Dual Income Earners and Productivity*

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PRELIMINARY AND INCOMPLETE - PLEASE DO NOT CIRCULATE

Abstract

The abstract

*We thank. All errors are our own.

1 Introduction

One of the most prominent common macroeconomic trends in advanced countries in the last decades has been a significant slowdown in labor productivity growth. In US for example, labor productivity growth declined during the 1990's and 2000's, only to recover in the 2010's (especially in the post pandemic years). The exact causes of these trends are still unclear, but the literature has pointed out the correlation between labor productivity indicators and trends in females labor force participation (Albanesi, 2022). Indeed, labor productivity growth slowed coinciding with the large inflow of women in the labor market, resulting in large increases in female Labor Force Participation (LFPR) and Employment-to-Population ratio (EPOP) during the 1990's. The argument is simple, and it is based on the idea that a large inflow of workers mechanically reduces labor productivity mechanically. In this paper, we argue that to understand the relationship between women labor market outcomes and aggregate productivity, we need to explore when and under which conditions women search for jobs that increase productivity, by favouring match-specific human capital accumulation.

2 Motivating Facts

Here we show the main facts motivating the analysis. We focus on three main facts. First, we present trends on labor productivity growth in US, whose slowdown (and sudden recovery) we aim to provide an explanation for. Second, we collect evidence on the behavior over time of US hours worked for female (vs males), focusing in particular on single females and females in joint households. The third set of facts relates to the *type* of job that females in joint (vs single) households choose over time.

Labor Productivity Trends The first fact we present relates to the behavior of labor productivity trends in the US over time, and is presented in Figure 1. Labor productivity growth in the US decline starting from the 1980's and remained well below the pre-1980 growth trend throughout the 1990's. Starting from the early 2000's, labor productivity growth increased significntly and consistently, with a peak after the 2020 pandemic. As we show in the next paragraph, the literature has proposed explanation for the decline that are connected to the large inflow of women in the labor force during the same years. Of course, labor productivity trends are influenced by a number of factors not directly related to the gender composition of the workforce, but the timing of the slowdown (and sudden increase) is very suggestive that trends in labor productivity and female labor supply are related.

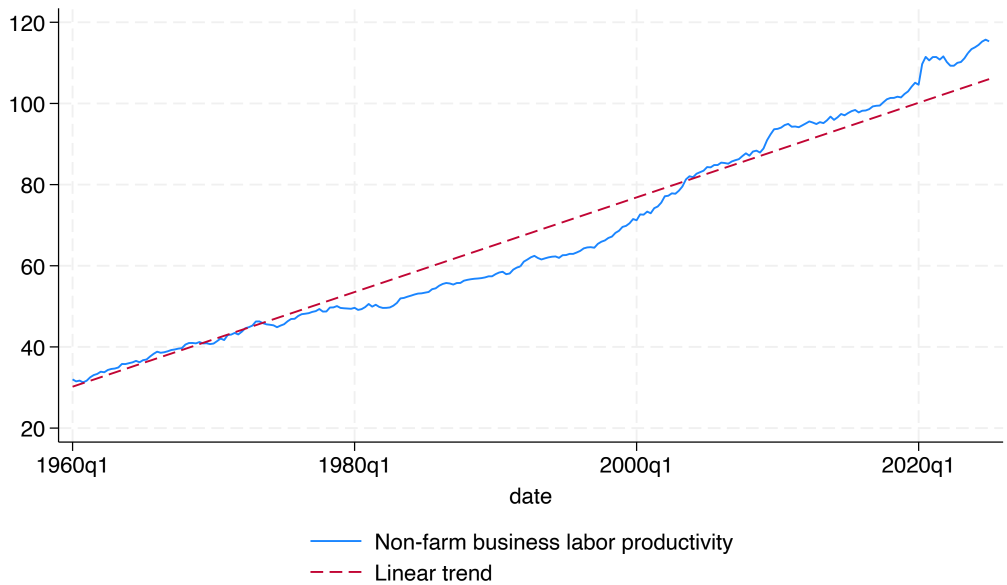

Figure 1: Labor Productivity Trend in the US.

Notes: Source: BLS and own calculations.

