Technology, Inequality, and the Labor Supply*

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Abstract

Aggregate evidence in the US points to long-run declines in hours per worker. Moreover, cross-sectional evidence indicates a positive (and increasing) correlation between hours and wages, coinciding in time with the well-documented increase in wage inequality. To jointly account for these facts, this paper proposes a matching model of the labor market where hours worked are endogenous. The theory characterizes the sorting patterns that emerge in equilibrium for general preferences and technology, and derives new implications for inequality in hours and wages, as well as for the aggregate patterns of hours worked. At the heart of the theory is the interaction of income effects in preferences and complementarities in production, giving rise to heterogenous returns to long hours worked across skills. When the latter are sufficiently strong, high skill work longer hours despite large income effects, therefore amplifying inequality. I use the theory to quantitatively analyze the impact of technological advancements on the labor-leisure trade-off and their effect on inequality and the aggregate labor supply in the US. I find that increasing complementarities in production are the main drivers of rising returns to long hours, and are an important driver of increasing inequality, as well as aggregate hours worked. The theory suggests that modeling technology - and its interaction with preferences - is key to understand the labor supply in the cross-section, and in the aggregate.

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1 Introduction

Over the last century, the nature of work has undergone significant transformations: the average american worker spends 10% less time at work than she used to do in the 70's. Moreover, *who* spends more time at work has significantly changed: high wage workers were spending less time at work in the 1970's, while low wage workers were working the longest hours. Nowadays, this correlation has reversed: it is the high skilled, high wage workers who work the most[1]. This widening gap in hours worked has coincided in time with the well-documented increase in wage inequality; the latter has led economists and policymakers to accept the view that technological change may be biased towards specific skills. However, the very same technological transformations behind the changes in the wage structure are likely to affect labor markets more broadly; for example, they may impact the labor-leisure trade-off that a worker faces, with implications for welfare that might go well beyond those solely based on wages.

Despite these considerations, few studies have jointly analyzed the evolution of hours worked with recent inequality trends. How can we reconcile the aggregate, long run evidence on hours worked with the more recent cross-sectional patterns? How do recent technological improvements shape the incentives to work, and what are the aggregate implications?

This project provides a new framework to answer these questions. In doing so, I make two contributions: first, I build a matching model of the labor market where workers sort in different jobs based on skill, and time input. I characterize, for general preferences and technology, the forces that shape the hours decision at the invididual level, and how this decision can amplify (or dampen) overall inequality and the sorting patterns that emerge in equilibrium. A key insight that the analysis reveals is that complementarities in production have first order implications for hours inequality, and for our understanding of the evolution of the aggregate labor supply. Second, I bring the model to the data and assess the impact of these complementarities on the evolution of inequality in hours and wages in US.

Through the estimated model, I find that in US, well-known forces - such as Skill Biased Technological Change - have been accompanied by further changes in the production function; in particular, hours-skills complementarities have increased (in addition to the well known increase in job-skill complementarity). This has made income non-linear in hours worked for the high skilled, thus counteracting the income effects and amplifying inequality, pushing up hours for the high skilled (and hence overall hours).

[1] The fact that the wealthy were consuming more leisure has attracted the attention of sociologists since at least the end of the 19th century: Veblen (1899) discusses how the wealthiest class was spending large fractions of their time in leisure activities, while low-class, working people were working the longest hours in an attempt to make a living. More recently, Jacobs and Gerson (2005) discuss the potential consequences of the growing 'time divide' between the increasingly working elite and the idle middle class.

I start the analysis by higlighting two key facts. The first is that from an aggregate, long-term perspective, hours per worker have declined in the US at a remarkably steady pattern. In the 1950's, the average American worker used to work 15% more than now (roughly 44 hours per week, compared to 38 hours per week nowadays). These facts can be well explained by models of the labor supply in which the utility function is such that income effects dominate substitution effects; when this is the case, rising productivity (and hence wages) for the typical household imply higher leisure time.[2] The key implication is that, other things being equal, higher wages imply lower hours worked.

While this preference-based explanation for hours worked does a very good job in explaining long run trends, the second cross-sectional fact of how hours relate to wages reveals there might be more to the story. During in the 1970's, low wage workers used to work the longest hours; nowadays, it is the highest wage workers who spend more time working. These patterns have been documented before, notably in Costa (2000) (see also Bick *et al.* (2018) for a cross-country analysis of these patterns), and I provide in this paper a systematic analysis of the evolving hours-wage correlation in the cross section. This changing correlation coincided in time with the well-documented increase in income inequality in the US. These facts are hard to reconcile in light of the aggregate evidence: if income effects dominate substitution effects, this would suggest that high skill workers work *less*, not more (and increasingly so).

To explain these facts, I propose a matching model of the labor market. The model features heterogeneous workers and firms, which match one to one in a competitive labor market. Crucially, the model departs from most existing assortative matching models in that it explicitly features an endogenous hours choice by the worker (for a summary of sorting models of the labor market, see Chade *et al.* (2017)). Hence, not only skills, but also time input are a determinant component of sorting in equilibrium. I characterize, for a general class of production functions and utility functions, the sorting pattern that arise in equilibrium, and the implications for wages and hours inequality.

At the heart of the model mechanism is the interaction between income effects (coming from preferences) and the heterogeneous substitution effects, coming from the properties of the production function: on the one hand, higher wages push toward working lower hours; on the other hand, due to complementarities in production, hours of work affect wages in a non-linear fashion. With respect to macroeconomic models that aim to explain aggregate patterns of hours worked, this model is new in that it allows for hours of work to impact wages differently for different workers. Hence not only preferences, but also technological

[2]For a thorough account of how income effects affects hours in balanced growth models, see Boppart and Krusell (2020)).

complementarities become a crucial determinant of aggregate hours worked. With respect to models of sorting (see among others Chade and Lindenlaub (2022), Eeckhout and Kircher (2018) and Eeckhout and Sepahsalari (2018)) this paper models explicitly the hours decision, which helps building a comprehensive picture of increasing inequality: not only in wages, but also in hours worked (and the interaction between the two). Thus, the framework nests several existing models as special cases.[3]

The model speaks to the motivating evidence precisely through the interaction of the preference channel and the technology channel: hours worked can amplify or dampen income inequality stemming from complementarities in production between skills and jobs. If income effects are sufficiently strong or hours enter the earnings function linearly (as in most existing models), high skill workers will be induced to choose lower hours. This will induce a negative relationship between hours worked and wages both in the cross-section as well as in the aggregate, similarly to the pre-1980's period in the US. If hours worked translate non linearly in earnings due to complementarities in production, high skill workers may decide to work longer hours, reverting the cross-sectional relationship between hours and wages, thus amplifying inequality and slowing down the aggregate hours decline (or even increasing it). A contribution of this paper is to explicitly characterize for which preferences and technology class each force dominates.

To quantitatively assess the relative importance of these channels, I structurally estimate the model using US data for the recent decades. The estimation results reveal that, in addition to the well know increase in complementarity between skills and jobs (commonly referred to as Skill Biased Technological Change), other technological changes have marked the U.S. experience. In particular, I estimate that hours and skills (in addition to hours and jobs) have become more complementary. I show through counterfactual exercises that these changes magnified the increase in inequality, and had first-order impact on the relationship between hours and wages, both in the cross section, and in the aggregate. In particular, SBTC hasincreased wage inequality, but has pushed the wage-hours elasticity down; the new complementarity patterns I highlight, however, have made hours worked higher for the high skilled, thus further magnifying inequality (although through a different channel than SBTC). Key for this result is to endogeneize the hours decision and study its implications for sorting, which is the main constribution of this paper.

The analysis reveals two further insights: the first is that in the cross section, it is fundamental to endogeneize the hours decision to assess the true impact of techonology on wage inequality. I show that in a similar model where hours are taken as an exogenous deci-

[3] In particular, the model allows for earnings to be non linear in hours worked, especially for high skilled workers. This can be considered as a generalization of typical labor supply models, where earnings are assumed to be linear in hours worked. These models have been used to both study labor supply in the aggregate as well as in the cross-section.

sion can bias our conclusions regarding the true determinants of technology on inequality, overstating the importance of skill-job complementarities and understating the skill-hours complementarities. Furthermore, I show that hours worked in the aggregate can rise or decline depending not only on preferences, but also on the tecnological properties of the production function, and how these properties changed over time. In particular, I show through counterfactual exercises the path of hours worked without the technological forces that I highlight in my model, finding that hours worked would have declined in a similar fashion as the pre-1980 era (and thus supporting the existence of strong income effects in the aggregate[4]). Thus, accounting for heterogeneity is key to understand aggregate outcomes and the future of hours worked.

I conclude by discussing the implications of my findings for the literature on income taxation and the gender gap. While fully developing these implications is beyond the scope of this paper [5], I show that the current framework can be used to explore other issues related to inequality. Therefore, the framework can provide a starting point to think about how the interaction of skills and hours (and the sorting pattern that emerge) can shed light on other related issues.

Literature Review In proposing a new force for increasing income inequality, as well as a new determinant for aggregate hours worked, the paper brings together two main literatures: a macro literature on the aggregate relationship between wages and hours, and a more micro literature, focused on the determinants of increasing sorting and inequality in labor market outcomes.

Among macroeconomic models of balanced growth, King *et al.* (1988) is a seminal contribution in specifying a preference class that implies constant hours worked along the growth path. The perceived need to work with preferences that imply constant hours was due to the fact that hours per capita are approximately constant in the US. However, as shown in several papers, the roughly constant level of hours worked in the post-war period masks significant heterogeneity, most notably reflecting increasing participation rates for women. Boppart and Krusell (2020) provide a theoretical analysis of aggregate hours worked, and a general preference class in which income effects dominate substitution effects, and they show it can account for the intensive margin of hours worked in the US and elsewhere. Relatedly, Bick *et al.* (2018) and Bick *et al.* (2022b) also present evidence pointing towards strong income effects in the aggregate, while highlighting the role of structural change in accounting for patterns of hours worked along the development spectrum. Relative to all

[4]It is precisely the relatively flat series of aggregate hours worked in US in the post-war period that first motivated the use of preferences where income and substitution effects cancel out, see King *et al.* (1988).

[5]I am pursuing this in separate projects.

these papers, this study proposes technological complementarities in production can shed new light on the evolution of the aggregate labor supply.

The sorting literature has studied how techonological complementarities are crucial in determining sorting patterns in equilibrium (see for example, Lindenlaub (2014), Chade and Lindenlaub (2022), Eeckhout and Kircher (2018), Eeckhout and Sepahsalari (2018)). Relative to these papers, my framework provides a general characterization of how hours worked affect the sorting patterns and the relative role of production and preferences in determining the equilibrium. The following contributions are closest to this framework: Calvo *et al.* (2021), Michelacci and Pijoan-Mas (2015) and Shao *et al.* (2021). The first studies how the interplay between marriage and labor market decisions, shaped by the endogenous hours decision, affects inequality and hours in equilibrium. The second provides a competitive growth model where the assignment of workers and jobs is andogenously affected by technological progress the hours decision. The third studies the role of hours complementarities in production on the size distribution of firms, and their impact on earnings inequality. Relative to these papers, I contribute theoretically by characterizing the sorting pattern for a general class of production and utility functions, and I show how the interplay between the two gives rise to new insights on the technological causes of inequality. I also differ in the application of the model, in that I estimate how the production technology has varied over time, and study the implications of hours and wages in the data. Another related paper is Erosa *et al.* (2022), which develop a Roy-style model of occupational choice to study the implications of gender differences in home production possibilities for the gender wage gap.

