Bon Dia!

BON DIA!

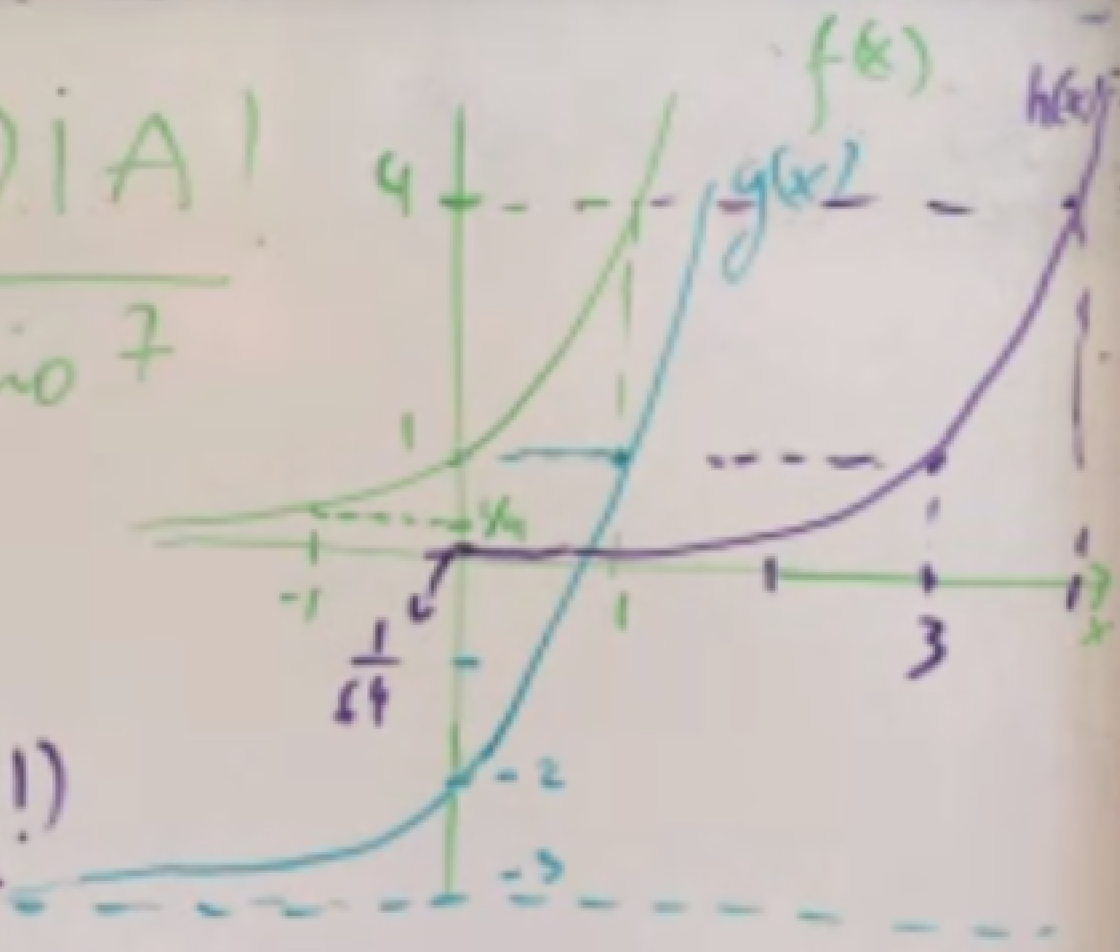
Pág. 38) exercício 7

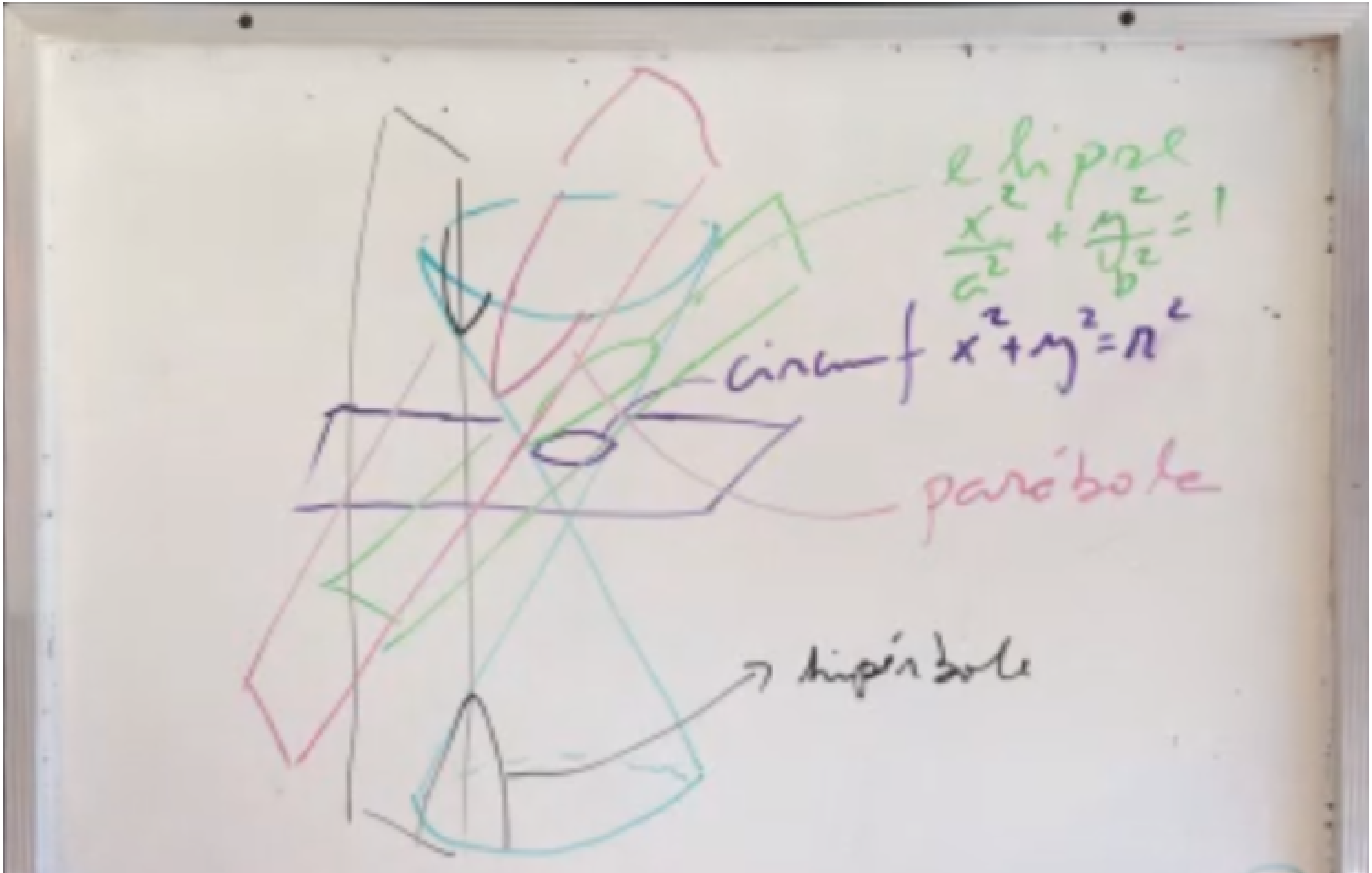
$f(x)=4^x$

$g(x)=f(x)-3=4^x-3$

$h(x)=f(x-3)=4^{x-3}$ (!!) $=$

$=4^x\cdot 4^{-3}=\frac{1}{64}4^x=\frac{1}{64}f(x)$

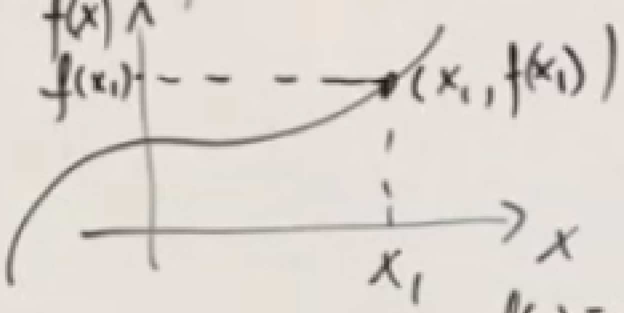


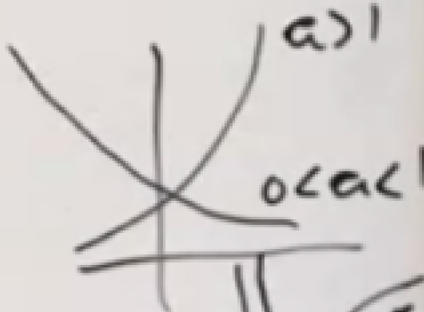

elipse
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
circunf $x^2 + y^2 = r^2$
parábola
hipérbole

$f(x)$ $\quad x \longmapsto f(x)$ $\quad f: D_f \subset \mathbb{R} \to CD_f = \mathbb{R}$

