Adding *Semantics* to Ontologies

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About

This talk is about the word *Semantic* in the "Semantic Web."

It will provides some basic theoretical notions for a better understanding of Semantic Web technologies (in particular OWL).

Topics

Some answers for the following questions will be provided:

- How can we represent knowledge? (ontologies)
- Which tools the Semantic Web framework offer to this end? (OWL)
- How representing knowledge with these tools can be useful? (reasoning)

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Notice

For the sake of readability and for historical reason, we will use notation and terminology from *Descriptio Logics*, which underpin Semantic Web languages (in particular OWL).

Representing Knowledge in Logic and Computer Science

An *ontology* is a (partial) description of the world.

Usually, it is a finite set of statements such as, for example,

- human beings are mortals,
- Alice is the Bob's mother,
- Federico II was an emperor.

Basics

A knowledge domain is described in terms of

- *individuals*, to indicate domain items (*Alice*, *Bob*, . . .),
- *concepts* (aka *classes*), which denote sets of domain objects (`Person`, `Male`, . . .), and
- *roles* (aka *properties*), which are relations on domain items (`parentOf`, `partnerOf`, . . .).

Vocabulary and Assertions

Each ontology is divided into two parts:

- the `TBox`(vocabulary), in which *vocabulary terms* for concepts (i.e. sets) and roles (i.e. relations) are defined, and
- the `ABox`, which contains statements (namely, *assertions*) about real world items.

`TBox`	*Concepts:* `Male`, `Female`,`Human`,`Woman`, `Man`. *Roles:* `relativeOf`, `childOf`, `grandChildOf`.
`ABox`	`Woman`(*Alice*), `Man`(*Bob*), *Alice* `relativeOf` *Bob*, *Alice* `childOf` *Charlie*.

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Open World Assumption

Facts not explicitly mentioned in the ontology are *not* assumed to be false, their truth value is just unknown.

In general, no one has complete knowledge.

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- What is the kinship relation between *Alice* and *Bob*? *Charlie* `childOf` *Alice*?
- Is *Charlie* a man? Is *Charlie* a human being?
- What about other persons in the world?

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Open World Assumption - Interpretations

All the *possible worlds* must be taken into account.

An *interpretation* associates sets, relations, and items of a given *domain* (i.e. Universe) Δ to concepts and roles in the vocabulary.

For example:

$$
\begin{aligned}
\Delta &:= \{Alice, Bob, Charlie, Dana, Edwige\} \\
\texttt{Human} &:= \{Alice, Bob, Charlie, Dana\} \\
\texttt{Female} &:= \{Alice, Dana, Edwige\} \\
\texttt{Male} &:= \{Bob, Charlie\} \\
\texttt{Woman} &:= \{Alice, Dana\} \\
\texttt{Man} &:= \{BobCharlie\}
\end{aligned}
$$

$Alice$ `childOf` Bob

$Charlie$ `relativeOf` Bob

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Alice `childOf` *Bob*
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\end{array}
$$

Open World Assumption - Models (1/2)

Roughly speaking, an interpretation is a *model* for an ontology if it does not contradicts the ontology itself

For example:

$$\begin{array}{rcl|l} \Delta & := & \{Alice, Bob, Charlie, Dana, Edwige\} & Alice\ \texttt{childOf}\ Charlie \\ \texttt{Human} & := & \{Alice, Bob, Charlie, Dana\} & Charlie\ \texttt{relativeOf}\ Bob \\ \texttt{Female} & := & \{Alice, Dana, Edwige\} & \\ \texttt{Male} & := & \{Bob, Charlie\} & \\ \texttt{Woman} & := & \{Alice, Dana\} & \\ \texttt{Man} & := & \{BobCharlie\} & \end{array}$$

is a model for

TBox	*Concepts:* Male, Female,Human,Woman, Man. *Roles:* relativeOf, childOf, grandChildOf.
ABox	Woman(*Alice*), Man(*Bob*), *Alice* relativeOf *Bob*, *Alice* childOf *Charlie*.

Open World Assumption - Models (2/2)

Wherease the following is not.

$$\begin{array}{rcl|l} \Delta & := & \{\mathit{Alice}, \mathit{Bob}, \mathit{Charlie}, \mathit{Dana}, \mathit{Edwige}\} & \mathit{Alice}\ \texttt{relativeOf}\ \mathit{Charlie} \\ \texttt{Human} & := & \{\mathit{Alice}, \mathit{Bob}, \mathit{Charlie}, \mathit{Dana}\} & \mathit{Charlie}\ \texttt{relativeOf}\ \mathit{Bob} \\ \texttt{Female} & := & \{\mathit{Alice}, \mathit{Dana}, \mathit{Edwige}\} & \\ \texttt{Male} & := & \{\mathit{Bob}, \mathit{Charlie}\} & \\ \texttt{Woman} & := & \{\mathit{Alice}, \mathit{Dana}\} & \\ \texttt{Man} & := & \{\mathit{BobCharlie}\} & \end{array}$$

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Constraints

How to to associate *meanings* to vocabulary terms:

by means of *constraints* (placed in the TBox) expressed in a mathematical fashion involving concepts, roles, individuals.

