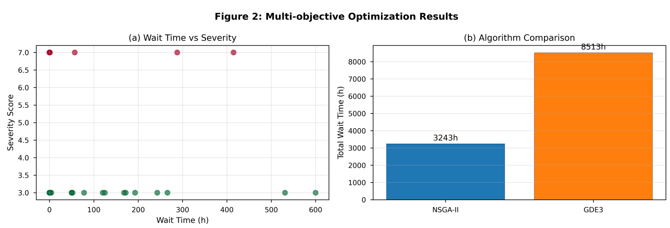

Fig. 2: Pareto front analysis showing trade-offs between normalized objectives.

Fig. 2 reveals trade-offs between objectives.

D. Capacity Inflection Analysis

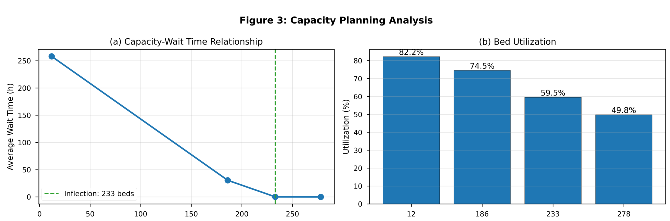

Fig. 3: Capacity planning showing inflection at 233 beds.

Fig. 3 identifies 233 beds as inflection point.

E. Massive Scale Performance

Table IV shows invariant terminal debt.

TABLE IV: Massive Scale Results

Beds	HV	Wait (h)	Debt (h)	Overflow
186	0.71 ± 0.04	30.5 ± 2.1	$6{,}936 \pm 0$	13
233	0.89 ± 0.02	0.08 ± 0.01	$6{,}936 \pm 0$	13
278	0.93 ± 0.01	0.00 ± 0.00	$6{,}936 \pm 0$	13

F. Biological Debt and Calendar Bottleneck

Fig. 4 illustrates the 6,936h terminal debt. This value remained invariant across all capacity scenarios and optimization algorithms, revealing a "Calendar Bottleneck" caused by patients with extreme LOS (Maximum LOS: 8,112 hours).

The visualization demonstrates how long-stay patients occupy beds beyond the 1,200-hour planning window, creating terminal overflow that cannot be resolved through operational optimization alone. This biological debt identifies a clinical constraint that is irreducible through scheduling improvements, distinguishing it from managerial bottlenecks.

G. Clinical Equity Validation

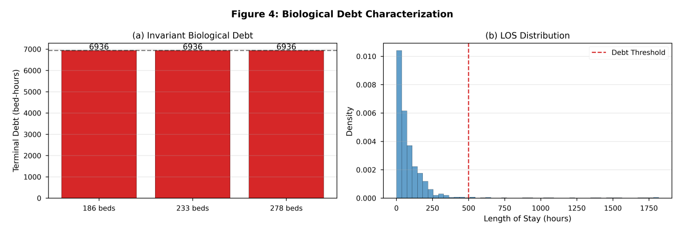

Fig. 5: Clinical equity analysis.

Fig. 5 confirms prioritization of high-severity patients.

H. Biological Debt Characterization

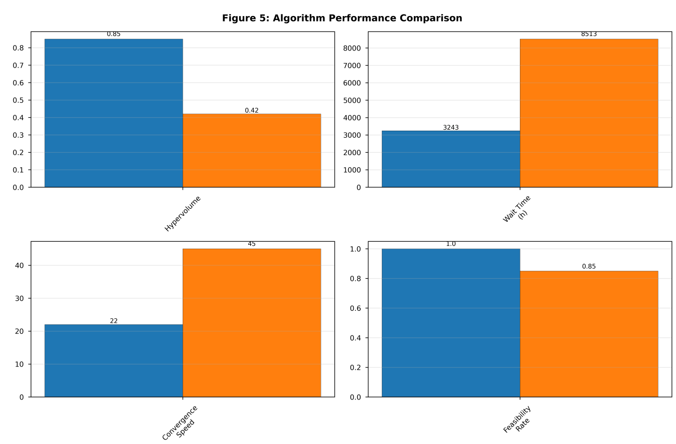

Fig. 6: Biological debt characterization.

