Hours-Biased Technological Change

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Job Market Paper

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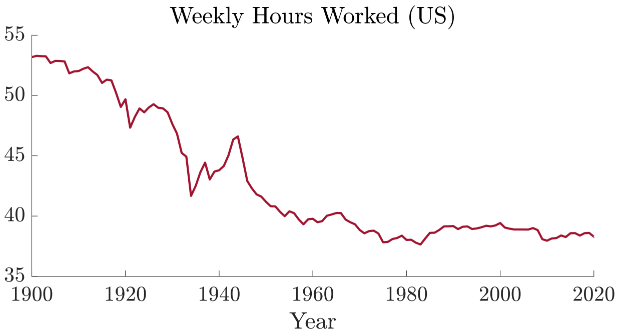
Intro: The Evolution of Working Time

How did work hours evolve over time?

Hours Decrease Over Time

- Hours per worker decline over the very long run.



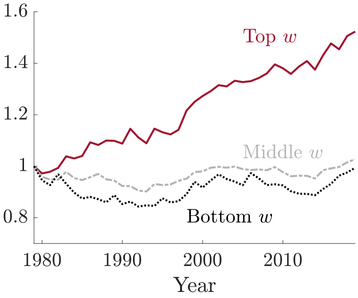
⏩ Cross-country ⏩ Cross-country: over time ⏩ Cross-country vs US ⏩ Hours/capita ⏩ By Gender ⏩ Extensive Margin

Intro: The Evolution of Working Time

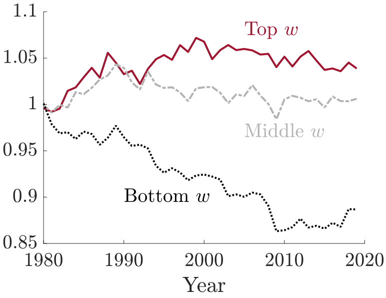
- Hours decline with wages in the long-run...
  - Leading explanation: income effect $>$ substitution effect (wage $\uparrow$, hours $\downarrow$)
  - Keynes(1930), Boppart and Krusell (2020)

Hours Increased For High-Wage Workers

- Cross-section: Wages increase relatively more at the top (wage inequality ↑)
- High-wage workers increased their hours



(a) Wages



(b) Hours

Figure: Wages and Hours By Wage Decile w (US)

⏩ ATUS ⏩ All Quintiles ⏩ Across Countries ⏩ Progressivity ⏩ ASEC

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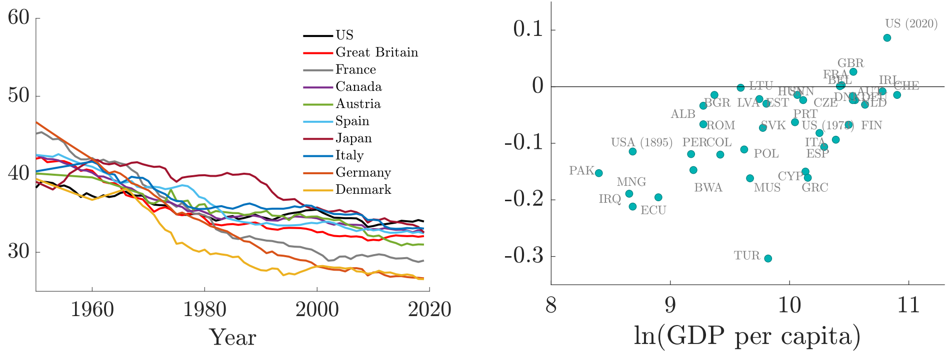
  - $\implies$ Hours-wage correlation turns from negative to positive ▸▸ Regression
  - Puzzle: if income effects dominates, we would expect the opposite

Common Patterns Across Countries

- Long-run declines in hours worked
- Cross-sectional hours-wage relationship increases with development



(a) Hours Worked (b) Hours-Wage Correlation (cross-section)

Figure: Cross-country patterns

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- How can we reconcile these patterns? Answer to this question is key for:
  - Tax Policy
  - Understanding inequality
  - Business cycles fluctuations

Hours-Biased Technological Change

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  - E.g. cheaper and more powerful computers
  - SBTC: technologies increase productivity of skilled workers (vs unskilled)

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Can HBTC reconcile patterns of hours worked? What are the implications?

This Paper

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    - Nests several known models as special cases

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- Quantitatively analyze technological change in US
  - Novel channel of technology on inequality
  - New driver of aggregate hours worked in US (dampen income effect)

Literature

- **Aggregate Hours**: King, Plosser, Rebelo (1988); Prescott (2004); Ramey and Francis (2009); Boppart and Krusell (2020); Rachel (2021); Bick et al. (2021).
  Take into account cross sectional heterogeneity

- **Leisure inequality**: Costa (2000); Aguiar and Hurst (2007); Kopitov et al. (2020); Boppart and Ngai (2021); Michelacci and Pijoan-Mas (2014); Doepke et al. (2018).
  Introduce sorting and endogenous wage inequality

- **Sorting**: Eeckhout and Kircher (2018); Eeckhout and Sepahsalari (2018); Calvo, Lindenlaub, Reynoso (2021); Michelacci and Pijoan-Mas (2014); Chade and Lindenlaub (2022).
  Characterize sorting for general preferences and technology

Model

Setup

- Population
  - Individuals with skill $x \sim G^x$
  - Jobs of type $y \sim G^y$
- Preferences
  - Choose hours h and consumption c
  - Utility function: $u(c, 1 - h)$
- Technology
  - Competitive labor market
  - Workers and jobs(firms) match one-to-one $\implies$ sorting
  - Output: $f(x, y, h)$
  - Note: $h = h(x)$ is endogenous and skill dependent

Setup

- Payoffs
  - Workers maximize: $u(c, 1-h)$ s.t. $c = e(x,h)$
  - Jobs maximize: $f(x,y,h) - e(x,h)$
  - In equilibrium, wages w are defined as $w = \frac{e}{h}$, where $w = w(x,y,h(x,y))$

- Market clearing
  - Equilibrium matching $y = \mu(x)$ assigns workers x to jobs y.
  - Under Positive Assortative Matching (PAM), market clearing requires:

$$\int_{\mu(x)}^{\bar{y}} g^y(s)ds = \int_{x}^{\bar{x}} g^x(s)ds \ ,$$

where g^x and g^y are the densities of workers and jobs, respectively. Details

- Competitive equilibrium = optimality + market clearing:
  - income $e(x)$; hours $h(x)$; matching $\mu(x)$ Full definition

Problem as Pareto Frontier

Pair's problem as Pareto Frontier

- Workers x and jobs y simultaneously solve, respectively:

$$\max_{h,y}\ u(c, 1-h) \quad \text{s.t.} \quad c = e$$

$$\max_{x}\ f(x, y, h) - e$$

- This can be re-written as pair x, y solving:

$$U(x, y, \pi) = \max_{h} u(f(x, y, h) - \pi, 1 - h)$$

where $U(x, y, .)$ defines the Pareto Frontier (for all profit values π).

Optimal Job Choice

Optimization

- Optimal job choice implies positive sorting iff:

$$U_{xy} - \frac{U_y}{U_\pi} U_{x\pi} \geq 0 \iff -\frac{U_y}{U_\pi} \text{ increasing in } x$$ ⏩ Proof

- Intuition: high x more willing to give more π (=less income) for a better y

- Trade-off between preferences (U_π) vs complementarity (U_y) across skills

Assortative Matching

Check $-\frac{U_y}{U_\pi} \uparrow x$

Proposition: *The equilibrium features Positive Assortative Matching iff:*

$$f_{xy} + f_{yh} h_x \geq 0$$

▸▸ Alternative Condition ▸▸ Decentralized eq.