Labor Supply & Extensive Margin A second fact we want to highlight is the behavior of the extensive margin of women's labor supply over time. We look at the Employment-to-Population ratio (EPOP) since our model does not feature a participation decision explicitly, but the literature has highlighted similar trends in Labor Force Participation (LFPR) for women over the same time period (see e.g. Albanesi and Prados, 2022; Albanesi 2017). EPOP in US rises strongly over time, and then plateaus (and declines) in the 2000's, with only a recent uptick post-2020 (which, as we will show later, represents an important clue for our mechanism). See Figure 2.

While the rise in EPOP (mirrored by the rise in LFPR, not shown here) has been interpreted as an opening up of the labor market (and the opportunities it offers) for women, the causes for the decline are still debated. Perhaps the most prominent explanation has to do with mechanism that, at a baseline level, have to do with added worker effect: with rising income for men, especially at the top of the income distribution, an income effect in women's labor supply predicts that women would supply less labor, thus potentially explaining the decline in EPOP for women following the 2000's (Albanesi and Prados, 2022). Interesingly, Figure 2 shows that trends in extensive labor supply (measured here by EPOP rate) are common across single and joint households. As we will show, this suggests there seems to be more to this story, since intensive margin supply (defined broadly) rises continuously for women, and even more importantly, intensive margin trends differ significantly between single and joint

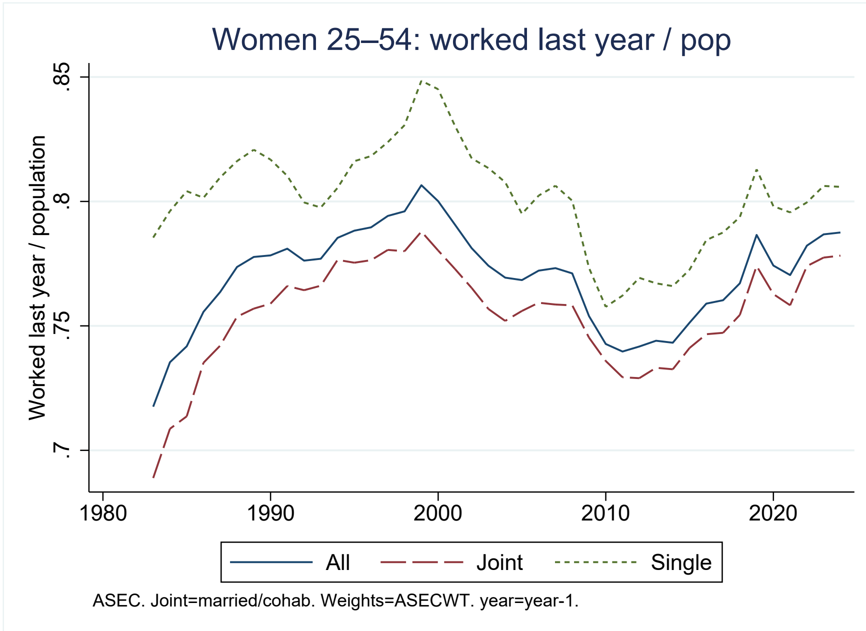

Figure 2: Female EPOP in the US, overall and by household status.

Notes: Source: CPS-ASEC.

households, as we show next.

Intensive Margin & Job Allocation The third set of facts is perhaps the most overlooked in the literature, and relates to the qualitative characteristics of women's work (conditional on working). We look at women's intensive margin of work and how it moved over time. Moreover, we are also interested in a broader notion of how intensively women work; this notion is captured by the type of jobs women perform, conditional on working (full time vs part time, Non-Routine Cognitive vs all others). The motivation behind this is to go beyond extensive margin trends over time and look at what kind of jobs women perform over time, as well as how many hours they put in these jobs. This is what our set of facts in this section try to capture.