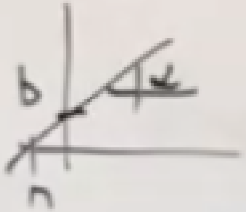
- Exclusiva ; - Exaustiva

$(x, f(x))$



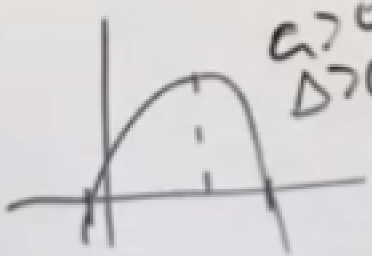


$f(x) = ax + b$



$tg\alpha = a$

$f(x) = ax^2 + bx + c$

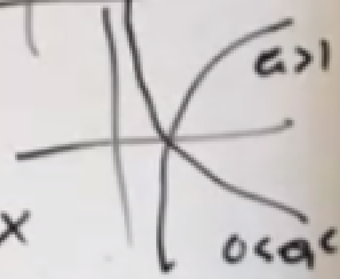


$P(x) = \sum_{i=0}^{n} a_i x^i$

$r(x) = \frac{P(x)}{Q(x)}$

$f(x) = a^x$

$f(x) = \log_a x$

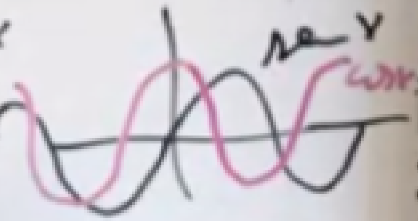


$a^{\log_a x} = \log_a a^x = x$

$f(x) = \text{sen}\, x$

$g(x) = \cos x$

$h(x) = tg\, x$



3

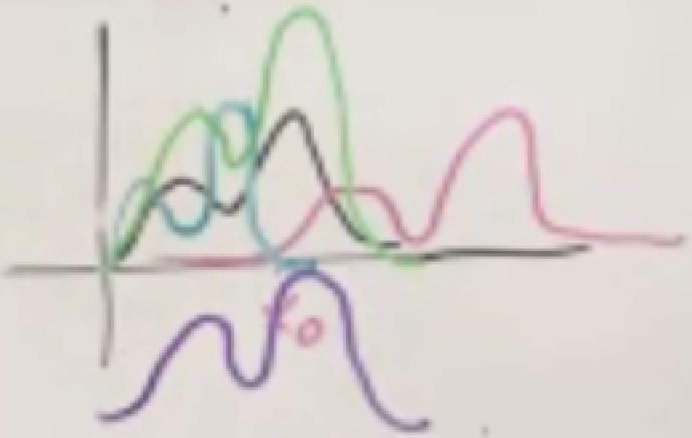
$f(x)$

$f(x-x_0)$

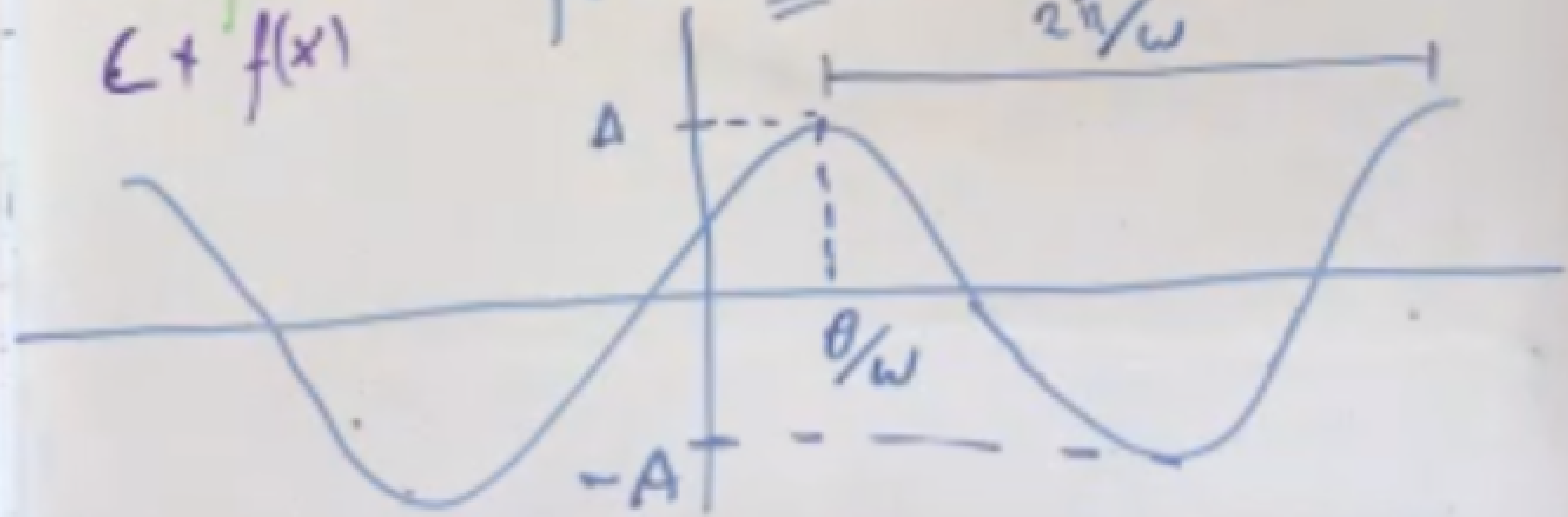
$f(ax)$

$Af(x)$

$c + f(x)$



$$f(x) = \underline{\underline{A}} \cos(\omega x - \theta) = a\cos\omega x + b\sin\omega x$$

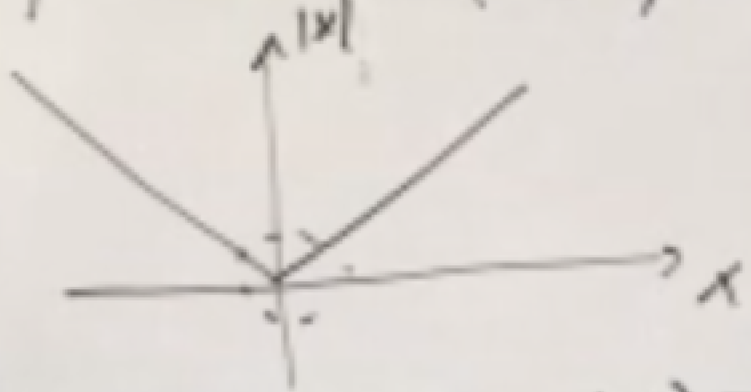


* Algumas funções a mais para o repertório

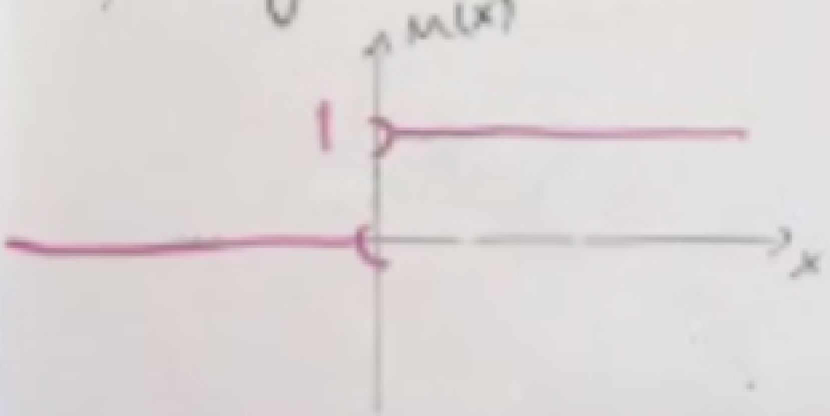
1) $f(x) = |x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$

ex: $f(3) = |3| = 3$

$f(-3) = |-3| = -(-3) = 3$



2) Degrau: $u(x) = \begin{cases} 0 & , x < 0 \\ 1 & , x > 0 \end{cases}$



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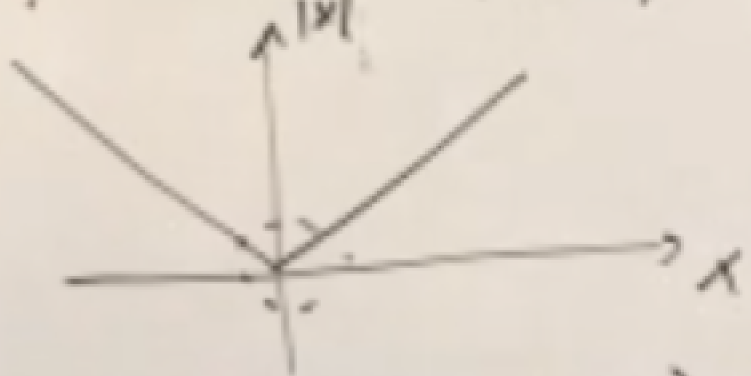

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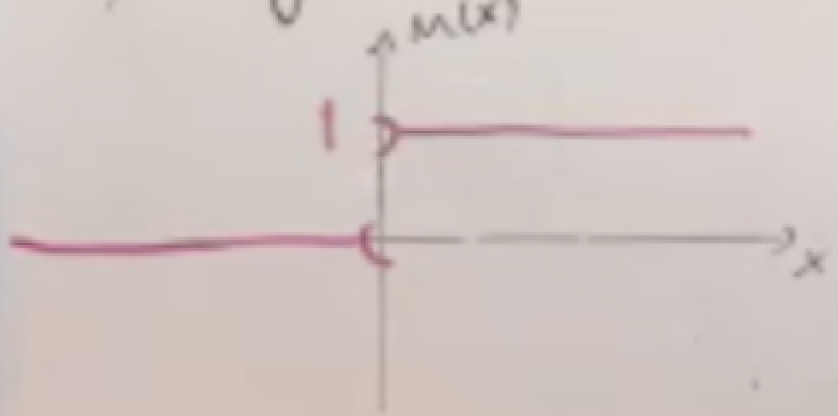
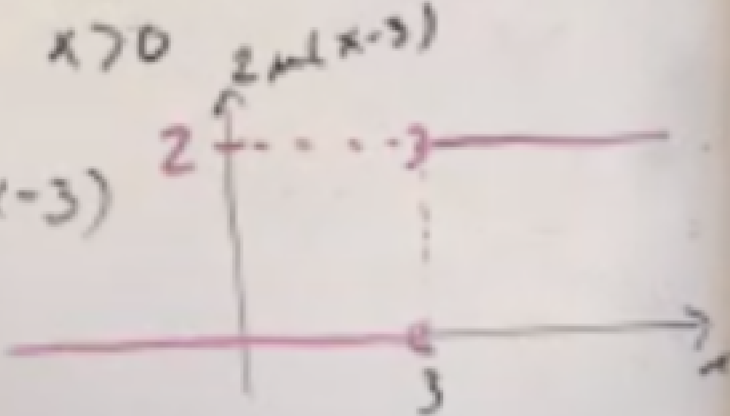
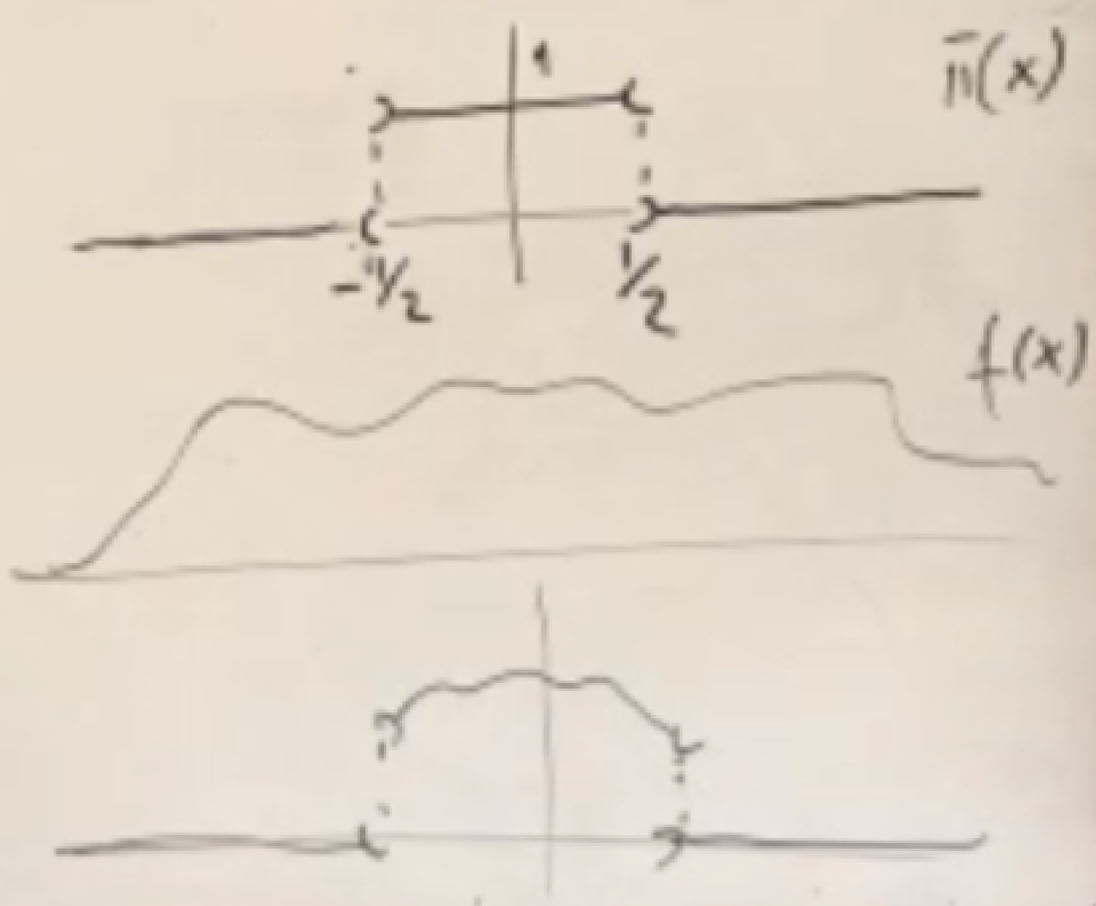