Outline Section 2 presents more in detail the main facts motivating the analysis. In section 3, I present and characterize the model implications, with an emphasis on the new insights and the how the model links to the previous literature. I estimate the in the model to US data in Section 4; Section 5 presents the main results of the quantitative application and the resulting implications. Section 6 concludes and presents some avenues for further research.

2 Motivating Evidence

In this section, I present two main facts (an aggregate fact and a cross-sectional fact) that motivate the theoretical analysis below[6].

Data

The main data source is CPS Ongoing Rotation Group (CPS-ORG), which contains detailed information on hours worked and wages for the period of interest. I complement hours and wage data from CPS with hours data from Kendrick (1961) and Kendrick (1973) to be able to trace total hours worked back in time. The latter datasets have been used extensively in the labor supply literature to capture long-run trends in hours worked (see e.g. Francis and Ramey (2009) and Cociuba *et al.* (2018)). In Appendix A, I show that the main messages delivered by the analysis below are very similar across other datasets of hours worked (e.g. ATUS).

Facts

Hours Worked in the Aggregate The first fact I highlight the long-run behavior of hours per worker[7]. The series is weekly hours worked per worker, and is plotted in Figure 1.
From the figure, it is evident that the data exhibit a clear downward trend, except for the most recent decades, where hours worked are relatively stable.
As noted before, this downward trend in average hours worked per worker is by no means a phenomenon specific to US: Bick *et al.* (2018) show that, along the development spectrum, hours per worker decline with GDP per capita, and that these pattern does not reflect systematic differences in age, educational attainment or sectoral composition across countries. Further evidence is also provided in Kopytov *et al.* (2021), where show that the overall decrease of hours as wages grow is a trend that characterizes all OECD countries in the post-war period.
These patterns, interpreted through the lens of neoclassical models of labor supply, point towards preferences where - along the balanced growth path - income effects are stronger than substitution effects, so that an increase in *average* wages due to productivity implies lower

[6]Since the quantitative application will use data from US, I will only present the facts for the latter; however, previous literature has shown that the main messages hold across several countries (see Appendix A).

[7]Since the focus of the paper is on the intensive margin, the relevant measure to be considered is hours per worker. Moreover, as noted in Boppart and Krusell (2020), long run movements in hours worked per worker (the *intensive* margin) are more important than participation rates (the *extensive* margin). Nevertheless, a similar picture emerges if we consider hours per capita over the very long term (see also Francis and Ramey (2009)).

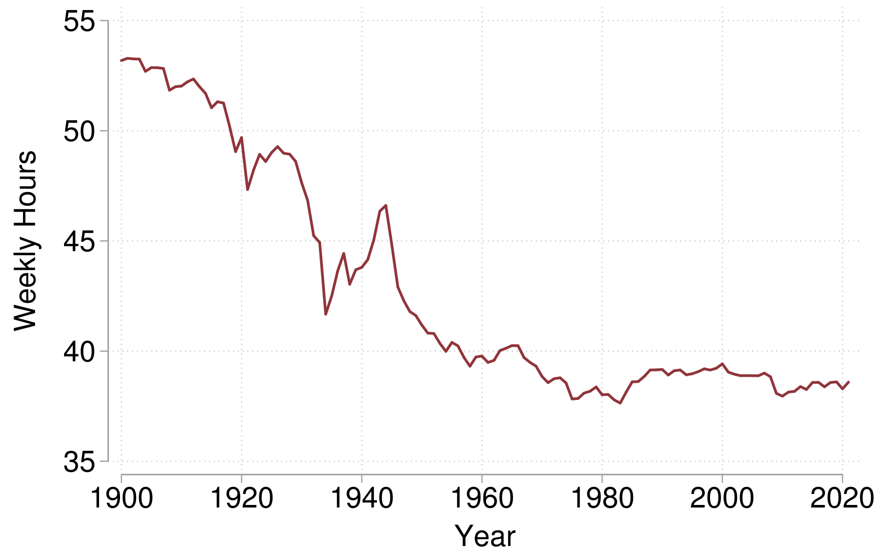

Figure 1: Hours per Worker (US).

Notes: Average weekly hours worked per employed worker in the US. Source: Kendrick (1961), Kendrick (1973), CPS and own calculations.

hours worked[8]. This is the main intuition behind models that aim to characterize the forces behind long-run declines in hours worked, as in Boppart and Krusell (2020) and Bick *et al.* (2022b)[9]. Overall, the main takeaway from this figure is that, from an aggregate perspective and over the long term, hours per worker have decreased in US, suggesting an aggregate *negative* relationship between wages and hours worked[10].

Hours Worked in the Cross-Section The second fact motivating the analysis concerns the crosssection data on hours worked. As first noted in Costa (2000), the US experience is characterized by a sign reversal in the hours-wage elasticity, from negative to positive: low wage workers used to work the longest hours in 1960's and 1970's; starting from the 1980's, however, high wage workers work significantly more hours than low wage workers.

I now update the analysis first conducted in Costa (2000) to more recent years, using the same specification as in her paper, and also adopted in Bick *et al.* (2018). In particular, I analyze the cross-sectional hours-wage elasticity by running the following regression year by year, in CPS data:

[8]Moreover, these trends are consistent with papers using time use data, which show that hours worked have declined (and leisure time increased) in US, as in Aguiar and Hurst (2007).

[9]The idea that, over long run periods of productivity and wages growth, people would dedicate more time to leisure activities traces back at least to Keynes (1930)

[10]In Appendix A, I provide further evidence that the pattern of declining hours per worker is not at all a phenomenon specific to US: using data from Bick *et al.* (2018), a clear pattern of declining hours per worker emerges when considering middle income and high income countries. See in particular Figure A.2.

$$log(h_i) = \alpha + \beta log(w_i) + X_i + \epsilon, \quad (1)$$

where h_i is individual hours worked, w_i is individual wages, and X_i are demographic controls[11]. The coefficient β describes the relationship between hours and wages in the cross-section. I plot the resulting coefficient in Figure 2. At the beginning of the sample, and consistent with Costa (2000)'s results, the coefficient is negative; starting from the 1980's, it turns significantly positive and increasing, with a mild decrease only in the most recent years in the sample[12].

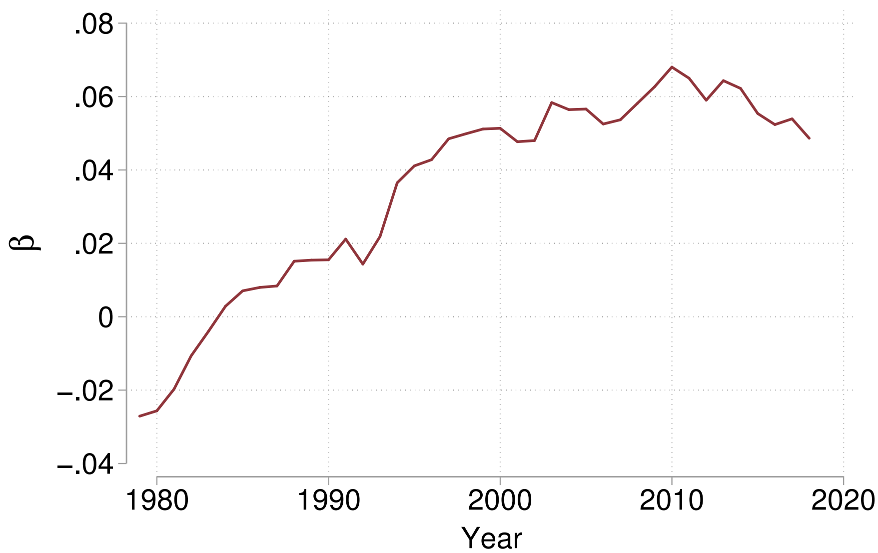

Figure 2: Hours-Wage Elasticity in the Cross Section (US).

Notes: The figure displays the coefficient β from regression (1), run separately for each year in the sample. Source: CPS-ORG and own calculations.

Importantly, Bick *et al.* (2018) note that across the development spectrum, the hours-wage elasticity is negative for low income countries, while it gets progressively smaller and even positive for high income countries; this is very much in line with the US experience that I uncover here. Moreover, data from time use sources (see in particular Aguiar and Hurst (2007)) are broadly consistent with the evidence presented thus far: starting from the mid-1980's, leisure time has increased for the low skilled workers more than for the high skilled workers (while importantly, it increased uniformly across skill categories between 1965 and 1985).

The key message from this evidence is that, starting from the 1980's, low wage workers have

[11]In the baseline specification, I consider age and age squared as controls to account for the systematic variation in hours worked across the life-cycle.

[12]In 2005, the coefficient β almost perfectly overlaps with the result in Bick *et al.* (2018).

worked progressively shorter hours than their high-wage counterparts. Thus, the negative aggregate hours-wage relationship that we highlighted in the previous paragraph does not fully reflect in the cross-section, where the relationship at the individual level has turned positive[13].

Summary and Implications The aggregate data presented so far, namely a decreasing trend in hours worked, suggests the need of using models of the labor supply where preferences are such that income effects are larger than substitution effects (as in Boppart and Krusell (2020)). This approach does a good job in describing aggregate data in the long run. However, cross-sectional data for more recent years seem to point to a richer picture; in fact, high wage workers working increasingly longer hours seem to suggest that, if we analyze a shorter time span, other forces might be at play.

It is important to note that, starting from roughly the same period, another phenomenon has been widely documented and studied: increasing wage inequality (see among many others Song *et al.* (2018)). In other words, leisure inequality has been accompanied by a widening wage inequality. The evidence presented thus far calls for a framework that takes into account aggregate, long run trend of decreasing hours worked and - at the same time - is able to account for increasing cross sectional dispersion in wages and hours worked in the last few decades.

The next section provides a parsimonious matching framework that accounts for these trends, and aims at shedding more light on the consequences of technological progress onthe *joint* determination of wages and hours worked across workers of different skills.

[13] I also plot hours worked by wage decile across years in Figure A.3, to visually inspect how hours moved over time across the wage distribution. From the figure it is clear that low wage workers have decreased hours significantly over time, while high wage workers have increased their hours, with most of the increase concentrated in the 1980's and 1990's. This is in line with the regression results just shown.

3 Theory

Motivated by the evidence of the previous section, this section develops an assignment framework, with the key feature that workers sort in the labor market based on skills and hours. After characterizing the theory, I show how it relates to the literature and in particular, how it includes several models of the labor supply as special cases. Finally, I do comparative statics to introduce the quantitative analysis carried in the next section.

3.1 Framework

Setup I consider a competitive labor market, composed of heterogeneous workers $x \sim H$ and firms $y \sim G$, where H and G are the distribution of workers and firms[14], respectively. Individuals are endowed with one unit of time to be allocated between market work and leisure (there is no home production)[15]. Workers and firms match in a one-to-one fashion to produce output $f(x, y, h)$.

