
What this code is about

The Padé approximant to a given formal power series expansion $\sum_{n=0}^{\infty} a_n \beta^n$, is given by

$$P_M^N(\beta) = \frac{\sum_{n=0}^N A_n \beta^n}{\sum_{n=0}^M B_n \beta^n}, \quad B_0 = 1, \quad (1)$$

where

$$\mathbf{M} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_M \end{bmatrix} = - \begin{bmatrix} a_{N+1} \\ a_{N+2} \\ \vdots \\ a_{N+M} \end{bmatrix}, \quad (2)$$

With $\mathbf{M}_{i,j} = a_{N+i-j}$ ($1 \leq i, j \leq M$). The coefficients in the numerator are

$$A_n = \sum_{j=0}^n a_{n-j} B_j, \quad 0 \leq n \leq N. \quad (3)$$

The `c++` code `constant.cpp` solves the system (2) for the coefficients B_j and writes to the file `Constant.txt`. The matrix \mathbf{M} and the vector at the right-hand side of (2) are populated by the coefficients a_n 's which are read-in from the file `../moments/moments.txt`. We apply the Pade approximant to the alternating divergent weak-field expansion for the Heisenberg-Euler Lagrangian in the case of a purely magnetic background given in equation (3.3),

$$f(\beta) = \sum_{n=2}^{\infty} a_n (-\beta)^n, \quad a_n = (-1)^n (2n-3)! c_n, \quad c_n = \frac{2-2^{2n}}{(2n)!} B_{2n}, \quad (4)$$

as $\beta \rightarrow 0$, where B_{2n} are the Bernoulli numbers. The file `run.sh` encapsulates commands to build and run the application using the `CMakeLists.txt` on a local machine running on `Ubuntu 24.04`.