$2u(x-3)$



3) filhotes da degrau

* Caixa: $\Pi(x) = u\left(x+\frac{1}{2}\right) - u\left(x-\frac{1}{2}\right)$



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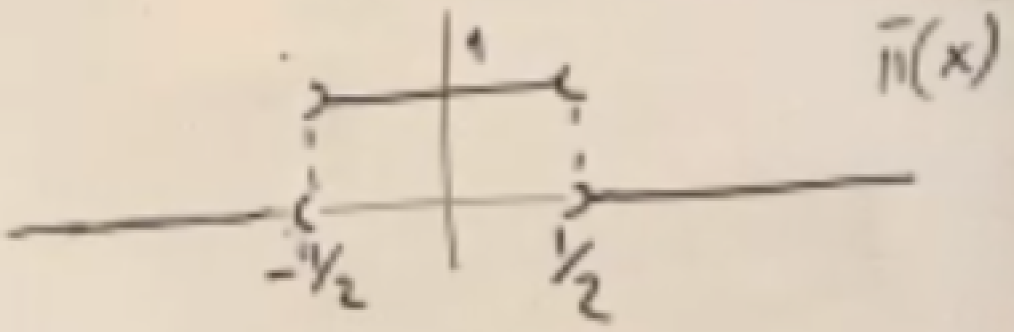
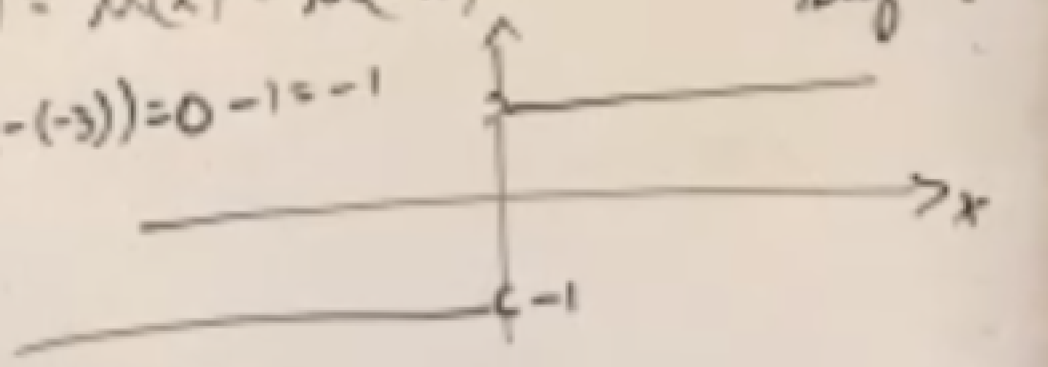


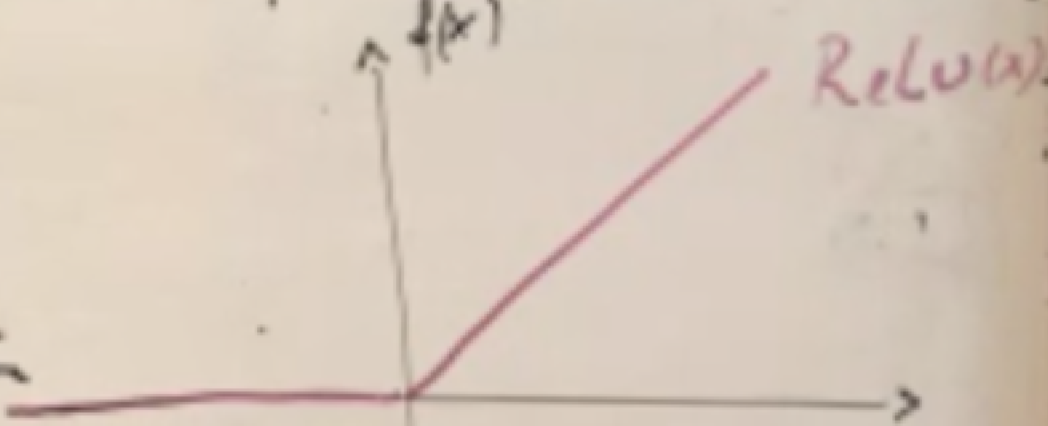
* Sinal: $\text{sign}(x) = u(x) - u(-x)$

$\text{sign}(-3) = u(-3) - u(-(-3)) = 0 - 1 = -1$



4) função ReLU (rectified linear unit)

$$f(x) = u(x) \cdot x = \max(0, x)$$



* função de ativação utilizada em redes neurais
* Não linear $f(ax_1 + bx_2) \neq a f(x_1) + b f(x_2)$

4) função ReLU (rectified linear unit)

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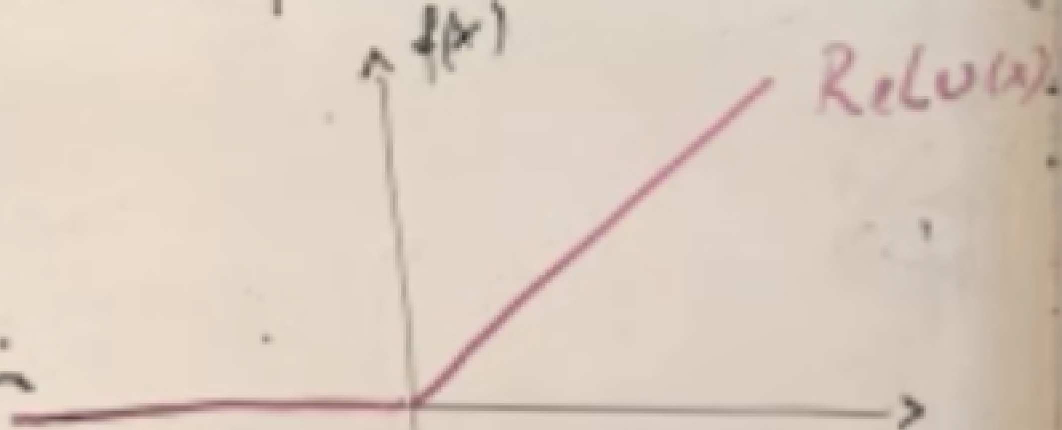
$= \max(0, x)$



* função de ativação utilizada em redes neurais
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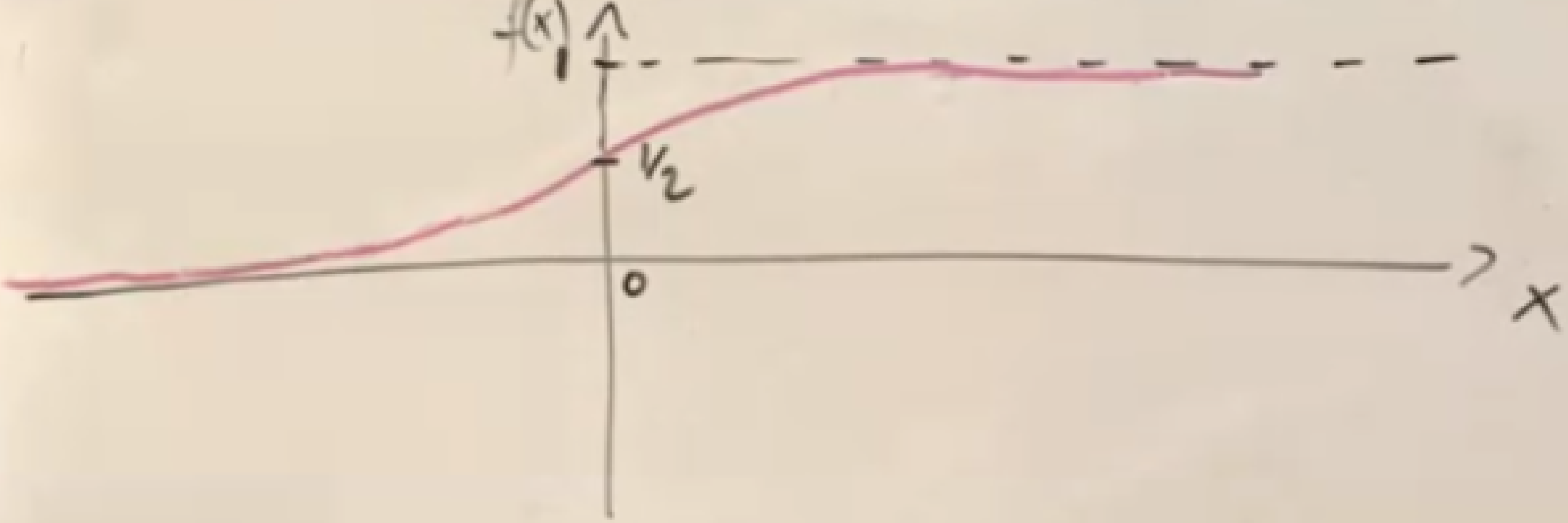
5) Logística (sigmoide)

* fç de ativação
* não linear

$$f(x) = \frac{1}{1+e^{-x}} \cdot \frac{e^x}{e^x} = \frac{e^x}{e^x+1}$$



5) Logística (sigmoide)

