CIAG - CALC 1VAR - 3° dia 30/03

→ Discussão dos exercícios!

→ Na pasta: notebook Resolucao_tombio

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→ Na pasta: notebook Resolucao_tombio

$$\frac{3^x}{9^{x-2}} = \frac{3^x}{(3^2)^{x-2}} = \frac{3^x}{3^{2x-4}} =$$

$$= 3^{x-(2x-4)} = 3^{-x+4} = 3^{4-x}$$

1

$$f(x) = \text{sen}\, x \quad ; \quad g(x) = x^3$$

$$f(g(x)) = f(x^3) = \text{sen}(x^3)$$

$$g(f(x)) = g(\text{sen}\, x) = (\text{sen}\, x)^3 \equiv \text{sen}^3 x$$

$f(x) = \operatorname{sen} x \quad ; \quad g(x) = x^3$

$f(g(x)) = f(x^3) = \operatorname{sen}(x^3)$

$g(f(x)) = g(\operatorname{sen} x) = (\operatorname{sen} x)^3 \equiv \operatorname{sen}^3 x$

$9^{\log_9 100} = 100$

$f(x) = 9^x$

$f^{-1}(x) = \log_9 x$

$f(f^{-1}(x)) = \underline{9^{\log_9 x}} =$

$= f^{-1}(f(x)) = \underline{\underline{\log_9 9^x}} = x$

2

$$e^{-\frac{1}{2}\ln\frac{1}{16}-\frac{2}{3}\ln 27} = e^{-\frac{1}{2}\ln 2^{-4}} \cdot e^{-\frac{2}{3}\ln 3^{3}} =$$

$$= e^{\ln(2^{-4})^{-\frac{1}{2}}} \cdot e^{\ln(3^{3})^{-\frac{2}{3}}} = (2^{-4})^{-\frac{1}{2}} \cdot (3^{3})^{-\frac{2}{3}} =$$

$$= 2^{2} \cdot 3^{-2} = \frac{4}{9}$$

2

$$e^{-\frac{1}{2}\ln\frac{1}{16}-\frac{2}{3}\ln 27} = e^{-\frac{1}{2}\ln 2^{-4}} \cdot e^{-\frac{2}{3}\ln 3^{3}} =$$

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$$= 2^{2} \cdot 3^{-2} = \frac{4}{9}$$

$$f(x) = \begin{cases} 0, & x \text{ irracional} \\ 1, & \text{c.c.} \end{cases}$$

$$e^{-\frac{1}{2}\ln\frac{1}{16}-\frac{2}{3}\ln 27} = e^{-\frac{1}{2}\ln 2^{-4}} \cdot e^{-\frac{2}{3}\ln 3^{3}} =$$

$$= e^{\ln(2^{-4})^{-\frac{1}{2}}} \cdot e^{\ln(3^{3})^{-\frac{2}{3}}} = (2^{-4})^{-\frac{1}{2}} \cdot (3^{3})^{-\frac{2}{3}} =$$

$$= 2^{2} \cdot 3^{-2} = \frac{4}{9}$$

$$f \circ g(x) = f(g(x)) = f(2x+1) = (2x+1)^2 - 1$$

$$g \circ f(x) = g(f(x)) = g(x^2-1) = 2(x^2-1)+1$$

2

$$\frac{2\pi}{R} = \frac{360^0}{G} \Rightarrow R = \frac{\pi G}{180}$$

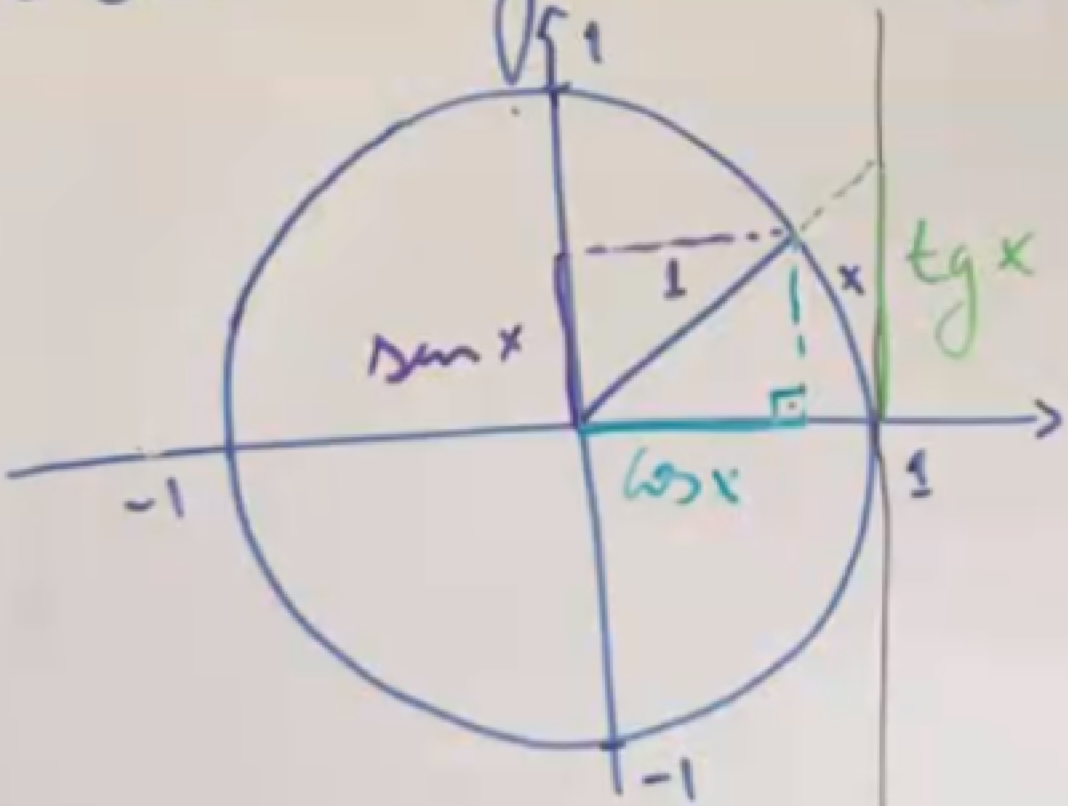
$$\frac{T_c - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} \Rightarrow T_F = 1{,}8\,T_c + 32$$

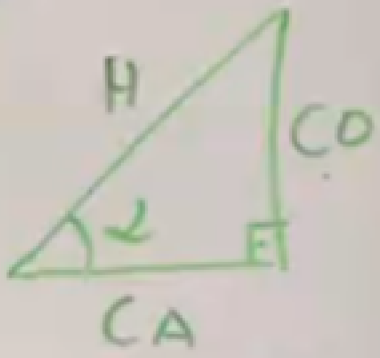
2

Continuando o repertório de funções:

FUNÇÕES TRIGONOMÉTRICAS

* O ciclo trigonométrico:





$\cos\alpha = \frac{CA}{H}$

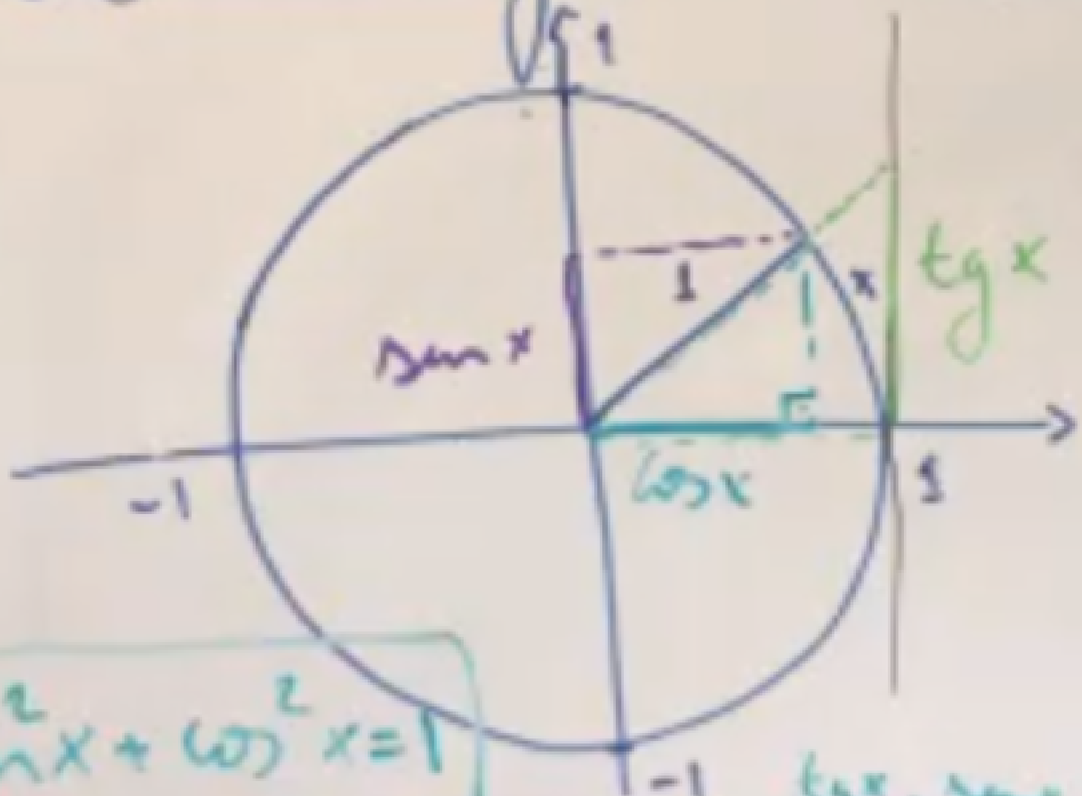
$\operatorname{sen}\alpha = \frac{CO}{H}$

