

Hours-Biased Technological Change

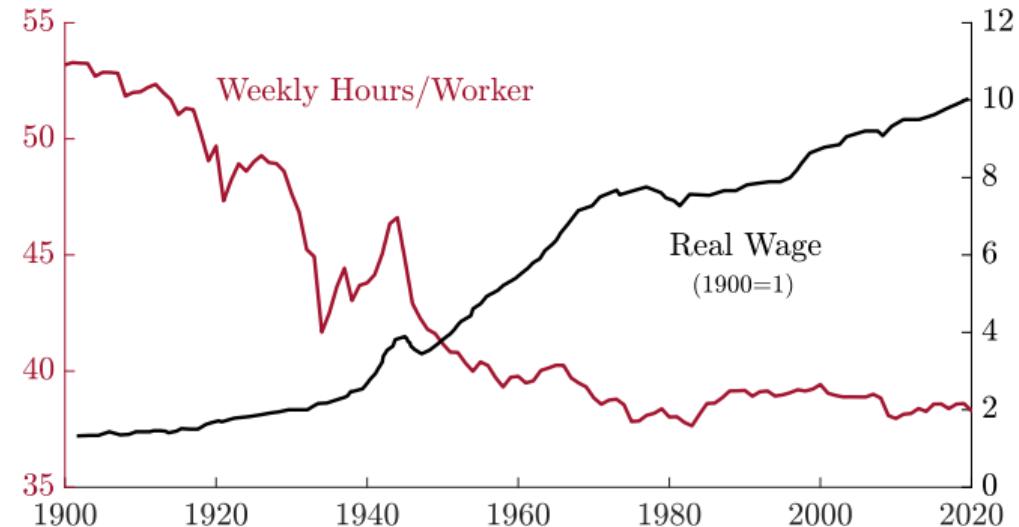
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April 1st, 2024

Motivation

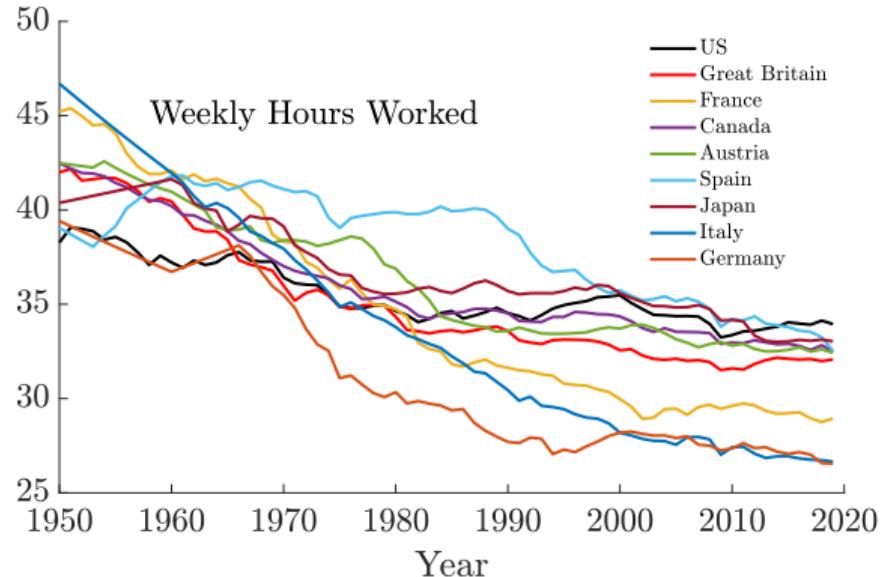
- Working hours decline in the long-run...

Hours Decrease Over Time: US



- ▶ Hours/worker by GDP
- ▶ Hours/capita by GDP
- ▶ Hours/capita over time
- ▶ Breakdown By Gender
- ▶ Extensive Margin

