Dual Income Earners and Productivity

Cristiano Mantovani
University of Melbourne

Shu Lin Wee
University of Melbourne

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Abstract

Post WW2, US labor productivity growth slowed but has recovered in recent years while female employment rose sharply before leveling off. Throughout this period, however, an increasing share of employed women – especially in joint households – moved into non-routine cognitive jobs. To examine how dual earner dynamics impact productivity trends, we develop a directed search model in which costly effort and differential opportunities to grow human capital affect job selection. Endogenous hours create a labor supply feedback that impacts vacancy creation, worker selectivity and skill accumulation. Household structures shape these hours worked and accordingly impact productivity dynamics. Quantitatively, we examine how secular changes affecting hours worked and labor market attachment contribute to productivity gains.

Mantovani: University of Melbourne (cristiano.l.mantovani@unimelb.edu.au); Wee: University of Melbourne (shulinwee.econ@gmail.com). Last updated: January 22nd, 2026.

1 Introduction

Labor productivity growth in the United States slowed markedly after the 1970s, remained subdued through the 1990s and 2000s, and then recovered in the late-2010s, with a further acceleration in the post pandemic period. Over this same period of time, female employment increased significantly before plateauing in the 2000s, and has only in recent years observed a resurgence following the pandemic. The literature has pointed to the negative correlation between labor productivity indicators and trends in female hours in the pre-2010 period (See Albanesi [2021] for example) as an explanation for the emergence of a productivity slowdown. So long as females were less productive than males, the rise female hours would contribute towards lower aggregate productivity. The recent resurgence in productivity growth post-pandemic, together with the strong labor market recovery led by females, and especially by females within joint households, suggest a reversal of this dynamic. In this paper, we argue that to understand the relationship between women labor market outcomes and aggregate productivity, we need to explore when and under which conditions women search for jobs that increase productivity, and why searching in these jobs is particularly pronounced for women in dual income earner households.

We make three contributions. First, we document that while employment rates evolve similarly for females in single and joint households, sorting into full-time non-routine cognitive work (FTNRCOG) rose disproportionately among women in joint households. Second, we develop a directed-search model of dual earners with endogenous hours and heterogeneous career prospects, which we characterize to study how income pooling affects job selectivity and upgrading. In our model, human capital is a risky asset and is non-transferable across jobs. Given the opportunity cost of hours worked and anticipated disruptions to their careers, individuals choose between *dead-end* jobs – that do not grow human capital – and *learning* jobs – which do offer the potential for human capital accumulation. Third, we use our model to quantify how secular changes in female labor supply translate into changes in job composition and long run labor productivity dynamics.

Using US data, we document three patterns. First, the rise and subsequent plateau in female Employment-to-Population (EPOP) is broadly similar for single females and females in joint households. Second, conditional on employment, growth in hours worked and the shares of females employed full-time is primarily concentrated among females in joint households, with a particularly large catch up for females in joint households aged 30 to 39 in the post COVID period. Third, conditional on working, females in joint households increasingly sort into full time non-routine cognitive jobs. Taken together, these patterns motivate the idea that to understand labor productivity trends and their relationship with female employment, we need to look at not only the extent to which females enter the labor market, but which jobs they take and how this is choice is affected by the household structure.

To account for these observations and study their implications, we develop a directed search model of the household in which spouses pool resources and choose hours after matching. In choosing where to direct their search, individuals select both the type of job – learning or dead-end job – as well as their desired wage share. Because hours worked responds to wage returns, the household's desired wage share affects vacancy creation not only through the firm's share of surplus, but also through the size of the surplus as more hours supplied raises total output. The hours feedback is heightened for joint households as the substitution effect dominates. As a consequence, females in joint households supply different hours and search in different sub-market relative to their single counterparts. Because hours worked by individual in joint households responds positively to higher wage returns, the hours feedback reduces the rate at which the firm's surplus is declining in the offered wage share, boosting job-finding rates for joint households. Higher job-finding rates in turn impact the worker's selectivity over what wage to search for and accept, as well as which type of job to select. As a result, the model can generate large changes in hours and job composition conditional on employment even when employment rates change little. Because the model features jobs that offer match specific productivity growth (learning jobs) as opposed to dead-end jobs, small shifts in selectivity can translate into large changes in the share of workers who climb the job ladder, providing a mechanism linking household labor supply and job allocation to labor productivity dynamics.

As in the data, the action in the model in terms of selection into dead-end vs learning jobs is driven by joint households. When spouses pool resources, the outside option and the marginal value of waiting depend on the partner's employment state and earnings. Because of the effect that hours have on firms' vacancy posting behavior, the household structure becomes a determinant of equilibrium market tightness and job allocation, complementing the joint-search literature [Guler et al., 2012, Mankart and Oikonomou, 2017, Pilossoph and Wee, 2021] while adding an explicit intensive margin channel through endogenous hours.

Our framework offers a new perspective on the productivity slowdown. In our model, the initial rise of female employment into lower intensity jobs (due to high costs of hours or social norms) exerts downward pressure on productivity growth. However, as the cost of providing hours falls (e.g., childcare availability), joint households can leverage their insurance capabilities to sort aggressively into high productivity career tracks. We use a quantitative version of the model to study the quantitative importance of this mechanism and to study how it can account for a significant portion of the observed long run productivity dynamics.

The household search literature has mostly focused on the cyclical properties of household search behavior. Existing work typically uses joint search models to explain the *added worker effect*, where spousal labor supply acts as an insurance mechanism against unemployment shocks [Lundberg, 1985, Mankart and Oikonomou, 2017]. We use the household search behavior to study the long run. We argue that the insurance provided by a dual income household allows households to shift the hours worked and undertake the risk associated with human capital accumulation in learning jobs. Consequently, the household structure determines the long term allocation of talent into high productivity occupations.[1]

The rest of the paper proceeds as follows. Section 3 documents the motivating facts on productivity growth, female labor supply, and job composition by household status. Section 4 presents the model and characterizes it. Section 5 derives the key comparative statics and clarifies how changes in household resources and the cost of hours shape selectivity and sorting into career jobs. Section 6 (in progress) quantifies the model and evaluates the contribution of changes in females's labor supply to long-run productivity trends.

2 Literature Review

Our paper connects three distinct strands of the macroeconomic literature: the literature of family economics and labor supply, the analysis of structural transformation and labor productivity trends, and the theory of frictional labor markets with joint search.

First, we contribute to the family macroeconomics literature, which has long emphasized that labor supply decisions are often made jointly within households rather than by isolated individuals. Doepke and Tertilt [2016] provide a comprehensive review of this approach, arguing that accounting for family dynamics is crucial for understanding aggregate economic outcomes. Attanasio et al. [2008] and Heathcote et al. [2010] document the life-cycle profiles and secular trends of female labor supply. We contribute a model where the insurance effect of spousal income (also known as added worker effect in the busuness cycle literature that models joint households explicitly) influences not just whether females work, but *where* they work a margin that is critical for human capital accumulation.

Second, our work relates to the literature on structural transformation and gender. Ngai and Petrongolo [2017] and Olivetti and Petrongolo [2016] have documented how the rise of the service economy and the marketization of home production have fundamentally shifted the demand for female labor. Closely related is the work by Goldin [2014], who highlights how the convexity of pay with respect to hours in certain greedy professions creates a wedge between men and women, particularly those in joint households facing time constraints. Our model captures this dynamic through the distinction between career jobs (which reward

[1] For another example of how added worker effects are studied to explain long run trends in female labor supply, see e.g. Albanesi and Prados [2022].

intensity with growth) and dead-end jobs. Furthermore, Manning and Petrongolo [2008] provide empirical evidence of the pay penalty associated with part-time work, supporting our assumption that low-intensity jobs offer lower returns to experience. Motivated by this work, our contribution is to theoretically and quantitatively explore the implications of these trends for the dynamics of labor productivity. The literature on business cycles (see below) has modeled the dynamics of joint housholds to study the cyclical implications. We study the long-term implications (over 40 years of so) of joint households search, with a particular emphasis on their long term productivity implications. Finally, our paper is complementary to recent work by Uniat [2025] who examines how various factors affecting female labor supply contributed to the decline in routine employment and rise in female employment in non-routine jobs. We add to this paper by highlighting the role of joint households and how secular changes in the opportunity cost of hours worked and labor market attachment can generate a larger response among dual income earners in terms of job selectivity and labor supply.

Third, we build on the theoretical literature on household search. The canonical framework by Guler et al. [2012] shows how joint income pooling creates dual search frictions that can lead to breadwinner cycles or hyper-selectivity. Mankart and Oikonomou [2017] extend this to business cycles while Pilossoph and Wee [2021] show how joint household search can impact the sign of wage premia. We depart from these frameworks by introducing an endogenous hours choice and match-specific human capital accumulation. This connects our work to Jang and Yum [2022] and Erosa et al. [2022], who investigate the relationship between hours worked, occupation types, and aggregate productivity. Our model provides a mechanism that connects labor productivity dynamics with household search, by showing how income pooling can translate into differential sorting into these high-intensity, high-return jobs.

Finally, we relate to the debate on the productivity slowdown and female employment. Albanesi [2021] and Albanesi and Prados [2022] who analyze how the leveling off of female hours has shaped aggregate business cycles and productivity. Additionally, Alon et al. [2022] highlight the unique impact of the pandemic recession on women's employment. We contribute to this debate by highlighting that the *composition* of jobs held by women determined by the household search friction is a key determinant of aggregate productivity growth, distinct from the pure extensive margin effect of entering the labor force.

3 Motivating Facts

Here we show the main facts motivating the analysis. We focus on three main facts. First, we present trends on labor productivity growth in US, whose slowdown (and sudden recovery) we aim to provide an explanation for. Second, we collect evidence on the behavior over time of US hours worked for female (vs males), focusing in particular on single females and females in joint households. The third set of facts relates to the *type* of job that females in joint (vs single) households choose over time.

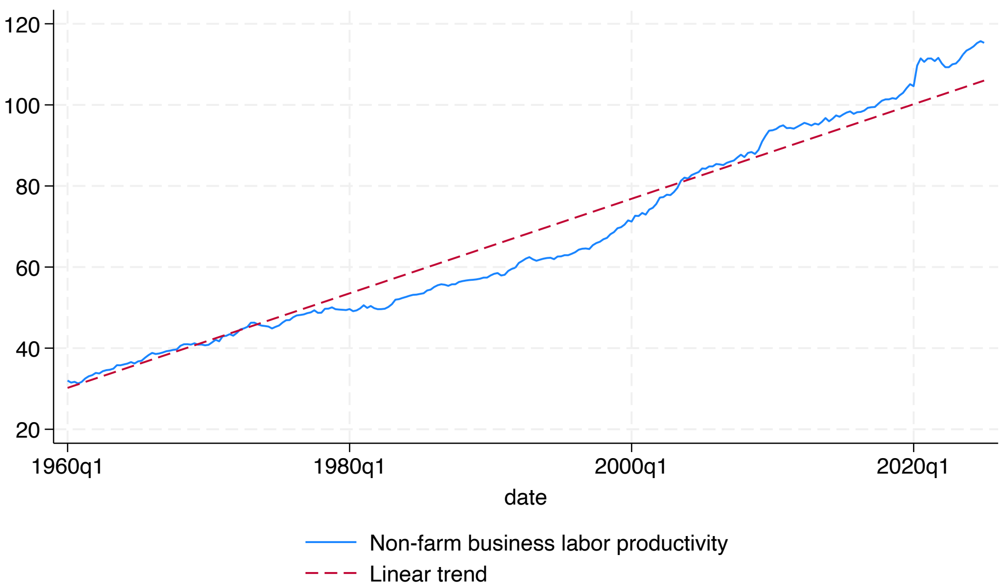

Figure 1: Labor Productivity Trend in the US.