- Intuition: two forces pushing towards positive sorting (PAM)
  - Skill-job complementarity f_{xy} (standard)
  - High skill working more ($h_x > 0$) + hours-job complementarity $f_{yh} > 0$ (new)

- Low skill working much more ($h_x << 0$) and $f_{yh} > 0$ can lead to NAM

Hours choice

- FOC for hours implies $u_c f_h + u_h = 0$. From this, we get that:

Proposition: *In equilibrium, high skill choose higher hours if:*

$$\underbrace{u_c f_{hx}}_{\text{subs. effects}} > \underbrace{-u_{cc} f_h f_x}_{\text{income effects}}$$

▸▸ Derivation ▸▸ CRRA

- Income effects (recall $u_{cc} < 0$)
  - Key force to explain long run behavior of hours (Boppart and Krusell, 2020)
- Hours complementarities: f_{xh}
  - If positive, raises marginal product of hours worked for high skill $\implies$ *HBTC*

Functional forms

- Production: $f(x,y,h) = A\left(\beta(\alpha x^{\rho} + (1-\alpha)y^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)h^{\gamma}\right)^{\frac{1}{\gamma}}$
  - Captures both SBTC (through α,ρ) and HBTC (through γ)
- Preferences (MaCurdy, 1981): $u(c,h) = \frac{c^{1-\sigma}}{1-\sigma} - \psi\frac{h^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}$
  - σ = strenght of income effects

Comparative Statics

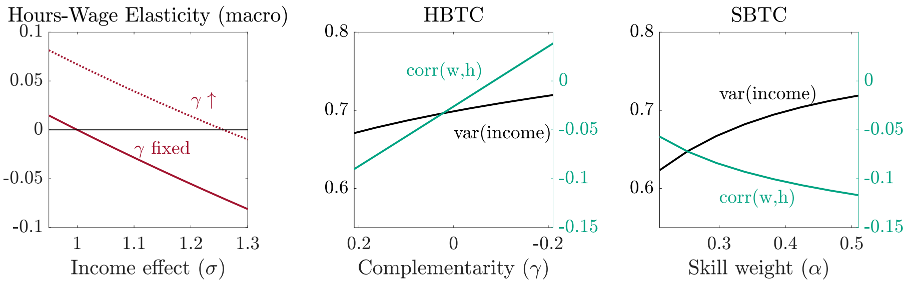


Figure: Comparative Statics.

Key takeaways:

- Hours complementarities (γ) matter for both *macro* and *micro* elasticities
- HBTC vs SBTC drive inequality up, but opposite effect on micro elasticity

Special Cases

- Becker (1973) sorting model
  - $f_{yh} = f_{xh} = 0$. Sorting only depends on f_{xy}
- Macro labor supply models with linear earnings
  - Hours affect earnings linearly (Boppart and Krusell, 2020)
  - Misses cross-sectional facts
- SBTC frameworks (katz-Murhpy, Acemoglu-Autor)
  - Capture increase in rel. productivity of skilled workers (extensive margin)
  - Silent about rel. productivity of hours (intensive margin)
- 'Effective types' models
  - Firms match with bundle $\tilde{x} = \tilde{x}(x, h) \Rightarrow f = f(\tilde{x}, y)$
  - Used to study sorting and optimal taxation (Scheuer and Werning, 2016)
- Non-linear earnings (Erosa et al., 2016; Bick et al., 2021)
  - hours choice depends on f_{xh}/u_{cc} (no matching effect)

Estimation

Overview

- Can changes in technology rationalize the data? Recall the prod. function:

$$f(x,y,h) = A\left(\beta(\alpha x^{\rho} + (1-\alpha)y^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)h^{\gamma}\right)^{\frac{1}{\gamma}} \tag{1}$$

- Estimate parameters year by year to match moments from the data (SMM).
  $\implies A = A_t,\ \alpha = \alpha_t,\ \beta = \beta_t,\ \gamma = \gamma_t,\ \rho = \rho_t$.

- Note: (1) is equivalent to

$$f(x,y,h) = \left((A_x x^{\rho} + A_y y^{\rho})^{\frac{\gamma}{\rho}} + A_h h^{\gamma}\right)^{\frac{1}{\gamma}}$$

- Goal: Assess changes in weights (A_t, α_t, β_t) and complementarities (ρ_t, γ_t)

Strategy

- Preferences: calibrate σ and θ
  - σ: mid-range of macro studies (1.4)
  - $\theta = 0.4$: consistent with micro-evidence (Violante et al, 2014).

- Distributions G^w and G^f
  - Assume log-normality: $x \sim \mathcal{LN}(\mu_x, \sigma_x)$; $y \sim \mathcal{LN}(\mu_y, \sigma_y)$
  - Set $\mu_x = \mu_y = 0$ and estimate ratio σ_x / σ_y

- Six parameters to be estimated (5 technology, 1 distributions)

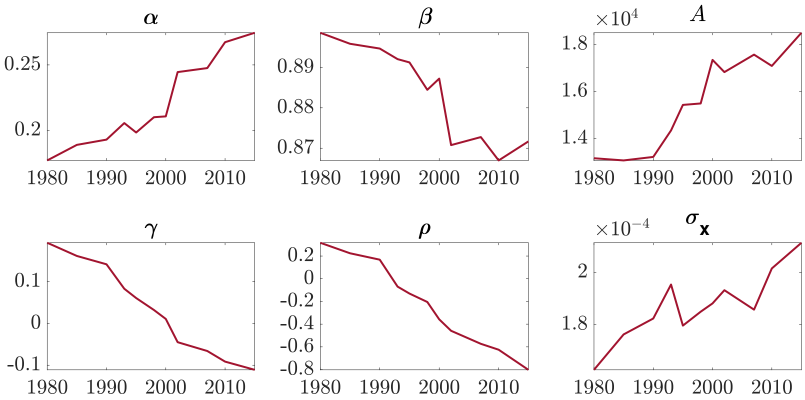
E(h)	Avg. Hours
E(w)	Avg. Wages
w90/w50	wage inequality (top)
w90/w10	wage inequality (overall)
Hours-wage Elasticity	Coeff. of reg. $log(h)$ on $log(w)$
std(w)	wage dispersion

Table: Summary of Targeted Moments

Estimates

- Both SBTC ($\alpha \uparrow$ and $\rho \downarrow$) and HBTC ($\gamma \downarrow$) Model Fit

$$f(x, y, h) = A\left(\beta(\alpha x^{\rho} + (1-\alpha)y^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)h^{\gamma}\right)^{\frac{1}{\gamma}}$$



Income Inequality

- Decomposite *income* inequality in:

$$var(i) = var(w) + var(h) + 2cov(w,h)$$

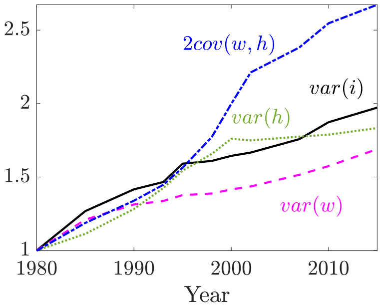
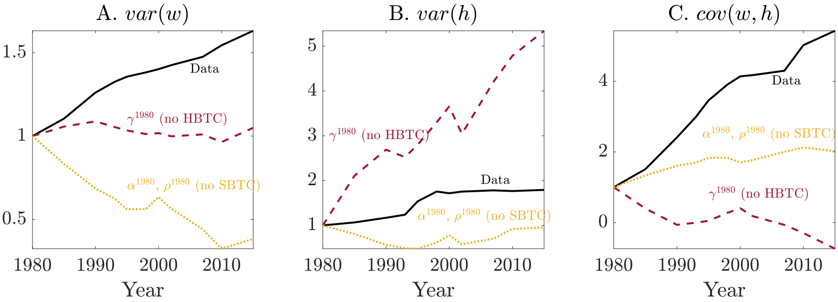


Figure: Drivers of Income Inequality (US)

- Different drivers of inequality. How does the model rationalize this?

