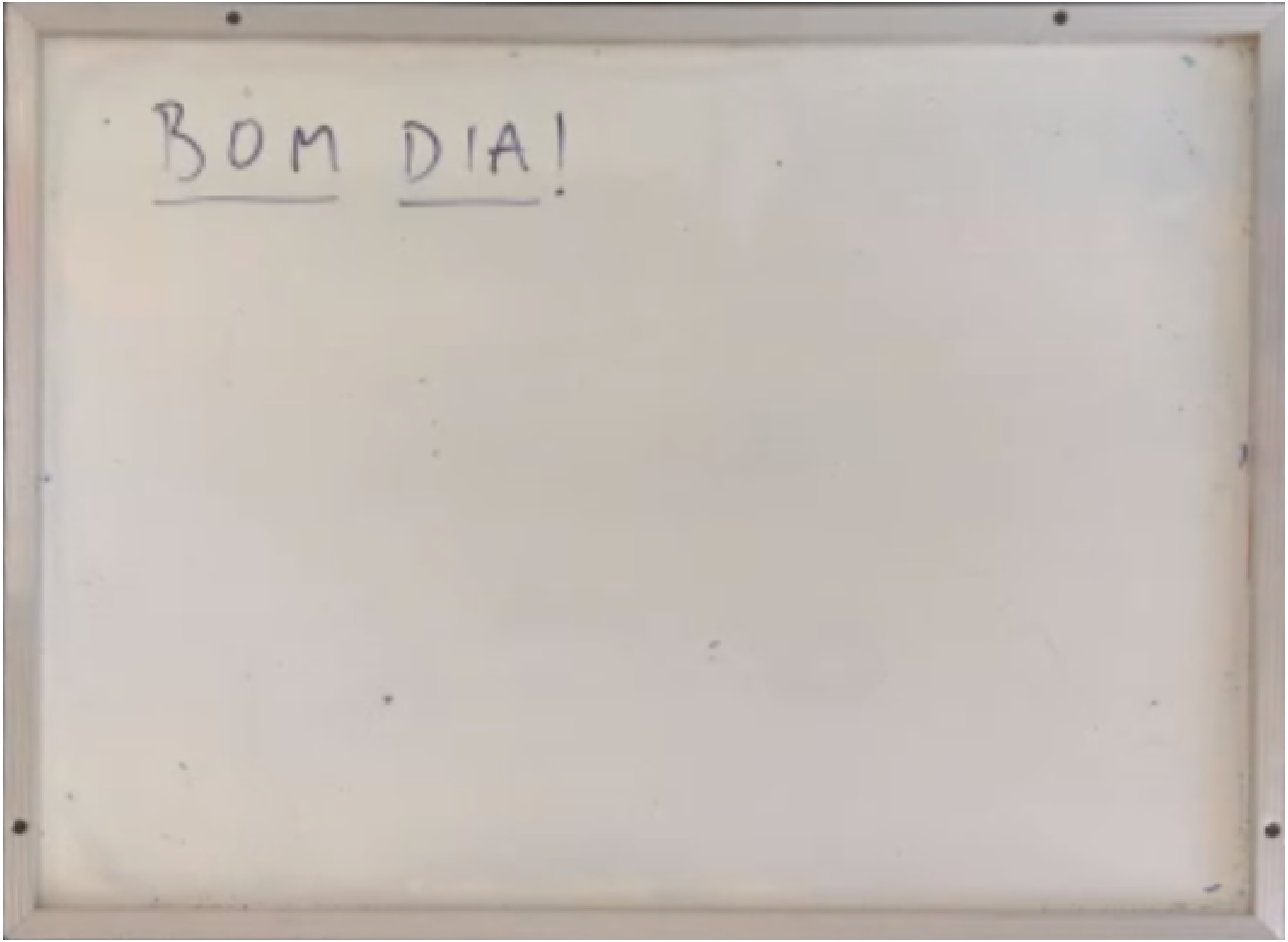

BOM DIA!

CIAG 2020 - CALC. 1 VAR - 2º DIA: 27/03

REVISÃO: CÁLCULO → função

* Associação
- Exclusividade $D_f \xrightarrow{f} CD_f \supset Im_f$,
- Exaustividade

* Conjuntos numéricos: $\mathbb{N} \to \dots \to \mathbb{R} (\to \mathbb{C} \to)$

CIAG 2020 - CALC. 1VAR - 2º DIA: 27/03

REVISÃO: CÁLCULO → função

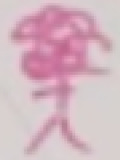
* Associação

- Exclusividade
- Exaustividade

$$D_f \xrightarrow{f} CD_f \supset Im_f$$

* Conjuntos numéricos: $\mathbb{N} \to \dots \to \mathbb{R}\ (\to \mathbb{C} \to)$

$\{0,1,2,3,\dots\}$



↳ Intervalos

$[0,1,2,\dots] \longleftrightarrow \quad \longrightarrow \mathbb{R}$

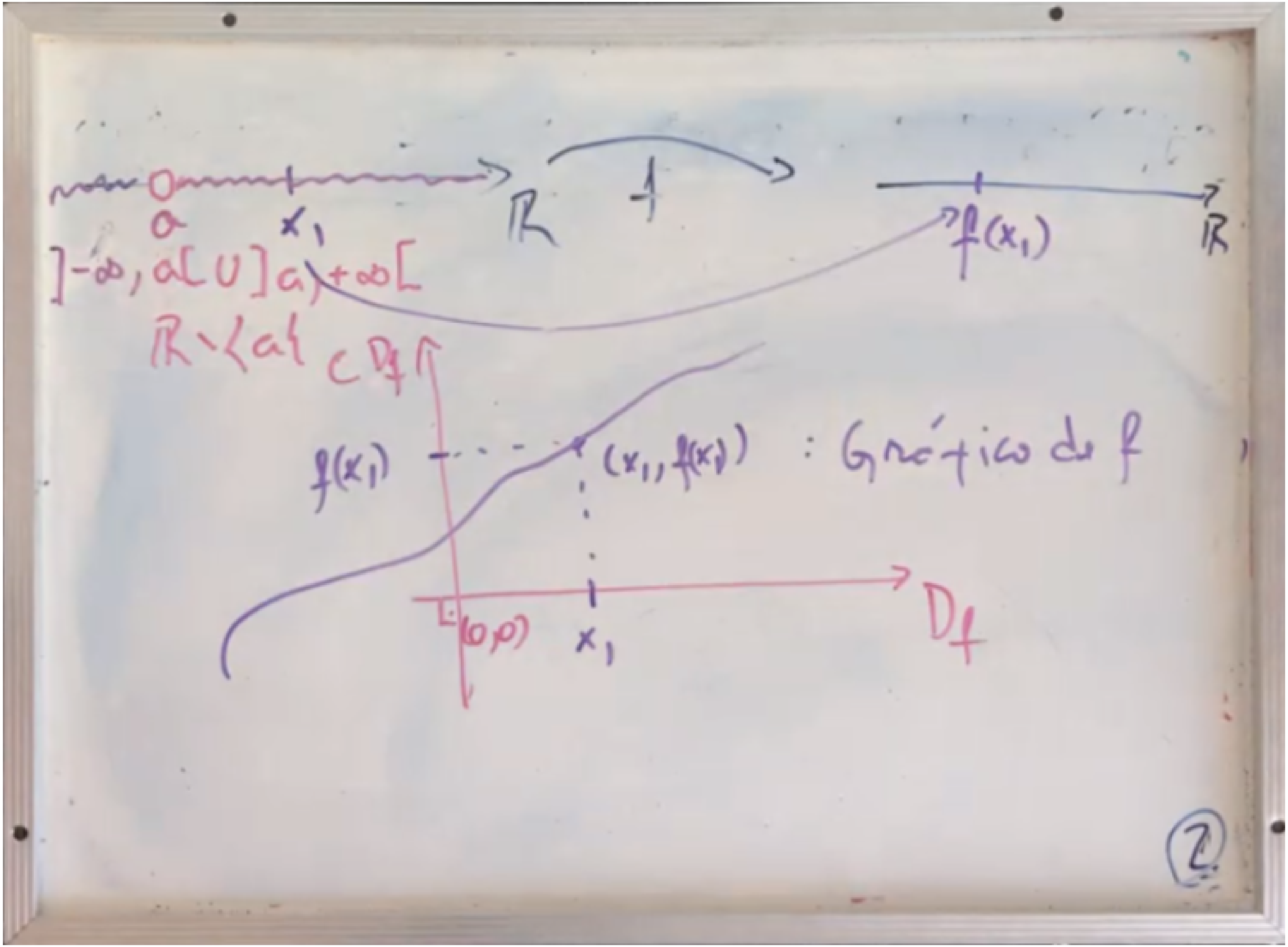
$[a,b] \subset \mathbb{R}$

$]a,b[$

$$\underset{a}{\bullet}\!\!-\!\!-\!\!-\!\!\underset{b}{\bullet} \longrightarrow \mathbb{R}$$

$$\underset{a}{\circ}\!\!-\!\!-\!\!-\!\!\underset{b}{\circ} \longrightarrow \mathbb{R}$$

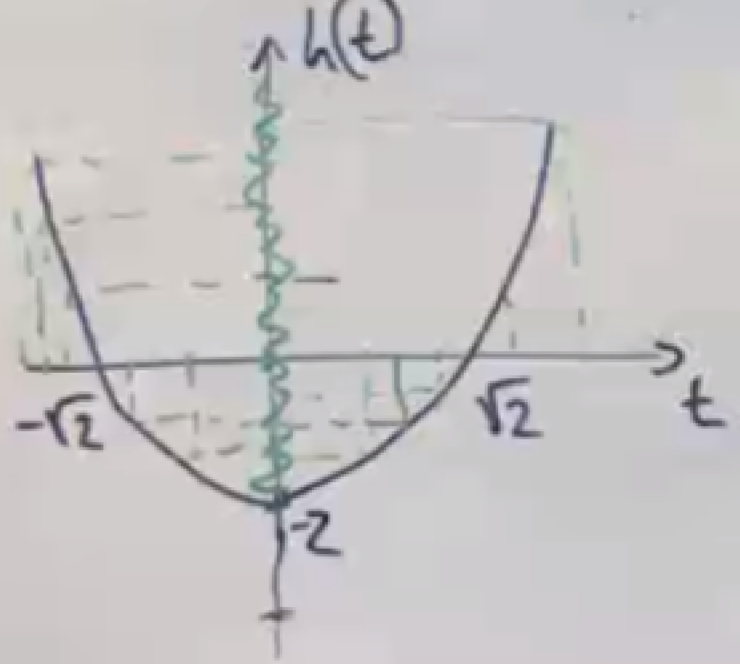
1



a
x₁
ℝ
f
f(x₁)
ℝ
]-∞, a[∪]a, +∞[
ℝ \ {a}
= D_f
f(x₁)
(x₁, f(x₁)) : Gráfico de f
(0,0)
x₁
D_f
2