We look at trends on the intensive margin in Figure 3. The figure shows hours worked conditional on working for prime age women (25-54) in US. The blue solid line shows that women in the US have increased hours worked, as phenomenon that has been noted elsewhere in the literature (Jang and Yum, 2022). Perhaps more surprisngly is that the increase in hours worked (so the intensive margin of womens' labor supply) is driven almost exclusively by women in joint households. Women in single households barely increased their hours since the 1980's. Women in joint households, on the other hand, increased their hours worked in a steady manner since the 1980's. The figure suggests that to understand trends in female

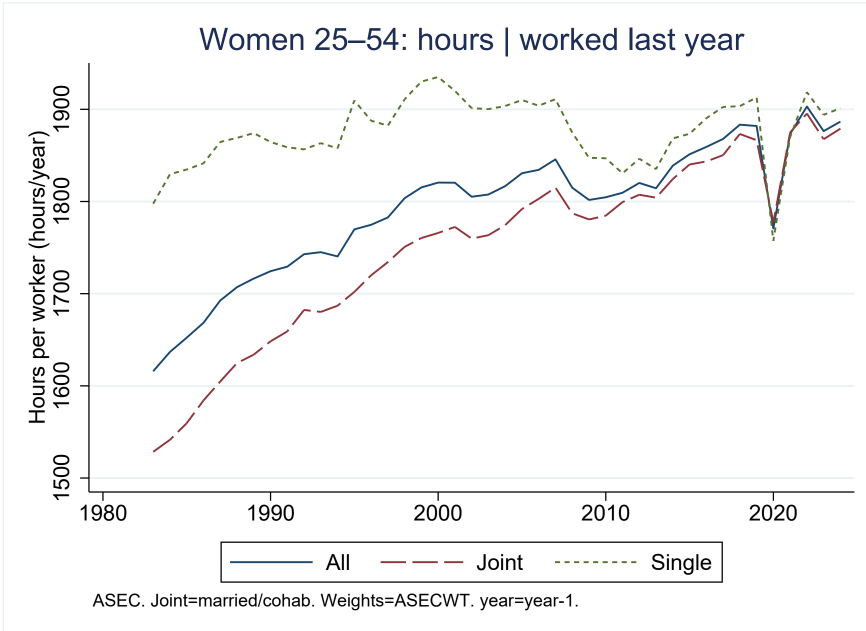

Figure 3: Female Hours Worked cond. on working (US), overall and by household status.

Notes: Source: CPS-ASEC.

labor supply, and in particular on the intensive margin, we need to look at women in joint households, which is what our model is meant to capture.

We explore next the proportion of prime-age women that are employed full-time (FT). Figure 4 shows that while females in both single and joint households observed similar trends in EPOP ratios, the fraction of females in joint household employed in full-time jobs severely lagged that of their single counterparts until the most recent post-COVID period.

Among employed women, the pre COVID shortfall in full-time work is concentrated among those in joint households aged 30 to 39, consistent with a higher (child-care related) opportunity cost of hours. If FT jobs build more productive human capital than part time (PT) jobs, this pattern could have contributed to weak labor productivity growth in the 1980s through the early 2000s.

To the best of our knowledge, the latter set of facts is new to the literature and we explore it further. We look, in particular, at whether women take on more Non-Routine Cognitive Full-time jobs (NRCOGFT). This is motivated by the idea that women can accumulate human capital, and therefore build productivity, in these jobs more than in jobs that are performed part-time, or where knowledge and skills are slowly accumulated. Figure5 shows that the probability of working in these jobs (left panel) has increased over time for women, and in particular for women in joint households, The middle and left panel demonstrate that this is not due to more women participating in the labor market, but due to the fact that