Results: Drivers of Inequality



Results:

- HBTC ($\gamma \downarrow$) constributes to income inequality through $cov(w, h)$
- α ($\approx$ *SBTC*) more important for $var(w)$ but pushes $cov(h, w)$ down

Results: Fixed Hours Model

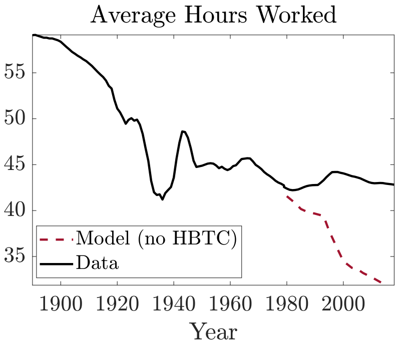
What is the role of endogenous hours response?

$$f(x, y, h) = A\left(\beta(\alpha x^{\rho} + (1-\alpha)y^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)h^{\gamma}\right)^{\frac{1}{\gamma}}$$

Estimated Change 1980-2015		
Parameter	Baseline	Fixed-hours
β	-60%	+2%
α	+199%	+102%
γ	-202%	-88%
ρ	+294%	+1200%

Fixed-h model overstates magnitude of SBTC (α, ρ) and understates HBTC ($\gamma \downarrow$)

Results: Aggregate Hours



- Explanation for relatively flat average hours worked in US post-1970
  - One explanation: income = substitution effect
  - This paper: income effect (Boppart and Krusell, 2020) + HBTC

- Across development path, relative strenght of these two forces vary

Robustness

On preferences:

- Income effects (different σ) ⏩ Results
- Idiosyncratic tastes for leisure ⏩ Results

On distributions:

- Take distributions from the data ⏩ Distributions ⏩ Results 1 ⏩ Results 2
- Assume Beta distributions (Lise, Meghir, Robin 2016) ⏩ Results
- Estimate only σ_x ⏩ Fit ⏩ Estimates

On technology/estimation:

- Different nest of CES function ⏩ Nest 2: Simulation ⏩ Nest 2: Results ⏩ Nest 3: Results ⏩ Nests: Comparison
- Simulate increase in tax progressivity (in progress)
- Standard errors + Sensitivity matrix (Shapiro et al, 2017 QJE) ✓

Conclusion

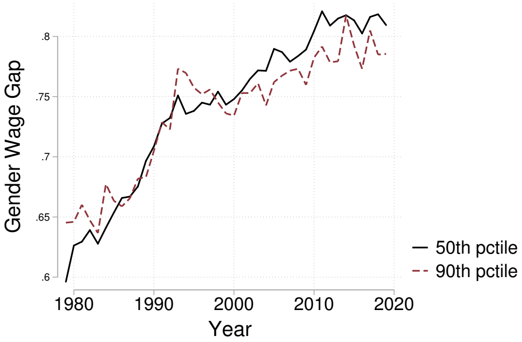
- Develop a theory that features effect of endogenous hours choice on sorting and inequality
  - Novel condition for assortative matching
- Novel mechanism: Hours-Biased Technological Change
  - New technologies raise returns to long hours for the high skill
- Estimate the framework: 1980-2015 in US
  - HBTC reconciles hours in the cross-section and time series

Future Work

- How do hours constraints (e.g. social norms) impact sorting?
  - Recall sorting condition: $f_{xy} + f_{yh}h_x$
  - If hours choice is distorted, this can affect misallocation and inequality
- Application: the (stalling) gender gap ▸▸ Empirics
  - Gender gap stalled for the high skilled since 1990's (Blau and Kahn, 2014)
  - Gap is higher in occupations where hours elasticity is higher (Goldin, 2014)
- Can HBTC provide an explanation for stalling gender gap? Two forces:
  - Over time, easier for women to provide longer hours (hours constraints $\downarrow$)
  - Technology amplifies effect of such constraints

Implications: Hours Constraints and the Gender Gap

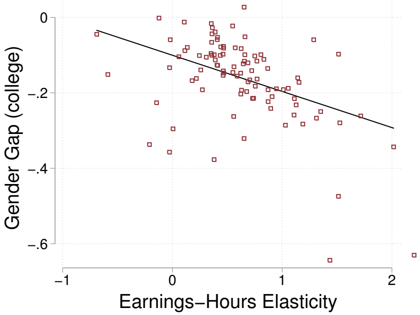
Gender gap has been decreasing across the wage distribution.



However, slower decline at the top since 1990's. ⏩ Back

Implications: Hours Constraints and the Gender Gap

Tight link between gender gap and how income responds to hours (Goldin, 2014).



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Distributions: From the Data

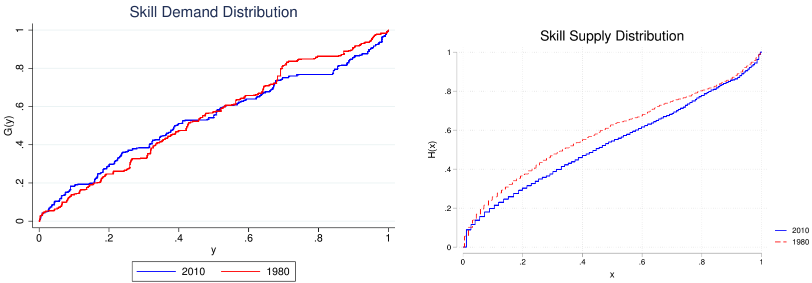

Figure: Estimated skill demand (left) and supply (right) distributions, for 1980 and 2010.

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Estimates: Fitted Distributions

Parameter	1980	2015	Meaning
β	0.91	0.83	weight of (x, y) in prod.
	(0.02)	(0.02)	
α	0.24	0.79	weight of skills in prod.
	(0.04)	(0.08)	
γ	0.22	-0.39	compl. $(h, (x, y))$
	(0.02)	(0.02)	
ρ	-1.58	-4.24	compl. (x,y)
	(0.27)	(1.0)	
A	183.1	127.1	TFP
	(27)	(1.0)	

Notes: Standard errors in parentheses.

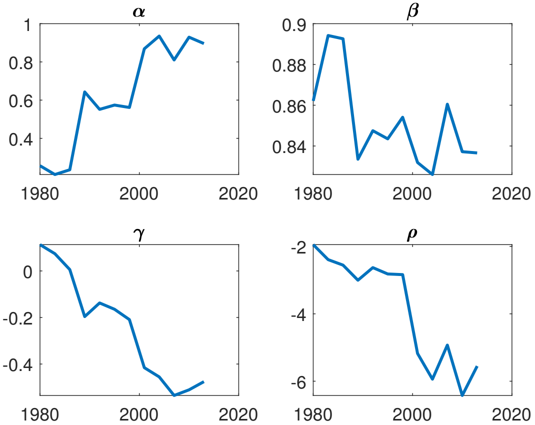
Key parameter changes:

- $\alpha \uparrow$ and $\rho \downarrow$: x, y more complementary
- $\gamma \downarrow$: hours/skills are more complementary ($f_{xh} \uparrow$)

Back

Estimates: Fitted Distributions

Parameters Estimated Over Time



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Estimates: Lower Income Effects

Parameter	1980	2015	Meaning
β	0.99	0.98	weight of (x, y) in prod.
α	0.05	0.13	weight of skills in prod.
γ	0.51	0.09	compl. $(h, (x, y))$
ρ	0.98	-0.57	compl. (x,y)
A	28,5179	26,677	TFP

Key parameter changes very similar to baseline specification:

- $\alpha \uparrow$ and $\rho \downarrow$: x, y more complementary
- $\gamma \downarrow$: hours/skills are more complementary ($f_{xh} \uparrow$)

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Estimates: Idiosyncratic Tastes for Leisure

Introduce idiosyncratic tastes for leisure ψ_i in the utility function to match dispersion in hours.
Captures preferences, amenities, health shocks..