ex: $h(t) = t^2 - 2$: $D_h = \mathbb{R}$; $CD_h = \mathbb{R}$

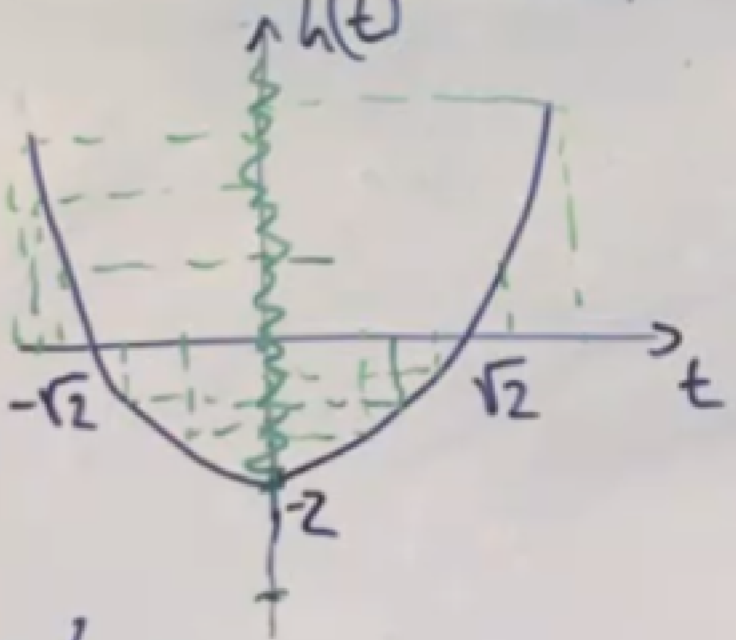
$Im_h = [-2, +\infty[$



3

ex: $h(t) = t^2 - 2$: $D_h = \mathbb{R}$; $CD_h = \mathbb{R}$

$Im_h = [-2, +\infty[$



$x^2 + 3x + 2 \neq 0$

$\Delta = b^2 - 4ac$

$x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

P.37 - 4

(a) $D_f = \{x \in \mathbb{R} \mid x \neq 1/3\} = \mathbb{R} \setminus \{1/3\} =]-\infty, \frac{1}{3}[\cup]\frac{1}{3}, \infty[$

$\sqrt{-4} = 2i$

np.sqrt(-4+0j) → 2j 3

$$e^{i\theta} = \cos\theta + i\,\mathrm{sen}\,\theta$$

$$\mathrm{Re}[e^{i\theta}] = \cos\theta$$

$$\mathrm{Im}[e^{i\theta}] = \mathrm{sen}\,\theta = \mathrm{Re}[e^{i(\theta+\pi/2)}]$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\mathrm{sen}\,\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

T. Hilbert

4

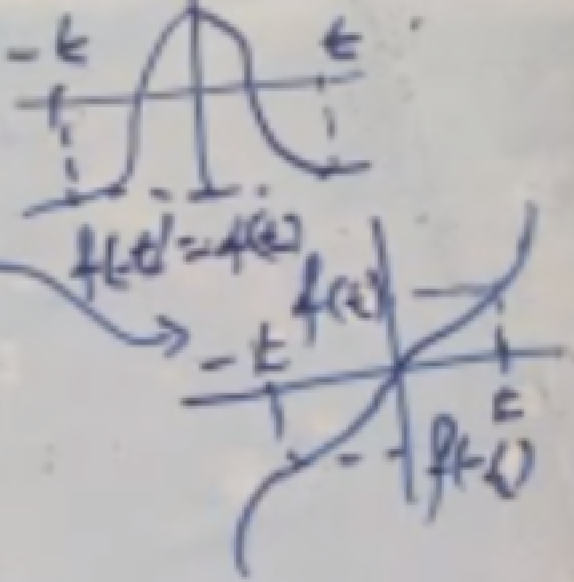
$$\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + \omega_0^2\, y = \boxed{f(t)}$$

$$y(t) = \underbrace{A\, e^{-\gamma t/2}}_{\text{parte absorção}}\ \underbrace{\cos(\omega t - \phi)}_{\text{parte oscilatória}} + f(t)$$

$$e^{(a+ib)}$$

$f(t) = f(-t)$: par

$f(t) = -f(-t)$: ímpar



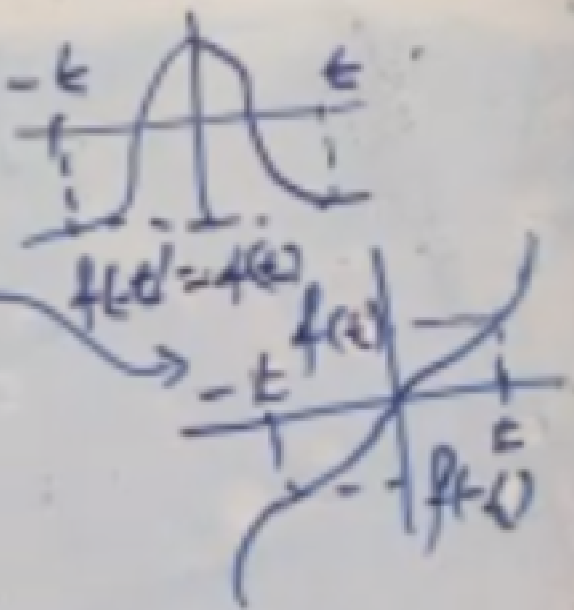
$$S(t) = S_0 + v_0 t + \frac{a t^2}{2}$$

$$S(-t) = S_0 - v_0 t + \frac{a t^2}{2}$$

$$\Rightarrow S(-t) = S(t) \Rightarrow v_0 = 0$$

$f(t) = f(-t)$: par

$f(t) = -f(-t)$: ímpar



$$S(t) = S_0 + v_0 t + \frac{a t^2}{2}$$

$$S(-t) = S_0 - v_0 t + \frac{a t^2}{2}$$

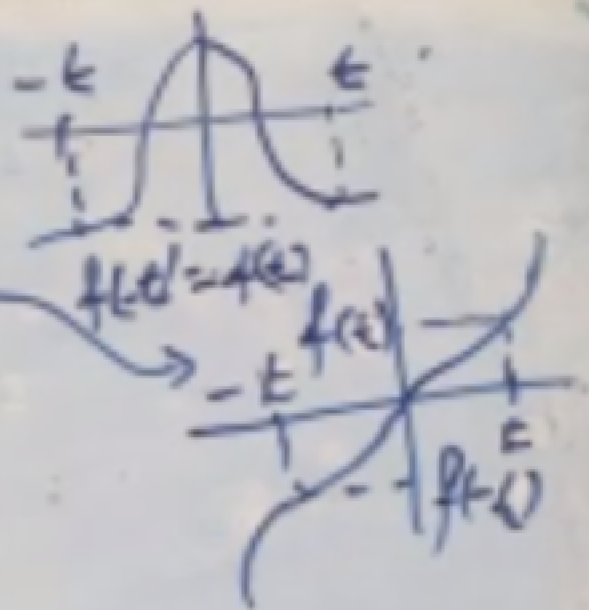
$$\Rightarrow \quad S(-t) = S(t) \Rightarrow v_0 = 0$$

$$S(-t) = -S(t) \Rightarrow S_0 = 0 \text{ e } a = 0$$

4

$f(t) = f(-t)$: par

$f(t) = -f(-t)$: ímpar



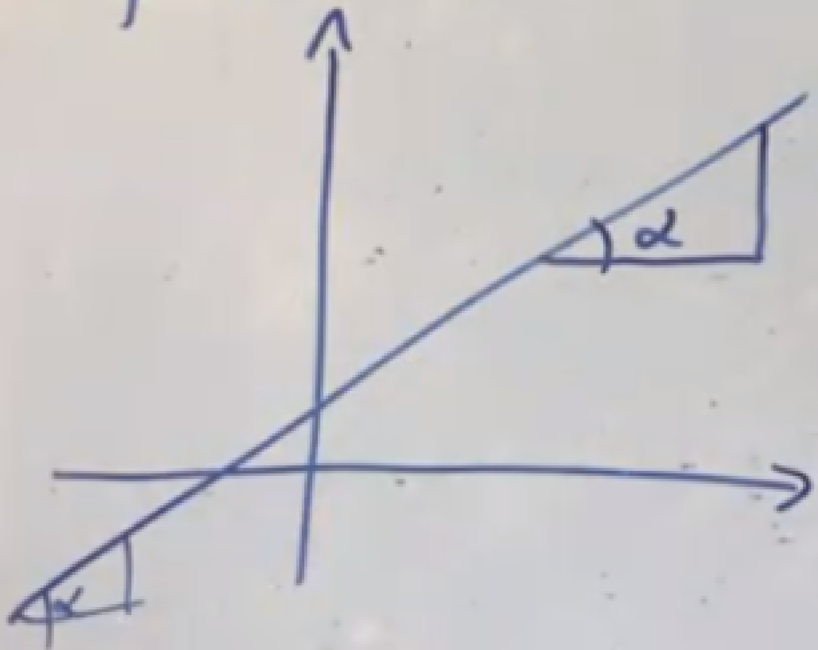
$$s(t) = s_0 t^0 + v_0 t^{1} + \frac{a t^{2}}{2}$$

$$s(-t) = s_0 - v_0 t + \frac{a t^2}{2}$$

$$\Rightarrow \begin{aligned} s(-t) &= s(t) \Rightarrow v_0 = 0 \\ s(-t) &= -s(t) \Rightarrow s_0 = 0 \text{ e } a = 0 \end{aligned}$$