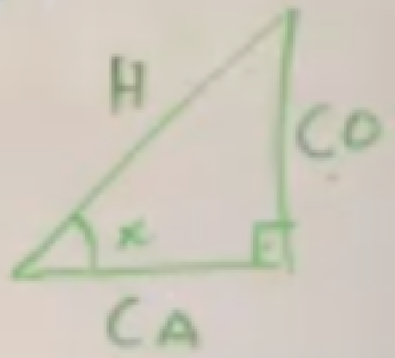
$\operatorname{tg}\alpha = \frac{CO}{CA}$

Continuando o repertório de funções:

FUNÇÕES TRIGONOMÉTRICAS

* O ciclo trigonométrico:





$\cos x = \frac{CA}{H}$

$\operatorname{sen} x = \frac{CO}{H}$

$\operatorname{tg} x = \frac{CO}{CA} = \frac{\operatorname{sen} x}{\cos x}$

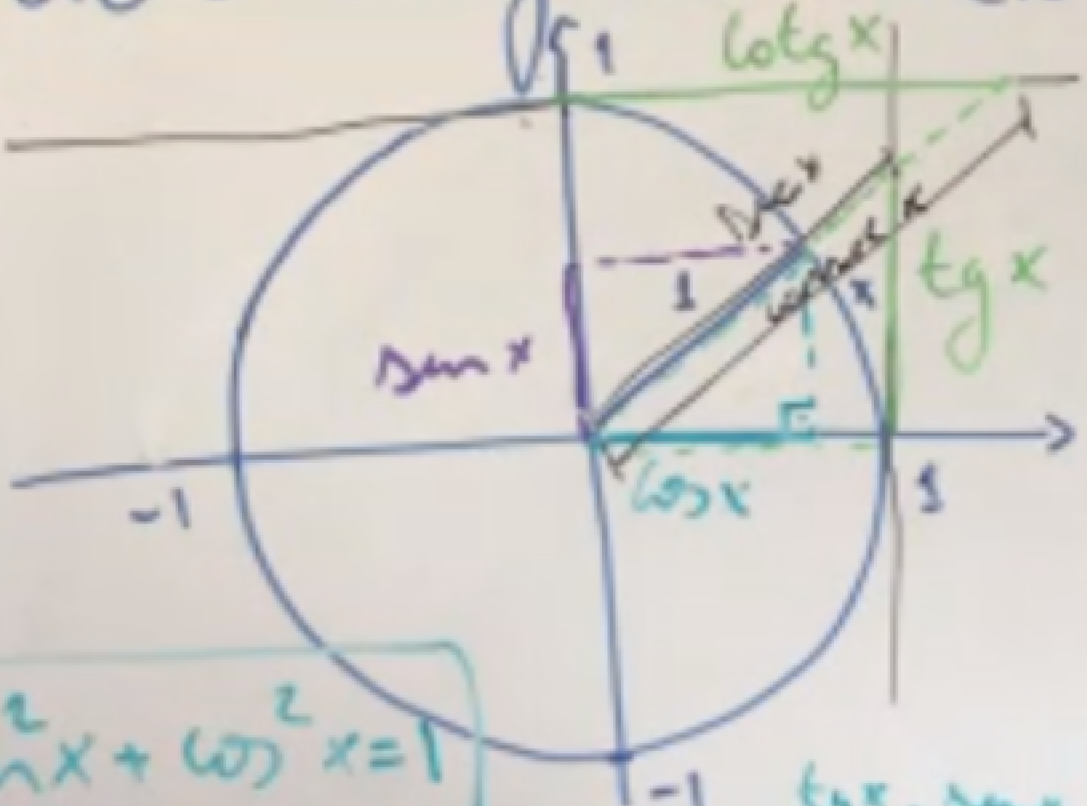
$\frac{\operatorname{tg} x}{1} = \frac{\operatorname{sen} x}{\cos x}$

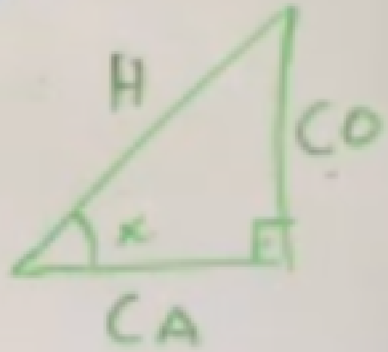
$\boxed{\operatorname{sen}^2 x + \cos^2 x = 1}$

Continuando o repertório de funções:

FUNÇÕES TRIGONOMÉTRICAS

* O ciclo trigonométrico:





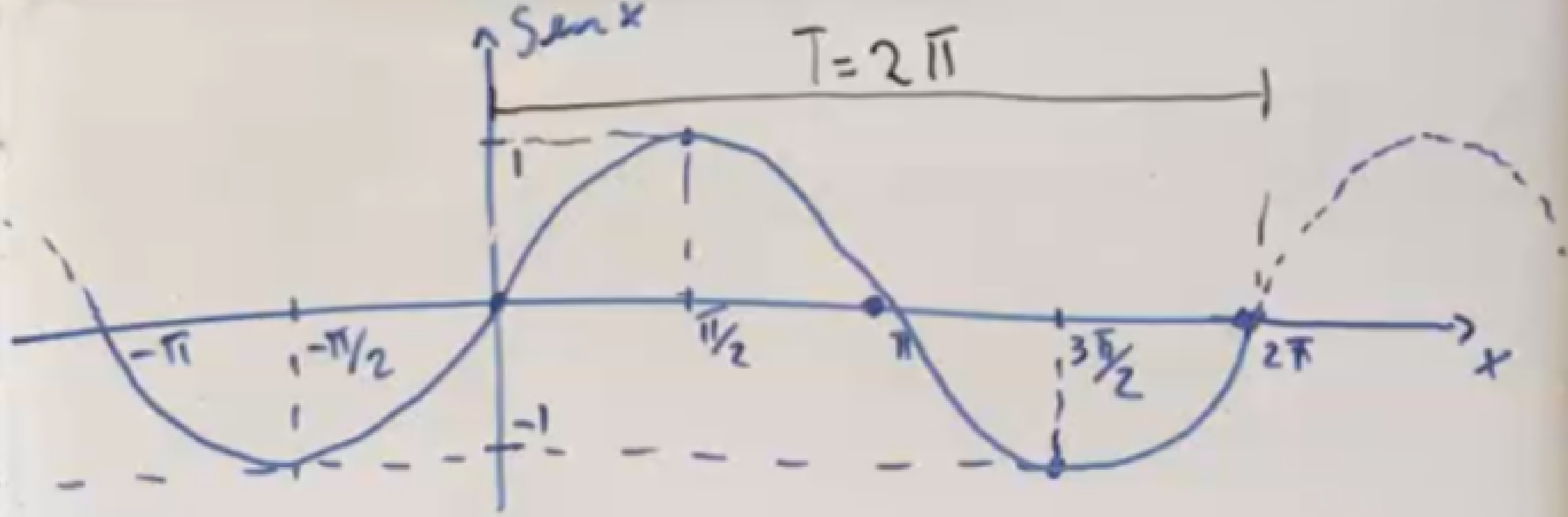
$\cos x = \dfrac{CA}{H}$

$\operatorname{sen} x = \dfrac{CO}{H}$

$\operatorname{tg} x = \dfrac{CO}{CA} = \dfrac{\operatorname{sen} x}{\cos x}$

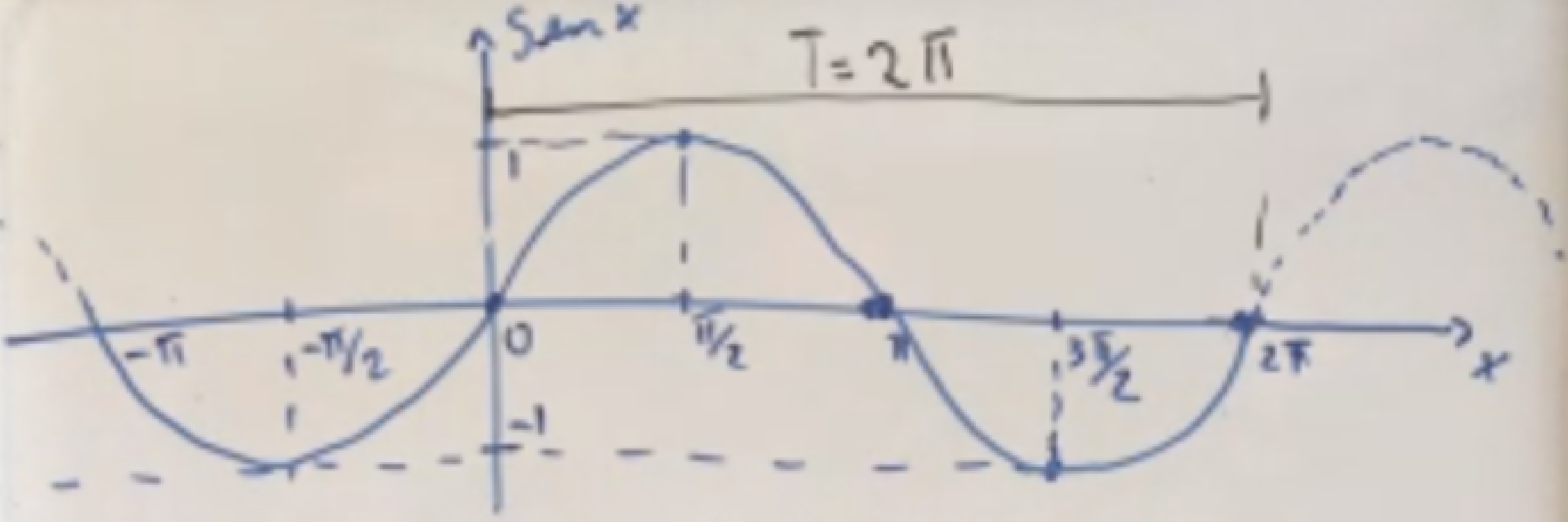
$\dfrac{\operatorname{tg} x}{1} = \dfrac{\operatorname{sen} x}{\cos x}$

