

CIAG - 2020 - 27/03 - TARDE - CÁLCULO

*COMO ASSOCIAR FUNÇÕES?

$$1) \quad h(x) = f(x) + g(x) = a^x + b^x \quad \left\{ \begin{array}{l} Aa^x + Bb^x + Ce^x \\ \end{array} \right.$$

$$2) \quad h(x) = Af(x) = A \cdot e^x$$

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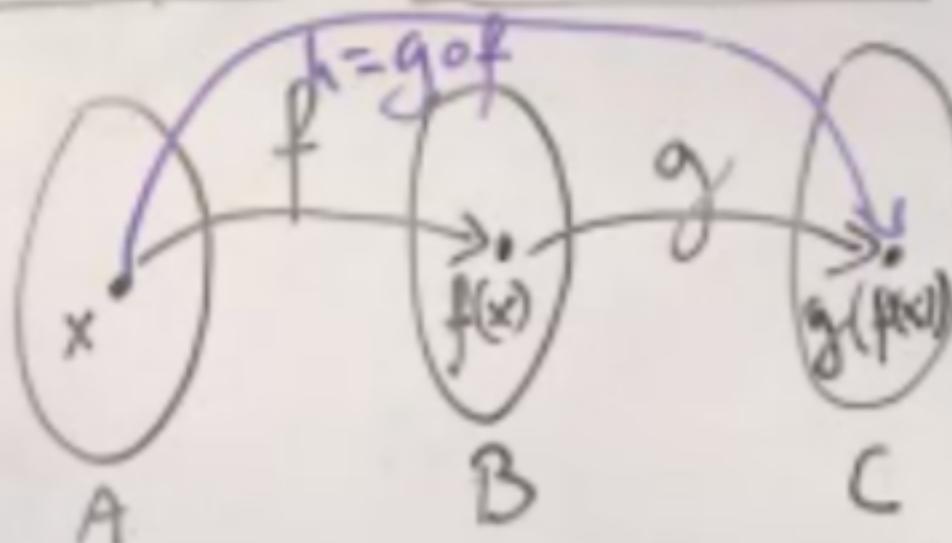
3) $h(x) = f(x)/g(x) = 1/e^x = (e^x)^{-1} = e^{-x} = \left(\frac{1}{e}\right)^x$

4) $h(x) = f(x)g(x) = a^x(1+Ax+Bx^2)$

5) $h(x) = f(g(x))$ Composição de funções

①

FUNÇÃO COMPOSTA



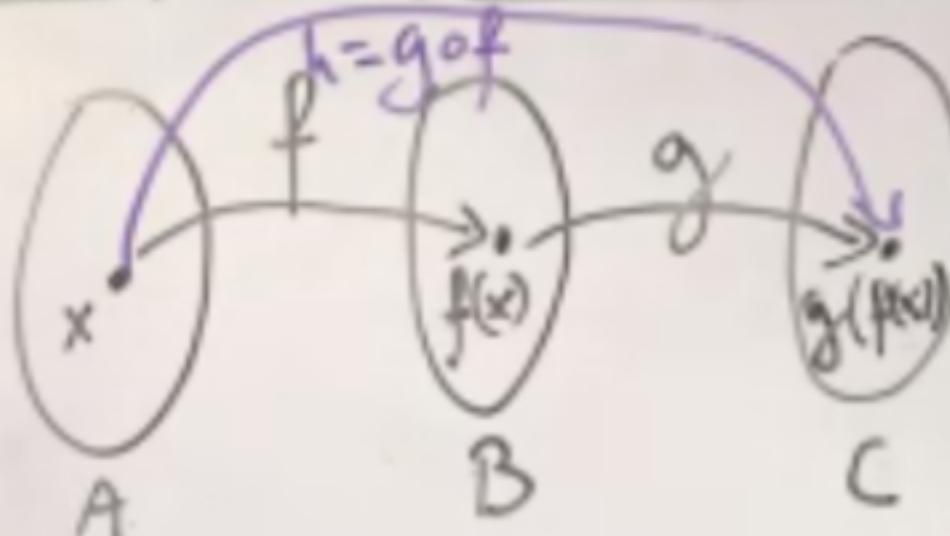
$$h = g \circ f$$
$$\Downarrow$$
$$h(x) = g(f(x))$$

$$f(x) = e^x$$
$$g(x) = -x^2$$

$$x \rightarrow \boxed{f} \rightarrow e^x$$
$$x \rightarrow \boxed{g} \rightarrow x^2$$
$$x \rightarrow \boxed{f} \rightarrow \boxed{g} \rightarrow -e^{2x}$$
$$g(f(x)) = -e^{2x}$$
$$-(e^x)^2 = -e^{2x}$$

②

FUNÇÃO COMPOSTA



$$h = g \circ f$$
$$\Downarrow$$
$$h(x) = g(f(x))$$

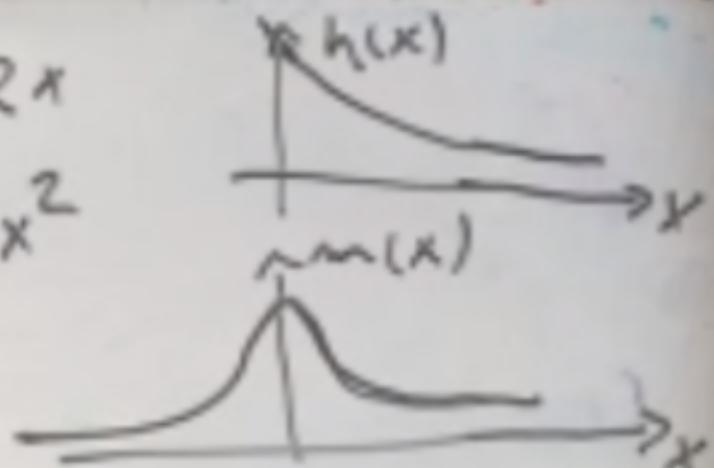
$$f(x) = e^x$$
$$g(x) = -x^2$$

$$x \rightarrow \boxed{f} \rightarrow e^x$$
$$x \rightarrow \boxed{g} \rightarrow x^2$$
$$x \rightarrow \boxed{f} \rightarrow \boxed{g} \rightarrow -e^{2x}$$
$$h(x) = g(f(x)) = -e^{2x}$$
$$x \rightarrow \boxed{g} \rightarrow \boxed{f} \rightarrow e^{-x^2}$$
$$m(x) = f(g(x)) = e^{-x^2}$$

