

---

## What this code is about

The Padé approximant to a given formal power series expansion  $\sum_{n=0}^{\infty} a_n \beta^n$ , is given by

$$P_M^N(\beta) = \frac{\sum_{n=0}^N A_n \beta^n}{\sum_{n=0}^M B_n \beta^n}, \quad B_0 = 1, \quad (1)$$

where

$$\mathbf{M} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_M \end{bmatrix} = - \begin{bmatrix} a_{N+1} \\ a_{N+2} \\ \vdots \\ a_{N+M} \end{bmatrix}, \quad (2)$$

With  $\mathbf{M}_{i,j} = a_{N+i-j}$  ( $1 \leq i, j \leq M$ ). The coefficients in the numerator are

$$A_n = \sum_{j=0}^n a_{n-j} B_j, \quad 0 \leq n \leq N. \quad (3)$$

The C++ code `constant.cpp` solves the system (2) for the coefficients  $B_j$  and writes to the file `Constant.txt`. The matrix  $\mathbf{M}$  and the vector at the right-hand side of (2) are populated by the coefficients  $a_n$ 's which are read-in from the file `moments.txt`. We apply the Pade approximant to the energy correction of the ground-state energy of the funnel potential,

$$E(\beta) = 1 + \sum_{k=1}^{\infty} e^{(k)} \beta^k = 1 + \beta \sum_{k=0}^{\infty} e^{(k+1)} \beta^k \quad (4)$$

The file `const.sh` is a shell script used to compile and run the code in an Ubuntu 22.04 local machine.