Notes: Source: BLS and own calculations.

Labor Productivity Trends The first fact we present relates to the behavior of labor productivity trends in the US over time, and is presented in Figure 1. Labor productivity growth slowed from the 1970s through to the mid-1990s. While the ICT boom and subsequent recessions have led to upticks and then declines in productivity growth, Figure 1 shows that from the mid-2000s, labor productivity has not only outpaced its linear time trend, but has further diverged from this trend following the 2020 pandemic.

The literature has argued that the 1970s productivity slowdown is connected to the large rise in female employment during the same years. Crucially, however, the recent labor market recovery and growth in labor productivity following the pandemic has been marked by a strong recovery in female employment and labor force attachment. Giving these opposing outcomes, it is important to understand the types of jobs females were employed in and how the allocation of female labor can impact productivity growth.

Labor Supply & Extensive Margin In what follows, we focus on the employment-to-population ratio (EPOP) of prime-age females. However, it should be noted that the literature has highlighted similar trends in female labor force participation rates (LFPR) over the same time period (See for example Albanesi and Prados [2022] and Albanesi [2021]). In particular, female EPOP in the US rose strongly over time, before peaking in the 2000s and then declining with the onset of the dot-com bust and Great Recession. Post pandemic, however, the US economy has seen a surprising uptick in female EPOP, as depicted in Figure 2. While a large part of the initial rise in female hours worked was driven by increased employment, as we show next, both the intensive margin and the types of jobs females – especially females in joint households – are employed in has continuously changed over time.

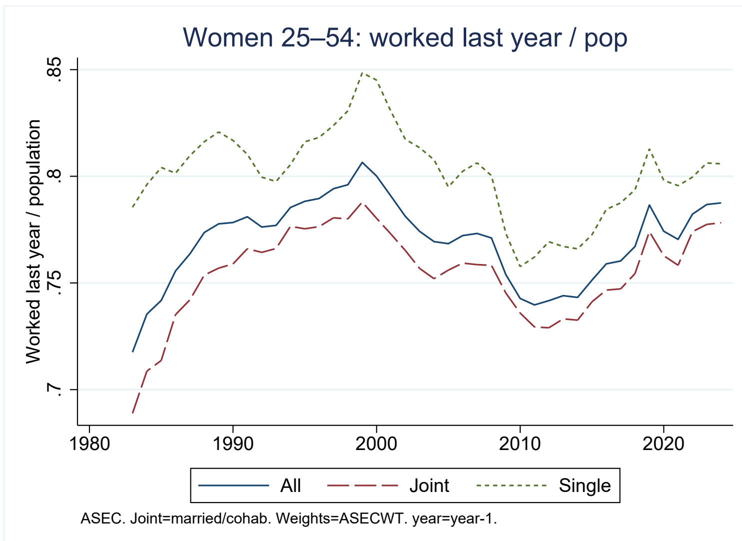

Figure 2: Female EPOP in the US, overall and by household status.

Notes: Source: CPS-ASEC.

Intensive Margin and Job Allocation The third set of facts is perhaps the most overlooked in the literature, and relates to how many hours females work conditional on being employed, and their allocation of jobs. To this end, we focus on the the intensive margin (hours worked). We augment our understanding of how hours worked among employed females have changed over time by examining the shares of employed females working in full-time vs. part-time jobs. Complementing this, we also examine the type of job they hold. In particular, we focus on how the share of employed females working in non-routine cognitive jobs has changed over time, and whether this is different for females in joint households. We argue that examining how the intensive margin and the allocation of jobs has changed over time is important for understanding productivity dynamics, and not all jobs provide the same potential for human capital growth nor are of the same productivity level.

Figure 3 shows how annual hours worked per employed prime-age female has changed over time. The blue solid line shows that women in the US have increased hours worked,

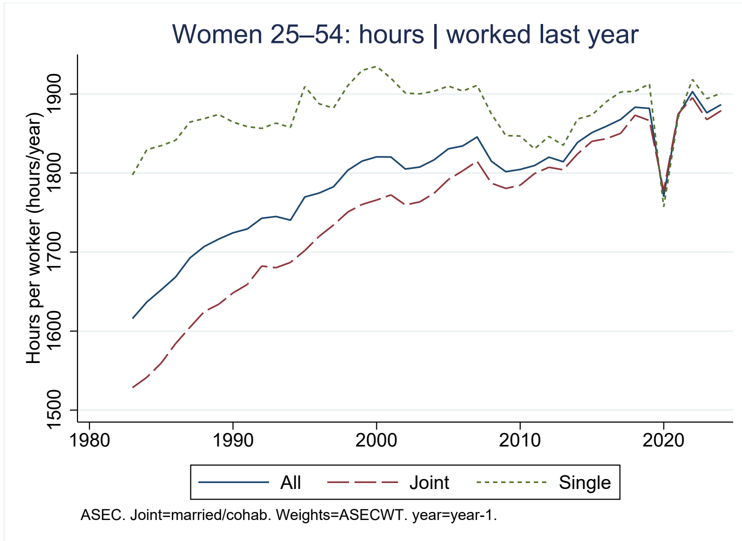

Figure 3: Female Hours Worked cond. on working (US), overall and by household status.

Notes: Source: CPS-ASEC.

a phenomenon also noted by Jang and Yum [2022]. Crucially, Figure 3 shows that the increase in hours worked (the intensive margin) is driven almost exclusively by females in joint households. Hours worked by females in single households have largely remained unchanged since the 1990s. Hours worked by females in joint households, on the other hand, has steadily increased since the 1980s, suggesting that the labor supply of females in joint households play an increasingly larger role in affecting aggregate female hours.

The rise in annual hours worked per employed female in a joint household can be rationalized through the types of jobs they increasingly occupy. Figure 4 shows that while females in both single and joint households observed similar trends in EPOP ratios, the fraction of females in joint households employed in full-time jobs severely lagged that of their single counterparts until the most recent post-pandemic period. Post-pandemic, the fraction of employed females in full-time jobs from both single and joint households is roughly similar. Given their initial smaller share, this suggest rapid catch-up in the fraction of employed females from joint households working full-time.

Further, among employed women, the pre-pandemic shortfall in full-time work is concentrated among females in joint households aged 30 to 39. Notably, this age group coincides with the period where families face child-care needs. To the extent that full-time permanent jobs are jobs which provide the potential for human capital growth, the more muted share of females in full-time jobs – especially for females in joint households – in the early 1980s and 1990s could contribute to the observed productivity slowdown.

To the best of our knowledge, the latter set of facts is new to the literature and we explore

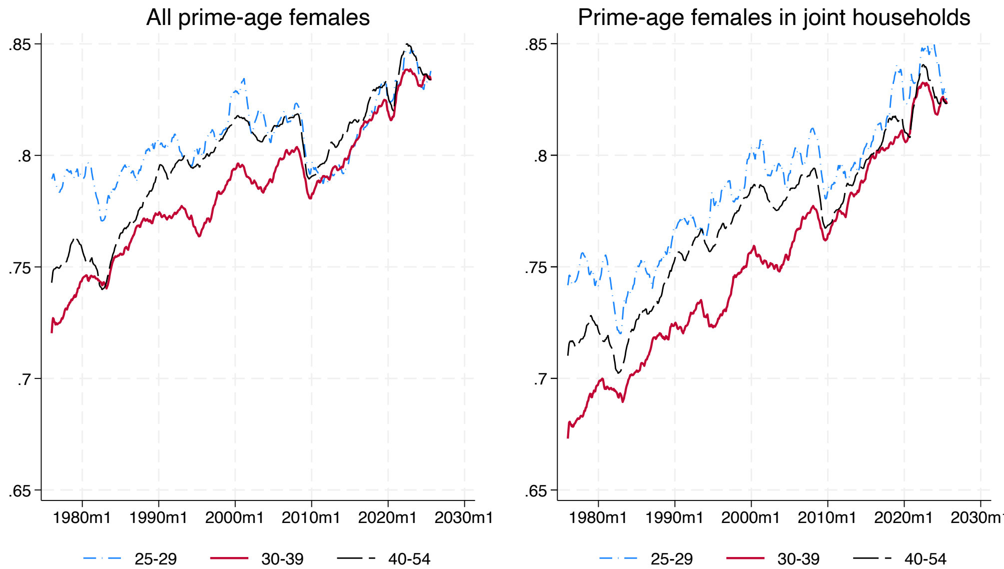

Figure 4: Fraction of employed females who are full-time employed (US).

Notes: Source: CPS.

it further. We look, in particular, at whether women take on more Non-Routine Cognitive Full-time jobs (NRCOGFT). Assuming that females can accumulate human capital and build productivity in these jobs more than in jobs that are performed part-time, or where tasks are routine in nature, Figure 5 shows that the probability of working in NRCOGFT jobs (left panel) has increased over time for females. Importantly, Figure 5 shows that conditional on employment, females in joint households are more likely to be represented in NRCOGFT jobs than their single counterparts. This result mirrors the findings in Figure 3. In summary, employed females in joint households have not only increased their annual hours worked over time, but increasingly, they are more likely than the average employed female to be employed in non-routine full-time jobs.

To the extent that the composition of workers across jobs of different productivity and with different potential for human capital growth matters for productivity gains, it is useful to understand what drives the differential trends in hours worked and job allocation between females in joint households and their single counterparts. To examine this question, we next introduce a simple 2 period model to illustrate the forces which drive differential choices between single females and females in joint households.

Figure 5: Female Hours Worked cond. on working (US), overall and in joint households only.

Notes: Source: CPS-ASEC.

4 A 2 Period Model

While our quantitative exercise will be based on an infinite horizon perpetual youth model, we derive a simple 2 period model in this section to outline how hours worked interacts with job selection and the accumulation of human capital.

4.1 Environment.

Time is discrete. There is a measure 1 of risk-averse single households as well as a measure 1 of risk-averse joint households who live for two periods, $t \in \{1, 2\}$. Joint households are households with exactly two members. We assume a unitary model of the joint household. For simplicity, we assume that all households observe log utility in consumption and discount the future with factor β. All individuals start period 1 as non-employed individuals.