$\boxed{\operatorname{sen}^2 x + \cos^2 x = 1}$



* As funções trigonomométricas são Periódicas: $\operatorname{sen}(x+Tm)=\operatorname{sen}(x)$, $\forall x$

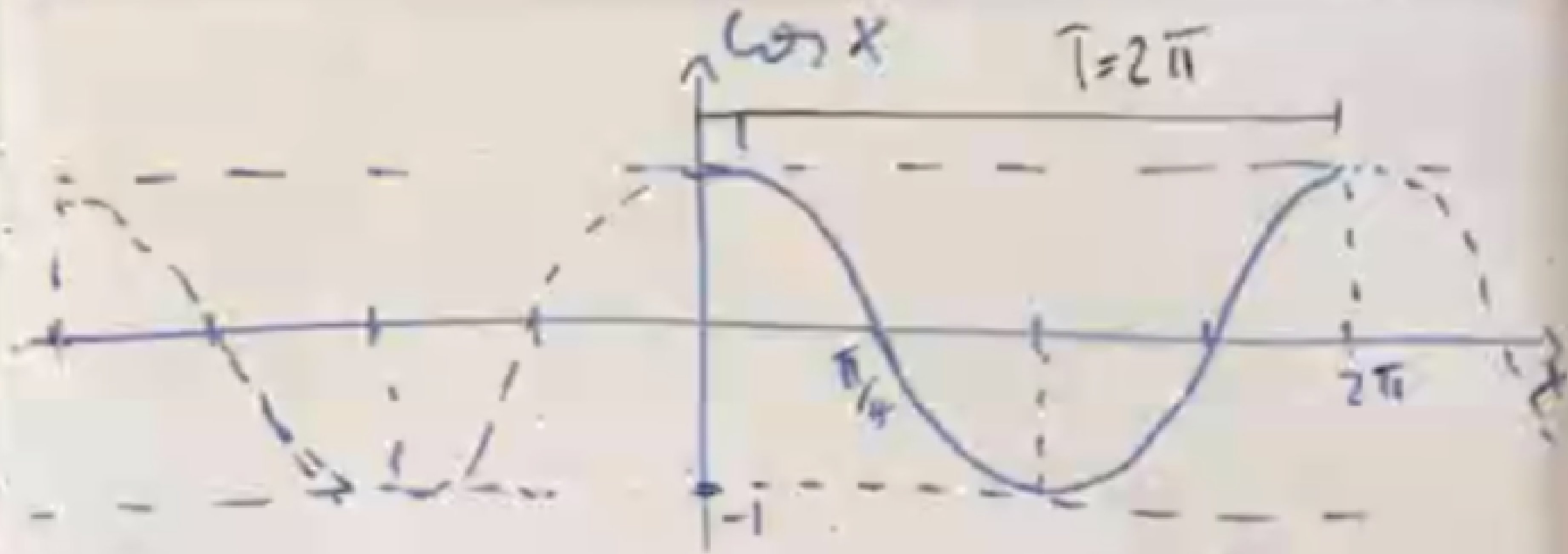
$$\operatorname{sen}(x+2\pi m) = \operatorname{sen}(x), \forall x$$



* As funções trigonomométricas são Periódicas:

$$\operatorname{sen}(x + Tm) = \operatorname{sen}(x), \ \forall x$$
$$\operatorname{sen}(x + 2\pi m) = \operatorname{sen}(x), \forall x$$

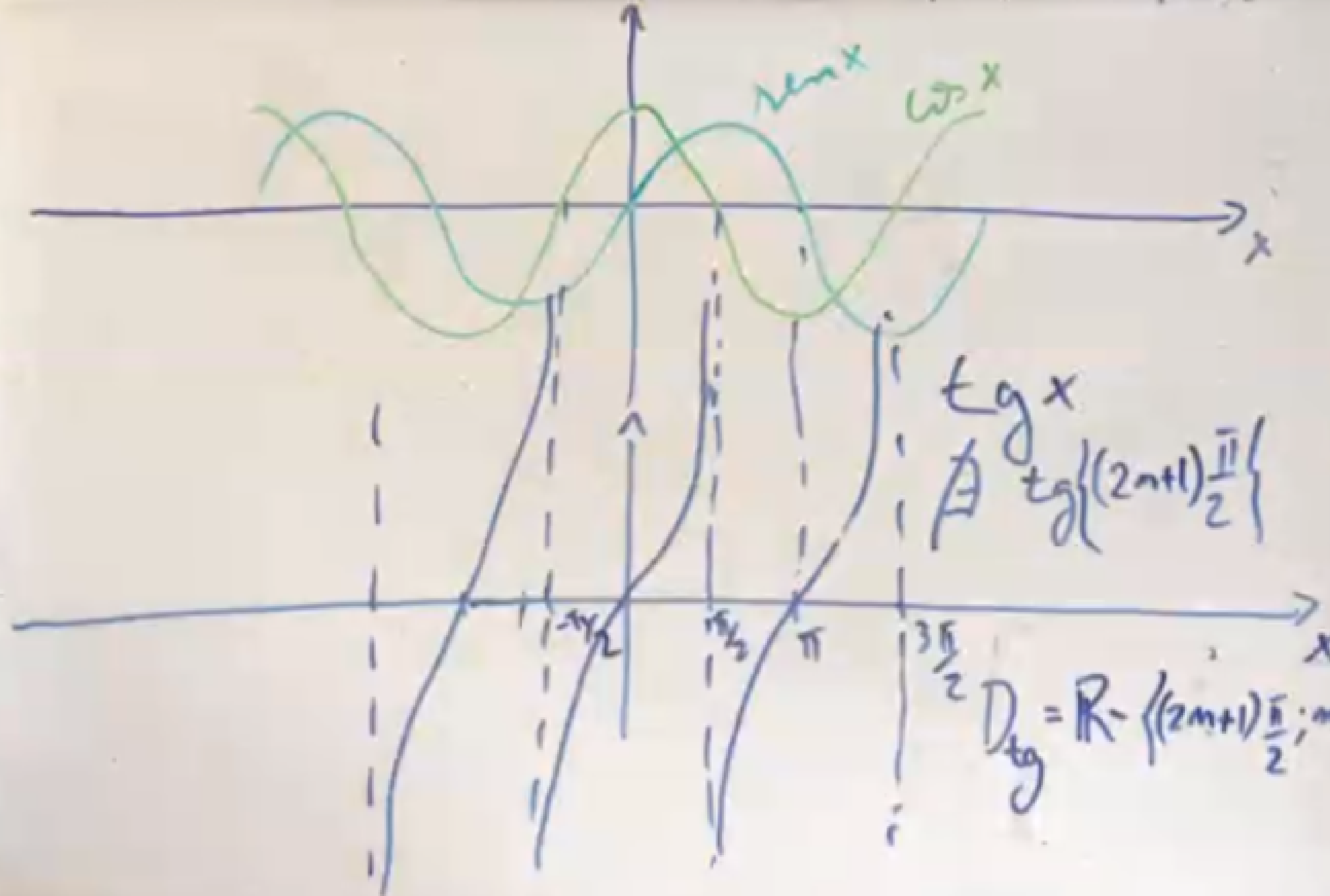
$$D_{sen} = \mathbb{R} = CD_{sen} \ ; \ Im_{sen} = [-1, 1]$$



$$\cos(x + mT) = \cos(x), \ \forall x$$

$$T = 2\pi$$

→ periódica!