Repertório

1) $f(x) = ax + b \rightsquigarrow (x, f(x)) = (x, ax+b)$

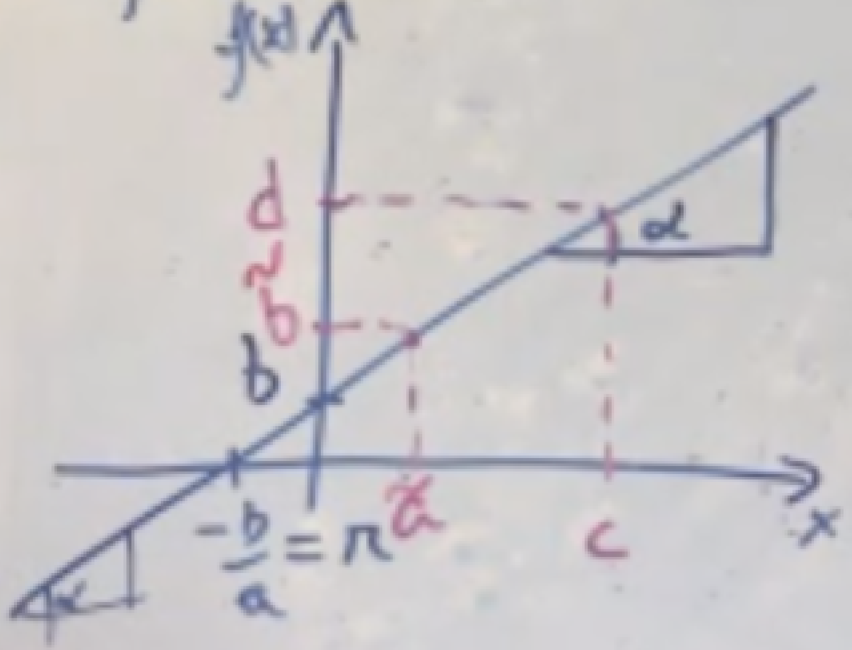


$tg\,\alpha = a$

5

Repertório

1) $f(x) = ax + b \rightsquigarrow (x, f(x)) = (x, ax+b)$



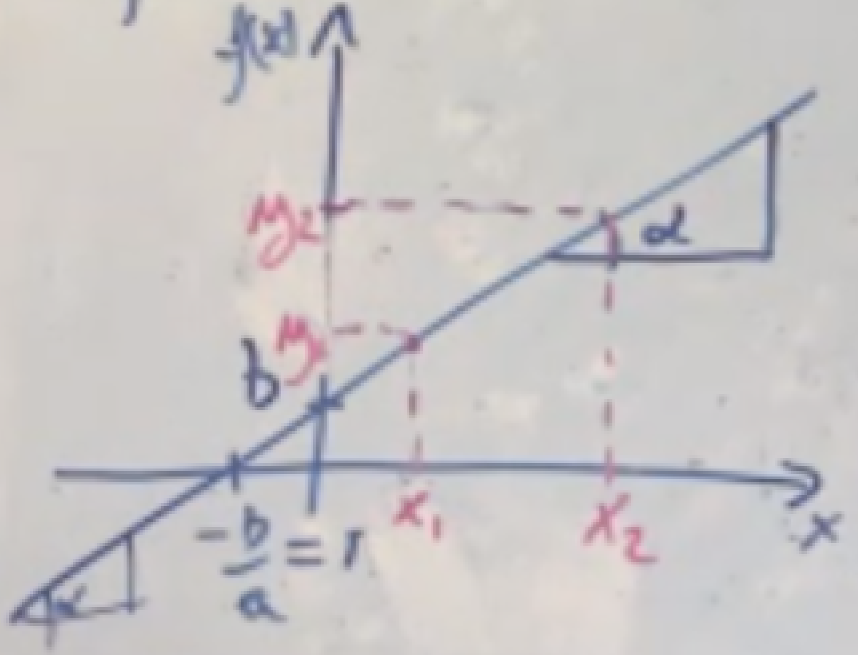
$tg\,\alpha = a$

$f(x) = ax + b$

$$\begin{cases} f(\tilde{a}) = a\tilde{a} + b = \tilde{b} \\ f(c) = ac + b = d \end{cases}$$

Repertório

1) $f(x) = ax + b \rightsquigarrow (x, f(x)) = (x, ax+b)$



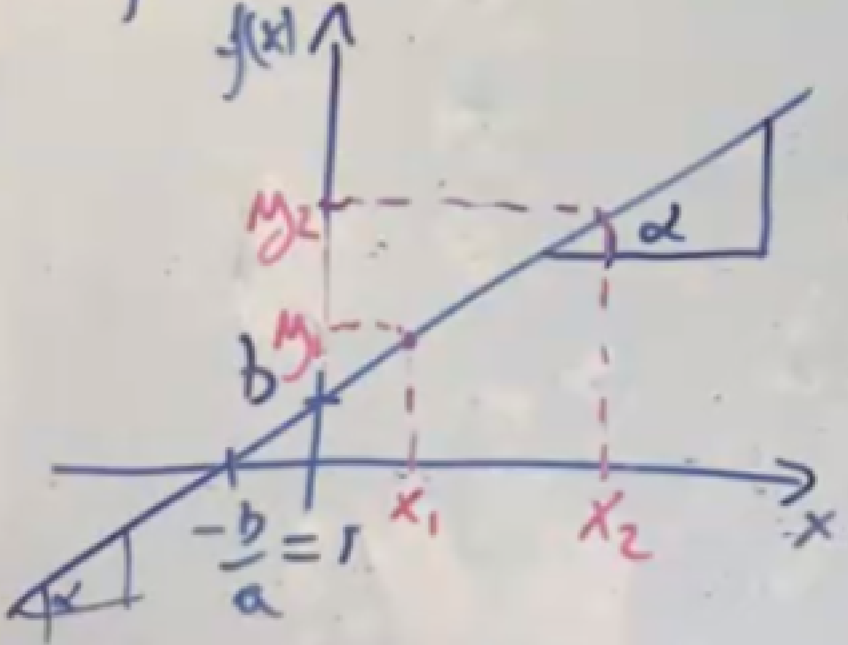
$$tg\,\alpha = a$$

$$f(x) = ax + b$$

$$\begin{cases} f(x_1) = ax_1 + b = y_1 \\ f(x_2) = ax_2 + b = y_2 \end{cases}$$

Repertório

1) $f(x) = ax + b \rightsquigarrow (x, f(x)) = (x, ax+b)$



$tg\,\alpha = a$

$f(x) = ax + b$

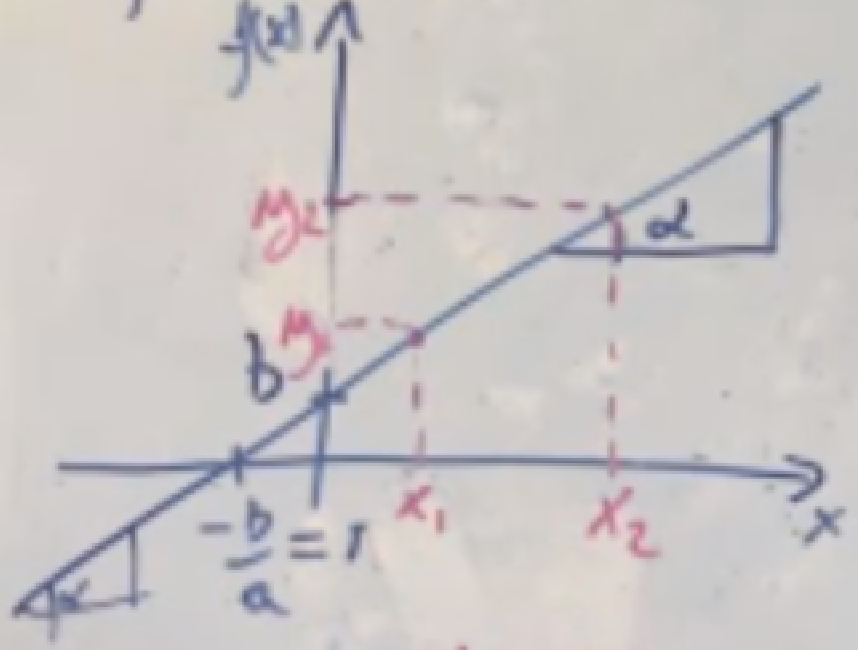
$$\begin{cases} f(x_1) = ax_1 + b = y_1 \\ f(x_2) = ax_2 + b = y_2 \end{cases} \; (-)$$

$a(x_1 - x_2) = y_1 - y_2$

$a = \frac{x_1 - x_2}{y_1 - y_2}$

Repertório

1) $f(x) = ax + b \rightsquigarrow (x, f(x)) = (x, ax+b)$



$tg\alpha = a$

$f(x) = ax + b$

$\begin{cases} f(x_1) = ax_1 + b = y_1 \quad (-) \\ f(x_2) = ax_2 + b = y_2 \end{cases}$

$a(x_1 - x_2) = y_1 - y_2$

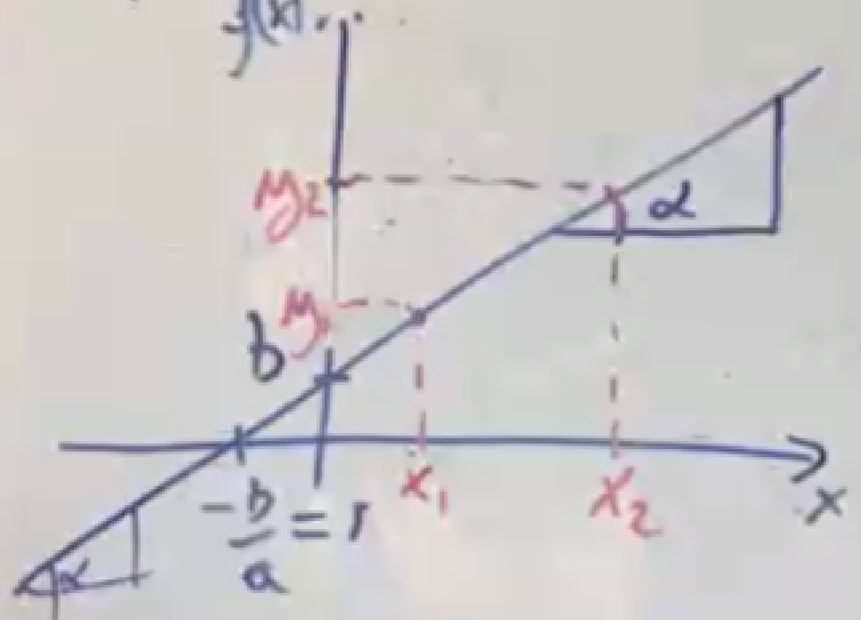
$a = \frac{x_1 - x_2}{y_1 - y_2}$

$(*) : \left(\frac{x_1 - x_2}{y_1 - y_2}\right) x_1 + b = y_1$

$b = y_1 - x_1 \left(\frac{x_1 - x_2}{y_1 - y_2}\right) =$

Rupe

1) f(x



$$\operatorname{tg}\alpha = a$$

$$f(x) = ax + b$$

$$\begin{cases} f(x_1) = a x_1 + b = y_1 \quad (-) \\ f(x_2) = a x_2 + b = y_2 \end{cases}$$

$$a(x_1 - x_2) = y_1 - y_2$$

$$a = \frac{y_1 - y_2}{x_1 - x_2}$$

$$(*): \left(\frac{y_1 - y_2}{x_1 - x_2}\right) x_1 + b = y_1$$

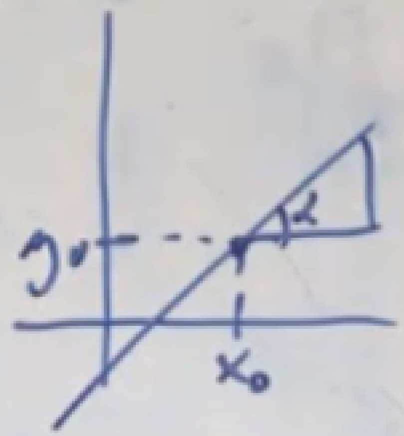
$$b = y_1 - x_1 \left(\frac{y_1 - y_2}{x_1 - x_2}\right) =$$