Parameter	1980	2015	Meaning
β	0.81	0.60	weight of (x, y) in prod.
α	0.23	0.67	weight of skills in prod.
γ	0.02	-0.13	compl. $(h, (x, y))$
ρ	0.98	-0.57	compl. (x,y)
A	6,5220	7,1405	TFP

Key parameter changes very similar to baseline specification.

Back

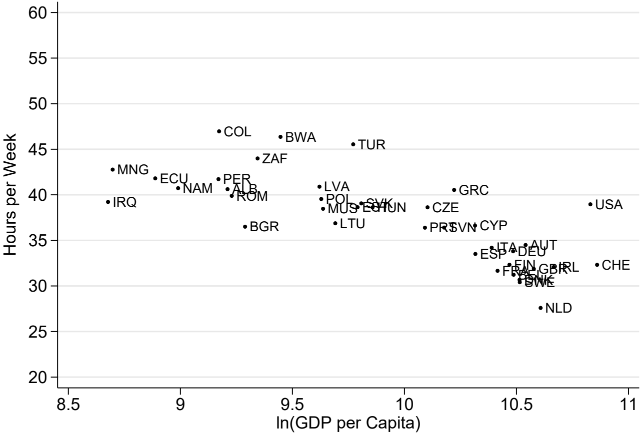
Estimates: Assume Beta Distributions

Parameter	1980	2015	Meaning
β	0.94	0.91	weight of (x, y) in prod.
α	0.08	0.14	weight of skills in prod.
γ	0.30	-0.07	compl. $(h, (x, y))$
ρ	0.93	-2.48	compl. (x,y)
A	16,5103	16,5703	TFP

Key parameter changes very similar to baseline specification.

Back

Additional evidence: Hours/worker over GDP



Source: Bick et al. (2018)

Hours/worker decline with GDPc in middle income and rich countries.

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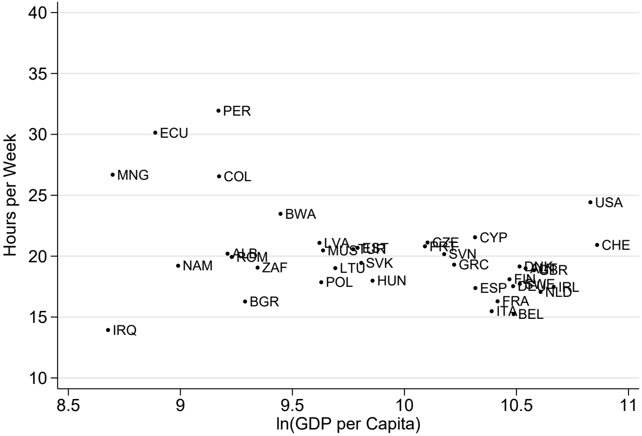
Additional evidence: Hours/capita over GDP



Source: Bick et al. (2018)

Hours/capita decline with GDPc in middle income and rich countries.

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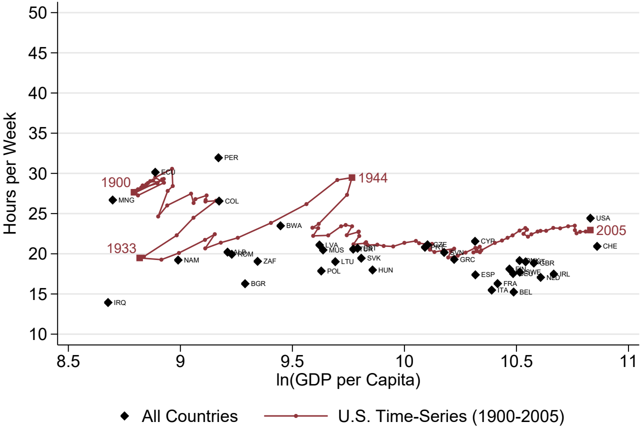
Additional evidence: US comparison

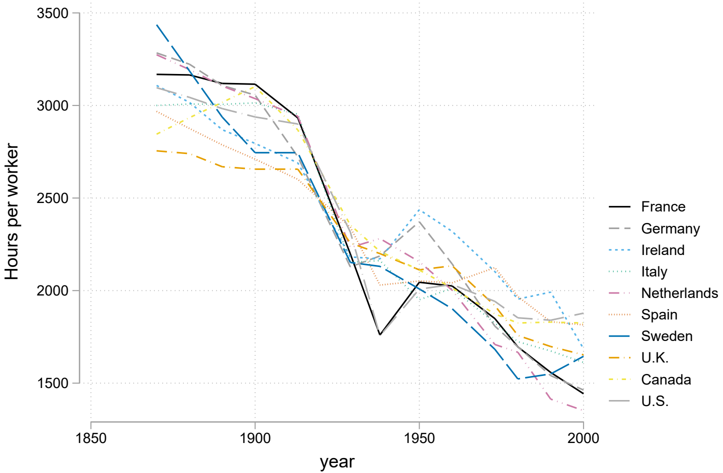


Source: Bick et al. (2018)

Hours/worker decline with GDPc in middle income and rich countries; focus on comparison with US.

▸▸ Back

Additional evidence: Across countries and Time



Hours/worker decline over time across countries.

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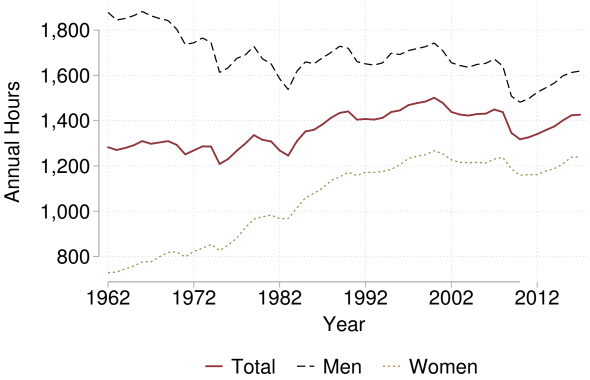
Additional evidence: Breakdown by Gender



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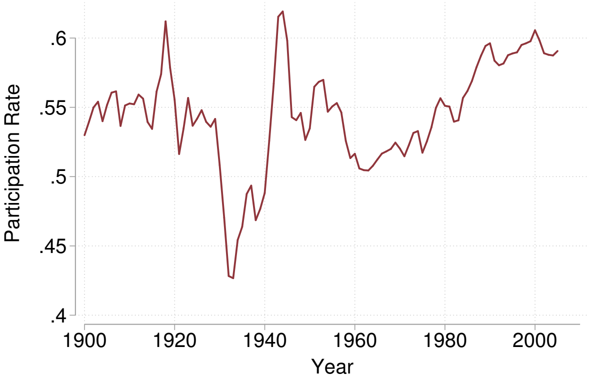
Additional evidence: Extensive Margin



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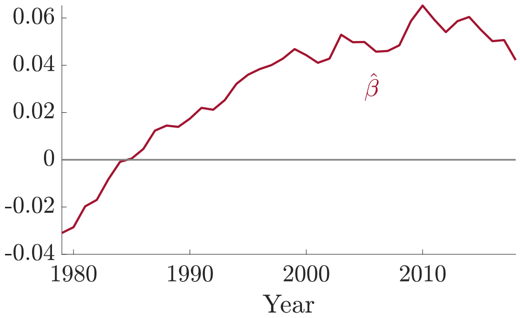
Additional evidence: Hours by Wage Decile

- Run the regression for every year 1978-2019 (CPS, males):

$$log(h_i) = \alpha + \beta log(w_i) + age_i + age_i^2 + \epsilon_i$$



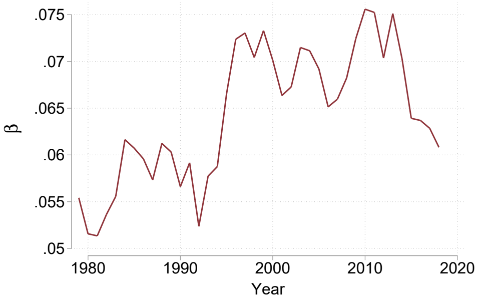
- Starting from 1980's, positive and increasing wage-hours relationship.