$tg\,x$

$\nexists\ tg\{(2n+1)\frac{\pi}{2}\}$

$D_{tg} = \mathbb{R} - \{(2n+1)\frac{\pi}{2};\ n \in \mathbb{Z}\}$

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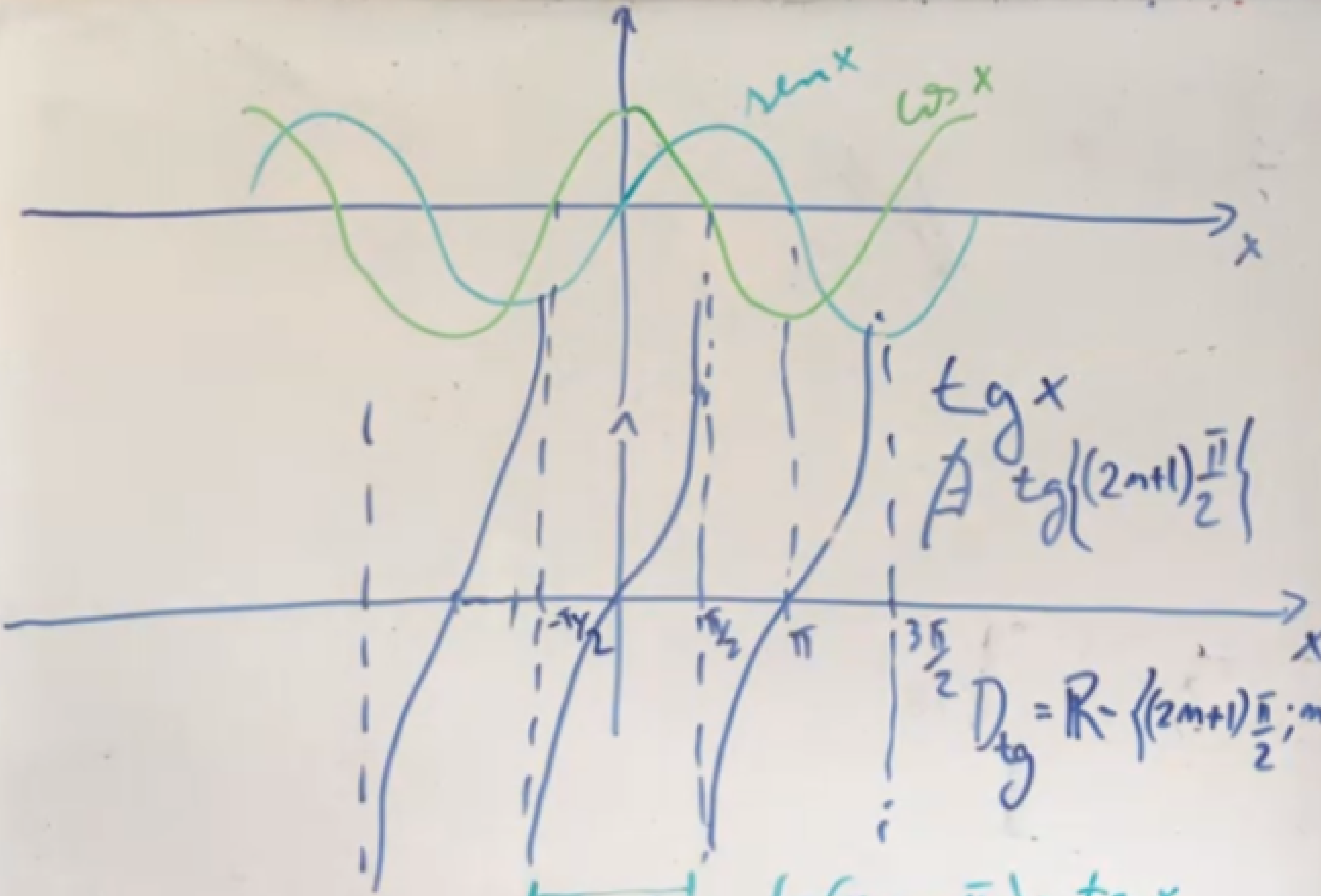

$\operatorname{tg} x$

$\nexists \operatorname{tg}\{(2n+1)\frac{\pi}{2}\}$

$D_{tg} = \mathbb{R} - \{(2n+1)\frac{\pi}{2} ; n \in \mathbb{Z}\}$

$T = \pi$

$\operatorname{tg}(x + n\pi) = \operatorname{tg} x$

6

$$\operatorname{sen}(A \pm B) = \operatorname{sen} A \cos B \pm \operatorname{sen} B \cos A$$

$$\cos(A \pm B) = \cos A \cos B \mp \operatorname{sen} A \operatorname{sen} B$$

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$$\begin{cases} \operatorname{sen}(A \pm B) = \operatorname{sen} A \cos B \pm \operatorname{sen} B \cos A \\ \cos(A \pm B) = \cos A \cos B \mp \operatorname{sen} A \operatorname{sen} B \end{cases}$$

$$\operatorname{sen}(A + 2\pi) = \operatorname{sen} A \cancelto{1}{\cos 2\pi} + \cancelto{0}{\operatorname{sen} 2\pi} \cos A = \operatorname{sen} A$$

$$\operatorname{sen}(A + \pi) = \operatorname{sen} A \cancelto{-1}{\cos \pi} + \cancelto{0}{\operatorname{sen} \pi} \cos A$$

$$\begin{cases} \operatorname{sen}(A \pm B) = \operatorname{sen} A \cos B \pm \operatorname{sen} B \cos A \\ \cos(A \pm B) = \cos A \cos B \mp \operatorname{sen} A \operatorname{sen} B \end{cases}$$

$$\operatorname{sen}(A + 2\pi) = \operatorname{sen} A \cancelto{1}{\cos 2\pi} + \cancelto{0}{\operatorname{sen} 2\pi} \cos A = \operatorname{sen} A$$

$$\operatorname{sen}(A + \pi) = \operatorname{sen} A \cancelto{-1}{\cos \pi} + \cancelto{0}{\operatorname{sen} \pi} \cos A = -\operatorname{sen} A$$

$$\operatorname{sen}(A + \pi/2) = \operatorname{sen} A \cancelto{0}{\cos \pi/2} + \cancelto{1}{\operatorname{sen} \pi/2} \cos A = \cos A$$

$$\begin{cases} \operatorname{sen}(A \pm B) = \operatorname{sen}A\cos B \pm \operatorname{sen}B\cos A \\ \cos(A \pm B) = \cos A\cos B \mp \operatorname{sen}A\operatorname{sen}B \end{cases}$$

$$\operatorname{sen}(A+2\pi) = \operatorname{sen}A\,\cancelto{1}{\cos 2\pi} + \cancelto{0}{\operatorname{sen}2\pi}\cos A = \operatorname{sen}A$$

$$\operatorname{sen}(A+\pi) = \operatorname{sen}A\,\cancelto{-1}{\cos\pi} + \cancelto{0}{\operatorname{sen}\pi}\cos A = -\operatorname{sen}A$$

$$\operatorname{sen}(A+\pi/2) = \operatorname{sen}A\,\cancelto{0}{\cos\pi/2} + \cancelto{1}{\operatorname{sen}\pi/2}\cos A = \cos A$$

$$\operatorname{sen}(-A) = \operatorname{sen}(0-A) = \cancelto{0}{\operatorname{sen}0}\cos A - \operatorname{sen}A\,\cancelto{1}{\cos 0} = -\operatorname{sen}A$$

$\rightarrow$ sen(x) é uma função ímpar!

$$\begin{cases} \operatorname{sen}(A \pm B) = \operatorname{sen} A \cos B \pm \operatorname{sen} B \cos A \\ \cos(A \pm B) = \cos A \cos B \mp \operatorname{sen} A \operatorname{sen} B \end{cases}$$

$$\operatorname{sen}(A + 2\pi) = \operatorname{sen} A \overset{1}{\cancel{\cos 2\pi}} + \overset{0}{\cancel{\operatorname{sen} 2\pi}} \cos A = \operatorname{sen} A$$

$$\operatorname{sen}(A + \pi) = \operatorname{sen} A \overset{-1}{\cancel{\cos \pi}} + \overset{0}{\cancel{\operatorname{sen} \pi}} \cos A = -\operatorname{sen} A$$