$$a = \frac{y_1 - y_2}{x_1 - x_2} \; ; \quad b = y_1 - x_1\left(\frac{y_1 - y_2}{x_1 - x_2}\right)$$

$$f(x) = ax + b = \left(\frac{y_1 - y_2}{x_1 - x_2}\right)x + y_1 - x_1\left(\frac{y_1 - y_2}{x_1 - x_2}\right) =$$

$$= \left(\frac{y_1 - y_2}{x_1 - x_2}\right)(x - x_1) + y_1$$

$$f(x) - y_1 = \left(\frac{y_1 - y_2}{x_1 - x_2}\right)(x - x_1)$$



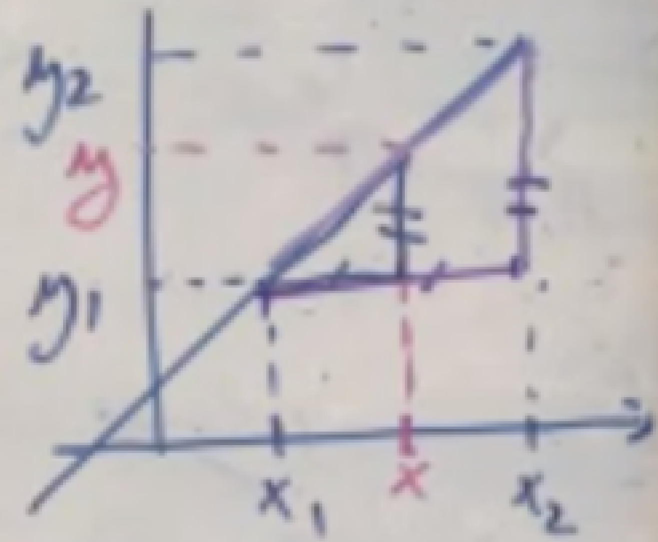
$$y - y_0 = m \quad (x - x_0)$$

$tg\alpha = m$

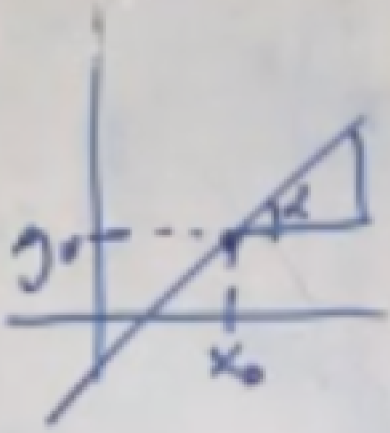
(6)

$$\frac{f(x)-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$$

$$\frac{\overset{y}{f(x)}-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$



$$f(x)-y_1 = \left(\frac{y_1-y_2}{x_1-x_2}\right)(x-x_1)$$

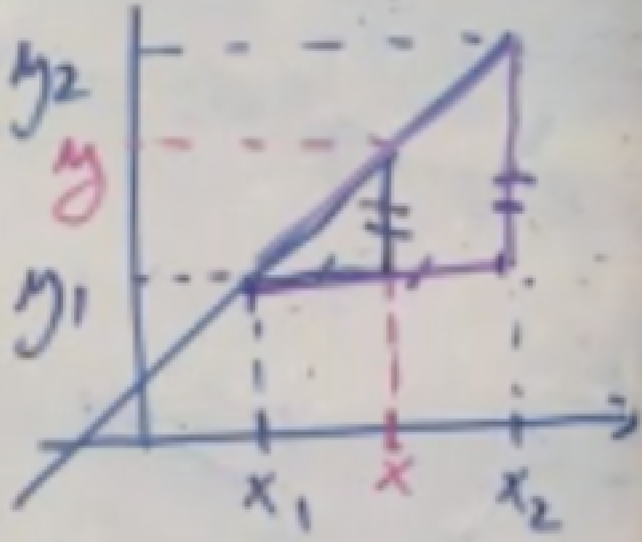


$$y-y_0 = m \qquad (x-x_0)$$

$\mathrm{tg}\,\alpha = m$

$$\frac{f(x)-y_1}{y_1-y_2}=\frac{x-x_1}{x_1-x_2}$$

$$\frac{f(x)-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

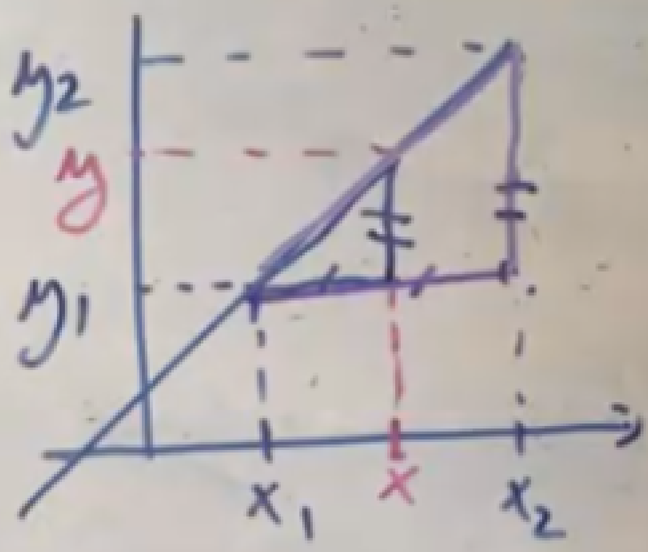


Ex 9: $f(x)=5^x$

$$\frac{f(x+h)-f(x)}{h}=\frac{5^{x+h}-5^x}{h}=\frac{5^x5^h-5^x}{h}=5^x\frac{(5^h-1)}{h}$$

$$\frac{f(x)-y_1}{y_1-y_2}=\frac{x-x_1}{x_1-x_2}$$

$$\frac{\overset{y}{f(x)}-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$



Ex 9: $f(x)=5^x$

$$\frac{f(x+h)-f(x)}{h}=\frac{5^{x+h}-5^x}{h}=\frac{5^x5^h-5^x}{h}=5^x\frac{(5^h-1)}{h}$$

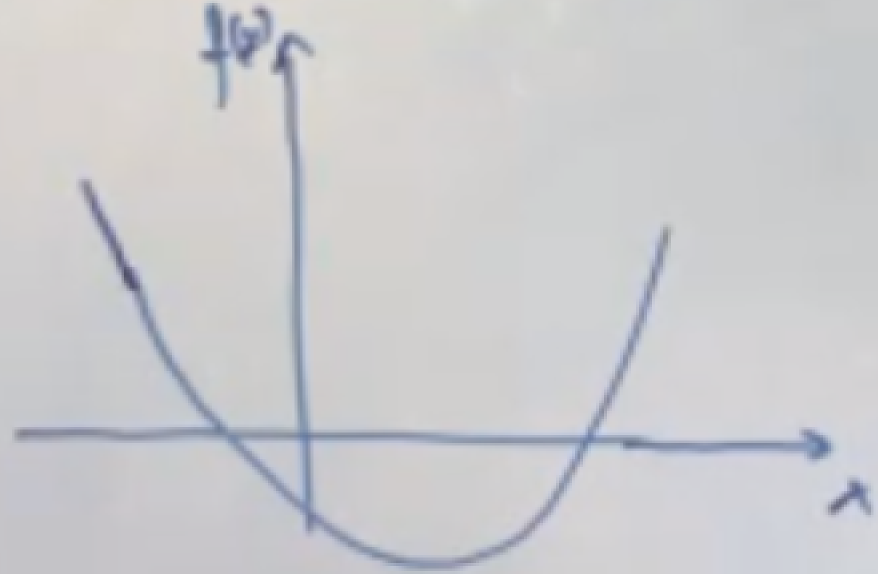
$$=5^x\left(\frac{5^h-5^0}{h-0}\right)$$