(2)

$$h(x) = g(f(x)) = -e^{2x}$$

$$m(x) = f(g(x)) = e^{-x^2}$$



$$f(x) = e^x$$

$$g(x) = -x^2$$

-

$$x \rightarrow \boxed{f} \rightarrow e^x$$

$$x \rightarrow \boxed{g} \rightarrow -x^2$$

$$h(x) = g(f(x)) = -e^{2x}$$

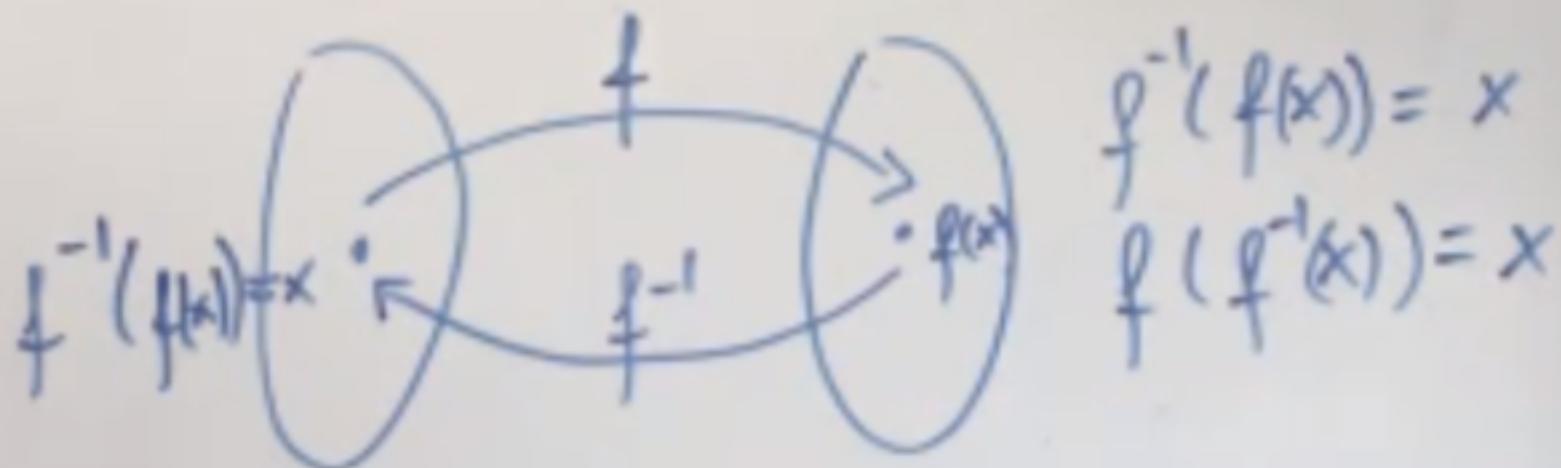
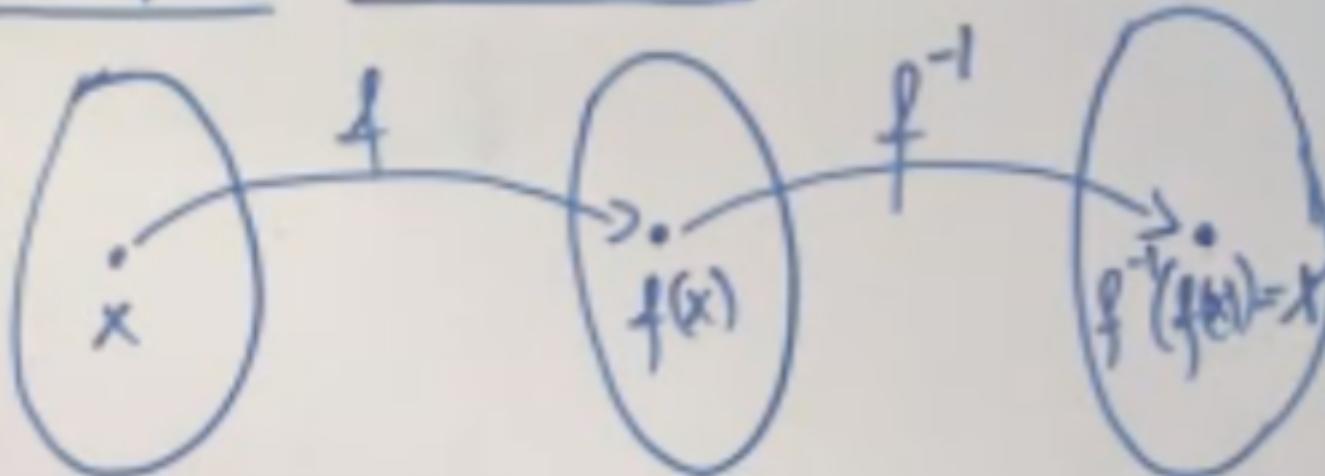
$$x \rightarrow \boxed{f} \rightarrow \boxed{g} \rightarrow -(-e^x)^2 = -e^{2x}$$

$$x \rightarrow \boxed{g} \rightarrow \boxed{f} \rightarrow e^{-x^2}$$

$$m(x) = f(g(x)) = e^{-x^2}$$

②

FUNÇÃO INVERSA



$$f^{-1}(f(x)) = x$$

$$f(f^{-1}(x)) = x$$

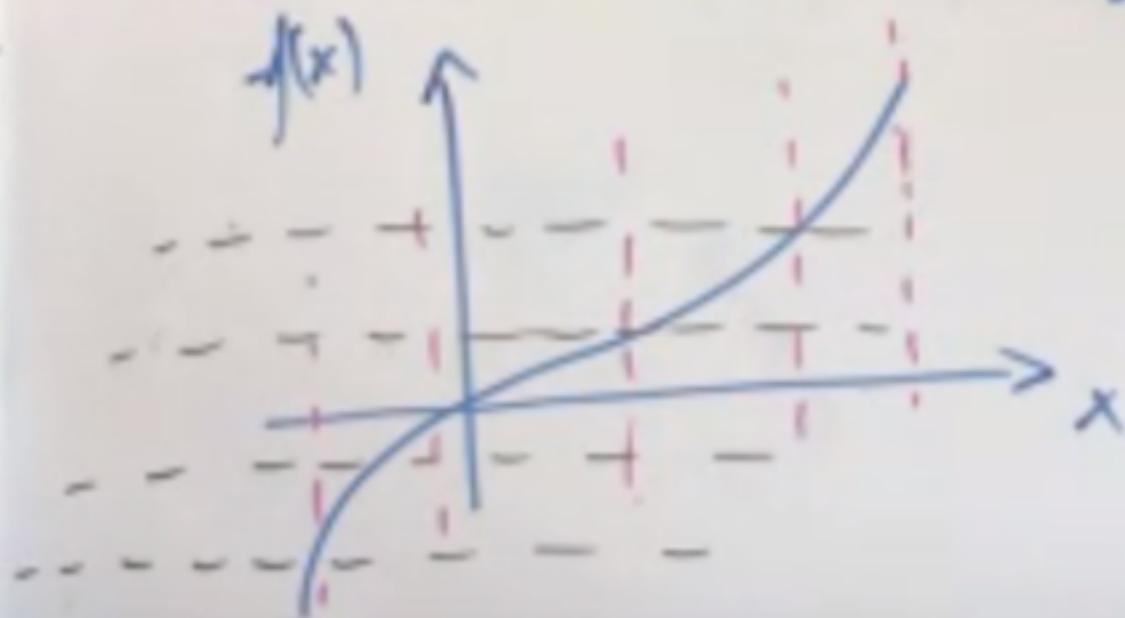
③

* Nem todo função é invertível!

