

Hours-Biased Technological Change

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Job Market Paper

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Intro: The Evolution of Working Time

- Hours decline with wages in the long-run...

Hours Decrease Over Time: US

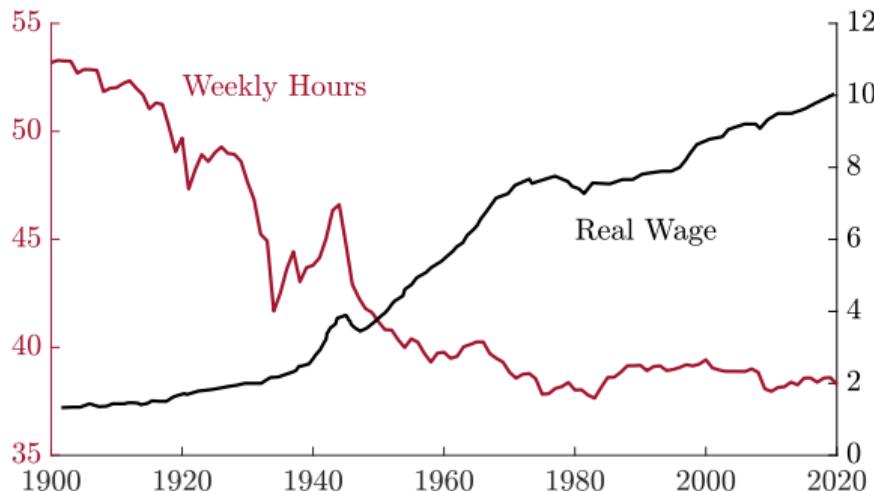


Figure: Hours per Worker and Wages in US

Notes: Weekly hours is weekly average hours worked from CPS, Kendrick (1961) and Kendrick (1973). Real wage is real gross national product divided by hours (1900-1928) from Kendrick (1961); Real labor productivity (FRED, 1929-2020). Real wages normalized so that 1900=1.

► Cross-country

► Cross-country: over time

► Cross-country vs US

► Hours/capita

► By Gender

► Extensive Margin

Hours Decrease Over Time: Cross-Country

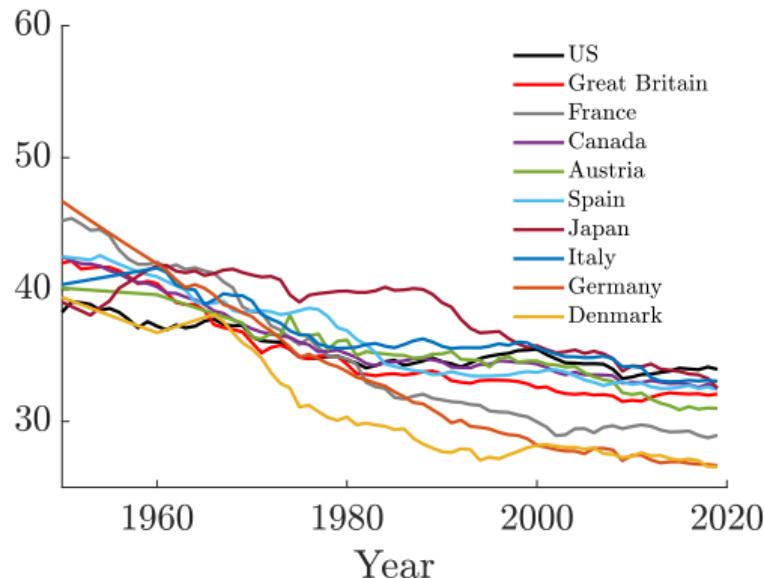


Figure: Hours per Worker: Selected Countries

Notes: Weekly hours is total annual hours worked from Penn World Tables 9.0, expressed weekly.

► Cross-country

► Cross-country: over time

► Cross-country vs US

► Hours/capita

► By Gender

► Extensive Margin

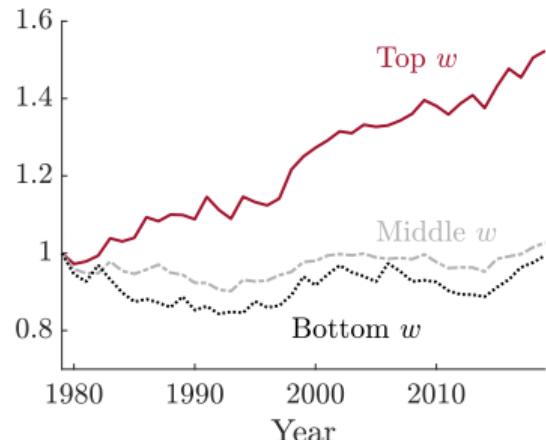
Intro: The Evolution of Working Time

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 - Leading explanation: income effect > substitution effect (wage ↑, hours ↓)
 - Keynes(1930), Boppart and Krusell (2020)

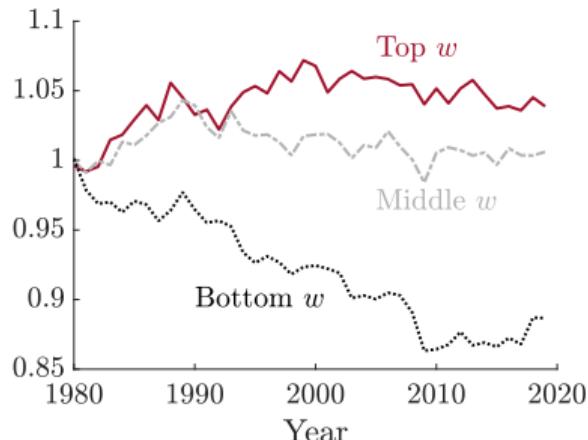
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- ...but not in the cross-section

Hours Increased For High-Wage Workers



(a) Wages



(b) Hours

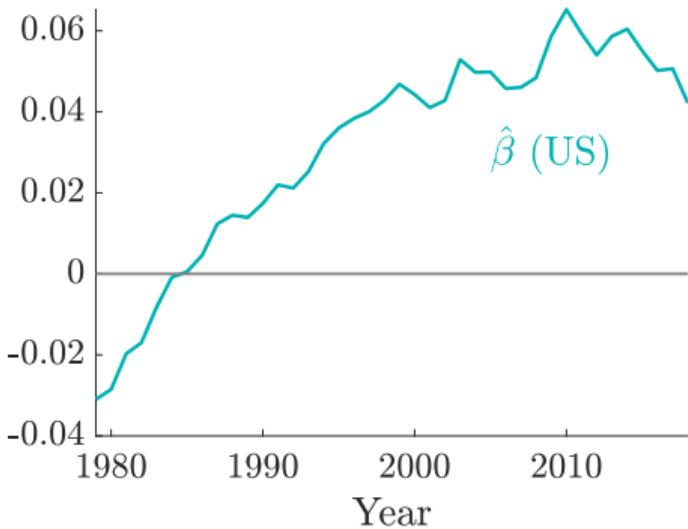
Figure: Wages and Hours By Wage Decile w (US)

Intro: The Evolution of Working Time

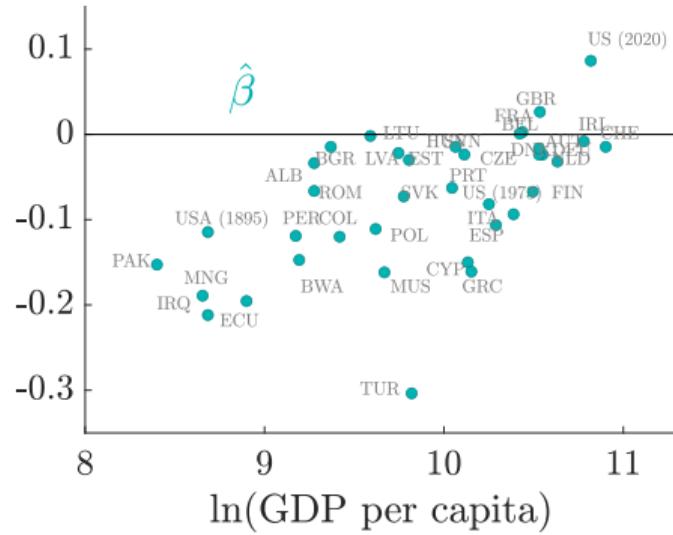
- Hours decline with wages in the long-run...
 - Leading explanation: income effect > substitution effect (wage ↑, hours ↓)
- ...but not in the cross-section
 - Workers who gained more also started working more
 - \Rightarrow Hours-wage correlation turns from negative to positive ► Regression
 - **Puzzle:** if income effects dominates, we would expect the opposite

Increasing Hours-Wage Correlation (Cross-Section)

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i, \quad i = \text{worker}$$



(a) Over Time (US)



(b) Across Countries

Figure: Hours-Wage Correlation

Intro: The Evolution of Working Time

- Hours decline with wages in the long-run ($IE > SE$)
 - Leading explanation: income effect > substitution effect (wage \uparrow , hours \downarrow)
- ...but not in the cross-section ($IE < SE$)
 - Workers who gained more also started working more
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- Increasing hours-wage correlation across development path

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 - **Puzzle:** if income effects dominates, we would expect the opposite
- Increasing hours-wage correlation across development path
- How can we reconcile these patterns? Answer to this question is key for:
 - Tax Policy
 - Understanding inequality

Hours-Biased Technological Change

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 - SBTC: technologies increase productivity of skilled workers (extensive margin)
 - Laptops, ICT..

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 - SBTC: technologies increase productivity of skilled workers (extensive margin)
 - Laptops, ICT..
- ... but also working time
 - More flexible hours (e.g. Uber), easier to work extra-time (e.g. Skype)
- New channel: Hours-Biased Technological Change
 - Extra-hour more productive for skilled
 - Changes *intensive* margin productivity

This Paper

- Build a theory of the labor market where **workers** are assigned to **jobs** based on skills and hours
 - Builds on SBTC frameworks but includes hours in technology

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- Mechanism: changes in technology (HBTC) \implies reward for long hours \uparrow
 - Higher incentives to work long hours for high skilled

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 - Builds on SBTC frameworks but includes hours in technology
- Mechanism: changes in technology (HBTC) \implies reward for long hours \uparrow
 - Higher incentives to work long hours for high skilled
- Quantitatively analyze technological change in US
 - Novel channel of technology on **inequality**
 - New driver of aggregate hours worked in US (offsets income effect)

Literature

- **Aggregate Hours:** King, Plosser, Rebelo (1988); Prescott (2004); Ramey and Francis (2009); Boppart and Krusell (2020); Rachel (2021); Bick et al. (2021).
Take into account cross sectional heterogeneity

- **Leisure inequality:** Costa (2000); Aguiar and Hurst (2007); Kopitov et al. (2020); Boppart and Ngai (2021); Michelacci and Pijoan-Mas (2014); Doepke et al. (2018).

Introduce sorting and endogenous wage inequality

- **Sorting:** Eeckhout and Kircher (2018); Eeckhout and Sepahsalari (2018); Calvo, Lindenlaub, Reynoso (2021); Michelacci and Pijoan-Mas (2014); Chade and Lindenlaub (2022), Shao et al. (2022).

Characterize sorting for general preferences and technology

Model

Setup

- Population
 - Individuals with skill $x \sim G^x$
 - Jobs of type $y \sim G^y$
- Preferences
 - Choose hours h and consumption c
 - Utility function: $u(c, 1 - h)$
- Technology
 - Competitive labor market
 - Workers and jobs(firms) match one-to-one \implies sorting
 - Output: $f(x, y, h)$
 - Note: $h = h(x)$ is endogenous and skill dependent

Setup

- Payoffs

- Workers maximize: $u(c, 1 - h)$ s.t. $c = e(x, h)$
- Jobs maximize: $f(x, y, h) - e(x, h)$
- In equilibrium, wages w are defined as $w = \frac{e}{h}$, where $w = w(x, y, h(x, y))$

- Market clearing

- Equilibrium matching $y = \mu(x)$ assigns workers x to jobs y .
- Under Positive Assortative Matching (PAM), market clearing requires:

$$\int_{\mu(x)}^{\bar{y}} g^y(s) ds = \int_x^{\bar{x}} g^x(s) ds ,$$

where g^x and g^y are the densities of workers and jobs, respectively.

▶ Details

- Competitive equilibrium = optimality + market clearing:

- income $e(x)$; hours $h(x)$; matching $\mu(x)$

▶ Full definition

Problem as Pareto Frontier

Pair's problem as Pareto Frontier

- Workers x and jobs y simultaneously solve, respectively:

$$\max_{h,y} u(c, 1 - h) \quad \text{s.t.} \quad c = e$$

$$\max_x f(x, y, h) - e$$

- This can be re-written as pair x, y solving:

$$U(x, y, \pi) = \max_h u(f(x, y, h) - \pi, 1 - h)$$

where $U(x, y, .)$ defines the Pareto Frontier (for all profit values π).

