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## What this code is about

The `c++` source code `constant.cpp` solves the system of  $d+1$  linear equations for the first  $d+1$  expansion coefficients  $c_m$

$$a_{n+2} = \sum_{m=0}^d c_m P(n, m) \quad (1)$$

where the matrix  $P(n, m)$  is given by

$$P(n, m) = m! 2^{2n+2} \sum_{k=0}^m \frac{(-2)^k (2n+k+1)!}{(k!)^2 (m-k)!}. \quad (2)$$

The matrix elements are read-in from the file `matrix_p.txt`. In the case of the Heisenberg-Euler Lagrangian in a purely electric background, the input  $a_{n+2}$  are the first  $d+1$  coefficients of the divergent weak magnetic field expansion given in equation (3.3),

$$f(\kappa) = \sum_{n=2}^{\infty} a_n \kappa^n, \quad \kappa \rightarrow 0. \quad (3)$$

These are read from the file `moments.txt`. The code writes to the file `constant.txt` the quantity  $c_m m!$  which will be relevant the reconstruction in equation (4.23)

$$g(x) = e^{-x/2} \sum_{m=0}^{\infty} c_m m! \sum_{k=0}^m \frac{(-x)^k}{(k!)^2 (m-k)!}. \quad (4)$$

and in the subsequent computations.

The file `run.sh` encapsulates commands to build and run the application using the `CMakeLists.txt` on a local machine running on Ubuntu 24.04.