$$\text{Im } f = \text{CD}_f$$

• não pode haver $x_1 \neq x_2 \mid f(x_1) = f(x_2)$

→ teste da reta horizontal

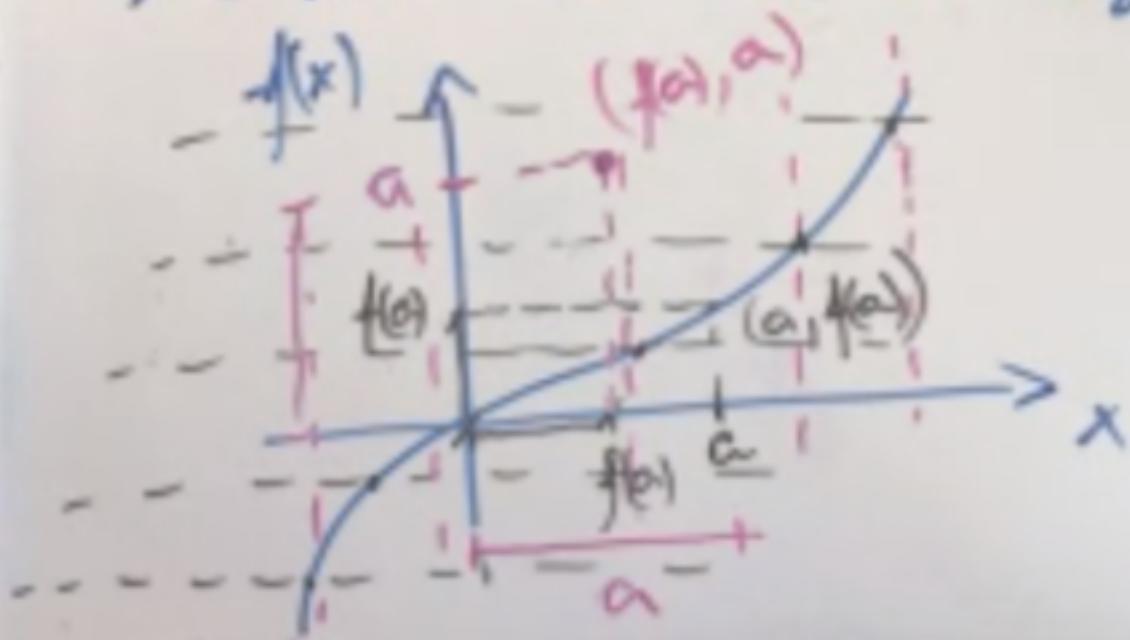


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→ teste da reta horizontal



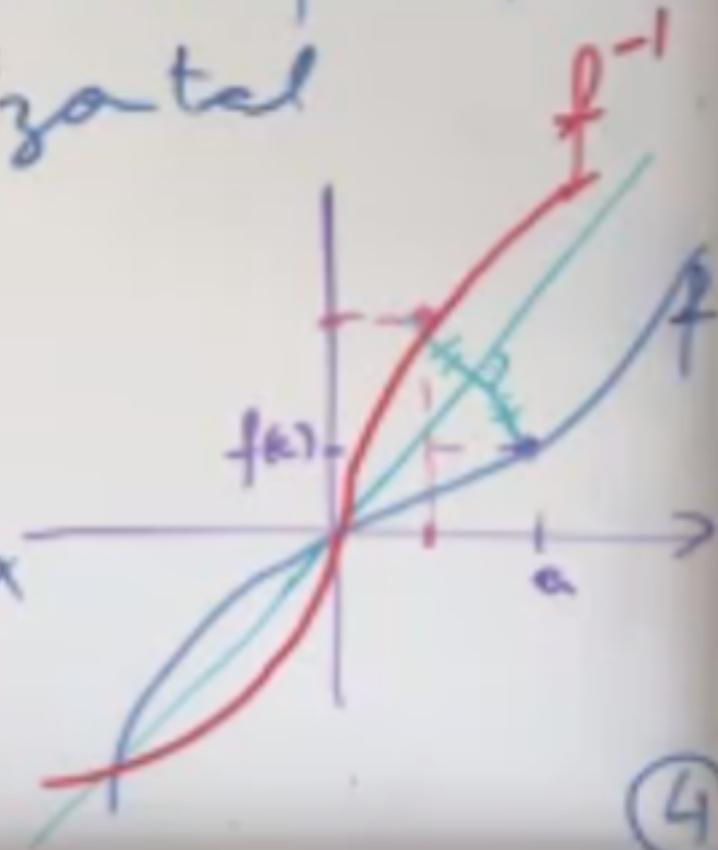
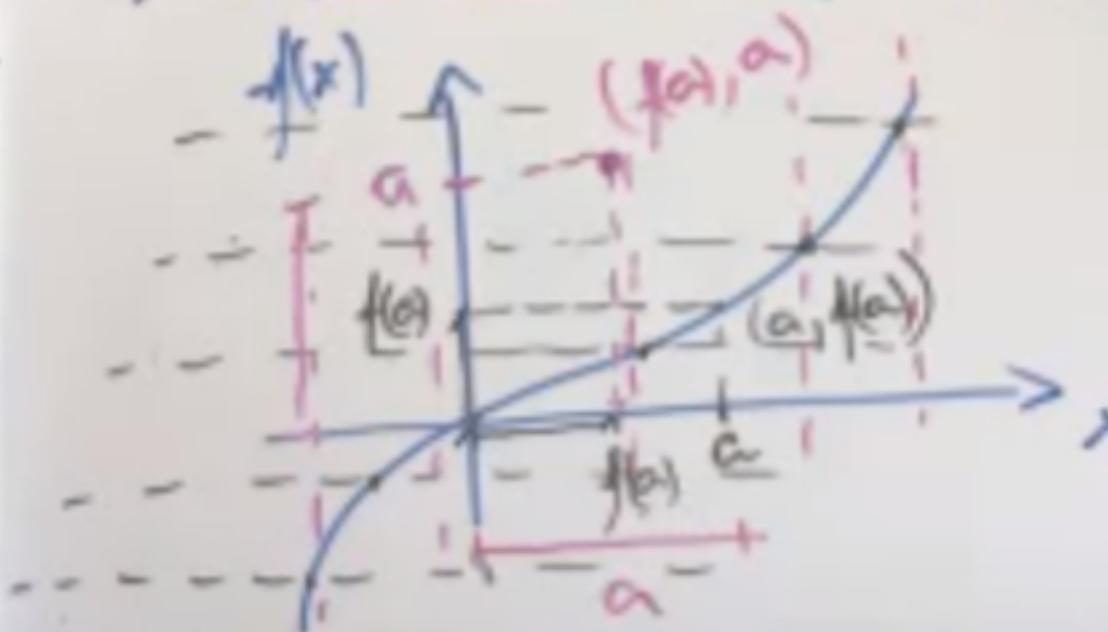
(4)