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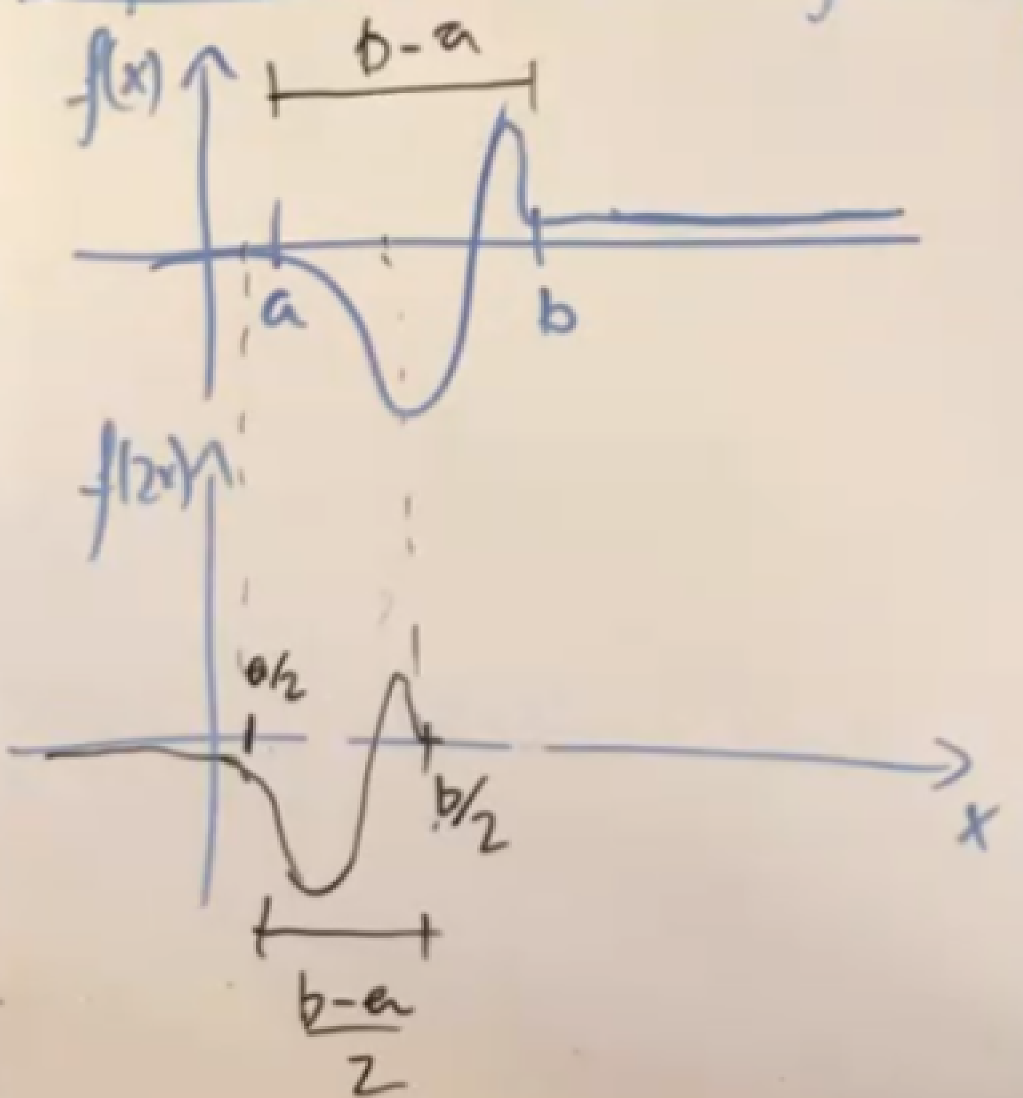
$$\operatorname{sen}(-A) = \operatorname{sen}(0 - A) = \overset{0}{\cancel{\operatorname{sen} 0}} \cos A - \operatorname{sen} A \overset{1}{\cancel{\cos 0}} = -\operatorname{sen} A$$

→ sen(x) é uma função ímpar!

$$\cos(-A) = \cos(0 - A) = \overset{1}{\cancel{\cos 0}} \cos A + \overset{0}{\cancel{\operatorname{sen} 0}} \operatorname{sen} A = \cos A$$

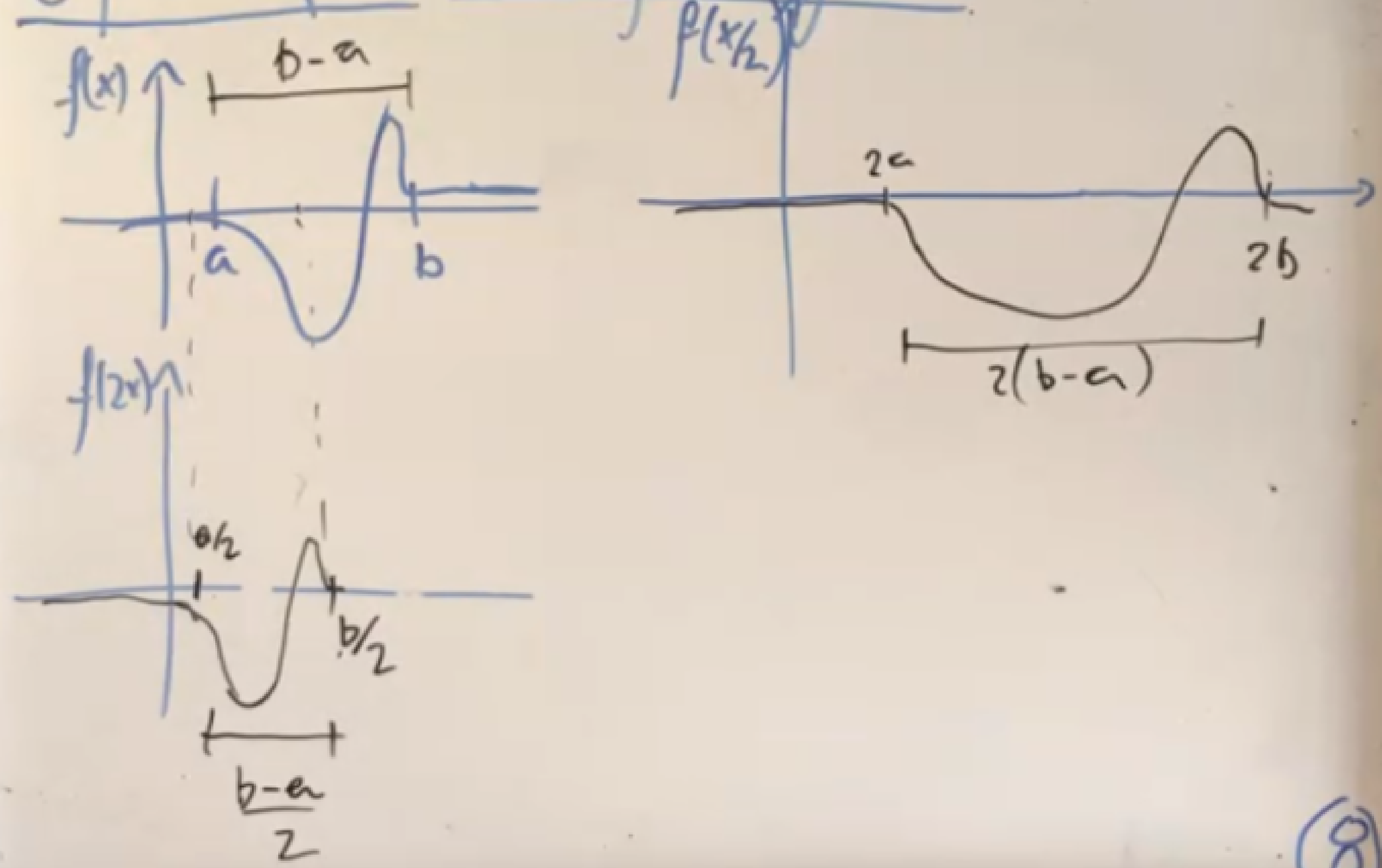
→ cos(x) é uma função par!