Firms' problem Firms choose type x to maximize output net of income paid to the worker:

$$\max_x f(x, y, h) - w \tag{2}$$

The choice of worker type x by the firm will determine, in equilibrium, the assignment function μ that maps workers to firms as well as wage and profit functions (commonly referred to as hedonic price schedules).

Household problem Households choose time allocation taking income w as given:

$$\max_h \quad u(c, h) \quad s.t. \quad c = w \tag{3}$$

This determines optimal choice of hours h as a function of skill, $h^*(x)$. Hours choice is the key link between worker problem and firms problem. With no hours choice and transferable utility (TU), this is a standard assignment game between workers and firms Becker (1973).

Market clearing The model is closed by specifying a market clearing condition, essentially requiring that the workers and firms match in a measure-preserving way. Market clearing

[14] For the theoretical section, I will refer to y as firms and jobs, interchengeably, as the model allows for both interpretations; when I take the model to the data, I will interpret y as jobs.

[15] While home production can be an important determinant of inequality, see e.g. Boerma and Karabarbounis (2021), I choose to abstract from home production as it does not display diverging trends across households. Moreover, doing so allows for a more direct comparison with macro models based on preferences, which typically abstract from this margin.

can be written as, under PAM[16]:

$$\int_{\mu(x)}^{\bar{y}} g(s)ds = \int_{x}^{\bar{x}} h(s)ds$$

Equilibrium We are now ready to define a competitive equilibrium of this economy.

Definition 1 *A competitive equilibrium of this economy is a tuple of functions (w, μ, h) such that:*

- *w and h solve problems (1) and (2)* *(optimality)*
- *equation (3) holds* *(market clearing)*

3.2 Assortative Matching

Towards a complete characterization of the equilibrium of this economy, we want to seek for conditions under which assortative matching arises. To do so, we can rewrite the *joint* maximization problem of the worker and the firm as a single maximization problem, by substituting the wage in the worker problem (3) using the definition of profits from (2). This becomes effectively a matching problem with non-linear Pareto frontiers (see Legros and Newman (2007)):

$$U(x, y, V) = \max_{y,h} \quad u\left(f(x, y, h) - V, h\right) \tag{4}$$

where V is the hedonic price schedule (in this case, profits) that arises in equilibrium and U is the value to a worker x matched to a job y to which he leaves the value V.
The FOCs for hours and firm choice are, respectively:

$$u_c f_h + u_h = 0 \tag{5}$$

$$u_c(f_y - V_y) = 0 \tag{6}$$

Equation (5) is akin to the standard labor-leisure choice, the main difference being that f_h is now allowed to depend on hours, implying that hours can have a non-linear effect on earnings; importantly, this effect varies across skills due to complementarities in production[17].

[16]Here and throughout the text, I refer to PAM as Positive Assortative Matching, indicating that better workers are matched in equilibrium with better firms (analogously for Negative Assortative Matching, NAM).

[17]In the textbook model of labor supply, f_h would be replaced by w, wage for efficiency units of labor. This is because earnings $w \cdot h$ are linear in hours.

Equation (6) is equivalent to one arising from the first order condition of the standard assignment model with transferable utility, where workers choose firms[18].
In this case, the sorting condition can be derived from the second order condition of (4) and it is equal to[19]:

$$U_{xy} - U_{Vx}\frac{U_y}{U_V} > 0 \tag{7}$$

In the case of TU, $U_{xy} = f_{xy}$ (i.e., complementarity between firm and worker type) is the only determinant of assortative matching; in this ITU setting, however, what matters for sorting is also how the surplus of the match varies across worker-firm pairs, which in turn depends on the complementarity between worker type and partner's (firm's) utility, captured by U_{Vx}. Intuitively, the easier it is for higher x to transfer utility to firms, the more likely it is they will match with these high type firms. In my setting, this will depend on both characteristics of preferences (income effects) and on technological complementarities between hours, firms and worker types. By writing explicitly the expressions U_{xy}, U_{Vx}, U_y and U_V in (7), the next proposition states this explicitly in terms of primitives of the model (preferences and technology)[20].

Proposition 1 *A necessary condition to have Positive Assortative Matching (PAM) for any distribution of types is*

$$f_{yx}\phi + f_{hy}(u_{cc}f_x f_h + u_c f_{hx}) > 0 \tag{8}$$

where $\phi > 0$. *The opposite inequality provides a condition for Negative Assortative Matching (NAM).*

Proof: see Appendix B. ■

Let us inspect this condition. From optimality, we know $(u_{cc}f_h f_h u_c f_{hh} u_{hh}) < 0$ implying $(-u_{cc}f_h f_h - u_c f_{hh} - u_{hh}) > 0$. It is clear that, to obtain PAM ($\mu_y > 0$), f_{yx} is not sufficient. We

[18]Typically, the standard assignment model with TU is solved from the perspective of the firm; however, it is easy to show that the conditions under which assortative matching arises are identical to those arising from the dual problem, in which workers choose firms. In this case, the FOC is precisely equal to (6).

[19]This condition is equivalent to the one used by Eeckhout (2018)- Section 2.2. There, the problem of the firm is: $\max_x \phi(x, y, u)$ where $u(x)$ is the utility of the worker. Analogously, we can interpret $\max_x \phi(x, y, u)$ as the problem of a worker y and a firm x where $u(x)$ is the utility of the firm (V in our case). It is easy to show that the condition under which $\mu'() > 0$ is simply that $\phi_{xy} > \frac{\phi_x}{\phi_u}\phi_{yu}$. In our case, this is equivalent to (7).

[20]Alternatively, the problem can be described as a joint, simultaneous choice of firms and hours by the workers, that gives rise to a multidimensional second order condition; then, PAM/NAM arises depending on conditions derived from the Hessian of the problem, as in Eeckhout and Kircher (2018). I provide the alternative proof in Appendix B, showing it gives rise to identical condition for sorting as described in the main text.

need the term $f_{hy}(u_{cc}f_xf_h + u_cf_{hx})$ to be positive as well, or not too negative, otherwise the term in parenthesis above will be negative, making it impossible for $\mu_y > 0$ to be an optimal outcome. Assuming f_{hy}, the key term becomes $(u_{cc}f_xf_h + u_cf_{hx})$, which captures the key income and substitution effects contained in the model, respectively. The first term $(u_{cc}f_xf_h$ is a by product of the skill and hours premium f_xf_h combined with income effects coming from preferences, u_{cc}, and is therefore negative. The second term captures the substution effect, which is skill-dependent in this model (u_cf_{hx}). This term is positive as long as f_{hx} is positive. Hence, we need this second term to outweight the income effect term for PAM to be an equilibrium outcome, *even if* we assume $f_{yx} > 0$. This discussion highlights the how key forces in the model (hours in production and preferences) play an additional role with respect to known forces in the standard sorting framework (Becker (1973), Eeckhout (2018)). Equation (8) expresses the PAM condition in terms of primitives; alternatively, we can summarize this condition by highlighting how the hours choice by worker skill affects sorting. Doing so leads us to the following corollary:

Corollary 1 *Condition (8) is equivalent to:*

$$f_{xy} > -f_{hy}h_x \tag{9}$$

where $h_x = \frac{\partial h}{\partial x}$.

Proof: see Appendix B. ■

If $f_{hy} > 0$, then $h_x > 0$ or $h_x < 0$ (but not too negative) implies PAM. The intuition here is that if high type jobs require longer hours, it can be that in equilibrium low skilled workers are matched with high type jobs (NAM), if they work sufficiently more than the high skilled to compensate for their lower skill. This can overturn the effect induced by f_{xy}, which pushes towards PAM. The opposite intuition is at work when $f_{yh} < 0$.

Hours Choice As the previous paragraph made clear, the key endogenous outcome that shapes equilibrium sorting is the hours choice. We thus analyze more in detail the determinant of hours choice and how sorting and hours affect each other in equilibrium.
We start with the following proposition:

Proposition 2 *In equilibrium, high skill workers choose higher hours* $(h_x > 0)$ *if*

$$\phi(u_{cc}f_hf_x + u_cf_{hx}) > -f_{xy}f_{yh}u_c \tag{10}$$

where $\phi = -(f_{yy} - V_{yy}) > 0$.

Proof: see Appendix B. ■

Proposition 2 makes it clear that there are several forces at work when it comes to the hours choice by skill type. First, let's analyze the term in parenthesis in the left-hand side of the inequality, $(u_{cc} f_h f_x + u_c f_{hx})$. This term is a combination of income effects, acting through $u_{cc} < 0$, which push towards $h_x < 0$ (since it will make the LHS smaller); and substitution effects, acting through f_{xh}, which push towards $h_x > 0$. These opposing forces arise in any static labor supply model, and the term $(u_{cc} f_h f_x + u_c f_{hx})$ summarizes them in the context of this framework.

The term on the RHS $-f_{xy} f_{yh} u_c$ is what I refer to as *matching effect*, to distinguish it from the previously considered income and substitution effects. If f_{yh} and f_{xy} are positive, this pushes towards h_x: high type firms are particularly productive with higher hours ($f_{yh} > 0$), and high type workers are particularly productive with high type firms ($f_{xy} > 0$); hence this creates an indirect skill-hours complementarity, reinforcing the direct complementarity given by f_{xh}. Both complementarity types make $h_x > 0$ more likely to happen in equilibrium. Notice that when $f_{yh} = 0$ or $f_{xy} = 0$, the RHS is zero, and the matching effect is shut down; it follows that the labor supply choice by skill boils down to the standard tension between income and substitution effects summarized by $(u_{cc} f_h f_x + u_c f_{hx})$.

Finally, notice that the term $(u_{cc} f_h f_x + u_c f_{hx})$, comprising the opposing forces of income and substitution effects is further multiplied by $-(f_{yy} - V_{yy})$; even though we can't characterize explicitly this term, we can sign it and put an upper bound on it. Using the previous propositions, it is easy to prove that:

$$(f_{yy} - V_{yy}) < \frac{f_{yh} u_c f_{hy}}{u_{cc} f_h f_h + u_c f_{hh} + u_{hh}} \tag{11}$$

The previous inequality further emphasizes how technological and preference forces affect hours choice; in fact, income effects u_{cc} and the curvature of effort disutility u_{hh}, as well as decreasing returns in hours f_{hh} all push towards a lower bound on $f_{yy} - V_{yy}$, which makes $h_x < 0$ more likely to occur (since ceteris paribus, it makes the LHS smaller, as dictated by Proposition 2).

3.3 Examples

In this section, I consider some pratical examples (assuming specific functional forms for production and utility functions, as well as distributions) to help clarifying the main forces at play.

Income Effects I first consider an example to highlight the role of income effects (and in general, or preferences) in driving the main outcomes of the model. I choose a widely used utility function in the macro literature and set $u(c,h,) = \frac{c^{1-\sigma}}{1-\sigma} - \psi\frac{h^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}$, where I allow σ to vary. Higher σ imply higher income effects. For production, I use$f(x,y,h) = A\left(\beta(\alpha x^{\rho} + (1-\alpha)y^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)h^{\gamma}\right)^{\frac{1}{\gamma}}$ with $\gamma, \rho < 1$.
Higher income effects imply two things; first, hours worked decrease for all workers, in line with the standard labor supply model. Second, hours worked decline *more* for high skilled workers: when $\sigma = 0.5$ (light grey line), hours worked are flat across skills. For high income effects ($\sigma = 2$, dark line), low skill worked work longer than high skilled workers. This is precisely due to the fact that the wage function - being increasing in skills - translates into higher income effects for higher skill workers, hence lower hours. For given complementarity, this effect is monotonic in σ.