Optimal Job Choice

Optimization

- Optimal job choice implies positive sorting iff:

$$U_{xy} - \frac{U_y}{U_\pi} U_{x\pi} \geq 0 \iff -\frac{U_y}{U_\pi} \text{ increasing in } x \quad \text{▶ Proof}$$

- **Intuition:** high x more willing to give more π (=less income) for a better y

Assortative Matching

Check $-\frac{U_y}{U_\pi} \uparrow x$

Proposition: *The equilibrium features Positive Assortative Matching iff:*

$$f_{xy} + f_{yh} h_x \geq 0$$

» Alternative Condition

» Decentralized eq.

- **Intuition:** two forces pushing towards positive sorting (PAM)
 - Skill-job complementarity f_{xy} (**standard**)
 - High skill working more ($h_x > 0$) + hours-job complementarity $f_{yh} > 0$ (**new**)
- Low skill working much more ($h_x << 0$) and $f_{yh} > 0$ can lead to NAM

Hours choice

- FOC for hours implies $u_c f_h + u_h = 0$. From this, we get that:

Proposition: *In equilibrium, high skill choose higher hours if:*

$$\underbrace{u_c f_{xh}}_{\text{subs. effects}} > \underbrace{-u_{cc} f_h f_x}_{\text{income effects}}$$

» Derivation

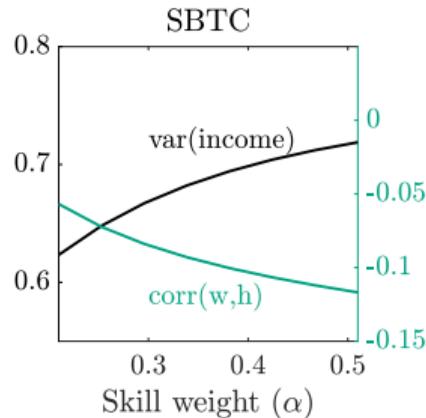
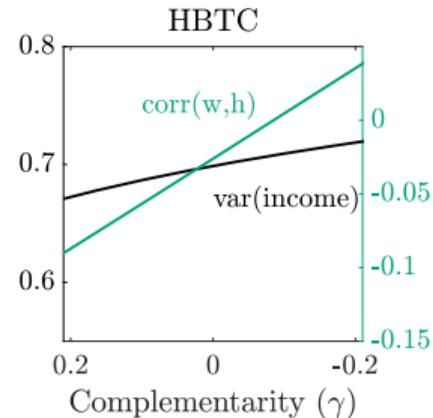
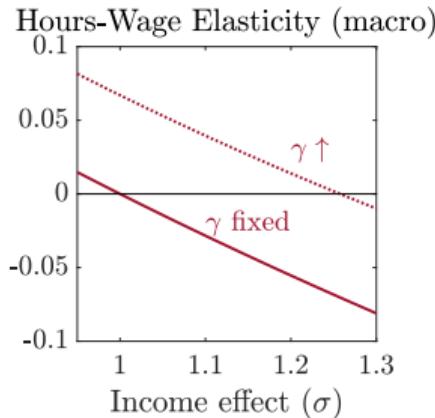
» CRRA

- **Income effects** (recall $u_{cc} < 0$)
 - Key force to explain long run behavior of hours (Boppart and Krusell, 2020)
- **Hours complementarities:** f_{xh}
 - If positive, raises marginal product of hours worked for high skill \implies HBTC

Functional forms

- Production: $f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)h^\gamma \right)^{\frac{1}{\gamma}}$
 - Captures both SBTC (through α, ρ) and HBTC (through γ) ▶ Nest 2: Simulation
- Preferences (MacCurdy, 1981): $u(c, h) = \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{h^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}$
 - σ = strength of income effects

Comparative Statics



Key takeaways:

- Hours complementarities (γ) matter for both *macro* and *micro* elasticities
- HBTC vs SBTC **drive up inequality**, but opposite effect on **micro elasticity**

Special Cases

- Becker (1973) sorting model
 - $f_{yh} = f_{xh} = 0$. Sorting only depends on f_{xy}
- Macro labor supply models with linear earnings
 - Hours affect earnings **linearly** (Boppart and Krusell, 2020)
 - Misses cross-sectional facts
- SBTC frameworks (Katz-Murphy, Acemoglu-Autor)
 - Only rel. productivity of skilled workers (extensive margin); no intensive margin
 - $f_{xh} = f_{yh} = 0$ ($\beta = 1$)
- 'Effective types' models
 - Firms match with **bundle** $\tilde{x} = \tilde{x}(x, h) \Rightarrow f = f(\tilde{x}, y)$
 - Used to study sorting and **optimal taxation** (Scheuer and Werning, 2016)
- Non-linear earnings (Erosa et al., 2016; Bick et al., 2021)
 - hours choice depends on f_{xh}/u_{cc} ($f_{xy} = 0$)

Estimation

Overview

- Can changes in **technology** rationalize the data? Recall the prod. function:

$$f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)h^\gamma \right)^{\frac{1}{\gamma}}$$

- Estimate parameters year by year to match moments from the data (SMM).
 $\implies A = A_t, \alpha = \alpha_t, \beta = \beta_t, \gamma = \gamma_t, \rho = \rho_t.$
- Note: $f(x, y, h)$ is equivalent to

$$f(x, y, h) = \left((A_x x^\rho + A_y y^\rho)^{\frac{\gamma}{\rho}} + A_h h^\gamma \right)^{\frac{1}{\gamma}}$$

- **Goal:** Assess changes in weights (A_t, α_t, β_t) and complementarities (ρ_t, γ_t)

Strategy

- **Preferences:** calibrate σ and θ
 - σ : mid-range of macro studies 1.21 (Bick et al., 22); 1.7 (Boppart Krusell, 20)
 - $\theta = 0.4$: consistent with micro-evidence (Violante et al, 2014).
- **Distributions** G^w and G^f
 - Assume log-normality: $x \sim \mathcal{LN}(\mu_x, \sigma_x)$; $y \sim \mathcal{LN}(\mu_y, \sigma_y)$
 - Set $\mu_x = \mu_y = 0$ and estimate ratio σ_x / σ_y
- Six parameters to be estimated (5 technology, 1 distributions)

E(h)	Avg. Hours
E(w)	Avg. Wages
w90/w50	wage inequality (top)
w90/w10	wage inequality (overall)
Hours-wage Elasticity	Coeff. of reg. $\log(h)$ on $\log(w)$
std(w)	wage dispersion

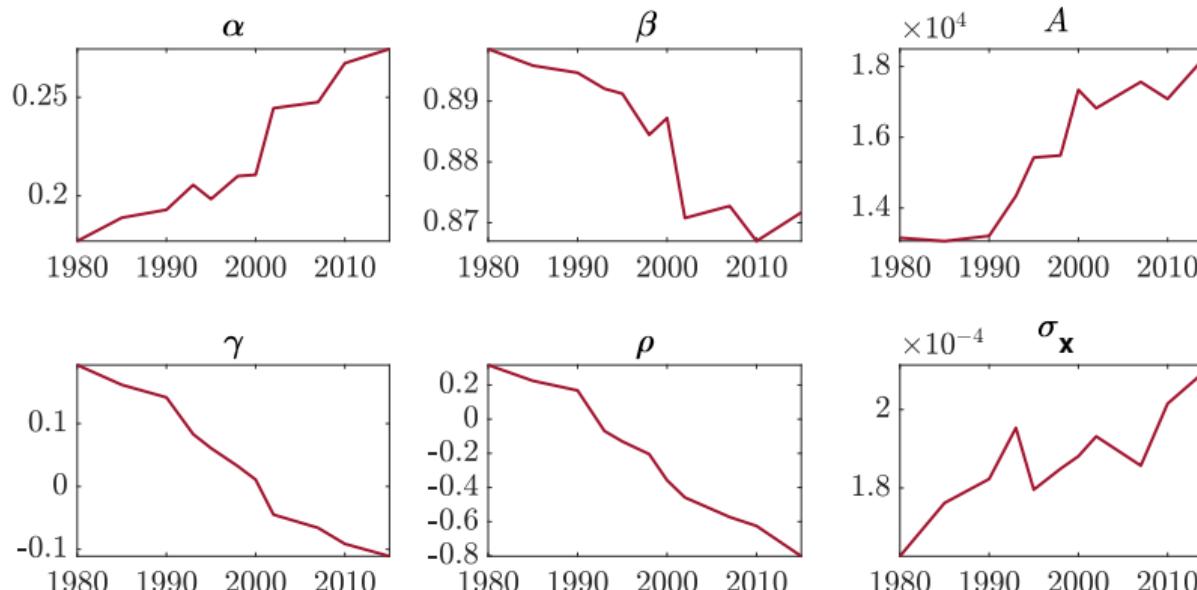
Estimates

- Both SBTC ($\alpha \uparrow$ and $\rho \downarrow$) and HBTC ($\gamma \downarrow$)

► Model Fit

► Fixed Hours Estimates

$$f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)h^\gamma \right)^{\frac{1}{\gamma}}$$



Income Inequality

- Decompose *income* inequality in:

$$\text{var}(e) = \text{var}(w) + \text{var}(h) + 2\text{cov}(w, h)$$

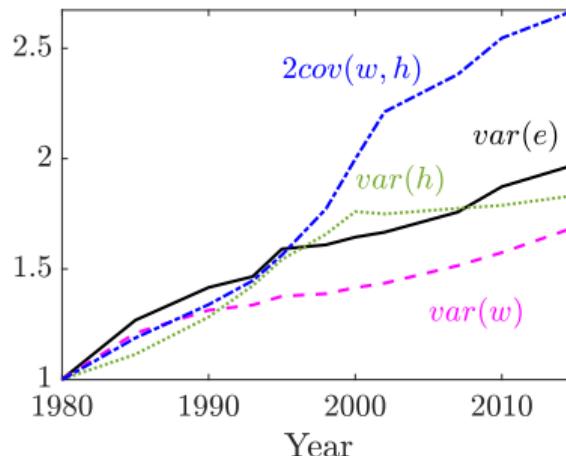
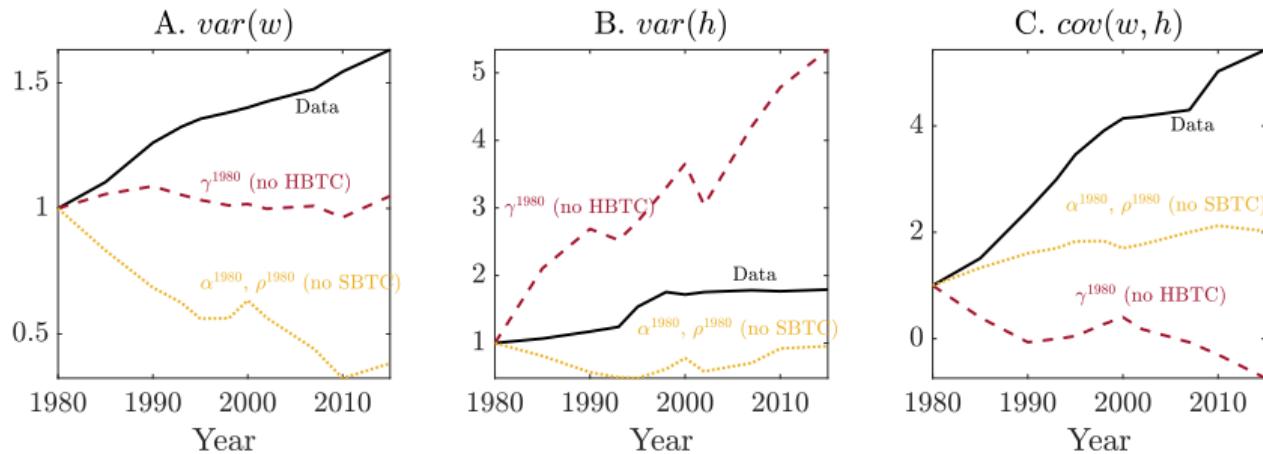


Figure: Drivers of Income Inequality (US)

- Different drivers of inequality. How does the model rationalize this?