Individuals are ex-ante heterogeneous in terms of their underlying type x which is drawn at date 0 from an exogenous distribution $F(x)$ with associated density $f(x)$. An individual's type affects her probability of becoming skilled and accumulating human capital. Specifically, denote $\gamma(x)$ as the probability that an individual of type x becomes skilled, where $\gamma'(x) > 0$. Human capital is a risky asset as it is match-specific. An individual can be either non-employed or employed. If non-employed, the individual produces b units of a home production good. If employed, the individual earns wage income, the level of which is dependent on her job, the contracted wage share and her hours worked. Employed individuals suffer disutility from labor; individuals who work h units incur disutility of the form $\nu(h) = \psi \frac{h^{1+\epsilon}}{1+\epsilon}$, where $1/\epsilon$ represents the Frisch elasticity of labor. Further, we assume that employed individuals can be employed at either *"dead-end"* jobs, i.e., jobs which have no potential for growing one's human capital – $\gamma(x) = 0$ for all x – or at *"learning"* jobs which provide individuals with the potential to accumulate human capital, i.e., $\gamma(x) \geq 0$. Employed individuals can also be exogenously separated from their job with some probability. To capture differential career interruptions faced by members within a joint household, we assume that secondary earners are primary care-givers and observe a higher job destruction rate, δ_s, then primary earners, δ_p, i.e., $\delta_s > \delta_p$. For simplicity, we assume that all single individuals observe separation rate δ_p.[2] Finally, there is no savings technology and all households consume their income at the end of the period.

Production A job is a single firm-worker pair. There are two types of jobs. A firm who hires a worker for a dead-end job produces y_D amounts of output per unit of labor supplied. A firm who hires a worker at the learning job produces $y_L < y_D$ amounts of output per unit of labor supplied until the worker becomes skilled. At this point, the firm with the skilled worker at the learning job produces $y_H > y_D$ amounts of output per unit of labor supplied. Because human capital is match-specific, all newly formed matches at learning jobs start out with producing y_L amounts of output per unit of labor supplied.

Search and matching Search is directed. To create a vacancy, all firms pay vacancy posting cost κ and advertise the wage share of output, η, that they are committed to transferring to the worker. In addition to the wage share η, each sub-market is characterized by the firm's desired productivity type x, the job-type, and by the household's characteristics. Collectively denote the advertised characteristics of a sub-market as $\mathbf{C}$.[3] Then within a sub-market, matching is random with total matches at time t following a Cobb-Douglas matching function:

$$M_t(\mathbf{C}) = \xi u_t(\mathbf{C})^{\alpha} v_t(\mathbf{C})^{1-\alpha}$$

where ξ refers to matching efficiency, $u_t(\mathbf{C})$ refers to the non-employed job-seekers in sub-market $\mathbf{C}$, $v_t(\mathbf{C})$ refers to the vacancies in that sub-market, and $0 < \alpha < 1$ is the elasticity of the matching function with respect to non-employed job-seekers. Denote $\theta_t(\mathbf{C})$ as the labor market tightness associated with sub-market of characteristics $\mathbf{C}$ at time t. Accordingly, the probability of finding a job and filling a job in that sub-market is given by:

$$p(\theta_t[\mathbf{C}]) = \frac{M_t(\mathbf{C})}{u_t(\mathbf{C})} = \xi \theta_t(\mathbf{C})^{1-\alpha}, \qquad q(\theta_t[\mathbf{C}]) = \frac{M_t(\mathbf{C})}{v_t(\mathbf{C})} = \xi \theta_t(\mathbf{C})^{-\alpha}.$$

[2] We relax this assumption in the quantitative section.

[3] We explicitly state the variables characterizing $\mathbf{C}$ when we talk about the search problems of singles and joint households.

Timing Finally the timing of our model is as follows: at the beginning of a period, firms post vacancies. Separation shocks occur next and all newly separated individuals wait one period before they can search the labor market. Next, non-employed individuals decide which markets to search in and individuals employed at a learning job observe some opportunity of becoming skilled. Matching then takes place. Given their contracted wage shares and skill-set, the employed choose how much labor to supply and finally production occurs.

Having described the model set-up, we now turn to describing the singles problem before turning to the joint household problem.

4.2 Single households

In what follows, it should be noted that all value functions for periods $t > 2$ are equal to zero as the economy only lasts for two periods. Further we make the simplifying assumption that only one event can happen within a period. In our quantitative exercises, we assume a period is of length Δ and take $\Delta \to 0$.

4.2.1 Households

At the end of period t, all households consume their income. Denote $U_t(x)$ as the value of a non-employed single and $W_t(\eta, x, y)$ as the value of an employed single of type x working with contracted wage share η and producing output $y \in \{y_L, y_D, y_H\}$.

The non-employed The end-of-period value of a non-employed single in period t can be written as:

$$U_t(x) = \log(b) + \beta \left[U_{t+1}(x) + R_{t+1}(x)\right]$$

where

$$\widetilde{R}_t(x) = \max\{\widetilde{R}_t^D(x), \widetilde{R}_t^L(x)\}$$

and

$$\widetilde{R}_t^k(x) = \max_{\eta} p\left[\theta_t\left(\eta, x, y_k\right)\right]\left[W_t\left(\eta, x, y_k\right) - U_t(x)\right] \quad \text{for } k \in \{D, L\}$$

s.t.

$$W_t\left(\eta, x, y_k\right) - U_t(x) \geq 0$$

The last condition is a participation constraint that says households will only search if their gain from matching is non-negative. The sub-script k denotes whether the job is a dead-end job, “D”, or a learning job “L”. In words, at the end of period t, the individual enjoys her current flow utility from home production. In the next period, the individual selects the type of job – dead-end or learning – as well as her desired wage share. Given her selected job type k and wage-share η, the individual finds a job with probability $p(\theta_t(\eta, x, y_k))$ and receives the worker's gain to matching $W_t(\eta, x, y_k) - U_t(x)$. Otherwise she continues with the value of non-employment in the next period.

The employed The end of period value of an employed single individual of type x working at a job which produces $y \in \{y_L, y_D, y_H\}$ can be written as:

$$\begin{aligned} W_t(\eta, x, y) &= \max_h \log \eta y h - \psi \frac{h^{1+\epsilon}}{1+\epsilon} \\ &\quad + \beta \Big\{ W_{t+1}(\eta, x, y) - \delta_p [W_{t+1}(\eta, x, y) - U_{t+1}(x)] \\ &\quad + \mathbb{I}(y = y_L) \gamma(x) [W_{t+1}(\eta, x, y_H) - W_{t+1}(\eta, x, y_L)] \Big\} \end{aligned}$$

The individual optimally chooses hours worked to maximize her current utility. With probability δ_p, the worker exits employment and observes the value of non-employment and with probability $1 - \delta_p$, she remains employed and receive the continuation value of $W_{t+1}(\eta, x, y)$. If the individual is an unskilled worker of type x in a learning job, then unlike the worker in a dead-end job or the skilled worker, she observes an additional probability $\gamma(x)$ of gaining human capital next period, becoming skilled and producing y_H output per labor supplied in the future. Because firms commit to sharing a constant wage share of output, wages are essentially back-loaded in a learning job for the same amount of labor supplied.

4.2.2 Firms

Matched firms For matched firms attached to workers of type x producing $y \in \{y_L, y_D, y_H\}$ per units of labor, the value of the matched firm who commits to wage share η can be characterized as:

$$\begin{aligned} J_t(\eta, x, y) &= (1-\eta) y h_t^s + \beta \Big\{ (1-\delta_p) J_{t+1}(\eta, x, y) \\ &\quad + \mathbb{I}(y = y_L) \gamma(x) [J_{t+1}(\eta, x, y_H) - J_{t+1}(\eta, x, y_L)] \Big\} \end{aligned}$$

where h_t^s is the optimal amount of hours chosen by the single individual at time t. The first term relates to the current profits of the firm. With probability $(1 - \delta_p)$, the firm retains the worker and receives continuation value $J_{t+1}(\eta, x, y)$. If the firm is matched to an unskilled worker in a learning job, then it also observes some probability $\gamma(x)$ of having their worker become skilled and producing y_H units of output per labor in the future. Since $\gamma'(x) > 0$, the higher the worker's x, the more likely she becomes skilled and produces higher output for the firm in the future.

Free entry Under free entry, firms post vacancies until the value of a vacancy is zero. Since human capital is match-specific, all newly matched learning jobs start with output per unit labor equal to y_L. As such, for $y \in \{y_L, y_D\}$, the following free entry condition for a sub-market $\mathbf{C} = (\eta, x, y)$ holds:

$$\kappa \geq q(\theta_t(\eta, x, y))J_t(\eta, x, y) \tag{1}$$

where the sub-market is deemed to be inactive if $\kappa > q(\theta_t(\eta, x, y))J_t(\eta, x, y)$.

4.3 Joint households

4.3.1 Households

The problem of joint households is similar to that of singles except now individuals can either be in a dual non-employed household, worker-searcher households where only one-member is employed and the job-type as well as the skill of the employed member can vary, dual employed households where the job-types and skills of the employed members can vary. As per the single household problem, we continue to assume that only event can occur within a period. Further, to allow for the fact that members within the joint household do not always face the same opportunity cost of hours worked nor the same rate of disruptions to their working life, we allow ψ the scaling factor on the disutility of labor and δ to be different across members. Accordingly, let a and b be the members in a joint household, and let ψ_a, ψ_b and $\delta_a, \delta_b \in \{\delta_s, \delta_p\}$ be the associated parameters that a and b face.

Dual non-employed Denote $\mathbf{x} = (x_a, x_b)$ where a and b refer to the individual members within the joint household. Then, the value of the dual non-employed joint household is given by:

$$\mathcal{U}_t(\mathbf{x}) = \log 2b + \beta \left[\mathcal{U}_{t+1}(\mathbf{x}) + \widetilde{\mathcal{R}}^{uu}_{a,t+1}(\mathbf{x}) + \widetilde{\mathcal{R}}^{uu}_{b,t+1}(\mathbf{x})\right]$$

where for $i \in \{a, b\}$

$$\widetilde{\mathcal{R}}^{uu}_{i,t}(\mathbf{x}) = \max\left\{\widetilde{\mathcal{R}}^{D,uu}_{i,t}(\mathbf{x}), \widetilde{\mathcal{R}}^{L,uu}_{i,t}(\mathbf{x})\right\}$$

and for $k \in \{D, L\}$

$$\widetilde{\mathcal{R}}^{k,uu}_{i,t}(\mathbf{x}) = \max_{\eta} p\left(\theta_t\left[\eta, \mathbf{x}, y_k\right]\right)\left[\mathcal{W}_t\left(\eta, \mathbf{x}, y_k\right) - \mathcal{U}_t(\mathbf{x})\right]$$

s.t.

$$\mathcal{W}_t\left(\eta, \mathbf{x}, y_k\right) - \mathcal{U}_t(\mathbf{x}) \geq 0$$

The value of the dual non-employed consists of their current utility from pooling their income from home production, the maximized values from search, failing which, they get the continuation value of non-employment in the next period. The joint household chooses for each member both the type of job as well as the sub-market to search in within a job-type.

