
What this code is about

The Padé approximant to a given formal power series expansion $\sum_{n=0}^{\infty} a_n \beta^n$, is given by

$$P_M^N(\beta) = \frac{\sum_{n=0}^N A_n \beta^n}{\sum_{n=0}^M B_n \beta^n}, \quad B_0 = 1, \quad (1)$$

where

$$\mathbf{M} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_M \end{bmatrix} = - \begin{bmatrix} a_{N+1} \\ a_{N+2} \\ \vdots \\ a_{N+M} \end{bmatrix}, \quad (2)$$

With $\mathbf{M}_{i,j} = a_{N+i-j}$ ($1 \leq i, j \leq M$). The coefficients in the numerator are

$$A_n = \sum_{j=0}^n a_{n-j} B_j, \quad 0 \leq n \leq N. \quad (3)$$

The C++ code `pade.cpp` computes the Padé approximant (1) for $\beta = 10^{-5} - 10^{23}$, 0.2 and $\beta = 4$. The coefficients B_j are read in from the file `Constant.txt` while the coefficients a_j are read in from the file `moments.txt`. The result for $P_M^N(\beta)$ are written to `pade.txt`. We apply the Pade approximant to the energy correction of the ground-state energy of the funnel potential,

$$E(\beta) = -\frac{1}{2} \left(1 - \beta \sum_{k=0}^{\infty} \mu_k (-\beta)^k \right) \quad (4)$$

The file `mpfr.sh` is a shell script used to compile and run the code in an Ubuntu 22.04 local machine.