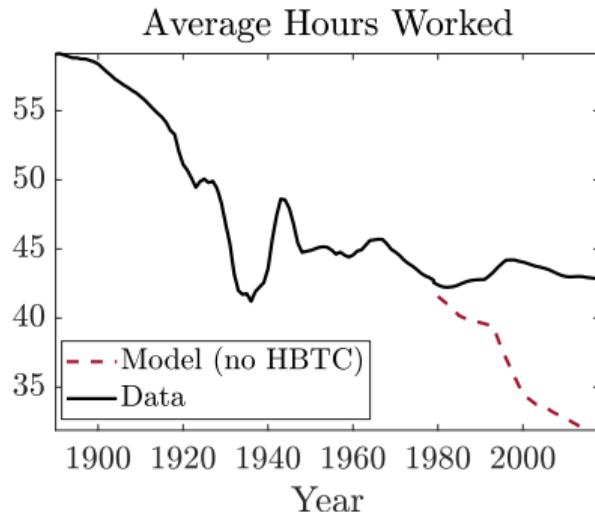
Results: Drivers of Inequality



Results:

- HBTC ($\gamma \downarrow$) contributes to income inequality through $\text{cov}(w, h)$
- α (\approx SBTC) more important for $\text{var}(w)$ but pushes $\text{cov}(h, w)$ down

Results: Aggregate Hours



- Explanation for relatively flat average hours worked in US post-1970
 - One explanation: income = substitution effect
 - This paper: income effect (Boppart and Krusell, 2020) + HBTC
- Across development path, relative strength of these two forces vary

Robustness

On preferences:

- Income effects (different σ) [» Results](#)
- Idiosyncratic tastes for leisure [» Results](#)

On distributions:

- Take distributions from the data [» Distributions](#) [» Results 1](#) [» Results 2](#)
- Assume Beta distributions (Lise, Meghir, Robin 2016) [» Results](#)
- Estimate only σ_x [» Fit](#) [» Estimates](#)

On technology/estimation:

- Different nest of CES function [» Nest 2: Simulation](#) [» Nest 2: Results](#) [» Nest 3: Results](#)
[» Nests: Comparison](#)
- Simulate increase in tax progressivity (in progress)
- Standard errors + Sensitivity matrix (Shapiro et al, 2017 QJE) ✓

Conclusion

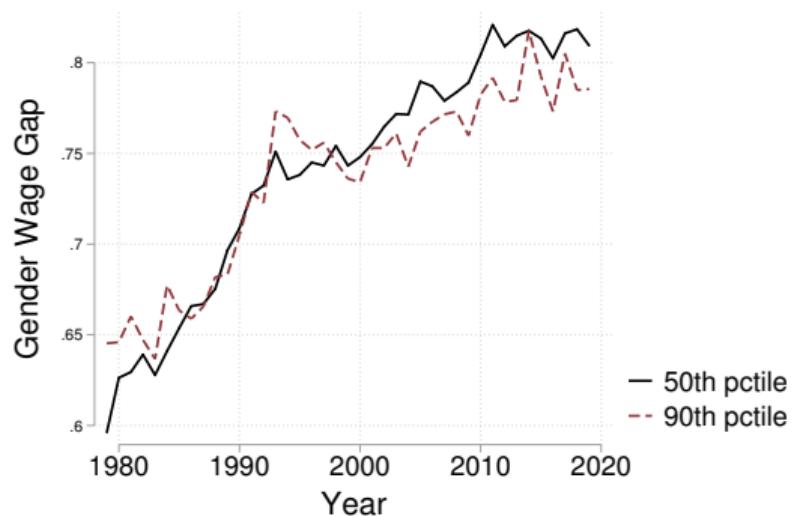
- Develop a theory that features effect of endogenous hours choice on sorting and inequality
 - Novel condition for assortative matching
- Novel mechanism: Hours-Biased Technological Change
 - New technologies raise returns to long hours for the high skill
- Estimate the framework: 1980-2015 in US
 - HBTC reconciles hours in the cross-section and time series
 - Accounts for roughly 30% of increase in income inequality

Future Work

- How do **hours constraints** (e.g. social norms) impact sorting?
 - Recall sorting condition: $f_{xy} + f_{yh} h_x$
 - If hours choice is **distorted**, this can affect misallocation and inequality
- Application: the (stalling) gender gap  **Empirics**
 - Gender gap stalled for the high skilled since 1990's (Blau and Kahn, 2014)
 - Gap is higher in occupations where hours elasticity is higher (Goldin, 2014)
- Can HBTC provide an explanation for stalling gender gap? Two forces:
 - Over time, easier for women to provide longer hours (hours constraints ↓)
 - Technology amplifies effect of such constraints

Implications: Hours Constraints and the Gender Gap

Gender gap has been decreasing across the wage distribution.

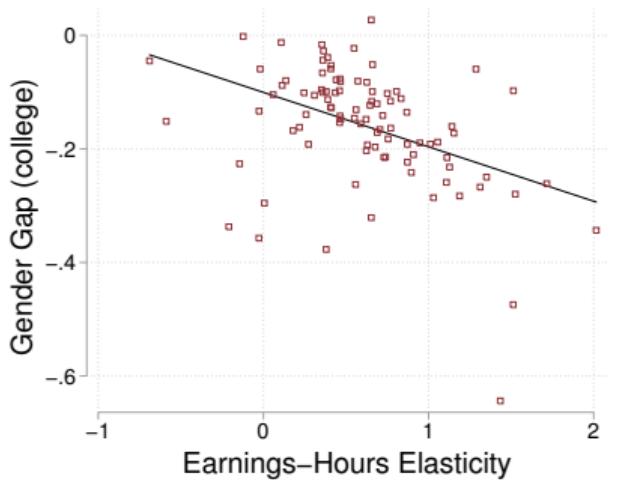


However, slower decline at the top since 1990's.

▶ Back

Implications: Hours Constraints and the Gender Gap

Tight link between gender gap and how income responds to hours (Goldin, 2014).



▶ Back

Distributions: From the Data

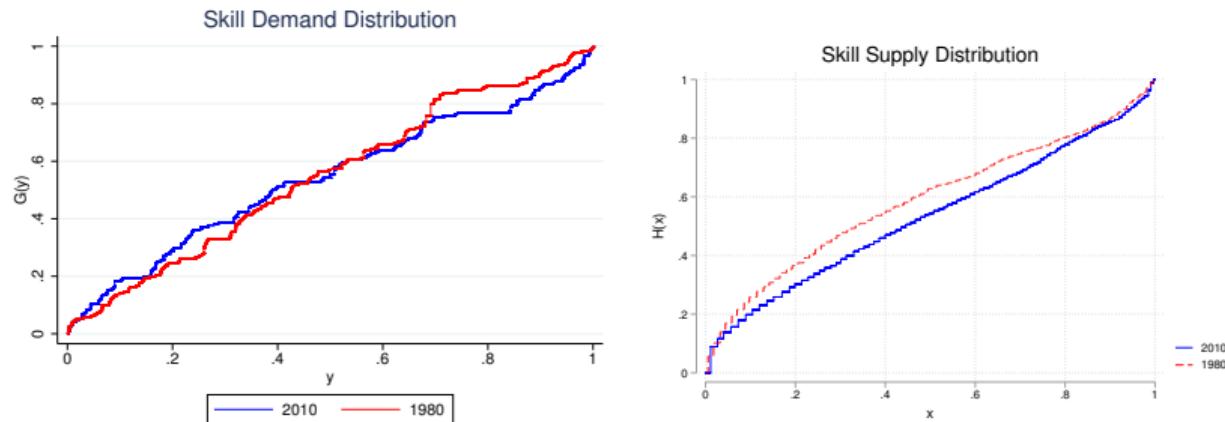


Figure: Estimated skill demand (left) and supply (right) distributions, for 1980 and 2010.

» Back

Estimates: Fitted Distributions

Parameter	1980	2015	Meaning
β	0.91 (0.02)	0.83 (0.02)	weight of (x, y) in prod.
α	0.24 (0.04)	0.79 (0.08)	weight of skills in prod.
γ	0.22 (0.02)	-0.39 (0.02)	compl. $(h, (x, y))$
ρ	-1.58 (0.27)	-4.24 (1.0)	compl. (x, y)
A	183.1 (27)	127.1 (1.0)	TFP

Notes: Standard errors in parentheses.

Key parameter changes:

- $\alpha \uparrow$ and $\rho \downarrow$: x, y more complementary
- $\gamma \downarrow$: hours/skills are more complementary ($f_{xh} \uparrow$)

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Results: Fixed Hours Model

What is the role of **endogenous** hours response?

$$f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)h^\gamma \right)^{\frac{1}{\gamma}}$$

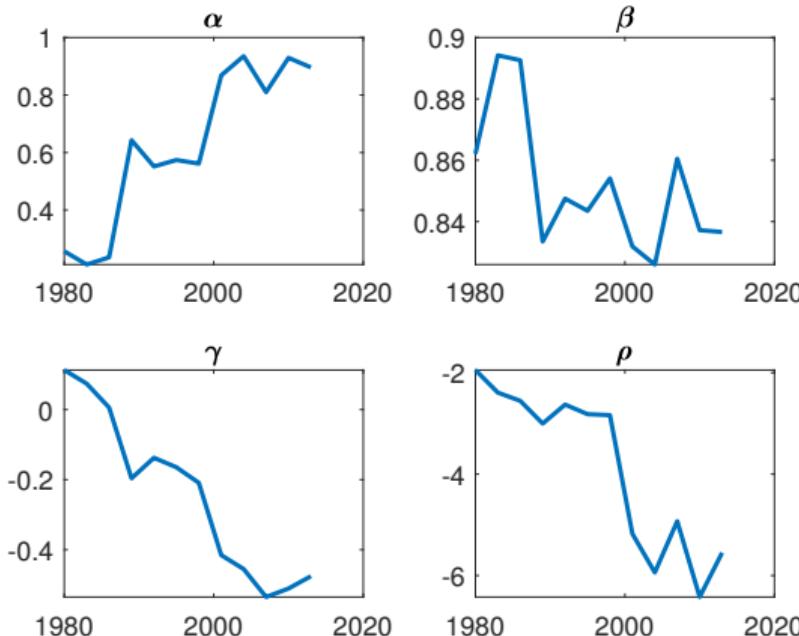
Estimated Change 1980-2015		
Parameter	Baseline	Fixed-hours
β	-60%	+2%
α	+199%	+102%
γ	-202%	-88%
ρ	+294%	+1200%

Fixed-h model **overstates** magnitude of SBTC (α, ρ) and **understates** HBTC ($\gamma \downarrow$)

» Back

Estimates: Fitted Distributions

Parameters Estimated Over Time



▶ Back

Estimates: Lower Income Effects

Parameter	1980	2015	Meaning
β	0.99	0.98	weight of (x, y) in prod.
α	0.05	0.13	weight of skills in prod.
γ	0.51	0.09	compl. $(h, (x, y))$
ρ	0.98	-0.57	compl. (x, y)
A	28,5179	26,677	TFP

Key parameter changes very similar to baseline specification:

- $\alpha \uparrow$ and $\rho \downarrow$: x, y more complementary
- $\gamma \downarrow$: hours/skills are more complementary ($f_{xh} \uparrow$)

» Back

Estimates: Idiosyncratic Tastes for Leisure

Introduce idiosyncratic tastes for leisure ψ_i in the utility function to match dispersion in hours.

Captures preferences, amenities, health shocks..

Parameter	1980	2015	Meaning
β	0.81	0.60	weight of (x, y) in prod.
α	0.23	0.67	weight of skills in prod.
γ	0.02	-0.13	compl. $(h, (x, y))$
ρ	0.98	-0.57	compl. (x, y)
A	6,5220	7,1405	TFP

Key parameter changes very similar to baseline specification.

» Back

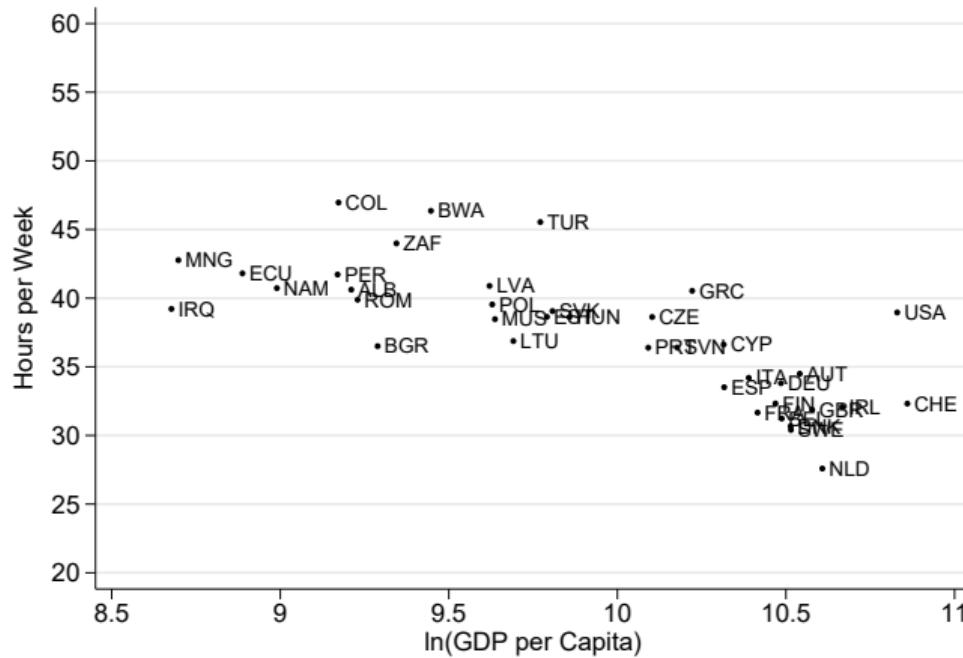
Estimates: Assume Beta Distributions

Parameter	1980	2015	Meaning
β	0.94	0.91	weight of (x, y) in prod.
α	0.08	0.14	weight of skills in prod.
γ	0.30	-0.07	compl. $(h, (x, y))$
ρ	0.93	-2.48	compl. (x, y)
A	16,5103	16,5703	TFP

Key parameter changes very similar to baseline specification.