* Nem toda função é inversível!

$$\cdot \text{Im } f = CD_f$$

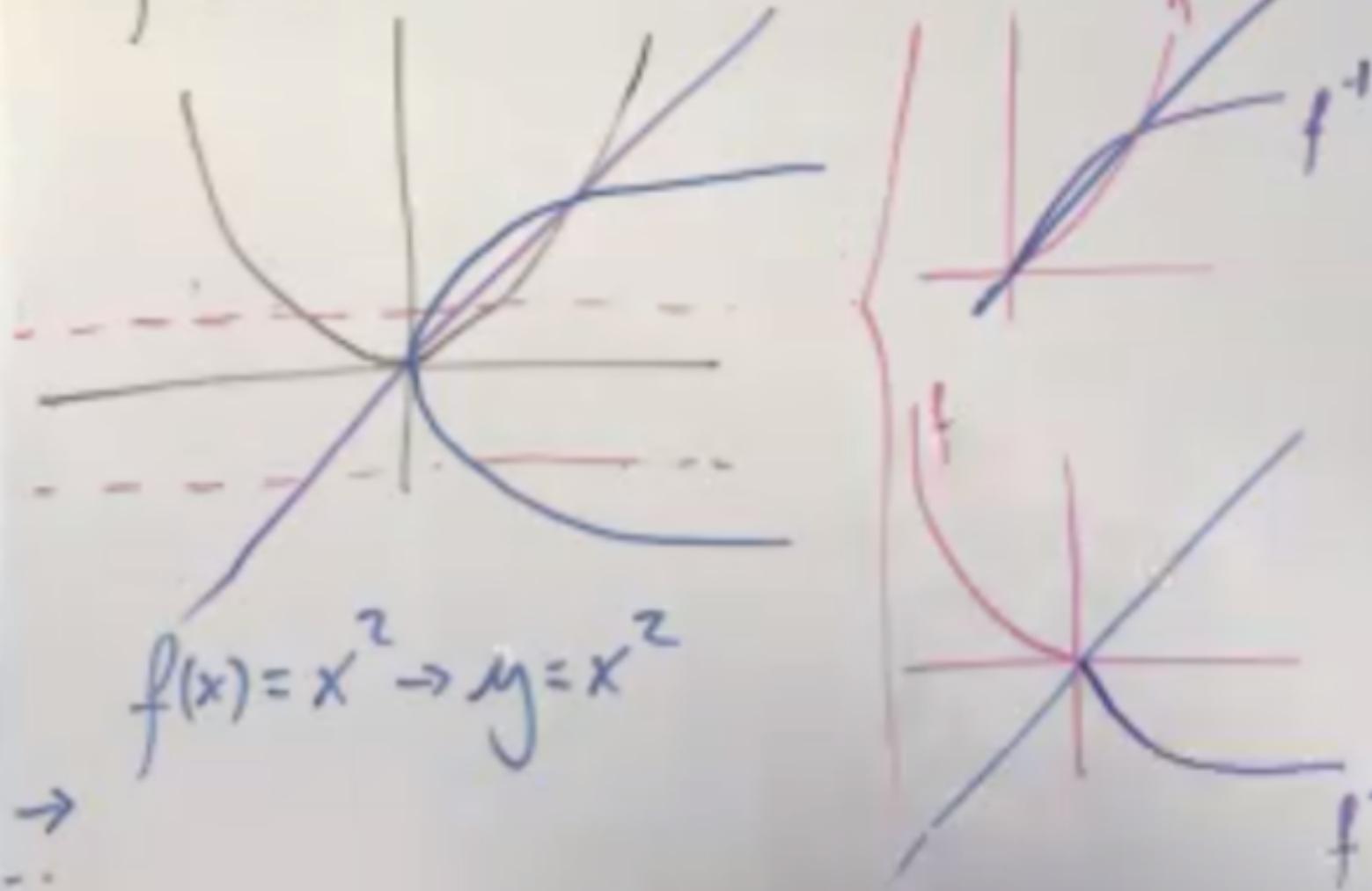
• não pode haver $x_1 \neq x_2 \mid f(x_1) = f(x_2)$