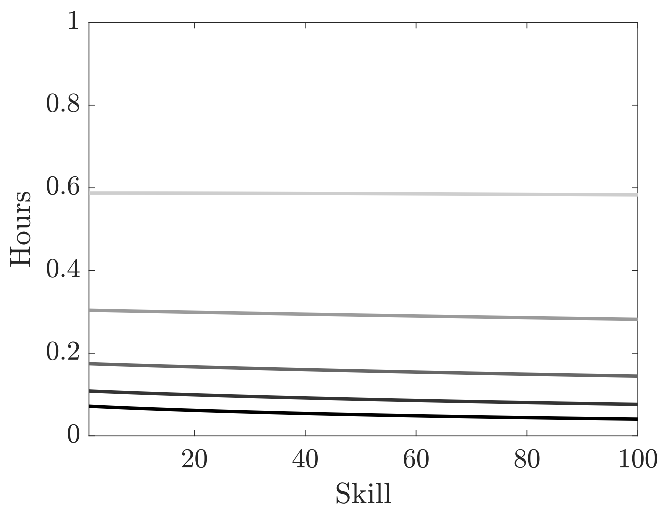

Figure 3: Hours with Varying Income Effects.

Notes: Hours function as function of skill. Utility function as $u(c,h,) = \frac{c^{1-\sigma}}{1-\sigma} - \psi\frac{h^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}$, with $\sigma = 0.8$ (light grey line), $\sigma = 1.8$ (black line). Production as $f(x,y,h) = A\left(\beta(\alpha x^{\rho} + (1-\alpha)y^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)h^{\gamma}\right)^{\frac{1}{\gamma}}$.

Production complementarities In this section I show a simple parametric example to illustrate how technological change can affect the hours decision and wages in equilibrium, taking as given the utility function. I fix σ to be slightly higher than 1, which implies strong income effects. I consider two types of comparative statics, meant to describe the different effects that technological change (understood as a change in the parameter governing the production function) can have in this economy. I consider as before an economy with utility and production function given respecitvely by $u(c,h,) = \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{h^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}$ and $f(x,y,h) = A\left(\beta(\alpha x^{\rho} + (1-\alpha)y^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)h^{\gamma}\right)^{\frac{1}{\gamma}}$, and vary α to simulate the effect of an increase in the importance of skills in production has in this economy. The simulatoin is reported in Figure 4. In this example, as α increases, hours worked become more negative in skills (left panel). This is due to the fact that wages are steeper in skills (right panel). As shown in the previous Section, this is at odds with the data: high wage, high skill workers have increased their hours compared to low skilled workers. Finally, note that aggregate hours go down, as all skills (except the very low skilled) decrease their hours worked.

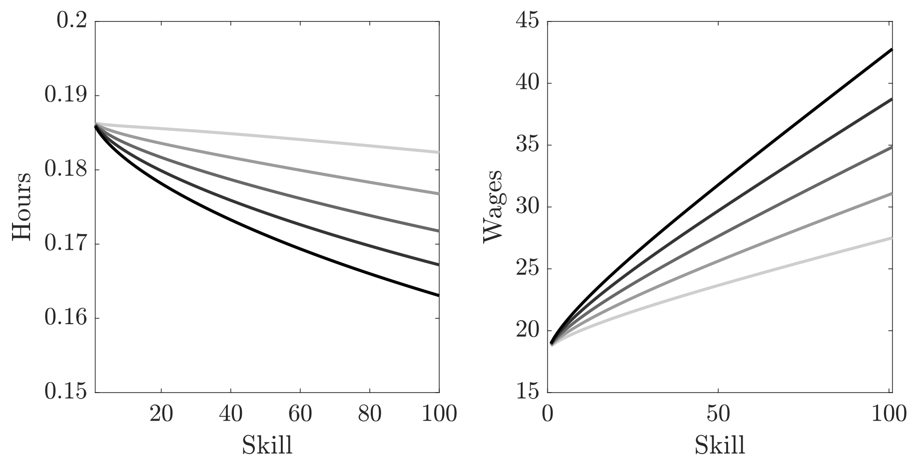

Figure 4: Wages and Hours with Varying Skill Share in Production.

Notes: Wage function as function of skill (left panel); Hours function as function of skill (right panel). Utility function as $u(c,h,) = \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{h^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}$, with $\sigma = 1.1$; . Production as $f(x,y,h) = A\left(\beta(\alpha x^{\rho} + (1-\alpha)y^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)h^{\gamma}\right)^{\frac{1}{\gamma}}$, with $\gamma = 0.8$, $\rho = 0.5$ and $\alpha = 0.1$ (brighter line) and $\alpha = 0.5$ (darker line).

I next consider another comparative static exercise, varying the parameter γ, which represents the complementarity between hours and the skill-job match. I report the exercise in Figure 5. I consider an increase in complementarity (represented by a decrease in γ). The main result is that, other things equal, the hours function turns from negatively sloped to positively sloped: following a decrease in γ, high skilled worker have a higher desire to work. In the example, hours worked rise for most skills (except at the very bottom), implying an increase in the aggregate labor supply.

In summary, we note two things: first, the way the hours profile reacts to technological change crucially depends on the type of technological change we consider. This, in turn, changes wage inequality in different ways. Second, and equally important, this can translate in an overall increase or decrease in the aggregate labor supply. Hence, this examples show that the evolution of the latter crucially on the properties of the production function, in addition to those of the utility function, which is the focus of a large part of the literature on labor supply (e.g. Boppart and Krusell (2020)).

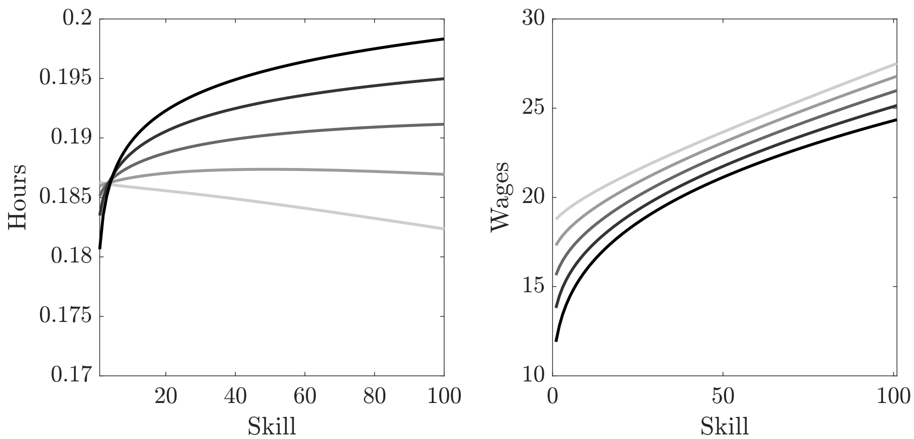

Figure 5: Wages and Hours with Varying Complementarities in Skill/Job and Hours.
Notes: Wage function as function of skill (left panel); Hours function as function of skill (right panel). Utility function as $u(c,h,) = \frac{c^{1-\sigma}}{1-\sigma} - \psi\frac{h^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}$, with $\sigma = 1.1$; . Production as $f(x,y,h) = A\left(\beta(\alpha x^{\rho} + (1-\alpha)y^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)h^{\gamma}\right)^{\frac{1}{\gamma}}$, with $\alpha = 0.3$, $\rho = 0.5$ and $\gamma = 0.7$ (bright red line) and $\gamma = 0.1$ (black line).

3.4 Special Cases

The purpose of this section is to discuss how my framework relates to existing models, deriving the latter as special cases whenever possible. Special emphasis will be put on the two extremes: the macro framework that focused on preferences (King *et al.* (1988), Boppart and Krusell (2020)), and the assignment framework that has largely abstracted from the labor supply decision (with important exceptions discussed in detail below).

Models without hours complementarities With few exceptions, models used to study inequality abstract from complementarities in production between hours and skills, or hours and jobs. Mathematically, this translates into $f_{yh} = f_{xh} = 0$. Earnings are therefore linear in hours worked ($e = w \cdot h$). The analysis in the previous section shows that these models, therefore, always predict PAM in equilibrium (this can be easily seen in Equation 8, where if we set $f_{yh} = 0$, positive sorting only depends on f_{xy}, as in the standard Beckerian framework). Thus, these models potentially miss the effects of hours on the equilibrium sorting pattern, and hence inequality.

Non-linear Earnings A key feature of this framework is that earnings are non-linear in hours worked. Thus, the model in this paper is in close connection to the works by French (2005) and its generalization in Bick *et al.* (2022a) and Erosa *et al.* (2022) . In these models, earnings $e(h)$ typically take the form $e(h) = x \cdot h^{\theta}$, where θ is the elasticity of earnings with respect to hours[21]. My framework can be seen as a more general version of these models, in that I allow for complementarities between skills and jobs. To see this, notice that the earnings function in these models[22] is $e(h) = x \cdot h^{\theta}$ can be microfounded in this framework by a production function where there is no skill-job complementarity $f_{xy} = 0$, but where jobs (or skills) and hours can potentially interact ($f_{yh} \neq 0, f_{xh} \neq 0$). Then, (10) becomes

$$u_{cc} f_h f_x + u_c f_{hx} > 0 \tag{12}$$

Assuming the functional form for earnings $e(h) = x \cdot h^{\theta}$ and $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ - as typically done in this class of models - and substituting terms in (12), it is easy to prove that high skill workers work longer hours if $\sigma = -c\frac{u_{cc}}{u_c} < 1$.

Hours Worked in Balanced Growth Models Balanced growth models have been used to describe long-run behavior of hours worked, specifying utility functions such that - together with rising productivity - a given pattern of hours worked is obtained along the balanced growth path. Leading examples are King *et al.* (1988) and Boppart and Krusell (2020), which specify general utility functions that imply constant and decreasing hours worked, respectively[23]. In these frameworks, the basic labor-leisure choice is summarized by the first order condition[24]:

[21] θ is typically estimated to match the empirical earnings function; French (2005) assumes this elasticity is constant across the hours distribution, i.e. $\theta_h = \bar{\theta}$; Bick *et al.* (2022a) allow for a more general specification, where θ_h can vary across the hours distribution.

[22] Or the more general version $e(h) = x \cdot g(h)$.

[23] The use of utility functions that imply constant hours worked along the balanced growth path is paramount in the literature, and was motivated by the fact that total hours worked in US have been roughly stable over the postwar period. However, as noted for example in Boppart and Krusell (2020) and Kopytov *et al.* (2021), constant hours worked are specific to US and Canada, and are mostly driven by the increase hours worked by women, possibly driven by their increase in the labor force participation. Hence, the need of specifying models that are consistent with decreasing work hours over time. The fact that constant hours worked in US reflect opposing trends between men and women hasl also been noted by Browning *et al.* (1999) and Attanasio *et al.* (2018).