» Back

Additional evidence: Hours/worker over GDP

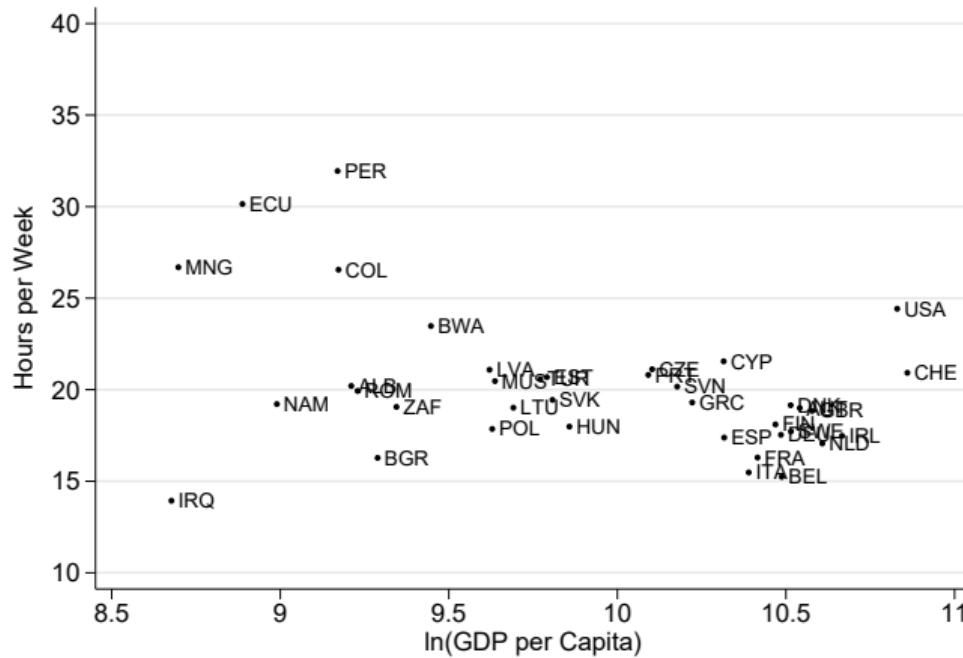


Source: Bick et al. (2018)

Hours/worker decline with GDPc in middle income and rich countries.

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Additional evidence: Hours/capita over GDP

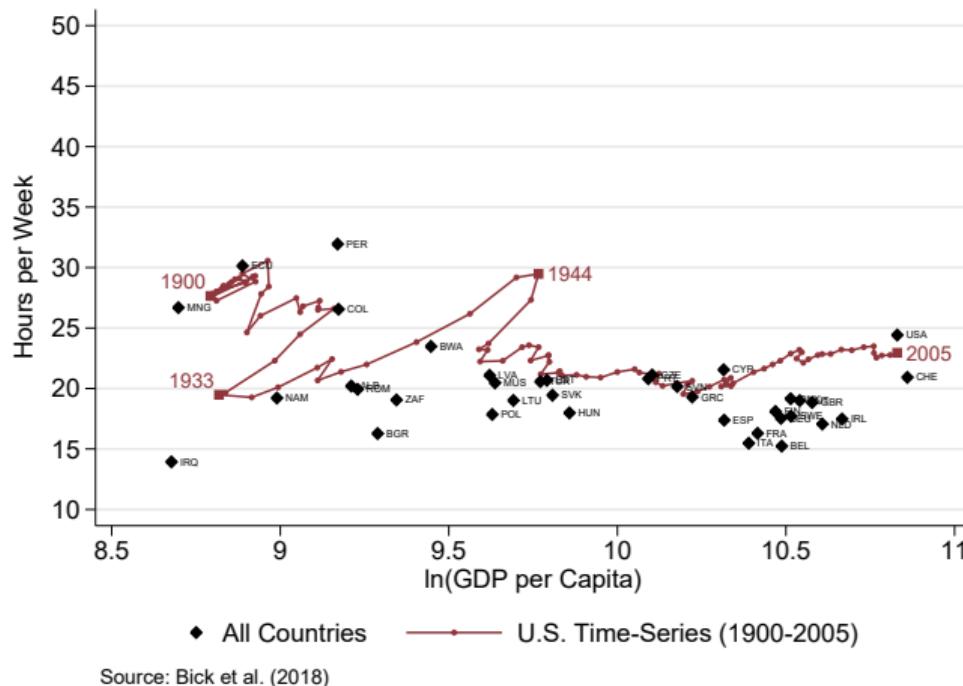


Source: Bick et al. (2018)

Hours/capita decline with GDPC in middle income and rich countries.

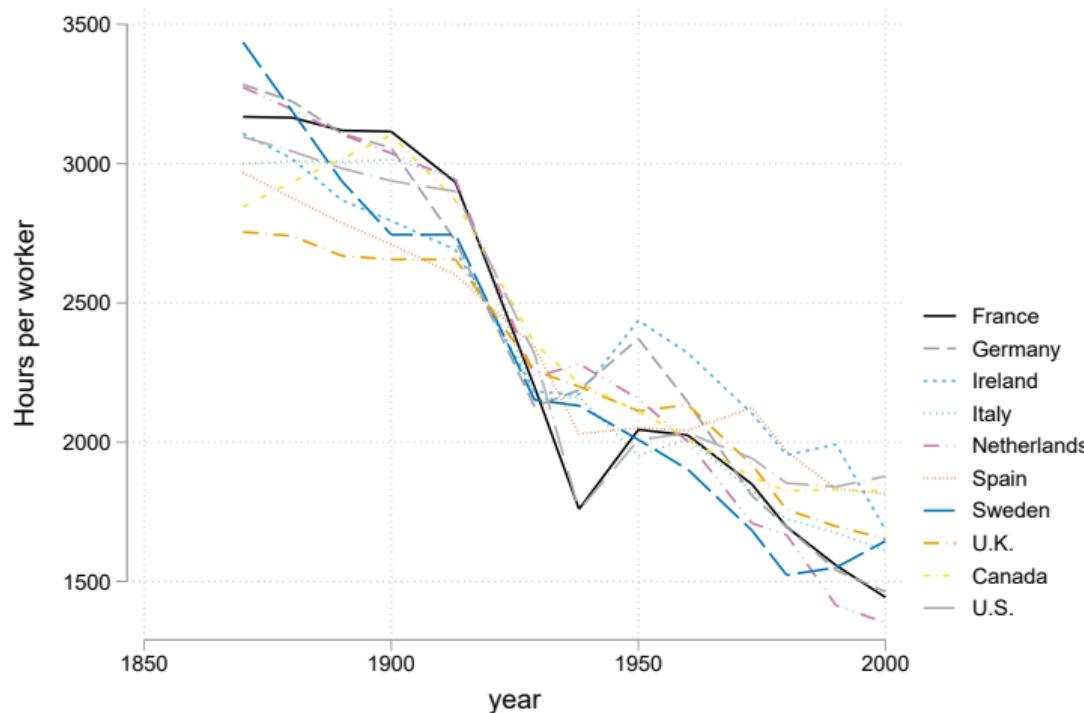
» Back

Additional evidence: US comparison



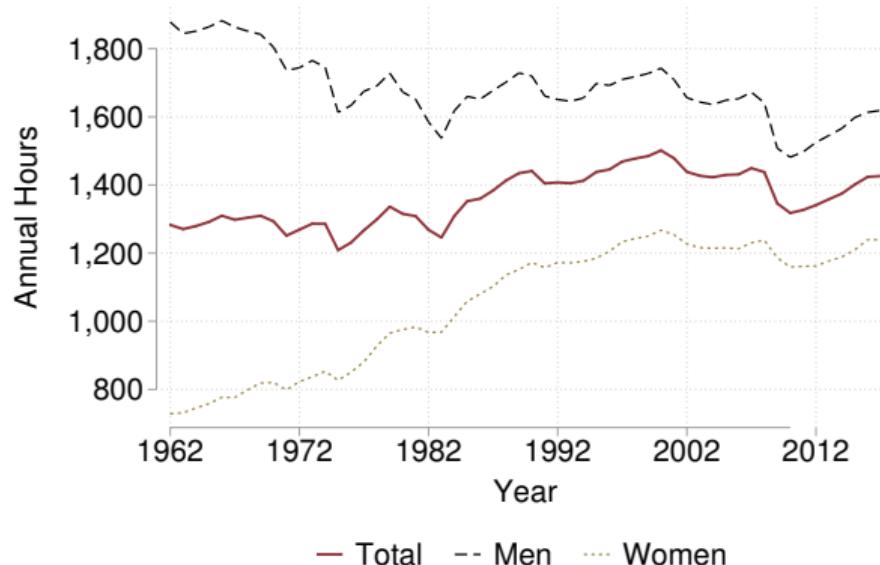
Hours/worker decline with GDPc in middle income and rich countries; focus on comparison with US.

Additional evidence: Across countries and Time



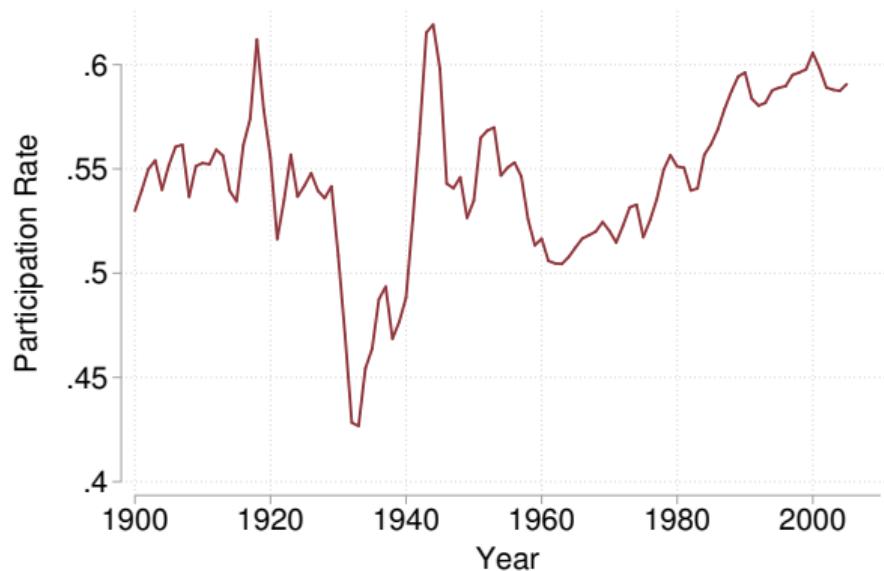
Hours/worker decline over time across countries.