Hours Decrease Over Time: Cross-Country



► Cross-country vs US

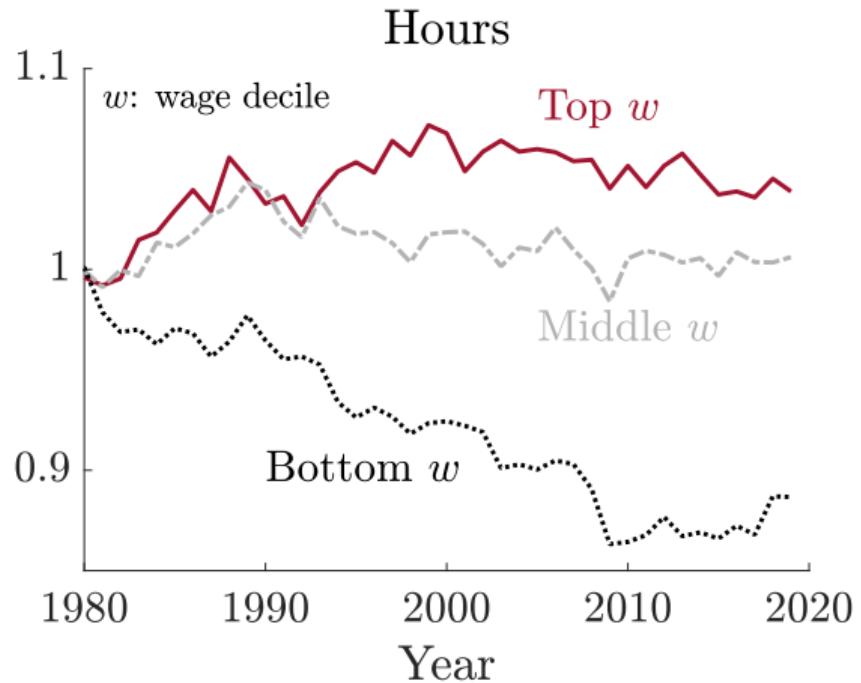
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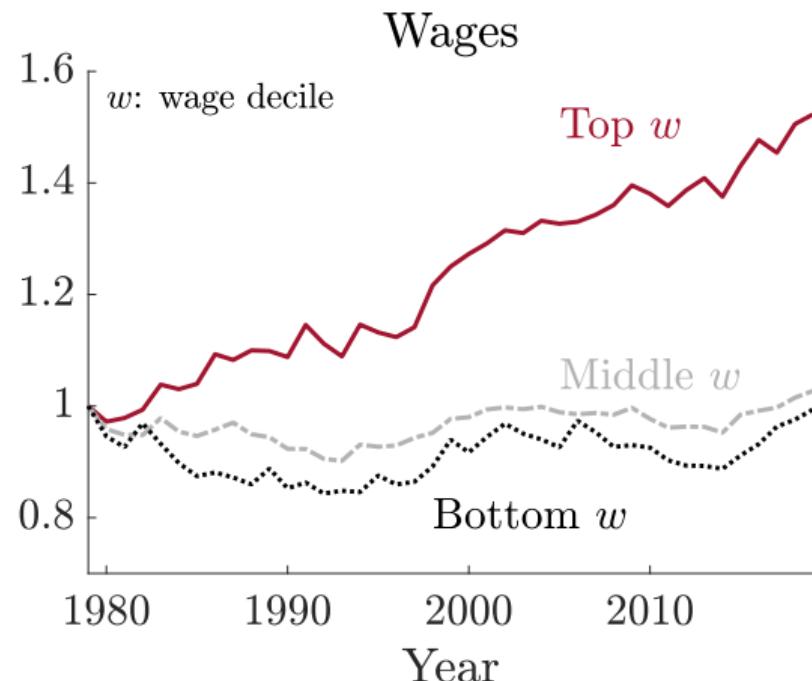
Hours Increased For High-Wage Workers



► ATUS

► All Quintiles

Wage Inequality Increased



Motivation

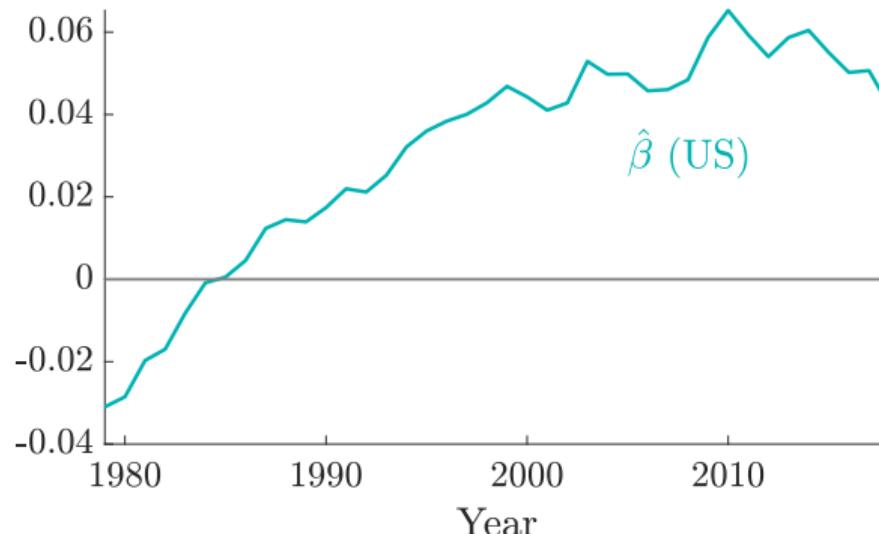
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- Hours-wage correlation turns from **negative** to **positive**