Example

$\mathtt{Human} \sqsubseteq \mathtt{Mortal}$	all persons are mortal;
$\mathtt{Woman} \equiv \mathtt{Human} \sqcap \mathtt{Female}$	women are female human beings;
$\mathtt{childOf} \circ \mathtt{childOf} \subseteq \mathtt{grandChildOf}$	the child of the child is the grandchild.

In general, the syntax of a *Knowledge Representation Language* is characterized by the types of constraints it allows, mainly in term of *operators* (eg. $\sqcap$, $\circ$, ...).

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Representing Knowledge the Semantic Web

Semantic Web Stack

Semantic Web technologies are *stratified*.

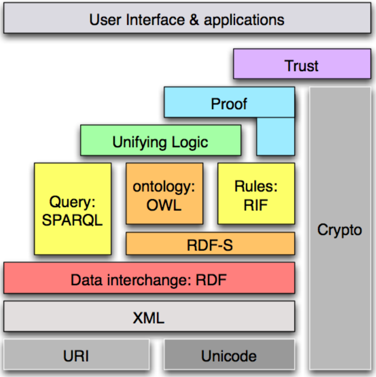

Figure : The Semantic Web stack

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Some of these are knowledge representation languages.

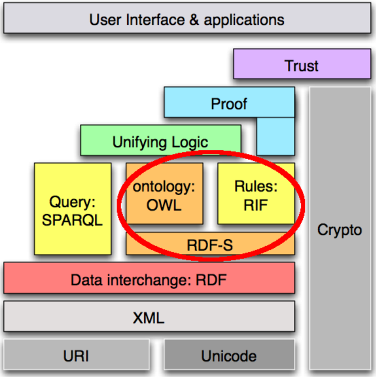

Figure : The Semantic Web stack

OWL is the concise name of the *Web Ontology Language*, currently at version 2.

`http://www.w3.org/TR/2012/REC-owl2-primer-20121211/`

Three different *profiles* (fragments) have been defined for it (see `http://www.w3.org/TR/2012/REC-owl2-profiles-20121211/`):

- *OWL-RL* quite good expressive power;
- *OWL-QL* for large `ABox`;
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OWL-EL and $\mathcal{EL}^{++}$

OWL-EL is underpinned by the description logic $\mathcal{EL}^{++}$. It has been devided in the biohealth domain (SNOMED-CT).

Concept constructors of $\mathcal{EL}^{++}$ are:

- $\top$ *(every)thing*;
- $\bot$ *nothing*;
- $\{a\}$ *nominal*, the set with only member a;
- $\exists R.C$ *existential restriction*, all those items related with some element in C;
- $C \sqcap D$ *conjunction*, all the members of both C and D;
- *concrete domains* (e.g. age > 18).

Such constructors can be used to form complex concepts, e.g.

$$\exists \texttt{childOf}.\{\mathit{Fido}\} := \{\mathit{Edwige}\}, \quad \exists \texttt{childOf}.(\texttt{Male} \sqcap \texttt{Human}).$$

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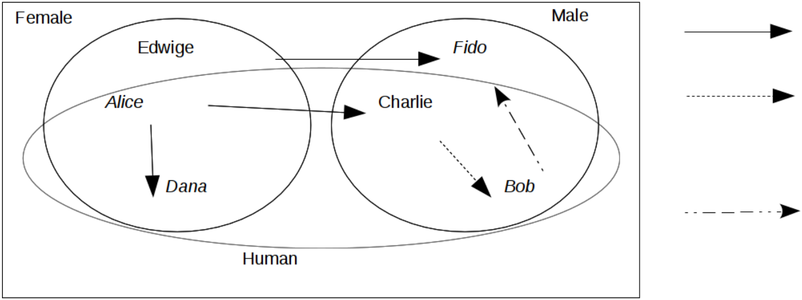
$\mathcal{EL}^{++}$ concepts - Thing

Let us consider the following *possible world* (interpretation)

$$
\begin{array}{rcl}
\Delta & := & \{Alice, Bob, Charlie, Dana, Edwige, Fido\} \\
\texttt{Human} & := & \{Alice, Bob, Charlie, Dana\} \\
\texttt{Female} & := & \{Alice, Dana, Edwige\} \\
\texttt{Male} & := & \{Bob, Charlie, Fido\}
\end{array}
$$

Alice `childOf` *Charlie*
Alice `childOf` *Dana*
Edwige `childOf` *Fido*
Charlie `relativeOf` *Bob*
Bob `ownerOf` *Fido*.

