

# Hours-Biased Technological Change

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BSE & UPF

Job Market Paper

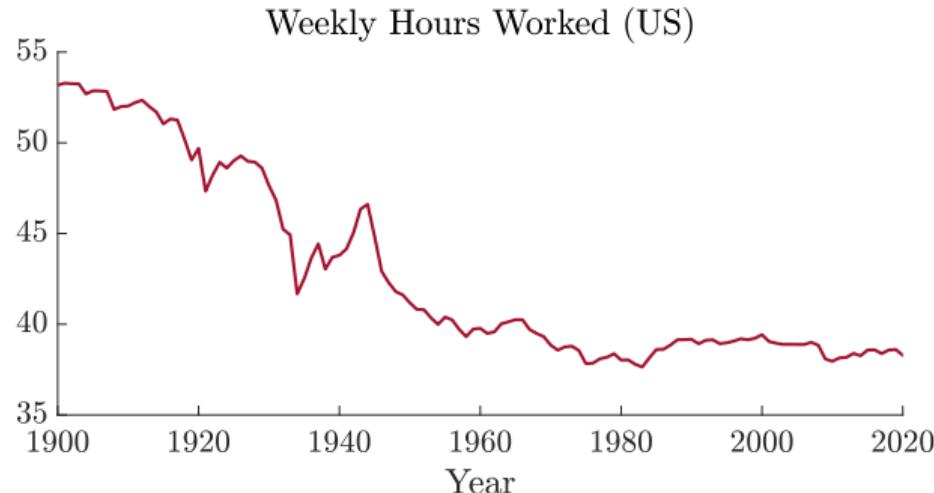
October 23, 2022

# Intro: The Evolution of Working Time

How did work hours evolve over time?

# Hours Decrease Over Time

- Hours per worker **decline** over the very long run.



► Cross-country

► Cross-country: over time

► Cross-country vs US

► Hours/capita

► By Gender

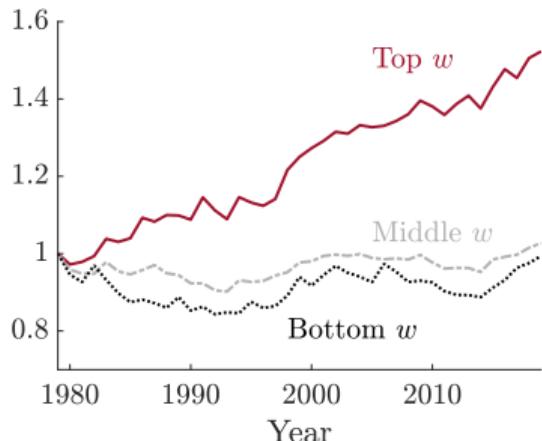
► Extensive Margin

# Intro: The Evolution of Working Time

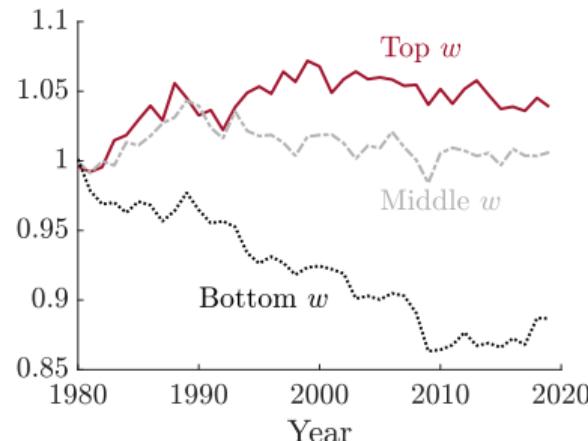
- Hours decline with wages in the long-run...
  - Leading explanation: income effect > substitution effect (wage ↑, hours ↓)
  - Keynes(1930), Boppart and Krusell (2020)

# Hours Increased For High-Wage Workers

- Cross-section: Wages increase relatively more at the top (wage inequality ↑)
- High-wage workers **increased** their hours



(a) Wages



(b) Hours

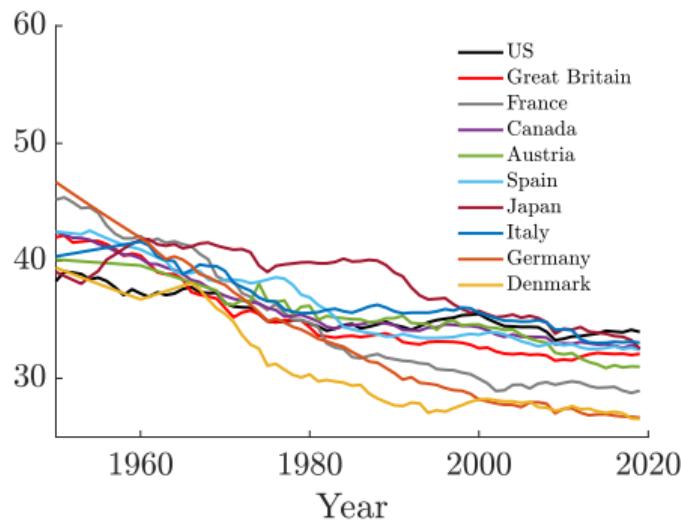
Figure: Wages and Hours By Wage Decile  $w$  (US)

## Intro: The Evolution of Working Time

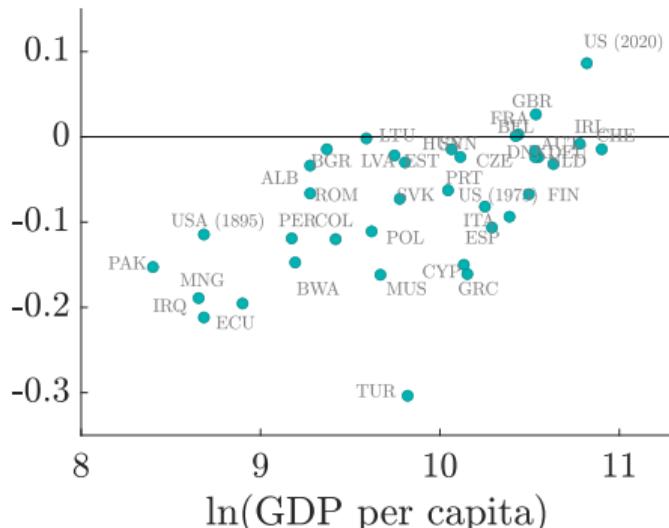
- Hours decline with wages in the long-run...
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- ...but not in the cross-section
  - Workers who gained more also started working more
  - $\Rightarrow$  Hours-wage correlation turns from negative to positive ► Regression
  - **Puzzle:** if income effects dominates, we would expect the opposite

# Common Patterns Across Countries

- Long-run declines in hours worked
- Cross-sectional hours-wage relationship increases with development



(a) Hours Worked



(b) Hours-Wage Correlation (cross-section)

Figure: Cross-country patterns

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  - **Puzzle:** if income effects dominates, we would expect the opposite
- Similar patterns occur across countries and time
- How can we reconcile these patterns? Answer to this question is key for:
  - Tax Policy
  - Understanding inequality
  - Business cycles fluctuations

## Hours-Biased Technological Change

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  - E.g. cheaper and more powerful computers
  - SBTC: technologies increase productivity of skilled workers (vs unskilled)

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  - Increase in rel. productivity of working hours for skilled
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Can HBTC reconcile patterns of hours worked? What are the implications?

## This Paper

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  - Can offset income effects from SBTC

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- Key mechanism: skill-hours complementarity  $\uparrow \implies$  reward for long hours  $\uparrow$ 
  - Endogenous higher work hours for skilled
  - Can offset income effects from SBTC
- Quantitatively analyze technological change in US
  - Novel channel of technology on **inequality**
  - New driver of aggregate hours worked in US (dampen income effect)

# Literature

- **Aggregate Hours:** King, Plosser, Rebelo (1988); Prescott (2004); Ramey and Francis (2009); Boppart and Krusell (2020); Rachel (2021); Bick et al. (2021).  
*Take into account cross sectional heterogeneity*

- **Leisure inequality:** Costa (2000); Aguiar and Hurst (2007); Kopitov et al. (2020); Boppart and Ngai (2021); Michelacci and Pijoan-Mas (2014); Doepke et al. (2018).

*Introduce sorting and endogenous wage inequality*

- **Sorting:** Eeckhout and Kircher (2018); Eeckhout and Sepahsalari (2018); Calvo, Lindenlaub, Reynoso (2021); Michelacci and Pijoan-Mas (2014); Chade and Lindenlaub (2022).

*Characterize sorting for general preferences and technology*

# Model

# Setup

- Population
  - Individuals with skill  $x \sim G^x$
  - Jobs of type  $y \sim G^y$
- Preferences
  - Choose hours  $h$  and consumption  $c$
  - Utility function:  $u(c, 1 - h)$
- Technology
  - Competitive labor market
  - Workers and jobs(firms) match one-to-one  $\implies$  sorting
  - Output:  $f(x, y, h)$
  - Note:  $h = h(x)$  is endogenous and skill dependent

# Setup

- Payoffs
  - Workers maximize:  $u(c, 1 - h)$  s.t.  $c = e(x, h)$
  - Jobs maximize:  $f(x, y, h) - e(x, h)$
  - In equilibrium, wages  $w$  are defined as  $w = \frac{e}{h}$ , where  $w = w(x, y, h(x, y))$
- Market clearing
  - Equilibrium matching  $y = \mu(x)$  assigns workers  $x$  to jobs  $y$ .
  - Under Positive Assortative Matching (PAM), market clearing requires:

$$\int_{\mu(x)}^{\bar{y}} g^y(s) ds = \int_x^{\bar{x}} g^x(s) ds ,$$

where  $g^x$  and  $g^y$  are the densities of workers and jobs, respectively. » Details

- Competitive equilibrium = optimality + market clearing:
  - income  $e(x)$ ; hours  $h(x)$ ; matching  $\mu(x)$  » Full definition

# Problem as Pareto Frontier

## Pair's problem as Pareto Frontier

- Workers  $x$  and jobs  $y$  simultaneously solve, respectively:

$$\max_{h,y} u(c, 1 - h) \quad \text{s.t.} \quad c = e$$

$$\max_x f(x, y, h) - e$$

- This can be re-written as pair  $x, y$  solving:

$$U(x, y, \pi) = \max_h u(f(x, y, h) - \pi, 1 - h)$$

where  $U(x, y, .)$  defines the Pareto Frontier (for all profit values  $\pi$ ).

# Optimal Job Choice

## Optimization

- Optimal job choice implies positive sorting iff:

$$U_{xy} - \frac{U_y}{U_\pi} U_{x\pi} \geq 0 \iff -\frac{U_y}{U_\pi} \text{ increasing in } x \quad \text{▶ Proof}$$

- **Intuition:** high  $x$  more willing to give more  $\pi$  (=less income) for a better  $y$
- Trade-off between **preferences** ( $U_\pi$ ) vs **complementarity** ( $U_y$ ) across skills

# Assortative Matching

Check  $-\frac{U_y}{U_\pi} \uparrow x$

**Proposition:** *The equilibrium features Positive Assortative Matching iff:*

$$f_{xy} + f_{yh} h_x \geq 0$$

» Alternative Condition

» Decentralized eq.