7

REPERTÓRIO DE FUNÇÕES

2) função do segundo grau

$$f(x) = ax^2 + bx + c$$

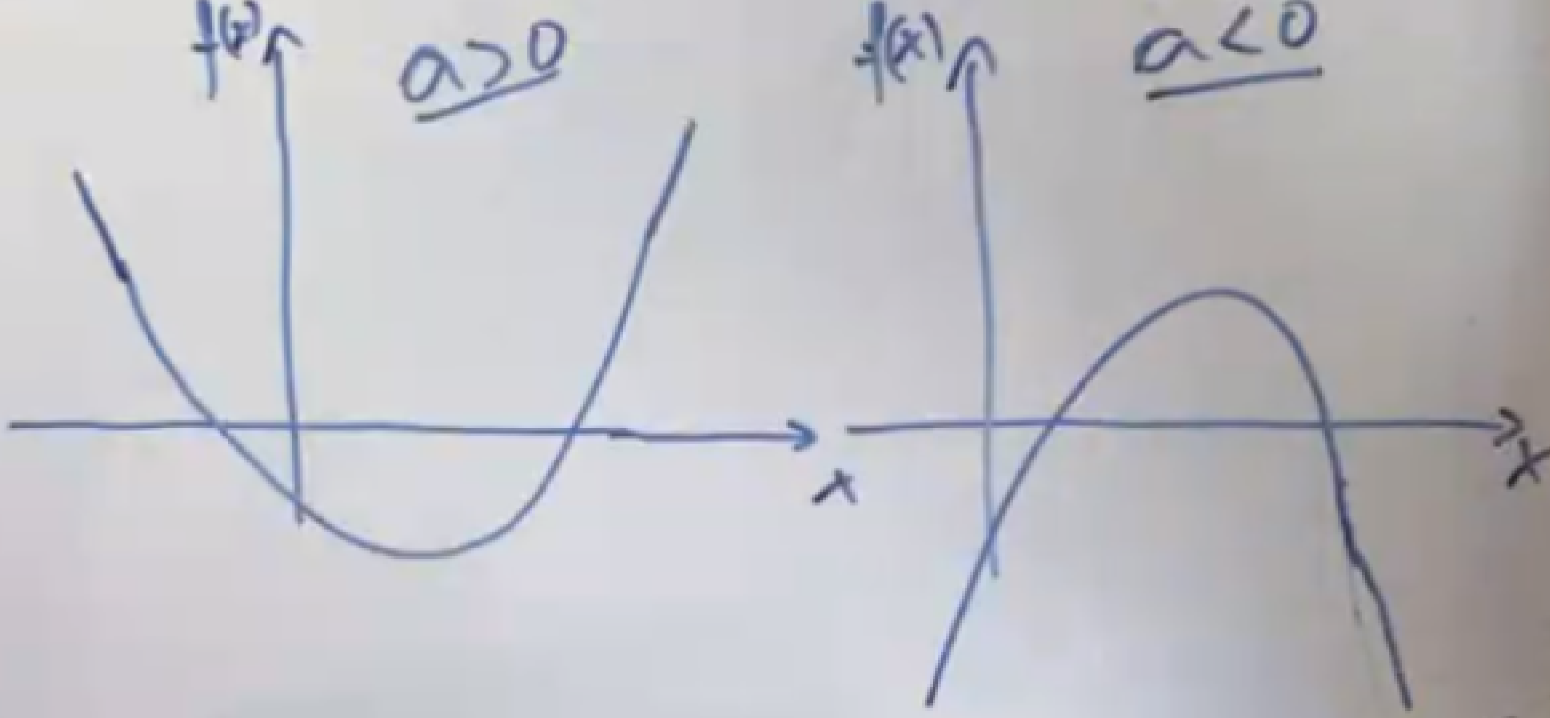


8

REPERTÓRIO DE FUNÇÕES

2) função do segundo grau

$f(x) = ax^2 + bx + c$

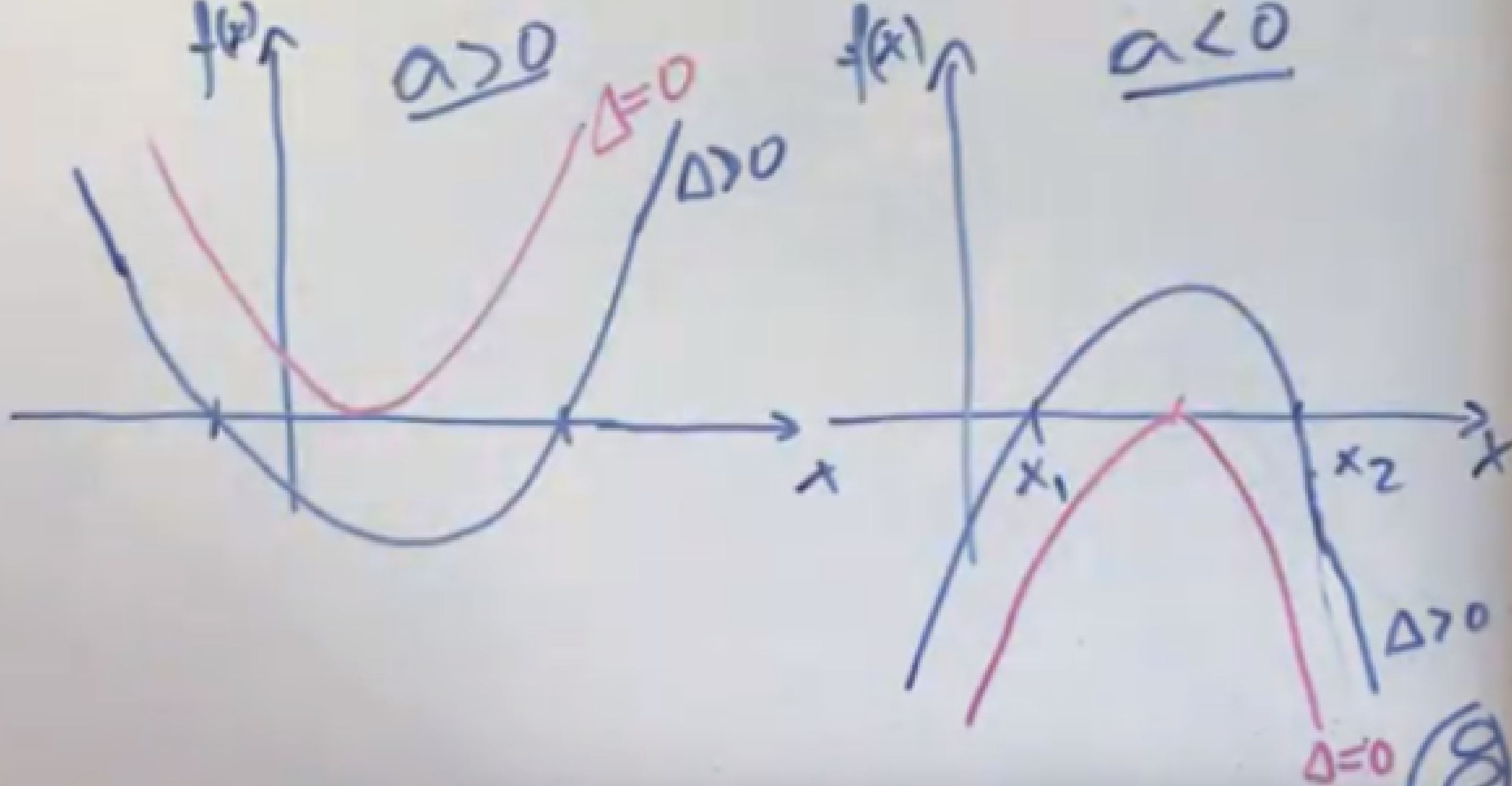


8

REPERTÓRIO DE FUNÇÕES

2) função do segundo grau

$f(x) = ax^2 + bx + c$; $\Delta = b^2 - 4ac$

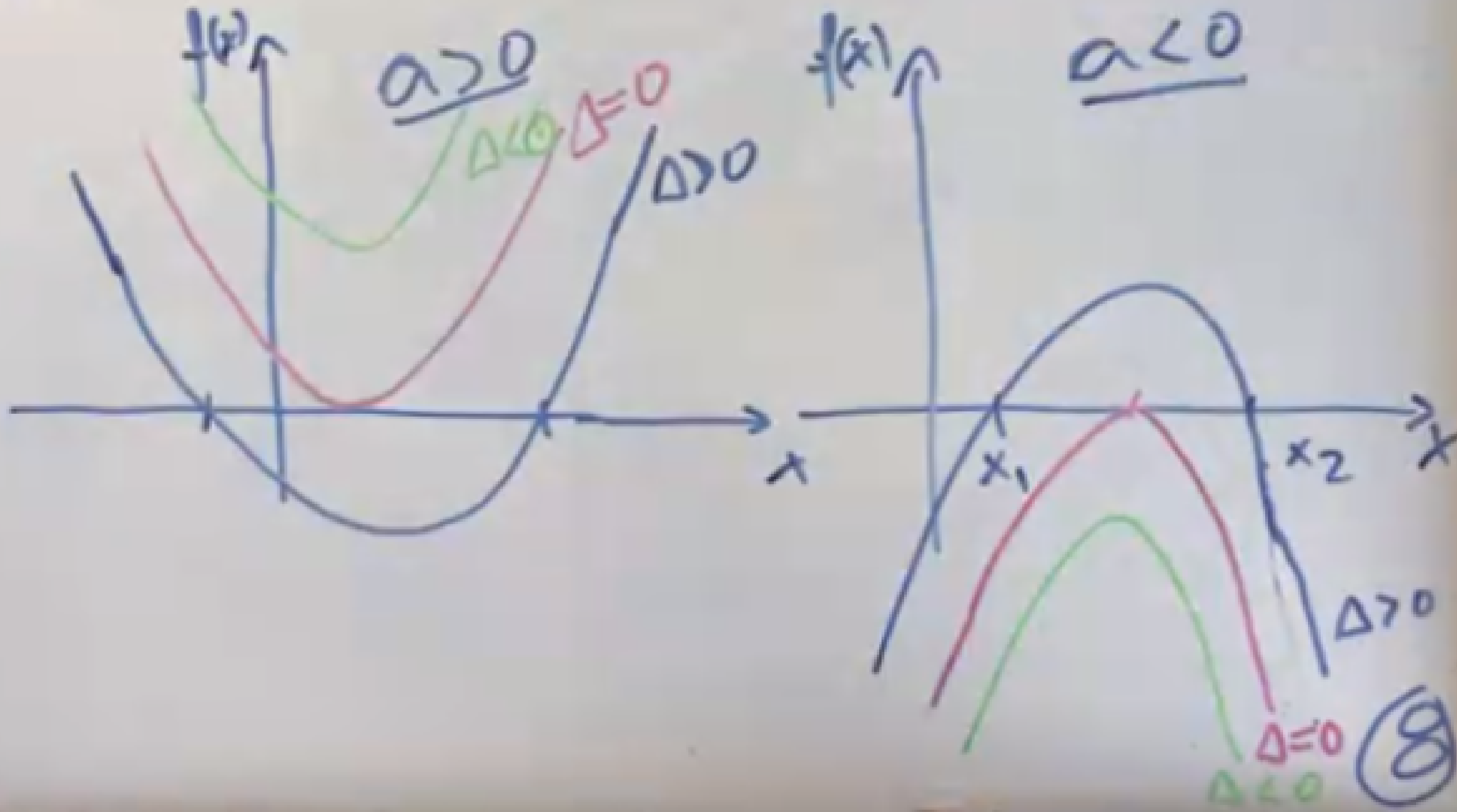


8

REPERTÓRIO DE FUNÇÕES

2) função do segundo grau

$f(x) = ax^2 + bx + c \quad ; \quad \Delta = b^2 - 4ac$

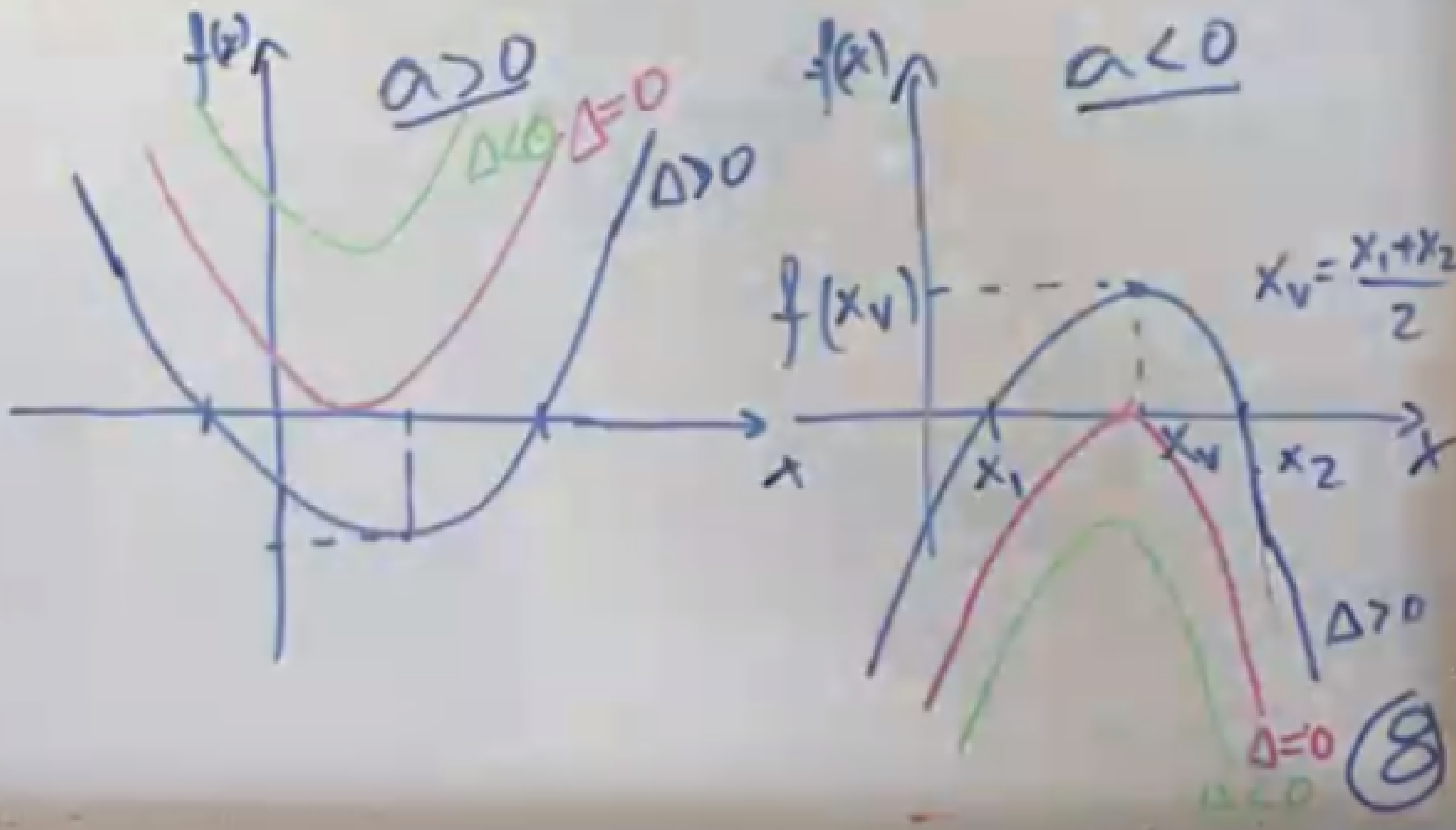


8

REPERTÓRIO DE FUNÇÕES

2) função do segundo grau

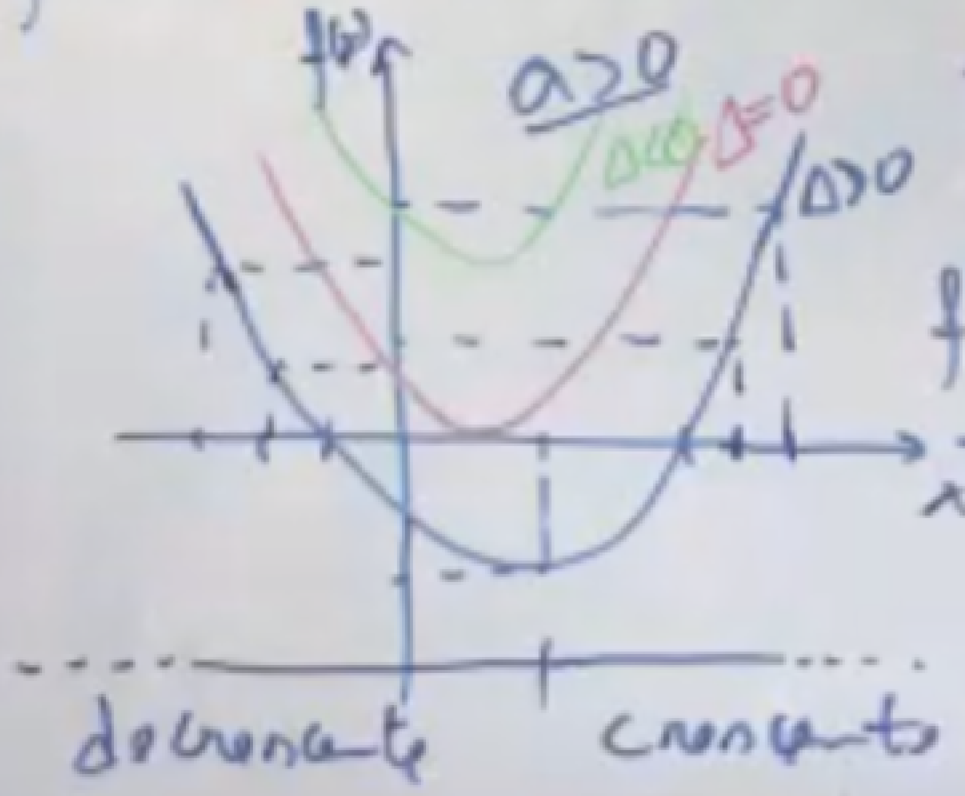
$f(x) = ax^2 + bx + c \quad ; \quad \Delta = b^2 - 4ac$

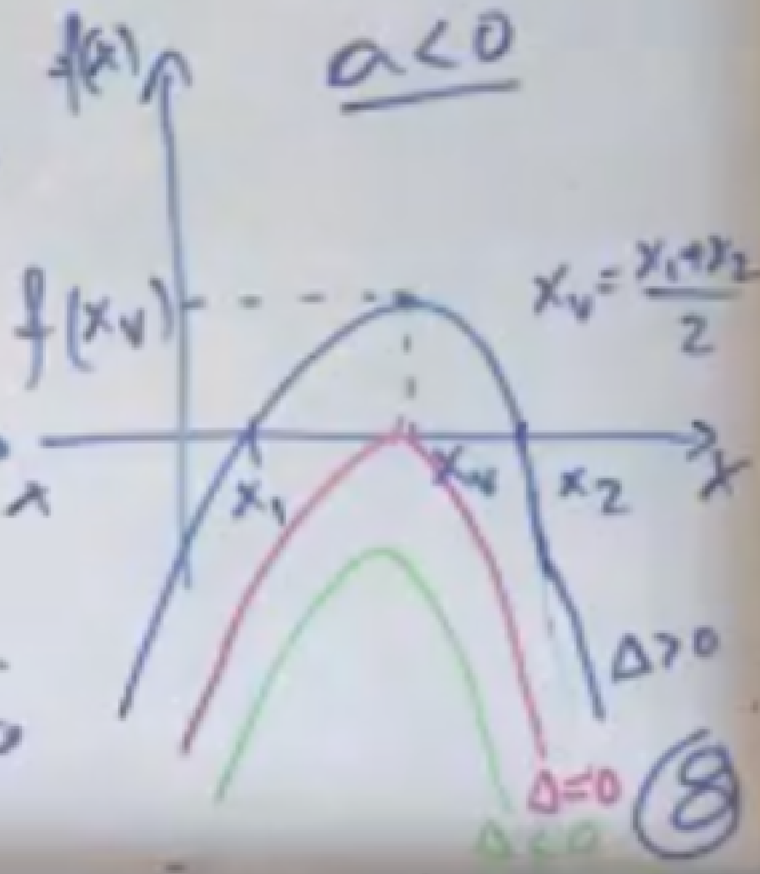


REPERTÓRIO DE FUNÇÕES

2) função do segundo grau

$f(x) = ax^2 + bx + c \quad ; \quad \Delta = b^2 - 4ac$





8

* Raízes: $r = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

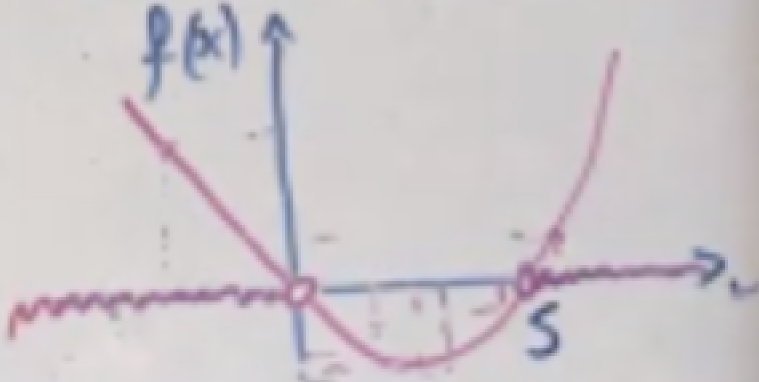
ex: p.37 4) e) $h(x) = \dfrac{1}{\sqrt[4]{x^2-5x}}$

C.E.: $x^2-5x>0$

$x^2-5x=0$

$\Delta = b^2-4ac = 5^2-4.1.0=25$

$r = \dfrac{+5\pm 5}{2.1} \begin{cases} +5 \\ 0 \end{cases}$



$D_h =]-\infty,0[\cup]5,+\infty[$

$= \mathbb{R} \setminus 0,5$

CE: $x^2-5x>0 \Rightarrow$

$\Rightarrow x<0$ ou $x>5$

$$x^2 - 5x = 0$$
$$x(x-5) = 0$$
$$x = 0$$
$$x - 5 = 0 \Rightarrow x = 5$$

$$x^2 - 4 = 0$$
$$x^2 = 4$$
$$x = 2$$
$$x = -2$$

$$x^2 - 5x = x(x-5) = (x-0)(x-5)$$

