What this code is about

The `c++` code `result.cpp` computes the Heisenberg-Euler Lagrangian in the case of a purely magnetic background,

$$f(\beta) = \sum_{k=0}^{\infty} \frac{(-1)^k}{\beta^{k-1}} \mu_{-(2k+2)} + \beta\Delta(\beta). \tag{1}$$

The first term in the right hand side above is computed as,

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{\beta^{k-1}} \mu_{-(2k+2)} = \sum_{k=0}^{\lfloor \frac{d-1}{2} \rfloor} \frac{(-1)^k}{\beta^{k-1}} \left(I_k + J_k + L_k\right) + \sum_{k=\lfloor \frac{d-1}{2} \rfloor + 1}^{\infty} \frac{(-1)^k}{\beta^{k-1}} M_k, \tag{2}$$

where

$$I_k = \sum_{m=0}^{2k} c_m m! \sum_{l=0}^{m} \frac{(-1)^l}{(l!)^2 (m-l)!} \fint_0^{\infty} \frac{e^{-x/2}}{x^{2k+1-l}} \mathrm{d}x, \tag{3}$$

$$J_k = \sum_{m=2k+1}^{d} c_m m! \sum_{l=0}^{2k} \frac{(-1)^l}{(l!)^2 (m-l)!} \fint_0^{\infty} \frac{e^{-x/2}}{x^{2k+1-l}} \mathrm{d}x, \tag{4}$$

$$L_k = \sum_{m=2k+1}^{d} c_m m! \sum_{l=2k+1}^{m} \frac{(-1)^l \, (l-2k-1)! \, 2^{l-2k}}{(l!)^2 (m-l)!}, \tag{5}$$

and

$$M_k = \sum_{m=0}^{d} c_m m! \sum_{l=0}^{m} \frac{(-1)^l}{(l!)^2 (m-l)!} \fint_0^{\infty} \frac{e^{-x/2}}{x^{2k+1-l}} \mathrm{d}x. \tag{6}$$

The finite part integrals appearing above are given by,

$$\fint_0^{\infty} \frac{e^{-x/2}}{x^{2k+1-l}} \mathrm{d}x = \frac{(-1)^{1-l} \left(\frac{1}{2}\right)^{2k-l}}{(2k-l)!} \left(\ln\left(\frac{1}{2}\right) - \psi(2k+1-l) \right). \tag{7}$$

Each of the terms in (2) are read-in from files `FIRST.txt`, `SECOND.txt`, `THIRD.txt`, `FOURTH.txt` for $\beta = 10^{-5} - 10^{23}, 0.2$ and $\beta = 4$.

The second term in the right hand side of (1) is computed using

$$\Delta(\beta) = \frac{\pi\sqrt{\beta}}{4} \left(\rho\left(\frac{i}{\sqrt{\beta}}\right) + \rho\left(\frac{-i}{\sqrt{\beta}}\right) \right) + \frac{\sqrt{\beta} \ln \beta}{4i} \left(\rho\left(\frac{i}{\sqrt{\beta}}\right) - \rho\left(\frac{-i}{\sqrt{\beta}}\right) \right), \tag{8}$$

where

$$\rho(x) = xg(x) = xe^{-x/2} \sum_{m=0}^{d} c_m m! \sum_{k=0}^{m} \frac{(-x)^k}{(k!)^2(m-k)!}. \tag{9}$$

The values for the same range of β is read from the file `FIFTH.txt`. $f(\beta)$ is then computed for the specified range of β and the results are written to the file `disectres.txt`.

The file `run.sh` encapsulates commands to build and run the application using the `CMakeLists.txt` on a local machine running on Ubuntu 24.04.