Additional evidence: Breakdown by Gender



▶ Back

Additional evidence: Extensive Margin

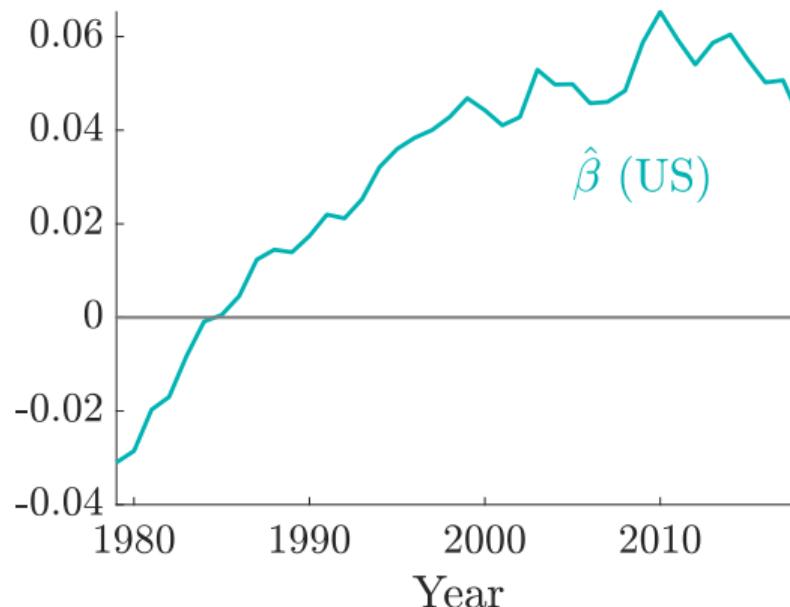


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Additional evidence: Hours by Wage Decile

- Run the regression for every year 1978-2019 (CPS, males):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$

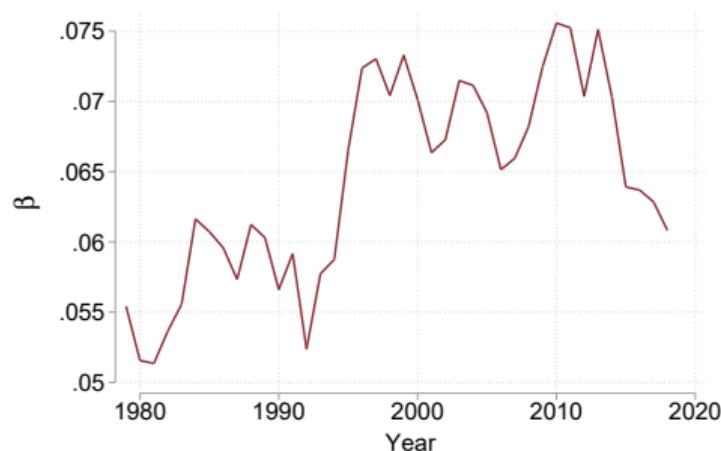


- Starting from 1980's, positive and increasing wage-hours relationship.

Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, both genders):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



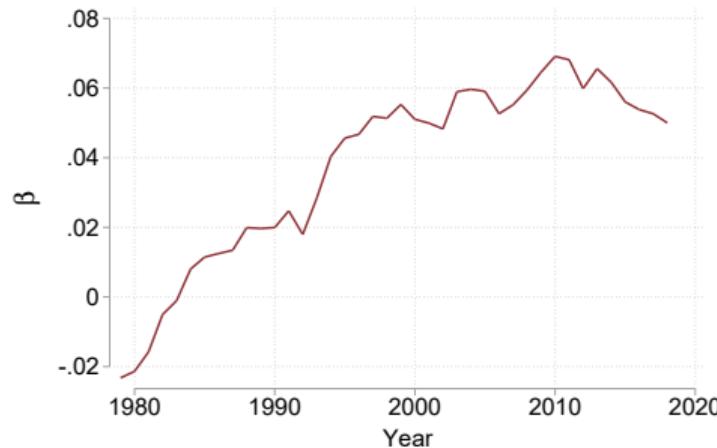
» Back to Main

» Back to Regression

Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, without mult. jobs):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



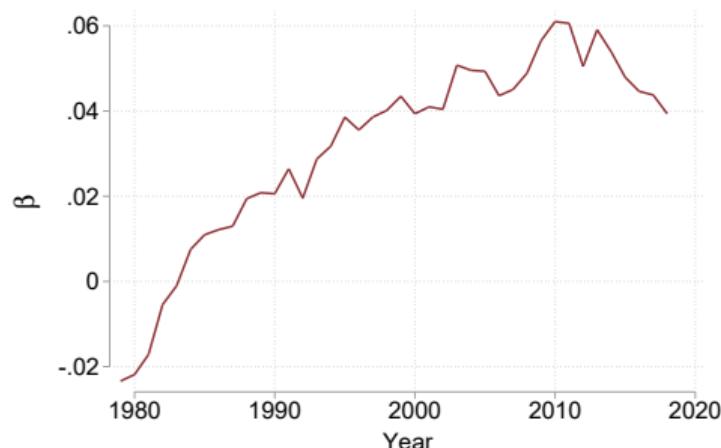
» Back to Main

» Back to Regression

Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, includes tips/commissions for hourly workers):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



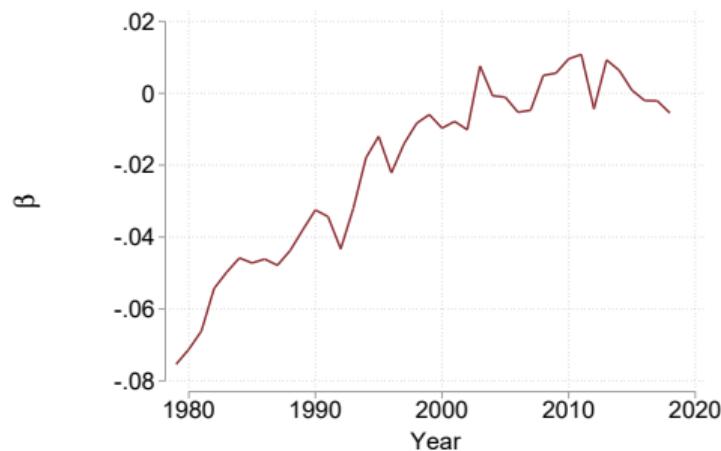
» Back to Main

» Back to Regression

Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, excludes hourly workers):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



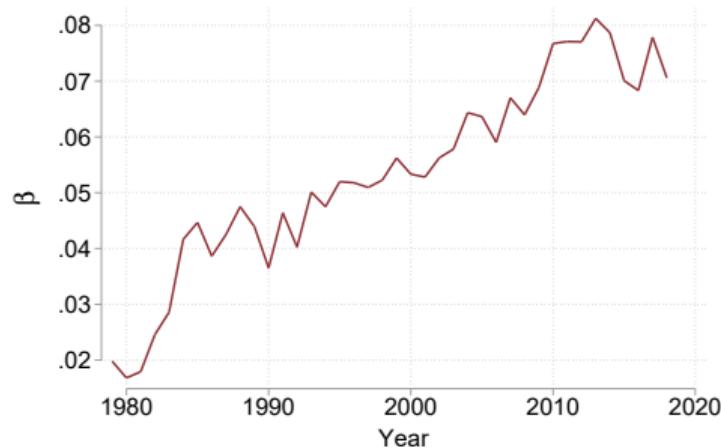
» Back to Main

» Back to Regression

Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, only hourly workers):

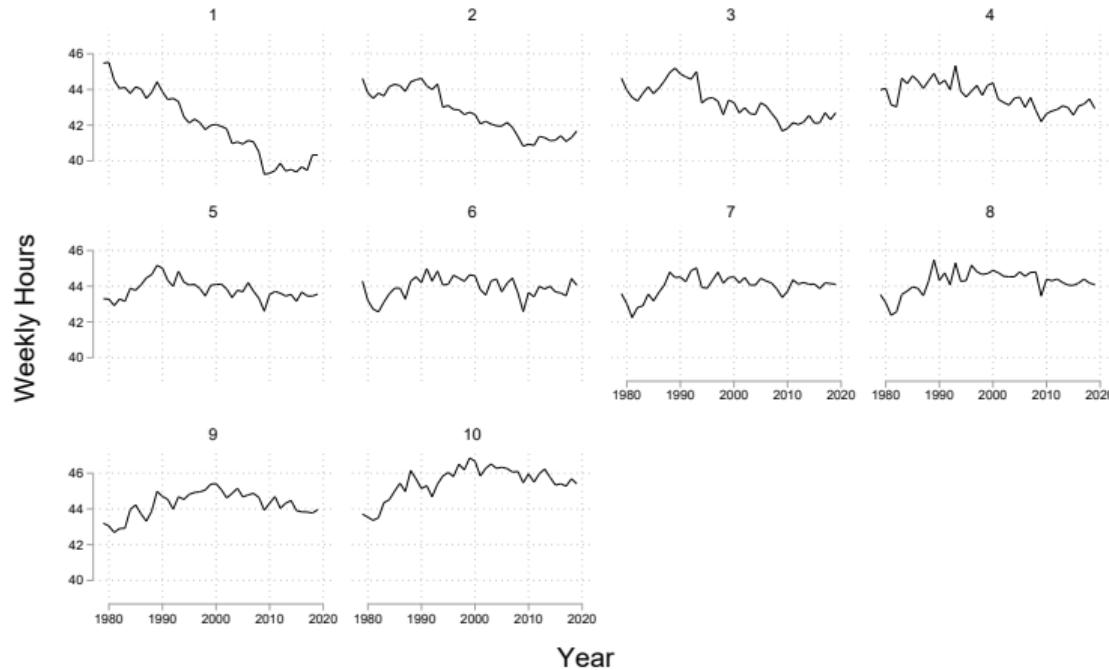
$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



» Back to Main

» Back to Regression

Additional evidence: Hours Worked by Wage P'ctile

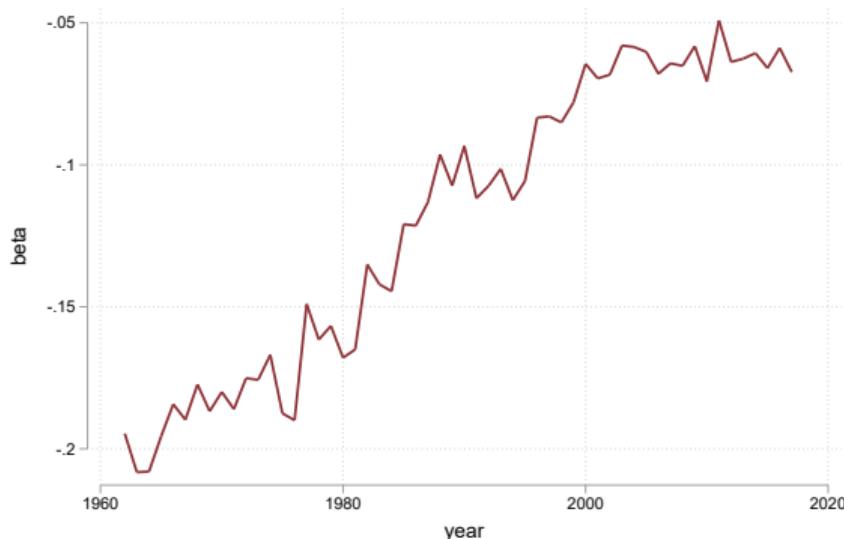


Graphs by hourw2

Additional evidence: ASEC

- Run the regression for every year 1978-2019 (ASEC, males, actual hours):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



» Back

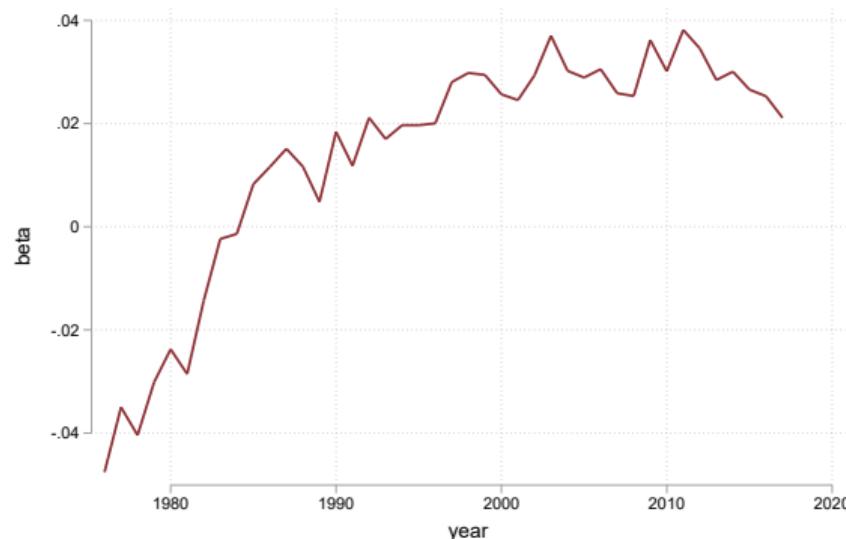
» Robustness: usual hours (M)

» Robustness: usual hours (F)

Additional evidence: ASEC

- Run the regression for every year 1978-2019 (ASEC, males, usual hours):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