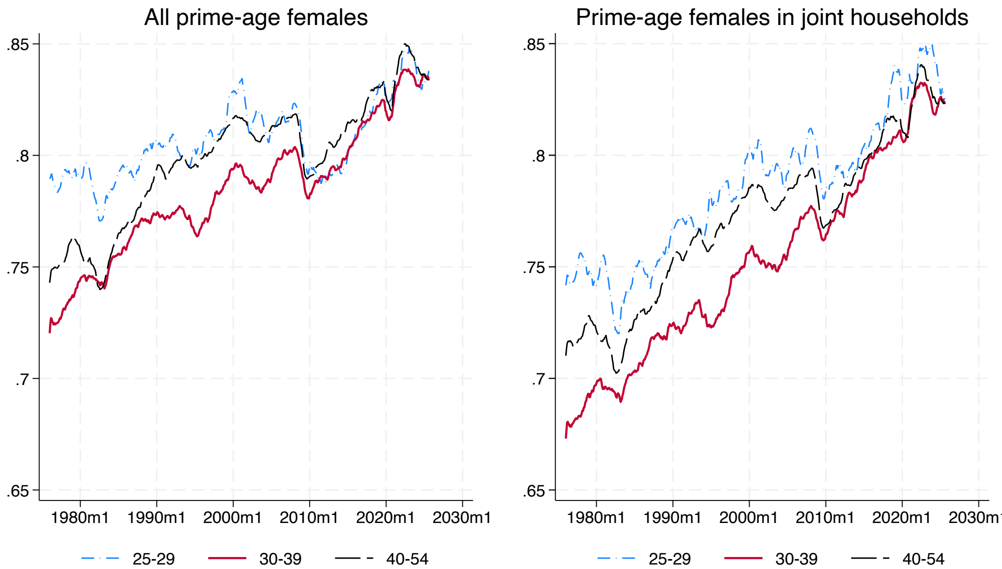

Figure 4: Fraction of employed females who are full-time employed (US).

Notes: Source: CPS.

conditional on participating, more women are represented in NRCOGFT jobs. This is once again particularly true for women in joint households, mirroring the results in Figure 3.

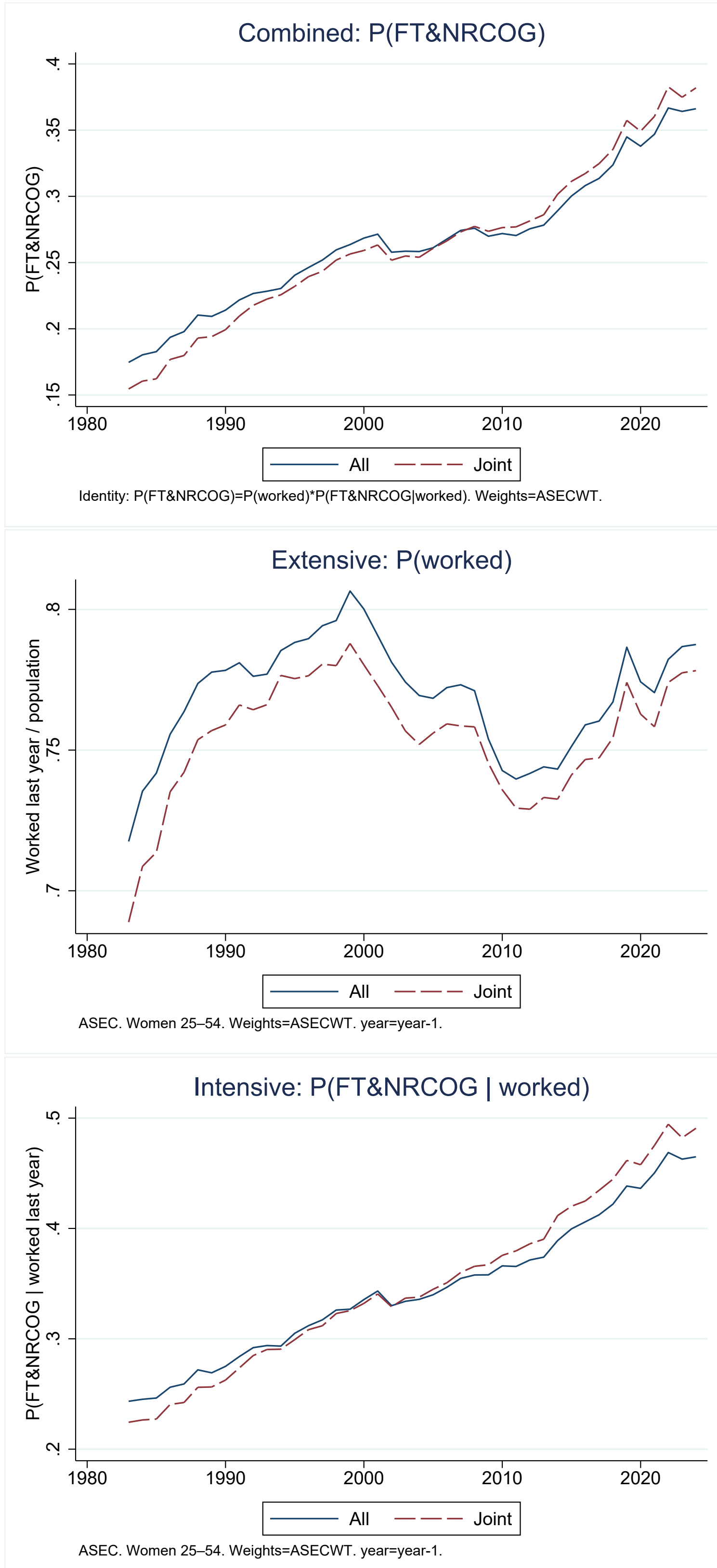

Figure 5: Female Hours Worked cond. on working (US), overall and by household status.