Increasing Hours-Wage Correlation (US)

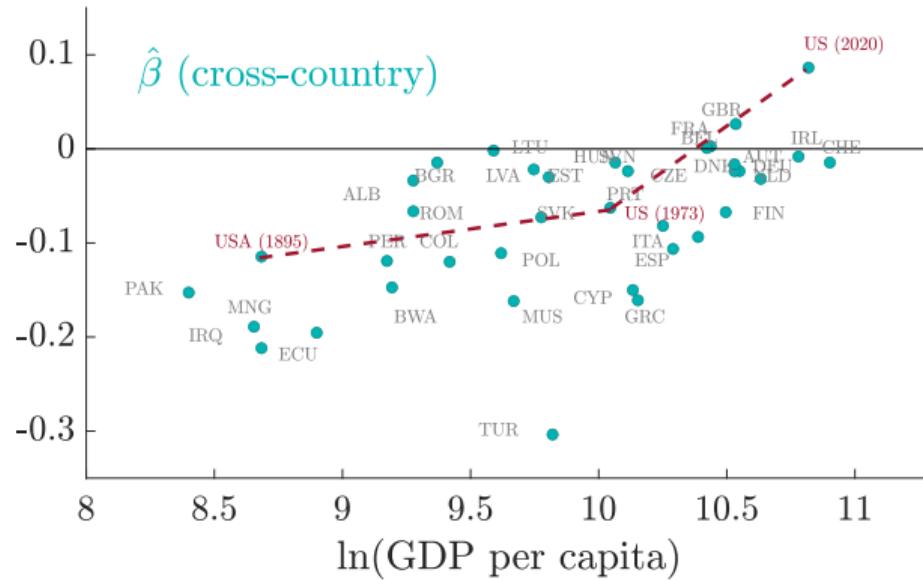
$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i, \quad i = \text{worker}$$



- Both Genders
- Excl. Mult. Jobs
- Overtime for Hourly Work.
- No Hourly Work.
- Only Hourly Work.
- ASEC

Increasing Hours-Wage Correlation (Across-Countries)

$$\log(h_i) = \alpha + \beta \log(w_i) + \text{age}_i + \text{age}_i^2 + \epsilon_i, \quad i = \text{worker}$$



► Progressivity

Motivation

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- Hours-wage correlation turns from **negative** to **positive**
 - Similar patterns across development path
- How can we reconcile these patterns? Answer to this question is key for:
 - Understanding inequality
 - Labor Supply Elasticity
 - Tax Policy

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The Role of Technological Change

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Technology also affects working time

- More flexible hours, easier to work extra-time (e.g. laptops, Skype..)

Hours-Biased Technological Change

Technology increases the marginal product of hours for skilled workers.

This Paper

- Theory of workers sorting into jobs based on skills and hours
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This Paper

- Theory of workers sorting into jobs based on skills and hours
 - Builds on SBTC frameworks but includes hours in technology
- Mechanism: changes in technology (HBTC) \implies reward for long hours \uparrow
 - Higher incentives to work long hours for high skilled
- Quantitatively analyze technological change in US
 - Novel channel of technology on inequality (HBTC)
 - New driver of aggregate hours worked in US (offsets income effect)

Literature

- **Aggregate Hours:** King, Plosser, Rebelo (1988); Prescott (2004); Ramey and Francis (2009); Boppart and Krusell (2020); Rachel (2021); Bick et al. (2021).
Take into account cross sectional heterogeneity
- **Leisure inequality:** Costa (2000); Aguiar and Hurst (2007); Heathcote et al. (2008); Kopitov et al. (2020); Boppart and Ngai (2021); Michelacci and Pijoan-Mas (2014); Doepke et al. (2018); Auray et al. (2019).
Introduce sorting and endogenous wage inequality
- **Sorting:** Eeckhout and Kircher (2018); Eeckhout and Sepahsalari (2018); Calvo, Lindenlaub, Reynoso (2021); Michelacci and Pijoan-Mas (2014); Chade and Lindenlaub (2022), Shao et al. (2022).
Characterize sorting for general preferences and technology