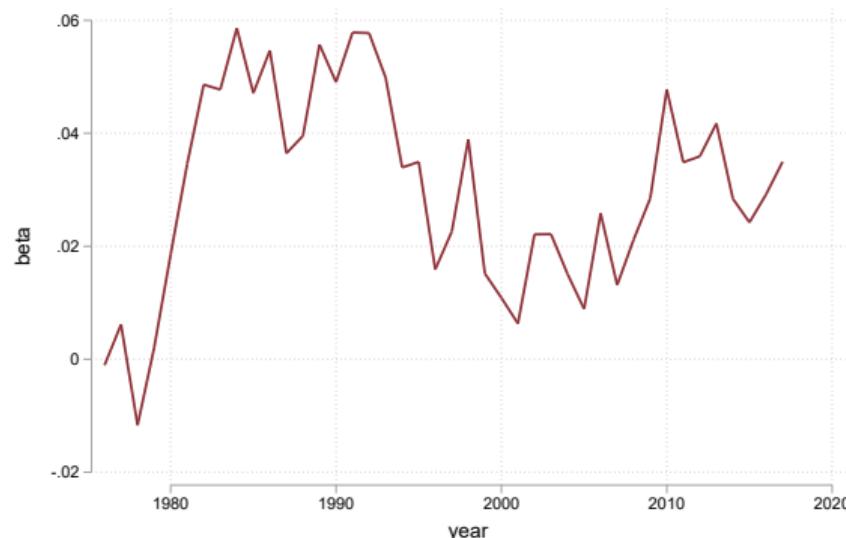
» Back to Main

» Back to Regression

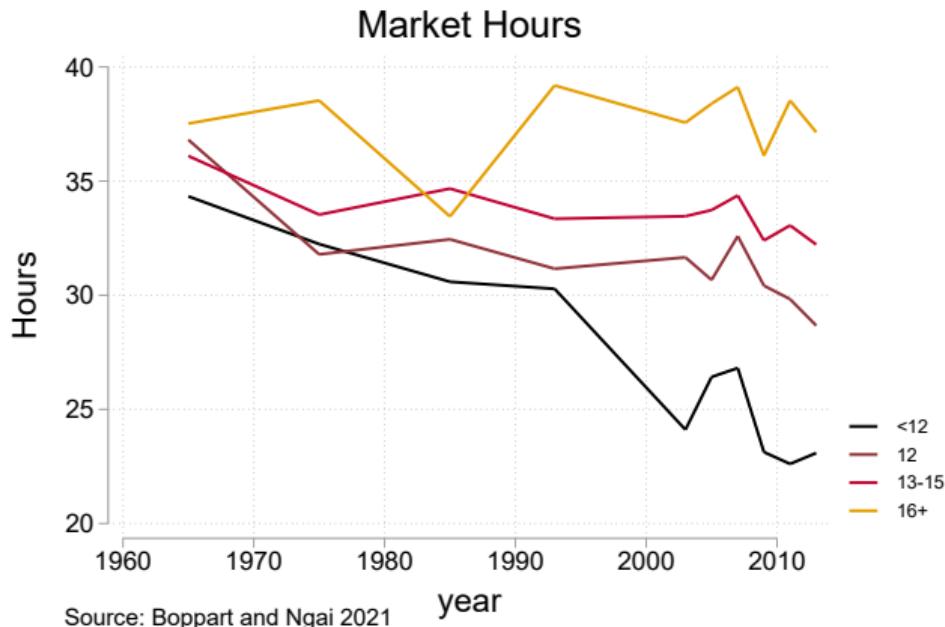
Additional evidence: ASEC

- Run the regression for every year 1978-2019 (ASEC, females, usual hours):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



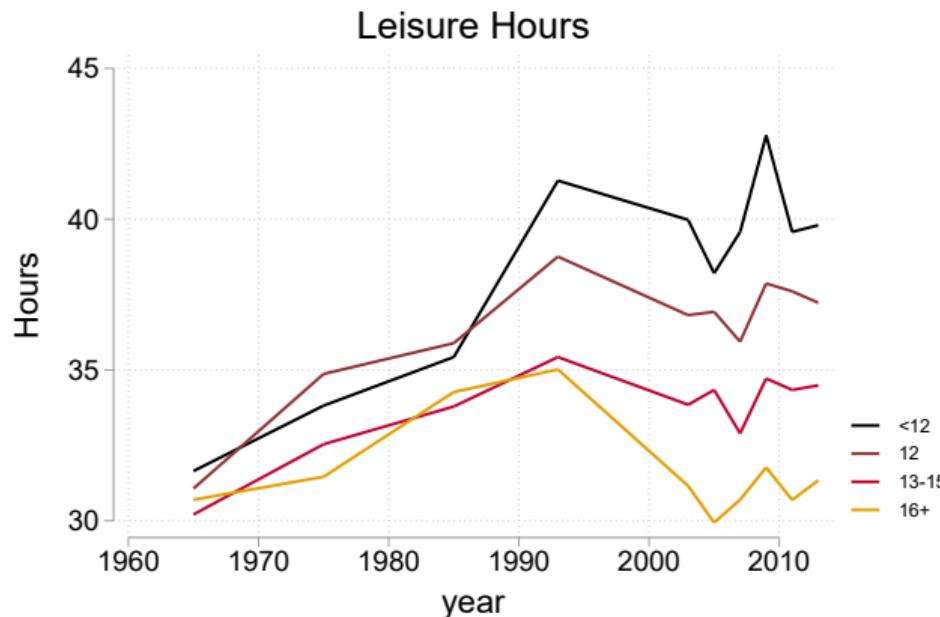
Additional evidence: ATUS data



Skilled workers people work roughly the same; low skilled work less.
Inequality in trends starts from 1980's.

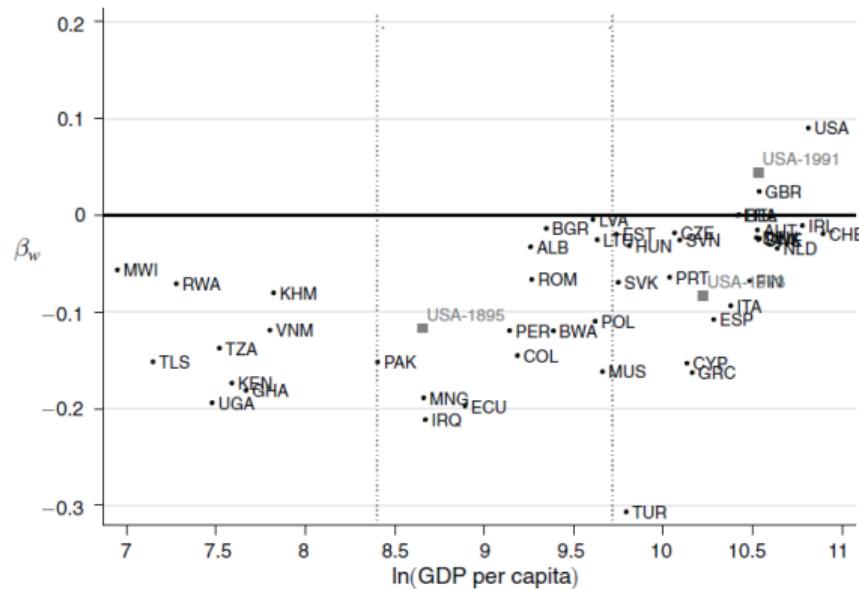
► Back

Additional evidence: ATUS data



Skilled workers consume less leisure, but starting from 1990's. Inequality in trends starts from 1980's. [➡ Back](#)

Additional evidence: β across countries



Additional evidence: progressivity across countries

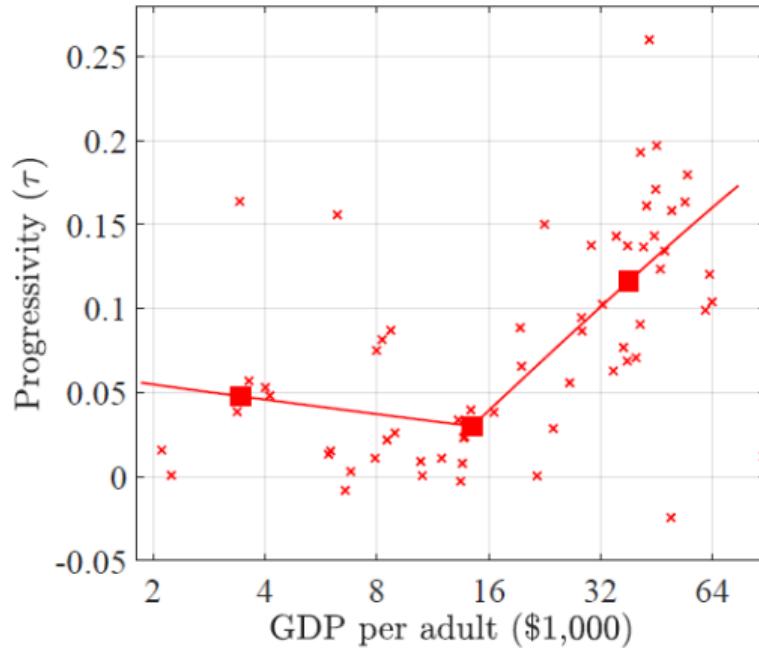


Figure: Fiscal progressivity across countries (source: Bick et al, 2021)

Market Clearing

Complete Definition

- Under Positive Assortative Matching (PAM), market clearing condition requires:

$$\int_{\mu(x)}^{\bar{y}} g^y(s) ds = \int_x^{\bar{x}} g^x(s) ds ,$$

where g^x and g^y are the densities of workers and jobs, respectively.

- Starting at the top, the highest type \bar{x} matches with the highest type \bar{y} .
- The type $x < \bar{x}$ matches with $y = \mu(x)$ if the measure of workers above x is equal to the measure of jobs above $\mu(x)$.
- This ensures that matching is measure-preserving.

Equilibrium definition

Market Clearing & Equilibrium

- Both households and firms take hedonic income function $e()$ as given.
- Market clearing determines matching function $\mu()$.
Under Positive Assortative Matching (PAM):

$$\int_{\mu(x)}^{\bar{y}} g^y(s) ds = \int_x^{\bar{x}} g^x(s) ds \quad (1)$$

Definition 1: Competitive equilibrium.

A competitive equilibrium of this economy is a tuple of functions (e, μ, h) such that:

- e and h maximize utility and profits (*optimality*)
- equation (1) holds (*market clearing*)

Derivation: Benchmark Model

- FOC:

$$f_x(x, y) - \frac{\partial w(x)}{\partial x} = 0$$

- Market clearing: $\mu()$ (measure preserving). PAM is defined as $\mu'(x) > 0$
- If $\mu'(x) > 0$, then we can write: $\Gamma(x) = \Phi(\mu(x)) \implies \mu(x) = \Phi^{-1}\Gamma(x)$.
- SOC:

$$f_{xx}(x, y) - w_{xx}(x) < 0$$

- To get rid of w_{xx} , we evaluate the FOC at $y = \mu(x)$:

$$f_{xx}(x, \mu(x)) + f_{xy}(x, \mu(x)) \cdot \frac{d\mu(x)}{dx} = w_{xx}(x)$$

- Substituting back, we get:

$$f_{xy}(x, \mu(x)) \cdot \frac{d\mu(x)}{dx} > 0$$

- Hence the sign of $\mu(x)$ is determined by the sign of f_{xy} (provided differentiability of $\mu()$)

Derivation: This Model

- Here we have two optimization problems (firm + hh). Hence FOC is a system of equations:

$$f_{\tilde{x}}(\tilde{x}, y) - \frac{\partial w(\tilde{x})}{\partial \tilde{x}} = 0 \quad (\text{firm})$$
$$u'_w \cdot w'_h + u'_h = 0 \quad (\text{hh})$$

- We repeat the previous derivation to understand properties of u and f that gives us PAM. The Hessian (SOC) is:

$$\begin{pmatrix} f_{xx}(x, y) - w_{xx}(x) & f_{xh}(x, y) - w_{xh}(x) \\ \frac{\partial u'_w \cdot w'_h}{\partial x} & \frac{\partial u'_w \cdot w'_h + u'_h}{\partial h} \end{pmatrix}$$

- Optimality requires $|H| \geq 0$. As before, we evaluate the FOC at $y = \mu(x)$ and substitute.
- Solving for $|H| \geq 0$ after some algebra we have that:

$$|H| \geq 0 \implies \text{the main condition.}$$

» Back

» Standard model

Hours choice: Derivation

The FOC for hours reads:

$$u_c f_h + u_h = 0$$

By monotone comparative statics, h_x increases in x if:

$$\frac{\partial (u_c f_h + u_h)}{\partial x} > 0$$

From which one gets:

$$\left(\underbrace{u_{cc} f_h f_x}_{\text{income effects}} + \underbrace{u_c f_{hx}}_{\text{subs. effects}} \right) > 0$$

Hours choice

Special Case: CRRA

- Hours choice:

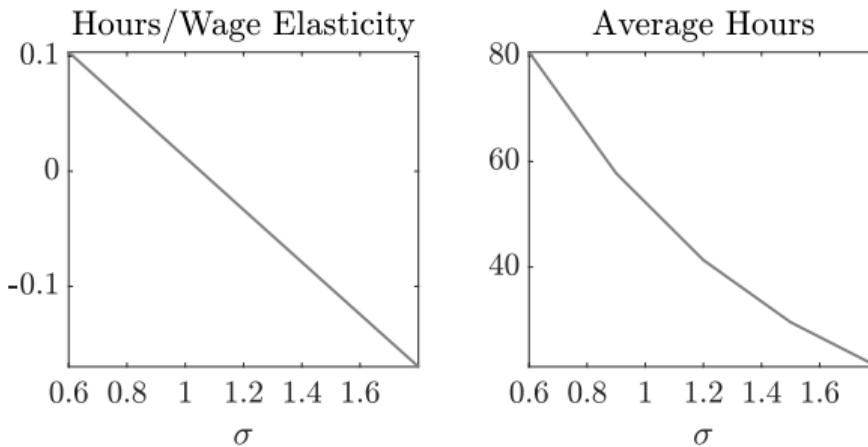
$$\left(\underbrace{u_{cc} f_h f_x}_{\text{income effects}} + \underbrace{u_c f_{hx}}_{\text{subs. effects}} \right) > 0$$

- **Sufficient condition:** relative risk aversion coefficient $\sigma = -c \frac{u_{cc}}{u_c} < 1$
 - σ governs strength of income vs substitution effects in macro models (King, Plosser, Rebelo 1988).
 - $\sigma > 1$ needed to match aggregate evidence (Boppart and Krusell, 2020)

► Back

Comparative Statics

Preferences



An increase in σ :

- **Decreases** hours wage elasticity (higher income effects) and average hours worked
- Elasticity turns negative for lower values of σ , if low complementarities.