- **Intuition:** two forces pushing towards positive sorting (PAM)
  - Skill-job complementarity  $f_{xy}$  (**standard**)
  - High skill working more ( $h_x > 0$ ) + hours-job complementarity  $f_{yh} > 0$  (**new**)
- Low skill working much more ( $h_x << 0$ ) and  $f_{yh} > 0$  can lead to NAM

## Hours choice

- FOC for hours implies  $u_c f_h + u_h = 0$ . From this, we get that:

**Proposition:** *In equilibrium, high skill choose higher hours if:*

$$\underbrace{u_c f_{hx}}_{\text{subs. effects}} > \underbrace{-u_{cc} f_h f_x}_{\text{income effects}}$$

» Derivation

» CRRA

- **Income effects** (recall  $u_{cc} < 0$ )
  - Key force to explain long run behavior of hours (Boppart and Krusell, 2020)
- **Hours complementarities:**  $f_{xh}$ 
  - If positive, raises marginal product of hours worked for high skill  $\implies$  HBTC

# Functional forms

- Production:  $f(x, y, h) = A \left( \beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)h^\gamma \right)^{\frac{1}{\gamma}}$ 
  - Captures both SBTC (through  $\alpha, \rho$ ) and HBTC (through  $\gamma$ )
- Preferences (MacCurdy, 1981):  $u(c, h) = \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{h^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}$ 
  - $\sigma$  = strength of income effects

# Comparative Statics

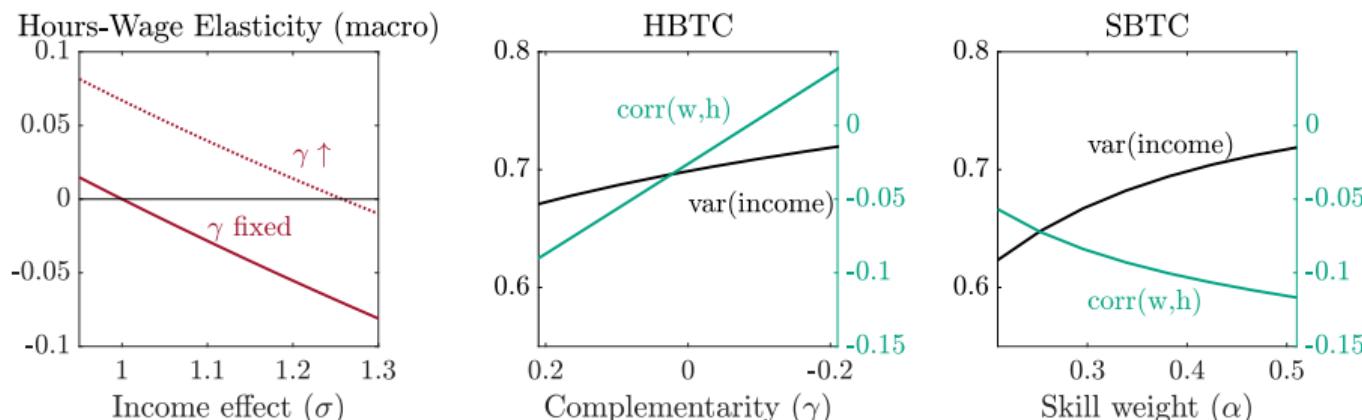


Figure: Comparative Statics.

Key takeaways:

- Hours complementarities ( $\gamma$ ) matter for both *macro* and *micro* elasticities
- HBTC vs SBTC drive inequality up, but opposite effect on micro elasticity

## Special Cases

- Becker (1973) sorting model
  - $f_{yh} = f_{xh} = 0$ . Sorting only depends on  $f_{xy}$
- Macro labor supply models with linear earnings
  - Hours affect earnings **linearly** (Boppart and Krusell, 2020)
  - Misses cross-sectional facts
- SBTC frameworks (Katz-Murphy, Acemoglu-Autor)
  - Capture increase in rel. productivity of skilled workers (extensive margin)
  - Silent about rel. productivity of hours (intensive margin)
- 'Effective types' models
  - Firms match with **bundle**  $\tilde{x} = \tilde{x}(x, h) \Rightarrow f = f(\tilde{x}, y)$
  - Used to study sorting and **optimal taxation** (Scheuer and Werning, 2016)
- Non-linear earnings (Erosa et al., 2016; Bick et al., 2021)
  - hours choice depends on  $f_{xh}/u_{cc}$  (no matching effect)

# Estimation

## Overview

- Can changes in **technology** rationalize the data? Recall the prod. function:

$$f(x, y, h) = A \left( \beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)h^\gamma \right)^{\frac{1}{\gamma}} \quad (1)$$

- Estimate parameters year by year to match moments from the data (SMM).  
 $\implies A = A_t, \alpha = \alpha_t, \beta = \beta_t, \gamma = \gamma_t, \rho = \rho_t.$
- Note: (1) is equivalent to

$$f(x, y, h) = \left( (A_x x^\rho + A_y y^\rho)^{\frac{\gamma}{\rho}} + A_h h^\gamma \right)^{\frac{1}{\gamma}}$$

- **Goal:** Assess changes in weights ( $A_t, \alpha_t, \beta_t$ ) and complementarities ( $\rho_t, \gamma_t$ )

# Strategy

- **Preferences:** calibrate  $\sigma$  and  $\theta$ 
  - $\sigma$ : mid-range of macro studies (1.4)
  - $\theta = 0.4$ : consistent with micro-evidence (Violante et al, 2014).
- **Distributions**  $G^w$  and  $G^f$ 
  - Assume log-normality:  $x \sim \mathcal{LN}(\mu_x, \sigma_x)$ ;  $y \sim \mathcal{LN}(\mu_y, \sigma_y)$
  - Set  $\mu_x = \mu_y = 0$  and estimate ratio  $\sigma_x / \sigma_y$
- Six parameters to be estimated (5 technology, 1 distributions)

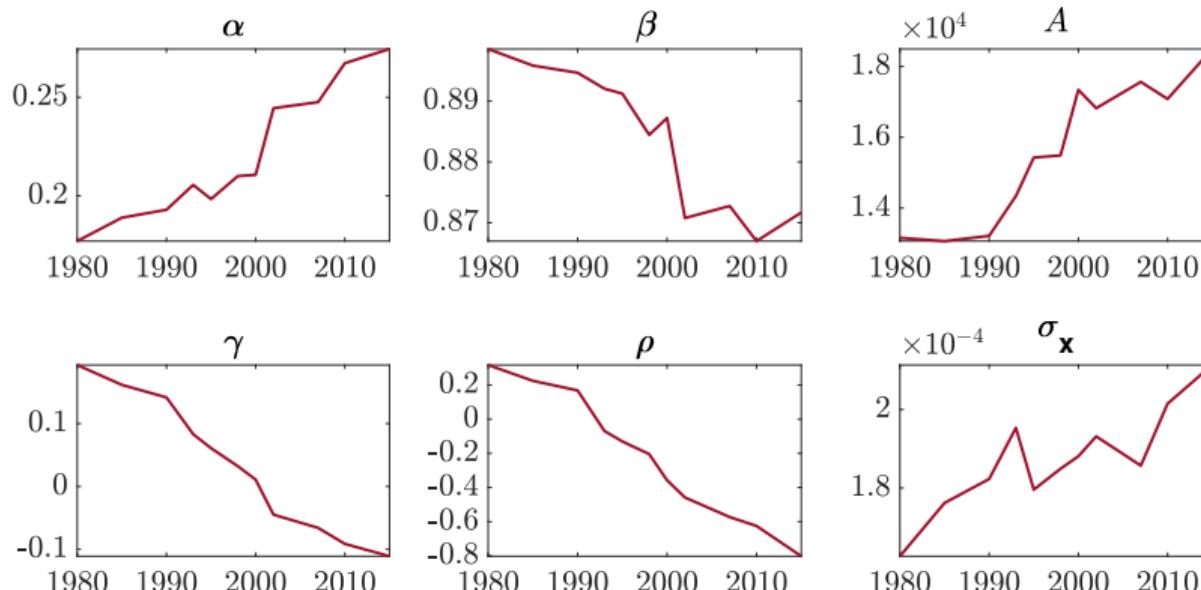
E(h)	Avg. Hours
E(w)	Avg. Wages
w90/w50	wage inequality (top)
w90/w10	wage inequality (overall)
Hours-wage Elasticity	Coeff. of reg. $\log(h)$ on $\log(w)$
std(w)	wage dispersion

Table: Summary of Targeted Moments

# Estimates

- Both SBTC ( $\alpha \uparrow$  and  $\rho \downarrow$ ) and HBTC ( $\gamma \downarrow$ ) ➡ Model Fit

$$f(x, y, h) = A \left( \beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)h^\gamma \right)^{\frac{1}{\gamma}}$$



# Income Inequality

- Decompose *income* inequality in:

$$\text{var}(e) = \text{var}(w) + \text{var}(h) + 2\text{cov}(w, h)$$

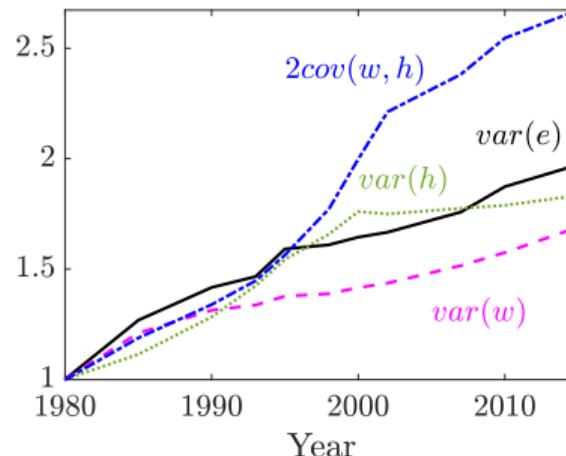
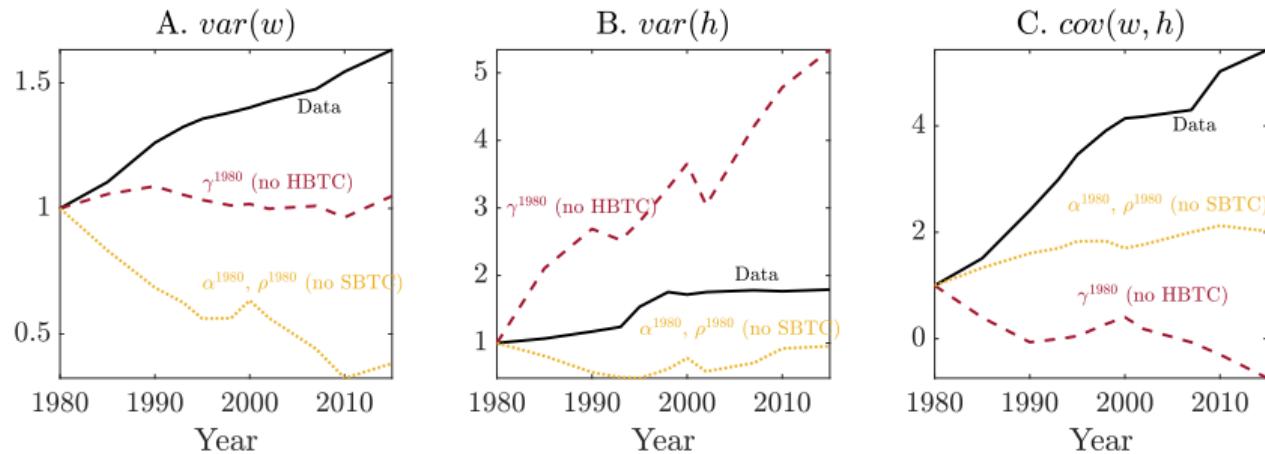


Figure: Drivers of Income Inequality (US)

- Different drivers of inequality. How does the model rationalize this?

# Results: Drivers of Inequality



## Results:

- HBTC ( $\gamma \downarrow$ ) contributes to income inequality through  $\text{cov}(w, h)$
- $\alpha$  ( $\approx$  SBTC) more important for  $\text{var}(w)$  but pushes  $\text{cov}(h, w)$  down

## Results: Fixed Hours Model

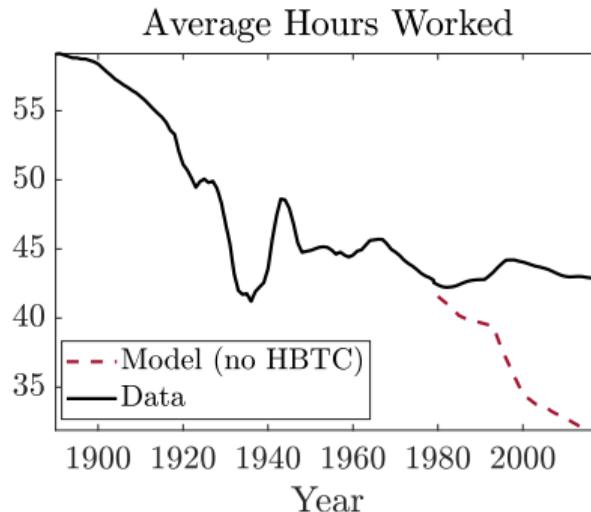
What is the role of **endogenous** hours response?

$$f(x, y, h) = A \left( \beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)h^\gamma \right)^{\frac{1}{\gamma}}$$

Estimated Change 1980-2015		
Parameter	Baseline	Fixed-hours
$\beta$	-60%	+2%
$\alpha$	+199%	+102%
$\gamma$	-202%	-88%
$\rho$	+294%	+1200%

Fixed-h model **overstates** magnitude of SBTC ( $\alpha, \rho$ ) and **understates** HBTC ( $\gamma \downarrow$ )

## Results: Aggregate Hours



- Explanation for relatively flat average hours worked in US post-1970
  - One explanation: income = substitution effect
  - This paper: income effect (Boppart and Krusell, 2020) + HBTC
- Across development path, relative strength of these two forces vary