Operações morfológicas

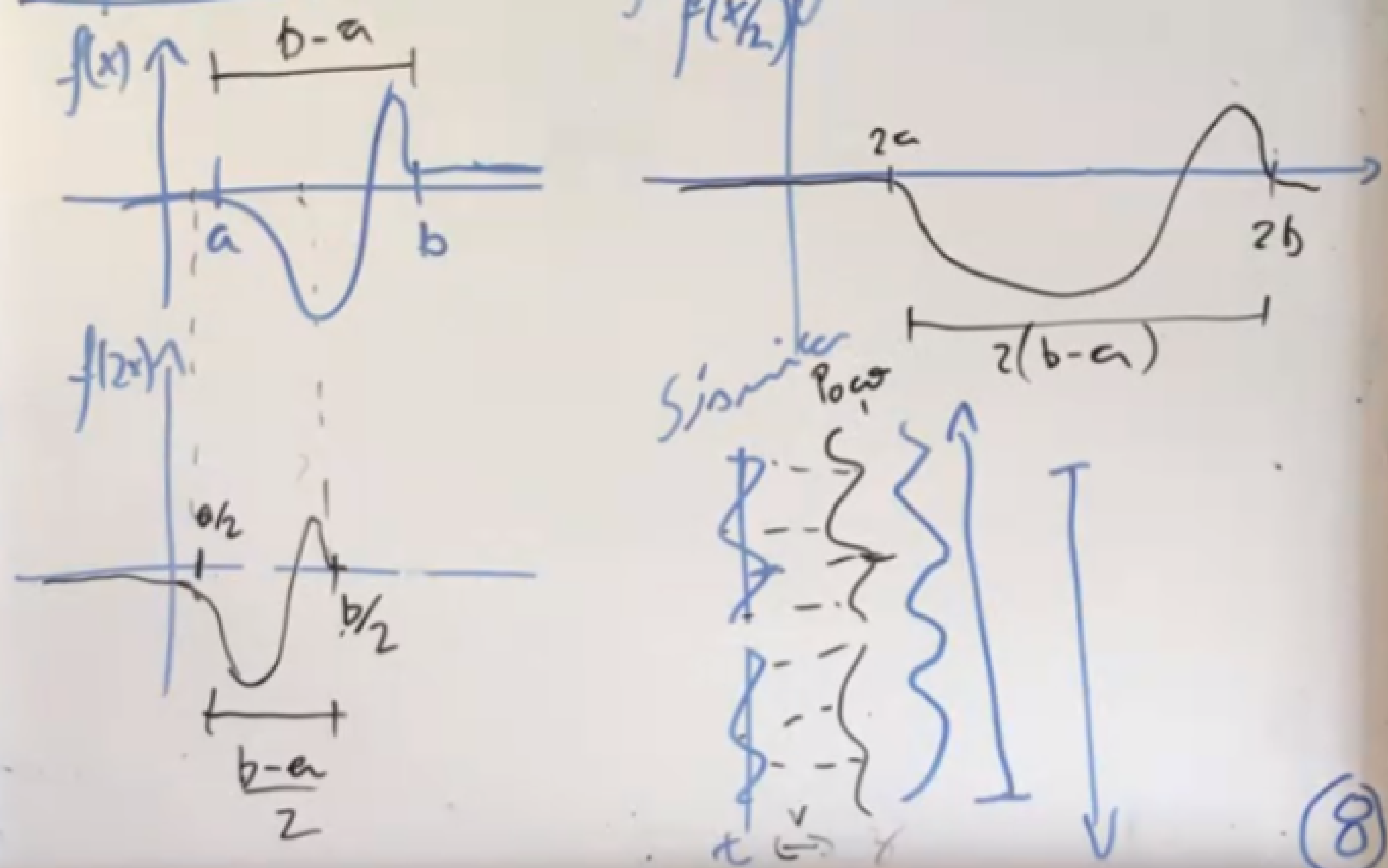


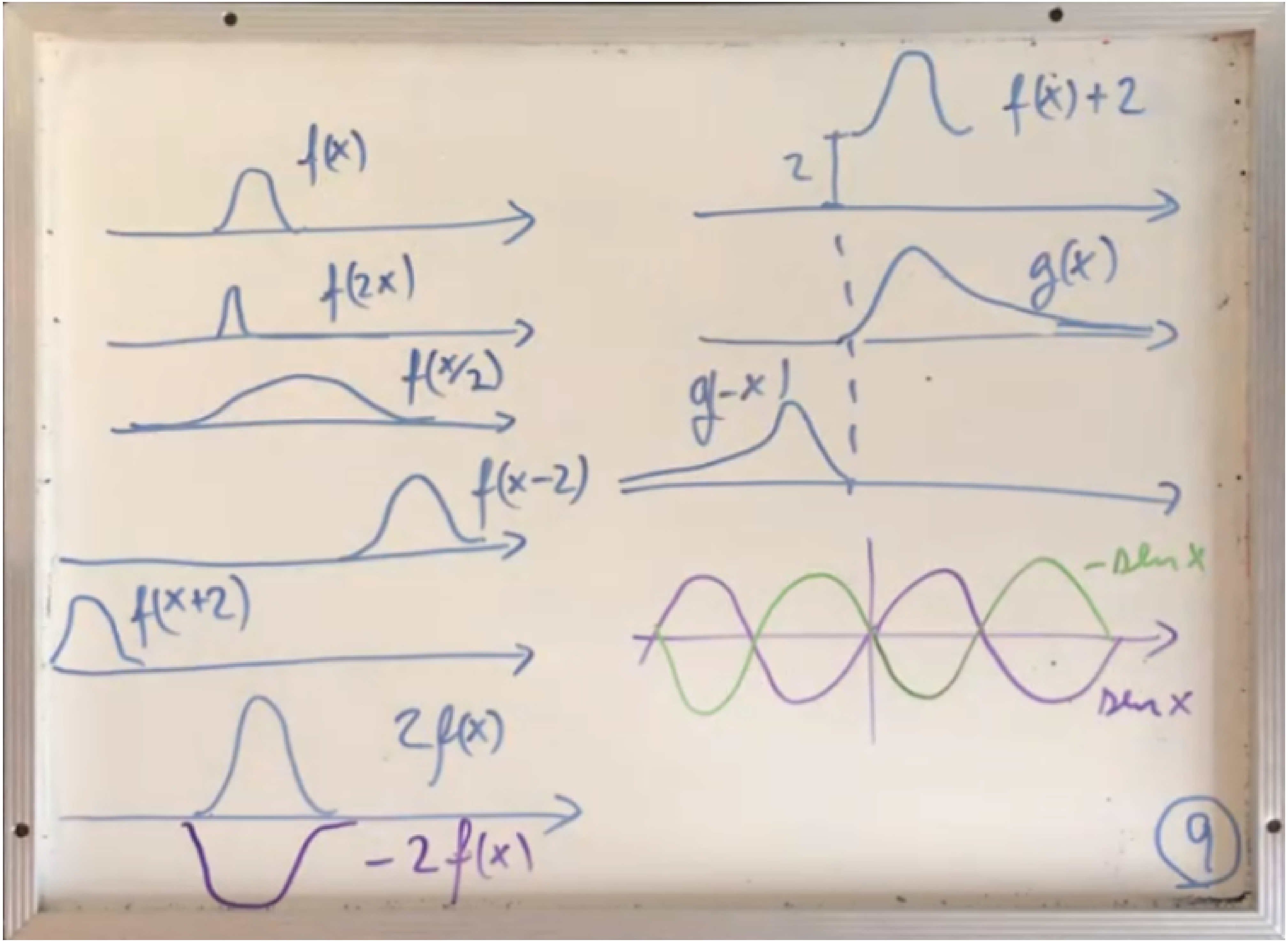
$$2x = a$$
$$x = \frac{a}{2}$$
$$2x = b$$
$$x = \frac{b}{2}$$

