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## What this code is about

The `c++` code `first.cpp` computes the highlighted term below in the first term of equation (4.39),

$$-\sum_{k=0}^{\infty} \frac{\mu_{-(2k+2)}}{\kappa^{k-1}} = -\sum_{k=0}^{\lfloor \frac{d-1}{2} \rfloor} \frac{1}{\kappa^{k-1}} (\textcolor{red}{I}_k + J_k + L_k) - \sum_{k=\lfloor \frac{d-1}{2} \rfloor + 1}^{\infty} \frac{M_k}{\kappa^{k-1}}, \quad (1)$$

where

$$I_k = \sum_{m=0}^{2k} c_m m! \sum_{l=0}^m \frac{(-1)^l}{(l!)^2 (m-l)!} \int_0^{\infty} \frac{e^{-x/2}}{x^{2k+1-l}} dx, \quad (2)$$

and

$$\int_0^{\infty} \frac{e^{-x/2}}{x^{2k+1-l}} dx = \frac{(-1)^{1-l} \left(\frac{1}{2}\right)^{2k-l}}{(2k-l)!} \left( \ln \left( \frac{1}{2} \right) - \psi(2k+1-l) \right). \quad (3)$$

The code requires the  $d+1$  numbers  $c_m$ 's as inputs. These are read-in from the file `../Constants/Constant.txt`. The code outputs values for  $\beta = 10^{-5} - 10^{23}$ ,  $0.2$  and  $\beta = 4$  to the file `../results/FIRST.txt`.

The file `run.sh` encapsulates commands to build and run the application using the `CMakeLists.txt` on local machine running on Ubuntu 24.04.