# Robustness

On preferences:

- Income effects (different  $\sigma$ ) [» Results](#)
- Idiosyncratic tastes for leisure [» Results](#)

On distributions:

- Take distributions from the data [» Distributions](#) [» Results 1](#) [» Results 2](#)
- Assume Beta distributions (Lise, Meghir, Robin 2016) [» Results](#)
- Estimate only  $\sigma_x$  [» Fit](#) [» Estimates](#)

On technology/estimation:

- Different nest of CES function [» Nest 2: Simulation](#) [» Nest 2: Results](#) [» Nest 3: Results](#)  
[» Nests: Comparison](#)
- Simulate increase in tax progressivity (in progress)
- Standard errors + Sensitivity matrix (Shapiro et al, 2017 QJE) ✓

# Conclusion

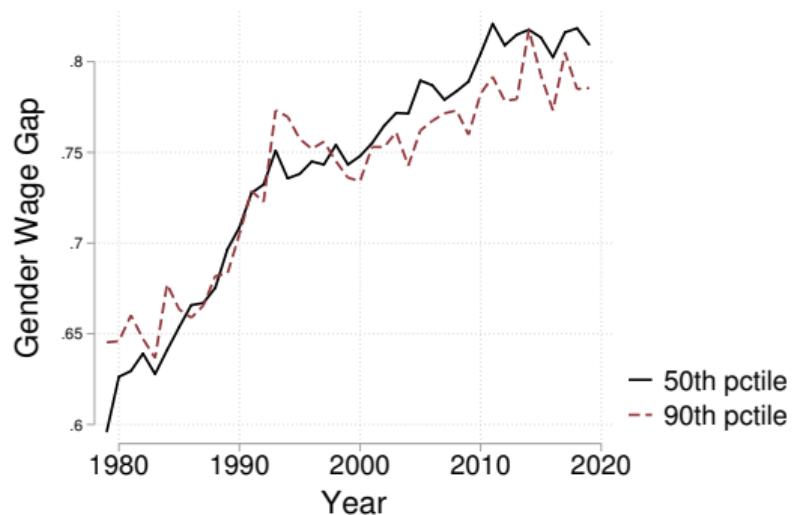
- Develop a theory that features effect of endogenous hours choice on sorting and inequality
  - Novel condition for assortative matching
- Novel mechanism: Hours-Biased Technological Change
  - New technologies raise returns to long hours for the high skill
- Estimate the framework: 1980-2015 in US
  - HBTC reconciles hours in the cross-section and time series
  - Accounts for roughly 30% of increase in income inequality

## Future Work

- How do **hours constraints** (e.g. social norms) impact sorting?
  - Recall sorting condition:  $f_{xy} + f_{yh} h_x$
  - If hours choice is **distorted**, this can affect misallocation and inequality
- Application: the (stalling) gender gap  **Empirics**
  - Gender gap stalled for the high skilled since 1990's (Blau and Kahn, 2014)
  - Gap is higher in occupations where hours elasticity is higher (Goldin, 2014)
- Can HBTC provide an explanation for stalling gender gap? Two forces:
  - Over time, easier for women to provide longer hours (hours constraints ↓)
  - Technology amplifies effect of such constraints

# Implications: Hours Constraints and the Gender Gap

Gender gap has been decreasing across the wage distribution.

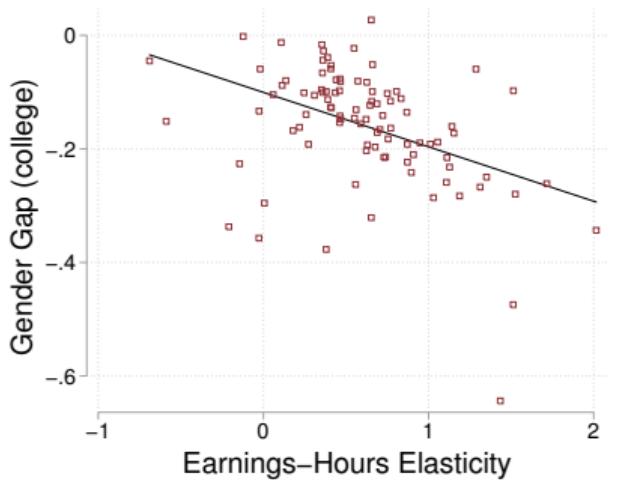


However, slower decline at the top since 1990's.

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# Implications: Hours Constraints and the Gender Gap

Tight link between gender gap and how income responds to hours (Goldin, 2014).



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# Distributions: From the Data

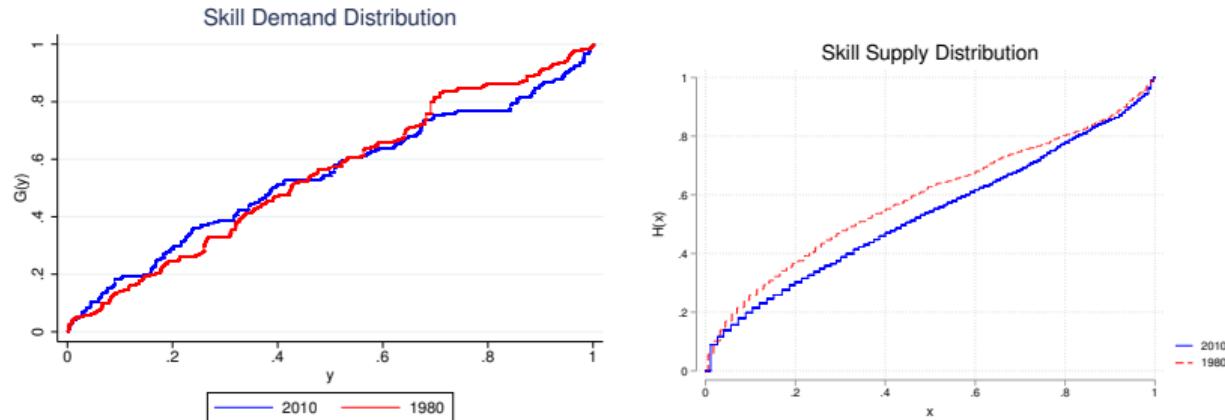


Figure: Estimated skill demand (left) and supply (right) distributions, for 1980 and 2010.

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## Estimates: Fitted Distributions

Parameter	1980	2015	Meaning
$\beta$	0.91 (0.02)	0.83 (0.02)	weight of $(x, y)$ in prod.
$\alpha$	0.24 (0.04)	0.79 (0.08)	weight of skills in prod.
$\gamma$	0.22 (0.02)	-0.39 (0.02)	compl. $(h, (x, y))$
$\rho$	-1.58 (0.27)	-4.24 (1.0)	compl. $(x, y)$
$A$	183.1 (27)	127.1 (1.0)	TFP

Notes: Standard errors in parentheses.

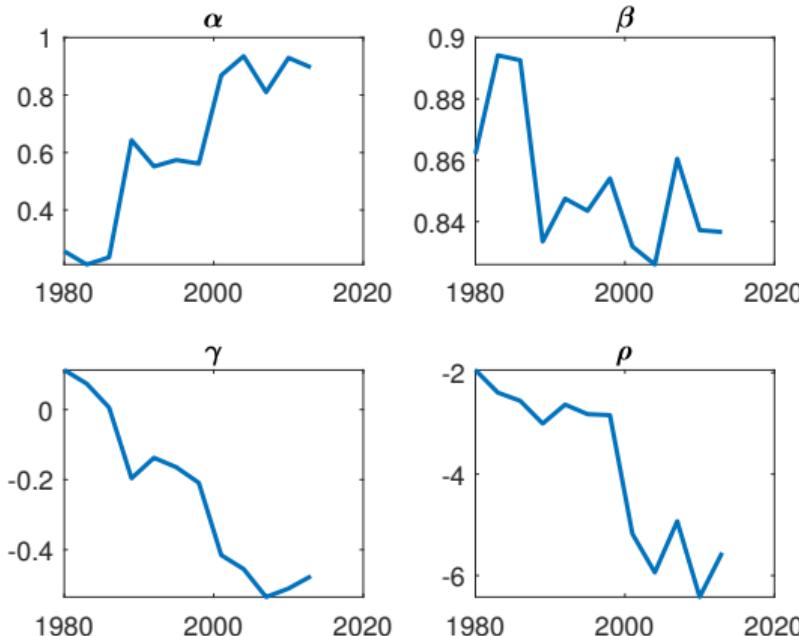
Key parameter changes:

- $\alpha \uparrow$  and  $\rho \downarrow$ :  $x, y$  more complementary
- $\gamma \downarrow$ : hours/skills are more complementary ( $f_{xh} \uparrow$ )

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# Estimates: Fitted Distributions

Parameters Estimated Over Time



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## Estimates: Lower Income Effects

Parameter	1980	2015	Meaning
$\beta$	0.99	0.98	weight of $(x, y)$ in prod.
$\alpha$	0.05	0.13	weight of skills in prod.
$\gamma$	0.51	0.09	compl. $(h, (x, y))$
$\rho$	0.98	-0.57	compl. $(x, y)$
$A$	28,5179	26,677	TFP

Key parameter changes very similar to baseline specification:

- $\alpha \uparrow$  and  $\rho \downarrow$ :  $x, y$  more complementary
- $\gamma \downarrow$ : hours/skills are more complementary ( $f_{xh} \uparrow$ )

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## Estimates: Idiosyncratic Tastes for Leisure

Introduce idiosyncratic tastes for leisure  $\psi_i$  in the utility function to match dispersion in hours.

Captures preferences, amenities, health shocks..

Parameter	1980	2015	Meaning
$\beta$	0.81	0.60	weight of $(x, y)$ in prod.
$\alpha$	0.23	0.67	weight of skills in prod.
$\gamma$	0.02	-0.13	compl. $(h, (x, y))$
$\rho$	0.98	-0.57	compl. $(x, y)$
$A$	6,5220	7,1405	TFP

Key parameter changes very similar to baseline specification.

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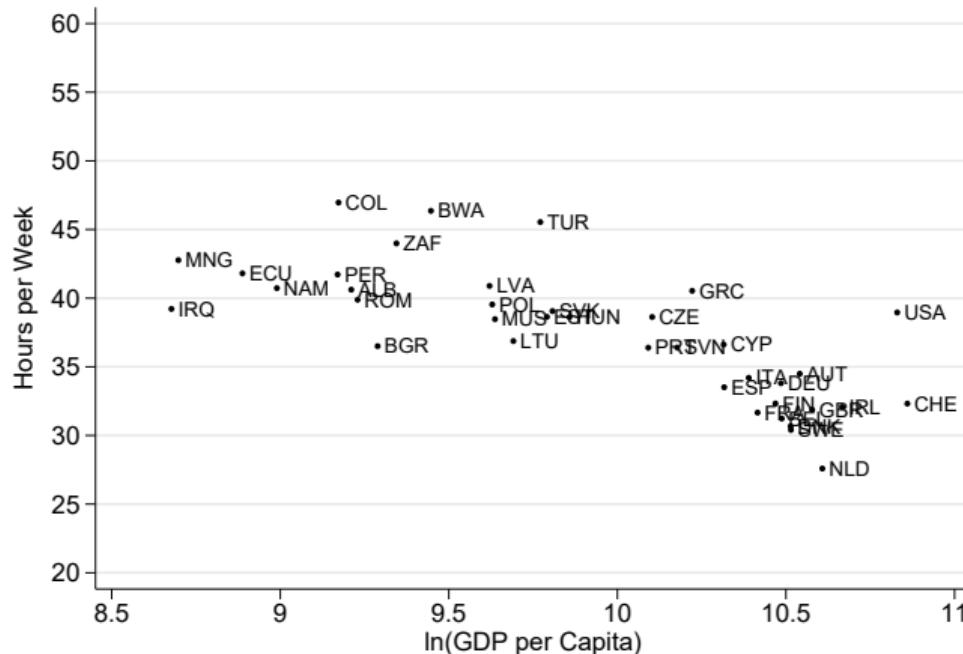
## Estimates: Assume Beta Distributions

Parameter	1980	2015	Meaning
$\beta$	0.94	0.91	weight of $(x, y)$ in prod.
$\alpha$	0.08	0.14	weight of skills in prod.
$\gamma$	0.30	-0.07	compl. $(h, (x, y))$
$\rho$	0.93	-2.48	compl. $(x, y)$
$A$	16,5103	16,5703	TFP

Key parameter changes very similar to baseline specification.