[24] Note that the present framework and frameworks such as Boppart and Krusell (2020) differ in several aspects; in particular, they use of models specified in BGP to speak to long-run data (post-war period or even more). Moreover, they specify not only an intratemporal choice between labor and leisure, but also an intratemporal allocation of consumption and savings. However, both my framework and theirs focus on the intensive margin of hours choice. For this reason, it is sensible to compare the optimal time allocation by workers and how they are related.

$$MRS = -\frac{u_c}{u_h} = w, \tag{13}$$

where w is the wage rate[25], to be equal to the Marginal Rate of Substitution (MRS). Boppart and Krusell (2020) derive a class of utility functions such that, in equilibrium:

$$\frac{u_c}{u_h} = c^{\frac{1}{1-\nu}} q(hc^{\frac{\nu}{1-\nu}}),$$

for some function $q()$. Note that the term $q(hc^{\frac{\nu}{1-\nu}})$ will be constant in the long run, which means the MRS in the long run is driven by the term $c^{\frac{1}{1-\nu}}$. The key implication of this model is that whenever $\nu > 0$, consumption will shrink and relatively more time will be devoted to leisure as an economy grows. Importantly, the rate at which this happens depends on ν, a constant[26].

In my framework, due to sorting, there is a tight linke between the $MRS = -\frac{u_c}{u_h}$ and technology. In other words, Equation (13) becomes:

$$-\frac{u_c}{u_h} = w_h, \tag{14}$$

where w_h is is the impact of one more unit of time on earnings; importantly, it crucially depends on the complementarities in production between x, y and h and the sorting patterns that obtain in equilibrium. This highlights the crucial point of this discussion: in addition to preferences, my framework makes clear how technology (and in particular, complementarities in production) can play a key role in describing hours worked by skill, and therefore in the aggregate. With respect to existing frameworks, it can therefore speak to both cross-sectional and aggregate patterns thanks to the explicit role of technology and preferences that is at the heart of the proposed framework. Therefore, there is the need of specifying and estimating a production function to capture salient aggregate and cross sectional data, which is what I do in the next section.

[25] I omit the time subscripts as my framework is static, unlike Boppart and Krusell (2020). In the absence of intertemporal decisions (e.g. savings choice), one can compare the two frameworks at a given point in time, i.e. for a given productivity level.

[26] When $\nu = 0$, time devoted to leisure is constant along the balanced growth path (and so are hours worked): this happens for example when preferences are of the form $u(c,h) = log(c) - \psi\frac{h^{1+1/\theta}}{1+1/\theta}$ (see King *et al.* (1988).

4 Quantitative Analysis

The goal of this section is to estimate whether and how the production function, as well as the skill and job types distributions have evolved in recent decades, and assess the quantitative relevance of the mechanism presented in the previous section. As I am primarily focused on the effects of technology on hours worked, I consider the period 1980-2015[27], during which the advances in computer and ICT technologies have advanced most rapidly. I will now make clear how I bring the three building blocks of the model to the data (distributions, preferences, production). The estimation will involve a mix between parameters set to match moments from the data, and parameters taken as input of the model.

4.1 Data

CPS has large sample size and is available for the whole period of interest (1980-2015). Moreover, it contains information on hours worked, earnings, and hourly wages. I therefore make use of CPS as the main dataset used in the estimation. When computing the estimation targets, I will retrict the focus to the population of full-time males, aged 25-64, and not self-employed[28]. I focus on the male population only because the analysis abstracts from participation margin[29]. All calculations use provided CPS sample weights.

4.2 Functional Forms

Production I assume a production function of the form:

$$f(x, y, h) = A\left(\beta(\alpha x^\rho + (1-\alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1-\beta)h^\gamma\right)^{\frac{1}{\gamma}}.$$

The advantage of this functional form is that it allows for complementarities between skills and jobs, captured by the parameter ρ as well as between skill/job and hours, captured by γ.[30] This production function can be thought of a generalization of a production function that takes as input skills x and jobs y, and that interprets the recent rise in wage inequality as captured by an increase in the parameter α (or similarly, a decrease in the parameter ρ).

[27] I will estimate the model year by year during this period, where parameters for each year are estimated separately using moments specified below.

[28] For a full description of the data used in the estimation, see Appendix A.

[29] Focusing on the male population is a common approach in the labor supply literature tat focuses on the intensive margin choice, see e.g. Bick *et al.* (2022a).

[30] Note that this is a slightly more general version of the multiplicatively separable production function, of the form $f(x, y, h) = A(\alpha x^\rho + (1-\alpha)y^\rho)^{\frac{\beta}{\rho}} h^{1-\beta}$. As noted for example in Chade and Lindenlaub (2022), the latter is perhaps one of the most commonly used production function for empirical applications, and it is a special case of the production function I employ (in particular, it can be obtained by letting $\gamma \to 0$)

Preferences I make use of the following utility function (MaCurdy (1981)):

$$u(c,h) = \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{h^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} \tag{15}$$

This preference formulation is a slightly more general version of the preferences typically used in balanced growth path models. This specification has several advantages for the purpose of this paper, most notably that the curvature in consumption is parametrized by σ, and hence from the macroeconomic literature on hours worked, we know that $\sigma > 1$ implies substitution effect being dominated by the income effect, other things being equal[31].

To calibrate this utility function, I take common values for σ and θ from the literature, and employ the normalization $\psi = 1$. In particular, I set $\sigma = 1.4$, which is in the mid-range of values from the literature that aims at matching the overall decline in hours worked, both across time (Boppart and Krusell (2020)) and across the development path (Bick *et al.* (2022b)). The latter study is particularly relevant for the calibration of the utility function employed in this paper, in that they estimate a utility function of the form in (15) to match aggregate data on hours worked. I then follow them and set $\theta = 0.49$; this value is also in the range of commonly used values to calibrate intensive-margin elasticities (see Keane (2011) for a survey).

Notice that the calibration I adopt for the utility function is very close to available estimates of the same functional form from cross-sectional studies; in particular, Heathcote *et al.* (2014) employ a similar specification as in (15) and find values of σ and θ of 1.71 and 0.46, respectively [32]. The important takeaway is that the calibration I use is roughly consistent with structural models that focus on *cross-sectional* data (e.g. Heathcote *et al.* (2014)), as well as very recent studies that aim at explaining long-run, *aggregate* data on hours worked (Bick *et al.* (2022b), Boppart and Krusell (2020)). This is important because it will be the focus of the present study, namely sorting on hours and skills, that will help reconcile cross-sectional data over the entire period considered (1980-2015) and shed new light on the patterns of inequality.

[31] This specification has the additional feature that leisure and consumption are separable, i.e. $u_{ch} = 0$. This formulation is convenient in this paper since - as the ultimate objective is to assess the role of technological complementarities in explaining patterns of hours inequality - it effectively shuts down complementarities between hours and consumption (or income) coming from the utility function, and hence isolates the role of technology in explaining the data.

[32] Of course, the latter study differs in other aspects, e.g. the heterogeneity in ψ (which I assume away) or the presence of ininsurable shocks, as well as the data used (PSID, CEX), and hence are not fully comparable with this paper.

Distributions In order to estimate the model, I need to provide skills and job distributions[33]. In the baseline estimation, I treat both skills and job distribution as unobserved, and hence to be estimated. I assume that skills x and jobs y are distributed according to log-normal distributions $log\mathcal{N}(a_x, b_x)$ and $log\mathcal{N}(a_y, b_y)$, respectively. A similar approach has been used extensively in the literature, see e.g. Lise *et al.* (2016). The advantage of this approach is that one does not need to treat worker and job types as observable, but it comes with additional computational cost since more parameters are to be estimated[34].

4.3 Moments and Identification

I estimate the model by Simulated Method of Moments (Pakes and Pollard (1989), McFadden (1989)). In practice, I pick a set of moments m to identify the set of model parameters θ. The estimation procedure uses a global search algorithm to search for the parameter vector θ that minimizes the weighted square distance between model moments $m(\theta)$ and data moments $\bar{m}$:

$$\min_{\theta} \quad (\bar{m} - m(\theta))'\Omega(\bar{m} - m(\theta)),$$

where Ω is a weighting matrix.

I now discuss how each moment is related to the parameter to be estimated. Of course, as typical in these models, each parameter is informed by more than one moment. The TFP level A informs the income level in the model, as it translates into a level shift of the earnings in the economy. Clearly, given the assumed preference specification, A will also inform the average hours worked in the economy. The parameter β informs, too, average hours worked, as it determines the importance of hourse in production (compared do job/skill). The parameters α, ρ and γ crucially determine the income and hours inequality in the model; however, key to identification is that they do so differently, as the comparative statics in the previous section have shown; in fact, while the former two have a negative effect on the wage/hours elasticity, the latter increases wage/hours elasticity in the cross section[35]; in sum, they have an opposite effect on the moment. Moreover, α and ρ inform inequality at different points in the income distribution. The former increases overall inequality, while the latter governs the convexity in the wage function (recall that the model features a com-

[33] I interpret y in my model as jobs rather than firms. This is motivated by the application of the model, and specifically the techonlogical changes at the center of the mechanism. For a similar approach, see e.g. Lindenlaub (2014).

[34] A common alternative approach is to estimate worker and job types from available data, as for example in Chade and Lindenlaub (2022) or in Calvo *et al.* (2021), and use them as input in the estimation of the remaining parameters. As a robustness check of the baseline estimation strategy, I follow this alternative approach trating distributions as observed as in Chade and Lindenlaub (2022). Details on the estimation results following this alternative approach are in **??**.

[35] Again, the assumed preference specification is crucial to this result.

petitive labor market, and hence it features a tight link between the production function and the shape of the income function). Finally, I estimate the worker and job distributions by assuming that they have mean 0, i.e. $a_x = a_y = 0$. With this identification strategy, there are in total seven parameters to be estimated: two from the production function (A, α, β, γ, ρ) and two from the distributions of worker and job types (b_x and b_y)[36]. Table 1 contains a summary of the parameters to be estimated.

Table 1: Endogenous, estimated parameters (time-varying)

Parameter	Meaning
A	Total Factor Productivity (TFP)
β	Importance of hours in production
α	Importance of skill in production (relative to job type)
γ	Elasticity of substitution between hours and skill/job
ρ	Elasticity of substitution between skill and job
b_x	Variance of distribution of skill types
b_y	Variance of distribution of job types

4.4 Estimates

I now present the estimates for the year at the beginning and at the end of the sample (1980 and 2015, respectively) [37]. The estimated parameters are presented in Table 2. The estimates reveal several interesting patterns. First, there has been Skill Biased Tehcnical Change, defined as an increase in the complementarity between skills x and jobs y (ρ has decreased). The marginal product of skill in production, capture by α, has increased. These patterns are at least qualitatively similar to what the literature on wage inequality in US has found.
However, the estimation reveals that other technological changes have taken place. In particular, the estimated model reveals that the complementarity between skills/jobs and hours (captured by γ) has also increased (i.e., γ decreased). Intuitively, this can be interpreted as an increase in the marginal product of one more hour of work for the high skilled workers (independently of the job they are matched with). Besides being conceptually different from parameters governing skill-biased technical change (α, ρ), a decrease in γ will have first order implications for our understanding of inequality, a result I will show in the next section through counterfactual experiments.