$$x^2 - 4 = (x-2)(x+2)$$

$$ax^2 + bx + c = 0$$

forma fatorada: $a(x - r_1)(x - r_2) = 0$

$$a\left(x - \frac{-b+\sqrt{\Delta}}{2a}\right)\left(x + \frac{-b-\sqrt{\Delta}}{2a}\right) = ax^2 + \frac{b}{a}x + \frac{c}{a}$$

10

3) função polinomial

$$p(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + \underbrace{a_3 x^3}_{\text{monômio}} + \dots + a_n x^n =$$

$$= \sum_{i=0}^{n} a_i x^i \; (= a_i x^i)$$

3) função polinomial

$$p_n(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + \underbrace{a_3 x^3}_{\text{monômio}} + \dots + a_n x^n =$$

$$= \sum_{i=0}^{n} a_i x^i$$

ex: $p_3(x) = -4 + 5x^2 + 7x^3$ $\quad \begin{cases} a_0 = -4 \\ a_1 = 0 \\ a_2 = +5 \\ a_3 = +7 \end{cases}$

$p_n(x) = 0 \rightarrow$ problema difícil

3) função polinomial

$$p_n(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + \underbrace{a_3 x^3}_{\text{monômio}} + \dots + a_n x^n =$$

$$= \sum_{i=0}^{n} a_i x^i$$

ex: $p_3(x) = -4 + 5x^2 + 7x^3$ $\begin{cases} a_0 = -4 \\ a_1 = 0 \\ a_2 = +5 \\ a_3 = +7 \end{cases}$

$p_n(x) = 0 \rightarrow$ problema difícil

$$p_n(x) = \sum_{i=0}^{n} a_i x^i = a_n \prod_{i=1}^{n} (x - r_i)$$

(17)


11

4) generalizações a partir de polinômios:

* funções potência $f(x) = x^{-1/2} + x^{3/4} + x^{1/\pi} =$

$= \frac{1}{x^{1/2}} + x^{3/4} + x^{1/\pi} = \frac{1}{\sqrt{x}} + \sqrt[4]{x^3} + \sqrt[\pi]{x}$

11

4) generalizações a partir de polinômios:

* funções potência $f(x) = x^{-1/2} + x^{3/4} + x^{1/\pi} =$
$$= \frac{1}{x^{1/2}} + x^{3/4} + x^{1/\pi} = \frac{1}{\sqrt{x}} + \sqrt[4]{x^3} + \sqrt[\pi]{x}$$

* funções racionais: $r(x) = \frac{p(x)}{q(x)}$ $(q(x) \neq 0)$

ex: $r(x) = \frac{x^2 - 1}{x^3 + 3}$

4) generalizações a partir de polinômios:

* funções potência $f(x) = x^{-1/2} + x^{3/4} + x^{1/\pi} =$

$$= \frac{1}{x^{1/2}} + x^{3/4} + x^{1/\pi} = \frac{1}{\sqrt{x}} + \sqrt[4]{x^3} + \sqrt[\pi]{x}$$

* funções racionais : $r(x) = \frac{p(x)}{q(x)}$ $(q(x) \neq 0)$

ex: $r(x) = \frac{x^2-1}{x^3+3}$ $\left(D_r = \mathbb{R} \setminus \{\sqrt[3]{-3}\} \right)$

11

5) funções exponenciais

$$f(x) = a^x \;;\; a > 0$$

ex: $\begin{cases} f(x) = 2^x \\ g(x) = \left(\frac{1}{2}\right)^x \end{cases}$

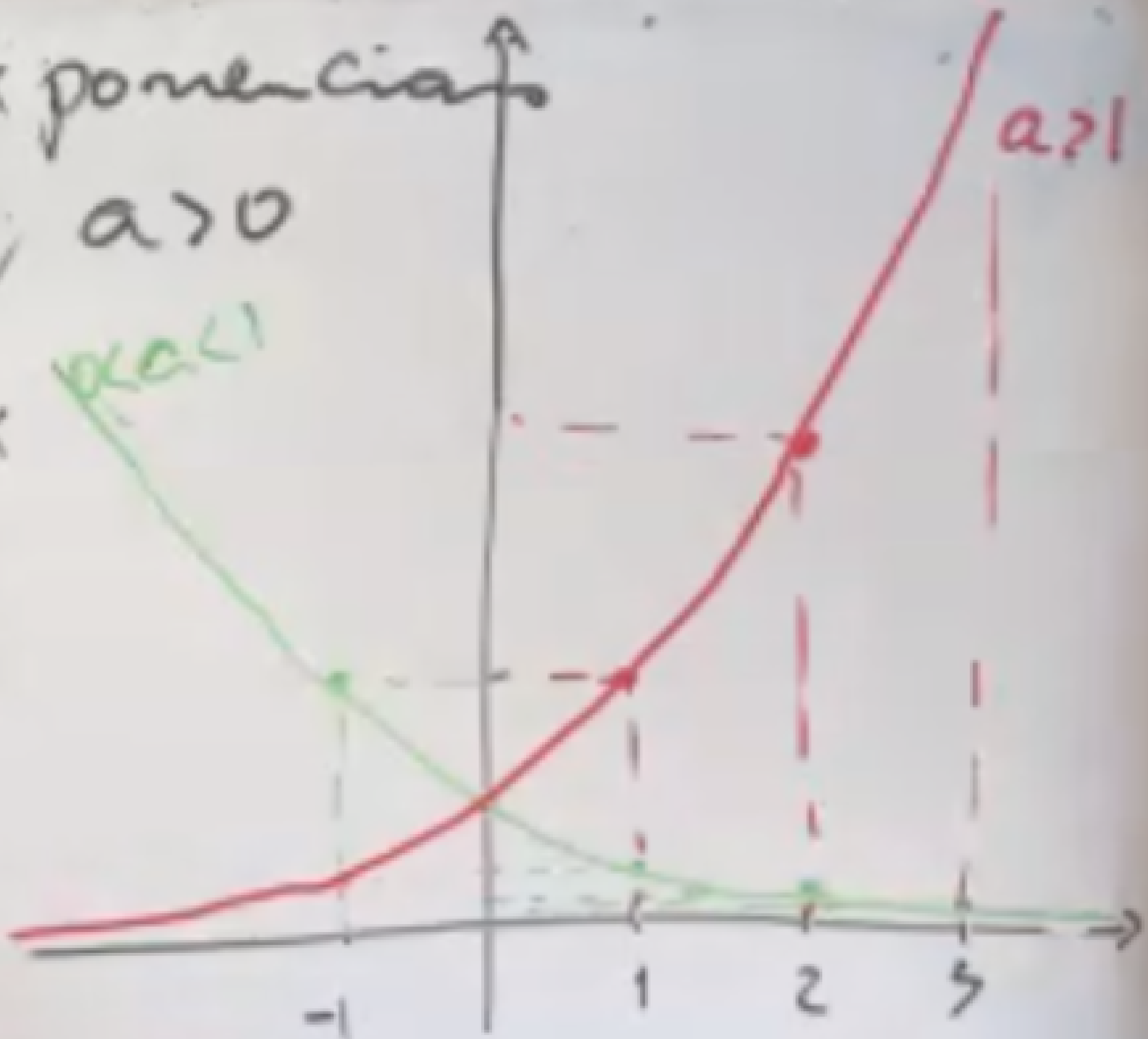
x	$f(x)$	$g(x)$
0	1	1

5) funções exponenciais

$$f(x) = a^x \; , \; a > 0$$