» back

Assortative Matching

- Pair x and y solve the problem:

$$U(x, y, V) = \max_h u(c, h) \quad \text{s.t.} \quad f(x, y, h) - c \geq V$$

where $U(x, y, .)$ traces a **Pareto frontier**.

- Hence we maximize U w.r.t. to job y to get the FOC:

$$U_y + U_\pi \pi_y = 0$$

- The equilibrium matching μ is optimal provided that the SOC < 0 , hence:

$$U_{yy} + 2U_{y\pi}\pi' + U_{\pi\pi}\pi'^2 + U_\pi\pi'' < 0$$

Assortative Matching

- Differentiate FOC along the candidate equilibrium $y = \mu(x)$:

$$U_{yy} + U_{xy}\mu' + U_{x\pi}\pi' + U_{x\pi}\pi' + U_{y\pi}\mu'\pi' + U_{\pi\pi}\pi'^2 + U_\pi\pi'' = 0$$

- Combining this condition with $SOC < 0$ gives:

$$\mu' \left[U_{xy} - \frac{U_y}{U_\pi} U_{x\pi} \right] \geq 0$$

- Hence PAM ($\mu' > 0$) obtains iff

$$U_{xy} - \frac{U_y}{U_\pi} U_{x\pi} \geq 0$$

Which completes the proof.

Assortative Matching

Alternative condition for PAM

- Corollary: A condition for PAM in terms of primitives is:

$$(f_{yx} \underbrace{(-u_{cc} f_h f_h - u_c f_{hh} - u_{hh})}_{>0} + f_{hy} (u_{cc} f_x f_h + u_c f_{hx})) > 0$$

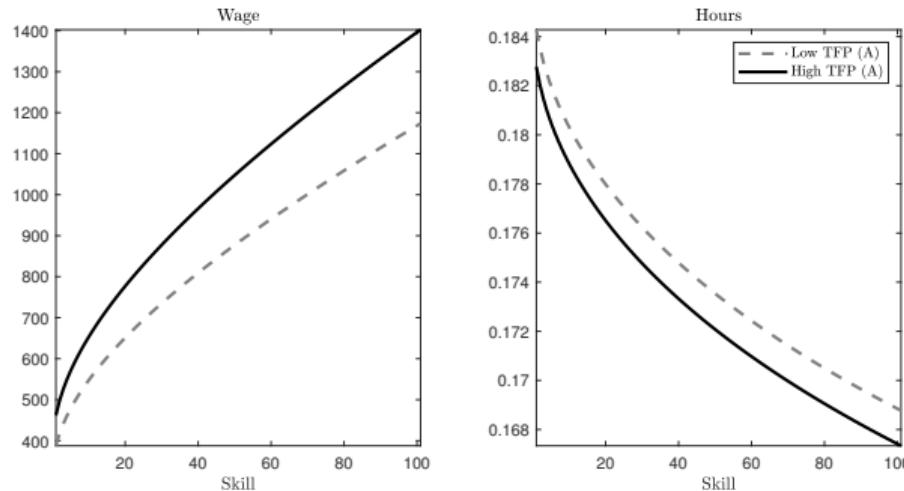
- Intuition:
 - f_{xy} : skill-job complementarity (Becker 1973)
 - f_{xh}, f_{yh} : hours complementarities (better workers produce more with more time)
 - $u_{cc} < 0$: income effects push towards NAM

► Proof

► Back

Prod. function: comparative statics

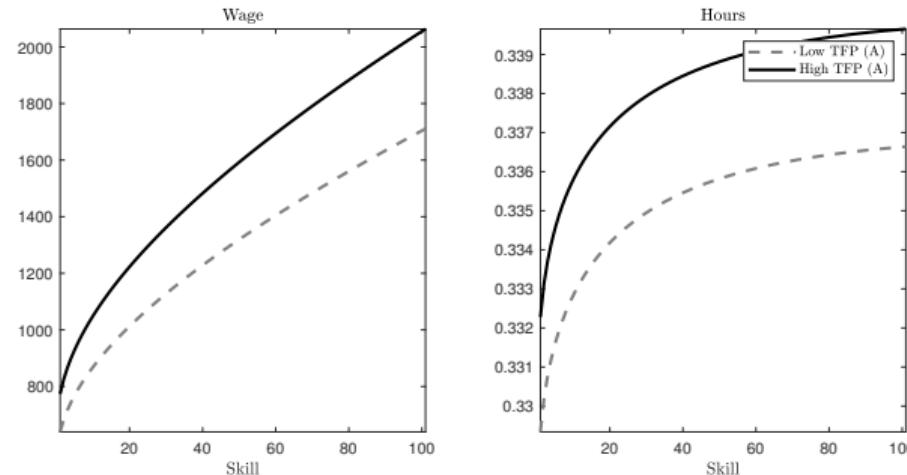
TFP: A, baseline



Increasing TFP has a level shift on wages (positive) and hours (negative, $\sigma > 1$).

Prod. function: comparative statics

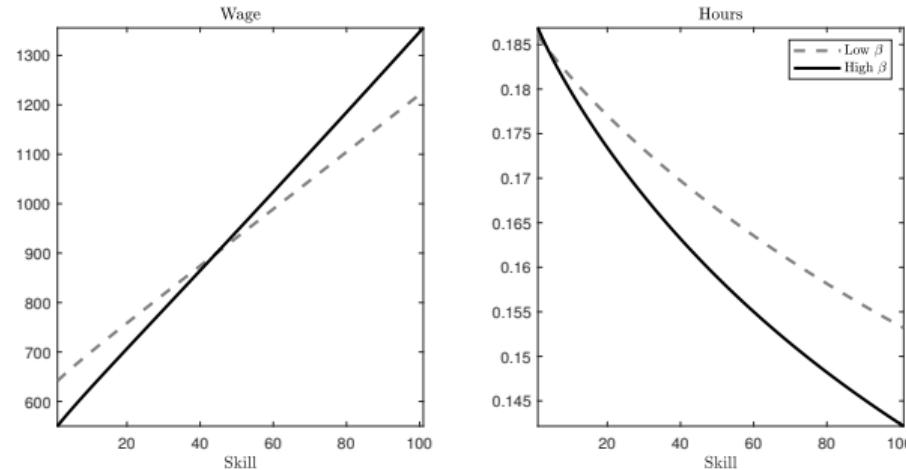
TFP: A, low σ



Increasing TFP has a level shift on wages (positive) and hours (positive, $\sigma < 1$).

Prod. function: comparative statics

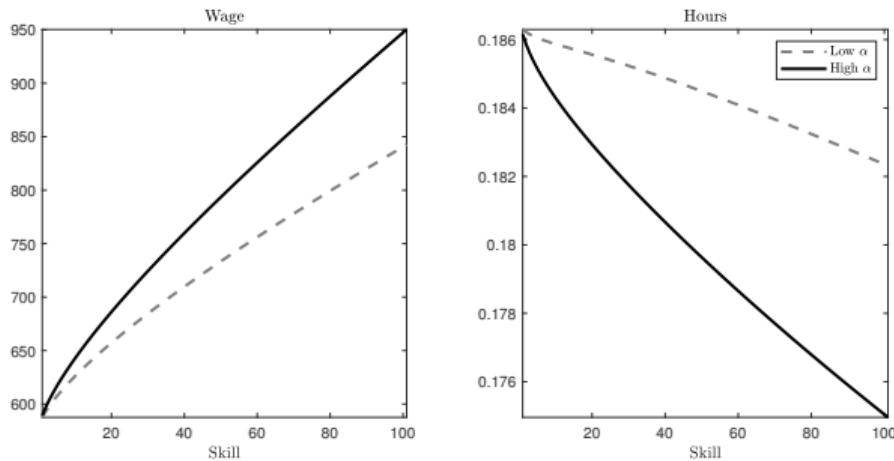
Hours/Match share: β



Increasing β decreases importance of hours (hence hours choice \downarrow).

Prod. function: comparative statics

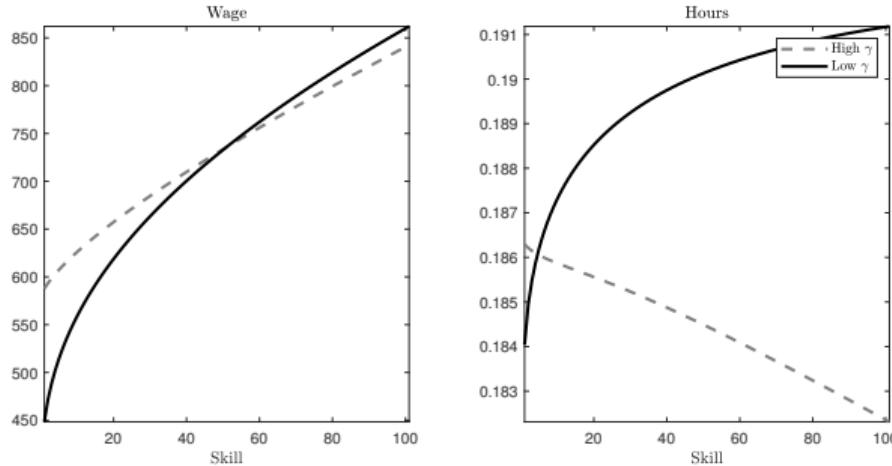
Skill importance: α



Increasing α makes wage steeper in skills; through income effects, hours less steep (hence $\partial h / \partial x \downarrow$).

Prod. function: comparative statics

Match/hours complementarity: γ



Increasing γ pushes towards $\partial h / \partial x \uparrow$. Also, it increases inequality. Key parameter to inform positive/increasing hours/wage elasticity! Missing in the previous specification ($\gamma = 0$).

Estimation

Jobs distribution

- Need distribution of worker and job types, $H(x)$ and $G(y)$, for 1980 and 2010
- Solution: estimate directly from the data
- $G(y) \approx$ cognitive skill requirement
 - 1980: DOT (Dictionary of occupational titles); $y \approx$ mathematical reasoning
 - 2010: O*NET: PCA on cognitive/math variables
- Final step: merge estimated y with CPS for both periods to get $G(y)$
- Same approach as in Lindenlaub(2014), Lindenlaub and Chade(2022)

Estimation

Skill/Ability Distributions

- $H(x) \approx$ test scores
 - 1980: NLSY79
 - 2010: NLSY97
- I rely on Altonji, Bharadwaj, and Lange (2009) criterion to make test scores comparable across the two periods¹
- (adjusted) distribution of test scores gives our measure of $H(x)$

¹e.g. test respondents were administered the test at different ages in the two cohorts

Estimation

Skill/Ability Distributions

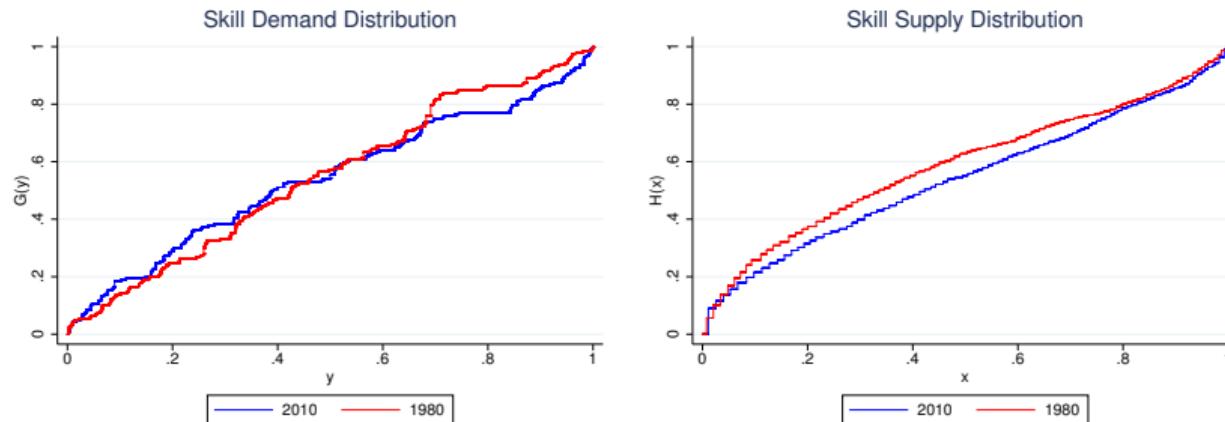


Figure: Estimated skill demand and supply distributions, for 1980 and 2015.