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## Additional evidence: Hours/worker over GDP

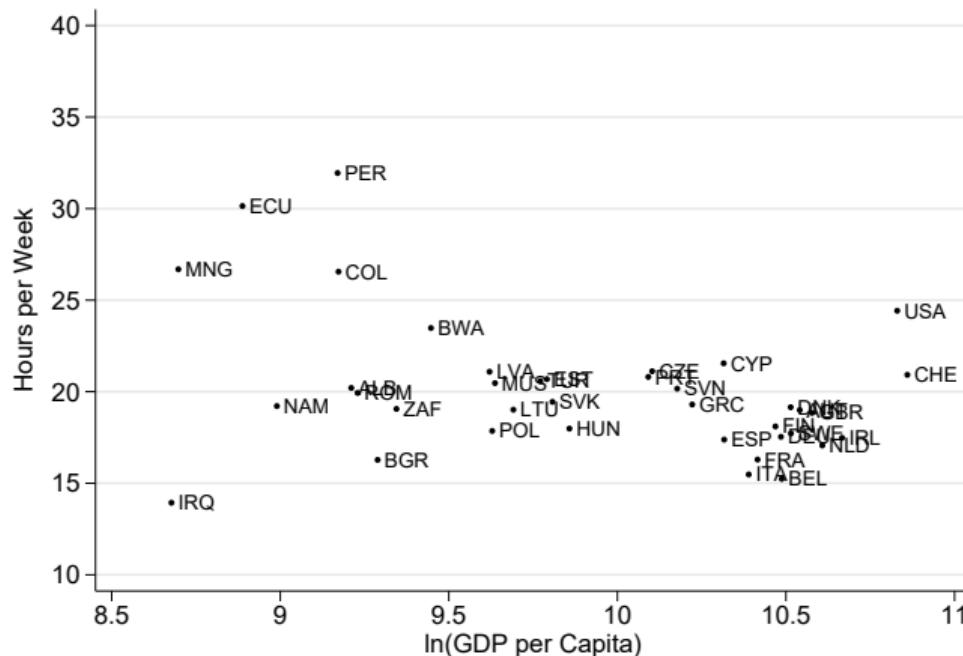


Source: Bick et al. (2018)

Hours/worker decline with GDPc in middle income and rich countries.

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## Additional evidence: Hours/capita over GDP

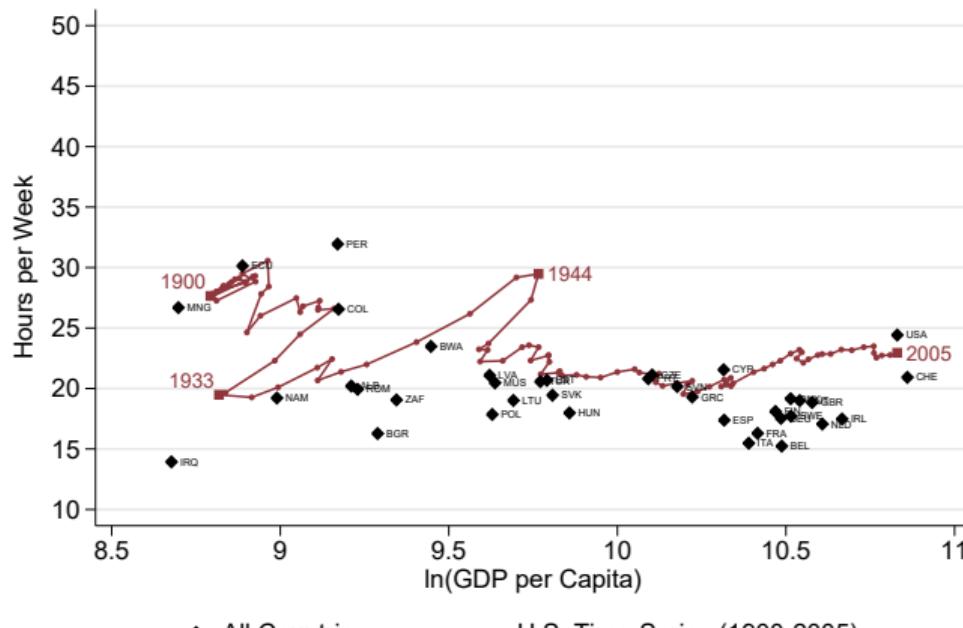


Source: Bick et al. (2018)

Hours/capita decline with GDPC in middle income and rich countries.

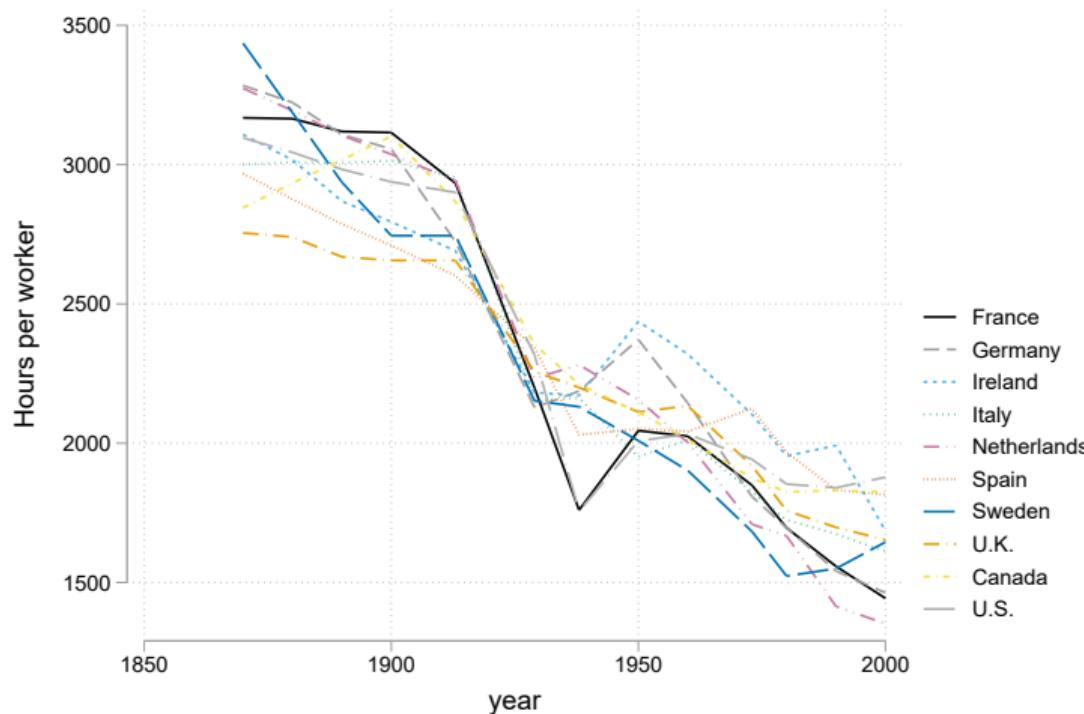
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## Additional evidence: US comparison



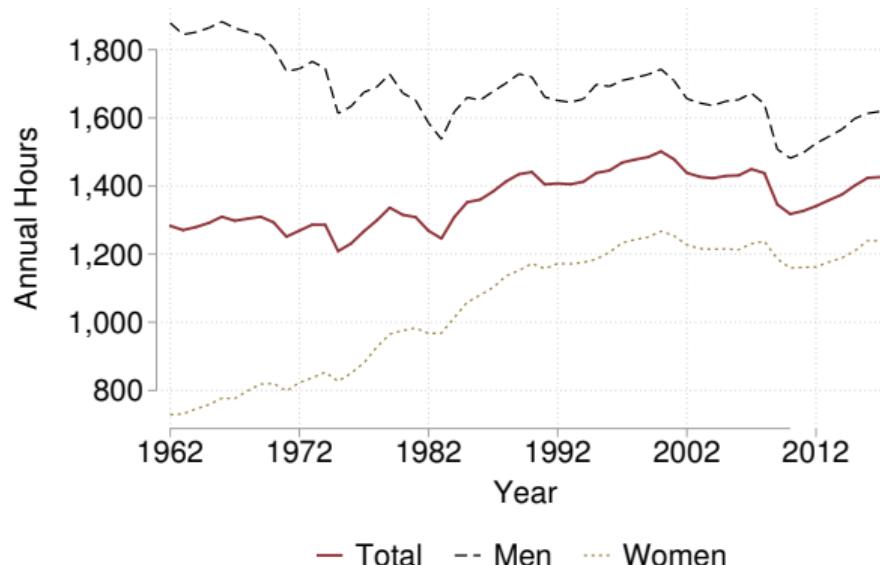
Hours/worker decline with GDPc in middle income and rich countries; focus on comparison with US.

## Additional evidence: Across countries and Time



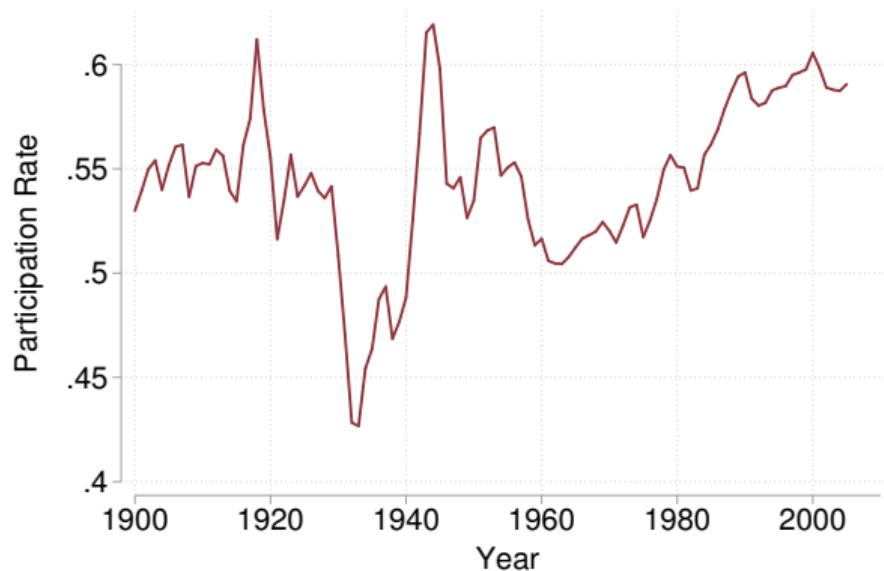
Hours/worker decline over time across countries.

## Additional evidence: Breakdown by Gender



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## Additional evidence: Extensive Margin

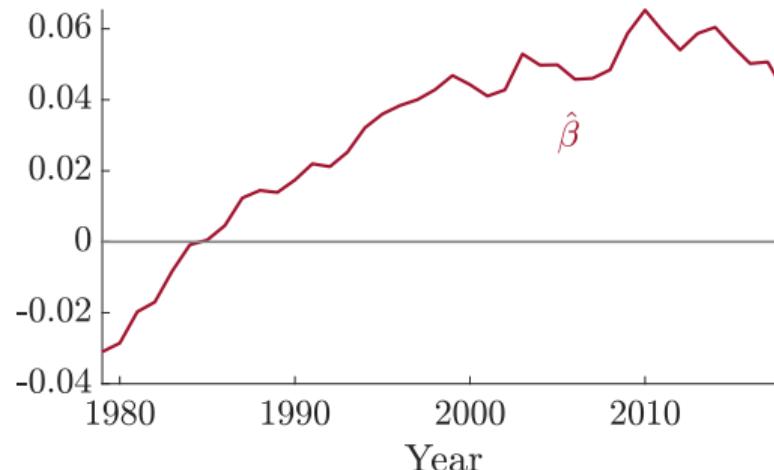


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## Additional evidence: Hours by Wage Decile

- Run the regression for every year 1978-2019 (CPS, males):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$

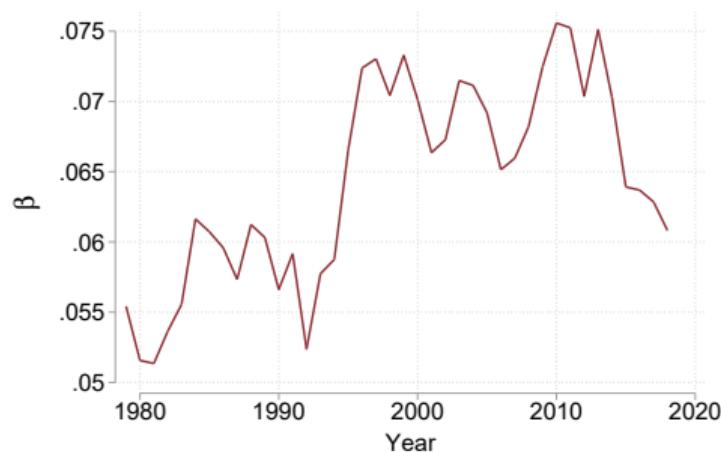


- Starting from 1980's, positive and increasing wage-hours relationship.

## Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, both genders):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



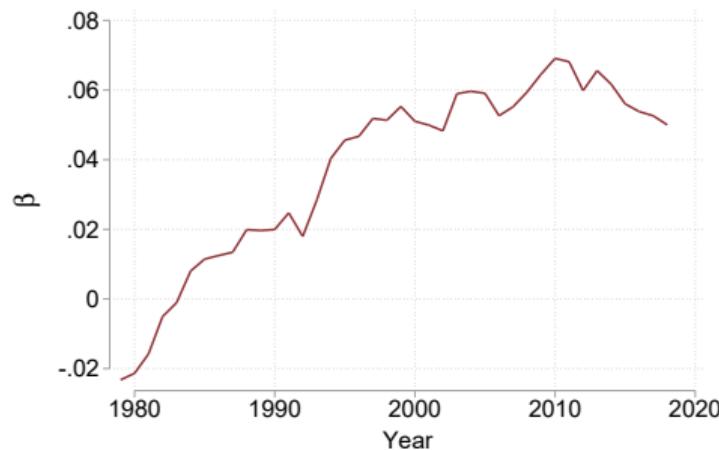
» Back to Main

» Back to Regression

## Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, without mult. jobs):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



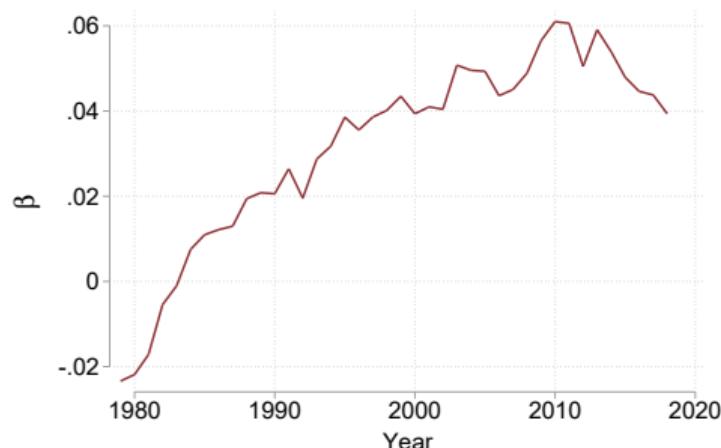
» Back to Main

» Back to Regression

## Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, includes tips/commissions for hourly workers):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



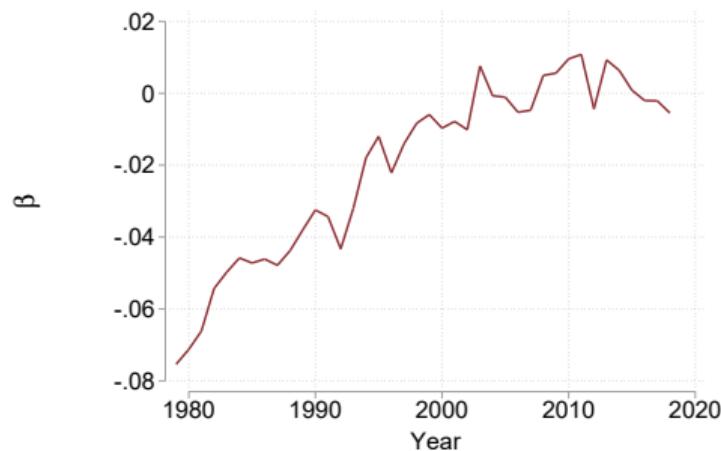
» Back to Main

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## Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, excludes hourly workers):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



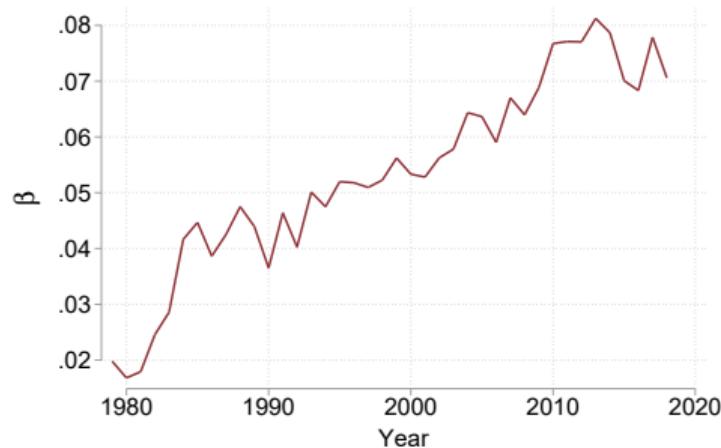
» Back to Main

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## Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, only hourly workers):

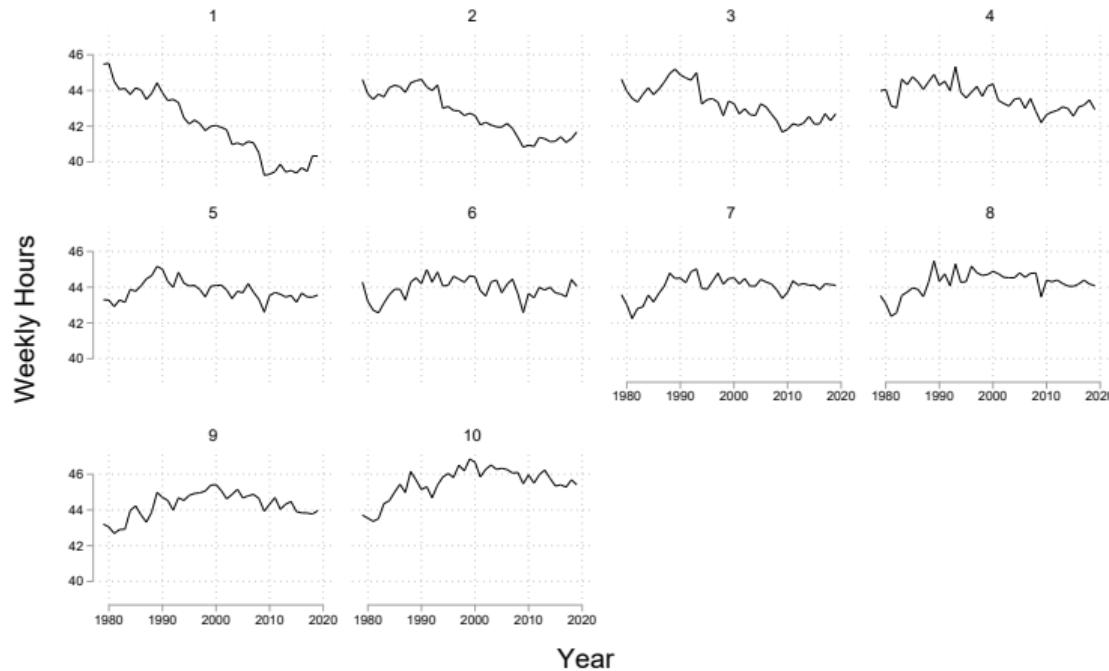
$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



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# Additional evidence: Hours Worked by Wage P'ctile

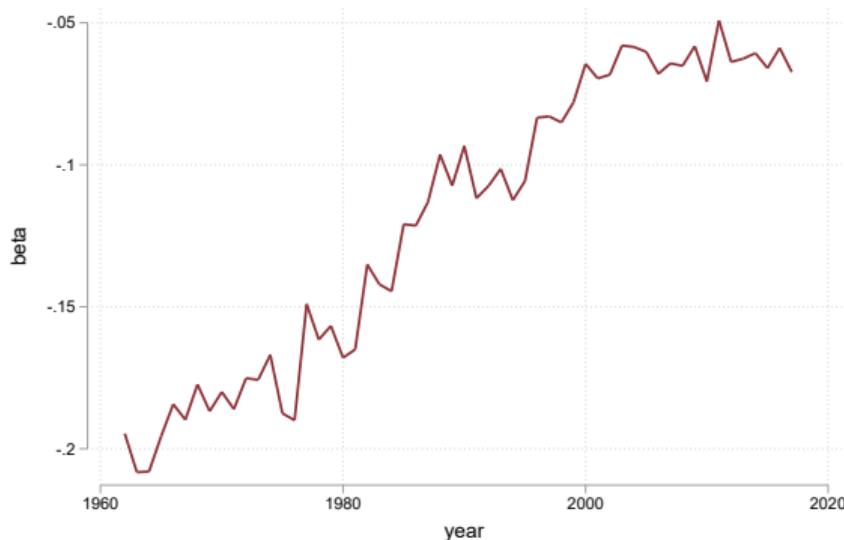


Graphs by hourw2

## Additional evidence: ASEC

- Run the regression for every year 1978-2019 (ASEC, males, actual hours):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



» Back

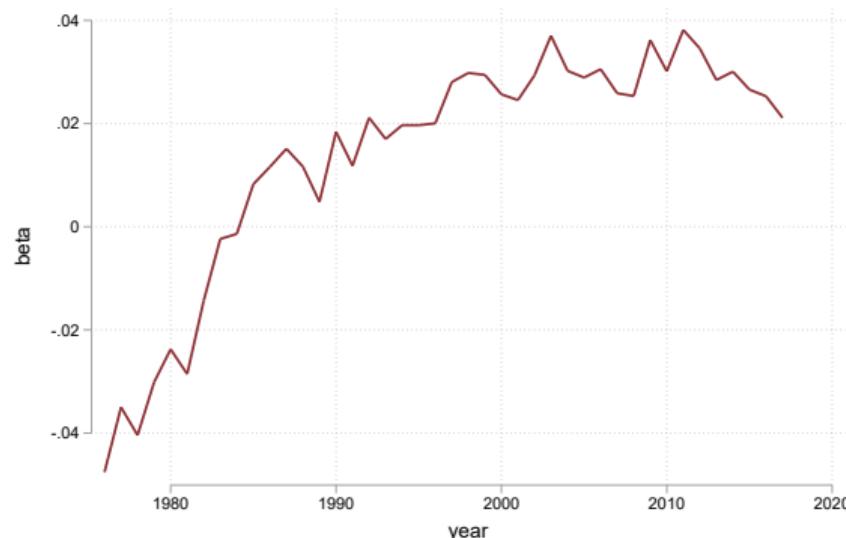
» Robustness: usual hours (M)

» Robustness: usual hours (F)

## Additional evidence: ASEC

- Run the regression for every year 1978-2019 (ASEC, males, usual hours):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



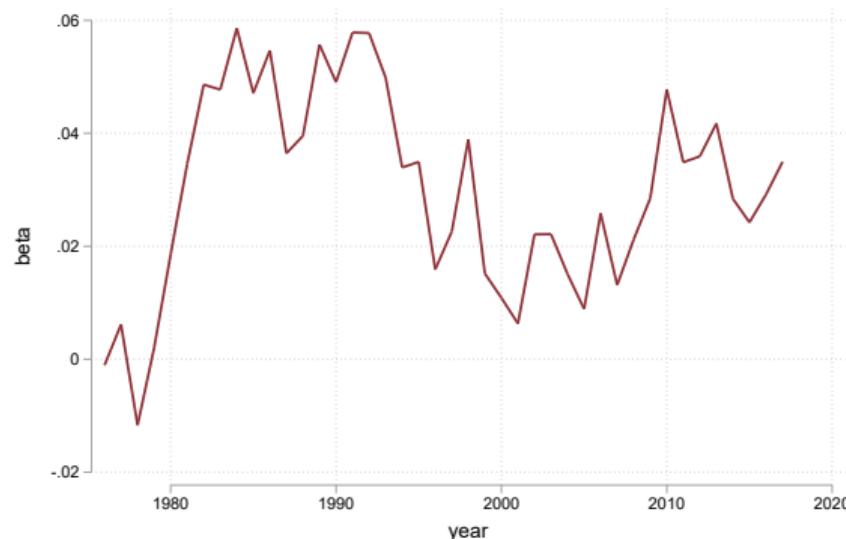
» Back to Main

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## Additional evidence: ASEC

- Run the regression for every year 1978-2019 (ASEC, females, usual hours):