ex: $\begin{cases} f(x) = 2^x \\ g(x) = \left(\frac{1}{2}\right)^x \end{cases}$

x	$f(x)$	$g(x)$
0	1	1
1	2	1/2
-1	1/2	2
2	4	1/4
3	8	1/8

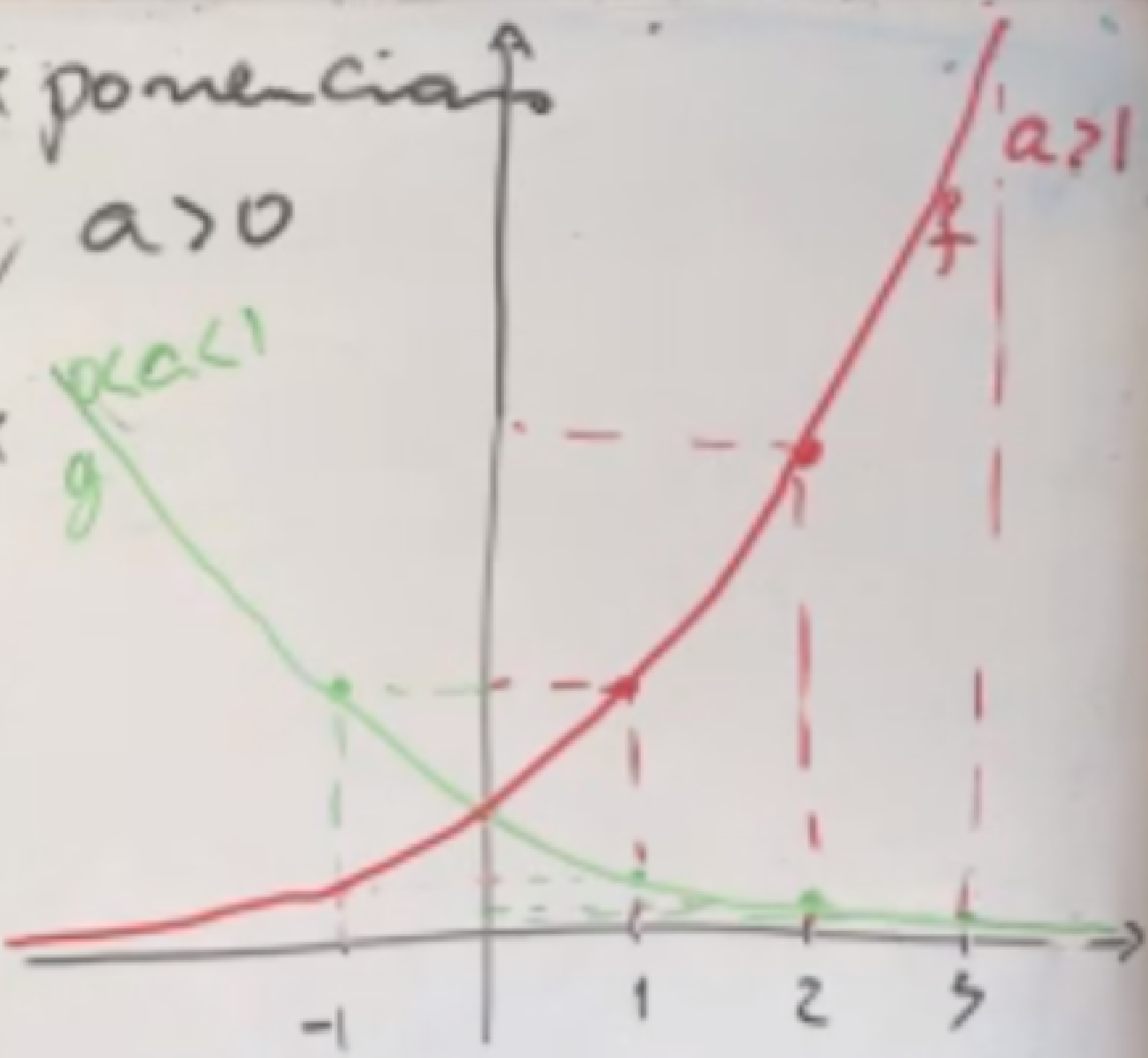


5) funções exponenciais

$$f(x) = a^x \ ; \ a > 0$$

ex: $\begin{cases} f(x) = 2^x \\ g(x) = \left(\frac{1}{2}\right)^x \end{cases}$

x	$f(x)$	$g(x)$
0	1	1
1	2	1/2
-1	1/2	2
2	4	1/4
3	8	1/8

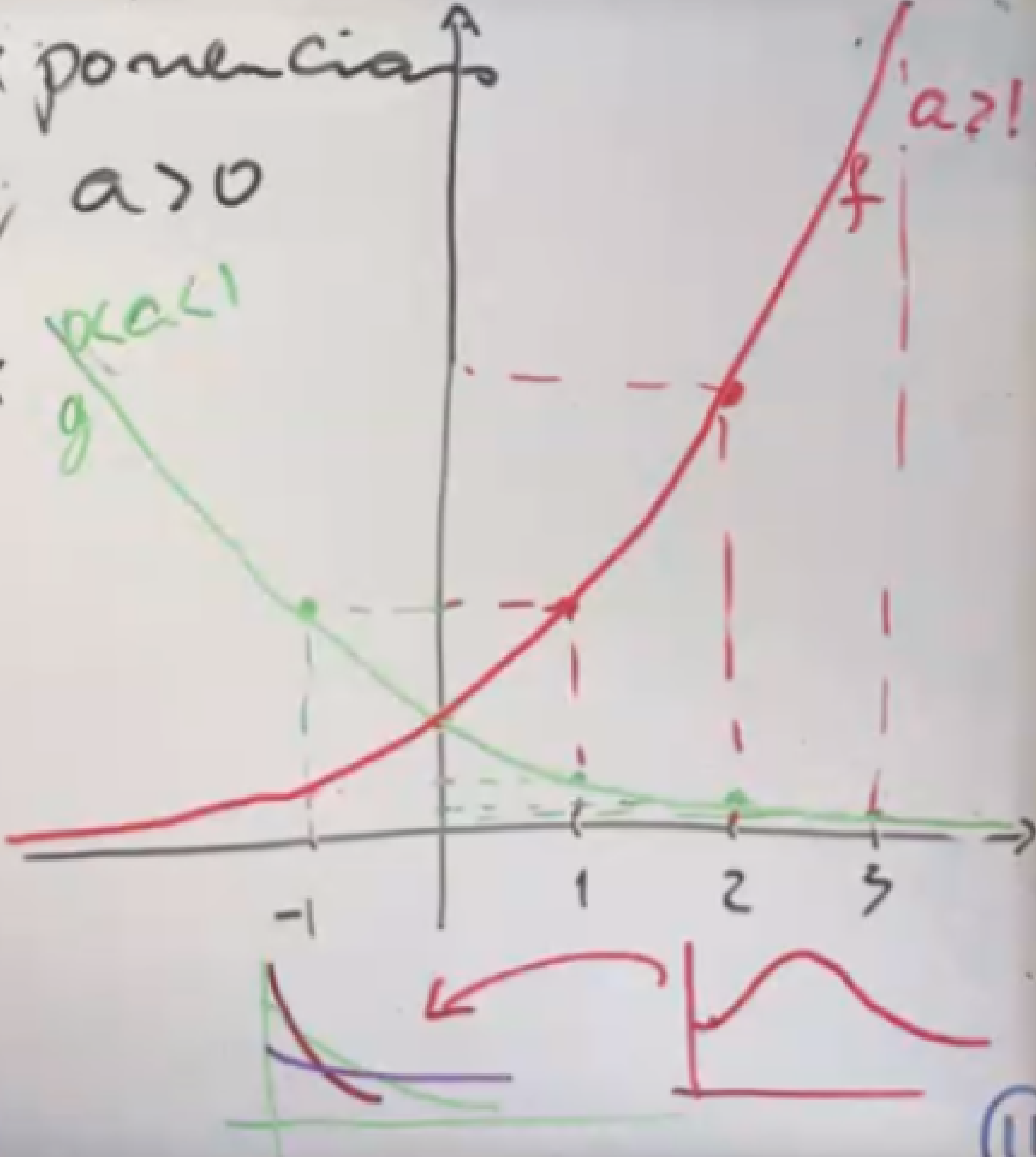


5) funções exponenciais

$$f(x) = a^x \; ; \; a > 0$$

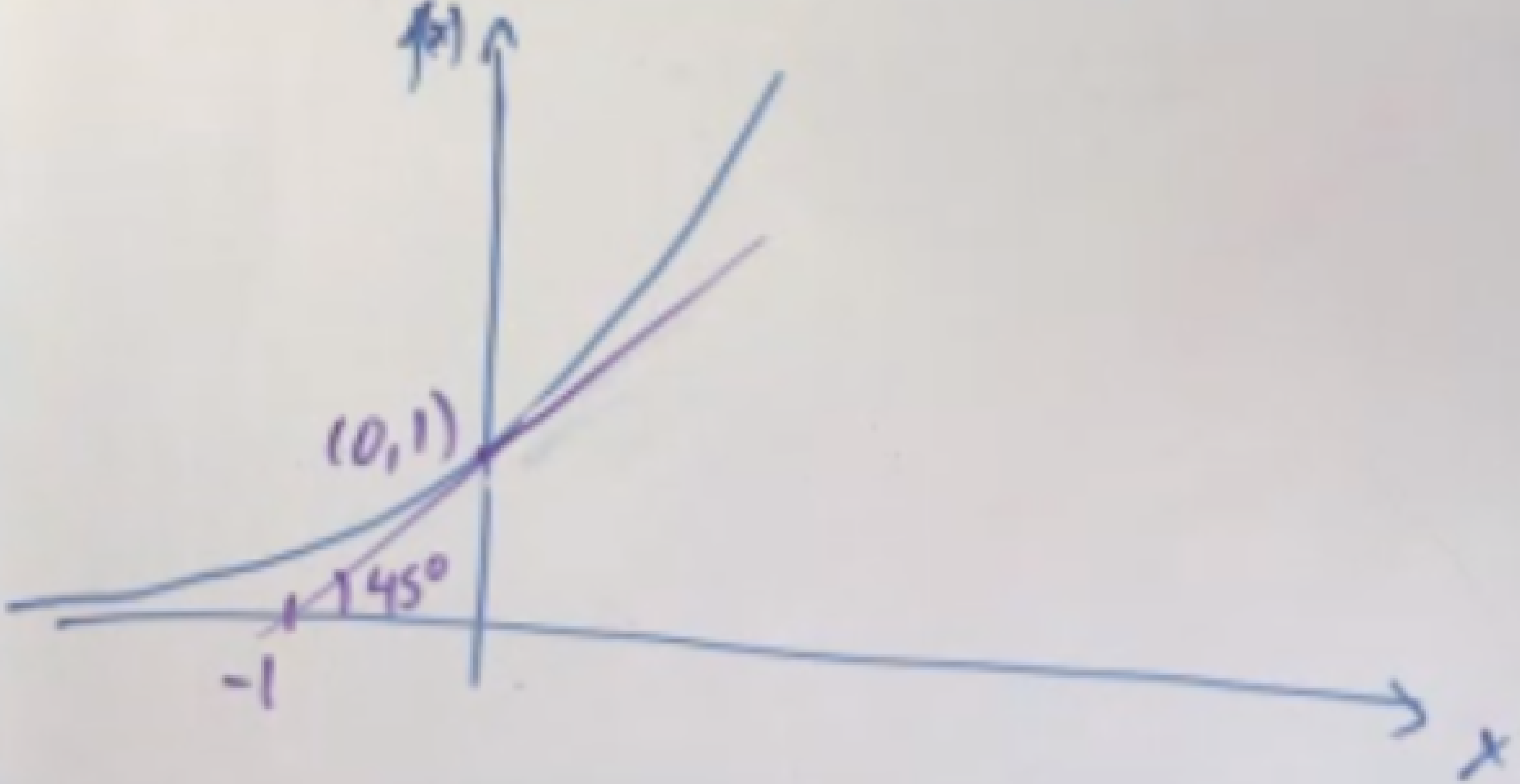
ex: $\begin{cases} f(x) = 2^x \\ g(x) = \left(\frac{1}{2}\right)^x \end{cases}$

x	$f(x)$	$g(x)$
0	1	1
1	2	$1/2$
-1	$1/2$	2
2	4	$1/4$
3	8	$1/8$



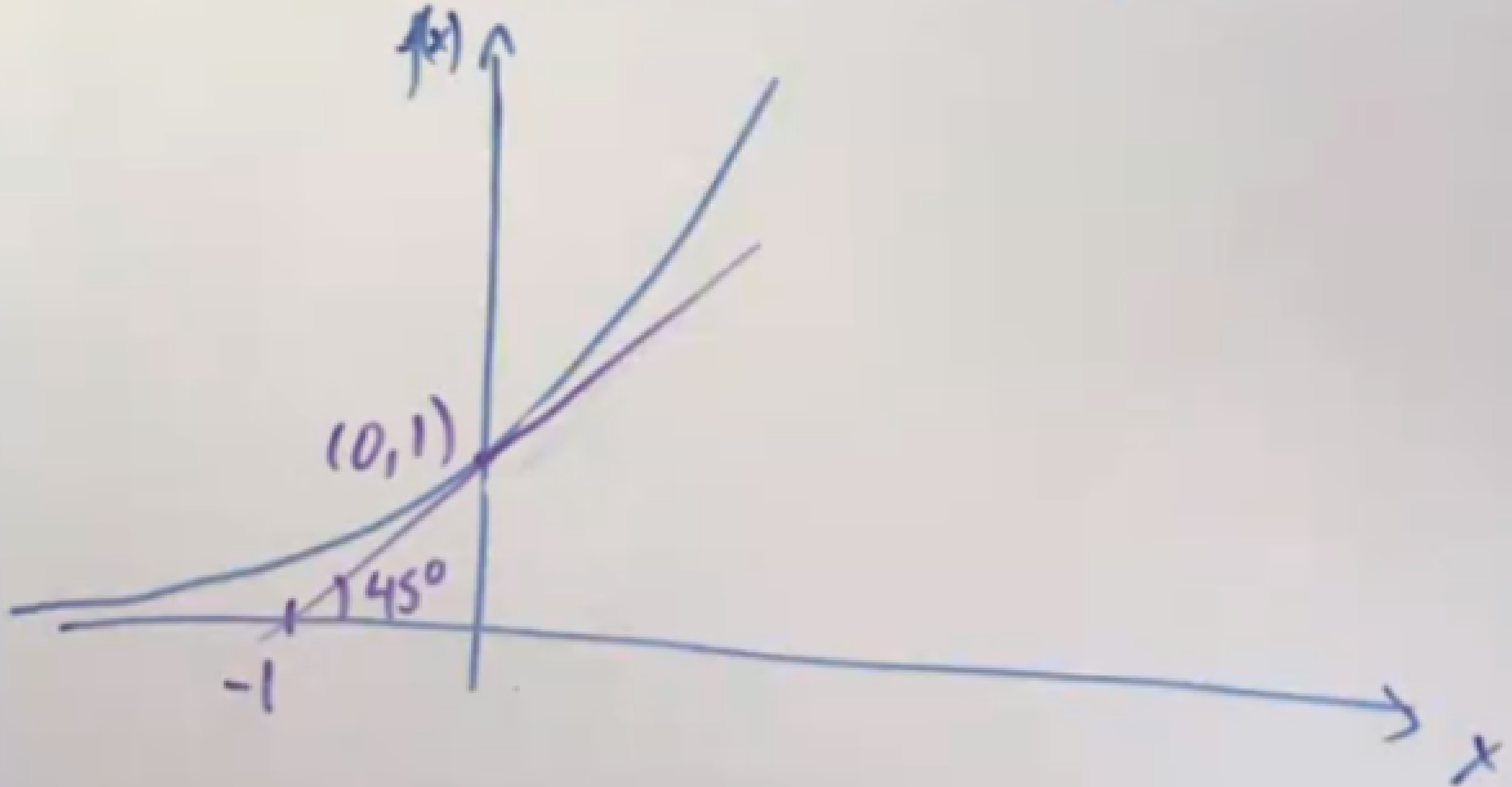
base "e" $\rightarrow$ número de Euler

$e \cong 2{,}71828 \longrightarrow f(x) = e^{x}$



base "e" → número de Euler

$e \cong 2,71828 \longrightarrow f(x) = e^x$



12