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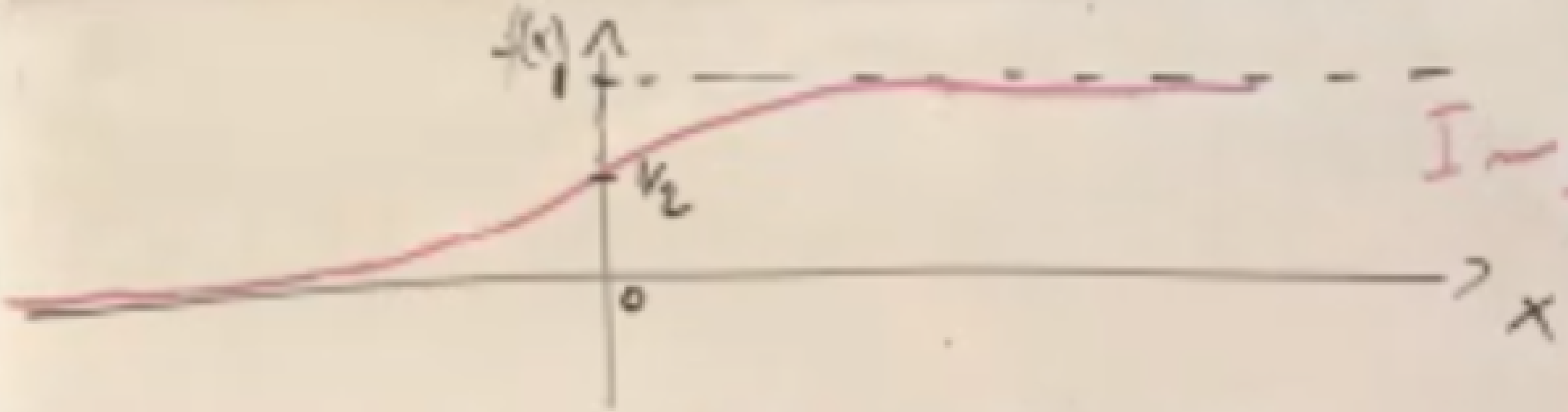
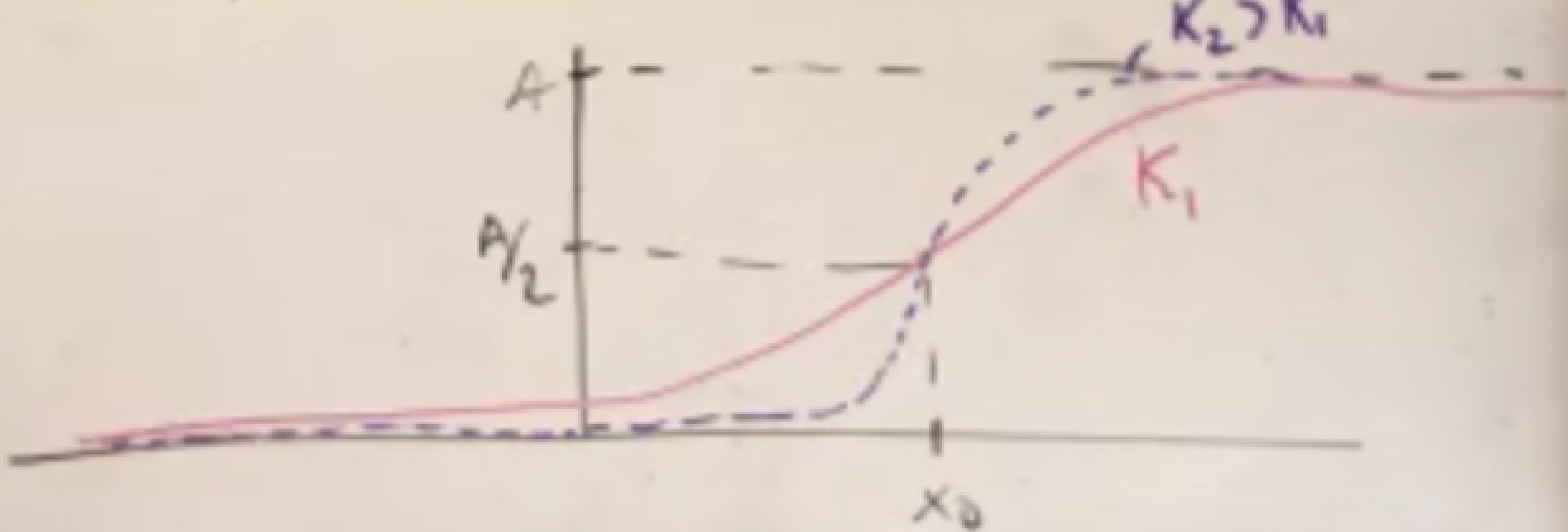
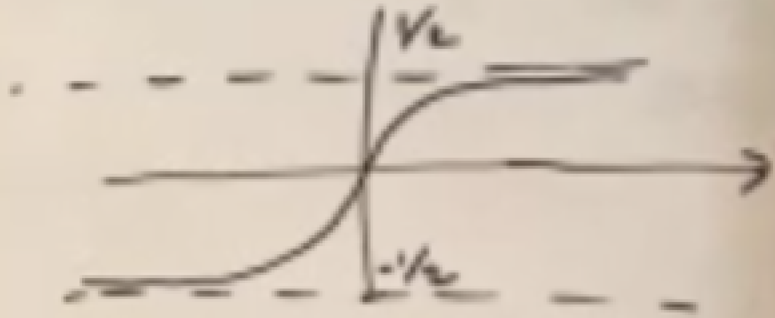


$I_f =]0,1[$

$$g(x) = A\,f(K(x-x_0)) = A\,\frac{1}{1+e^{-K(x-x_0)}}$$



$$f(x) = \frac{1}{1+e^{-x}}$$



$$g(x) = f(x) - \frac{1}{2} = \frac{1}{1+e^{-x}} - \frac{1}{2}$$

$$g(-x) = \frac{1}{1+e^{x}} - \frac{1}{2} = \frac{2-1-e^{x}}{2(1+e^{x})} = \frac{1-e^{x}}{2(1+e^{x})}$$

$$g(x) = \frac{1}{1+e^{-x}} - \frac{1}{2} = \frac{2-1-e^{-x}}{2(1+e^{-x})} = \frac{1-e^{-x}}{2(1+e^{-x})} \cdot \frac{e^{x}}{e^{x}} =$$

$$= \frac{e^{x}-1}{2(e^{x}+1)} = \frac{-1(1-e^{x})}{2(1+e^{x})} = -g(-x)$$

$g(x)$ é ímpar $\Rightarrow$ $f(x)$ é ímpar a menos de uma constante

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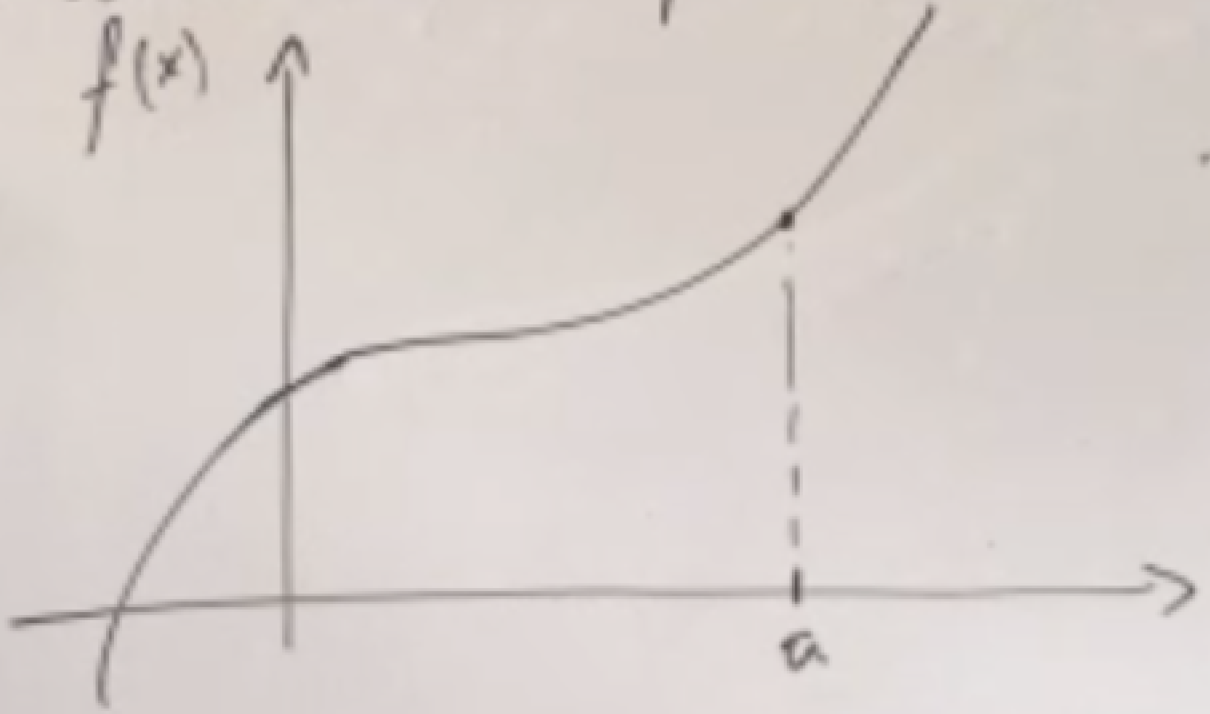
LIMITES:

• Qual o comportamento da função quando a variável se aproxima de determinado valor?



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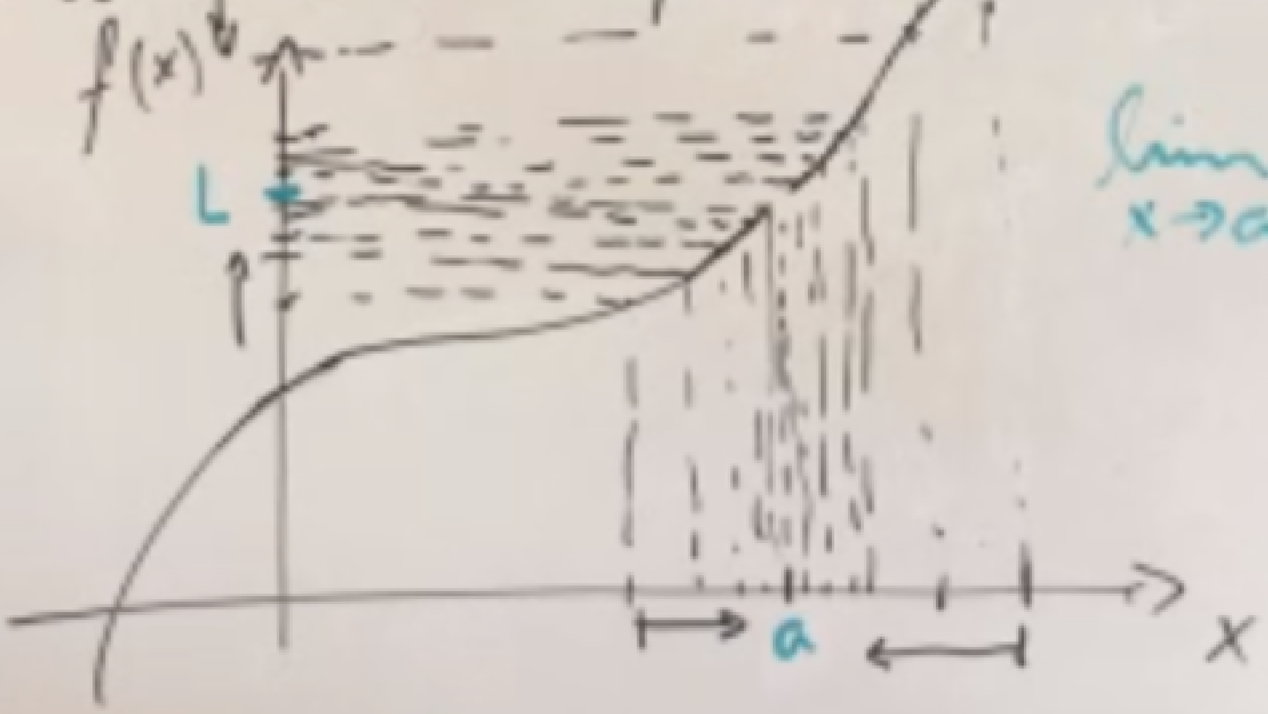

$$\lim_{x \to a} f(x) = L$$

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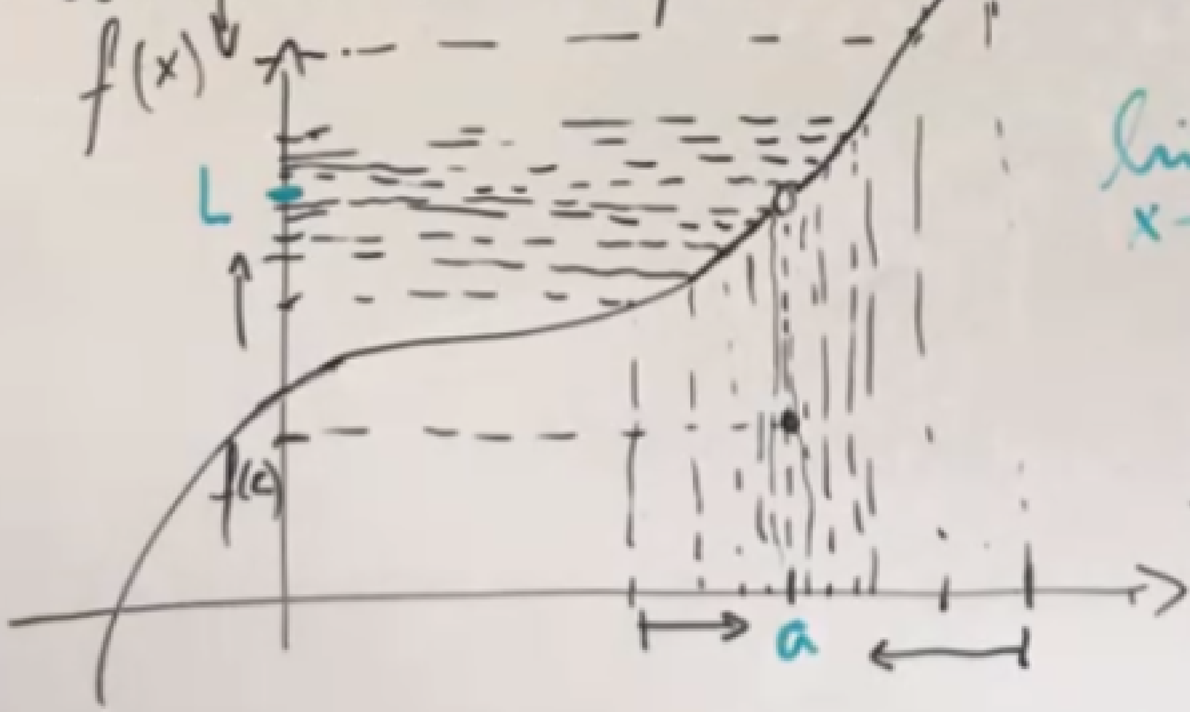