Notes: Source: CPS-ASEC.

3 A Simple Model

3.1 Directed search with household labor supply

Environment. Time is discrete and there are two periods, $t \in \{1, 2\}$. There is no savings and households consume contemporaneously. A continuum of workers is indexed by type $x \in \mathcal{X}$ drawn from a distribution F. Types govern the probability of accumulating match-specific human capital. Workers live either as singles or as members of a two-adult household ("joint household"). Within a joint household, one member is designated the primary earner ($i = p$) and the other the secondary earner ($i = s$); exogenous job destruction rates satisfy $\delta_s > \delta_p$. For tractability, at most one household member can form a new match within a period.

Production and learning. A filled job produces yh, where h denotes hours supplied and y denotes output per hour. New matches start at productivity y_L. If the match survives into period 2, the worker becomes skilled with probability $\gamma(x)$, in which case per-hour productivity becomes $y_H > y_L$. Human capital is match-specific: if the job is destroyed, productivity resets to y_L in the next match.

Preferences, home production, and household states. Non-employed adults produce a home good b. Joint households pool income (unitary model). In a given period, a joint household can be in state (U, U), (E, U), or (E, E). Household consumption is

$$C = b \cdot (\text{non-employed}) + \sum_{\ell \in \text{employed}} \eta_\ell y_\ell h_\ell,$$

where $\eta_\ell \in (0, 1)$ is the wage share received by employed member ℓ. Period utility is

$$u(C, \{h_\ell\}) = \log(C) - \sum_{\ell \in \text{employed}} \frac{\psi}{1 + \varepsilon} h_\ell^{1+\varepsilon},$$

with discount factor $\beta \in (0, 1)$. The Frisch elasticity is $1/\varepsilon$.

Directed search and matching. Labor markets are segmented by promised wage share $\eta \in (0, 1)$ and (in period 1) by worker characteristics (x, i). Let $\theta \equiv v/u$ denote market tightness in a submarket. Matches are formed according to a Cobb–Douglas matching function $M(u,v) = \xi u^{\alpha} v^{1-\alpha}$, implying the job-finding and vacancy-filling probabilities

$$p(\theta) = \frac{M(u,v)}{u} = \xi\theta^{1-\alpha}, \qquad q(\theta) = \frac{M(u,v)}{v} = \xi\theta^{-\alpha}.$$

Posting a vacancy costs κ. Free entry implies that, in any active submarket,

$$\kappa = q(\theta_t)\, J_t,$$

where J_t is the firm's value of a filled match in that submarket.

Timing. In each period: (i) firms post vacancies; (ii) existing matches are destroyed with probability δ_i; (iii) in period 1, surviving employed workers upgrade with probability $\gamma(x)$; (iv) unemployed workers direct their search by choosing a wage share η; (v) matching occurs; (vi) households choose hours; (vii) production and consumption occur.

Hours choice. Given a contract (η, y) and household resources, hours are chosen after matching. For a single employed worker, $C = \eta y h$ and the optimal hours satisfy

$$\frac{1}{h} = \psi h^{\varepsilon},$$

so hours depend only on (ψ, ε) and are independent of (η, y) under log utility. In a joint household with one employed adult, consumption is $C = b + \eta y h$ and optimal hours $h(\eta, y)$ satisfy

$$\frac{\eta y}{b + \eta y h} = \psi h^{\varepsilon}, \qquad \frac{\partial h(\eta, y)}{\partial \eta} > 0,$$

so hours increase with the wage share when there are pooled resources (here b). With two employed adults, (h_i, h_j) solve

$$\frac{\eta_i y_i}{C} = \psi h_i^{\varepsilon}, \qquad \frac{\eta_j y_j}{C} = \psi h_j^{\varepsilon}, \qquad C = \eta_i y_i h_i + \eta_j y_j h_j,$$

so hours depend on partner earnings through C.