Back | Robustness: Both Genders | Robustness: Excl. Mult. Jobs | Robustness: Overtime for Hourly Work.
Robustness: No Hourly Work. | Robustness: Only Hourly Work.

Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, both genders):

$$log(h_i) = \alpha + \beta log(w_i) + age_i + age_i^2 + \epsilon_i$$

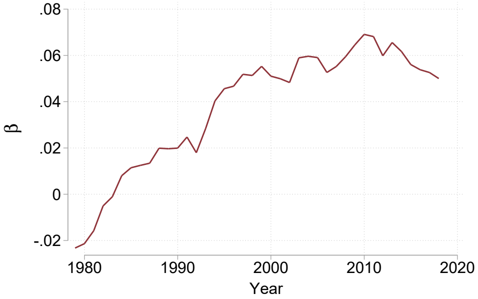


Back to Main
Back to Regression

Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, without mult. jobs):

$$log(h_i) = \alpha + \beta log(w_i) + age_i + age_i^2 + \epsilon_i$$

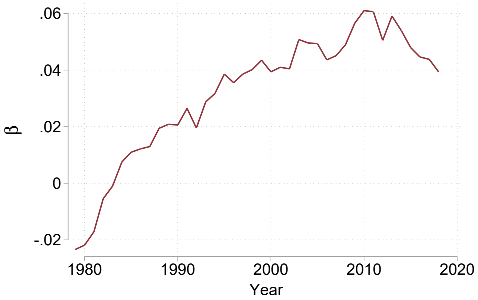


⏩ Back to Main ⏩ Back to Regression

Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, includes tips/commissions for hourly workers):

$$log(h_i) = \alpha + \beta log(w_i) + age_i + age_i^2 + \epsilon_i$$



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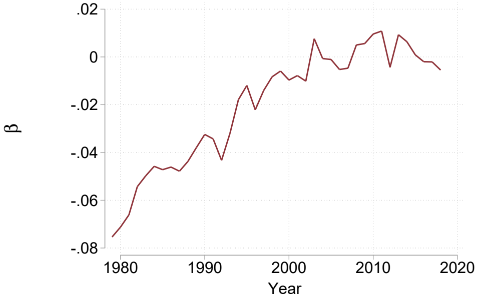
Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, excludes hourly workers):

$$log(h_i) = \alpha + \beta log(w_i) + age_i + age_i^2 + \epsilon_i$$



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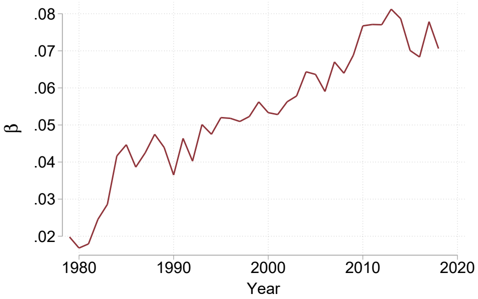
Back to Regression

Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, only hourly workers):

$$log(h_i) = \alpha + \beta log(w_i) + age_i + age_i^2 + \epsilon_i$$



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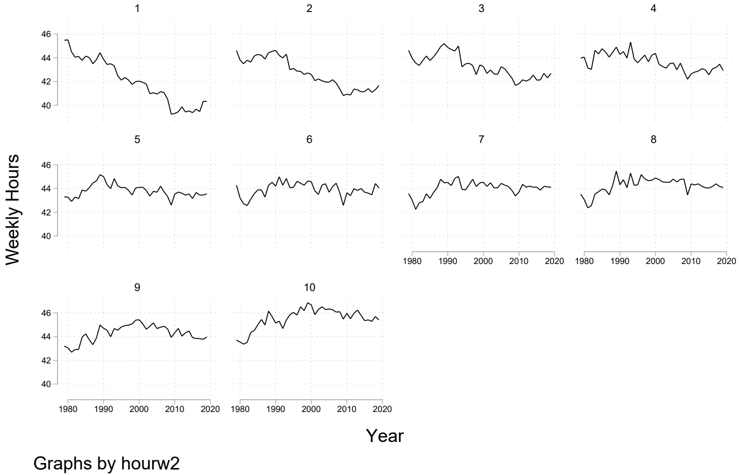
Additional evidence: Hours Worked by Wage P'ctile

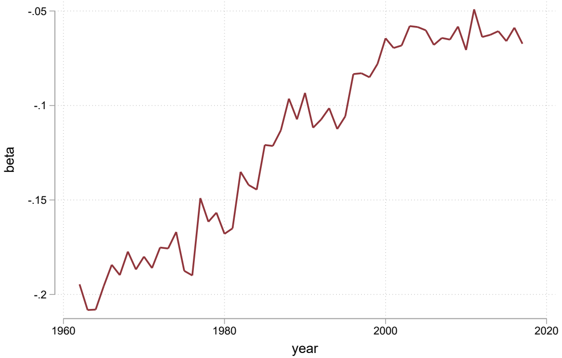


⏩ Back

Additional evidence: ASEC

- Run the regression for every year 1978-2019 (ASEC, males, actual hours):

$$log(h_i) = \alpha + \beta log(w_i) + age_i + age_i^2 + \epsilon_i$$

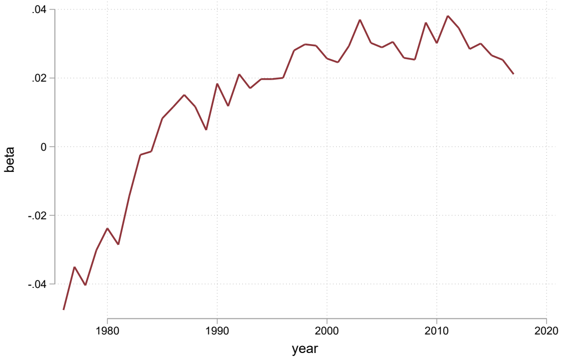


Back | Robustness: usual hours (M) | Robustness: usual hours (F)

Additional evidence: ASEC

- Run the regression for every year 1978-2019 (ASEC, males, usual hours):

$$log(h_i) = \alpha + \beta log(w_i) + age_i + age_i^2 + \epsilon_i$$

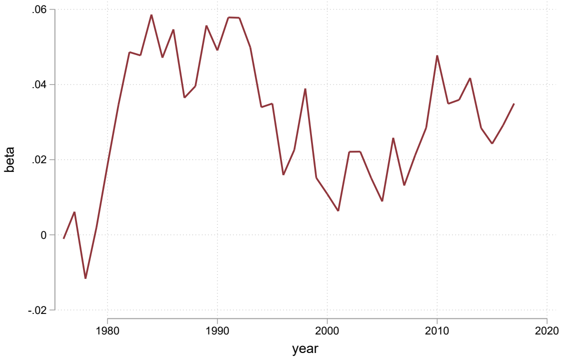


▸▸ Back to Main ▸▸ Back to Regression

Additional evidence: ASEC

- Run the regression for every year 1978-2019 (ASEC, females, usual hours):