→ teste da reta horizontal



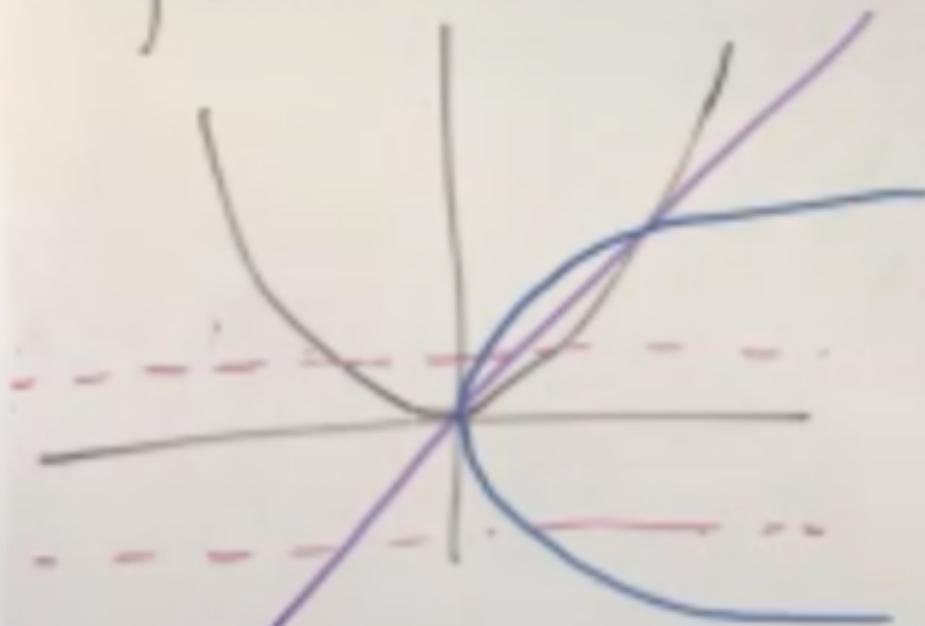
④

$f(x) = x^2 \rightsquigarrow$ non invertibile



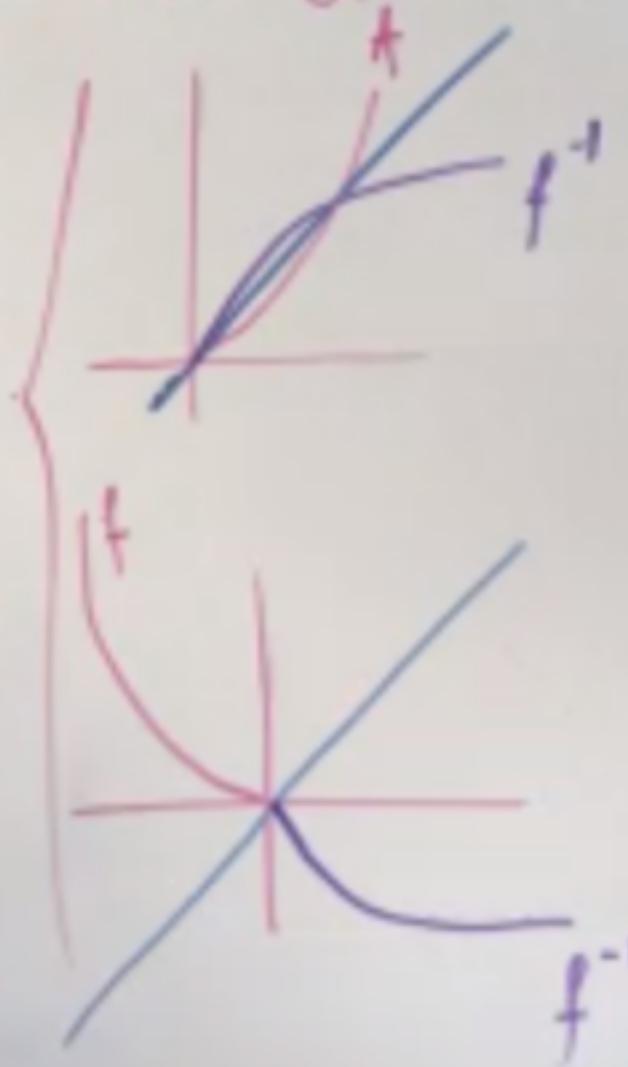
⑤

$f(x) = x^2$ \rightsquigarrow non invertibile



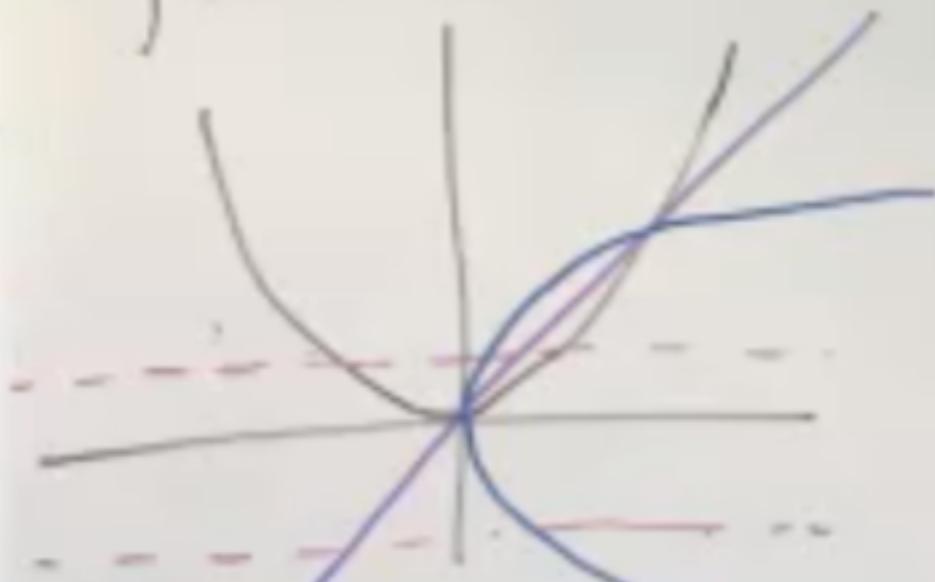
$$f(x) = x^2 \rightarrow y = x^2$$

$$\rightarrow x = y^2 \rightarrow y = \sqrt{x}$$



⑤

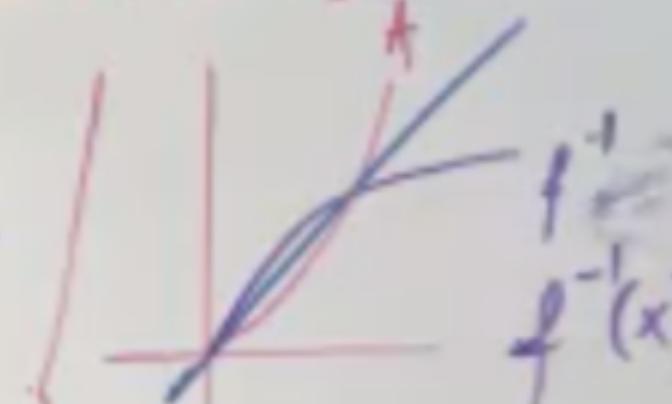
$f(x) = x^2$ → não é invertível,



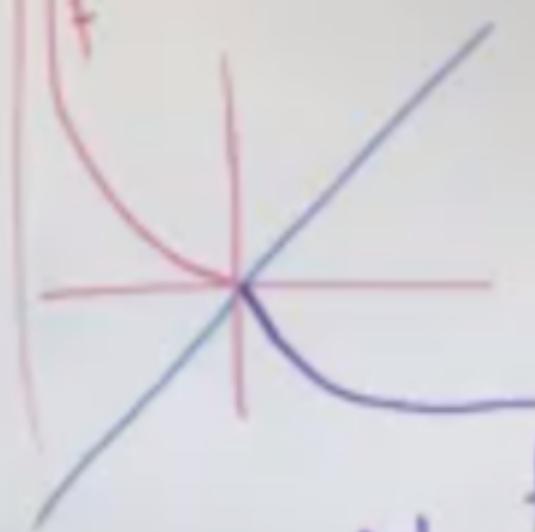