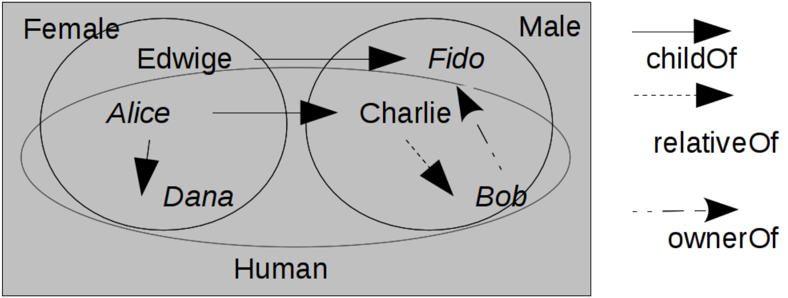


$\mathcal{EL}^{++}$ concepts - Thing

Let us consider the following *possible world*

$$\begin{array}{rcl|l} \Delta & := & \{Alice, Bob, Charlie, Dana, Edwige, Fido\} & Alice\ \texttt{childOf}\ Charlie \\ \texttt{Human} & := & \{Alice, Bob, Charlie, Dana\} & Alice\ \texttt{childOf}\ Dana \\ \texttt{Female} & := & \{Alice, Dana, Edwige\} & Edwige\ \texttt{childOf}\ Fido \\ \texttt{Male} & := & \{Bob, Charlie, Fido\} & Charlie\ \texttt{relativeOf}\ Bob \\ & & & Bob\ \texttt{ownerOf}\ Fido. \end{array}$$

$$\top := \{Alice, Bob, Charlie, Dana, Edwige, Fido\}$$



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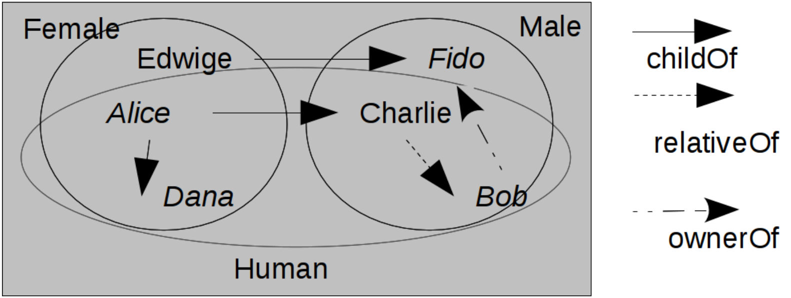
 & & & Bob\ \texttt{ownerOf}\ Fido.
\end{array}
$$

$$\bot := \emptyset$$

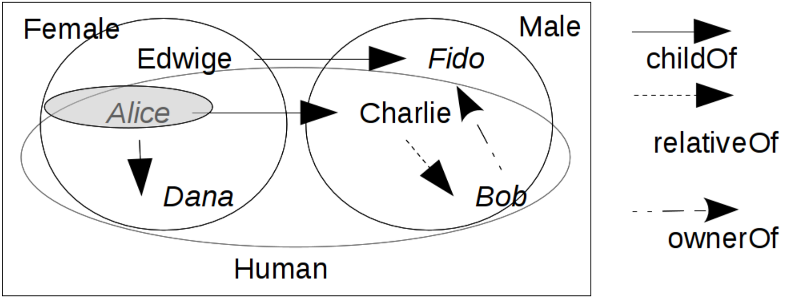


$\mathcal{EL}^{++}$ concepts - Nominal

Let us consider the following *possible world*

Δ	$:=$	$\{Alice, Bob, Charlie, Dana, Edwige, Fido\}$	*Alice* `childOf` *Charlie*
`Human`	$:=$	$\{Alice, Bob, Charlie, Dana\}$	*Alice* `childOf` *Dana*
`Female`	$:=$	$\{Alice, Dana, Edwige\}$	*Edwige* `childOf` *Fido*
`Male`	$:=$	$\{Bob, Charlie, Fido\}$	*Charlie* `relativeOf` *Bob*
			Bob `ownerOf` *Fido*.