Model

Setup

- Population
 - Individuals with skill $x \sim G^x$
 - Jobs of type $y \sim G^y$
- Preferences
 - Choose hours h and consumption c
 - Utility function: $u(c, 1 - h)$
- Technology
 - Competitive labor market
 - Workers and jobs(firms) match one-to-one \implies sorting
 - Output: $f(x, y, h)$
 - Note: $h = h(x)$ is endogenous and skill dependent

Setup

- Payoffs

- Workers maximize: $u(c, 1 - h)$ s.t. $c = e(x, y, h)$
- Jobs maximize: $f(x, y, h) - e(x, y, h)$
- In equilibrium, wages w are defined as $w = \frac{e}{h}$, hence $w = w(x, y, h)$

- Market clearing

- Equilibrium matching $y = \mu(x)$ assigns workers x to jobs y .
- Under Positive Assortative Matching (PAM), market clearing requires:

$$\int_{\mu(x)}^y g^y(s) ds = \int_x^{\bar{x}} g^x(s) ds ,$$

where g^x and g^y are the densities of workers and jobs, respectively. ► Details

- Competitive equilibrium = optimality + market clearing:

- income $e(x)$; hours $h(x)$; matching $\mu(x)$ ► Full definition

Problem as Pareto Frontier

Pair's problem as Pareto Frontier

- Workers x and jobs y simultaneously solve, respectively:

$$\max_{h,y} u(c, 1 - h) \quad \text{s.t.} \quad c = e$$

$$\max_x f(x, y, h) - e$$

- This can be re-written as pair x, y solving:

$$U(x, y, \pi) = \max_h u(f(x, y, h) - \pi, 1 - h)$$

where $U(x, y, .)$ defines the Pareto Frontier (for all profit values π).

Optimal Job Choice

Optimization

- Optimal job choice implies positive sorting iff:

$$U_{xy} - \frac{U_y}{U_\pi} U_{x\pi} \geq 0 \iff -\frac{U_y}{U_\pi} \text{ increasing in } x \quad \text{▶ Proof}$$

- **Intuition:** high x more willing to give more π (=less income) for a better y

Assortative Matching

Check $-\frac{U_y}{U_\pi} \uparrow x$

Proposition: *The equilibrium features Positive Assortative Matching iff:*

$$f_{xy} + f_{yh} h_x \geq 0$$

► Alternative Condition

► Decentralized eq.

- **Intuition:** two forces pushing towards positive sorting (PAM)
 - Skill-job complementarity f_{xy} (**standard**)
 - High skill working more ($h_x > 0$) + hours-job complementarity $f_{yh} > 0$ (**new**)
- Low skill working much more ($h_x << 0$) and $f_{yh} > 0$ can lead to NAM

Hours choice

- FOC for hours implies $u_c f_h + u_h = 0$. From this, we get that:

Proposition: *In equilibrium, high skill choose higher hours if:*

$$\underbrace{u_c f_{xh}}_{\text{subs. effects}} > \underbrace{-u_{cc} f_h f_x}_{\text{income effects}}$$

» Derivation

» CRRA

- Income effects (recall $u_{cc} < 0$)
 - Key force to explain long run behavior of hours (Boppart and Krusell, 2020)
- Hours complementarities: f_{xh}
 - If positive, raises marginal product of hours worked for high skill \implies HBTC

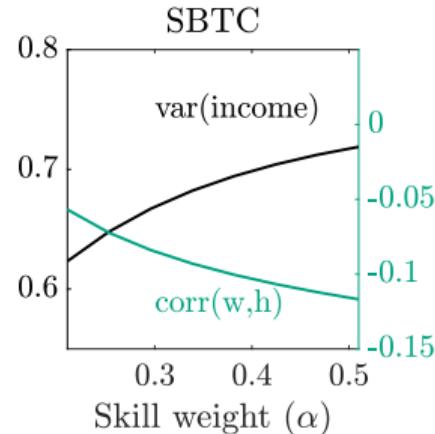
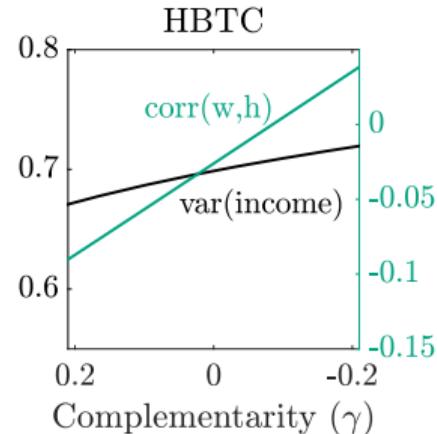
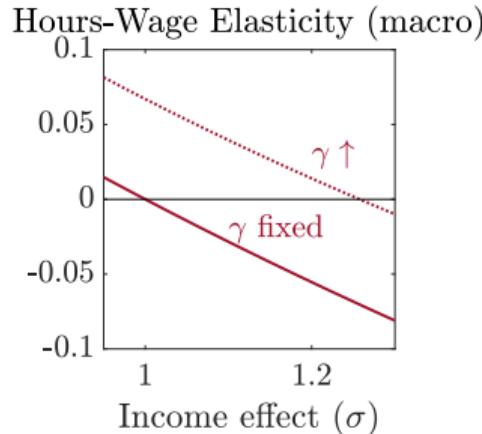
Functional forms