Worker-searcher Suppose member a is employed, and member b is non-employed. Denote $\mathbf{n} = (\eta_a, \eta_b)$, and $\mathbf{y} = (y_a, y_b)$. Then for $y_a \in \{y_L, y_D, y_H\}$, the value of a worker-searcher household where member a is employed is given by:

$$\begin{aligned}\mathcal{W}_t\left(\eta_a, \mathbf{x}, y_a\right) &= \max_{h_a} \log\left(b + \eta_a y_a h_a\right) - \psi_a \frac{h_a^{1+\epsilon}}{1+\epsilon} \\ &\quad + \beta \Big\{ \mathcal{W}_{t+1}\left(\eta_a, \mathbf{x}, y_a\right) + \widetilde{\mathcal{R}}_{t+1}^{eu}\left(\eta_a, \mathbf{x}, y_a\right) - \delta_a[\mathcal{W}_{t+1}\left(\eta_a, \mathbf{x}, y_a\right) - \mathcal{U}_{t+1}(\mathbf{x})] \\ &\quad + \mathbb{I}(y_a = y_L)\gamma(x_a)[\mathcal{W}_{t+1}\left(\eta_a, \mathbf{x}, y_H\right) - \mathcal{W}_{t+1}\left(\eta_a, \mathbf{x}, y_L\right)] \Big\}\end{aligned}$$

where

$$\widetilde{\mathcal{R}}_t^{eu}\left(\eta_a, \mathbf{x}, y_a\right) = \max\left\{\widetilde{\mathcal{R}}_t^{D,eu}\left(\eta_a, \mathbf{x}, y_a\right), \widetilde{\mathcal{R}}_t^{L,eu}\left(\eta_a, \mathbf{x}, y_a\right)\right\}$$

and for $k \in \{D, L\}$

$$\widetilde{\mathcal{R}}_t^{k,eu}\left(\eta_a, \mathbf{x}, y_a\right) = \max_{\eta_b} p(\theta_t[\mathbf{n}, \mathbf{x}, \mathbf{y}])\left[\mathcal{T}(\mathbf{n}, \mathbf{x}, \mathbf{y}) - \mathcal{W}(\eta_a, \mathbf{x}, y_a)\right]$$

s.t.

$$\mathcal{T}(\eta_a, \eta_b, \mathbf{x}, y_a, y_k) - \mathcal{W}(\eta_a, \mathbf{x}, y_a) \geq 0$$

where the last condition represents the participation constraint of the joint household. Different from the employed singles problem, the worker-searcher household problem also features the search decision of its non-employed member, $\widetilde{\mathcal{R}}^{eu}(\eta_a, \mathbf{x}, y_a)$. Notably, the non-employed member's decision of what job to select and which sub-market to search depends not just on the individual's type, but also their partner's characteristics, i.e., her type x_a as well as her current income.

Analogous expressions exist for worker-searcher households where member b is employed.

Dual employed The value of a dual employed household for $y_a, y_b \in \{y_L, y_D, y_H\}$ is:

$$\begin{aligned}\mathcal{T}(\mathbf{n}, \mathbf{x}, \mathbf{y}) &= \max_{h_a, h_b} \log(\eta_a y_a h_a + \eta_b y_a h_b) - \psi_a \frac{h_a^{1+\epsilon}}{1+\epsilon} - \psi_b \frac{h_b^{1+\epsilon}}{1+\epsilon} \\ &\quad + \beta \Big\{ \mathcal{T}(\mathbf{n}, \mathbf{x}, \mathbf{y}) - \delta_a[\mathcal{T}(\mathbf{n}, \mathbf{x}, \mathbf{y}) - \mathcal{W}(\eta_b, \mathbf{x}, y_b)] - \delta_b[\mathcal{T}(\mathbf{n}, \mathbf{x}, \mathbf{y}) - \mathcal{W}(\eta_a, \mathbf{x}, y_a)] \\ &\quad + \mathbb{I}(y_a = y_L)\gamma(x_a)[\mathcal{T}(\mathbf{n}, \mathbf{x}, y_H, y_b) - \mathcal{T}(\mathbf{n}, \mathbf{x}, \mathbf{y})] \\ &\quad + \mathbb{I}(y_a = y_L)\gamma(x_b)[\mathcal{T}(\mathbf{n}, \mathbf{x}, y_a, y_H) - \mathcal{T}(\mathbf{n}, \mathbf{x}, \mathbf{y})] \Big\}\end{aligned}$$

The first line represents the current utility of the household less the disutility from both members supplying labor. The second line captures the expected change in values should any member of the household lose their job. The third and fourth lines capture how if either member is an unskilled worker in a learning job, they observe some probability of gaining human capital and becoming skilled at their job.

4.3.2 Firms

Value of firm matched to worker in a worker-searcher household Denote $s = (\eta_a, \mathbf{x}, y_a)$ as the current state variables of the worker-searcher household. The value of the a firm matched to member a in a worker-searcher households is given by:

$$
\begin{aligned}
\mathcal{J}_t(\eta_a, \mathbf{x}, y_a) \quad = \quad & (1-\eta_a)\, y_a h^*_{j,t}(s) + \beta \Big\{ (1-\delta_a)\, \mathcal{J}_{t+1}(\eta_a, \mathbf{x}, y_a) \\
& + \mathbb{I}(D[s]) p\left(\theta_{t+1}\left[\eta_a, \eta_b^*(s), \mathbf{x}, y_a, y_D\right]\right)\left[\mathcal{J}_{t+1}(\mathbf{n}, \mathbf{x}, \mathbf{y}) - \mathcal{J}_{t+1}(\eta_a, \mathbf{x}, y_a)\right] \\
& + (1-\mathbb{I}(D[s])) p\left(\theta_{t+1}\left[\eta_a, \eta_b^*(s), \mathbf{x}, y_a, y_L\right]\right)\left[\mathcal{J}_{t+1}(\mathbf{n}, \mathbf{x}, \mathbf{y}) - \mathcal{J}_{t+1}(\eta_a, \mathbf{x}, y_a)\right] \\
& + \mathbb{I}(y_a = y_L)\gamma(x_a)\left[\mathcal{J}_{t+1}(\eta_a, \mathbf{x}, y_H) - \mathcal{J}_{t+1}(\eta_a, \mathbf{x}, y_L)\right] \Big\}
\end{aligned}
$$

The first line captures the firm's current profits given optimal worker hours $h^*_{j,t}(s)$ and the continuation value the firm gets if the match continues next period. The second and third line captures how the firm's value changes if the joint households transitions from being a worker-searcher household to a dual employed household. The last line captures the change in value if the firm is attached to an unskilled worker in a learning job, and that worker becomes skilled with probability $\gamma(x_a)$. Crucially, the employment status of the worker's partner matters for the matched firm's value as the worker's chosen hours worked can vary with the income situation of the household. We elaborate on this further in Section 4.4.

Value of firm matched to worker in a dual-employed household Denote $\mathbf{s} = (\mathbf{n}, \mathbf{x}, \mathbf{y})$, i.e., the current state variables of the dual employed household. Then the value of a firm matched to member a from a dual-employed household is given by:

$$
\begin{aligned}
\mathcal{J}_t(\mathbf{n}, \mathbf{x}, \mathbf{y}) \quad = \quad & (1-\eta_a) y_a h^*_{a,t}(\mathbf{s}) + \beta \Big\{ (1-\delta_a)\mathcal{J}_{t+1}(\mathbf{n}, \mathbf{x}, \mathbf{y}) \\
& - \delta_b\left[\mathcal{J}_{t+1}(\mathbf{n}, \mathbf{x}, \mathbf{y}) - \mathcal{J}_t(\eta_a, \mathbf{x}, y_a)\right] \\
& + \mathbb{I}(y_a = y_L)\gamma(x_a)\left[\mathcal{J}_{t+1}(\mathbf{n}, \mathbf{x}, y_H, y_b) - \mathcal{J}_{t+1}(\mathbf{n}, \mathbf{x}, \mathbf{y})\right] \\
& + \mathbb{I}(y_b = y_L)\gamma(x_b)\left[\mathcal{J}_{t+1}(\mathbf{n}, \mathbf{x}, y_a, y_H) - \mathcal{J}_{t+1}(\mathbf{n}, \mathbf{x}, \mathbf{y})\right] \Big\}
\end{aligned}
$$

In words, the matched firm attached to worker a in a dual employed household receives current profits given worker a's optimal hours, $h^*_{a,t}(\mathbf{s})$. With probability $1-\delta_a$, he retains the worker and has continuation value $\mathcal{J}_{t+1}(\mathbf{n}, \mathbf{x}, \mathbf{y})$. With probability δ_b, the worker's partner loses a job, and the firm receives a change in value from becoming a firm attached to an individual in a worker-searcher household. The last two lines show the change in the firm's value if either the worker or the worker's partner is unskilled in a learning job and gains human capital with probability.

Free entry Under free entry, all firms post vacancies until the value of a vacancy is zero. In this case, $\mathbf{C} = \{\eta, \mathbf{x}, y\}$ if the firm advertises for a worker from a dual non-employed household, and $\mathbf{C} = \{\eta, \eta_b, \mathbf{x}, y, y_b\}$ if the firm advertises for worker from a worker-searcher household where b is already employed. Accordingly, the following free entry conditions hold for the two types of firms advertising for workers in joint households:

$$\kappa \geq q(\theta_t[\eta, \mathbf{x}, y])\mathcal{J}_t(\eta, \mathbf{x}, y) \tag{2}$$

and

$$\kappa \geq q(\theta_t[\eta, \eta_b, \mathbf{x}, y, y_b])\mathcal{J}_t(\mathbf{n}, \mathbf{x}, \mathbf{y}) \tag{3}$$

Having characterized the values of the households and firms in our model, we now move towards discussing how hours worked can affect the search behavior of the joint household

4.4 Optimal hours

In both the joint household and single household problems, the choice of how much hours to supply each period is a static decision. Given the household's state variables (ηs and ys) as well as their individual disutility costs, the household optimally chooses the hours worked for each member.

Lemma 1. *Given the household's state variables, the following equations determine equilibrium labor supplied for each household of type x and $\mathbf{x}$.*

Single households:

$$h^s_t = h^s = \left(\frac{1}{\psi}\right)^{1/(1+\epsilon)} \tag{4}$$

Worker-searcher households where a is employed:

$$\left(h^*_a\left[\eta, \mathbf{x}, y\right]\right)^{\epsilon}\left[\frac{b}{\eta y} + h^*_a\left[\eta, \mathbf{x}, y\right]\right] = \frac{1}{\psi_a} \tag{5}$$

Dual-employed households:

$$h_a^*(\mathbf{n},\mathbf{x},\mathbf{y}) = \left[\frac{\eta_a y_a}{\eta_b y_b}\frac{\psi_b}{\psi_a}\right]^{\frac{1}{\epsilon}} h_b^*(\mathbf{n},\mathbf{x},\mathbf{y}) \tag{6}$$

and

$$h_b^*(\mathbf{n},\mathbf{x},\mathbf{y}) = \left(\psi_b \left[\frac{\eta_a y_a}{\eta_b y_b}\right] \left[\frac{\eta_a y_a}{\eta_b y_b}\frac{\psi_b}{\psi_a}\right]^{\frac{1}{\epsilon}} + 1\right)^{-1/(1+\epsilon)} \tag{7}$$

Moreover, while hours worked for single individuals are insensitive to wage income, workers in joint households who observe relatively higher returns to hours worked supply more labor. Holding all else constant,

$$\frac{dh^s}{d\eta} = 0, \qquad \frac{dh_a}{d\eta_a} > 0, \quad \frac{dh_a}{d\eta_b} < 0 \qquad \frac{dh_b}{d\eta_b} > 0, \quad \frac{dh_b}{d\eta_a} < 0$$

Proof. See Appendix A. □

Intuitively, under log utility, income and substitution effects completely cancel each other out in the single's problem, leaving single hours worked to be constant given parameters of the model. While this result for singles arises due to our specification of log utility, it serves as a useful benchmark relative to the hours chosen by joint households. Notably, the substitution effect dominates for workers in joint households. A higher wage share induces individuals to supply more labor. Moreover, Equation 6 reveals that the ratio of hours within the dual employed joint household depends on the relative earning powers of members within the household and the relative cost of supplying hours as can be seen below:

$$\frac{h_a^*(\mathbf{n},\mathbf{x},\mathbf{y})}{h_b^*(\mathbf{n},\mathbf{x},\mathbf{y})} = \left[\frac{\eta_a y_a}{\eta_b y_b}\frac{\psi_b}{\psi_a}\right]^{\frac{1}{\epsilon}}$$

Clearly, if the returns to hours for a, $\eta_a y_a$, are higher than the returns to hours for b, $\eta_b y_b$, then member a would supply more labor relative to b. Similarly, if the disutility cost to labor is higher for b than for a, i.e., $\psi_b > \psi_a$, then member a would work longer hours than b, holding all else constant. The interaction of returns to working and costs of hours allows our model to rationalize differential hours worked within the joint household. Finally, η_a and η_b are endogenous objects, suggesting that members in joint households when selecting which sub-market to search take into account how both members' hours worked would be affected.