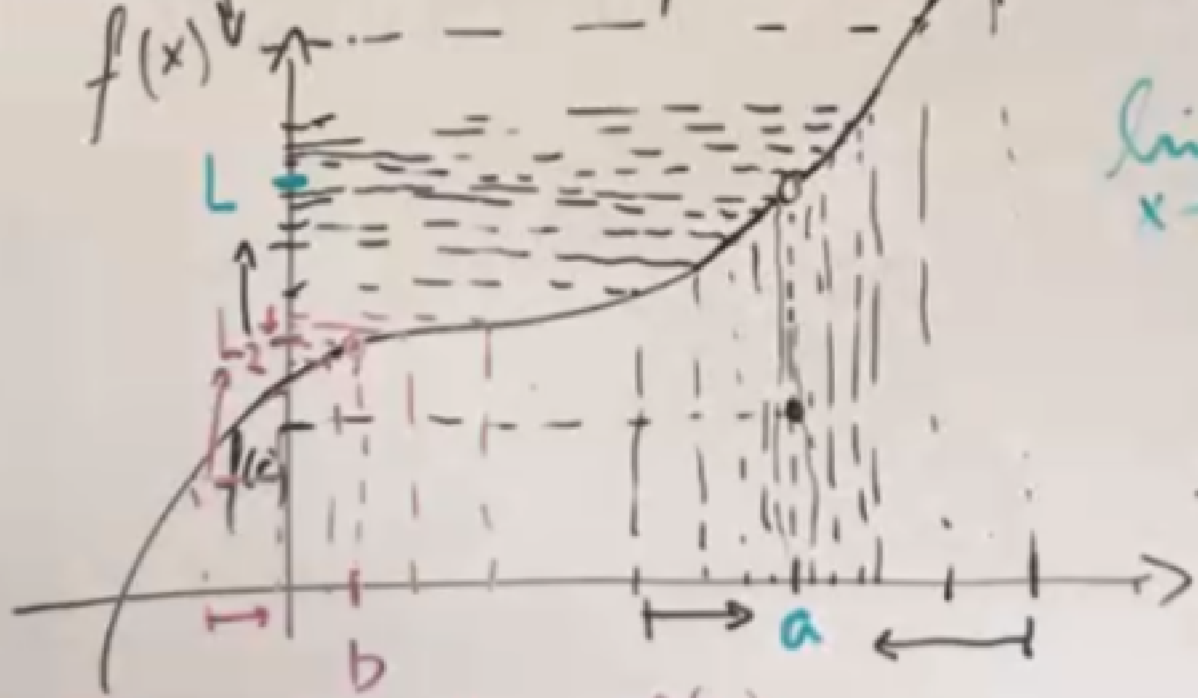
$$\lim_{x \to a} f(x) = L$$

neste caso
$f(a)$ existe e
$f(a) \neq L$

$$f(x) = \begin{cases} x^3 & ; x \neq 2 \\ 7 & ; x = 2 \end{cases}$$

LIMITES:

* Qual o comportamento da função quando a variável se aproxima de determinado valor?



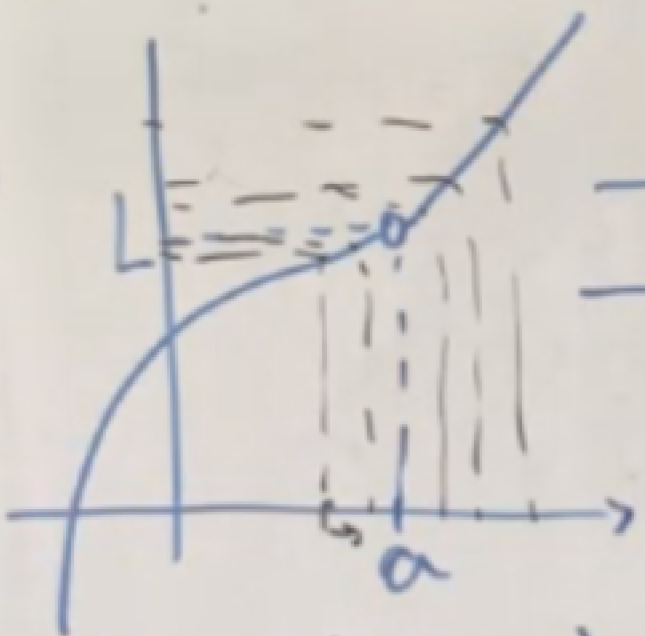
$$\lim_{x \to a} f(x) = L$$

neste caso $f(a)$ existe e

$$f(a) \neq L$$

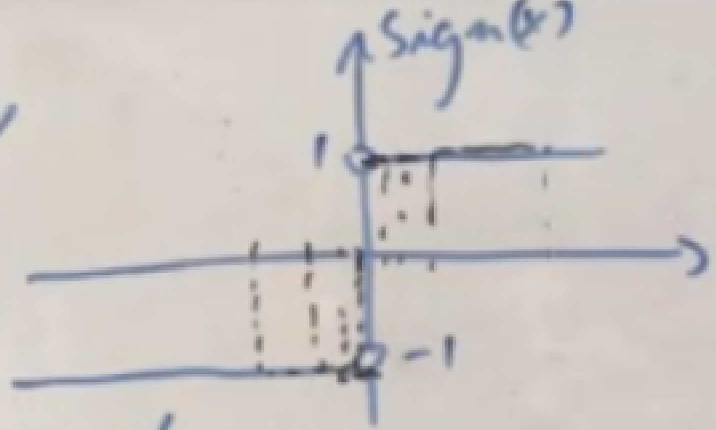
$$\lim_{x \to b} f(x) = L_2 = f(b)$$

$$f(x) = \begin{cases} x^3 ; & x \neq 2 \\ 1-7 ; & x = 2 \end{cases}$$



$\nexists\, f(a) \quad (a \notin D_f)$

$$\lim_{x \to a} f(x) = L$$

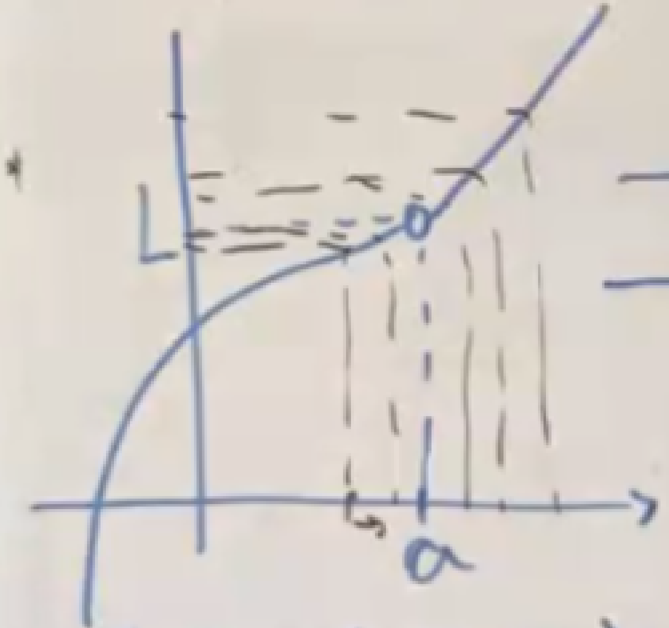


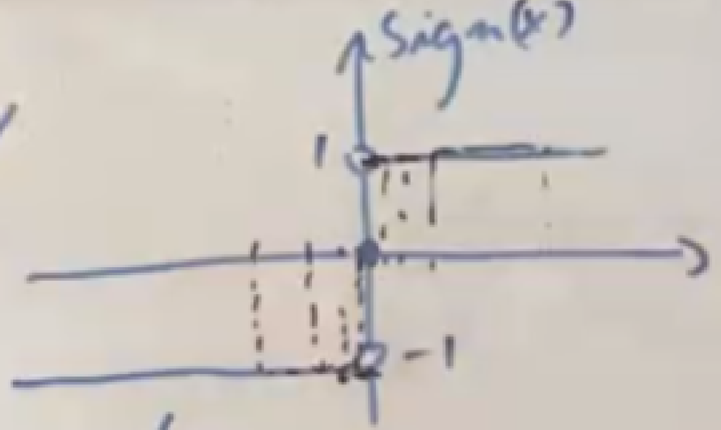
$$\nexists \lim_{x \to 0} \operatorname{sign}(x)$$

Limites laterais

$$\lim_{x \to 0^-} \operatorname{sign}(x) = -1$$

$$\lim_{x \to 0^+} \operatorname{sign}(x) = +1$$





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$\lim_{x \to 0^-} \text{sign}(x) = -1$

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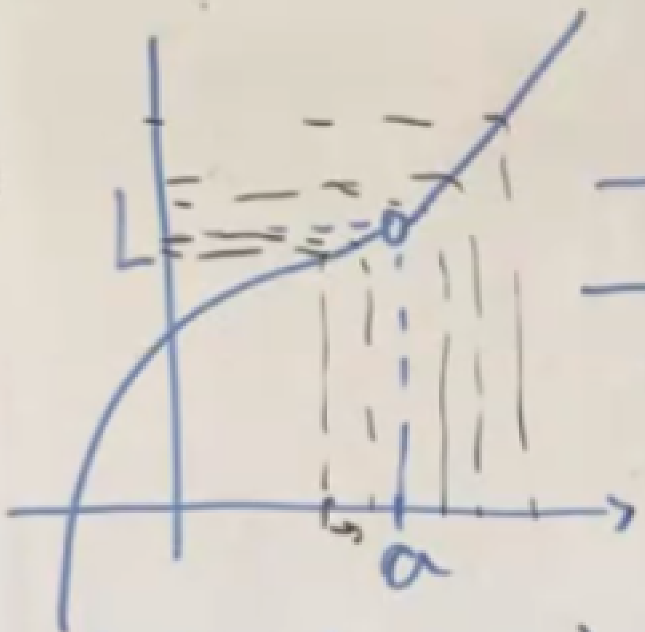
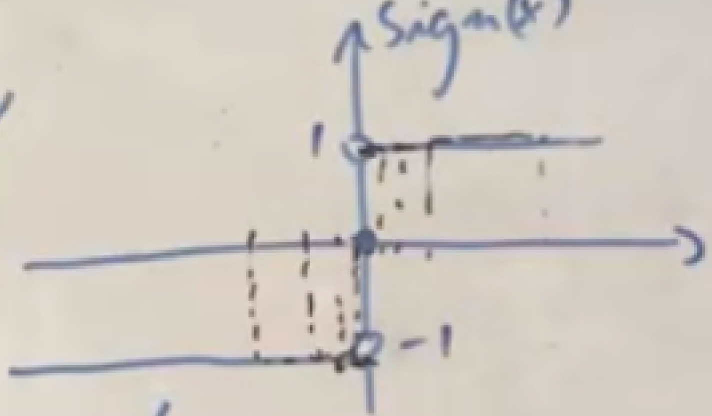
$\nexists\, f(a) \; (a \notin D_f)$