$$log(h_i) = \alpha + \beta log(w_i) + age_i + age_i^2 + \epsilon_i$$



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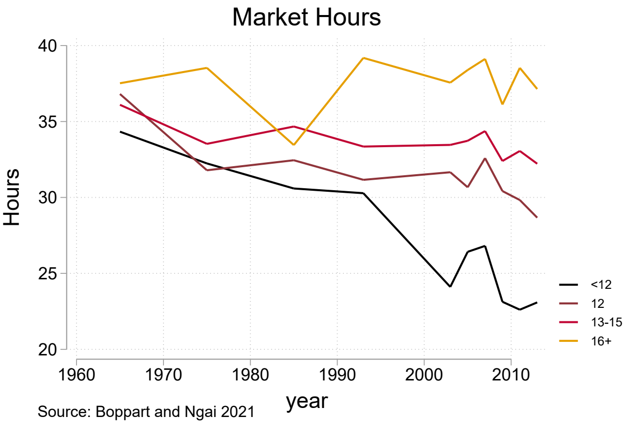
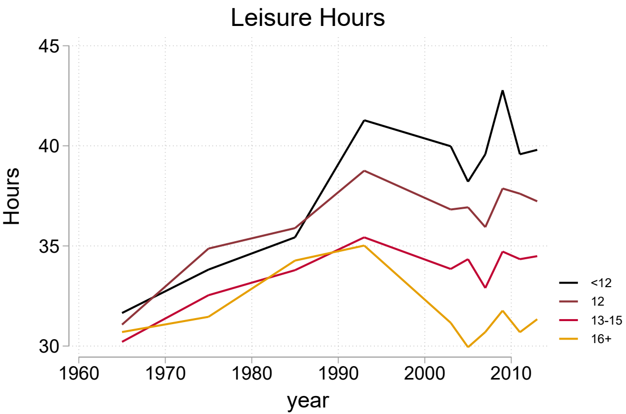
Additional evidence: ATUS data



Source: Boppart and Ngai 2021

Skilled workers people work roughly the same; low skilled work less.
Inequality in trends starts from 1980's. Back

Additional evidence: ATUS data



Skilled workers consume less leisure, but starting from 1990’s. Inequality in trends starts from 1980’s. Back

Additional evidence: β across countries

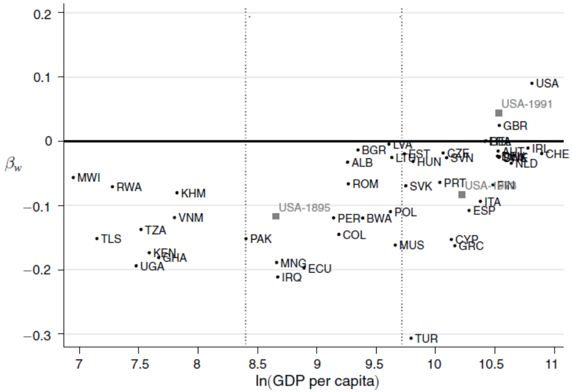


Figure: Cross-sectional relationship of hours and wages (source: Bick et al, 2021)

Similarly to US: β increasing with GDP for developed countries. Back

Additional evidence: progressivity across countries

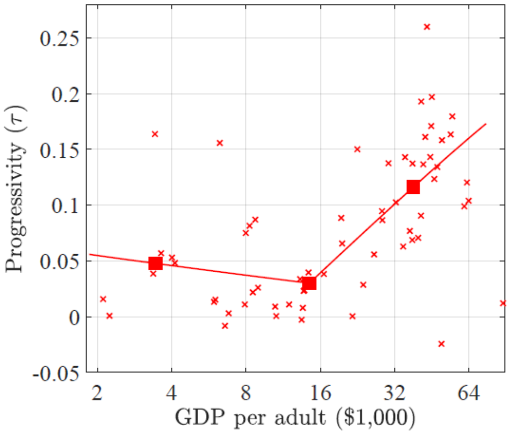


Figure: Fiscal progressivity across countries (source: Bick et al, 2021)

Back

Market Clearing

Complete Definition

- Under Positive Assortative Matching (PAM), market clearing condition requires:

$$\int_{\mu(x)}^{\bar{y}} g^y(s)ds = \int_x^{\bar{x}} g^x(s)ds \,,$$

  where g^x and g^y are the densities of workers and jobs, respectively.

- Starting at the top, the highest type $\bar{x}$ matches with the highest type $\bar{y}$.

- The type $x < \bar{x}$ matches with $y = \mu(x)$ if the measure of workers above x is equal to the measure of jobs above $\mu(x)$.

- This ensures that meatching is measure-preserving.

Back

Equilibrium definition

Market Clearing & Equilibrium

- Both households and firms take hedonic income function $e()$ as given.
- Market clearing determines matching function $\mu()$.
  Under Positive Assortative Matching (PAM):

$$\int_{\mu(x)}^{\bar{y}} g^y(s)ds = \int_{x}^{\bar{x}} g^x(s)ds \tag{2}$$

Definition 1: Competitive equilibrium.
A competitive equilibrium of this economy is a tuple of functions (e, μ, h) such that:

- e and h maximize utility and profits *(optimality)*
- equation (1) holds *(market clearing)*

Back

Derivation: Benchmark Model

- FOC:

$$f_x(x,y) - \frac{\partial w(x)}{\partial x} = 0$$

- Market clearing: $\mu()$ (measure preserving). PAM is defined as $\mu'(x) > 0$
- If $\mu'(x) > 0$, then we can write: $\Gamma(x) = \Phi(\mu(x)) \implies \mu(x) = \Phi^{-1}\Gamma(x)$.
- SOC:

$$f_{xx}(x,y) - w_{xx}(x) < 0$$

- To get rid of w_{xx}, we evaluate the FOC at $y = \mu(x)$:

$$f_{xx}(x,\mu(x)) + f_{xy}(x,\mu(x)) \cdot \frac{d\mu(x)}{dx} = w_{xx}(x)$$

- Substituting back, we get:

$$f_{xy}(x,\mu(x)) \cdot \frac{d\mu(x)}{dx} > 0$$

- Hence the sign of $\mu(x)$ is determined by the sign of f_{xy} (provided differentiability of $\mu()$

Back

Derivation: This Model

- Here we have two optimization problems (firm + hh). Hence FOC is a system of equations:

$$f_{\tilde{x}}(\tilde{x}, y) - \frac{\partial w(\tilde{x})}{\partial \tilde{x}} = 0 \quad \text{(firm)}$$

$$u'_w \cdot w'_h + u'_h = 0 \quad \text{(hh)}$$

- We repeat the previous derivation to understand properties of u and f that gives us PAM. The Hessian (SOC) is:

$$\begin{pmatrix} f_{xx}(x, y) - w_{xx}(x) & f_{xh}(x, y) - w_{xh}(x) \\ \frac{\partial u'_w \cdot w'_h}{\partial x} & \frac{\partial u'_w \cdot w'_h + u'_h}{\partial h} \end{pmatrix}$$

- Optimality requires $|H| \geq 0$. As before, we evaluate the FOC at $y = \mu(x)$ and subtitute.
- Solving for $|H| \geq 0$ after some algebra we have that:

$$|H| \geq 0 \implies \text{the main condition.}$$

Back Standard model

Hours choice: Derivation

The FOC for hours reads:

$$u_c f_h + u_h = 0$$

By monotone comparative statics, h_x increases in x if:

$$\frac{\partial \left(u_c f_h + u_h \right)}{\partial x} > 0$$

From which one gets:

$$\Big(\underbrace{u_{cc} f_h f_x}_{\text{income effects}} + \underbrace{u_c f_{hx}}_{\text{subs. effects}} \Big) > 0$$

Back

Hours choice

Special Case: CRRA

- Hours choice:

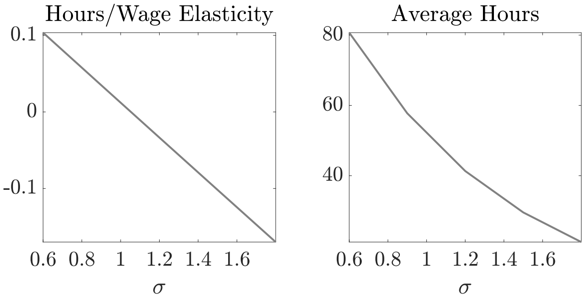
$$\Big(\underbrace{u_{cc} f_h f_x}_{\text{income effects}} + \underbrace{u_c f_{hx}}_{\text{subs. effects}}\Big) > 0$$

- Sufficient condition: relative risk aversion coefficient $\sigma = -c\frac{u_{cc}}{u_c} < 1$
  - σ governs strength of income vs substitution effects in macro models (King, Plosser, Rebelo 1988).
  - $\sigma > 1$ needed to match aggregate evidence (Boppart and Krusell, 2020)

Back

Comparative Statics

Preferences



An increase in in σ:

- Decreases hours wage elasticity (higher income effects) and average hours worked
- Elasticity turns negative for lower values of σ, if low complementarities.

back

Assortative Matching

- Pair x and y solve the problem:

$$U(x, y, V) = \max_h u(c, h) \quad \text{s.t.} \quad f(x, y, h) - c \geq V$$

where $U(x, y, .)$ traces a Pareto frontier.

- Hence we maximize U w.r.t. to job y to get the FOC:

$$U_y + U_\pi \pi_y = 0$$

- The equilibrium matching μ is optimal provided that the SOC $<$0, hence:

$$U_{yy} + 2U_{y\pi}\pi' + U_{\pi\pi}\pi'^2 + U_\pi \pi'' < 0$$

Assortative Matching

- Differentiate FOC along the candidate equilibrium $y = \mu(x)$:

$$U_{yy} + U_{xy}\mu' + U_{x\pi}\pi' + U_{x\pi}\pi' + U_{y\pi}\mu'\pi' + U_{\pi\pi}\pi'^2 + U_{\pi}\pi'' = 0$$

- Combining this condition with $SOC < 0$ gives:

$$\mu'\left[U_{xy} - \frac{U_y}{U_\pi}U_{x\pi}\right] \geq 0$$

- Hence PAM ($\mu' > 0$) obtains iff

$$U_{xy} - \frac{U_y}{U_\pi}U_{x\pi} \geq 0$$

Which completes the proof.

Back

Assortative Matching

Alternative condition for PAM

- Corollary: *A condition for PAM in terms of primitives is:*

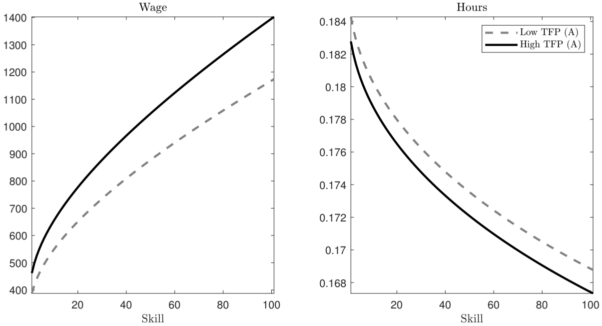
$$\left(f_{yx}\left(\underbrace{-u_{cc}f_h f_h - u_c f_{hh} - u_{hh}}_{>0}\right) + f_{hy}\left(u_{cc}f_x f_h + u_c f_{hx}\right)\right) > 0$$

- Intuition:
  - f_{xy}: skill-job complementarity (Becker 1973)
  - f_{xh}, f_{yh}: *hours* complementarities (better workers produce more with more time)
  - $u_{cc} < 0$: income effects push towards NAM

Proof Back

Prod. function: comparative statics

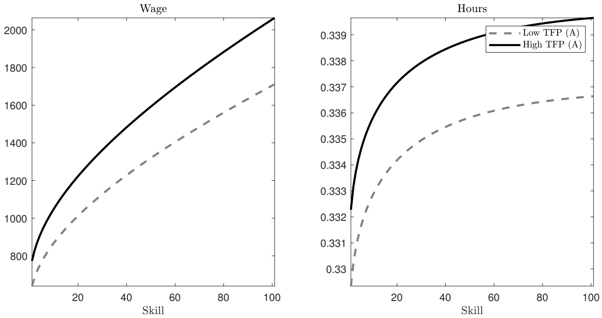
TFP: A, baseline



Increasing TFP has a level shift on wages (positive) and hours (negative, $\sigma > 1$).

Prod. function: comparative statics

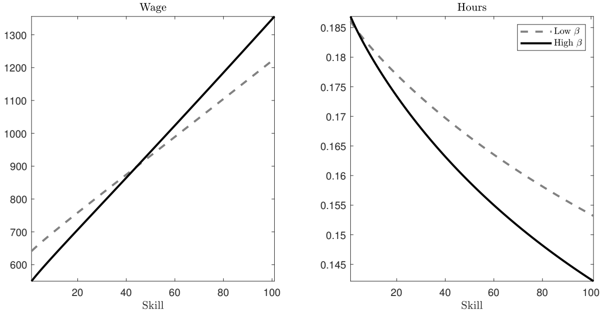
TFP: A, low σ



Increasing TFP has a level shift on wages (positive) and hours (positive, $\sigma < 1$).

Prod. function: comparative statics

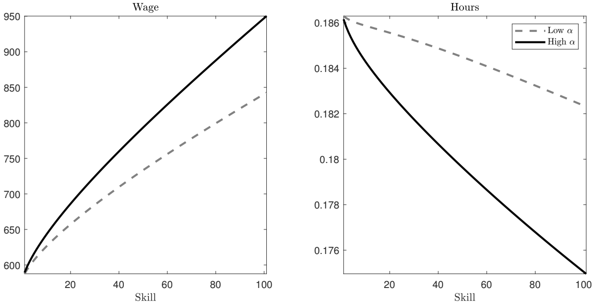
Hours/Match share: β



Increasing β decreases importance of hours (hence hours choice $\downarrow$).

Prod. function: comparative statics

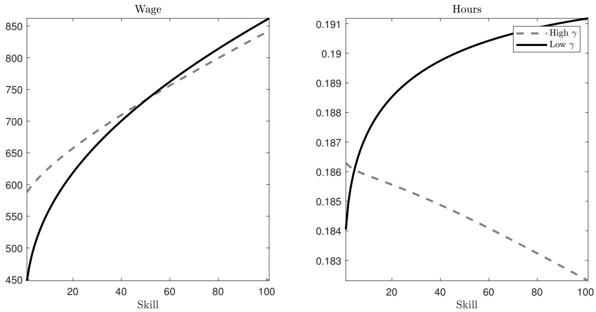
Skill importance: α



Increasing α makes wage steeper in skills; through income effects, hours less steep (hence $\partial h / \partial x \downarrow$).

Prod. function: comparative statics

Match/hours complementarity: γ



Increasing γ pushes towards $\partial h/\partial x \uparrow$. Also, it increases inequality. Key parameter to inform positive/increasing hours/wage elasticity! Missing in the previous specification ($\gamma = 0$).