- Assume technology

$$f(x, y, h) = A \left(\underbrace{\beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}}}_{\text{SBTC}} + \underbrace{(1 - \beta)h^\gamma}_{\text{HBTC}} \right)^{\frac{1}{\gamma}}$$

- parameters ρ, γ govern strength of SBTC/HBTC
- Preferences (MacCurdy, 1981): $u(c, h) = \frac{c^{1-\sigma}}{1-\sigma} - \psi \frac{h^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}$
 - σ = strength of income effects

Comparative Statics



Key takeaways:

- Hours complementarities (γ) matter for both *macro* and *micro* elasticities
- HBTC vs SBTC **drive up inequality**, but opposite effect on *micro elasticity*

Estimation

Overview

- Can changes in **technology** rationalize the data? Recall the prod. function:

$$f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)h^\gamma \right)^{\frac{1}{\gamma}}$$

- Estimate parameters year by year to match moments from the data (SMM).
 $\implies A = A_t, \alpha = \alpha_t, \beta = \beta_t, \gamma = \gamma_t, \rho = \rho_t.$
- Data: CPS (males, 24-65). Yearly data 1979-2018.
- **Goal:** Assess changes in weights (A_t, α_t, β_t) and complementarities (ρ_t, γ_t)

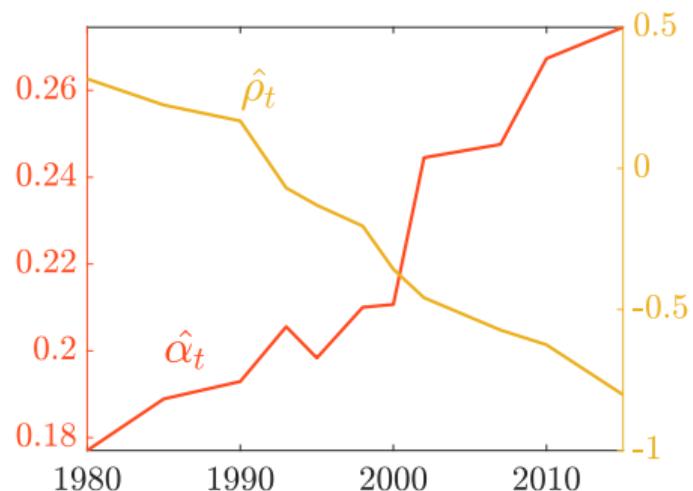
Strategy

- Preferences: calibrate σ and θ
 - σ : mid-range of macro studies 1.21 (Bick et al., 22); 1.7 (BK 2020) ➔ Robustness
 - $\theta = 0.4$: consistent with micro-evidence (Violante et al, 2014).
- Distributions G^w and G^f
 - Assume log-normality: $x \sim \mathcal{LN}(\mu_x, \sigma_x)$; $y \sim \mathcal{LN}(\mu_y, \sigma_y)$
 - Set $\mu_x = \mu_y = 0$ and estimate ratio σ_x/σ_y ➔ Fitted Distr. ➔ Beta Distr.
- Six parameters to be estimated (5 technology, 1 distributions); six moments:

$E(h)$	Avg. Hours
$E(w)$	Avg. Wages
w90/w50	Wage inequality (top)
w90/w10	Wage inequality (overall)
Hours-wage Elasticity	Coeff. of reg. $\log(h)$ on $\log(w)$
std(w)	Wage dispersion