$$f(x) = x^2 \rightarrow y = x^2$$

$$\rightarrow x = y^2 \rightarrow y = \pm\sqrt{x}$$

$$f^{-1}(x) = \pm\sqrt{x}$$



$$f^{-1}(x) = +\sqrt{x}$$



$$f^{-1}(x) = -\sqrt{x}$$

⑤

Ex.: $f(x) = 2x - 3$

$$y = 2x - 3$$

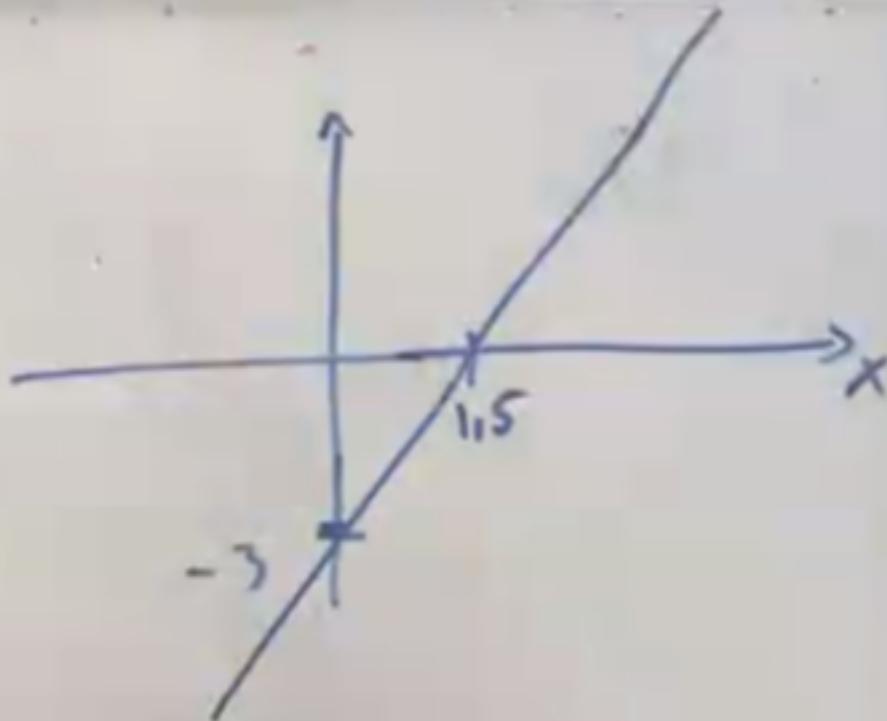
inverse:

$$x = 2y - 3$$

$$2y = x + 3$$

$$y = \frac{1}{2}x + \frac{3}{2}$$

$$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$$



⑥

Ex.: $f(x) = 2x - 3$

$$y = 2x - 3$$

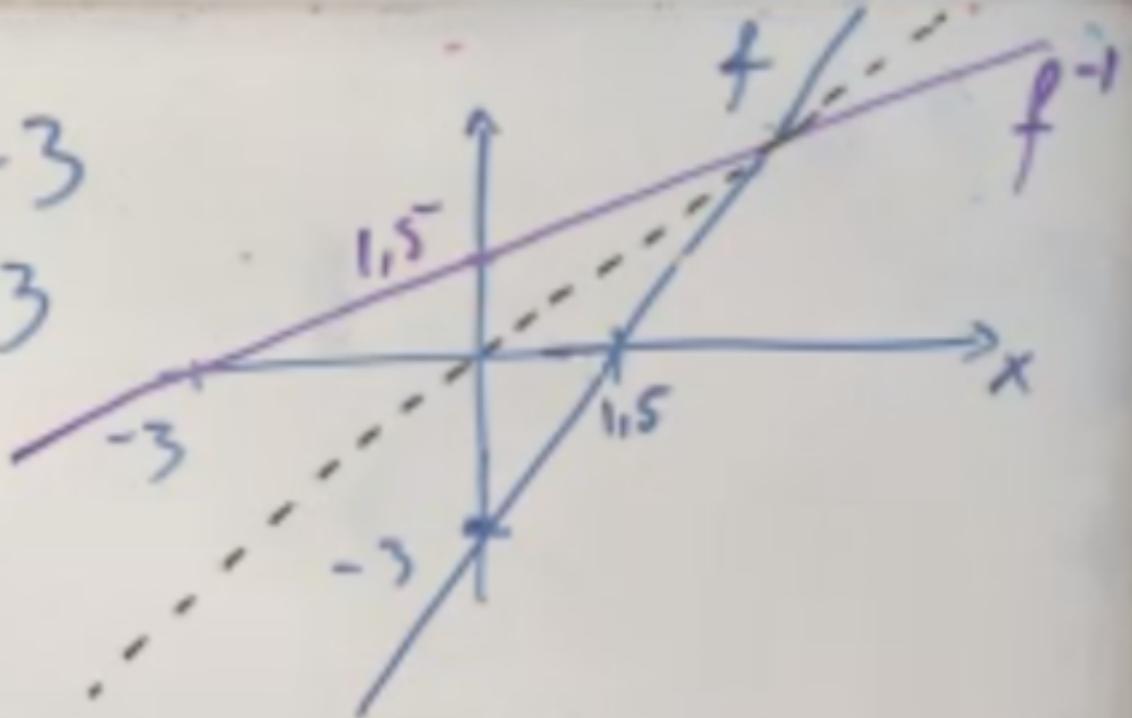
inversen:

$$x = 2y - 3$$

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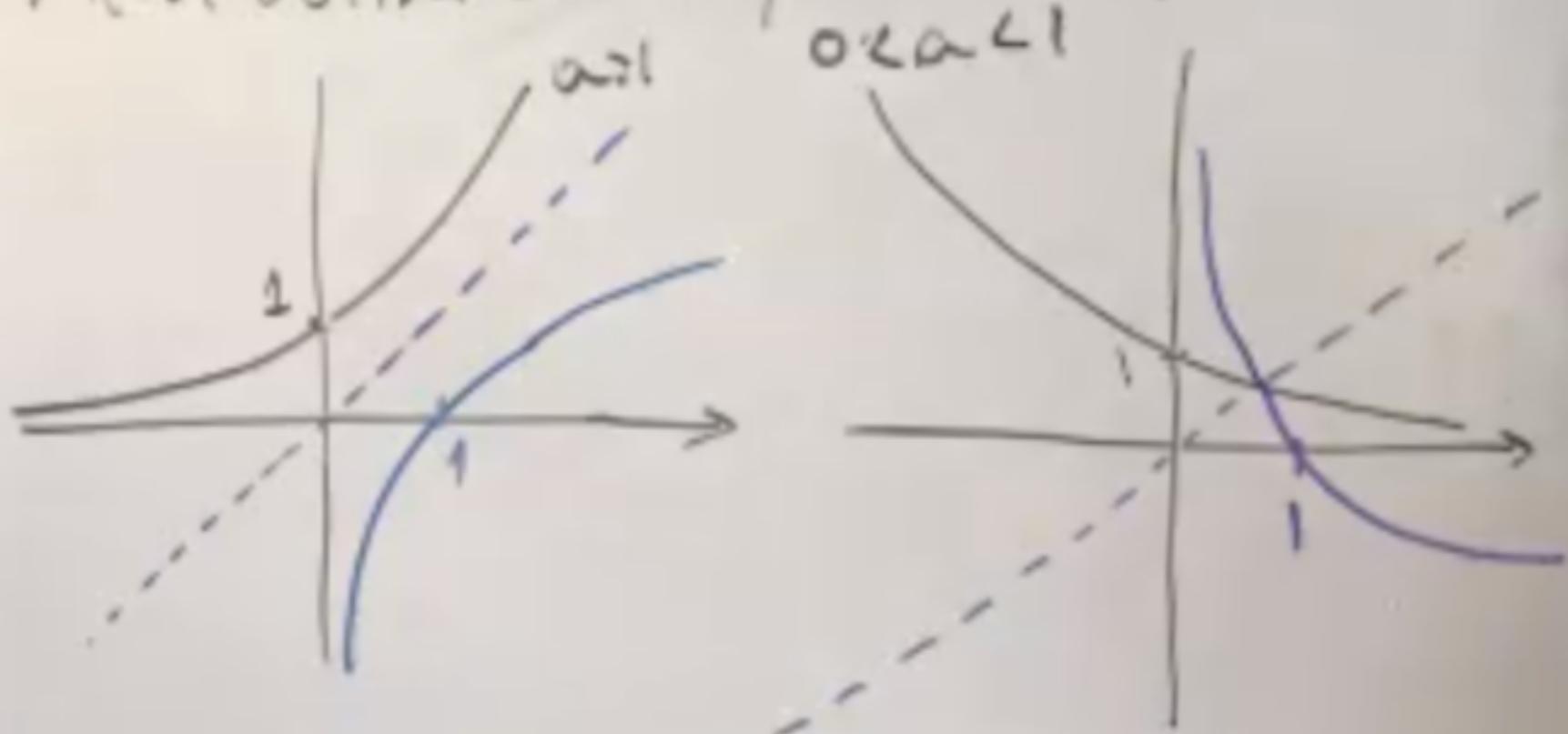
$$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$$



⑥

6) FUNÇÃO LOGARITMO

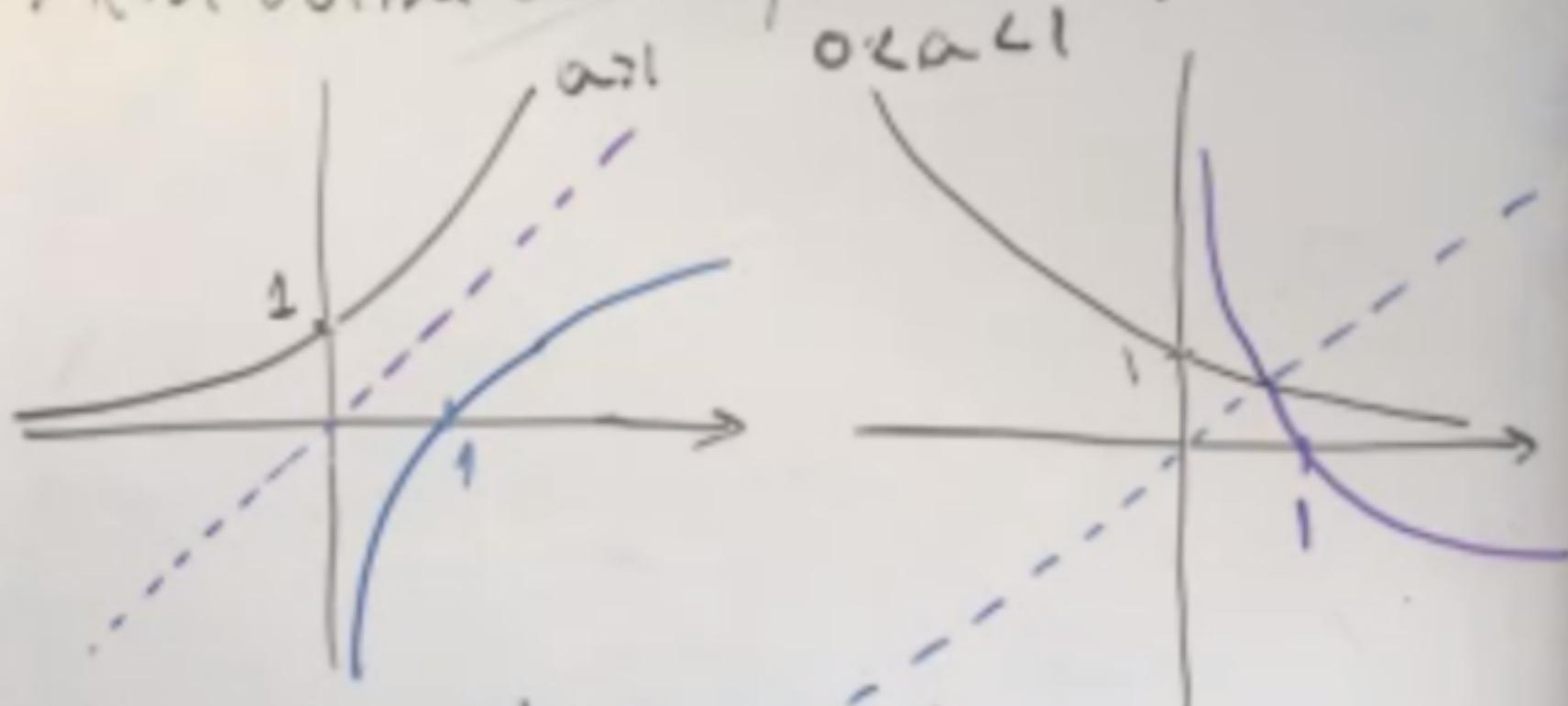
→ inversa da exponencial



⑦

6) FUNÇÃO LOGARÍTMICO

→ inversa da exponencial



$$f(x) = a^x$$

$$f^{-1}(f(x)) = f^{-1}(a^x) = x$$

$$f^{-1}(x) \equiv \log_a x$$

⑦

$$f(x) = a^x \quad f^{-1}(x) = \log_a x$$
$$\log_a a^x = x ; \quad a^{\log_a x} = x$$

$$f(x) = a^x \quad f^{-1}(f(x)) = f^{-1}(a^x) = x$$
$$f^{-1}(x) \equiv \log_a x$$