$\lim_{x \to a} f(x) = L$

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$





$$f(x)=\begin{cases}1, & x\in\mathbb{Q}\\ 0, & c.c.\end{cases}$$

$$f(3/4)=1$$

$$\nexists \lim_{x\to 0} \mathrm{sign}(x)$$

$$\nexists \lim_{x\to 3/4} f(x)$$

Límits laterals

$$\nexists \lim_{x\to x_0} f(x); \quad \forall x_0\in\mathbb{R}$$

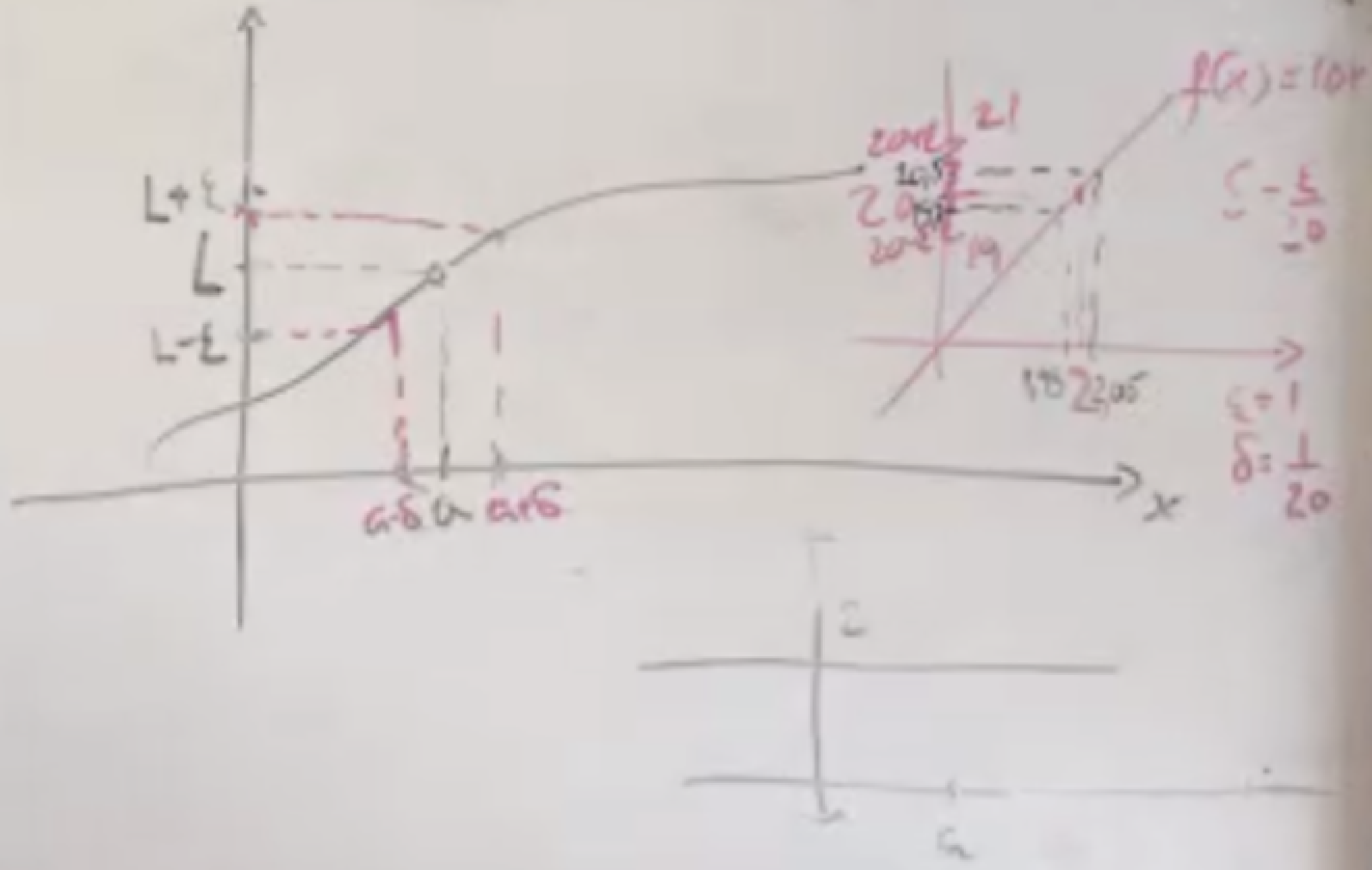
$$\nexists f(a) \ (a\notin D_f)$$

$$\lim_{x\to a} f(x) = L$$

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$$

$$\lim_{x\to 0^-} \mathrm{sign}(x) = -1$$

$$\lim_{x\to 0^+} \mathrm{sign}(x) = +1$$


L+ε
L
L−ε
a−δ
a
a+δ
x
f(x) = 10x
21
20
19
ε = 1
δ = 1/20

* Se $\lim_{x \to a} f(x) = f(a)$: f é contínua em a
* Se $f(x)$ for contínua $\forall x \in [a, b]$; f é contínua em $[a, b]$

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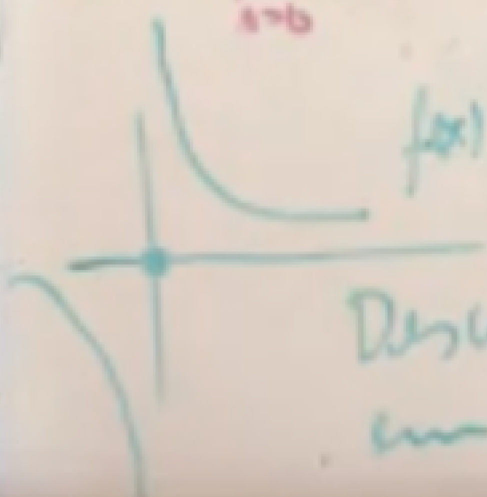
* se $\lim\limits_{x \to a} f(x) = f(a)$: f é contínua em a

* se $f(x)$ for contínua $\forall x \in [a, b]$; f é contínua em $[a, b]$

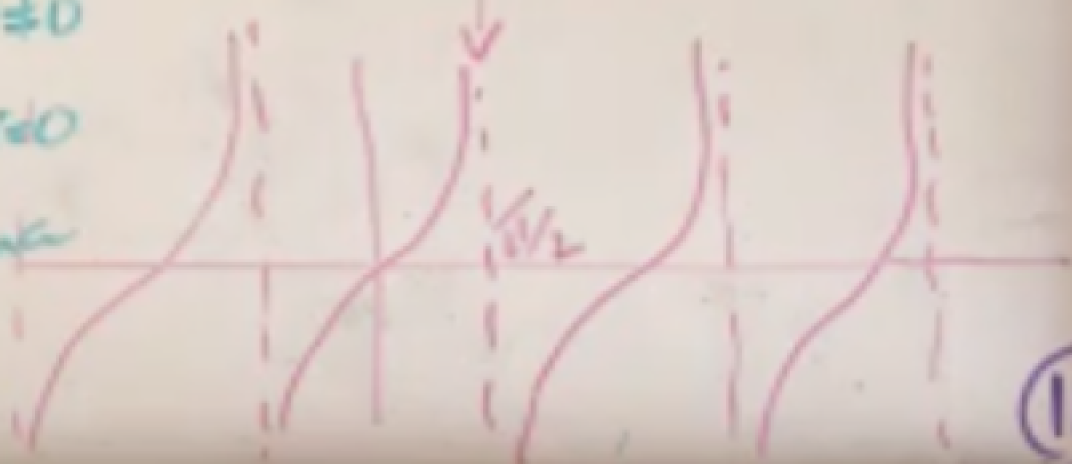
* se $f(x)$ for contínua $\forall x \in D_f$: f é contínua

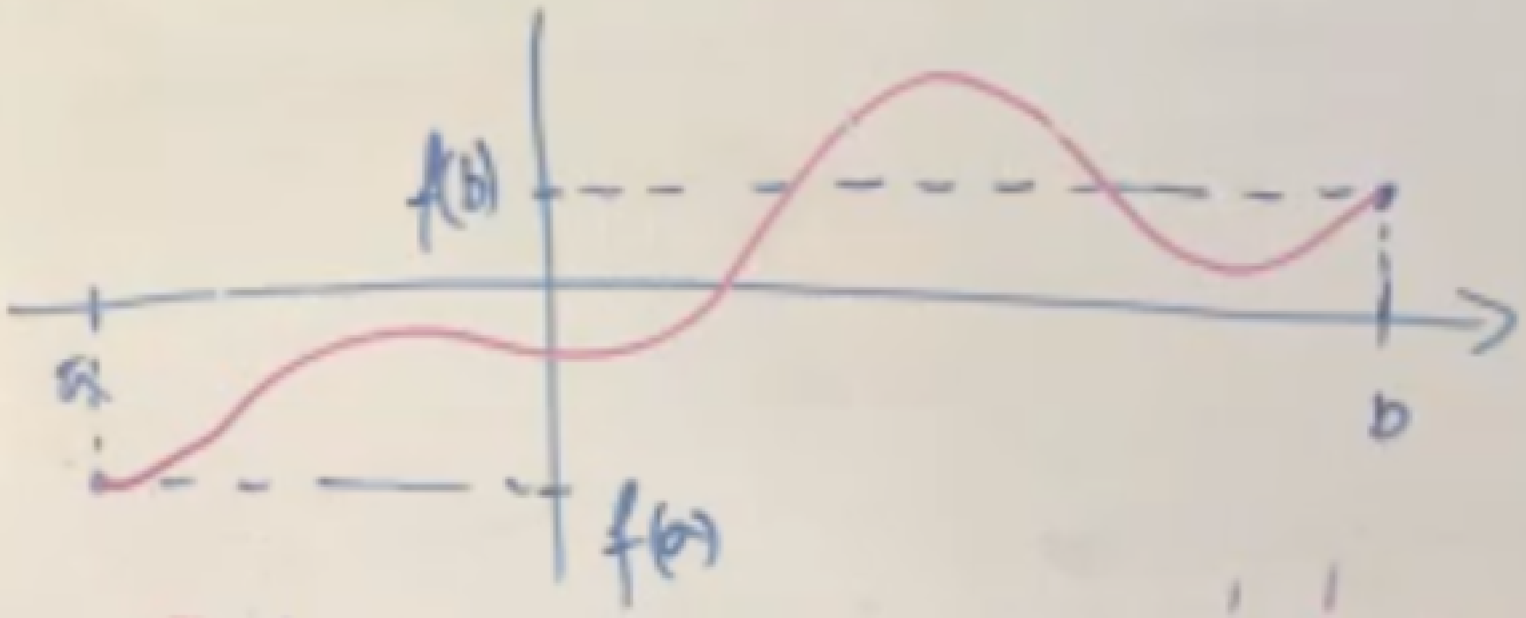
ex: $\sum_{i=0}^{n} a_i x^i$, $\operatorname{sen} x$, $\cos x$, $\operatorname{tg} x$, e^x, $\ln x$, $\frac{1}{x}$