Estimates: SBTC

$$f(x, y, h) = A \left(\beta(\alpha x^{\hat{\rho}} + (1 - \alpha)y^{\hat{\rho}})^{\frac{\gamma}{\hat{\rho}}} + (1 - \beta)h^{\gamma} \right)^{\frac{1}{\gamma}}$$



- $\hat{\alpha}$ increased (skill more important)
- $\hat{\rho}$ decreased (x, y more complementary)
- SBTC: productivity of good workers in good jobs has increased

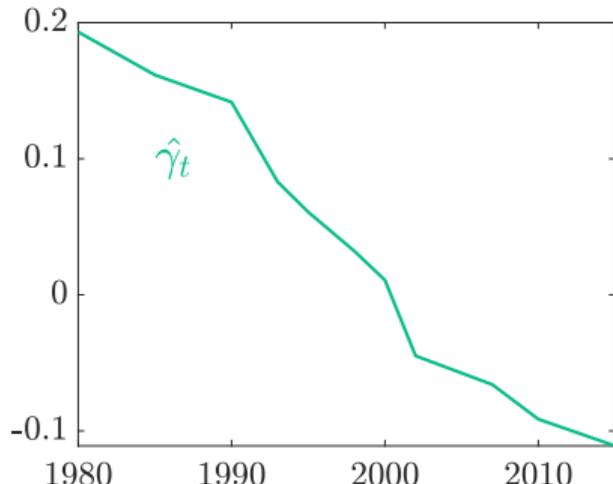
► Model Fit

► Full Estimates

► Fixed Hours Estimates

Estimates: HBTC

$$f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{1}{\rho}} + (1 - \beta)h^{\gamma} \right)^{\frac{1}{\gamma}}$$



- $\hat{\gamma}$ decreased (h and x/y more complementary)
- HBTC: good workers/jobs produce more with extra hours

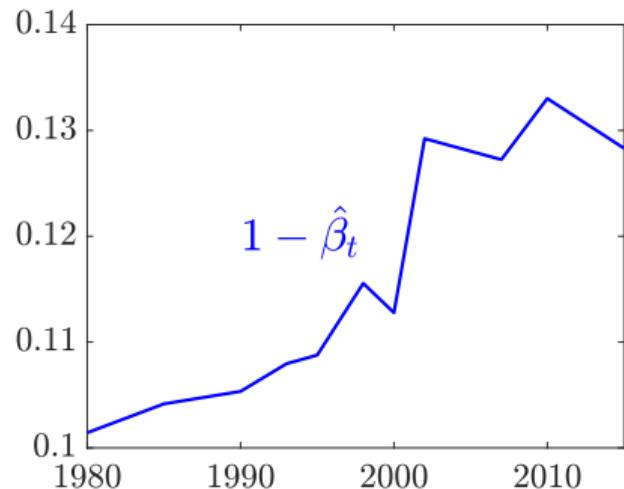
► Model Fit

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Estimates: Weight of Hours

$$f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)h^\gamma \right)^{\frac{1}{\gamma}}$$



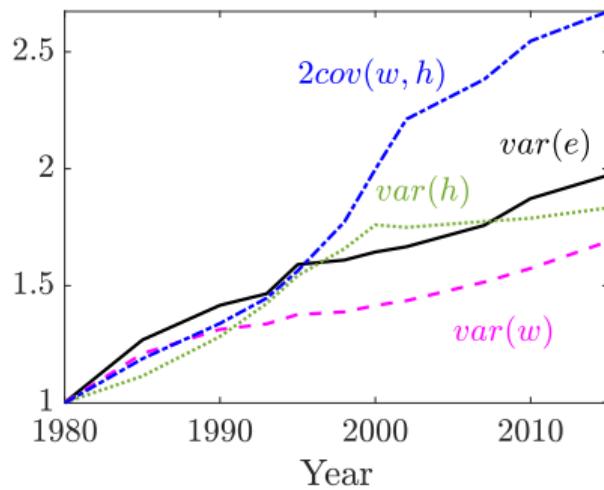
- $1 - \hat{\beta}$ increased
- Hours are more productive, regardless of skills