Joint-household static problems in period 2. Because period 2 is terminal, the joint-household values are given by within-period optimization problems. In state (U, U), both adults are non-employed and consume home production,

$$U_2^{UU} = \log(2b).$$

In state (E,U), one adult is employed at contract (η, y) and the other produces b. The household chooses hours h and obtains

$$W_2^{EU}(\eta, y) = \max_{h \geq 0} \left\{ \log(b + \eta y h) - \psi \frac{h^{1+\varepsilon}}{1+\varepsilon} \right\}, \tag{1}$$

$$\text{s.t.} \qquad \frac{\eta y}{b + \eta y h} = \psi h^{\varepsilon}. \tag{2}$$

In state (E,E), both adults are employed at contracts (η_a, y_a) and (η_b, y_b) and the household chooses (h_a, h_b),

$$T_2^{EE}(\eta_a, y_a; \eta_b, y_b) = \max_{h_a, h_b \geq 0} \left\{ \log(\eta_a y_a h_a + \eta_b y_b h_b) - \psi_a \frac{h_a^{1+\varepsilon}}{1+\varepsilon} - \psi_b \frac{h_b^{1+\varepsilon}}{1+\varepsilon} \right\}, \tag{3}$$

$$\text{s.t.} \qquad \frac{\eta_a y_a}{C} = \psi_a h_a^{\varepsilon}, \qquad \frac{\eta_b y_b}{C} = \psi_b h_b^{\varepsilon}, \qquad C = \eta_a y_a h_a + \eta_b y_b h_b. \tag{4}$$

These period 2 values depend on productivity realizations through y (e.g. $y \in \{y_L, y_H\}$ for matches formed in period 1), but not directly on x beyond its effect on the distribution of y through learning.

Properties of joint-household hours. The worker–searcher problem (1)–(2) implies that hours respond to the wage share when there are pooled resources. Applying the implicit function theorem to (2) yields

$$\frac{\partial h(\eta, y)}{\partial \eta} > 0, \qquad \frac{\partial h(\eta, y)}{\partial b} < 0. \tag{5}$$

In the dual-employed problem (3)–(4), each spouse's hours are increasing in own wage share and decreasing in the partner's wage share (the latter through the household consumption aggregator C). In particular, holding $(y_a, y_b, \psi_a, \psi_b)$ fixed,

$$\frac{\partial h_a}{\partial \eta_a} > 0, \quad \frac{\partial h_a}{\partial \eta_b} < 0, \qquad \frac{\partial h_b}{\partial \eta_b} > 0, \quad \frac{\partial h_b}{\partial \eta_a} < 0. \tag{6}$$

These sign restrictions formalize that, in joint households, hours are an endogenous intensive margin that responds to contractual terms and household resources.

Regularity and gains from employment. Introducing disutility of hours implies that employment is not mechanically preferred to non-employment. We impose parameter restrictions such that, at the equilibrium wage shares, the gains from employment are positive:

$$W_2^{EU}(\eta, y) - U_2^{UU} > 0, \qquad T_2^{EE}(\eta_a, y_a; \eta_b, y_b) - W_2^{EU}(\eta_a, y_a) > 0, \tag{7}$$

and that the value of a match is increasing in the wage share, $\partial W_2^{EU}(\eta, y)/\partial\eta > 0$ and $\partial T_2^{EE}(\eta_a, y_a; \eta_b, y_b)/\partial\eta_\ell > 0$ for $\ell \in \{a, b\}$.

Well-posedness of wage-share choice and the Frisch elasticity. With endogenous hours, firm flow profit in period 2 is $(1-\eta)\, y\, h^*(\eta, \cdot)$, so free entry implies

$$\theta_2(\eta) \propto \left((1-\eta)\, y\, h^*(\eta, \cdot)\right)^{1/\alpha}.$$

A convenient sufficient condition for the directed-search problem over wage shares to be well behaved is that tightness is decreasing in η in the relevant household states, i.e.,

$$\frac{d\log\theta_2(\eta)}{d\eta} = \frac{1}{\alpha}\left(-\frac{1}{1-\eta} + \frac{d\log h^*(\eta, \cdot)}{d\eta}\right) < 0 \qquad \Longleftrightarrow \qquad \frac{d\log h^*(\eta, \cdot)}{d\eta} < \frac{1}{1-\eta}. \tag{8}$$

In the worker–searcher state (E, U), the hours elasticity can be bounded as

$$\frac{d\log h}{d\eta} \leq \frac{1}{\eta\varepsilon},$$

so (8) is ensured whenever

$$\eta > \frac{1}{1+\varepsilon}. \tag{9}$$

We therefore calibrate a low Frisch elasticity (large ε) so that hours respond to wage shares, but not so strongly as to overturn the basic job-finding versus pay tradeoff that pins down interior wage-share choices.