$$\{Alice\} := \{Alice\}$$



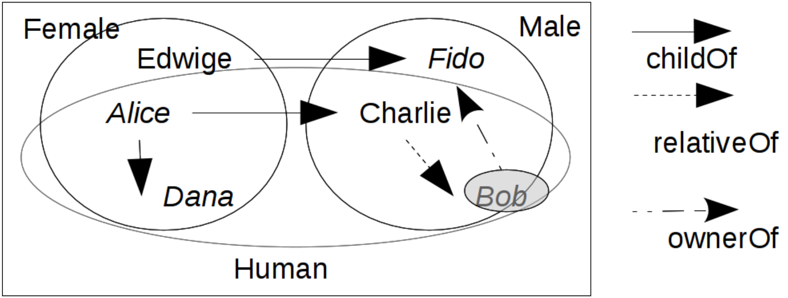
$\mathcal{EL}^{++}$ concepts - existential restriction

Let us consider the following *possible world*

$$\begin{array}{rcl} \Delta & := & \{Alice, Bob, Charlie, Dana, Edwige, Fido\} \\ \texttt{Human} & := & \{Alice, Bob, Charlie, Dana\} \\ \texttt{Female} & := & \{Alice, Dana, Edwige\} \\ \texttt{Male} & := & \{Bob, Charlie, Fido\} \end{array}$$

Alice `childOf` *Charlie*
Alice `childOf` *Dana*
Edwige `childOf` *Fido*
Charlie `relativeOf` *Bob*
Bob `ownerOf` *Fido*.

$$\exists \texttt{ownerOf}.\texttt{Male} := \{Bob\}$$



$\mathcal{EL}^{++}$ concepts - conjunction (intersection)

Let us consider the following *possible world*

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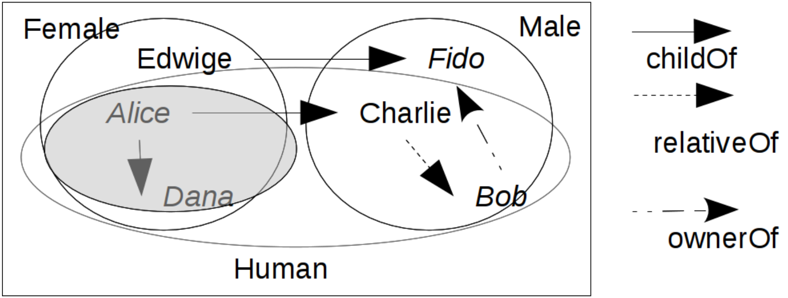
Alice `childOf` *Charlie*
Alice `childOf` *Dana*
Edwige `childOf` *Fido*
Charlie `relativeOf` *Bob*
Bob `ownerOf` *Fido*.

$$\texttt{Female} \sqcap \texttt{Human} := \{Alice, Dana\}$$

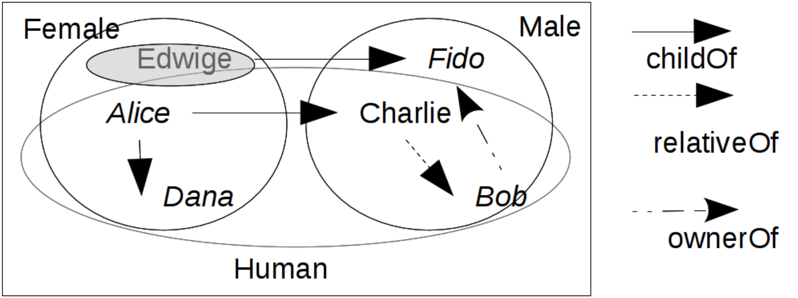


$\mathcal{EL}^{++}$ concepts - complex concepts

Let us consider the following *possible world*

Δ	$:=$	$\{Alice, Bob, Charlie, Dana, Edwige, Fido\}$	*Alice* `childOf` *Charlie*
`Human`	$:=$	$\{Alice, Bob, Charlie, Dana\}$	*Alice* `childOf` *Dana*
`Female`	$:=$	$\{Alice, Dana, Edwige\}$	*Edwige* `childOf` *Fido*
`Male`	$:=$	$\{Bob, Charlie, Fido\}$	*Charlie* `relativeOf` *Bob*
			Bob `ownerOf` *Fido*.

$$\exists \texttt{childOf}.\{Fido\} := \{Edwige\}$$



$\mathcal{EL}^{++}$ constraints

Constraints allowed in $\mathcal{EL}^{++}$ `TBox` are of the types:

- $C \sqsubseteq D$ every member of C is a member of D as well;
- $C \equiv D$ the sets C and D coincide;
- $R_1 \circ \ldots \circ R_n \sqsubseteq S$ the *composition* of $R_1, \ldots, R_n$ is a subrelation of S;
- $\mathrm{dom}(R) \sqsubseteq C$ if x is related to y by R, then x must be a member of C;
- $\mathrm{range}(R) \sqsubseteq C$ if x is related to y by R, then y must be a member of C.