[36] I also employ, as a robustness check, a different strategy for identifying skills and job distribution, finding similar parameter estimates for the production function. See footnote 34 and **??** for details.

[37] To support the identification argument in the previous seciton, I also estimate the model parameters separately, year by year for the period 1980-2015.

Table 2: Estimates

Moment	1980	2015	Meaning
A	5002	5620	TFP
β	0.41	0.49	weight of hours in prod.
α	0.22	0.31	weight of skills in prod.
γ	0.31	-0.29	compl. $(h,(x,y))$
ρ	0.78	0.75	compl. (x,y)

5 Results

In light with the estimated technology parameters, I focus on three sets of counterfactuals: the first related to the change in inequality; the second, on the change in the hours-wage elasticity. The third counterfactual relates to the aggregate labor supply: we ask how much aggregate hours would have evolved absent the observed technological changes in production.

5.1 Counterfactuals

Inequality Table 3 show the counterfactual results on wage inequality in the model; I focus on the standard deviation of wages, since it is a targeted moment and hence well captured by the model at both points in time. The table shows the observed change in inequality in the data and in the model (second and third column) and the predicted change in inequality obtained by feeding two parameters according to their estimated value - first α, and then γ - to the esimated model in the 1980. I focus on these two parameters because they represent the larget changes in the estimated values in the two points in time considered.

The main result is that both forces in the model, α and γ, significantly contribute the the observed wage inequality. Thus, the model interprets the increase in inequality in a new fashion - not purely attributed to skills, but rather to the (endogenous) hours choice. Thus, this paper joins contributes to the literature on inequality in pushing for the need to explore the equilibrium sorting patterns from a broad perspective, not uniqueliy related to the sorting of skills and jobs.

Table 3: Counterfactuals: inequality

	Δ Data in %	Δ Model in %	$\Delta\alpha$	$\Delta\gamma$
Wage inequality	21%	21%	4%	7%

Hours-wage elasticity The second counterfactual that we analyze is the change in the cross-sectional hours wage elasticity. Table 4 show the results[38]. In line with the comparative statics exercises in the previous sections, this counterfactual shows that α and γ push the elasticity in the model in opposite ways: the model-implied elasticity would have increased even more, absent changes in α. This is because α pushes towards more inequality, but through income effects, it also pushes towards lower hours worked for the high skilled. Viceversa, if we only feed the model the estimated change in γ, we notice that the hours wage elasticity would have increased even more. The reason is that α increases the marginal product of skills but leaves unchanged the marginal product of one hour worked for the high skilled. On the opposite, γ increases how much a high skill worker can produce if she works longer hours (in terms of complementarities, this would be captured by f_{xh}). Hence, it incentivizes longer hours worked for the high skilled, counteracting the income effect due to higher wages, and pushes up the hours-wage elasticity.

Table 4: Counterfactuals: wage-hours elasticity

	Δ Data	Δ Model	$\Delta\alpha$	$\Delta\gamma$
Hours-wage elasticity	+0.08	+0.07	-0.04	0.10

Total Hours Worked I analyze the third counterfactual on the aggregate labor supply in the economy. This exercise is motivated by the fact that, as explained in the previous sections, models of the aggregate labor supply are traditionally focused on preferences specifications to rationalize the behavior of aggregate hours worked. Here I show that, first, technology is an important determinant of hours worked in the aggregate. Second, which type of technological change matters quantitatively and qualitatively for this conclusion. Table 5 shows the result.

Table 5: Counterfactuals: aggregate hours worked

	Δ Data	Δ Model	$\Delta\alpha$	$\Delta\gamma$
Avg. hours worked	+1%	+1%	-4%	+8%

Once again, the counterfactual predictions depend heavily on the parameter that we consider: when α (γ) is changed, hours worked decline (increase) extensively even in the aggregate. Thus, the cross-sectional patterns reflect in a quantitatively relevant way even in

[38] To facilitate comparison, I report the changes in the elasticity in levels. Thus, for example, the estimate elasticity has changed from 0.002 to 0.085 in the data. Hence, the table reports a change of +0.08.

the aggregate, thus leading us to conclude that technology plays a key role in describing aggregate hours changes in the economy.

These last set of counterfactuals closely relate to those in, for example, Bick *et al.* (2022b). Like in this paper, they consider what forces could determine aggregate hours worked in addition to preferences, and highlight structural transformation as one such force; this paper joins this literature in concluding that that the prediction on the future of hours worked (tracing back at least to Keynes (1930)) rely heavily on other forces (namely, technology), and that such forces are crucial to formulate predictions on the future of work. Importantly, this counterfactual also provides a potential explanation for why hours worked in US have become flat after the 1970's; rather than interpreting hours worked with models where income and substitution effects cancel out, this paper shows that an alternative interpretation is one where income effects prevailed in the 1960's and 1970's; starting from the 1980's, technological change increased the importance of substitution effects, and these two roughly cancel out in the aggregate. In other words, the patterns of hours worked in the aggregate mask substantial heterogeneity.

5.2 Fixed Hours

The purpose of this section is to show that the model mechanism provides new insights about how the endogenous hours decision shape our understanding of increasing inequality. In order to do so, I follow in spirit the exercise in Chade and Lindenlaub (2022), namely I estimate a version of the model in which the hours decision is exogenous. In particular, I feed the model with the estimated endogenous hours decision in the previous section, and re-estimate the model fully (with the same functional forms). The spirit of the exercise is precisely to show how endogeneizing the hours decision - the key new ingredient of my framework - changes our conclusions about the drivers of technological change. Results are shown in Table 6.

The first column reports the estimated parameters in the baseline model, while the second

Table 6: Estimated parameters - model with fixed hours

Parameter	2015 (baseline)	2015 (fixed h)	Meaning
β	0.89	0.91	weight of (x, y) in prod.
α	0.83	0.80	weight of skills in prod.
γ	-0.47	-0.04	compl. $(h, (x, y))$
ρ	-5.5	-5.6	compl. (x,y)

reports the estimated parameters in the model where hours worked are no longer a choice. The key result that emerges is that the complementarity parameter γ is significantly lower, while the parameters governing skill-job complementarity are of similar magnitude. There is a clear intuition behind this: when hours are not a choice, γ does not have to decrease to 'incentivize' the higher hours worked by high skilled, offsetting the stronger income effects they face due to increasing inequality. On the other hand, small changes occur to the other parameters, which drive up inequality in the model according to the data. The implication is that, by not taking into account the endogeneity of hours decision, our understanding of the driving forces causing technological change and inequality would be biased.

5.3 Implications

I now briefly comment on two potential implications that the model results have, beyond thos already discussed. Fully developing on exploring these implications is well beyond the scope of this paper, but I am developing them as separate projects.

Income taxation The labor-leisure trade off, and its determinants, are at the heart of the literature on income taxation. In particular, a key question the literature is trying to answer is whether and how income progressivity should respond in US to the widening income inequality. While the model in this paper does not provide an answer to it, I argue that the view proposed in this paper can be potentially useful on this matter.
To see why, notice that the classic equity-efficiency tradeoff that determines the optimal level of progressivity is typically governed by preference parameters, and in particular the elasticity of labor supply. This framework potentially adds to this literature by showing that a complete understanding of how hours enter in production is an equally important; in fact, complementarities in production of different nature determine how sensitive the hours choice of each worker is and, as such, they may impact the level of progressivity for a given social welfare function. In other words, understanding whether rising income inequality is ultimately driven by x, y or x, h complementarities may matter for the responsiveness of hours to tax progressivity and hence, for the optimal determination of taxes.

Gender Gap/Flexibility The second potential application of the framework I develop in this paper concerns the gender gap, defined as wage differential between men and women. A number of authors have emphasized how the gender gap has been declining at a slower rate, especially for the high skilled (see Goldin (2014) and Cortés and Pan (2019)). The same literature has noted how measures of gender gap are clearly correlated with how earnings respond to hours in the cross-section, as Figure 6 shows.
The fundamental insight of this paper - that technlogical change shape how income responds to hours worked - may help us understand why the gender wage gap has stalled in the

1990's, especially for the high skilled- high wage workers. The intuition is that the adverse effects on wages of social norms or external constraints that prevent women for working longer hours might be have been magnified by technological change. In other words, one lower hour worked for a woman in a top occupation might result in bigger output and earning losses than for a woman in a low-skill occupation, thus preventing the forces that point towards convergence in earnings between men and women to fully close the gap. The additional implication is that misallocation and output costs of hours constrains can be severely understated if we do not consider the non-linear nature of income in hours worked.

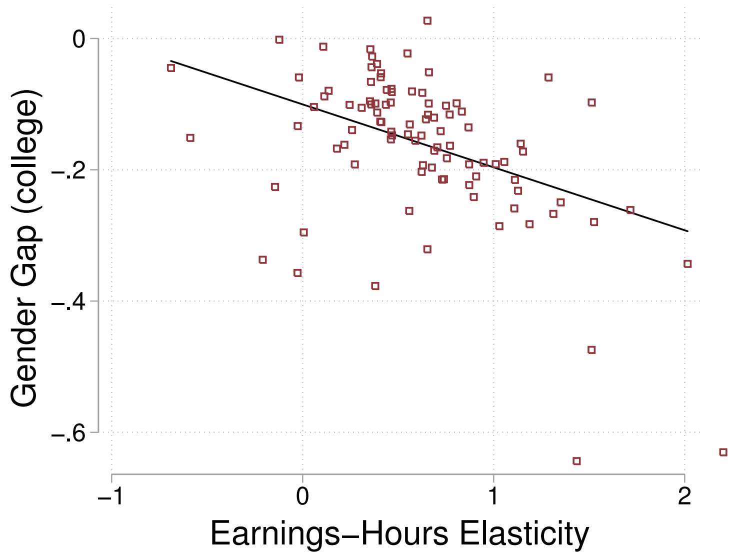

Figure 6: Gender Wage Gap and Hours Elasticity (US).

Notes: See Goldin (2014) for details in the consntruction of the variables.

6 Concluding Remarks

This paper provides a new framework to study the allocation of workers and hours across jobs, and uses it to study the aggregate, as well as the cross sectional relationship between hours worked and wages. I obtain the following results: on the theory side, I show that hours, sorting and wages depend on the properties of the production function as well as the utility function, and I characterize such forces interact in equilibrium. I then analyze how such forces might have contributed to the cross-sectional relationship between hours and wages, and what are the aggregate implications. I conclude that the techological properties of the production function are crucial to have a more complete picture of wage inequality, as well as the future of hours worked. I plan to extend the model to understand the potential impact of progressive taxation on the forces highlighted in the model, and discuss how my findings relate to the literature on the gender gap.

References

AGUIAR, M. and HURST, E. (2007). Measuring trends in leisure: The allocation of time over five decades. *Quarterly Journal of Economics*, **122** (3), 969–1006.

ALTONJI, J. G., BHARADWAJ, P. and LANGE, F. (2009). *Constructing AFQT Scores that are comparable Across the NLSY79 and the NLSY97*. Tech. rep.