4.5 Optimal search

Having described the behavior hours in single and joint households, we now turn to examining the search behavior of individuals within these different types of households. Proposition 1 below shows that under the following conditions, the optimal sub-market the worker searches for is unique, and can be described by the following optimality conditions:

Proposition 1. *Under free entry, the response of market tightness to variations in the wage share can be characterized as:*

$$\frac{d\log\theta_t(\eta)}{d\eta} = \frac{1}{\alpha}\Bigg(- \frac{1}{1-\eta} + \Omega_t(\eta)\Bigg). \tag{8}$$

where $\Omega_t(\eta)$ represents the feedback from how hours worked responds to the choice of η and thus how it affects the firm's value. Further, for large enough ϵ, the following optimality conditions characterize the unique sub-market individuals search for within a job-type:

Single households

$$p'(\theta_t[\eta,x,y])\frac{d\theta_t(\eta,x,y)}{d\eta}\left[W_t\left(\eta,x,y_k\right)-U_t\right]+\frac{dW_t\left(\eta,x,y_k\right)}{d\eta}=0 \tag{9}$$

Dual non-employed households

$$p'(\theta_t[\eta,\mathbf{x},y])\frac{d\theta_t(\eta,\mathbf{x},y)}{d\eta}\left[\mathcal{W}_t\left(\eta,\mathbf{x},y_k\right)-\mathcal{U}_t\right]+\frac{d\mathcal{W}_t\left(\eta,\mathbf{x},y_k\right)}{d\eta}=0 \tag{10}$$

Worker-searcher households

$$p'(\theta_t[\mathbf{n},\mathbf{x},\mathbf{y}])\frac{d\theta_t[\mathbf{n},\mathbf{x},\mathbf{y}]}{d\eta_b}\left[\mathcal{T}(\mathbf{n},\mathbf{x},\mathbf{y})-\mathcal{W}(\eta_a,\mathbf{x},y_a)\right]+\frac{d\mathcal{T}(\mathbf{n},\mathbf{x},\mathbf{y})}{d\eta_b}=0 \tag{11}$$

Proof. See Appendix B. □

The independence of single hours worked with respect to the wage share implies that for firms matched to single workers, the firm's value is always declining in the wage share. That is, $\frac{dJ_t(\eta,x,y)}{d\eta} < 0$ for all $y \in \{y_L, y_D, y_H\}$. Intuitively, a higher η implies that firms give a larger share of output to single workers without inducing any variation to hours worked. As such, for firms attached to single workers, the firm's value is strictly declining in η. From equation 1, this in turn implies that θ is also declining in η for singles as $\Omega_t(\eta) = 0$ under log utility. Given that employment values are increasing in η and conditional on the participation constraint satisfied, an interior solution to the non-employed single's search problem within a particular job-type (dead-end or learning job) exists and is characterized by Equation 9.

For joint households, hours do vary with the contracted wage share, $\Omega_t(\eta) > 0$. Pooled resources within the joint household make an individual's own hours increase with her wage share. This implies that there are two opposing forces that affect the matched firm's value when choosing η. While a higher η implies that the firm gets a lower share of output produced, a higher η also induces the worker from the joint household to supply more labor. Appendix B shows a sufficient condition for the matched firm's value to be always declining in η is for $\eta > \frac{1}{1+\epsilon}$. That is, for ϵ large enough, the first effect dominates the second effect from the hours feedback. Then so long as the matched firm's value is declining in η, the free entry conditions in equations 2 and 3 imply that $\frac{d\theta(\eta,\mathbf{x},y)}{d\eta} < 0$ and $\frac{d\theta(\mathbf{n},\mathbf{x},\mathbf{y})}{d\eta} < 0$, and there exists a unique sub-market that individuals search for given their particular state. In our quantitative exercises, $1/\epsilon$ corresponds to the Frisch elasticity of labor, which in micro-estimates is typically small, implying a fairly large ϵ.[4]

Having discussed the optimal sub-market choice within a job-type, we now elaborate on the worker's optimal job selection. Proposition 2 describes the optimal job choice

Proposition 2. *Denote s as the household's state. Given the household's state variables, there exists a threshold $x^*(s)$ for each household state, above which non-employed workers select into learning jobs.*

For singles, the marginal x-type is characterized by the following indifference condition:

$$\widetilde{R}_t^D(x) = \widetilde{R}_t^L(x)$$

Focusing on the problem of worker a, for the dual non-employed, the indifference condition that characterizes $x^*(\mathbf{x})$:

$$\widetilde{\mathcal{R}}_{a,t}^{D,uu}(\mathbf{x}) = \widetilde{\mathcal{R}}_{a,t}^{L,uu}(\mathbf{x})$$

Focusing on the problem of non-employed worker b, for the worker-searcher household, the indifference condition that characterizes $x^*(\eta_a, \mathbf{x}, y_a)$ is:

$$\widetilde{\mathcal{R}}_{b,t}^{D,eu}(\eta_a, \mathbf{x}, y_a) = \widetilde{\mathcal{R}}_{b,t}^{L,eu}(\eta_a, \mathbf{x}, y_a)$$

[4] Because we distinguish between intensive and extensive margins of labor supply in our model, we argue that targeting $1/\epsilon$ to micro-estimates of the Frisch elasticity is more pertinent as ϵ largely affects the intensive margin of hours worked in our model.

Moreover, individuals who anticipate fewer job disruptions (lower δ) are more likely to select into learning jobs.

Proof. See Appendix C. $\square$

Intuitively, learning jobs are the only jobs that provide the potential to increase one's human capital and output at a job. The likelihood that one does observe an increase in human capital is dependent on the worker's x, as $\gamma'(x) > 0$. Appendix C shows that in our two period model, employment values and matched firm values in the dead-end job are independent of x. Conversely, employment values and matched firm values in the learning job are increasing in x as workers with higher x have an increased propensity to gain human capital and thus, produce more output per labor in the future. Since the value of search in the learning job is increasing in x while the value of search in the dead-end job is independent of x, there exists a cut-off x given the household's state above which the member of the household selects into the learning job.

5 Comparative Statics

Endogenous hours are included for three reasons. First, they provide a direct mapping from the model to the intensive margin in the data: employment is an extensive-margin outcome generated by search and matching, while hours are chosen conditional on employment. This allows the model to speak to changes in the intensive margin and, how variations in hours worked can impact the worker selectivity over jobs.

Second, hours generate a state-dependent feedback from wage-share demands to vacancy creation that differentiates singles from joint households. Under log utility, singles' optimal hours are independent of (η, y), so the standard competitive-search logic applies: higher η reduces firm surplus one-for-one through $(1 - \eta)$ and tightness falls steeply with η. In joint households, larger outside options from pooling resources cause the substitution effect to dominate and thus for hours worked to increase with one's wage share. As a result, when a household targets a higher wage share it also supplies more hours upon matching, partially restoring firm surplus. This partially dampens the rate at which the firm's value is declining in the wage share, boosting vacancy creation and thus job-finding rates for joint households.

Third, endogenous hours create a "selection through search" channel that is absent if hours are fixed. Let z denote household resources (e.g. spouse income). Equilibrium hours satisfy $h^*(z) = h(\eta^*(z), z)$, so

$$\frac{dh^*}{dz} = \underbrace{\frac{\partial h}{\partial \eta}\frac{d\eta^*}{dz}}_{\text{selection through wage-share choice}} + \underbrace{\frac{\partial h}{\partial z}}_{\text{direct income effect}}.$$

The direct term is typically negative, but the selection term is positive in joint households because higher resources increase desired selectivity and $\partial h/\partial \eta > 0$. This is the key mechanism through which the model can generate sizable movements in intensive outcomes conditional on employment even when employment rates change little.

5.1 Partners Income and Selection

In this subsection, we show the main equilibrium implications of the model, as well as some comparative statics in light of the quantitative section (in progress).

Household selectivity and spouse resources. The key difference between singles and joint households is that for joint households, the wage share affects both pay and hours worked. This generates an additional term in the wage share first order condition, which shifts the optimum toward higher wage shares for joint households relative to a fixed hours benchmark. Moreover, the strength of this feedback depends on household resources.

To illustrate, in period 2, consider the worker-searcher household where the non-employed member searches for a wage share η taking the partner's income as given. The optimality condition for η can be written as an equality between a job finding term and a marginal value term, where the latter includes an additional hours feedback component that is increasing in partner income. In particular, the worker searcher wage share choice loads on a term of the form:

$$\frac{1}{\eta} \cdot \frac{(\eta_j y_j)^{1+\varepsilon}}{(y\,\eta)^{1+\varepsilon} + (\eta_j y_j)^{1+\varepsilon}},$$

Larger partner earnings $\eta_j y_j$ strengthen the feedback and increase the worker's choice of η. In other words, enlarged outside options lead the worker to hold out for better-paying jobs.

Job search and spousal income Further, endogenous hours make joint households even more selective. Relative to a fixed hours benchmark, the optimal wage share is such that $\eta^*_{\text{HH}} > \eta^*_{\text{no hours}}$. In addition, in the worker-searcher household, the choice of wage share searched for by the non-employed partner, η^*, is increasing in partner earnings $\eta_j y_j$, because higher partner income increases the magnitude of the hours feedback that flattens the wage share versus job finding frontier. This result is illustrated in Figure 6.