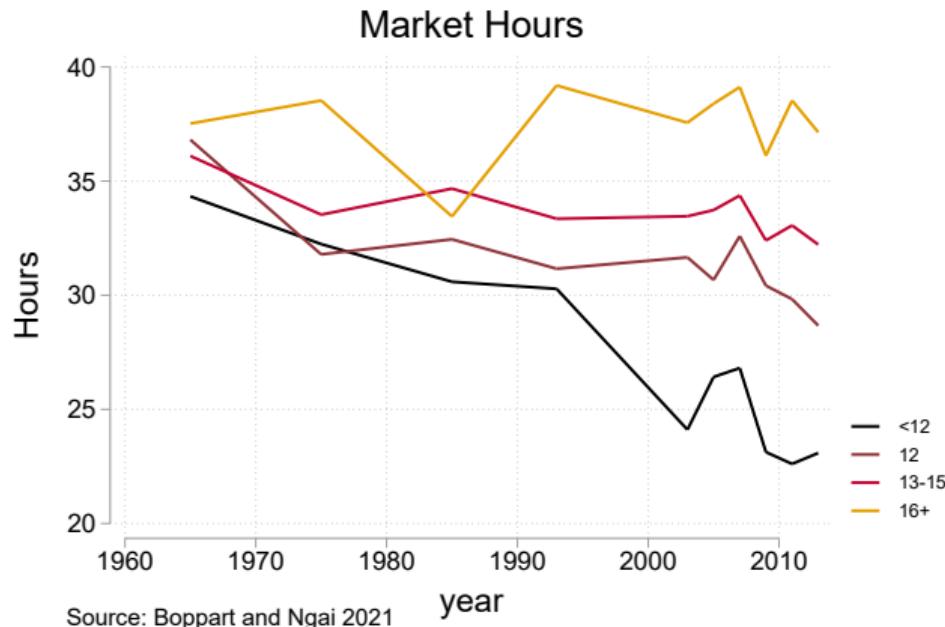
$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



» Back to Main

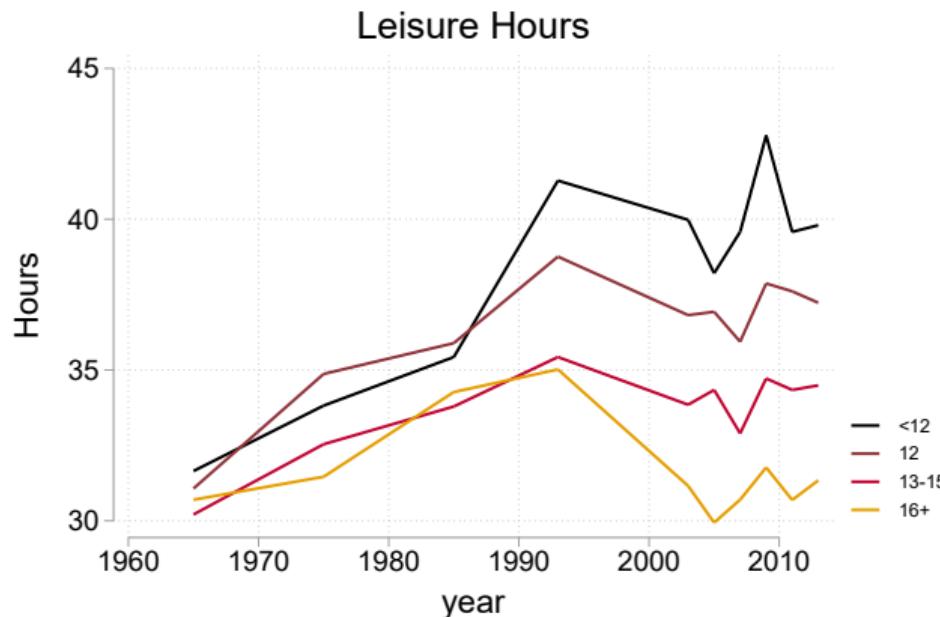
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## Additional evidence: ATUS data



Skilled workers people work roughly the same; low skilled work less.  
Inequality in trends starts from 1980's. [► Back](#)

## Additional evidence: ATUS data



Skilled workers consume less leisure, but starting from 1990's. Inequality in trends starts from 1980's. [➡ Back](#)

## Additional evidence: $\beta$ across countries

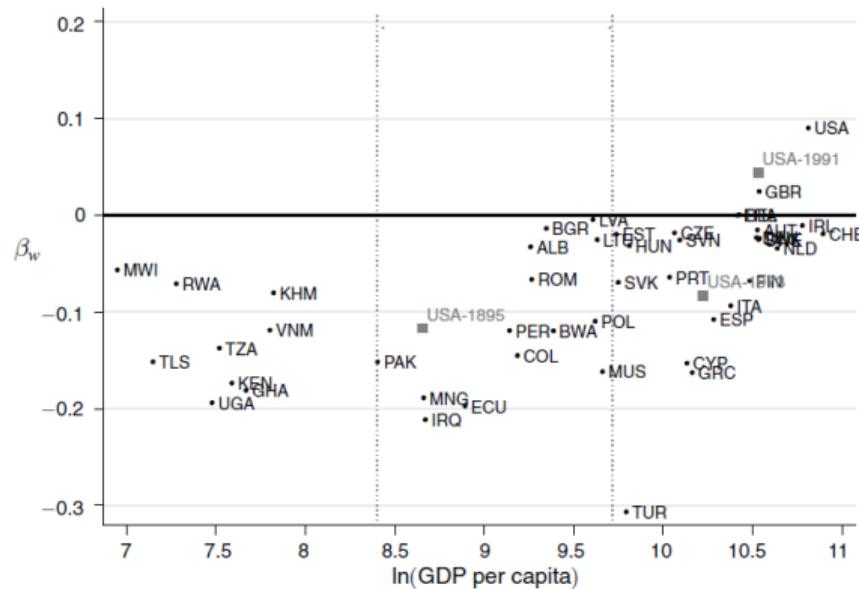


Figure: Cross-sectional relationship of hours and wages (source: Bick et al, 2021)

Similarly to US:  $\beta$  increasing with GDP for developed countries.

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## Additional evidence: progressivity across countries

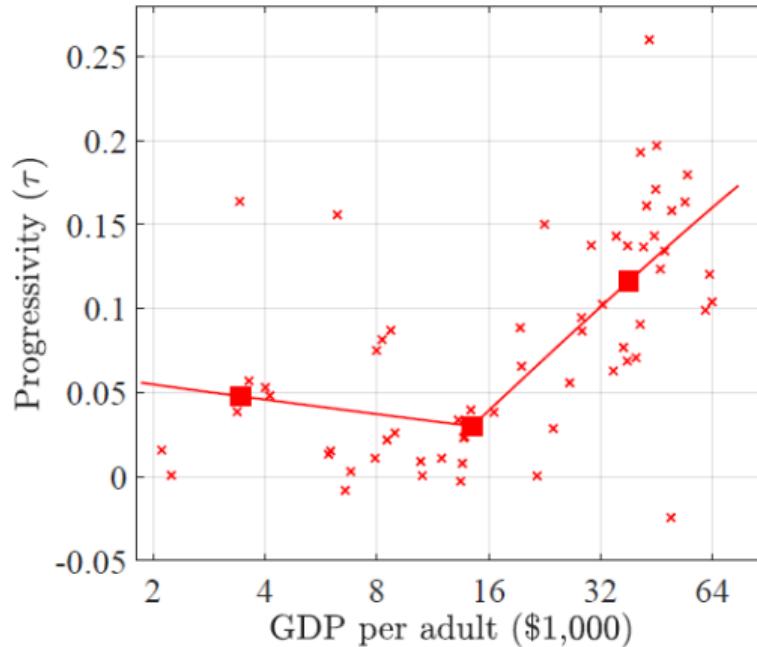


Figure: Fiscal progressivity across countries (source: Bick et al, 2021)

# Market Clearing

## Complete Definition

- Under Positive Assortative Matching (PAM), market clearing condition requires:

$$\int_{\mu(x)}^{\bar{y}} g^y(s) ds = \int_x^{\bar{x}} g^x(s) ds ,$$

where  $g^x$  and  $g^y$  are the densities of workers and jobs, respectively.

- Starting at the top, the highest type  $\bar{x}$  matches with the highest type  $\bar{y}$ .
- The type  $x < \bar{x}$  matches with  $y = \mu(x)$  if the measure of workers above  $x$  is equal to the measure of jobs above  $\mu(x)$ .
- This ensures that matching is measure-preserving.

# Equilibrium definition

## Market Clearing & Equilibrium

- Both households and firms take hedonic income function  $e()$  as given.
- Market clearing determines matching function  $\mu()$ .  
Under Positive Assortative Matching (PAM):

$$\int_{\mu(x)}^{\bar{y}} g^y(s) ds = \int_x^{\bar{x}} g^x(s) ds \quad (2)$$

### Definition 1: Competitive equilibrium.

A competitive equilibrium of this economy is a tuple of functions  $(e, \mu, h)$  such that:

- $e$  and  $h$  maximize utility and profits      (*optimality*)
- equation (1) holds    (*market clearing*)

## Derivation: Benchmark Model

- FOC:

$$f_x(x, y) - \frac{\partial w(x)}{\partial x} = 0$$

- Market clearing:  $\mu()$  (measure preserving). PAM is defined as  $\mu'(x) > 0$
- If  $\mu'(x) > 0$ , then we can write:  $\Gamma(x) = \Phi(\mu(x)) \implies \mu(x) = \Phi^{-1}\Gamma(x)$ .
- SOC:

$$f_{xx}(x, y) - w_{xx}(x) < 0$$

- To get rid of  $w_{xx}$ , we evaluate the FOC at  $y = \mu(x)$ :

$$f_{xx}(x, \mu(x)) + f_{xy}(x, \mu(x)) \cdot \frac{d\mu(x)}{dx} = w_{xx}(x)$$

- Substituting back, we get:

$$f_{xy}(x, \mu(x)) \cdot \frac{d\mu(x)}{dx} > 0$$

- Hence the sign of  $\mu(x)$  is determined by the sign of  $f_{xy}$  (provided differentiability of  $\mu()$ )

## Derivation: This Model

- Here we have two optimization problems (firm + hh). Hence FOC is a system of equations:

$$f_{\tilde{x}}(\tilde{x}, y) - \frac{\partial w(\tilde{x})}{\partial \tilde{x}} = 0 \quad (\text{firm})$$
$$u'_w \cdot w'_h + u'_h = 0 \quad (\text{hh})$$

- We repeat the previous derivation to understand properties of  $u$  and  $f$  that gives us PAM. The Hessian (SOC) is:

$$\begin{pmatrix} f_{xx}(x, y) - w_{xx}(x) & f_{xh}(x, y) - w_{xh}(x) \\ \frac{\partial u'_w \cdot w'_h}{\partial x} & \frac{\partial u'_w \cdot w'_h + u'_h}{\partial h} \end{pmatrix}$$

- Optimality requires  $|H| \geq 0$ . As before, we evaluate the FOC at  $y = \mu(x)$  and substitute.
- Solving for  $|H| \geq 0$  after some algebra we have that:

$$|H| \geq 0 \implies \text{the main condition.}$$

» Back

» Standard model

## Hours choice: Derivation

The FOC for hours reads:

$$u_c f_h + u_h = 0$$

By monotone comparative statics,  $h_x$  increases in  $x$  if:

$$\frac{\partial (u_c f_h + u_h)}{\partial x} > 0$$

From which one gets:

$$\left( \underbrace{u_{cc} f_h f_x}_{\text{income effects}} + \underbrace{u_c f_{hx}}_{\text{subs. effects}} \right) > 0$$

# Hours choice

Special Case: CRRA

- Hours choice:

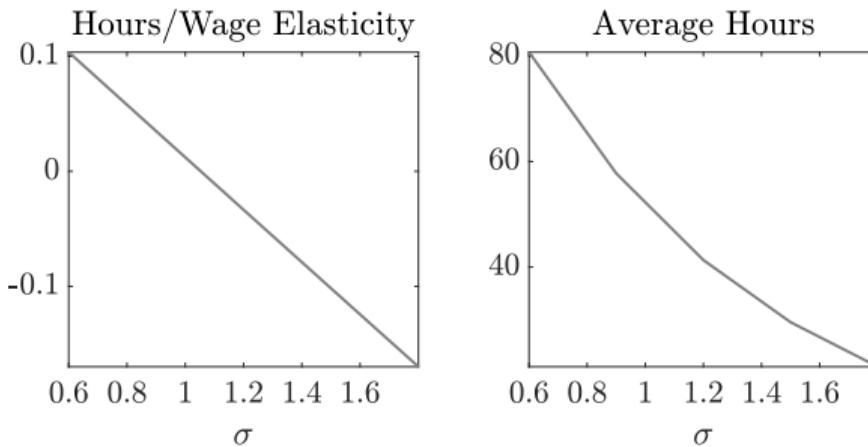
$$\left( \underbrace{u_{cc} f_h f_x}_{\text{income effects}} + \underbrace{u_c f_{hx}}_{\text{subs. effects}} \right) > 0$$

- **Sufficient condition:** relative risk aversion coefficient  $\sigma = -c \frac{u_{cc}}{u_c} < 1$ 
  - $\sigma$  governs strength of income vs substitution effects in macro models (King, Plosser, Rebelo 1988).
  - $\sigma > 1$  needed to match aggregate evidence (Boppart and Krusell, 2020)

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# Comparative Statics

## Preferences



An increase in  $\sigma$ :

- **Decreases** hours wage elasticity (higher income effects) and average hours worked
- Elasticity turns negative for lower values of  $\sigma$ , if low complementarities.