ATTANASIO, O., LEVELL, P., LOW, H. and SANCHEZ-MARCOS, V. (2018). Aggregating elasticities: Intensive and extensive margins of women's labor supply. *Econometrica*, **86** (6), 2049–2082.

AUTOR, D. H., LEVY, F. and MURNANE, R. J. (2003). The skill content of recent technological change: An empirical exploration. *Quarterly Journal of Economics*, **118** (4), 1279–1334.

BECKER, G. (1973). A theory of marriage l. *Journal of Political Economy*, **81**, 813–846.

BICK, A., BLANDIN, A. and ROGERSON, R. (2022a). Hours and wages. *Quarterly Journal of Economics*.

—, FUCHS-SCHÜNDELN, N. and LAKAGOS, D. (2018). How do hours worked vary with income? cross-country evidence and implications. *American Economic Review*, **108** (1), 170–199.

—, FÜCHS-SCHUNDELN, N., LAGAKOS, D. and TSUJIYAMA, H. (2022b). *Structural Change in Labor Supply and Cross-Country Differences in Hours Worked*. Tech. rep.

BOERMA, J. and KARABARBOUNIS, L. (2021). Inferring inequality with home production. *Econometrica*, **89** (5), 2517–2556.

BOPPART, T. and KRUSELL, P. (2020). Labor supply in the past, present, and future: A balanced-growth perspective. *Journal of Political Economy*, **128** (1), 118–157.

BROWNING, M., HANSEN, L. P. and HECKMAN, J. J. (1999). Chapter 8 micro data and general equilibrium models. *Handbook of Macroeconomics*, vol. 1, Elsevier, pp. 543–633.

CALVO, P., LINDENLAUB, I. and REYNOSO, A. (2021). *Marriage Market and Labor Market Sorting*. Tech. rep.

CHADE, H., EECKHOUT, J. and SMITH, L. (2017). Sorting through search and matching models in economics. *Journal of Economic Literature*, **49** (4), 961–1075.

— and LINDENLAUB, I. (2022). Risky matching. *Review of Economic Studies*, **forthcoming**.

COCIUBA, S. E., PRESCOTT, E. C. and UEBERFELDT, A. (2018). Us hours at work. *Economics Letters*, **169**, 87–90.

CORTÉS, P. and PAN, J. (2019). When time binds: Substitutes for household production, returns to working long hours, and the skilled gender wage gap. *Journal of Labor Economics*, **37** (2).

COSTA, D. L. (2000). The wage and the length of the work day: From the 1890s to 1991. *Journal of Labor Economics*, **18** (1), 156–181.

EECKHOUT, J. (2018). Sorting in the labor market. *Annual Review of Economics*, **10**.

— and KIRCHER, P. (2018). Assortative matching with large firms. *Econometrica*, **86** (1), 85–132.

— and SEPAHSALARI, A. (2018). *The Effect of Asset Holdings on Worker Productivity*. Tech. rep.

EROSA, A., FUSTER, L., KAMBOUROV, G. and ROGERSON, R. (2022). Hours, occupations, and gender differences in labor market outcomes. *American Economic Journal: Macroeconomics*, **14** (3), 543–590.

FRANCIS, N. and RAMEY, V. (2009). A century of work and leisure. *American Economic Journal: Macroeconomics*, **1** (2).

FRENCH, E. (2005). The effects of health, wealth, and wages on labour supply and retirement behaviour. *Review of Economic Studies*, **72** (2), 395–427.

GOLDIN, C. (2014). A grand gender convergence: Its last chapter. *American Economic Review*, **104** (4).

HEATHCOTE, J., STORESLETTEN, K. and VIOLANTE, G. L. (2014). Consumption and labor supply with partial insurance: An analytical framework. *American Economic Review*, **104** (7), 2075–2126.

JACOBS, J. A. and GERSON, K. (2005). The time divide - work, family, and gender inequality.

KEANE, M. P. (2011). Labor supply and taxes: A survey. *Journal of Economic Literature*.

KENDRICK, J. W. (1961). *Productivity Trends in the United States*. National Bureau of Economic Research, Inc.

— (1973). *Postwar Productivity Trends in the United States, 1948–1969*. National Bureau of Economic Research, Inc.

KEYNES, J. M. (1930). Economic possibilities for our grandchildren.

KING, R. G., PLOSSER, C. I. and REBELO, S. T. (1988). Production, growth and business cycles: I. the basic neoclassical model. *Journal of Monetary Economics*, **21** (2-3), 195–232.

KOPYTOV, A., ROUSSANOV, N. and TASCHERAU-DUMOUCHEL, M. (2021). *Cheap Thrills: the Price of Leisure and the Global Decline in Work Hours*. Tech. rep.

LEGROS, P. and NEWMAN, A. F. (2007). Beauty is a beast, frog is a prince: Assortative matching with nontransferabilities. *Econometrica*, **75** (4), 1073–1102.

LINDENLAUB, I. (2014). Sorting multidimensional types: Theory and application. *Review of Economic Studies*, **84** (2), 718–789.

LISE, J., MEGHIR, C. and ROBIN, J.-M. (2016). Matching, sorting, and wages. *Review of Economic Dynamics*, **19**, 63–87.

MACURDY, T. E. (1981). An empirical model of labor supply in a life-cycle setting. *Journal of Political Economy*, **89** (6), 1059–85.

MCFADDEN, D. (1989). A method of simulated moments for estimation of discrete response models without numerical integration. *Econometrica*, **57** (5), 995–1026.

MICHELACCI, C. and PIJOAN-MAS, J. (2015). Labor supply with job assignmentunder balanced growth. *Journal of Economic Theory*.

PAKES, A. and POLLARD, D. D. (1989). Simulation and the asymptotics of optimization estimators. *Econometrica*, **57** (5), 1027–1057.

SHAO, L., SOHAIL, F. and YURDAGUL, E. (2021). *Labor Supply and Establishment Size*. Tech. rep.

SONG, J., PRICE, D., GUVENEN, F., BLOOM, N. and VON WACHTER, T. (2018). Firming up inequality. *Quarterly Journal of Economics*, **134** (1), 1–50.

VEBLEN, T. (1899). The theory of the leisure class.

A Data Appendix

A.1 Additional Evidence

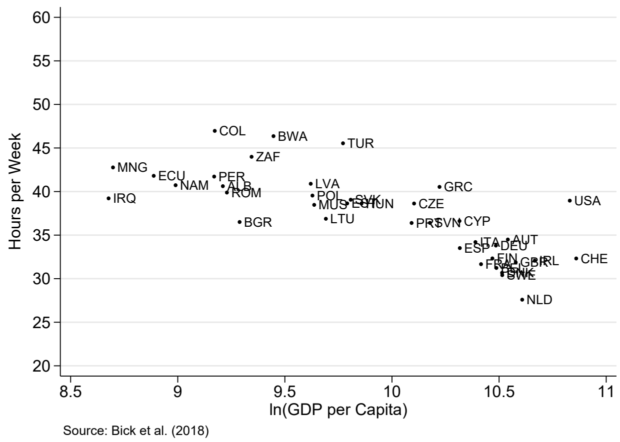

Figure A.1: Hours per worker in middle and rich countries. Source: Bick *et al.* (2018) database.

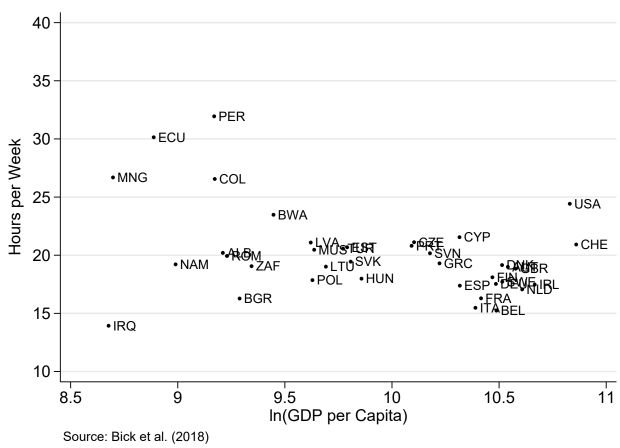

Figure A.2: Hours per adult in middle and rich countries. Source: Bick *et al.* (2018) database.

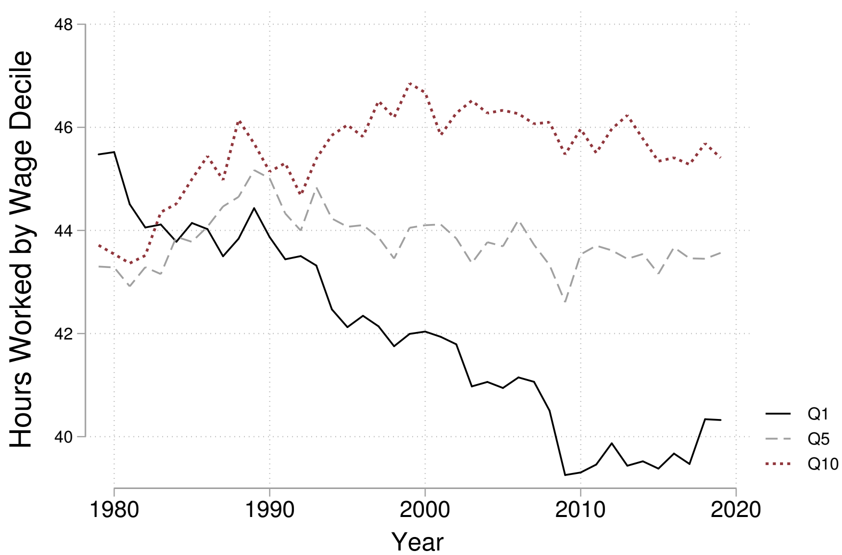

Figure A.3: Hours worked by wage decile (Q1 = decile with lowest wage). Source: CPS and own calculations.