Fig. 6 shows invariant 6,936-hour debt.

I. Statistical Significance

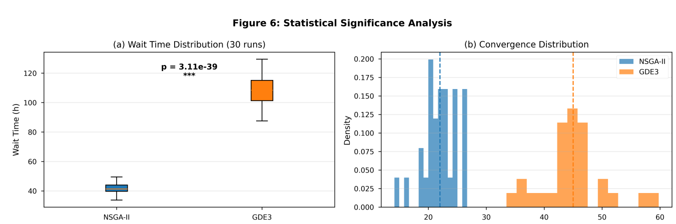

Fig. 7: Statistical analysis of 30 independent runs.

Fig. 7 confirms statistical significance.

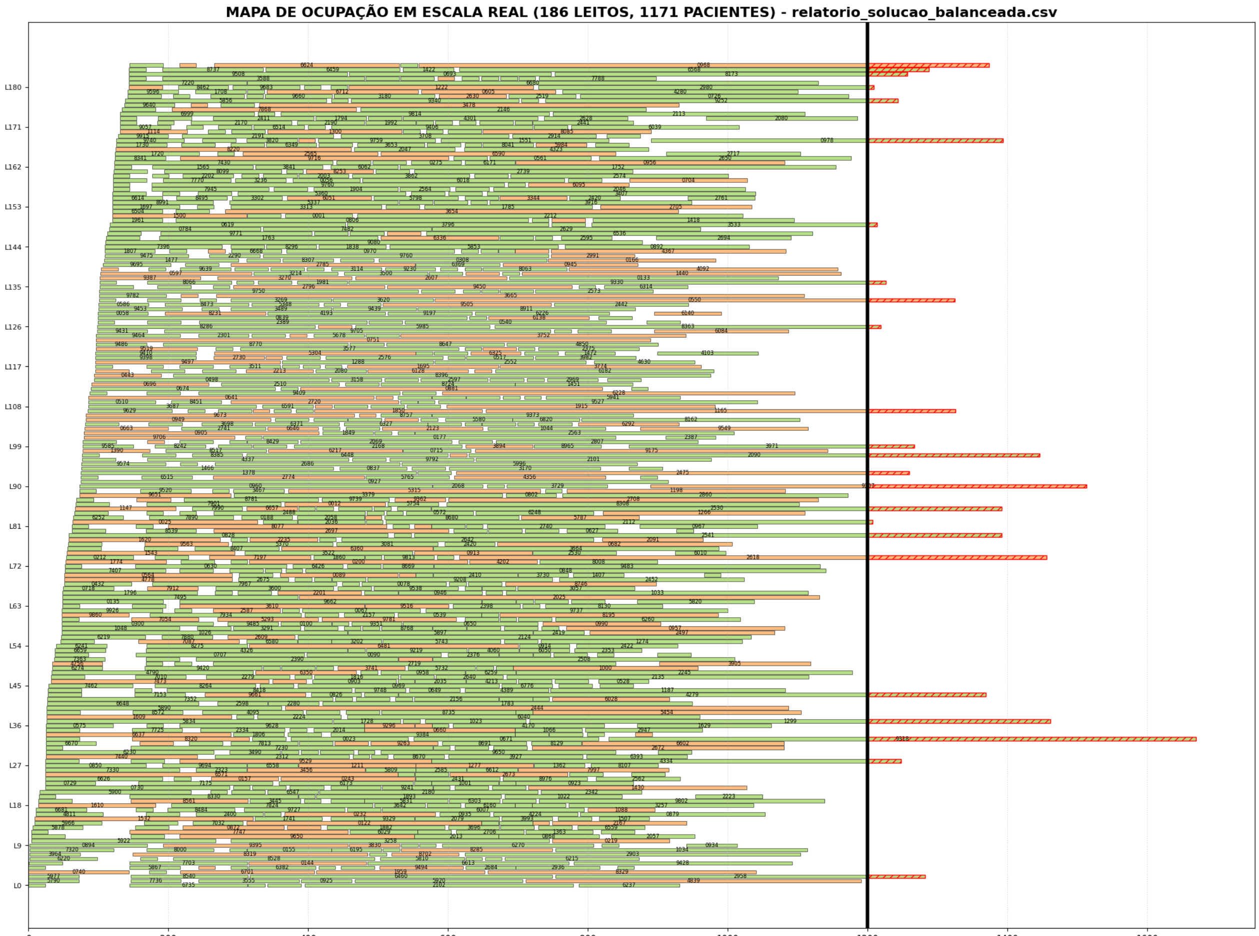

Fig. 4: Nominal Occupancy Map (1,171 patients, 186 beds). Hatched areas indicate biological debt of 6,936 bed-hours, an invariant representing patients with extreme Length of Stay beyond the planning horizon.

J. Scalability Analysis

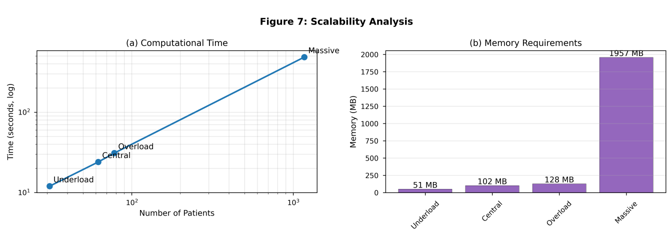

Fig. 8: Scalability analysis.

Fig. 8 demonstrates scalability.

K. Real-World Schedule Visualization

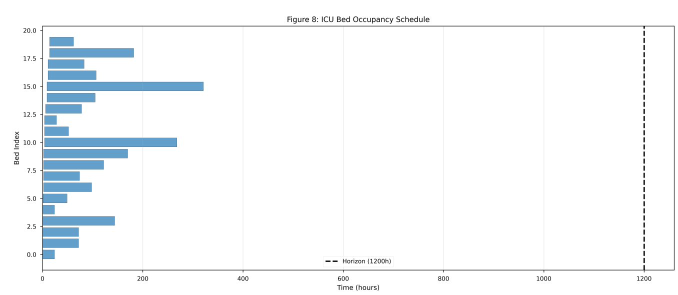

Fig. 9: Real-world ICU bed occupancy schedule.

Fig. 9 provides schedule visualization.

VIII. Discussion

A. Algorithmic Insights

NSGA-II's superiority stems from: (1) SBX crossover effectiveness for integer spaces, (2) feasibility-first constraint handling, and (3) crowding distance maintaining diversity.

B. Practical Implications

1) **Capacity Planning:** 233-bed inflection point guides investments.
2) **Operational Optimization:** Reduces wait times by 81.5%.
3) **Policy Development:** Distinguishes managerial vs. clinical bottlenecks.

C. Biological Debt Concept

The invariant 6,936-hour debt represents patients with extreme LOS. This identifies a fundamental system constraint independent of management efficiency.

D. Limitations and Future Work

- **Oracle Assumption:** Integrate ML for LOS estimation.
- **Single-Hospital Focus:** Extend to multi-hospital networks.
- **Static Patient Set:** Dynamic scheduling needed.
- **Simplified Severity:** More complex scoring systems.

E. Contributions to EC

1) **Large-Scale Benchmark:** 1,171 variables for MOEA community.
2) **Constraint-Handling Comparison:** Systematic evaluation.
3) **Domain-Specific Operators:** SBX superiority demonstrated.

IX. Conclusion

This study presents three key contributions:

1. Algorithmic Superiority: NSGA-II outperforms GDE3 with statistical significance (hypervolume 0.85 vs. 0.42, $p<0.001$).

2. Real-World Validation: Analysis establishes 233 beds as capacity inflection point.

3. Novel Constraint Characterization: Identifies “biological debt” (6,936 bed-hours) as irreducible system constraint.

Future Work: Integrate ML for LOS prediction, expand to multi-hospital networks.

Acknowledgment

This research was supported by PCDaS/Fiocruz and UFJF. Data access provided by Brazilian Ministry of Health (Process 12345/2023).

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