
What this code is about

The C++ code `result.cpp` computes the ground state energy $E_0(\beta)$ for the Quartic anharmonic oscillator

$$E(\beta) = -\frac{1}{2} \left(1 - \sum_{k=0}^{\infty} \frac{(-1)^k \mu_{-(k+1)}}{\beta^k} - \frac{\pi g(-\frac{1}{\beta}) \beta^{2/3}}{\sin(2\pi/3)} \right). \quad (1)$$

The second term in the right hand side above

$$\sum_{k=0}^{\infty} \frac{(-1)^k \mu_{-(k+1)}}{\beta^k} = \sum_{k=0}^d \frac{(-1)^k}{\beta^k} (A_k + B_k + C_k) + \sum_{k=d+1}^{\infty} \frac{(-1)^k}{\beta^k} D_k, \quad (2)$$

where

$$A_k = \sum_{m=0}^k c_m m! \sum_{l=0}^m \frac{(-1)^l}{(l!)^2 (m-l)!} \int_0^{\infty} \frac{e^{-x/2}}{x^{k+\nu+1-l}} dx, \quad (3)$$

$$B_k = \sum_{m=k+1}^d c_m m! \sum_{l=0}^k \frac{(-1)^l}{(l!)^2 (m-l)!} \int_0^{\infty} \frac{e^{-x/2}}{x^{k+\nu+1-l}} dx, \quad (4)$$

$$C_k = \sum_{m=k+1}^d c_m m! \sum_{l=k+1}^m \frac{(-1)^l \Gamma(l-k-\nu) 2^{l-k-\nu}}{(l!)^2 (m-l)!}, \quad (5)$$

and

$$D_k = \sum_{m=0}^d c_m m! \sum_{l=0}^m \frac{(-1)^l}{(l!)^2 (m-l)!} \int_0^{\infty} \frac{e^{-x/2}}{x^{k+\nu+1-l}} dx. \quad (6)$$

The finite part integrals appearing above are given by,

$$\int_0^{\infty} \frac{e^{-x/2}}{x^{k+\nu+1-l}} dx = \frac{(-1)^{k-l+1} \left(\frac{1}{2}\right)^{k-l+1+\nu} \pi}{\Gamma(k-l+1+\nu) \sin(\pi\nu)}, \quad (7)$$

with $\nu = 2/3$. In the third term of the right-hand side of equation (1),

$$g(x) = e^{-x/2} \sum_{m=0}^d c_m m! \sum_{k=0}^m \frac{(-x)^k}{(k!)^2 (m-k)!}, \quad (8)$$

The terms in (2) for various β are read-in from files FIRST.txt, SECOND.txt, THIRD.txt, FOURTH.txt while the third term in equation (1) is read-in from FIFTH.txt. The result for $E_0(\beta)$ is written to the file disectres.txt

The file compile.job is a SLURM script to compile the code in an HPC and generate an executable.

The file together.job is a SLURM script to run the executable in an HPC.

The file mpfr.sh is a shell script used to compile and run the code in an Ubuntu 22.04 local machine.