Estimation

Jobs distribution

- Need distribution of worker and job types, $H(x)$ and $G(y)$, for 1980 and 2010
- Solution: estimate directly from the data
- $G(y) \approx$ cognitive skill requirement
  - 1980: DOT (Dictionary of occupational titles); $y \approx$ mathematical reasoning
  - 2010: O*NET: PCA on cognitive/math variables
- Final step: merge estimated y with CPS for both periods to get $G(y)$
- Same approach as in Lindenlaub(2014), Lindenlaub and Chade(2022)

Estimation

Skill/Ability Distributions

- $H(x) \approx$ test scores
  - 1980: NLSY79
  - 2010: NLSY97
- I rely on Altonji, Bharadwaj, and Lange (2009) criterion to make test scores comparable across the two periods[1]
- (adjusted) distribution of test scores gives our measure of $H(x)$

[1] e.g. test respondents were administered the test at different ages in the two cohorts

Estimation

Skill/Ability Distributions

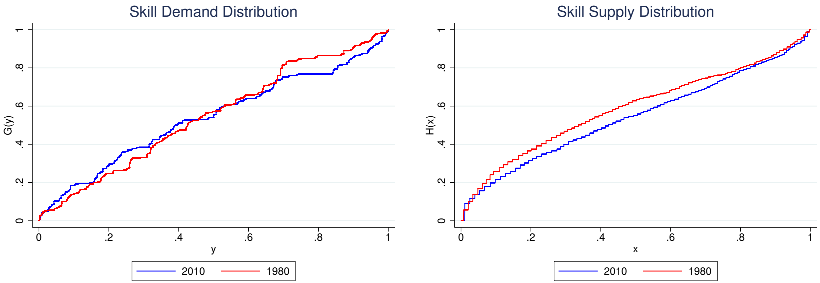


Figure: Estimated skill demand and supply distributions, for 1980 and 2015.

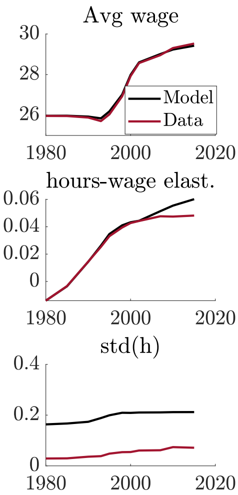
- Jobs (left): slight change in dispersion
- Workers (right): 'better' distribution of skills

Targets

Results						
Moment	Data 1980	Data 2015	Model 1980	Model 2015	Data Δ	Model Δ
E(w)	19.6	23.9	19.6	23.9	21%	21%
E(h)	37.9	38.4	37.9	38.4	+1%	+1%
W75/W25	1.86	2.29	1.86	2.29	23%	23%
W50/W10	1.92	2.03	1.92	2.03	6%	6%
h/w elast.	-0.012	0.062	-0.012	0.062	7%	7%

back

Model Fit



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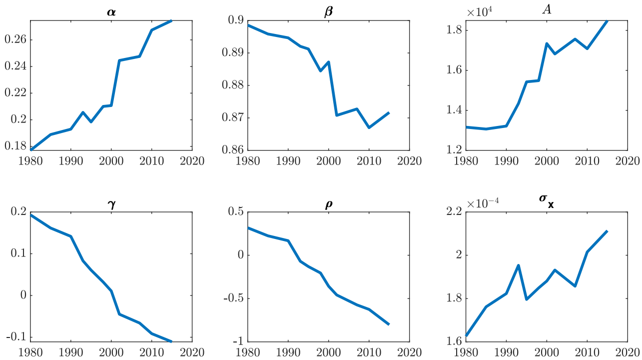
Estimates



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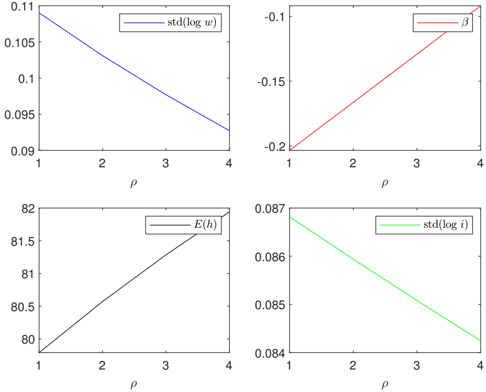
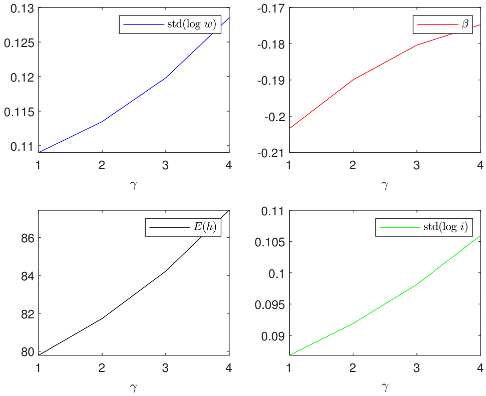
Different CES nesting

$$f(x, y, h) = A\left(\beta(\alpha x^{\rho} + (1-\alpha)h^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)y^{\gamma}\right)^{\frac{1}{\gamma}}$$



Different CES nesting

$$f(x,y,h) = A\left(\beta(\alpha x^{\rho} + (1-\alpha)h^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)y^{\gamma}\right)^{\frac{1}{\gamma}}$$



Back

Different CES nesting

Assume a different nest for the CES: workers and hours form a 'bundle', $\tilde{x} = \tilde{x}(x, h)$.

$$f(x, y, h) = A\left(\beta(\alpha x^{\rho} + (1-\alpha)h^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)y^{\gamma}\right)^{\frac{1}{\gamma}}$$

Parameter	1980	2015	Meaning
β	0.09	0.17	weight of (x, h) in prod.
α	0.80	0.85	weight of skills in prod.
γ	1.01	-0.34	compl. ($\tilde{x}$,y)
ρ	0.17	0.11	compl. (h, x)
A	32,209	27,276	TFP

Back

Different CES nesting

Assume a different nest for the CES: firms/jobs and hours nested as $CES((y,h),x)$.

$$f(x,y,h) = A\left(\beta(\alpha y^{\rho} + (1-\alpha)h^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)x^{\gamma}\right)^{\frac{1}{\gamma}}$$

Parameter	1980	2015	Meaning
β	0.40	0.90	weight of (y,h) in prod.
α	0.73	0.97	weight of jobs y in prod.
γ	-3.3	-4.5	compl. (x,y/h)
ρ	0.56	0.04	compl. (h,y)
A	4,489	27,597	TFP

Back

Different CES nesting

$$\text{Nest1} \quad f(x,y,h) = A\left(\beta(\alpha x^{\rho} + (1-\alpha)y^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)h^{\gamma}\right)^{\frac{1}{\gamma}}$$

$$\text{Nest2} \quad f(x,y,h) = A\left(\beta(\alpha x^{\rho} + (1-\alpha)h^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)y^{\gamma}\right)^{\frac{1}{\gamma}}$$

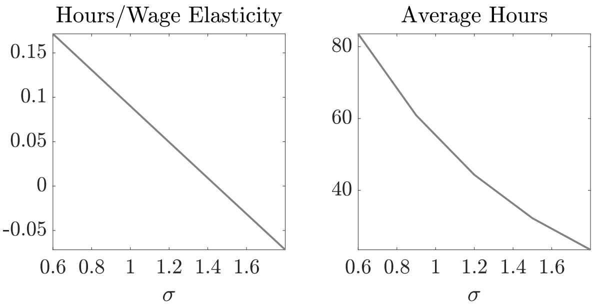
$$\text{Nest3} \quad f(x,y,h) = A\left(\beta(\alpha y^{\rho} + (1-\alpha)h^{\rho})^{\frac{\gamma}{\rho}} + (1-\beta)x^{\gamma}\right)^{\frac{1}{\gamma}}$$

	Nest 1		Nest 2		Nest 3	
Parameter	1980	2015	1980	2015	1980	2015
β	0.90	0.87	0.09	0.17	0.40	0.90
α	0.13	0.21	0.80	0.85	0.73	0.97
γ	0.32	-0.07	0.98	-0.34	-3.3	-4.5
ρ	0.22	-1.20	0.17	0.11	0.56	0.04
A	12,871	22,218	32,209	27,276	4,489	27,597

Back

Comparative Statics

Preferences



An increase in in σ:

- Decreases hours wage elasticity (higher income effects) and average hours worked

▸▸ Low compl. ▸▸ Back