What this code is about

The `c++` code `result.cpp` computes the real part of the complex Heisenberg-Euler Lagrangian in the case of a purely electric background,

$$f(\kappa) = -\sum_{n=0}^{\infty} \frac{\mu_{-(2n+2)}}{\kappa^{n-1}} - \kappa\Lambda(\kappa) + \frac{i\pi}{2}\kappa^{3/2}\rho\left(\frac{1}{\sqrt{\kappa}}\right), \tag{1}$$

The first term in the right hand side above is computed as,

$$-\sum_{k=0}^{\infty} \frac{\mu_{-(2k+2)}}{\kappa^{k-1}} = -\sum_{k=0}^{\lfloor\frac{d-1}{2}\rfloor} \frac{1}{\kappa^{k-1}}\left(I_k + J_k + L_k\right) - \sum_{k=\lfloor\frac{d-1}{2}\rfloor+1}^{\infty} \frac{M_k}{\kappa^{k-1}}, \tag{2}$$

where

$$I_k = \sum_{m=0}^{2k} c_m m! \sum_{l=0}^{m} \frac{(-1)^l}{(l!)^2(m-l)!} \fint_0^{\infty} \frac{e^{-x/2}}{x^{2k+1-l}} \mathrm{d}x, \tag{3}$$

$$J_k = \sum_{m=2k+1}^{d} c_m m! \sum_{l=0}^{2k} \frac{(-1)^l}{(l!)^2(m-l)!} \fint_0^{\infty} \frac{e^{-x/2}}{x^{2k+1-l}} \mathrm{d}x, \tag{4}$$

$$L_k = \sum_{m=2k+1}^{d} c_m m! \sum_{l=2k+1}^{m} \frac{(-1)^l\,(l-2k-1)!\,2^{l-2k}}{(l!)^2(m-l)!}, \tag{5}$$

and

$$M_k = \sum_{m=0}^{d} c_m m! \sum_{l=0}^{m} \frac{(-1)^l}{(l!)^2(m-l)!} \fint_0^{\infty} \frac{e^{-x/2}}{x^{2k+1-l}} \mathrm{d}x. \tag{6}$$

The finite part integrals appearing above are given by,

$$\fint_0^{\infty} \frac{e^{-x/2}}{x^{2k+1-l}} \mathrm{d}x = \frac{(-1)^{1-l}\left(\frac{1}{2}\right)^{2k-l}}{(2k-l)!}\left(\ln\left(\frac{1}{2}\right) - \psi(2k+1-l)\right). \tag{7}$$

Each of the terms in (2) are read-in from the files `FIRST.txt`, `SECOND.txt`, `THIRD.txt`, `FOURTH.txt` for $\kappa = 10^{-5} - 10^{23}, 0.2$ and $\kappa = 4$.

The second term in the right hand side of (1) is computed using

$$\Lambda(\kappa) = \frac{\sqrt{\kappa}}{2}\ln\left(\sqrt{\kappa}\right)\left(\rho\left(\frac{1}{\sqrt{\kappa}}\right) - \rho\left(-\frac{1}{\sqrt{\kappa}}\right)\right). \tag{8}$$

The values for the same range of κ is read from the file `FIFTH.txt`. Finally, the imaginary part in the right hand side of (1) is computed using,

$$\rho(x) = xg(x) = xe^{-x/2} \sum_{m=0}^{d} c_m m! \sum_{k=0}^{m} \frac{(-x)^k}{(k!)^2(m-k)!}. \tag{9}$$

The values for the same range of κ is given in the file `SIXTH.txt`. The real part is then computed for the specified range of κ and the results are written to the file `disectres.txt`.

The file `run.sh` encapsulates commands to build and run the application using the `CMakeLists.txt` on a local machine running on Ubuntu 24.04.