- Model Fit
- Full Estimates
- Fixed Hours Estimates

Income Inequality

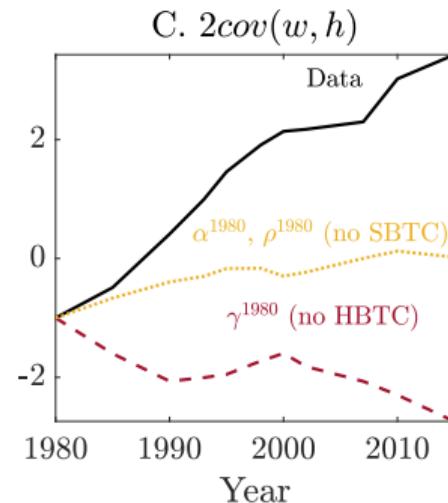
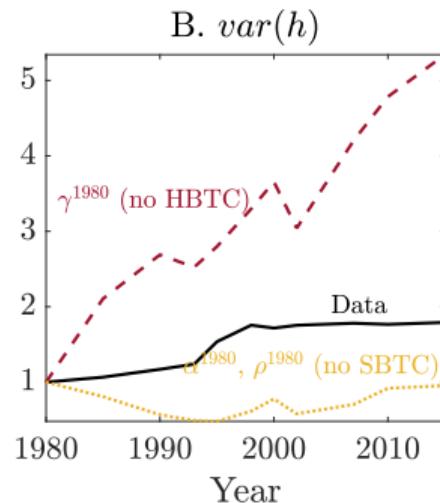
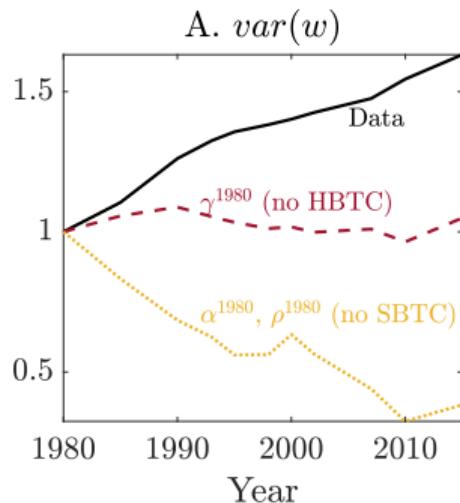
- Decompose *income* inequality in:

$$\text{var}(e) = \text{var}(w) + \text{var}(h) + 2\text{cov}(w, h)$$



Counterfactuals: Drivers of Inequality

- Need both **HBTC** and **SBTC** to explain increase in $\text{var}(w)$ and $\text{cov}(w, h)$

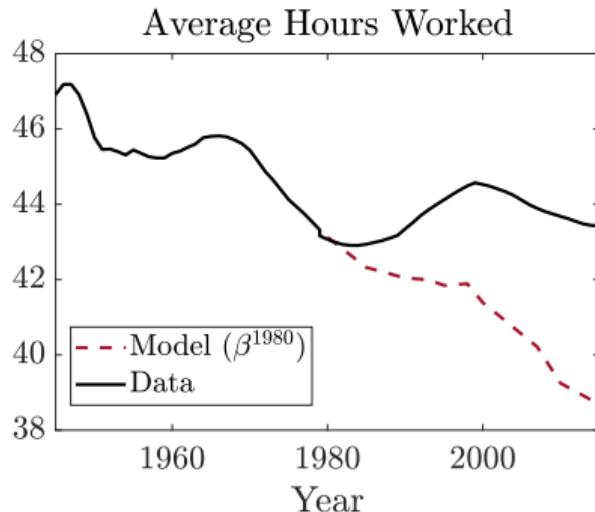


► Only one param. 1/2

► Only one param. 2/2

► Cumulative

Counterfactuals: Aggregate Hours



- Explanation for relatively flat average hours worked in US post-1970
 - One explanation: income = substitution effect
 - This paper: income effect (Boppart and Krusell, 2020) + tech. change (β)
- Across development path, relative strength of these two forces vary