Firm values and tightness. In the last period, the firm obtains flow profit $(1-\eta)yh^*$, hence (suppressing state arguments for readability)

$$J_2(\eta, y) = (1-\eta)y\, h^*(\eta, y), \qquad \theta_2(\eta) = \left[\frac{\xi(1-\eta)y_L\, h^*(\eta, y_L)}{\kappa}\right]^{1/\alpha}.$$

In period 1, firm values incorporate survival, learning, and continuation into period 2:

$$J_1(\eta, x, i) = (1-\eta)y_L h^*(\eta, y_L) + \beta(1-\delta_i)\Big(J_2(\eta, y_L) + \gamma(x)\big[J_2(\eta, y_H) - J_2(\eta, y_L)\big]\Big),$$

where, in joint households, J_2 depends on whether the partner is employed because hours depend on household resources.

Household search. Let U_t denote the value of non-employment and $W_t(\eta, \cdot)$ the value upon matching at wage share η. At the search stage, the searching unit chooses η to maximize expected gains from employment:

$$R_t = \max_{\eta \in (0,1)} p\big(\theta_t(\eta)\big) \left[W_t(\eta) - U_t\right].$$

Hours feedback and the role of the Frisch elasticity. Differentiating the free-entry mapping in the last period yields

$$\frac{d \log \theta_2(\eta)}{d\eta} = \frac{1}{\alpha}\left(-\frac{1}{1-\eta} + \underbrace{\frac{d \log h^*(\eta, y_L)}{d\eta}}_{\Omega(\eta)\ \text{(hours feedback)}} \right).$$

For singles, $\Omega(\eta) = 0$ under log utility, so tightness is strictly decreasing in η. For joint households, $\Omega(\eta) > 0$ because pooled resources make hours increase with the wage share; this flattens the η–tightness trade-off and increases optimal selectivity. We calibrate a low Frisch elasticity (large ε) to keep $\Omega(\eta)$ modest so that the directed-search problem remains well behaved and admits interior wage-share choices, while preserving the key state dependence of the search trade-off in joint households.

3.2 Introducing discrete job choice

This extension adds a discrete choice of job type at the search stage. The baseline environment is unchanged unless noted.

Job types. There are two job types, $k \in \{d, h\}$. A dead-end job ($k = d$) produces $y_D h$ each period and does not generate human-capital growth. A career job ($k = h$) starts at productivity y_L and, conditional on match survival into period 2, upgrades to $y_H > y_L$ with probability $\gamma(x)$. We allow $y_H > y_D > y_L$ to capture that dead-end jobs can pay more immediately than entry-level career jobs but lack growth.

Directed search by job type and wage share. Markets are now segmented by (k, η) and, in period 1, by (x, i). Let $\theta_{t,k}(\eta)$ denote tightness in submarket (k, η) at time t. Matching within each submarket is as in the baseline with meeting probabilities $p(\theta) = \xi\theta^{1-\alpha}$ and $q(\theta) = \xi\theta^{-\alpha}$. Free entry implies

$$\kappa = q\big(\theta_{t,k}(\eta)\big)\ J_{t,k}(\eta, \cdot), \qquad \theta_{t,k}(\eta) = \left[\frac{\xi\ J_{t,k}(\eta, \cdot)}{\kappa}\right]^{1/\alpha}.$$

Firm values. In period 2, a newly created match in submarket (k, η) yields

$$J_{2,k}(\eta) = (1-\eta)\, y_k\, h_k^*(\eta), \qquad y_k = \begin{cases} y_D, & k = d, \\ y_L, & k = h, \end{cases}$$

where $h_k^*(\eta)$ is the household's optimal hours choice given job type k and household resources. In period 1, the dead-end job has no learning option, while the career job includes match-specific upgrading:

$$J_{1,d}(\eta, i) = (1-\eta) y_D h_d^*(\eta) + \beta(1-\delta_i) J_{2,d}(\eta),$$
$$J_{1,h}(\eta, x, i) = (1-\eta) y_L h_h^*(\eta) + \beta(1-\delta_i)\Big(J_{2,h}(\eta) + \gamma(x)\big[J_{2,H}(\eta) - J_{2,h}(\eta)\big]\Big),$$

where $J_{2,H}(\eta)$ denotes the period 2 value of an incumbent career match that has upgraded to y_H.