The following is a *model* for

$$\texttt{Man} \sqsubseteq \texttt{Human}$$

$$\begin{array}{rcl|l}
\Delta & := & \{Alice, Bob, Charlie, Dana, Edwige, Fido\} & Alice\ \texttt{childOf}\ Charlie \\
\texttt{Human} & := & \{Alice, Bob, Charlie, Dana\} & Alice\ \texttt{childOf}\ Dana \\
\texttt{Female} & := & \{Alice, Dana, Edwige\} & Edwige\ \texttt{childOf}\ Fido \\
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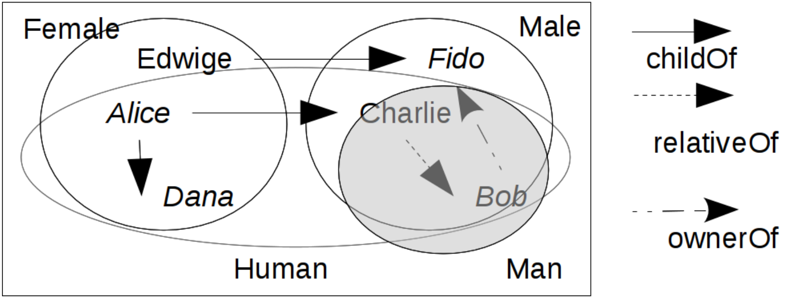

$\mathcal{EL}^{++}$ constraints - equivalence

The same is a model for

$$\texttt{Man} \equiv \texttt{Human} \sqcap \texttt{Male}$$

$$\begin{array}{rcl|l}
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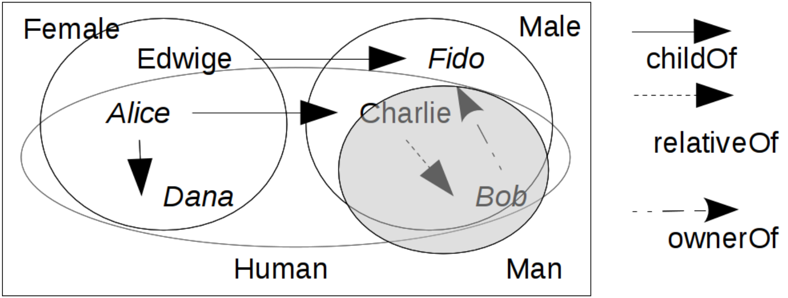

$\mathcal{EL}^{++}$ constraints - composition (1/2)

Let us consider

$$\texttt{childOf} \circ \texttt{relativeOf} \subseteq \texttt{relativeOf} \qquad \text{the child of a relative is a relative as well.}$$

The following is *not* a model for this constraint (*Alice* should be relative of *Charlie*):

$$\begin{array}{rcl|l}
\Delta & := & \{\mathit{Alice}, \mathit{Bob}, \mathit{Charlie}, \mathit{Dana}, \mathit{Edwige}, \mathit{Fido}\} & \mathit{Alice}\ \texttt{childOf}\ \mathit{Charlie} \\
\texttt{Human} & := & \{\mathit{Alice}, \mathit{Bob}, \mathit{Charlie}, \mathit{Dana}\} & \mathit{Alice}\ \texttt{childOf}\ \mathit{Dana} \\
\texttt{Female} & := & \{\mathit{Alice}, \mathit{Dana}, \mathit{Edwige}\} & \mathit{Edwige}\ \texttt{childOf}\ \mathit{Fido} \\
\texttt{Male} & := & \{\mathit{Bob}, \mathit{Charlie}, \mathit{Fido}\} & \mathit{Charlie}\ \texttt{relativeOf}\ \mathit{Bob} \\
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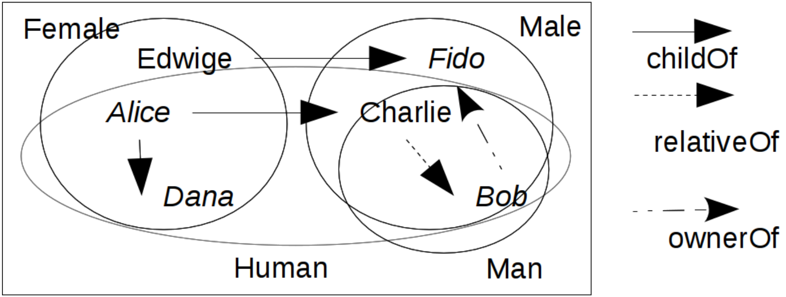