⑦

$$f(x) = a^x \quad f^{-1}(x) = \log_a x$$
$$\log_a a^x = x ; \quad a^{\log_a x} = x$$

$$\log \equiv \log_{10} ; \quad \ln \equiv \log_e$$

$$f(x) = e^x \Rightarrow f^{-1}(x) = \ln x$$
$$\ln e^x = e^{\ln x} = x$$

⑧

$$f(x) = a^x \quad f^{-1}(x) = \log_a x$$
$$\log_a a^x = x ; \quad a^{\log_a x} = x$$

$$\log \equiv \log_{10} ; \quad \ln \equiv \log_e$$

$$f(x) = e^x \Rightarrow f^{-1}(x) = \ln x$$
$$\ln e^x = e^{\ln x} = x$$

$$[\text{* Obs: } \text{mp.log}(x) = \ln(x)$$
$$\text{mp.log10}(x) = \log(x)]$$

⑧

$$\log(a \cdot b) = \log a + \log b$$

$$\log(a/b) = \log a - \log b$$

$$\log(a^b) = b \log a$$

$$\log_b a = \frac{\log a}{\log b}$$

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$$\log(a^b) = b \log a$$

$$\log_b a = \frac{\log a}{\log b}$$

$$\log x = \frac{\ln x}{\ln 10} \Rightarrow \ln x = \ln 10 \cdot \log x$$

$$a^x = y \Rightarrow \ln a^x = \ln y \quad \left| \begin{array}{l} x \ln a = \ln y \\ a^x = e^{x \ln a} = y \end{array} \right.$$

(9)

$$\log(a \cdot b) = \log a + \log b$$

$$\log(a/b) = \log a - \log b$$

$$\log(a^b) = b \log a$$

$$\log_b a = \frac{\log a}{\log b}$$

O que é isso
mais gente
dá errado
é só agora!

$$\log_a 1 = 0$$

$$\log_a x = y \\ \log_a \leftarrow \\ a^y = x$$

$$\log x = \frac{\ln x}{\ln 10} \Rightarrow \ln x = \ln 10 \cdot \log x$$

$$a^x = y \Rightarrow \ln a^x = \ln y \\ x \ln a = \ln y$$

$$l = l$$

$$a^x = y$$

⑨

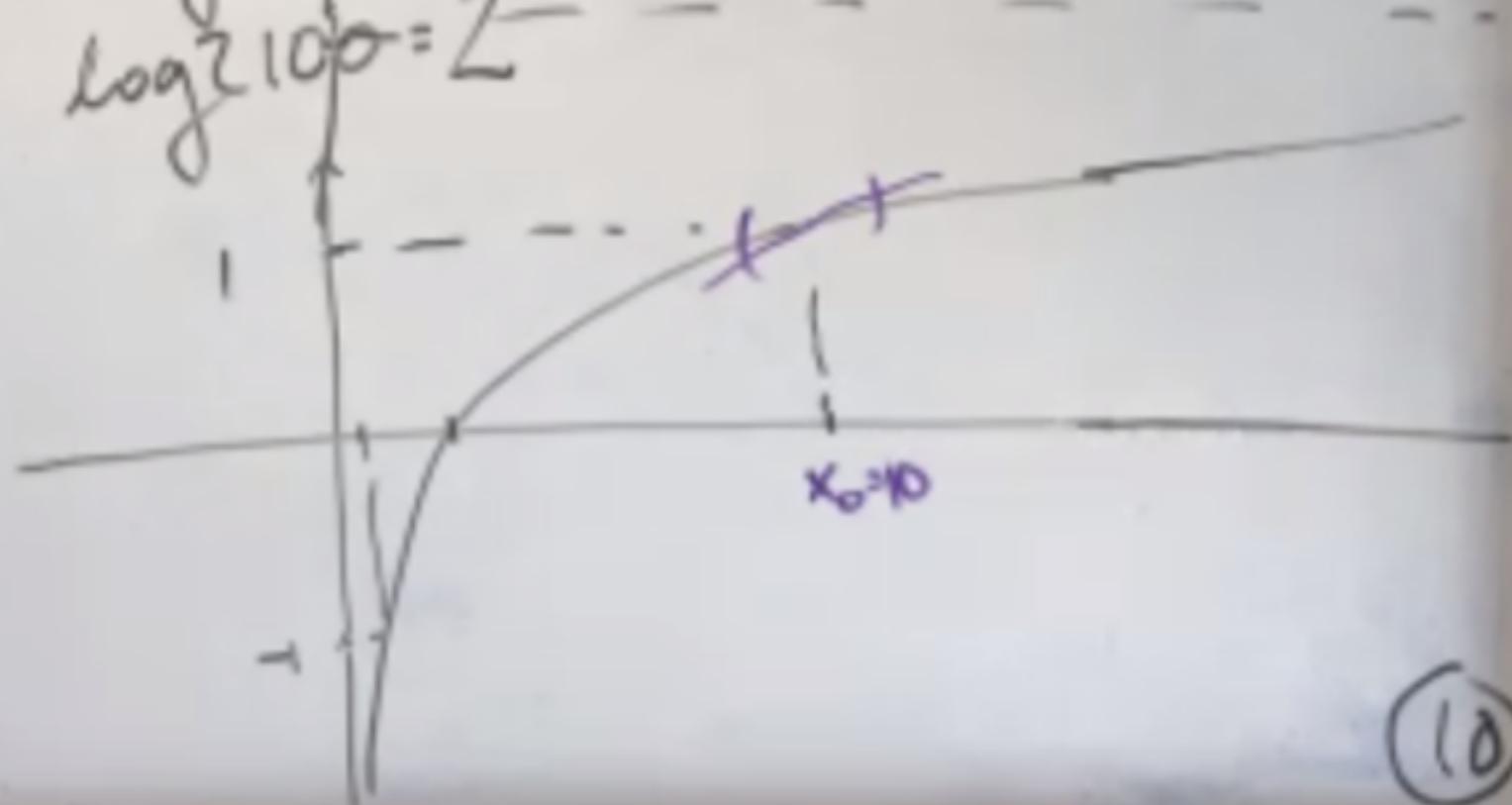
$$\log_{10} 0,1 = \log_{10} 10^{-1} = -1$$

$$\log_{10} 1 = 0$$

$$\log_{10} 10 = 1$$

$$\log_{10} 100 = 2$$

$\ln(1+x)$



(10)