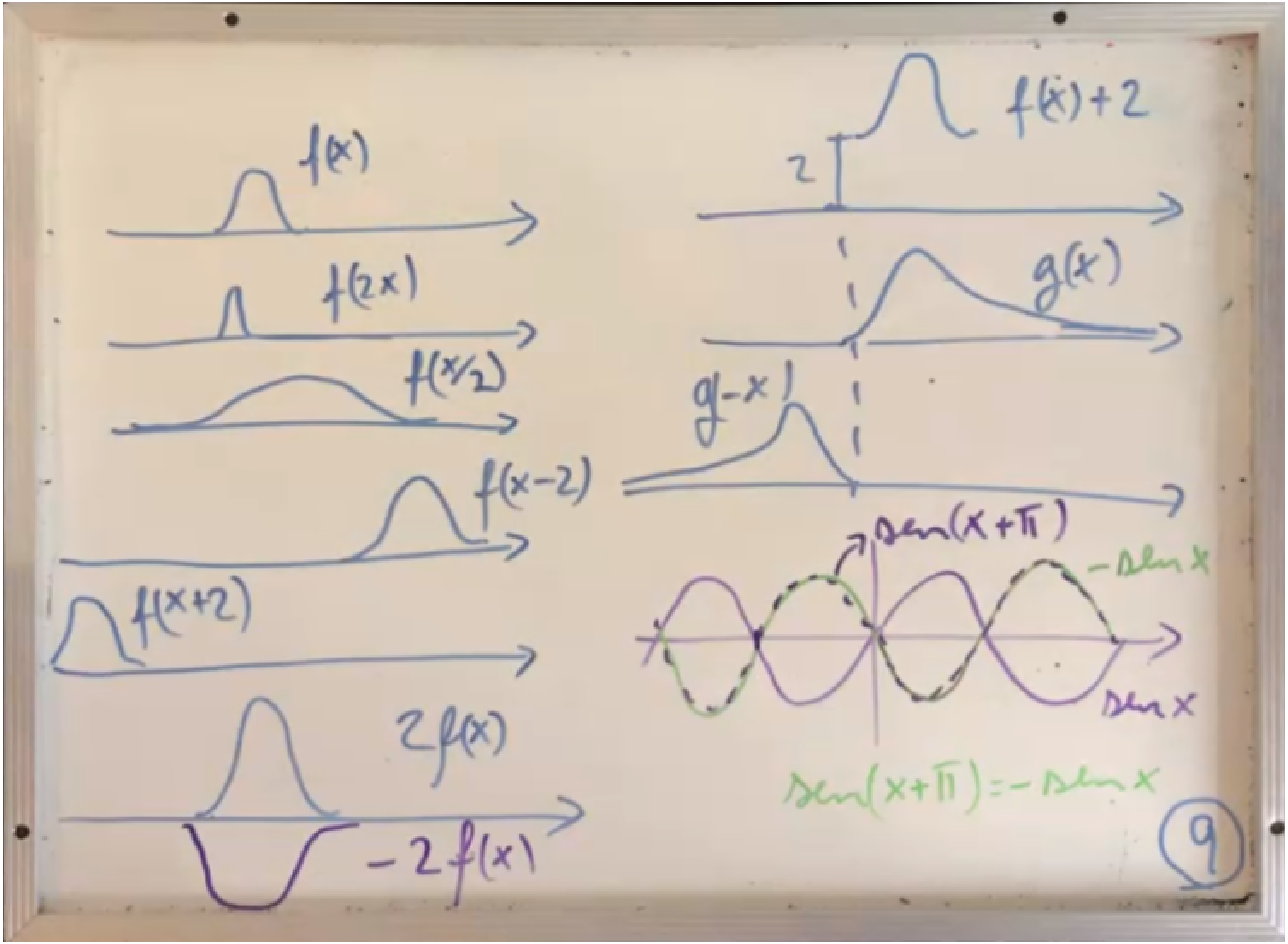
Operações morfológicas

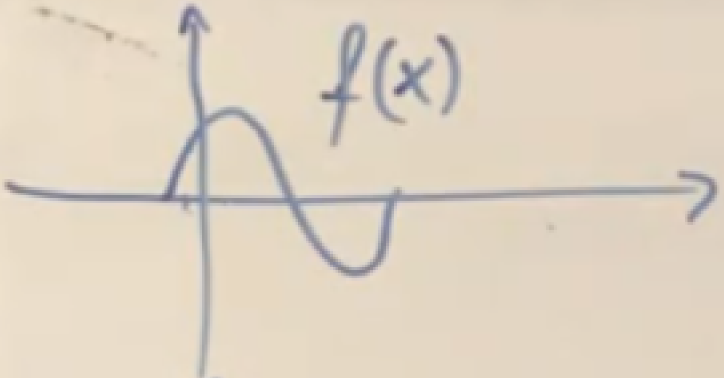


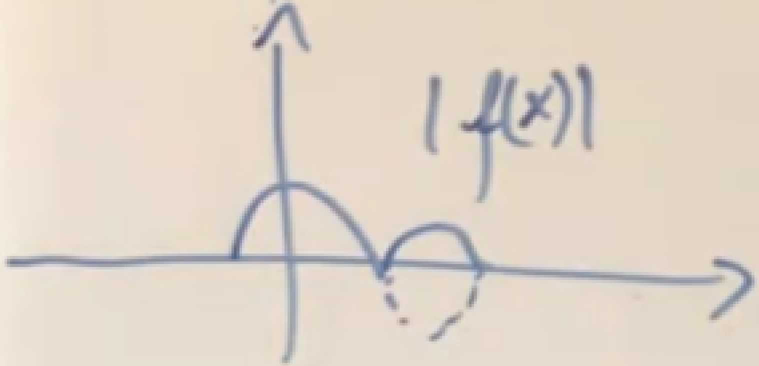
Operações morfológicas

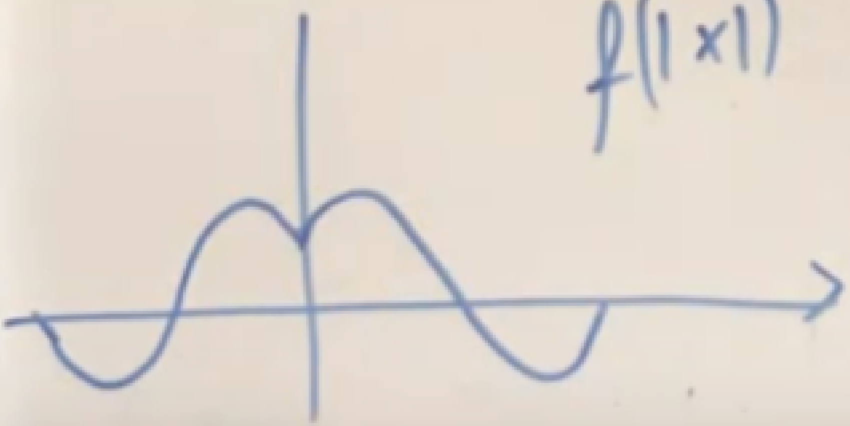



f(x)
f(2x)
f(x/2)
f(x-2)
f(x+2)
2f(x)
-2f(x)
f(x)+2
2
g(x)
g-x)
-sen x
sen x
9


f(x)
f(2x)
f(x/2)
f(x-2)
f(x+2)
2f(x)
-2f(x)
f(x)+2
2
g(x)
g-x)
sen(x+π)
-sen x
sen x
sen(x+π) = -sen x
9







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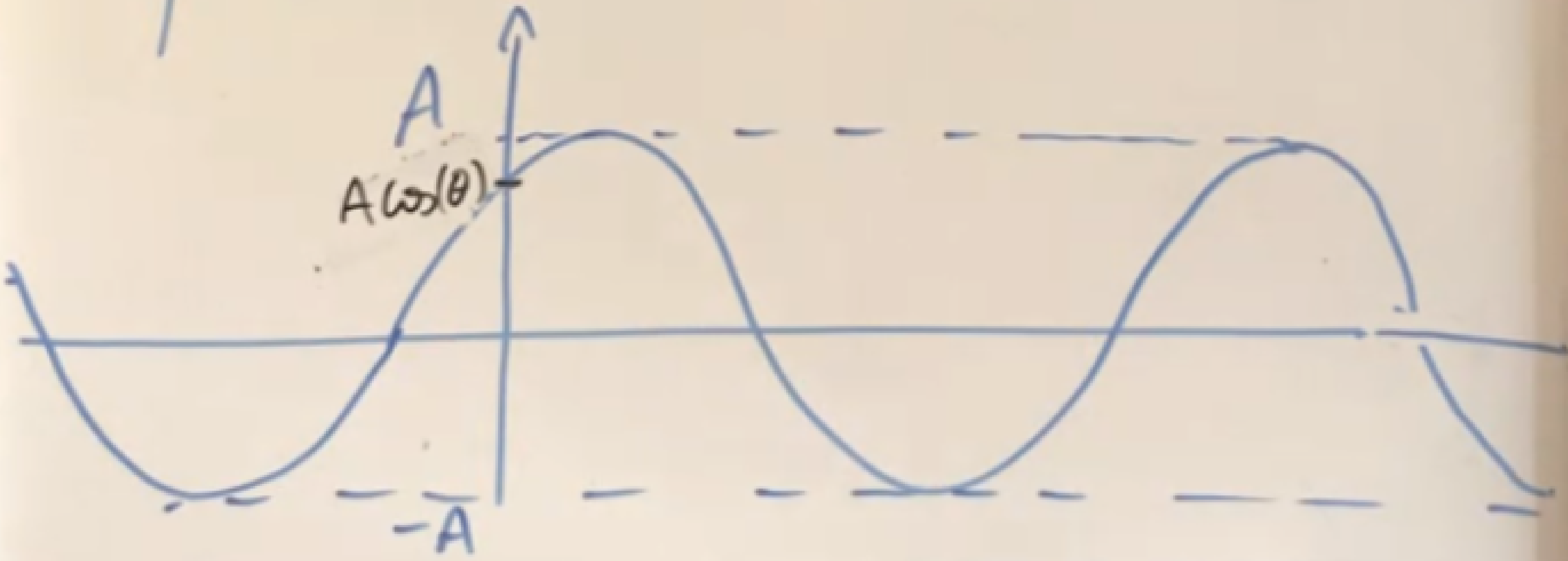
Um exemplo: Senoide

$$f(x) = A\cos(\omega t - \theta)$$



Se T é o período de $f(x) \Rightarrow f(x+T) = f(x) \Rightarrow A\cos(\omega(x+T) - \theta) = A\cos(\omega x - \theta)$

$$\cos(\underbrace{\omega x - \theta}_{A} + \underbrace{\omega T}_{B}) = \cos(\omega x - \theta)$$

$$\cos(\omega x - \theta)\underbrace{\cos(\omega T)}_{1} - \operatorname{sen}(\omega x - \theta)\underbrace{\operatorname{sen}(\omega T)}_{0} = \cos(\omega x - \theta) \Rightarrow \begin{cases} \cos \omega T = 1 \\ \operatorname{sen} \omega T = 0 \end{cases}$$

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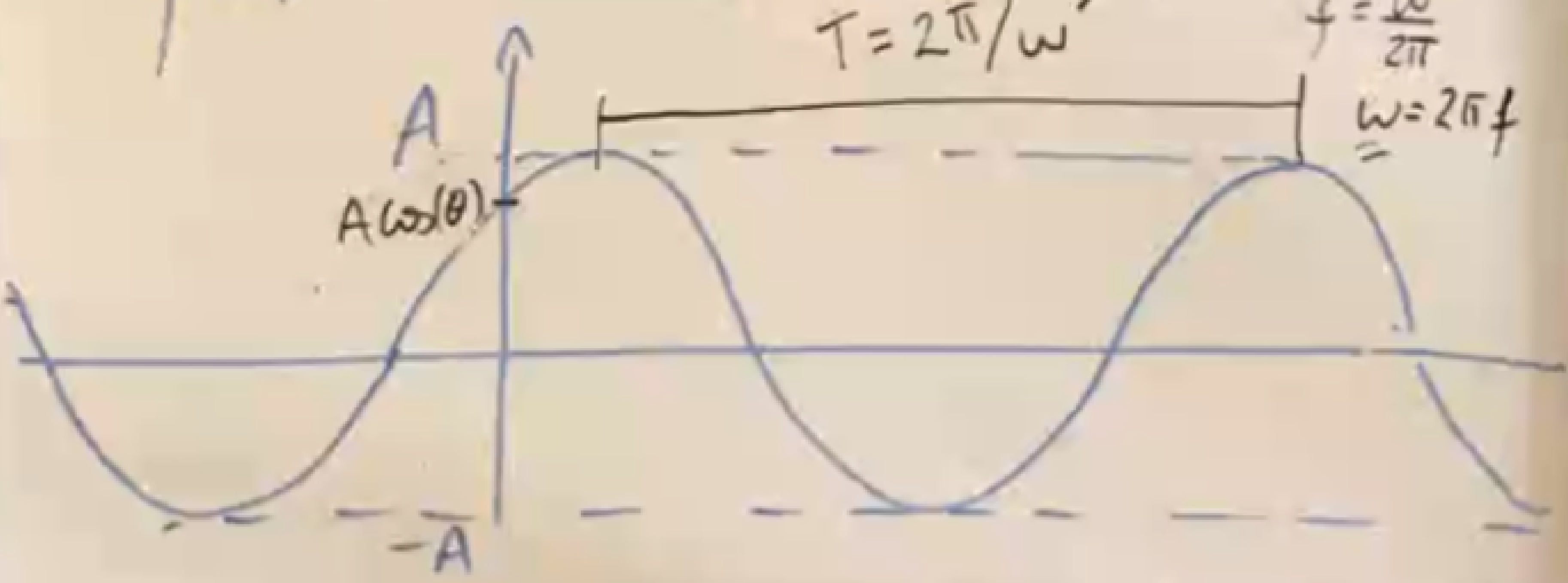
Um exemplo: Senoide

$$f(x) = A\cos(\underline{\underline{\omega}}\, x - \theta)$$

$\rightarrow$ freq. $f = \dfrac{1}{T}$

$f = \dfrac{\omega}{2\pi}$

$\underline{\underline{\omega}} = 2\pi f$



$\rightarrow$ T é o período de $f(x) \Rightarrow f(x+T) = f(x) \Rightarrow A\cos(\omega(x+T)-\theta) = A\cos(\omega x-\theta)$

$$\cos(\omega x - \theta + \omega T) = \cos(\omega x - \theta)$$

$$\cos(\omega x-\theta)\underbrace{\cos(\omega T)}_{1} - \operatorname{sen}(\omega x-\theta)\underbrace{\operatorname{sen}(\omega T)}_{0} = \cos(\omega x - \theta) \Rightarrow \begin{cases} \cos \omega T = 1 \\ \operatorname{sen} \omega T = 0 \end{cases} \Rightarrow \boxed{\omega T = 2\pi}$$