$\mathcal{EL}^{++}$ constraints - composition (2/2)

A model for $\texttt{childOf} \circ \texttt{relativeOf} \sqsubseteq \texttt{relativeOf}$ is:
The following is *not* a model for this constraint (*Alice* should be relative of *Bob*):

Δ	:=	$\{Alice, Bob, Charlie, Dana, Edwige, Fido\}$	*Alice* `childOf` *Charlie*
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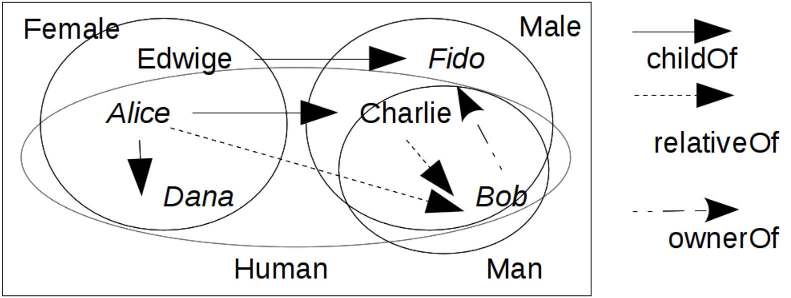
`Female`	:=	$\{Alice, Dana, Edwige\}$	*Edwige* `childOf` *Fido*
`Male`	:=	$\{Bob, Charlie, Fido\}$	*Charlie* `relativeOf` *Bob*
`Man`	:=	$\{Bob, Charlie\}$	*Bob* `ownerOf` *Fido*
			Alice `relativeOf` *Bob*.



$\mathcal{EL}^{++}$ constraints - domain

It is a model for

$$\mathrm{dom}(\mathtt{ownerOf}) \sqsubseteq \mathtt{Human} \quad \text{all owners are humans.}$$

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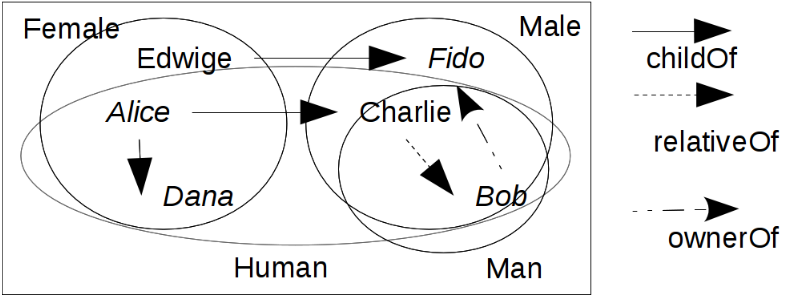
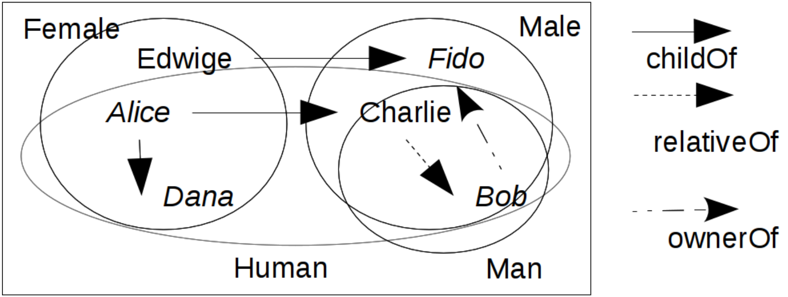


$\mathcal{EL}^{++}$ constraints - range

But this is *not* a model for

$$\mathrm{range}(\texttt{ownerOf}) \sqsubseteq \texttt{Human} \quad \text{all } \textit{owned} \text{ are humans.}$$

Δ	:=	{*Alice*, *Bob*, *Charlie*, *Dana*, *Edwige*, *Fido*}	*Alice* `childOf` *Charlie*
`Human`	:=	{*Alice*, *Bob*, *Charlie*, *Dana*}	*Alice* `childOf` *Dana*
`Female`	:=	{*Alice*, *Dana*, *Edwige*}	*Edwige* `childOf` *Fido*
`Male`	:=	{*Bob*, *Charlie*, *Fido*}	*Charlie* `relativeOf` *Bob*
`Man`	:=	{*Bob*, *Charlie*}	*Bob* `ownerOf` *Fido*.