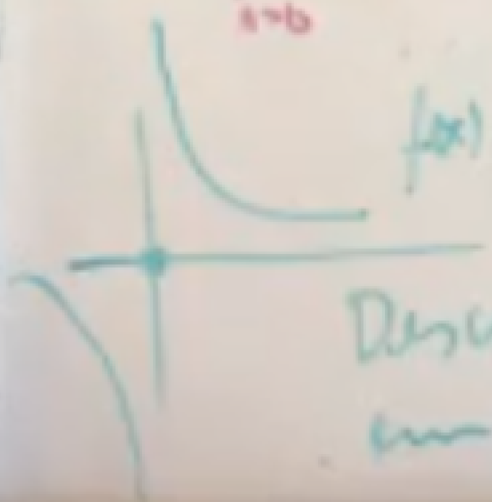
$$f(x) \begin{cases} = \frac{1}{x}, & x \neq 0 \\ = 0, & x = 0 \end{cases}$$

Descontínua em $x = 0$



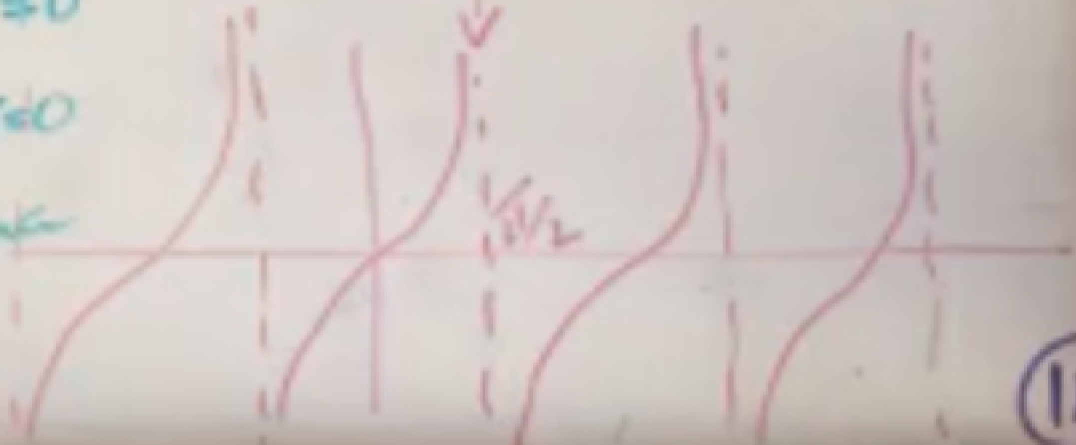


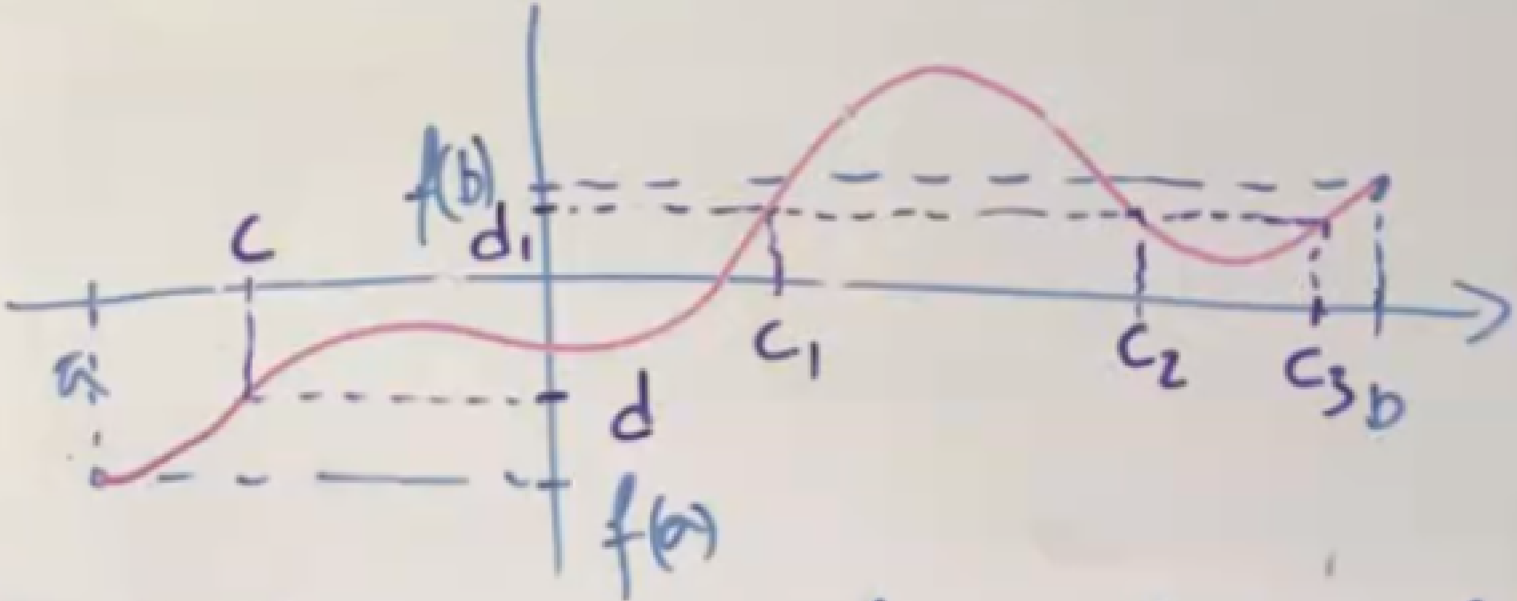
ex: $\sum_{i=0}^{n} a_n x^i$, sen x, cos x, tg x, e^x, ln x, $\frac{1}{x}$



$$f(x)\begin{cases} = \frac{1}{x}, & x \neq 0 \\ = 0, & x = 0 \end{cases}$$

Descontínua em $x = 0$



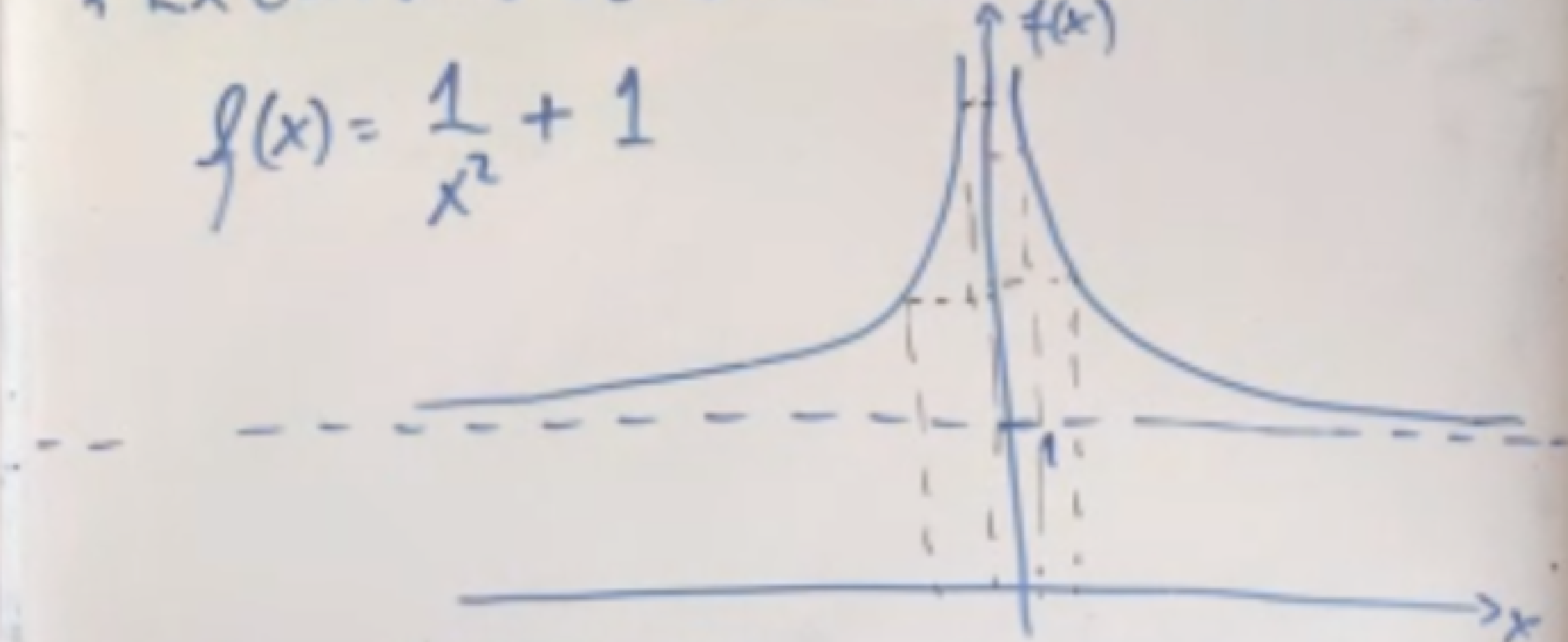


Teorema do valor intermediário (T.V.I.)

$$f(a) < d < f(b) \Longrightarrow \exists\, c \in [a, b] \mid f(c) = d$$

* Extensões do conceito de limite

$$f(x) = \frac{1}{x^2} + 1$$



$$\lim_{x \to 0} f(x) = +\infty$$

$$\lim_{x \to +\infty} f(x) = 1$$

$$\lim_{x \to -\infty} f(x) = 1$$

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$$f(x) = \frac{x^2 - x - 12}{x-4} \quad ; \quad D_f = \mathbb{R} \setminus \{4\}$$

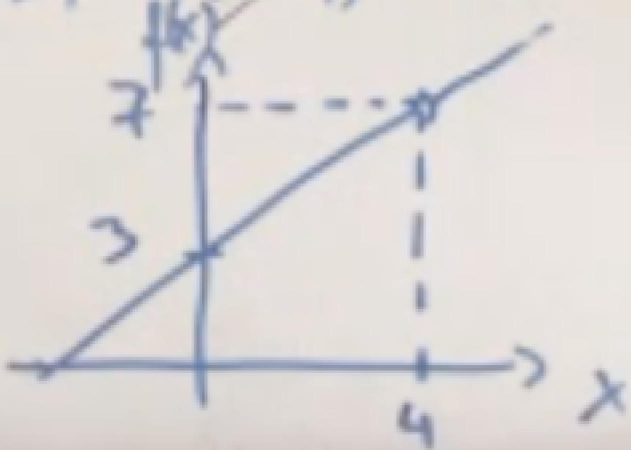
$$\lim_{x \to 4} f(x) = \lim_{x \to 4} \frac{x^2 - x - 12}{x-4}$$

$$f(x) = x^2 - x - 12 = 0 \quad ; \quad \Delta = 1 - 4(-12) = 49 \quad r = \frac{1 \pm 7}{2} \begin{cases} 4 \\ -3 \end{cases}$$

$$x^2 - x - 12 = (x-4)(x+3)$$

$$\lim_{x \to 4} f(x) = \lim_{x \to 4} \frac{\cancel{(x-4)}(x+3)}{\cancel{(x-4)}} = \lim_{x \to 4} (x+3) = 7$$

$$\frac{1}{1+e^{-x}} \cdot \frac{e^x}{e^x} = \frac{e^x}{e^x + 1}$$



$$\frac{x(x-4)}{(x-4)} = x \; ?$$

NÃO!