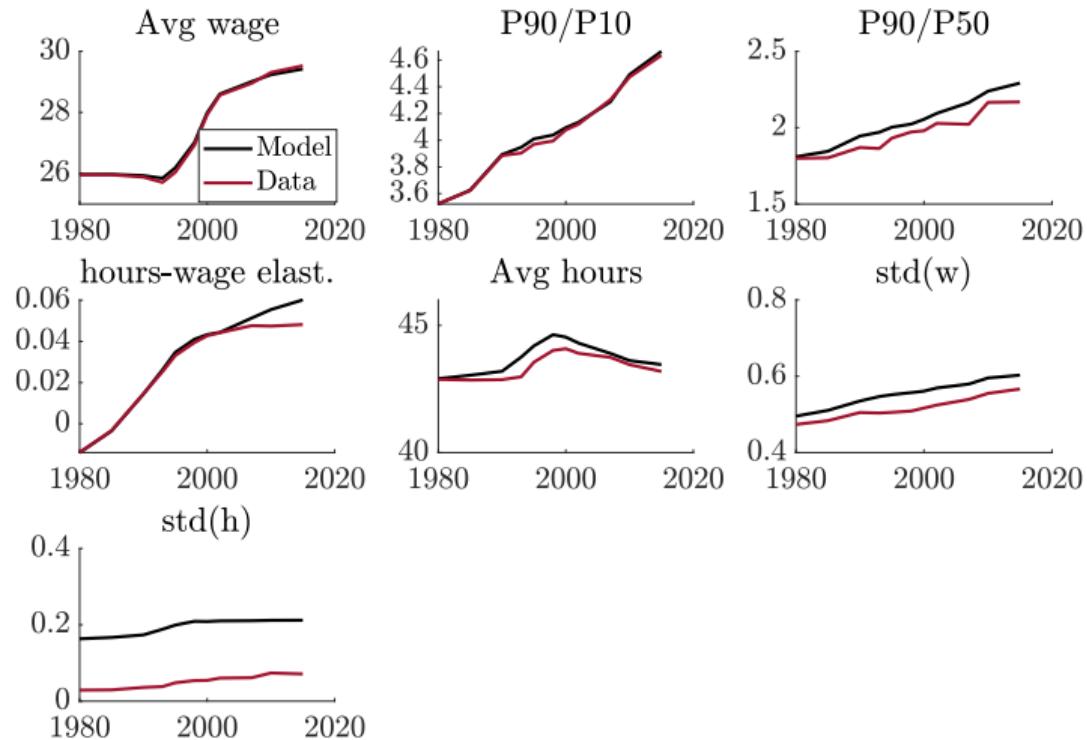
- Jobs (left): slight change in dispersion
- Workers (right): 'better' distribution of skills

Targets

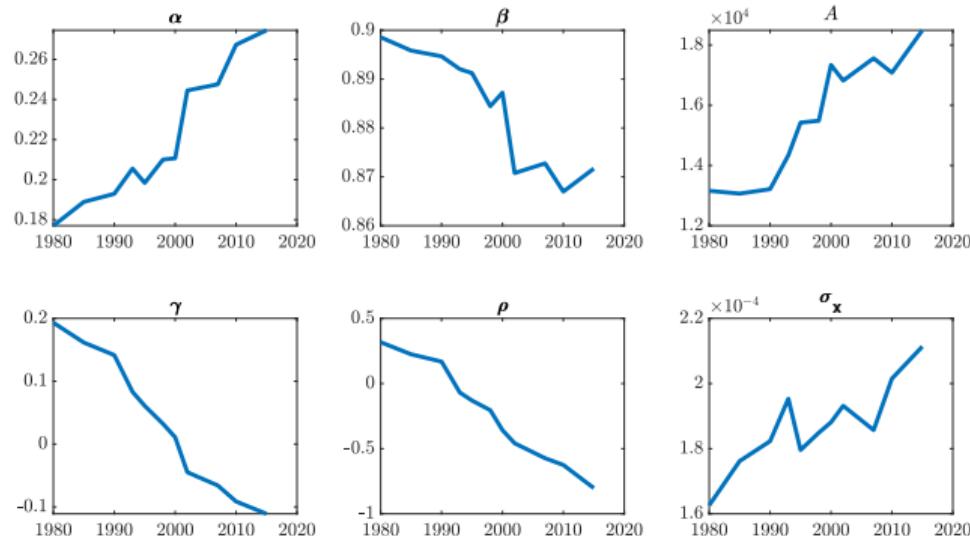
Results						
Moment	Data 1980	Data 2015	Model 1980	Model 2015	Data Δ	Model Δ
E(w)	19.6	23.9	19.6	23.9	21%	21%
E(h)	37.9	38.4	37.9	38.4	+1%	+1%
W75/W25	1.86	2.29	1.86	2.29	23%	23%
W50/W10	1.92	2.03	1.92	2.03	6%	6%
h/w elast.	-0.012	0.062	-0.012	0.062	7%	7%

» back

Model Fit



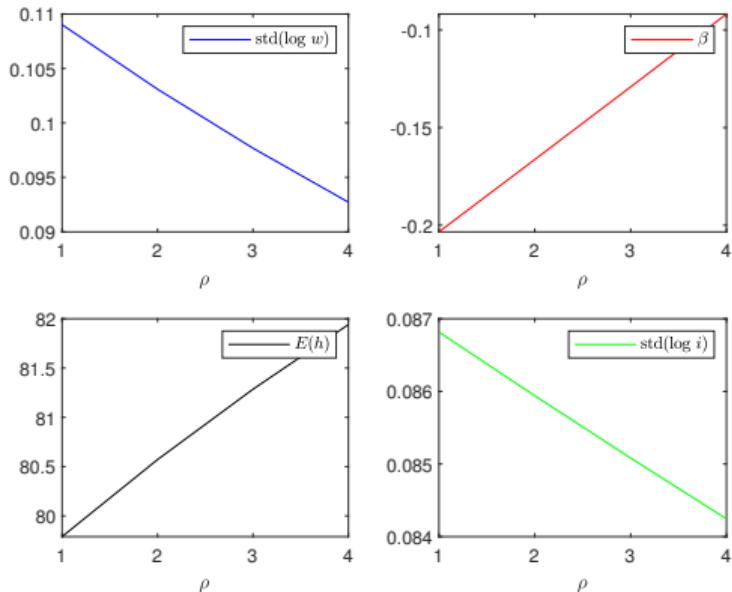
Estimates



► Back

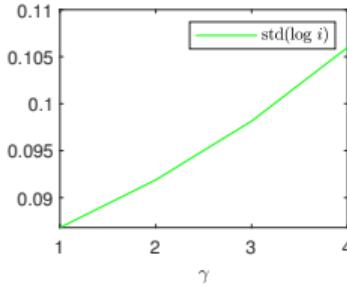
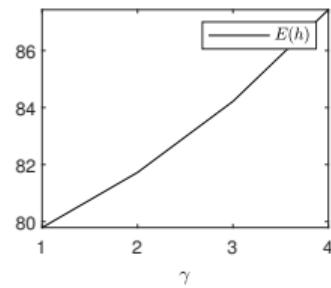
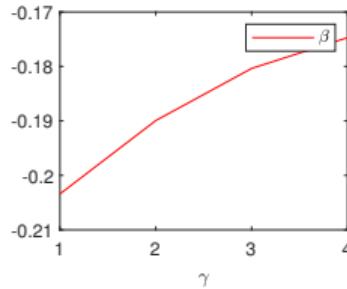
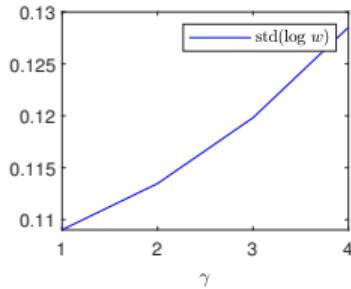
Different CES nesting

$$f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)y^\gamma \right)^{\frac{1}{\gamma}}$$



Different CES nesting

$$f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)y^\gamma \right)^{\frac{1}{\gamma}}$$



▶ Back

Different CES nesting

Assume a different nest for the CES: workers and hours form a 'bundle',
 $\tilde{x} = \tilde{x}(x, h)$.

$$f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)y^\gamma \right)^{\frac{1}{\gamma}}$$

Parameter	1980	2015	Meaning
β	0.09	0.17	weight of (x, h) in prod.
α	0.80	0.85	weight of skills in prod.
γ	1.01	-0.34	compl. (\tilde{x}, y)
ρ	0.17	0.11	compl. (h, x)
A	32,209	27,276	TFP

► Back

Different CES nesting

Assume a different nest for the CES: firms/jobs and hours nested as $CES((y, h), x)$.

$$f(x, y, h) = A \left(\beta(\alpha y^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)x^\gamma \right)^{\frac{1}{\gamma}}$$

Parameter	1980	2015	Meaning
β	0.40	0.90	weight of (y, h) in prod.
α	0.73	0.97	weight of jobs y in prod.
γ	-3.3	-4.5	compl. $(x, y/h)$
ρ	0.56	0.04	compl. (h, y)
A	4,489	27,597	TFP

» Back

Different CES nesting

Nest1 $f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)h^\gamma \right)^{\frac{1}{\gamma}}$

Nest2 $f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)y^\gamma \right)^{\frac{1}{\gamma}}$

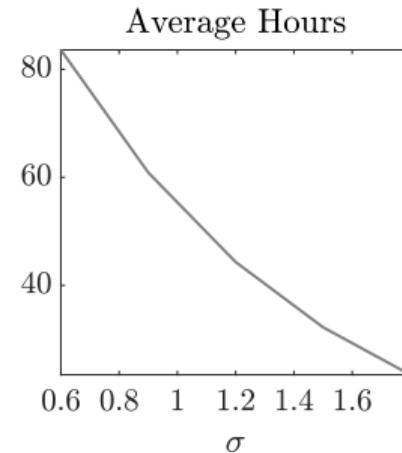
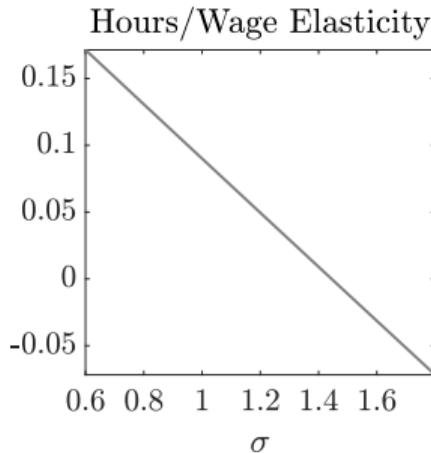
Nest3 $f(x, y, h) = A \left(\beta(\alpha y^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)x^\gamma \right)^{\frac{1}{\gamma}}$

	Nest 1		Nest 2		Nest 3	
Parameter	1980	2015	1980	2015	1980	2015
β	0.90	0.87	0.09	0.17	0.40	0.90
α	0.13	0.21	0.80	0.85	0.73	0.97
γ	0.32	-0.07	0.98	-0.34	-3.3	-4.5
ρ	0.22	-1.20	0.17	0.11	0.56	0.04
A	12,871	22,218	32,209	27,276	4,489	27,597

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Comparative Statics

Preferences



An increase in in σ :

- **Decreases** hours wage elasticity (higher income effects) and average hours worked

» Low compl.

» Back