Using Knowledge encoded in $\mathcal{EL}^{++}$

Reasoning

Defining a formal semantics enables some *reasoning* task to be performed automatically.
Here we will see

- *consistency checking*, to check the absence of logical errors,
- *inferencing*, to extract hidden knowledge.

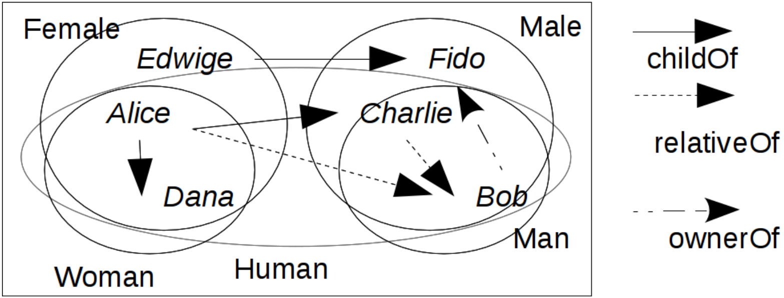
Reasoning - Consistency

An ontology is *consistent* if a possible world which *matches* it exists.

Let us consider the following ontology:

TBox	ABox
$\texttt{Woman} \equiv \texttt{Female} \sqcap \texttt{Human}$	$\texttt{Woman}(\mathit{Alice})$
$\texttt{Man} \equiv \texttt{Male} \sqcap \texttt{Human}$	$\texttt{Man}(\mathit{Charlie})$
$\texttt{childOf} \circ \texttt{relativeOf} \subseteq \texttt{relativeOf}$	$\mathit{Alice}\ \texttt{childOf}\ \mathit{Charlie}$
$\text{dom}(\texttt{ownerOf}) \sqsubseteq \texttt{Human}$	$\mathit{Charlie}\ \texttt{relativeOf}\ \mathit{Bob}$
	$\mathit{Bob}\ \texttt{ownerOf}\ \mathit{Fido}$

It is consistent (see before for a model).



Reasoning - Consistency - example 2

The following one is *not* consistent.

TBox	ABox
$\texttt{Woman} \equiv \texttt{Female} \sqcap \texttt{Human}$	$\texttt{Woman}(Alice)$
$\texttt{Man} \equiv \texttt{Male} \sqcap \texttt{Human}$	$\texttt{Man}(Charlie)$
$\texttt{childOf} \circ \texttt{relativeOf} \subseteq \texttt{relativeOf}$	$Alice\ \texttt{childOf}\ Charlie$
$\texttt{dom}(\texttt{ownerOf}) \sqsubseteq \texttt{Human}$	$Charlie\ \texttt{relativeOf}\ Bob$
$\texttt{Male} \sqcap \texttt{Female} \equiv \bot$	$Bob\ \texttt{ownerOf}\ Fido$
	$\texttt{Female}(Bob)$

Bob must be `Male`, as it is a `Man`, all men are `Male` and `Male` and `Female` are disjoint.

Reasoning - Consistency - example 3

The following one is *not* consistent.

TBox	ABox
$\texttt{Woman} \equiv \texttt{Female} \sqcap \texttt{Human}$ $\texttt{Man} \equiv \texttt{Male} \sqcap \texttt{Human}$ $\texttt{childOf} \circ \texttt{relativeOf} \subseteq \texttt{relativeOf}$ $\texttt{dom(ownerOf)} \sqsubseteq \texttt{Human}$ $\texttt{Human} \sqcap \mathit{Edwige} \equiv \bot$	$\texttt{Woman}(\mathit{Alice})$ $\texttt{Man}(\mathit{Charlie})$ $\mathit{Alice}\ \texttt{childOf}\ \mathit{Charlie}$ $\mathit{Charlie}\ \texttt{relativeOf}\ \mathit{Bob}$ $\mathit{Bob}\ \texttt{ownerOf}\ \mathit{Fido}$ $\mathit{Edwige}\ \texttt{ownerOf}\ \mathit{Fido}$

Edwige can't be the owner of *Fido* because *Edwige* is not a member of Human.

Reasoning - Inference

Inferences retrieves all the assertions which hold in *all* the possible models.