Household values and search. Let U_t be the value of non-employment and $W_{t,k}(\eta)$ the value of matching in submarket (k, η). The search value associated with job type k is

$$R_{t,k} = \max_{\eta \in (0,1)} p\big(\theta_{t,k}(\eta)\big)\, \big[W_{t,k}(\eta) - U_t\big],$$

and the searching unit chooses the job type with the highest value:

$$R_t = \max\{R_{t,d}, R_{t,h}\}.$$

Type cutoff and interpretation. In period 1, $R_{1,h}$ is increasing in x through $\gamma(x)$, while $R_{1,d}$ is independent of x absent learning. Under standard regularity conditions, there exists a cutoff $\bar{x}_i$ such that workers with $x \geq \bar{x}_i$ search for career jobs and workers with $x < \bar{x}_i$ search for dead-end jobs. The cutoff depends on job destruction (δ_i) because a higher separation rate lowers the value of the career ladder.

Hours feedback and the Frisch elasticity. Within each job type k, hours enter tightness through free entry:

$$\theta_{2,k}(\eta) = \left[\frac{\xi(1-\eta) y_k\, h_k^*(\eta)}{\kappa}\right]^{1/\alpha}, \qquad \frac{d \log \theta_{2,k}(\eta)}{d\eta} = \frac{1}{\alpha}\left(-\frac{1}{1-\eta} + \frac{d \log h_k^*(\eta)}{d\eta}\right).$$

As in the baseline, singles have $d \log h_k^*(\eta)/d\eta = 0$ under log utility, whereas joint households have $d \log h_k^*(\eta)/d\eta > 0$ because pooled resources make hours increasing in the wage share.

We calibrate a low Frisch elasticity (large ε) to keep this feedback modest so that wage-share choices remain interior and tightness schedules remain well behaved. Even when the hours response is small, the discrete job choice can amplify its implications: small changes in search values induced by the hours feedback can shift the cutoff $\bar{x}_i$ and generate sizable changes in job composition conditional on employment.

3.3 The Role of Endogenous Hours

Endogenous hours are included for three reasons. First, they provide a direct mapping from the model to the intensive margin in the data: employment is an extensive-margin outcome generated by search and matching, while hours are chosen conditional on employment. This allows the model to speak to changes in full-time intensity and, in the extension, to changes in job composition among employed workers.

Second, hours generate a state-dependent feedback from wage-share demands to vacancy creation that differentiates singles from joint households. Under log utility, singles' optimal hours are independent of (η, y), so the standard competitive-search logic applies: higher η reduces firm surplus one-for-one through $(1-\eta)$ and tightness falls steeply with η. In joint households, pooled resources break this cancellation and imply $\partial h/\partial\eta > 0$; as a result, when a household targets a higher wage share it also supplies more hours upon matching, partially restoring firm surplus. This flattens the wage-share versus job-finding frontier for joint households relative to singles.

Third, endogenous hours create a "selection through search" channel that is absent if hours are fixed. Let z denote household resources (e.g. spouse income). Equilibrium hours satisfy $h^*(z) = h(\eta^*(z), z)$, so

$$\frac{dh^*}{dz} = \underbrace{\frac{\partial h}{\partial \eta}\frac{d\eta^*}{dz}}_{\text{selection through wage-share choice}} + \underbrace{\frac{\partial h}{\partial z}}_{\text{direct income effect}}.$$

The direct term is typically negative, but the selection term is positive in joint households because higher resources increase desired selectivity and $\partial h/\partial\eta > 0$. This is the key mechanism through which the model can generate sizable movements in intensive outcomes conditional on employment even when employment rates change little.

4 Quantitative

In progress, to be completed soon.