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## Assortative Matching

- Pair  $x$  and  $y$  solve the problem:

$$U(x, y, V) = \max_h u(c, h) \quad \text{s.t.} \quad f(x, y, h) - c \geq V$$

where  $U(x, y, .)$  traces a **Pareto frontier**.

- Hence we maximize  $U$  w.r.t. to job  $y$  to get the FOC:

$$U_y + U_\pi \pi_y = 0$$

- The equilibrium matching  $\mu$  is optimal provided that the SOC  $< 0$ , hence:

$$U_{yy} + 2U_{y\pi}\pi' + U_{\pi\pi}\pi'^2 + U_\pi\pi'' < 0$$

## Assortative Matching

- Differentiate FOC along the candidate equilibrium  $y = \mu(x)$ :

$$U_{yy} + U_{xy}\mu' + U_{x\pi}\pi' + U_{x\pi}\pi' + U_{y\pi}\mu'\pi' + U_{\pi\pi}\pi'^2 + U_\pi\pi'' = 0$$

- Combining this condition with  $SOC < 0$  gives:

$$\mu' \left[ U_{xy} - \frac{U_y}{U_\pi} U_{x\pi} \right] \geq 0$$

- Hence PAM ( $\mu' > 0$ ) obtains iff

$$U_{xy} - \frac{U_y}{U_\pi} U_{x\pi} \geq 0$$

Which completes the proof.

# Assortative Matching

Alternative condition for PAM

- Corollary: A condition for PAM in terms of primitives is:

$$(f_{yx} \underbrace{(-u_{cc} f_h f_h - u_c f_{hh} - u_{hh})}_{>0} + f_{hy} (u_{cc} f_x f_h + u_c f_{hx})) > 0$$

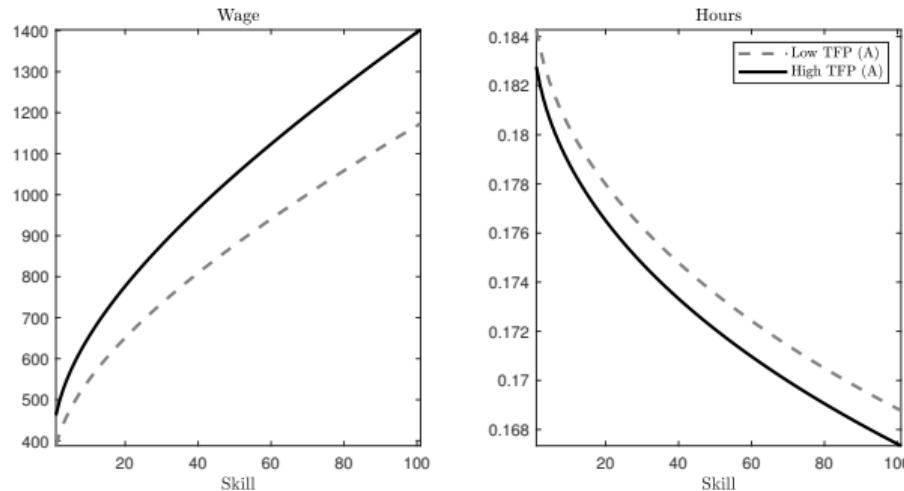
- Intuition:
  - $f_{xy}$ : skill-job complementarity (Becker 1973)
  - $f_{xh}, f_{yh}$ : hours complementarities (better workers produce more with more time)
  - $u_{cc} < 0$ : income effects push towards NAM

► Proof

► Back

# Prod. function: comparative statics

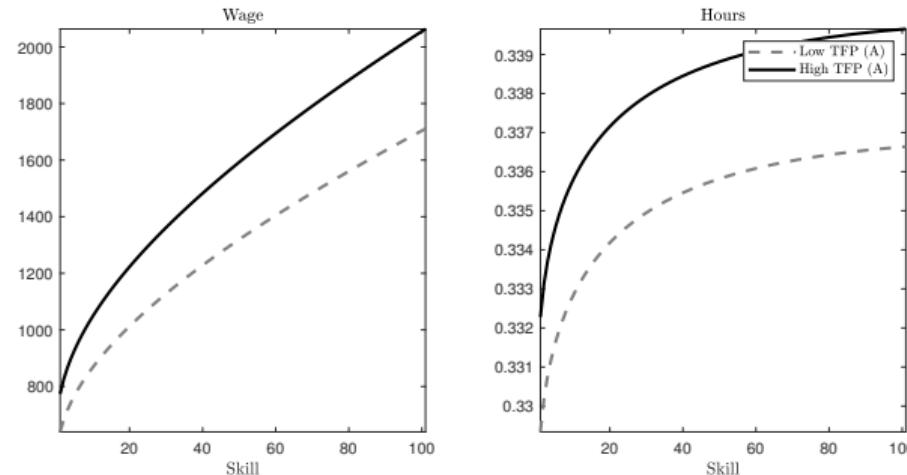
TFP: A, baseline



Increasing TFP has a level shift on wages (positive) and hours (negative,  $\sigma > 1$ ).

# Prod. function: comparative statics

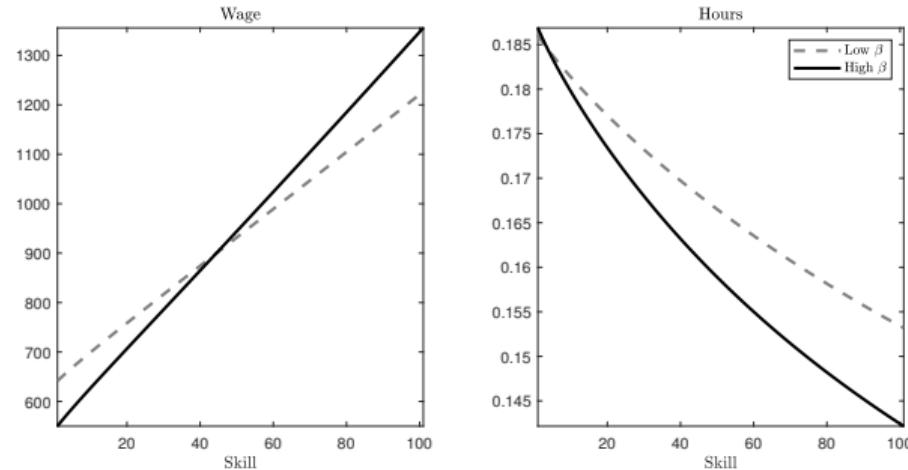
TFP: A, low  $\sigma$



Increasing TFP has a level shift on wages (positive) and hours (positive,  $\sigma < 1$ ).

# Prod. function: comparative statics

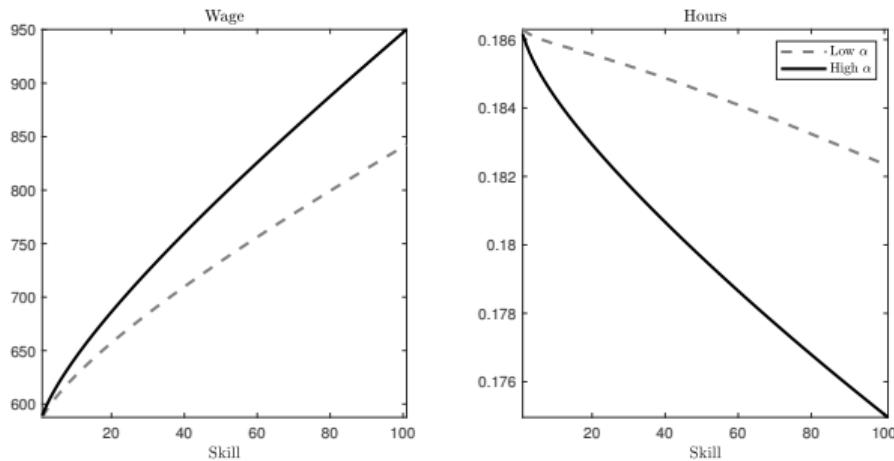
Hours/Match share:  $\beta$



Increasing  $\beta$  decreases importance of hours (hence hours choice  $\downarrow$ ).

# Prod. function: comparative statics

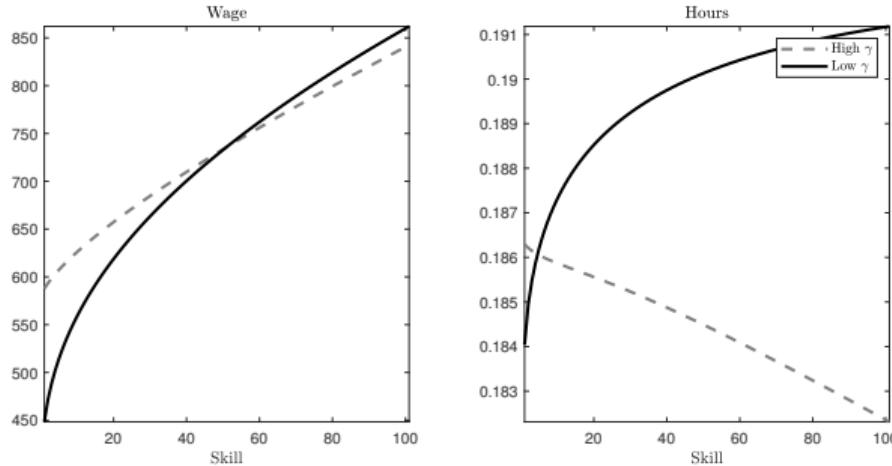
Skill importance:  $\alpha$



Increasing  $\alpha$  makes wage steeper in skills; through income effects, hours less steep (hence  $\partial h / \partial x \downarrow$ ).

# Prod. function: comparative statics

Match/hours complementarity:  $\gamma$



Increasing  $\gamma$  pushes towards  $\partial h / \partial x \uparrow$ . Also, it increases inequality. Key parameter to inform positive/increasing hours/wage elasticity! Missing in the previous specification ( $\gamma = 0$ ).

# Estimation

## Jobs distribution

- Need distribution of worker and job types,  $H(x)$  and  $G(y)$ , for 1980 and 2010
- Solution: estimate directly from the data
- $G(y) \approx$  cognitive skill requirement
  - 1980: DOT (Dictionary of occupational titles);  $y \approx$  mathematical reasoning
  - 2010: O\*NET: PCA on cognitive/math variables
- Final step: merge estimated  $y$  with CPS for both periods to get  $G(y)$
- Same approach as in Lindenlaub(2014), Lindenlaub and Chade(2022)

# Estimation

## Skill/Ability Distributions

- $H(x) \approx$  test scores
  - 1980: NLSY79
  - 2010: NLSY97
- I rely on Altonji, Bharadwaj, and Lange (2009) criterion to make test scores comparable across the two periods<sup>1</sup>
- (adjusted) distribution of test scores gives our measure of  $H(x)$

---

<sup>1</sup>e.g. test respondents were administered the test at different ages in the two cohorts

# Estimation

## Skill/Ability Distributions

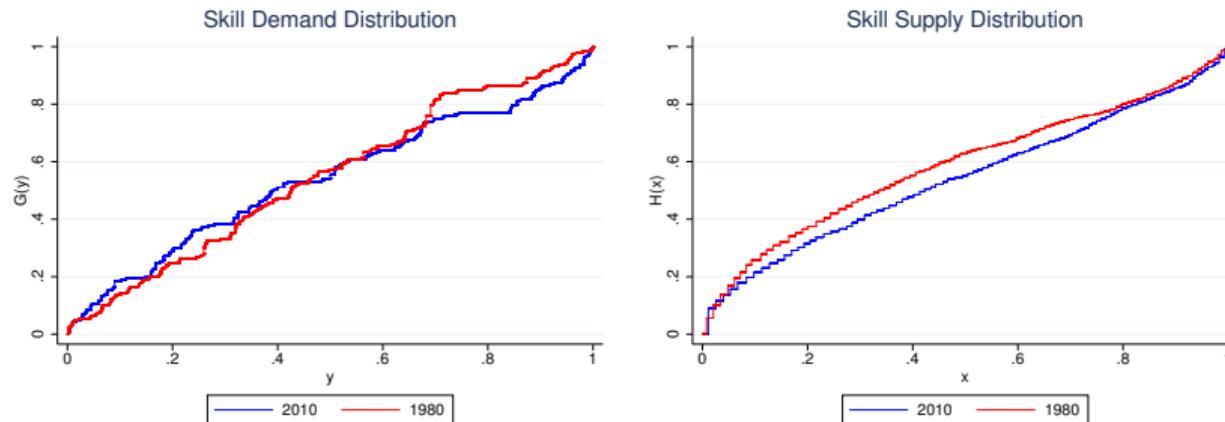


Figure: Estimated skill demand and supply distributions, for 1980 and 2015.