TBox	ABox
$\texttt{Woman} \equiv \texttt{Female} \sqcap \texttt{Human}$ $\texttt{Man} \equiv \texttt{Male} \sqcap \texttt{Human}$ $\texttt{childOf} \circ \texttt{relativeOf} \subseteq \texttt{relativeOf}$ $\mathsf{dom}(\texttt{ownerOf}) \sqsubseteq \texttt{Human}$	$\texttt{Woman}(\textit{Alice})$ $\texttt{Man}(\textit{Charlie})$ $\textit{Alice}\ \texttt{childOf}\ \textit{Charlie}$ $\textit{Charlie}\ \texttt{relativeOf}\ \textit{Bob}$ $\textit{Bob}\ \texttt{ownerOf}\ \textit{Fido}$
Inferred Statements	

Inferences retrieves all the assertions which hold in *all* the possible models.

TBox	ABox
$\texttt{Woman} \equiv \texttt{Female} \sqcap \texttt{Human}$ $\texttt{Man} \equiv \texttt{Male} \sqcap \texttt{Human}$ $\texttt{childOf} \circ \texttt{relativeOf} \subseteq \texttt{relativeOf}$ $\text{dom}(\texttt{ownerOf}) \sqsubseteq \texttt{Human}$	$\texttt{Woman}(\mathit{Alice})$ $\texttt{Man}(\mathit{Charlie})$ $\mathit{Alice}\ \texttt{childOf}\ \mathit{Charlie}$ $\mathit{Charlie}\ \texttt{relativeOf}\ \mathit{Bob}$ $\mathit{Bob}\ \texttt{ownerOf}\ \mathit{Fido}$
Inferred Statements $\texttt{Female}(\mathit{Alice})$ $\texttt{Human}(\mathit{Alice})$	

Inferences retrieves all the assertions which hold in *all* the possible models.

TBox	ABox
$\texttt{Woman} \equiv \texttt{Female} \sqcap \texttt{Human}$ $\texttt{Man} \equiv \texttt{Male} \sqcap \texttt{Human}$ $\texttt{childOf} \circ \texttt{relativeOf} \subseteq \texttt{relativeOf}$ $\text{dom}(\texttt{ownerOf}) \sqsubseteq \texttt{Human}$	$\texttt{Woman}(\mathit{Alice})$ $\texttt{Man}(\mathit{Charlie})$ $\mathit{Alice}\ \texttt{childOf}\ \mathit{Charlie}$ $\mathit{Charlie}\ \texttt{relativeOf}\ \mathit{Bob}$ $\mathit{Bob}\ \texttt{ownerOf}\ \mathit{Fido}$
Inferred Statements $\texttt{Female}(\mathit{Alice})$ $\texttt{Human}(\mathit{Alice})$ $\texttt{Male}(\mathit{Charlie})$ $\texttt{Human}(\mathit{Charlie})$	

Reasoning - Inference

Inferences retrieves all the assertions which hold in *all* the possible models.

TBox	ABox
$\texttt{Woman} \equiv \texttt{Female} \sqcap \texttt{Human}$ $\texttt{Man} \equiv \texttt{Male} \sqcap \texttt{Human}$ $\texttt{childOf} \circ \texttt{relativeOf} \subseteq \texttt{relativeOf}$ $\mathrm{dom}(\texttt{ownerOf}) \sqsubseteq \texttt{Human}$	$\texttt{Woman}(\mathit{Alice})$ $\texttt{Man}(\mathit{Charlie})$ *Alice* childOf *Charlie* *Charlie* relativeOf *Bob* *Bob* ownerOf *Fido*
Inferred Statements $\texttt{Female}(\mathit{Alice})$ $\texttt{Human}(\mathit{Alice})$ $\texttt{Male}(\mathit{Charlie})$ $\texttt{Human}(\mathit{Charlie})$ *Alice* relativeOf *Bob*	

Reasoning - Inference

Inferences retrieves all the assertions which hold in *all* the possible models.

TBox	ABox
$\mathtt{Woman} \equiv \mathtt{Female} \sqcap \mathtt{Human}$ $\mathtt{Man} \equiv \mathtt{Male} \sqcap \mathtt{Human}$ $\mathtt{childOf} \circ \mathtt{relativeOf} \subseteq \mathtt{relativeOf}$ $\mathrm{dom}(\mathtt{ownerOf}) \sqsubseteq \mathtt{Human}$	$\mathtt{Woman}(Alice)$ $\mathtt{Man}(Charlie)$ $Alice\ \mathtt{childOf}\ Charlie$ $Charlie\ \mathtt{relativeOf}\ Bob$ $Bob\ \mathtt{ownerOf}\ Fido$
Inferred Statements $\mathtt{Female}(Alice)$ $\mathtt{Human}(Alice)$ $\mathtt{Male}(Charlie)$ $\mathtt{Human}(Charlie)$ $Alice\ \mathtt{relativeOf}\ Bob$ $\mathtt{Human}(Bob)$	

Thank You!

(questions?)