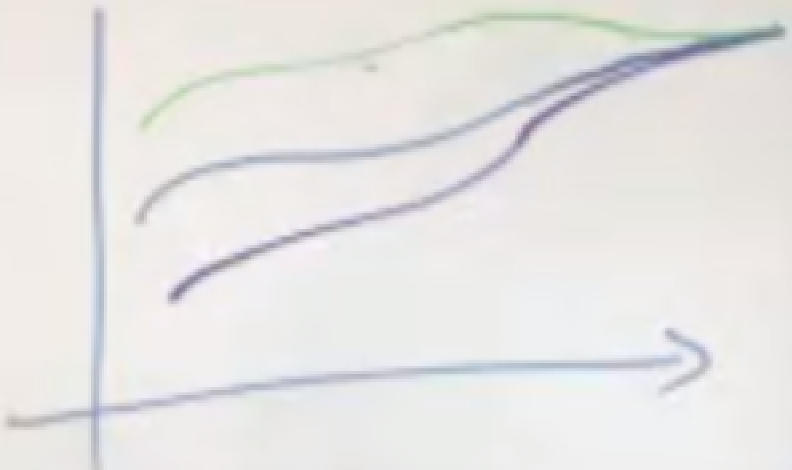
TEOREMA DO CONFRONTO

$f(x), g(x), h(x)$ contínuas em $[a, b]$ e

$f(x) \leq g(x) \leq h(x)$, então se $c \in]a, b[$

$\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L$, então

$\lim_{x \to c} g(x) = L$



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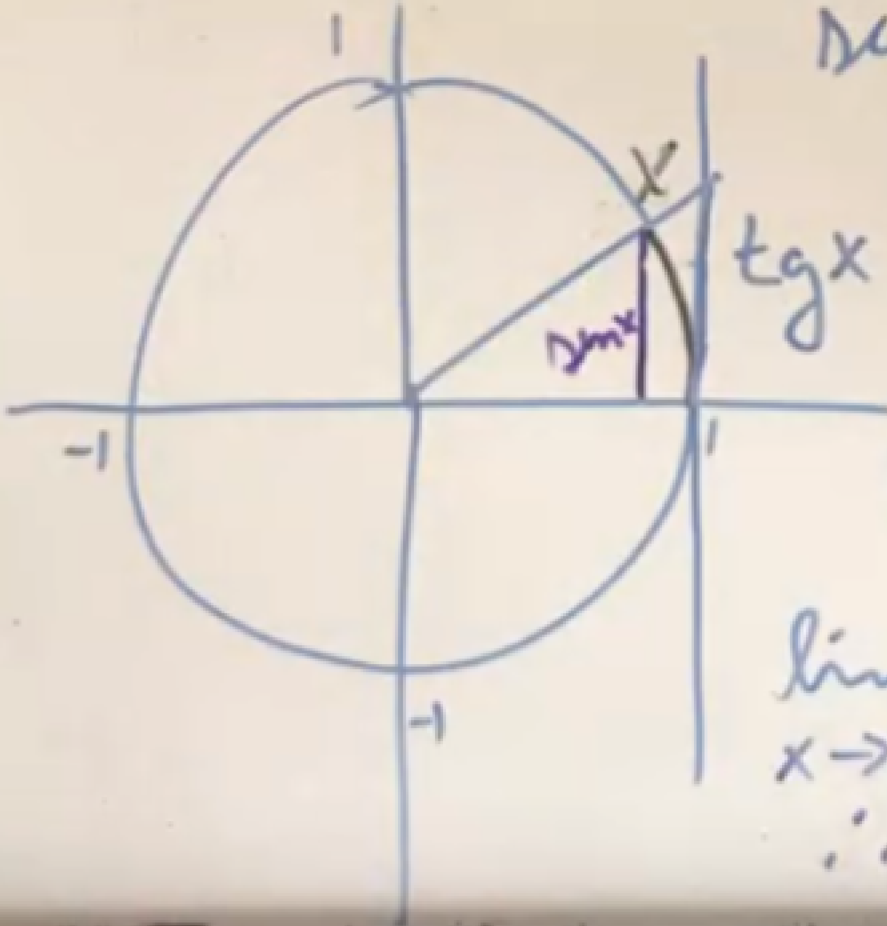
Ex: o limite fundamental

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{x}$$



$$\operatorname{sen} x \leq x \leq \operatorname{tg} x \quad (\div \operatorname{sen} x)$$

$$1 < \frac{x}{\operatorname{sen} x} < \frac{1}{\cos x} \quad (\,)^{-1}$$

$$\cos x < \frac{\operatorname{sen} x}{x} < 1$$

$$\lim_{x \to 0} \cos x = 1 = \lim_{x \to 0} 1$$

$$\therefore \boxed{\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1}$$

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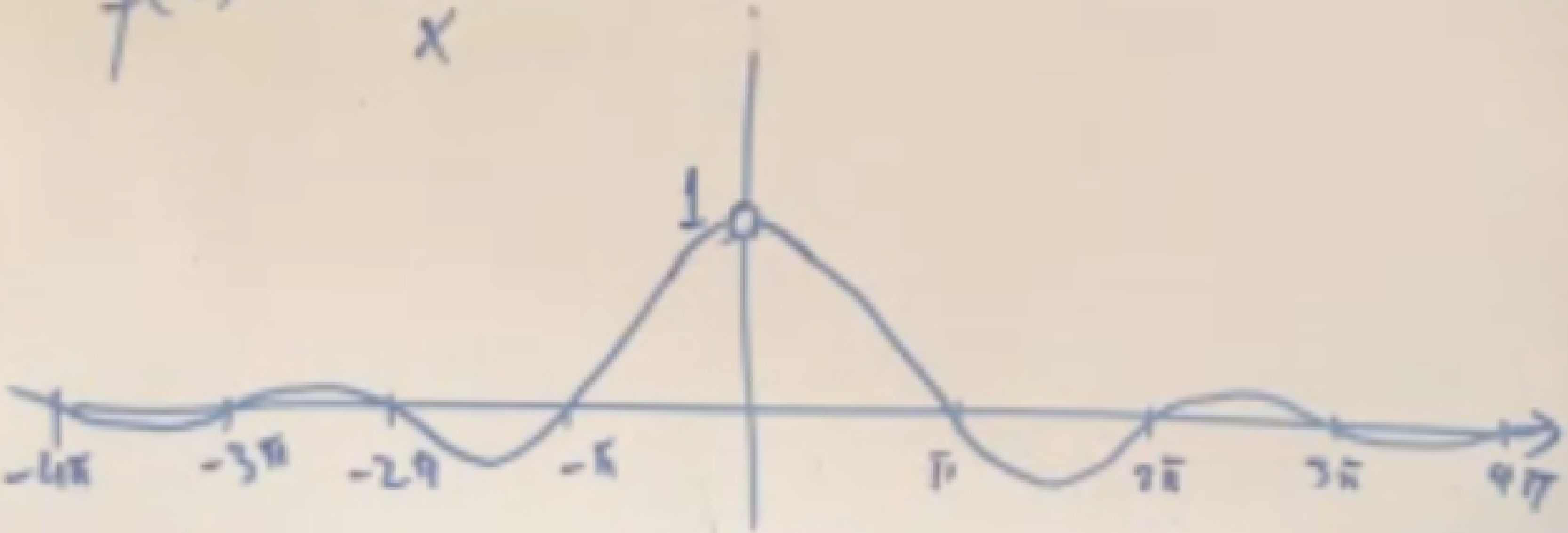
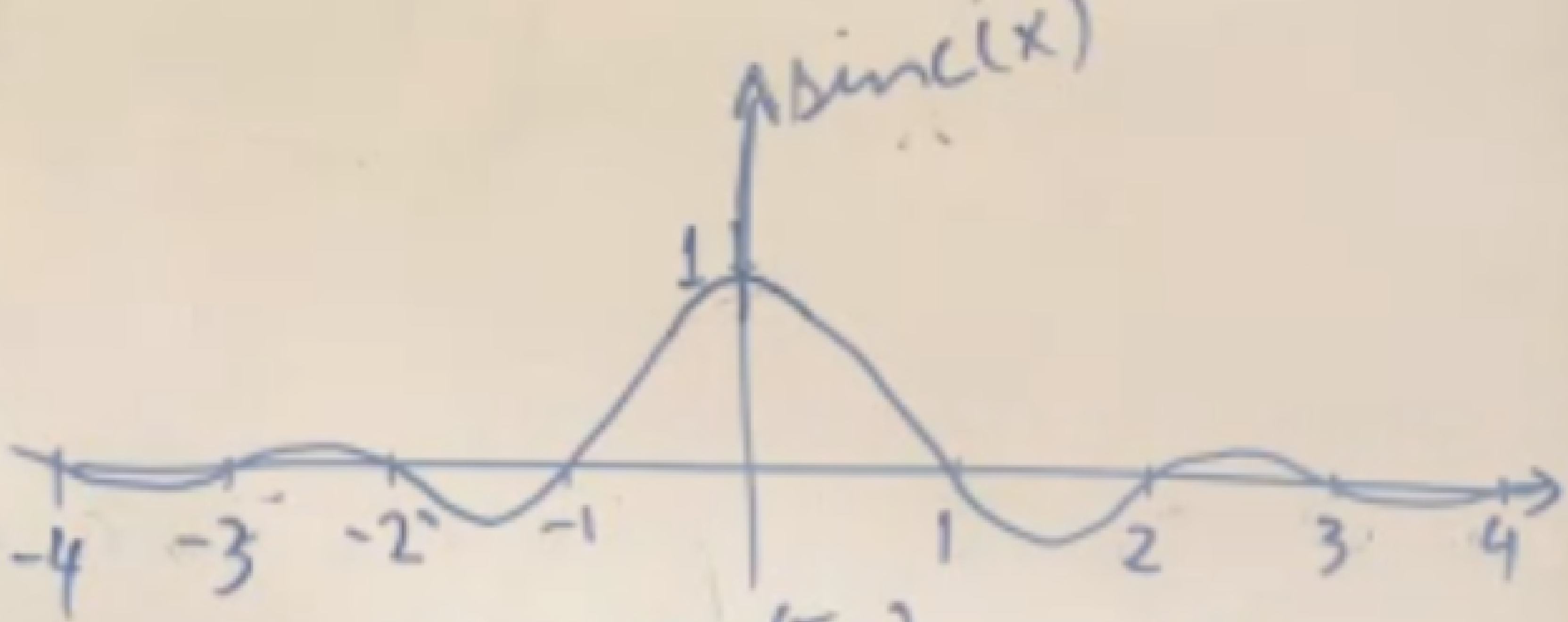
$$f(x) = \frac{\operatorname{sen} x}{x}$$



$$* \ \operatorname{sinc}(x) = \begin{cases} \dfrac{\operatorname{sen}(\pi x)}{\pi x} & , x \neq 0 \\ 1 & , x = 0 \end{cases}$$



$$* \; \mathrm{sinc}(x) = \begin{cases} \dfrac{\sin(\pi x)}{\pi x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

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