B Theory Appendix

B.1 Conditions for Assortative Matching

Proof of Proposition 1 We write the terms in (8) explicitly as follows:

$$U_y = u_c \cdot f_y + U_h \cdot \frac{\partial h}{\partial y} = u_c \cdot f_y \tag{B.1}$$

$$U_x = u_c \cdot (f_x + f_h h_x) + u_h h_x + U_h \cdot \frac{\partial h}{\partial y} = u_c \cdot (f_x + f_h h_x) + u_h h_x \tag{B.2}$$

$$U_V = -u_c \tag{B.3}$$

And notice that $U_h = 0$ by the envelope theorem since U is maximized with respect to h.

$$U_{xy} = \left(u_{cc} f_y + u_{cc} f_h h_y\right) f_x + u_c \left(f_{xy} + f_{xh} h_y\right) + \left(u_{cc} f_y + u_{cc} f_h h_y\right) f_h h_x + u_c h_x \left(f_{hy} + f_{hh} h_y\right) + u_{hh} h_x h_y \tag{B.4}$$

$$U_{Vx} = \left(-u_{cc} + u_{cc} f_h h_V\right) f_x + u_c \left(f_{xh} h_V\right) + \left(-u_{cc} + u_{cc} f_h h_V\right) f_h h_x + u_c \left(f_{hh} h_x h_V\right) + u_{hh} h_V h_x \tag{B.5}$$

The sorting condition (7) becomes:

$$\begin{aligned} &\underbrace{u_{cc}\left(f_y + f_h h_y\right)\left(f_x + f_h h_x\right) + u_c\left(h_x(f_{hy} + f_{hh} h_y) + f_{xy} + f_{xh} h_y\right) + u_{hh} h_x h_y}_{U_{xy}} - \\ &\underbrace{\left[u_{cc}\left(-1 + f_h h_V\right)\left(f_x + f_h h_x\right) + u_c\left(f_{xh} h_V + f_{hh} h_x h_V\right) + u_{hh} h_V h_x\right]}_{U_{Vx}} \cdot \underbrace{\left(-f_y\right)}_{\frac{U_y}{U_V}} > 0 \end{aligned} \tag{B.6}$$

We can simplify this expression further, by getting explicit expressions for h_y and h_V using the implicit function theorem. First, notice that the first order condition of the household with respect to hours h is:

$$u_c(\underbrace{f - V}_{w}) w_h + u_h = 0 \tag{B.7}$$

Denote $F(y, V, h(y, V)) = u_c w_h + u_h$. By virtue of the theorem applied to (B.7) we can write:

$$\frac{\partial F}{\partial V} + \frac{\partial F}{\partial h} h_V = 0$$

from which we have that:

$$\underbrace{-u_{cc} w_h}_{\frac{\partial F}{\partial V}} + \underbrace{(u_{cc} f_h w_h + u_c w_{hh} + u_{hh})}_{\frac{\partial F}{\partial h}} h_V = 0 \tag{B.8}$$

Rearranging and multiplying by h_x, and noting that f_h and w_h are interchangeable since $w = f - V$:

$$h_x u_{cc} f_h = h_x h_V (u_{cc} f_h f_h + u_c f_{hh} + u_{hh}) \tag{B.9}$$

Analogously, by noting that $\frac{\partial F}{\partial y} + \frac{\partial F}{\partial h} h_y = 0$ by the implicit function theorem applied to (B.7), we get:

$$\underbrace{f_h u_{cc} w_h}_{\frac{\partial F}{\partial y}} + \underbrace{(u_{cc} f_h w_h + u_c w_{hh} + u_{hh})}_{\frac{\partial F}{\partial h}} h_y = 0 \tag{B.10}$$

Hence, similarly to before we can rearrange and multiply by h_x to get:

$$f_h h_x u_{cc} f_h = h_x h_y (u_{cc} f_h f_h + u_c f_{hh} + u_{hh}) \tag{B.11}$$

Use (B.9) and (B.11) in the sorting condition above (B.6) to get:

$$\begin{aligned} &u_{cc}(f_y + f_h h_y) f_x + u_c(f_{xh} h_y) + u_c(f_{xy} + f_{hy} h_x) - \\ &\quad (u_{cc}(-1 + f_h h_V) f_x + u_c(f_{xh} h_V)) \cdot (-f_y) > 0 \end{aligned} \tag{B.12}$$

Notice that (B.8) and (B.10) imply that:

$$\begin{aligned} -1 &= -F_h h_V \frac{1}{u_{cc} w_h} \\ f_y &= -F_h h_y \frac{1}{u_{cc} w_h} \end{aligned}$$

where $F_h = \frac{\partial F}{\partial h}$. Rearranging the latter two equations, we have that:

$$\frac{-1}{h_V} = \frac{f_y}{h_y} \implies -f_y h_V = h_y$$

Using $-f_y h_V = h_y$ in (B.12), we are simply left with:

$$f_{xy} > -f_{hy} h_x \tag{B.13}$$

since u_c is assumed to be positive.

This is the condition expressed in Corollary 1. To fully express this in terms of primitives, we further write h_x explicitly as follows. Using the implicit function theorem, we have that

$$h_x = -\frac{\begin{vmatrix} \frac{\partial U_h}{\partial x} & \frac{\partial U_h}{\partial y} \\ \frac{\partial U_y}{\partial x} & \frac{\partial U_y}{\partial y} \end{vmatrix}}{|H|}$$

The determinant of the Hessian of the problem, appearing in the denominator, is equivalent to B.15. We can write explicitly the terms in the numerator as:

$$\begin{aligned}
U_{xh} &= u_{cc} f_x f_h + u_c f_{xh} \\
U_{yx} &= u_{cc} f_x (f_y - V_y) + u_c f_{xy} = u_c f_{xy} \\
U_{hy} &= u_{cc} f_h (f_y - V_y) + u_c f_{hy} = u_c f_{hy} \\
U_{yy} &= u_{cc} (f_y - V_y)(f_y - V_y) + u_c (f_{yy} - V_{yy}) = u_c (f_{yy} - V_{yy})
\end{aligned}$$

We can plug the resulting expression for h_x in B.13 to get:

$$f_{xy} > -f_{hy} \left[\frac{-u_c (f_{yy} - V_{yy})(u_{cc} f_x f_h + u_c f_{xh}) - u_c f_{hy} u_c f_{yx}}{u_c (f_{yy} - V_{yy})(u_{cc} f_h f_h + u_c f_{hh} + u_{hh}) - u_c f_{hy} u_c f_{yh}} \right]$$

which becomes (simplifying u_c and bringing the denominator to the left hand side):

$$f_{xy} \left[(f_{yy} - V_{yy})(u_{cc} f_h f_h + f_{hh} + u_{hh}) - u_c f_{hy} u_c f_{yh} \right] > -f_{hy} \left[(f_{yy} - V_{yy})(u_{cc} f_x f_h + u_c f_{xh}) - u_c f_{hy} u_c f_{yx} \right]$$

which simplifies to

$$f_{xy} \left[(f_{yy} - V_{yy})(u_{cc} f_h f_h + f_{hh} + u_{hh}) \right] > -f_{hy} \left[(f_{yy} - V_{yy})(u_{cc} f_x f_h + u_c f_{xh}) \right]$$

We can finally divide both the rhs and lhs of the previous equation (note that $f_{yy} - V_{yy} < 0$

so we switch sign):

$$f_{xy}\left[(u_{cc}f_hf_h + f_{hh} + u_{hh})\right] > -f_{hy}\left[(u_{cc}f_xf_h + u_cf_{xh})\right]$$

which is the condition in the main text. ■

Alternative Proof of Proposition 1 We can repeat the derivation before using a similar method. This derivation makes use of the Hessian of the second order condition of the problem to derive the PAM (NAM) condition, similarly to Eeckhout and Kircher (2018). Notice that we start with the same problem:

$$U(x,y,V) = \max_{y,h} \quad u\left(f(x,y,h) - V, h\right) \tag{B.14}$$

This time, we don't make use of U as a matching problem (and the solution method in Eeckhout (2018) and Eeckhout and Sepahsalari (2018)), but rather derive PAM/NAM conditions based on the Hessian of the problems. We take the FOCs, which are:

$$\begin{aligned} U_h = 0 &\implies u_cf_h + u_h = 0 \\ U_y = 0 &\implies u_c(f_y - V_y) = 0 \end{aligned}$$

The second order condition of the problem require that the Hessian $\mathbf{H}$ is negative definite. In this case, the Hessian $\mathbf{H}$ is:

$$\mathbf{H} = \begin{bmatrix} \frac{\partial U_h}{\partial h} & \frac{\partial U_h}{\partial y} \\ \frac{\partial U_y}{\partial h} & \frac{\partial U_y}{\partial y} \end{bmatrix}$$

For $\mathbf{H}$ to be negative definite, we require the determinants of the principal minors to have alternating signs, starting with negative sign. This is equivalent to say that we need $\frac{\partial U_h}{\partial h} < 0$ and $|\mathbf{H}| > 0$, where $|\mathbf{H}|$ is the determinant of $\mathbf{H}$. We can write each term composing $|\mathbf{H}|$ as follows [39]:

[39] We use the notation U_{hh} to express $\frac{\partial U_h}{\partial h}$, and similarly for other terms.

$$U_{hh} = u_{cc} f_h f_h + u_c f_{hh} + u_{hh}$$
$$U_{hy} = u_{cc}(f_y - V_y) f_h + u_c f_{hy}$$
$$U_{yh} = u_{cc}(f_y - V_y) f_h + u_c f_{yh}$$
$$U_{yy} = u_{cc}(f_y - V_y)(f_y - V_y) + u_c(f_{yy} - V_{yy})$$

hence for optimality, we require $U_{hh} = u_{cc} f_h f_h + u_c f_{hh} + u_{hh} < 0$ and $U_{hh}U_{yy} - U_{hy}U_{yh} > 0$. Using the explicit expressions derived just above, we can write the latter inequality as:

$$\begin{aligned} U_{hh}U_{yy} - U_{hy}U_{yh} = (u_{cc} f_h f_h + u_c f_{hh} + u_{hh}) \left(f_{yy} - V_{yy} \right) \\ - (u_c f_{hy})(f_{yh}) > 0, \end{aligned} \tag{B.15}$$

where we divded both terms in B.15 by u_c since it is assumed to be positive. We can get more intuition if we write the term $f_{yy} - V_{yy}$ explicitly. To do so, we differentiate the second FOC ($f_y - V_y$) with respect to y, along the equilibrium allocation:

$$f_{yx}\mu_y + f_{yy} + f_{hy}h_y - V_{yy} = 0$$

which implies

$$f_{yy} - V_{yy} = -f_{yx}\mu_y - f_{hy}h_y$$

Use this into B.15 to have:

$$\begin{aligned} &= -u_{cc} f_h f_h f_{yx}\mu_y - u_c f_{hh} f_{yx}\mu_y - u_{hh} f_{yx}\mu_y \\ &- u_{cc} f_h f_h f_{hy} h_y - u_c f_{hh} f_{hy} h_y - u_{hh} f_{hy} h_y - u_c f_{hy} f_{yh} > 0 \end{aligned} \tag{B.16}$$

Next, we want to write h_y in the explicitly in the previous expression. To do so, differentiate the first FOC ($U_h = 0$) with respect to worker type, to get:

$$u_{cc}(f_x\mu_y + f_Y + f_h h_y - V_y)f_h + u_c f_{hx}(\mu_y + f_{hy} + f_{hh}h_y) + u_{hh}h_y = 0$$
$$u_{cc}(f_x\mu_y + f_h h_y)f_h + u_c(f_{hx}\mu_y + f_{hy} + f_{hh}h_y) + u_{hh}h_y = 0$$

Rearrange and multiply both sides by $h_y f_{hy}$ to get:

$$f_{hy}(u_{cc}f_x f_h \mu_y + u_c f_{hx}\mu_y + u_c f_{hy}) = (-u_{cc}f_h f_h - u_c f_{hh} - u_{hh})h_y f_{hy} \tag{B.17}$$

Use the terms on the right hand side of (B.17) into (B.16) and rearrange to get:

$$\mu_y(f_{yx}(-u_{cc}f_h f_h - u_c f_{hh} - u_{hh}) + f_{hy}(u_{cc}f_x f_h + u_c f_{hx})) > 0 \tag{B.18}$$

which is the condition expressed in the main text. ■

Proof of Proposition 2 [tbc]