Wage share choice by type x and the sign of $d\eta^*/dx$. In period 1, learning generates an option value that is increasing in x through $\gamma(x)$. Let $W_1(\eta, x, y)$ denote the value of matching at wage share η for type x and output per labor y, and let $U_1(x)$ denote the value of remaining unmatched. The directed search problem is

$$R_1^k(x) = \max_{\eta \in (0,1)} p\big(\theta_1(\eta, x, y_k)\big)\big[W_1(\eta, x, y_k) - U_1(x, i)\big].$$

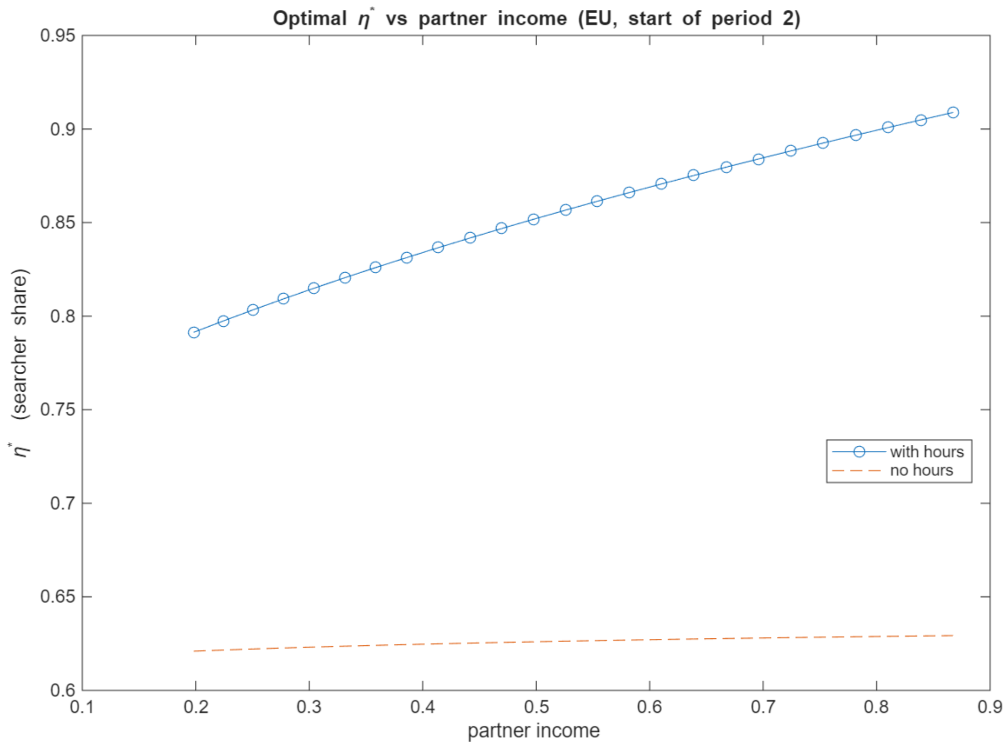

Figure 6: Optimal wage share in joint households as a function of partner income (illustration).

where k refers to the job type, $k \in \{D, L\}$. When hours are independent of (η, y) for singles, the wage share first order condition can be rearranged into a simple tradeoff between job finding and pay. Since the surplus term $W_1 - U_1$ increases with x via the learning option value, the optimal policy $\eta_k^*(x)$ is decreasing in x: high x workers accept a lower wage share in exchange for higher tightness and faster matching, which is valuable because it increases the probability of reaching the high productivity state in period 2. In joint households, endogenous hours shift the level of η^* upward, and can steepen the decline of $\eta^*(x)$ at low x because the hours feedback is stronger when the expected productivity is low (and low x workers are more likely to remain at y_L). We summarize these results below.

Selection: In the benchmark illustration, $\eta^*(x)$ is decreasing in skill x. Endogenous hours raise the level of η^* for joint households relative to singles and can make the mapping $x \mapsto \eta^*(x)$ more concave, with a steeper decline at low x when the hours feedback is strongest.

5.2 Opportunity Cost of hours

We now run some comparative statics with the model. In particular, we study what are the model implications of a decrease in the cost of working, capturing e.g. the availability of affordable childcare or, alternatively, the widespread increase in work from home policies

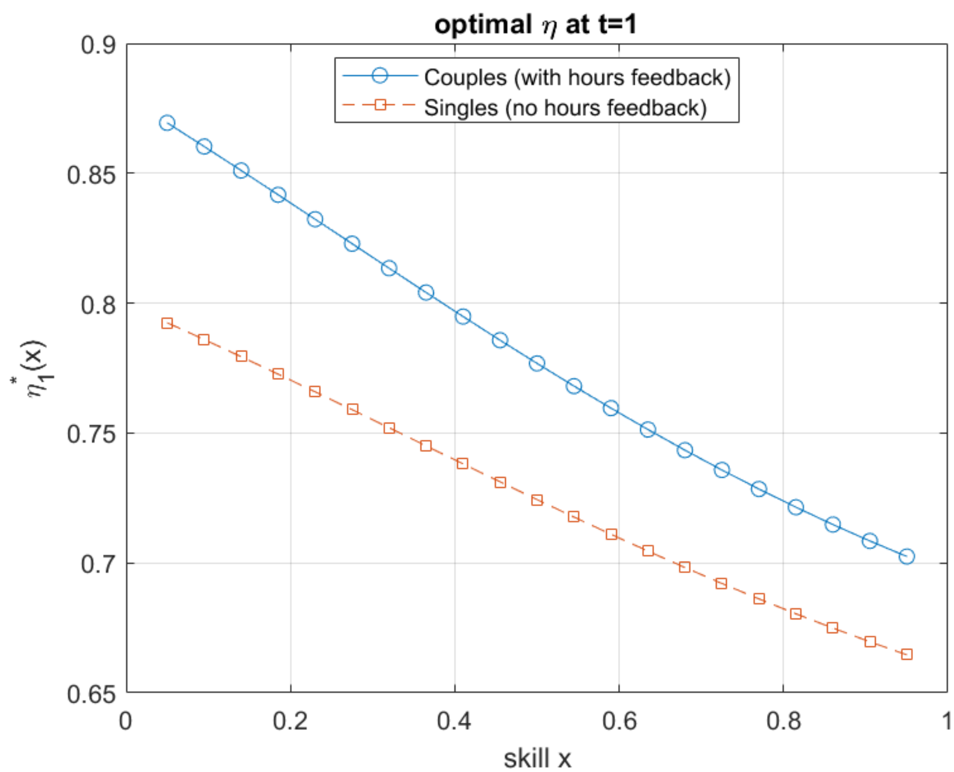

Figure 7: Optimal wage share in period 1 as a function of skill: singles versus joint households (illustration).

and availability post-pandemic. The ultimate goal is to understand what are the macroeconomic implications, in particular for productivity growth, of these exogenous changes in the economic environment (from the model point of view).

Opportunity cost of hours We characterize the effects of a change in disutility cost of hours ψ, emphasizing the difference that such a change has for singles relative to joint households. As we know the values of employment and of the firm at the end of period 2, it is useful to focus on the search problems in period 2. To simplify the math, we further impose the elasticity of the matching function with respect to unemployment to be $\alpha = 0.5$, which is in the range of estimates found for the matching function as documented in Petrongolo and Pissarides [2001]. This is the value we also impose in our quantitative exercises.

For singles, we have the following result:

$$\frac{d\eta^S}{d\psi} = \frac{\frac{1}{1-\eta^S}\frac{1}{1+\epsilon}\frac{1}{\psi}}{\frac{1}{(\eta^S)^2} + \left(1 + \frac{\eta^S[W_2(\eta^S)-U_2]}{1-\eta^S}\right)\frac{1}{\eta^S(1-\eta^S)}} \; > \; 0. \tag{12}$$

Thus, for singles, increasing the opportunity cost of hours raises the optimal wage share. The expression for singles make it particularly clear that ψ_b, through hours, affects both the value of employment (in the numerator) as well as the wage share η. Intuitively, when the opportunity cost of hours worked is high (ψ large), individuals require greater compensation for the supply of labor. As such, $d\eta^S/\psi > 0$.

Suppose member a is employed, member b is non-employed and searching for job. Then for this worker-searcher household, the optimal search problem is:

$$-\frac{1-\alpha}{\alpha}\left\{\frac{1}{1-\eta_b} - \frac{1}{h^*_{b2}}\frac{dh^*_{b2}}{d\eta_b}\right\}\Big[\mathcal{T}_2(\eta_a, y_a, \eta_b, y) - \mathcal{W}_2(\eta_a, y_a)\Big] + \frac{d\mathcal{T}_2(\eta_a, y_a, \eta_b, y)}{d\eta_b} = 0, \tag{13}$$

Differentiating (13) with respect to ψ_b reveals that several additional terms appear in the expression for joint households that were not present in the expression for singles (see Appendix D for the additional terms in the joint household problem). Here, we provide the basic intuition for the emergence of these additional terms. Unlike single households, the level of the wage share positively affects the labor supply decision not just of the individual but also of their partner. This impacts total hours worked within the joint household, which in turn affects both the gain to matching and the marginal value of dual employment. Further, while a higher ψ_b depresses labor supplied by member b, variations in η_b in response to ψ_b further affect member b's hours worked. This dual impact on the response of hours worked affects how job-finding rates vary with the choice of wage share. Proposition **??** characterizes how the choice of wage share varies with ψ for joint households.

Proposition 3. *Suppose the following conditions hold:*

$$\frac{dh_{b2}}{d\eta_b}\frac{\eta_b}{h_{b2}} < \frac{1}{(1+\epsilon)} \qquad \textit{and} \qquad \left|\frac{dh_{b2}}{d\psi_b}\frac{\psi_b}{h_{b2}}\right| < \frac{1}{1+\epsilon} \tag{14}$$

Then $\frac{d\eta_b}{d\psi_b} > 0$.

Proof. See Appendix D □

Proposition 3 says that if hours are not too elastic, then the desired wage share of the non-employed member in a joint household is increasing in the opportunity cost of hours. Intuitively, if hours are not too responsive such that the worker cannot significantly reduce her labor supplied when the opportunity cost of hours worked is high, then the worker requires greater compensation to be even employed in the first place and to supply labor.

What is left to characterize is whether $\frac{d\eta_b}{d\psi_b}$ is more or less positive for single than for joint households. To this end, we show this result numerically.

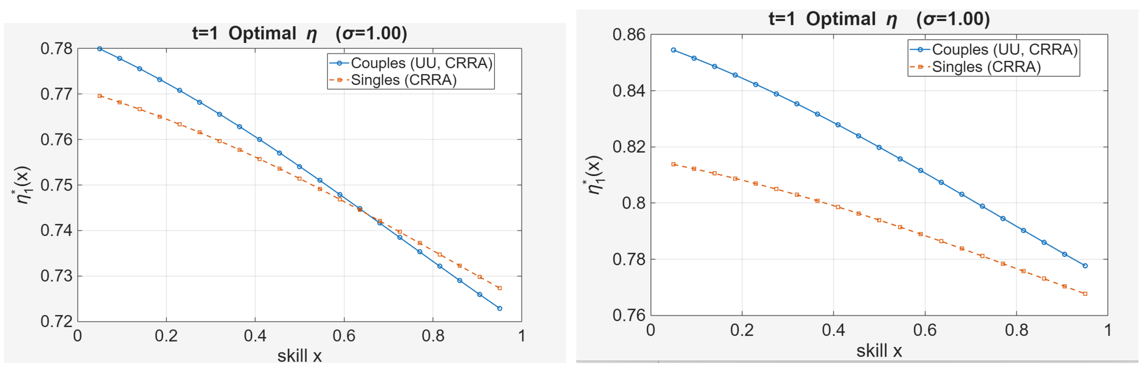

Figure 8: Optimal wage share in period 1 as a function of skill: singles versus joint households; low cost of hours ψ (left panel), high cost of hours ψ (right panel) - illustration.

