What this code is about

The Padé approximant to a given formal power series expansion $\sum_{n=0}^{\infty} a_n \beta^n$, is given by

$$P_M^N(\beta) = \frac{\sum_{n=0}^{N} A_n \beta^n}{\sum_{n=0}^{M} B_n \beta^n}, \quad B_0 = 1, \tag{1}$$

where

$$\mathbf{M} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_M \end{bmatrix} = - \begin{bmatrix} a_{N+1} \\ a_{N+2} \\ \vdots \\ a_{N+M} \end{bmatrix}, \tag{2}$$

With $\mathbf{M}_{i,j} = a_{N+i-j} (1 \leq i, j \leq M)$. The coefficients in the numerator are

$$A_n = \sum_{j=0}^{n} a_{n-j} B_j, \quad 0 \leq n \leq N. \tag{3}$$

The `c++` code `pade.cpp` computes the Padé approximant (1) for $\beta = 10^{-5} - 10^{23}, 0.2$ and $\beta = 4$. The coefficients B_j are read in from the file `../Constants/Constant.txt` while the coefficients a_j are read in from the file `../moments/moments.txt`. The result for $P_M^N(\beta)$ are written to the file `pade.txt`. We apply the Pade approximant to the alternating divergent weak-field expansion for the Heisenberg-Euler Lagrangian in the case of a purely magnetic background given in equation (3.3),

$$f(\beta) = \sum_{n=2}^{\infty} a_n (-\beta)^n, \qquad a_n = (-1)^n (2n-3)! c_n, \qquad c_n = \frac{2 - 2^{2n}}{(2n)!} B_{2n}, \tag{4}$$

as $\beta \to 0$, where B_{2n} are the Bernoulli numbers. The file `run.sh` encapsulates commands to build and run the application using the `CMakeLists.txt` on a local machine running on `Ubuntu 24.04`.