Um exemplo: Senoide

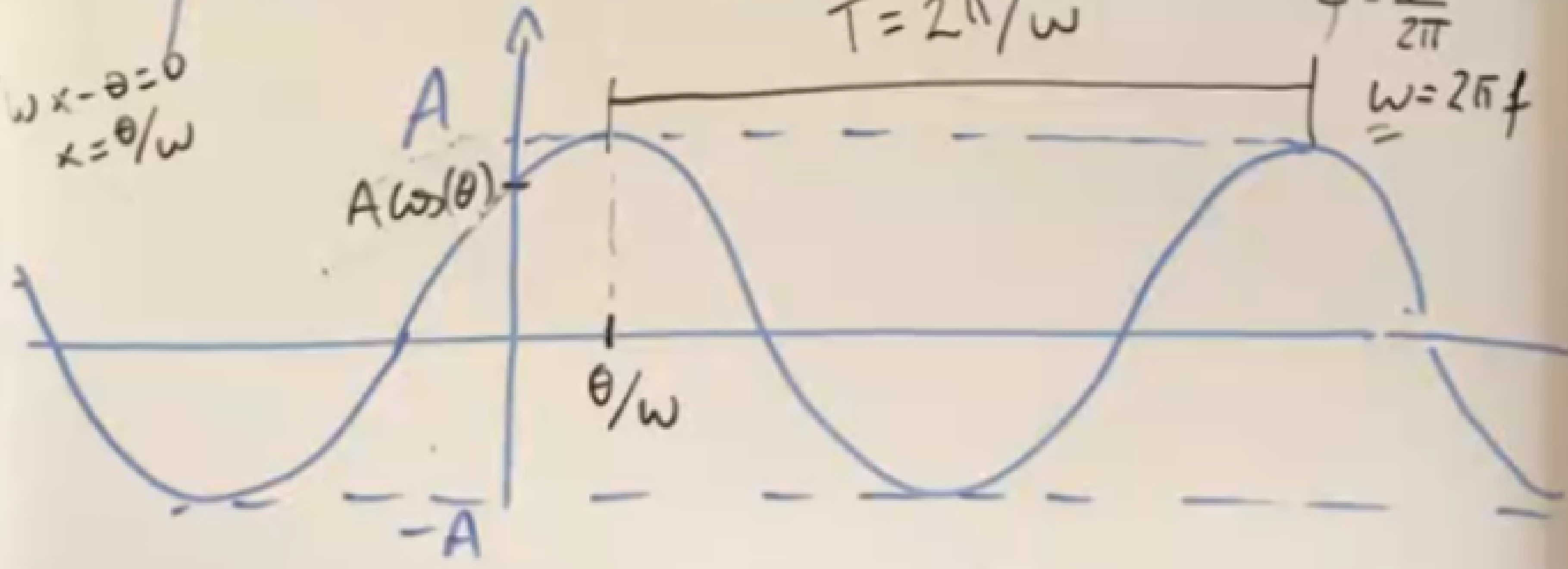
$$f(x) = A\cos(\omega x - \theta)$$

$\omega x - \theta = 0$
$x = \theta/\omega$

freq. $f = \dfrac{1}{T}$

$f = \dfrac{\omega}{2\pi}$

$\underline{\omega} = 2\pi f$



$\Rightarrow T$ é o período de $f(x) \Rightarrow f(x+T) = f(x) \Rightarrow A\cos(\omega(x+T)-\theta) = A\cos(\omega x - \theta)$

$\cos(\underbrace{\omega x - \theta}_{A} + \underbrace{\omega T}_{B}) = \cos(\omega x - \theta)$

$\cos(\omega x - \theta)\underbrace{\cos(\omega T)}_{1} - \operatorname{sen}(\omega x - \theta)\underbrace{\operatorname{sen}(\omega T)}_{0} = \cos(\omega x - \theta) \Rightarrow \begin{cases} \cos \omega T = 1 \\ \operatorname{sen} \omega T = 0 \end{cases}$

$\Rightarrow \boxed{\omega T = 2\pi}$

$$f(x) = A \cos(\omega x - \theta)$$

$$f_n(x) = A_n \cos(n\omega_0 x - \theta_n)$$

$$f(x) = \sum_{n=1}^{\infty} \underline{\underline{A_n}} \cos(\underline{n}\, \omega_0 x - \underline{\underline{\theta_n}})$$

↳ Série de Fourier

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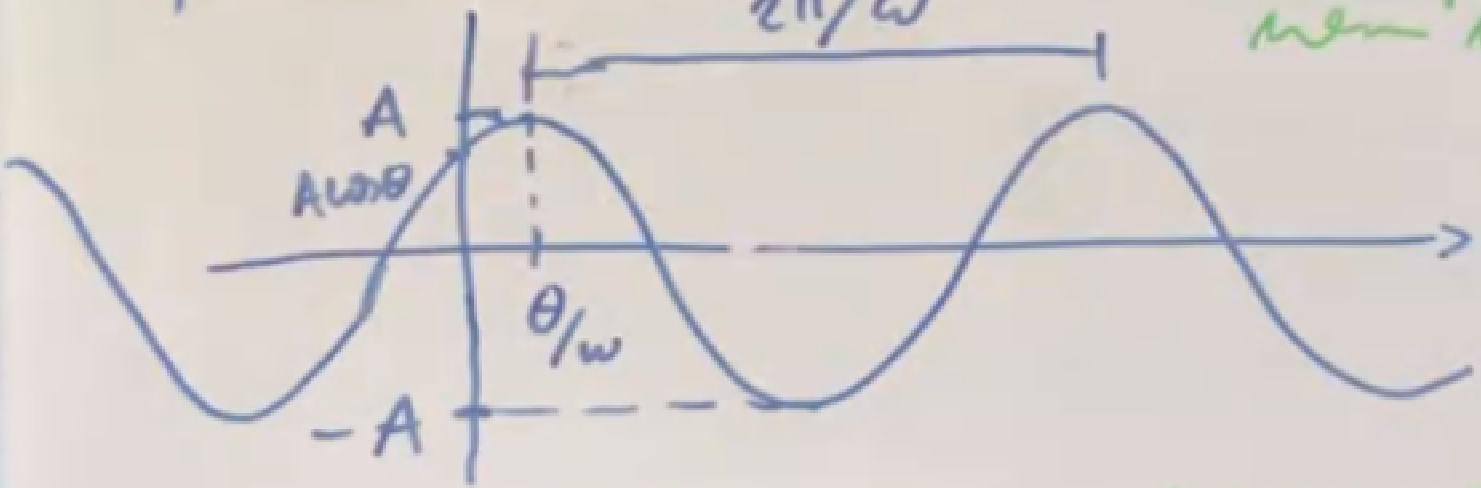
$f(x) = A\cos(\omega x - \theta)$ é par ou ímpar?

nem par
nem ímpar



$$f(x) = A\cos(\omega x - \theta) = A\cos\omega x\cos\theta + A\,\mathrm{sen}\,\omega x\,\mathrm{sen}\,\theta =$$

$$= \underbrace{A\cos\theta}\cos\omega x + \underbrace{A\,\mathrm{sen}\,\theta}\,\mathrm{sen}\,\omega x =$$

$$= \underbrace{a\cos(\omega x)}_{\text{par}} + \underbrace{b\ \mathrm{sen}(\omega x)}_{\text{ímpar}}$$

$$f_p(x) = \underbrace{\frac{f(x)+f(-x)}{2}}_{A\cos\theta\cos(wx)} \quad ; \quad f_I(x) = \underbrace{\frac{f(x)-f(-x)}{2}}_{A\,\mathrm{sen}\,\theta\,\mathrm{sen}(wx)}$$

$$f(x) = A\cos(wx-\theta) = A\cos wx\cos\theta + A\,\mathrm{sen}\,wx\,\mathrm{sen}\,\theta =$$

$$= \underbrace{A\cos\theta}\cos wx + \underbrace{A\,\mathrm{sen}\,\theta}\,\mathrm{sen}\,wx =$$

$$= \underbrace{a\ \cos(wx)}_{\text{par}} + \underbrace{b\ \ \mathrm{sen}(wx)}_{\text{ímpar}}$$

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