Figure 8 show the comparative statics of changing the disutility cost of hours, captured by the parameter ψ, to complement Result 4 whenever the analytical characterization is not possible. Decreasing ψ (from right to left panel) has a level effect on the optimal submarkets chosen by both singles and households, as well as a slope effect. With lower ψ workers choose lower η to take advantage of the lower cost of working, and hence the relatively more attractive job opportunities. Interesintly, though, the job search strategies of singles and couples become more similar by skill. In other words, when the cost of hours is high, low skill workers choose a much higher wage share (and therefore are pickier) than low skill single workers. This result is much attenuated and even reverses for high skill households when ψ is sufficiently low: high skill couples trade-off a higher probability of getting employed against a lower wage share. This is intuitive and is at the core of the model mechanism: when working is less costly, it is precisely high skill joint households who start searching more aggresively to take advantage of relatively more attractive job opportunities, and in particular those where finding a job and accumulating human capital is easier. This exercise shows that the cost of providing hours in this model (and its changes) can affect job search and selection by skill among single and joint households, with potentially interesting macroeconomic implications to the extent that - as we posit - some jobs provide more opportunities to accumulate human capital than others.

Summary The central implication for the empirical patterns is that the household state affects search selectivity primarily through the equilibrium mapping $\eta \mapsto \theta(\eta)$, while hours are chosen after matching. Therefore, the model naturally generates movements in hours and job composition *conditional on employment* without requiring large movements in employment rates. In particular, increases in spousal resources can raise desired selectivity and thereby dampen job finding, while simultaneously increasing intensive margin. The objective of the next section is to quantitatively evaluate these forces and their implication for labor productivity trends, the central question of the paper.

6 Quantitative

TBA.

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A Proof for Lemma 1

Single households Because the household's hours decision is a static problem, we can focus on solving the following problem for singles:

$$\max_{h^s} \log \eta y h - \psi \frac{h^{1+\epsilon}}{1+\epsilon}$$

Taking first order conditions, optimal hours for singles can be written as:

$$h^s = \left(\frac{1}{\psi}\right)^{1/(1+\epsilon)}$$

which is the same as Equation 4. It is clear from the above that hours worked for singles only depend on parameters of the model, that is $\frac{dh^s}{d\eta} = 0$.

Worker-searcher households Worker-searcher households solve the following problem:

$$\max_{h_a} \log\left(b + \eta_a y_a h_a\right) - \psi_a \frac{h_a^{1+\epsilon}}{1+\epsilon}$$

Taking first order conditions, one can show that

$$h_a^{\epsilon}\left[\frac{b}{\eta_a y_a} + h_a\right] = \frac{1}{\psi_a}$$

which is the same as equation 5. By implicit function theorem, one can show that:

$$\frac{dh_a}{d\eta_a} = \frac{b y_a}{(\eta_a y_a)^2 + (b + \eta_a y_a h_a)^2 \epsilon \psi_a h_a^{\epsilon-1}} > 0$$

Thus, for worker-searcher households, the substitution effect dominates, workers supply more hours as their wage share increases.

Dual employed households Finally, for dual employed joint households, they solve the following problem:

$$\max_{h_a, h_b} \log(\eta_a y_a h_a + \eta_b y_a h_b) - \psi_a \frac{h_a^{1+\epsilon}}{1+\epsilon} - \psi_b \frac{h_b^{1+\epsilon}}{1+\epsilon}$$

Taking first order conditions, we arrive at:

$$h_a^{1+\epsilon}\left\{1 + \frac{\eta_b y_b}{\eta_a y_a}\frac{h_{b2}}{h_{a2}}\right\} = \frac{1}{\psi_a}$$

and

$$h_{b2}{}^{1+\epsilon}\left\{1 + \frac{\eta_a y_a}{\eta_b y_b}\frac{h_{a2}}{h_{b2}}\right\} = \frac{1}{\psi_b}$$

Denote $A = \frac{\eta_b y_b}{\eta_a y_a}$ and $z = \frac{h_a}{h_b}$ Then we can re-write the first order conditions as:

$$z^{\epsilon} h_b^{1+\epsilon}\left\{z + A\right\} = \frac{1}{\psi_a}$$

and

$$h_b^{1+\epsilon}\left\{\frac{z+A}{A}\right\} = \frac{1}{\psi_b}$$

We can divide first equation (the first order condition wrt h_a) by the second equation (the first order condition wrt h_b). This gives us:

$$z = \left[\frac{1}{A}\frac{\psi_b}{\psi_a}\right]^{\frac{1}{\epsilon}}$$

We can plug the form of z back into the FOC wrt h_b:

$$h_{b2} = \left(\psi_b \left[\frac{\eta_a y_a}{\eta_b y_b}\right]\left[\frac{\eta_a y_a}{\eta_b y_b}\frac{\psi_b}{\psi_a}\right]^{\frac{1}{\epsilon}} + 1\right)^{-1/(1+\epsilon)}$$

which is the same as equation 7. Finally, we can use the fact that $h_a = zh_b$ which gives us back equation 6.

Given these closed form solutions, one can easily show that taking derivatives with respect to η_a and η_b, we arrive at:

$$\frac{dh_a}{d\eta_a} = \frac{1}{\epsilon\eta_a}\psi_b h_b^{2+\epsilon}\left[\frac{\eta_a y_a}{\eta_b y_b}\frac{\psi_b}{\psi_a}\right]^{\frac{1}{\epsilon}} > 0 \tag{15}$$

and

$$\frac{dh_a}{d\eta_b} = -\frac{1}{\epsilon\eta_b}\psi_b h_b^{2+\epsilon}\left[\frac{\eta_a y_a}{\eta_b y_b}\frac{\psi_b}{\psi_a}\right]^{\frac{1}{\epsilon}} < 0 \tag{16}$$

and

$$\frac{dh_b}{d\eta_a} = -\frac{1}{\epsilon\eta_a}\psi_b h_b^{2+\epsilon}\left[\frac{\eta_a y_a}{\eta_b y_b}\right]^{\frac{1+\epsilon}{\epsilon}}\left[\frac{\psi_b}{\psi_a}\right]^{\frac{1}{\epsilon}} < 0 \tag{17}$$

and

$$\frac{dh_b}{d\eta_b} = \frac{1}{\epsilon\eta_b}\psi_b h_b^{2+\epsilon}\left[\frac{\eta_a y_a}{\eta_b y_b}\right]^{\frac{1+\epsilon}{\epsilon}}\left[\frac{\psi_b}{\psi_a}\right]^{\frac{1}{\epsilon}} > 0 \tag{18}$$

For dual employed households, hours worked is increasing in one's own wage share but declining in their partner's wage share.

B Proof for Proposition 1

In what follows, we consider the firm's value at the end of period 2. Note that in our simple 2 period model, the firm's value at the end of period 1 can always be characterized in terms of period 2 value functions and parameters of the model. As such, it is without loss of generality that we only show our results using the firm's value at the end of period 2.

Singles Consider the firm attached to a single worker at the end of period 2. This firm's value is given by:

$$J_2(\eta, x, y) = (1-\eta) y h^s$$

Then from equation 1, we can derive the labor market tightness in an active sub-market for singles as:

$$\theta_2(\eta, x, y) = \left(\frac{\xi}{\kappa}(1-\eta)\, y h^s\right)^{1/\alpha}$$

then using equation 4, independence of hours with respect to wage income implies $\Omega_2(\eta) = 0$ and

$$\frac{d \log \theta_2(\eta, x, y)}{d\eta} = -\frac{1}{\alpha}\left(\frac{1}{1-\eta}\right) < 0$$

Then since tightness is declining in η, employment values are increasing in η, the single non-employed's search problem admits a unique interior solution so long as the household's participation constraint is satisfied. The unique sub-market searched for given the household's state is characterized by equation 9.

Joint households For ease of exposition, we consider the problem of a firm matched to member b from a dual employed household at the end of period 2. Note that the same arguments below also apply to a firm attached to member a from a worker-searcher household. At the end of period 2, the firm matched to member b from a dual employed household has value given by:

$$\mathcal{J}_2(\mathbf{n}, \mathbf{x}, \mathbf{y}) = (1-\eta_b) y_b h_b(\mathbf{n}, \mathbf{x}, \mathbf{y}) \tag{19}$$

Differentiating this value w.r.t to η, we get:

$$\frac{d\mathcal{J}_2(\mathbf{n}, \mathbf{x}, \mathbf{y})}{d\eta_b} = -y_b h_b(\mathbf{n}, \mathbf{x}, \mathbf{y}) + (1-\eta_b)\, y_b \frac{dh_b(\mathbf{n}, \mathbf{x}, \mathbf{y})}{d\eta_b}$$

Under free-entry, we know that

$$\theta_2(\mathbf{n}, \mathbf{x}, \mathbf{y}) = \left[\frac{\xi}{\kappa}(1-\eta_b) y_b h_b(\mathbf{n}, \mathbf{x}, \mathbf{y})\right]^{1/\alpha}$$

And thus:

$$\frac{d \log \theta_2(\mathbf{n}, \mathbf{x}, \mathbf{y})}{d\eta_b} = \frac{1}{\alpha}\left(-\frac{1}{1-\eta} + \underbrace{\frac{dh_b(\mathbf{n}, \mathbf{x}, \mathbf{y})}{d\eta_b}}_{\Omega(h_b)}\right)$$

From equation 18, we know $\frac{dh_b(\mathbf{n},\mathbf{x},\mathbf{y})}{d\eta_b} > 0$ and thus $\Omega(h_b) > 0$. Because the substitution effect dominates, workers in joint households respond to increases in η by supplying more labor. This in turn gives rise to the positive hours feedback, $\Omega(h)$, as firms are more willing to create vacancies at higher wage shares if workers are willing to supply more labor.

To understand when the worker's search problem permits an interior solution, observe that equation 19 is strictly declining in η if the first term is larger in absolute terms than the second term on the RHS, i.e., we require

$$1 > \frac{(1-\eta_b)}{\eta_b}\frac{dh_b(\mathbf{n},\mathbf{x},\mathbf{y})}{d\eta_b}\frac{\eta_b}{h_b(\mathbf{n},\mathbf{x},\mathbf{y})} = \frac{1-\eta_b}{\eta_b}\frac{1}{\epsilon}\left(1-\psi_b h_b^{1+\epsilon}\right)$$

where the last equality substitute in the information from equations 7 and 18. Further, since from equation 7, one can show that

$$0 < \psi_b h_b^{1+\epsilon} = \left(\left[\frac{\eta_a y_a}{\eta_b y_b}\right]\left[\frac{\eta_a y_a}{\eta_b y_b}\frac{\psi_b}{\psi_a}\right]^{\frac{1}{\epsilon}} + 1\right)^{-1} < 1$$

a sufficient condition for the firm's value to be declining in η is given by:

$$\frac{\eta_b}{1-\eta_b} > \frac{1}{\epsilon} > \frac{1}{\epsilon}\left(1-\psi_b h_b^{1+\epsilon}\right)$$

Equivalently, the sufficient condition above can also be re-expressed as:

$$\eta > \frac{1}{1+\epsilon}$$

Thus, for large enough ϵ, the firm's value is declining in η, which implies that under the free entry condition 3, tightness is declining in η.

$$\frac{d\theta_2(\mathbf{n},\mathbf{x},\mathbf{y})}{d\eta_b} = -\frac{1}{\alpha\eta_b}\theta_2(\mathbf{n},\mathbf{x},\mathbf{y})\left[\frac{\eta_b}{(1-\eta_b)} - \frac{dh_b(\mathbf{n},\mathbf{x},\mathbf{y})}{d\eta_b}\frac{\eta_b}{h_b(\mathbf{n},\mathbf{x},\mathbf{y})}\right] < 0$$

Then given that tightness is declining in η, and dual employment values are increasing in η, the non-employed member's search problem in a worker-searcher household also admits a unique interior solution so long as the household's participation constraint is satisfied. The unique sub-market is characterized by equation 11.

