

---

## What this code is about

The `c++` code `fourth.cpp` computes the fourth term in equation (4.26) in the paper,

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{\beta^{k-1}} \mu_{-(2k+2)} = \sum_{k=0}^{\lfloor \frac{d-1}{2} \rfloor} \frac{(-1)^k}{\beta^{k-1}} (I_k + J_k + L_k) + \sum_{k=\lfloor \frac{d-1}{2} \rfloor + 1}^{\infty} \frac{(-1)^k}{\beta^{k-1}} M_k, \quad (1)$$

where

$$M_k = \sum_{m=0}^d c_m m! \sum_{l=0}^m \frac{(-1)^l}{(l!)^2 (m-l)!} \int_0^{\infty} \frac{e^{-x/2}}{x^{2k+1-l}} dx. \quad (2)$$

and

$$\int_0^{\infty} \frac{e^{-x/2}}{x^{2k+1-l}} dx = \frac{(-1)^{1-l} \left(\frac{1}{2}\right)^{2k-l}}{(2k-l)!} \left( \ln \left( \frac{1}{2} \right) - \psi(2k+1-l) \right). \quad (3)$$

The code requires the  $d+1$  numbers  $c_m$ 's as inputs. These are read-in from the file `../Constants/Constant.txt`. The code outputs values for  $\beta = 10^{-5} - 10^{23}$ ,  $0.2$  and  $\beta = 4$  to the file `../results/FOURTH.txt`.

The file `run.sh` encapsulates commands to build and run the application using the `CMakeLists.txt` on local machine running on Ubuntu 24.04.