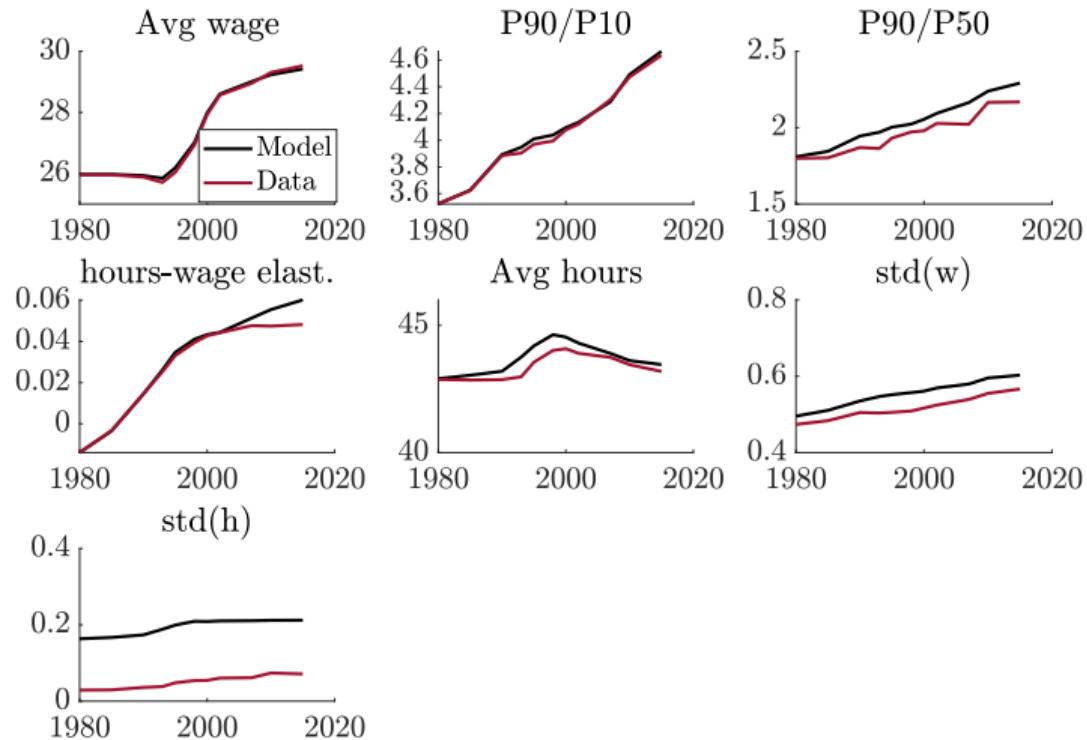
- Jobs (left): slight change in dispersion
- Workers (right): 'better' distribution of skills

# Targets

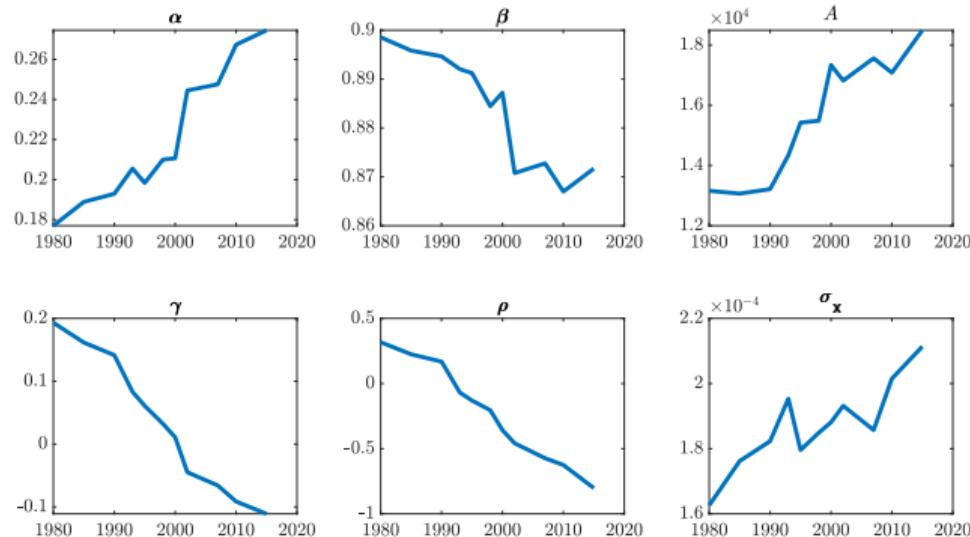
Results						
Moment	Data 1980	Data 2015	Model 1980	Model 2015	Data $\Delta$	Model $\Delta$
E(w)	19.6	23.9	19.6	23.9	21%	21%
E(h)	37.9	38.4	37.9	38.4	+1%	+1%
W75/W25	1.86	2.29	1.86	2.29	23%	23%
W50/W10	1.92	2.03	1.92	2.03	6%	6%
h/w elast.	-0.012	0.062	-0.012	0.062	7%	7%

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# Model Fit



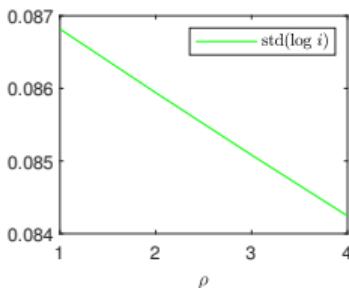
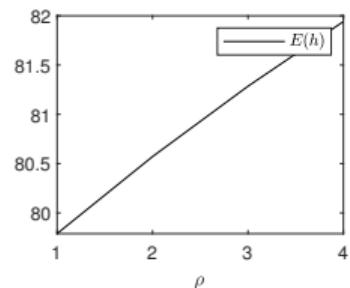
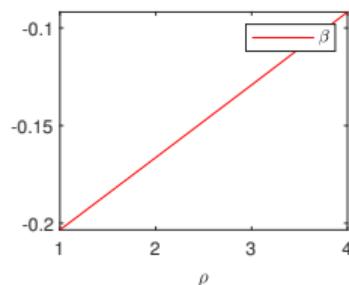
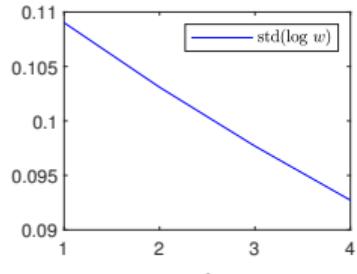
# Estimates



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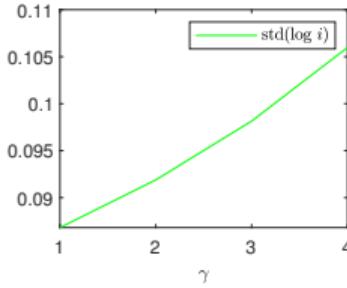
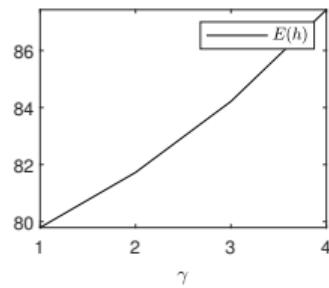
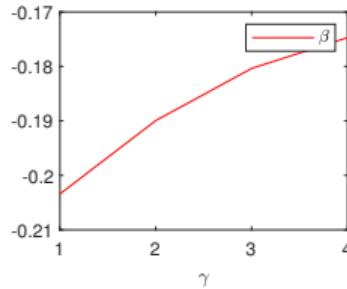
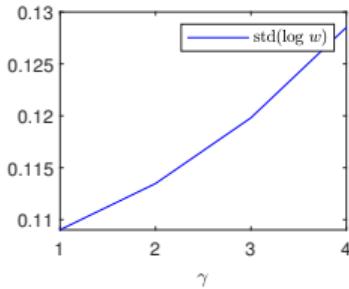
# Different CES nesting

$$f(x, y, h) = A \left( \beta(\alpha x^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)y^\gamma \right)^{\frac{1}{\gamma}}$$



# Different CES nesting

$$f(x, y, h) = A \left( \beta(\alpha x^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)y^\gamma \right)^{\frac{1}{\gamma}}$$



## Different CES nesting

Assume a different nest for the CES: workers and hours form a 'bundle',  
 $\tilde{x} = \tilde{x}(x, h)$ .

$$f(x, y, h) = A \left( \beta(\alpha x^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)y^\gamma \right)^{\frac{1}{\gamma}}$$

Parameter	1980	2015	Meaning
$\beta$	0.09	0.17	weight of $(x, h)$ in prod.
$\alpha$	0.80	0.85	weight of skills in prod.
$\gamma$	1.01	-0.34	compl. $(\tilde{x}, y)$
$\rho$	0.17	0.11	compl. $(h, x)$
$A$	32,209	27,276	TFP

► Back

## Different CES nesting

Assume a different nest for the CES: firms/jobs and hours nested as  $CES((y, h), x)$ .

$$f(x, y, h) = A \left( \beta(\alpha y^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)x^\gamma \right)^{\frac{1}{\gamma}}$$

Parameter	1980	2015	Meaning
$\beta$	0.40	0.90	weight of $(y, h)$ in prod.
$\alpha$	0.73	0.97	weight of jobs $y$ in prod.
$\gamma$	-3.3	-4.5	compl. $(x, y/h)$
$\rho$	0.56	0.04	compl. $(h, y)$
$A$	4,489	27,597	TFP

» Back

## Different CES nesting

Nest1  $f(x, y, h) = A \left( \beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)h^\gamma \right)^{\frac{1}{\gamma}}$

Nest2  $f(x, y, h) = A \left( \beta(\alpha x^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)y^\gamma \right)^{\frac{1}{\gamma}}$

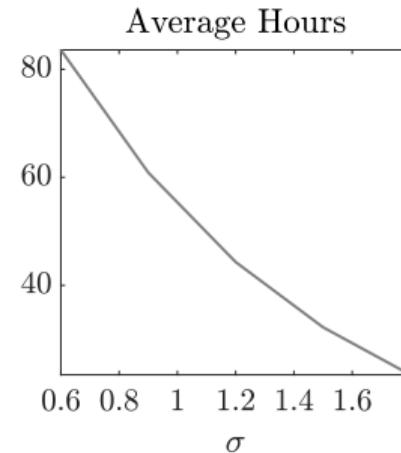
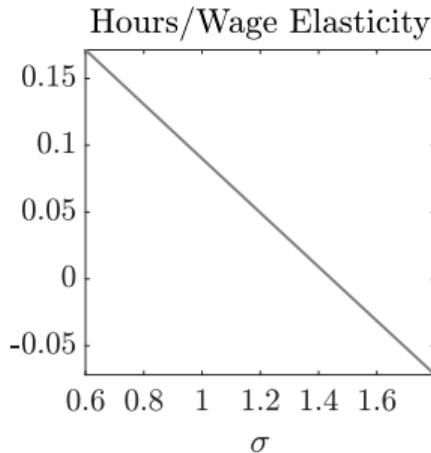
Nest3  $f(x, y, h) = A \left( \beta(\alpha y^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)x^\gamma \right)^{\frac{1}{\gamma}}$

	Nest 1		Nest 2		Nest 3	
Parameter	1980	2015	1980	2015	1980	2015
$\beta$	0.90	0.87	0.09	0.17	0.40	0.90
$\alpha$	0.13	0.21	0.80	0.85	0.73	0.97
$\gamma$	0.32	-0.07	0.98	-0.34	-3.3	-4.5
$\rho$	0.22	-1.20	0.17	0.11	0.56	0.04
$A$	12,871	22,218	32,209	27,276	4,489	27,597

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# Comparative Statics

## Preferences



An increase in  $\sigma$ :

- Decreases hours wage elasticity (higher income effects) and average hours worked

» Low compl.

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