C Proof of Proposition 2

In our 2 period model, end of period 2 values are independent of x since households consume their realized income and firms take home their realized profits. Accordingly, the only period where job-selection matters is in period 1. This is because individuals who select into learning jobs in period 1, observe some probability to increase their human capital in period 2.

For the single household, the value of employment in a learning job at the end of period 1 can be written as:

$$\begin{aligned} W_1(x,\eta,y_L) &= \ln \eta y_L h^s - \psi \frac{(h^s)^{1+\epsilon}}{1+\epsilon} \\ &\quad +\beta \Big\{ W_2(\eta,y_L) - \delta \left[W_2(\eta,y_L) - U_2 \right] \\ &\quad +\gamma(x) \left[W_2(\eta,y_H) - W_2(\eta,y_L) \right] \Big\} \end{aligned}$$

where we have used the fact that $U_2(x) = U_2 = \log b$ and $W_2(\eta,x,y) = W_2(\eta,y) = \log \eta y h^s$. The value of employment at a dead-end job at the end of period 1 can be written as:

$$W_1(\eta,x,y_D) = W_1(\eta,y_D) = \ln \eta y_D h^s - \psi \frac{(h^s)^{1+\epsilon}}{1+\epsilon} + \beta W_2(\eta,y_D) + \beta\delta \left[U_2 - W_2(\eta,y_D) \right]$$

In our two period model, the value of employment in a dead-end job is independent of x. This in turn implies that the value of search in the dead-end job is independent of x, i.e., $\widetilde{R}^D(x) = \widetilde{R}^D$. In contrast, the employment value in a learning job does depend on x. Differentiating $W_1(\eta,x,y_L)$ wrt x, one can show that

$$\frac{dW_1(x,\eta,y_L)}{dx} = \beta\gamma'(x) \log \frac{y_H}{y_L} > 0$$

Further, job-finding rates are also increasing in x as the value of a firm attached to an unskilled single worker in a learning job in period 1 is also increasing in x:

$$J_1(x,\eta,y_L) = (1-\eta)\left(\left[1 + \beta(1-\delta) \right] y_L h^s + \beta\gamma(x) h^s \left[y_H - y_L \right] \right)$$

and thus $\frac{dJ_1(x,\eta,y_L)}{dx} = \beta\gamma'(x) h^s \left[y_H - y_L \right] > 0$. Given that both job finding rates and employment values are increasing in x for the learning job, this implies the search value of a learning job, $\widetilde{R}^L(x)$ is increasing in x. Since $\widetilde{R}^L(x)$ while $\widetilde{R}^D$ is constant for all x, there exists a unique cut-off x for single households, above which they always select into learning jobs.

A similar argument pertains to joint households where one can show that because end-of-period 2 values are independent of x, the value of searching in a dead-end job in period

1 does not vary in x, $\widetilde{\mathcal{R}}_{i,1}^{D,uu}(\mathbf{x}) = \widetilde{\mathcal{R}}_{i,1}^{D,uu}$ and $\widetilde{\mathcal{R}}_{i,1}^{D,eu}(\eta, \mathbf{x}, y) = \widetilde{\mathcal{R}}_{i,1}^{D,eu}(\eta, y)$ for $i \in \{a, b\}$. Conversely, job-finding rates and employment values in a learning job are increasing in x through the probability that the worker gains human capital, $\gamma(x)$. Consequently, given a household state $\mathbf{s}$, since the search value in a dead-end job is independent of x while the search value in a learning job is increasing in x, there exists a cut-off $x^*(\mathbf{s})$ above which that member of the household selects into a learning job.

D Proof of Proposition 3

Differentiating 13 with respect of ψ_b, we have three separate terms (we highlight differences relative to the expression for singles):

Derivative related to job-finding:

$$\frac{d}{d\psi_b}\left\{\frac{1}{1-\eta_b} - \frac{1}{h_b}\frac{dh_b}{d\eta_b}\right\} = - \underbrace{\frac{h_b^{1+\epsilon}}{\epsilon\eta_b}\frac{dh_b}{d\eta_b}\frac{\eta_b}{h_b}}_{\text{additional term in numerator}} + \left[\frac{1}{(1-\eta_b)^2} + \overbrace{\frac{(1-\psi_b h_b^{1+\epsilon})}{\epsilon\eta_b^2}\left(1 + \frac{(1+\epsilon)\psi_b h_b^{1+\epsilon}}{\epsilon}\right)}^{\text{additional term in denominator}}\right]\frac{d\eta_b}{d\psi_b} \tag{20}$$

In relation to job finding, an increase in ψ_b induces a direct impact on hours worked (first line of RHS), as well as an indirect impact on hours worked through its impact on the desired wage share (second term in square brackets in second line of RHS).

Derivative related to gain in matching

$$\frac{d}{d\psi_b}\Big[\mathcal{T}_2 - \mathcal{W}_2\Big] = \left(\frac{1}{\eta_b} \overbrace{-\frac{\epsilon}{\eta_b}\frac{dh_b}{d\eta_b}\frac{\eta_b}{h_b}}^{\text{additional term in denominator}}\right)\frac{d\eta_b}{d\psi_b} - \frac{1}{\psi_b}\left(1 + \epsilon \underbrace{\frac{dh_b}{d\psi_b}\frac{\psi_b}{h_b}}_{\text{additional term in numerator}}\right), \tag{21}$$

In relation to gains to matching, a rise in ψ impacts the selected wage share through its additional impact on how hours worked of both the individual *and* the individual's partner vary with η_b (2nd term on first line of RHS). A rise in ψ also changes directly affects the hours worked of the individual (2nd line of RHS).

Derivative related to marginal value of dual employment

$$\frac{d}{d\psi_b}\left[\frac{d\mathcal{T}_2}{d\eta_b}\right] = \overbrace{\frac{h_b^{1+\epsilon}}{\eta_b}\left[1+(1+\epsilon)\frac{dh_b}{d\psi_b}\frac{\psi_b}{h_b}\right]}^{\text{additional term in numerator}} + \underbrace{\left[(1+\epsilon)\frac{dh_b}{d\eta_b}\frac{\eta_b}{h_b}-1\right]\frac{\psi_b h_b^{1+\epsilon}}{\eta_b^2}}_{\text{different denominator}}\frac{d\eta_b}{d\psi_b}. \tag{22}$$

An increase in ψ_b impacts the marginal value of dual employment by changing the hours worked of both members directly (1st line of RHS). The second line in the RHS shows how the marginal value of dual employment is also impacted because hours also respond to variations in η_b.

Using these expressions in 13, and given $\alpha = 0.5$, the LHS of the derivative of equation 13 with respect to ψ_b becomes:

$$\begin{aligned} LHS = & \left\{ \left[\frac{1}{[1-\eta_b]^2} + \frac{\left(1-\psi_b h_b^{1+\epsilon}\right)}{\epsilon\eta_b^2} + \frac{(1+\epsilon)\,\psi_b h_b^{1+\epsilon}}{\epsilon\eta_b^2}\frac{dh_b}{d\eta_b}\frac{\eta_b}{h_b}\right]\left[\mathcal{T}_2(\cdot) - \mathcal{W}_2(\cdot)\right] \right. \\ & + \left[\frac{\eta_b}{(1-\eta_b)} - \frac{dh_b}{d\eta_b}\frac{\eta_b}{h_b}\right]\frac{1}{\eta_b^2}\left[1-\epsilon\frac{dh_b}{d\eta_b}\frac{\eta_b}{h_b}\right] \\ & \left. + \left[1-(1+\epsilon)\frac{dh_b}{d\eta_b}\frac{\eta_b}{h_b}\right]\frac{\psi_b h_b^{1+\epsilon}}{\eta_b^2}\right\}\frac{d\eta_b}{d\psi_b} \end{aligned}$$

The second summand of the LHS is positive given the results in Proposition 1, i.e., as long as

$$\frac{\eta_b}{1-\eta_b} > \frac{1}{\epsilon} > \frac{1}{\epsilon}\left(1-\psi_b h_b^{1+\epsilon}\right) = \frac{dh_b}{d\eta_b}\frac{\eta_b}{h_b} > 0$$

The third summand of the LHS is positive if

$$\left[1-(1+\epsilon)\frac{dh_b}{d\eta_b}\frac{\eta_b}{h_b}\right] > 0 \implies \frac{1}{(1+\epsilon)} > \frac{dh_b}{d\eta_b}\frac{\eta_b}{h_b}$$

which implies the additional restriction that

$$\frac{dh_b}{d\eta_b}\frac{\eta_b}{h_b} < \frac{1}{(1+\epsilon)}$$

Turning to the RHS of the derivative:

$$
\begin{aligned}
RHS &= \frac{h_b^{1+\epsilon}}{\epsilon\eta_b}\frac{dh_b}{d\eta_b}\frac{\eta_b}{h_b}\left[\mathcal{T}_2\left(\cdot\right)-\mathcal{W}_2\left(\cdot\right)\right] \\
&\quad +\frac{1}{\eta_b}\frac{1}{\psi_b}\left\{\frac{\eta_b}{\left(1-\eta_b\right)}-\frac{dh_b}{d\eta_b}\frac{\eta_b}{h_b}\right\}\left(1+\epsilon\frac{dh_b}{d\psi_b}\frac{\psi_b}{h_b}\right) \\
&\quad +\frac{h_b^{1+\epsilon}}{\eta_b}\left[1+\left(1+\epsilon\right)\frac{dh_b}{d\psi_b}\frac{\psi_b}{h_b}\right]
\end{aligned}
$$

The first line of the RHS is positive so long as the household's participation constraint is satisfied. For the RHS, note that if

$$
\left|\frac{dh_b}{d\psi_b}\frac{\psi_b}{h_b}\right| < \frac{1}{\left(1+\epsilon\right)} < \frac{1}{\epsilon}
$$

then the conditions above guarantee that:

$$
\left[1+\left(1+\epsilon\right)\frac{dh_b}{d\psi_b}\frac{\psi_b}{h_b}\right] > 0 \quad \text{and} \quad \left(1+\epsilon\frac{dh_b}{d\psi_b}\frac{\psi_b}{h_b}\right)>0
$$

Then RHS is positive. Together with positive LHS, this implies $\frac{d\eta_b}{d\psi_b}$ is also positive for the joint households.