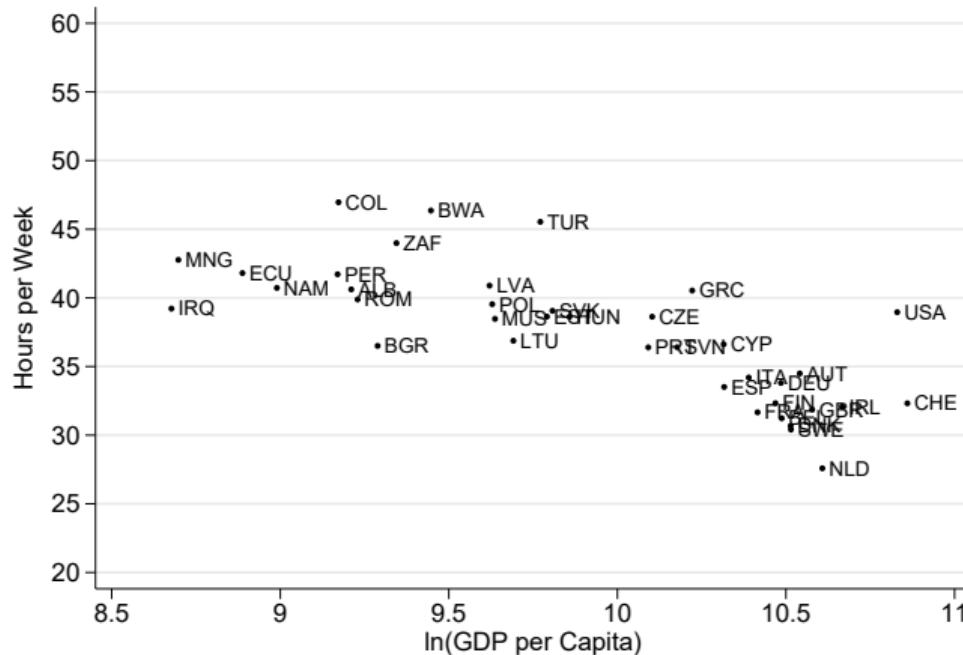
Conclusion

- Develop a theory that features effect of endogenous hours choice on sorting and inequality
 - Novel condition for assortative matching
- Novel mechanism: Hours-Biased Technological Change
 - New technologies raise returns to long hours for the high skill
- Estimate the framework: 1980-2015 in US ► Robustness
 - HBTC reconciles hours in the cross-section and time series
 - Accounts for roughly **30%** of increase in income inequality

Future Work

- How do **hours constraints** (e.g. social norms) impact sorting? Misallocation?
 - Recall sorting condition: $f_{xy} + f_{yh} h_x$
- Application: the (stalling) gender gap  **Empirics**
 - Gender gap stalled for the high skilled since 1990's (Blau and Kahn, 2014)
 - Gap is higher in occupations where hours elasticity is higher (Goldin, 2014)
- Can HBTC provide an explanation for stalling gender gap? Two forces:
 - Over time, easier for women to provide longer hours (hours constraints \downarrow)
 - Technology amplifies effect of such constraints

Additional evidence: Hours/worker over GDP

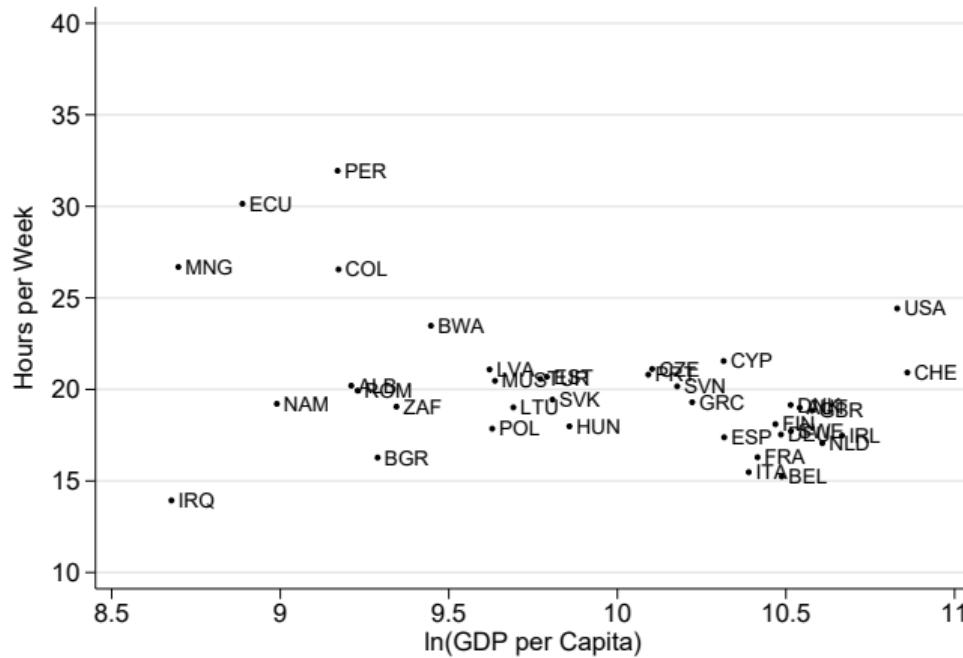


Source: Bick et al. (2018)

Hours/worker decline with GDPC in middle income and rich countries.

► Back

Additional evidence: Hours/capita over GDP

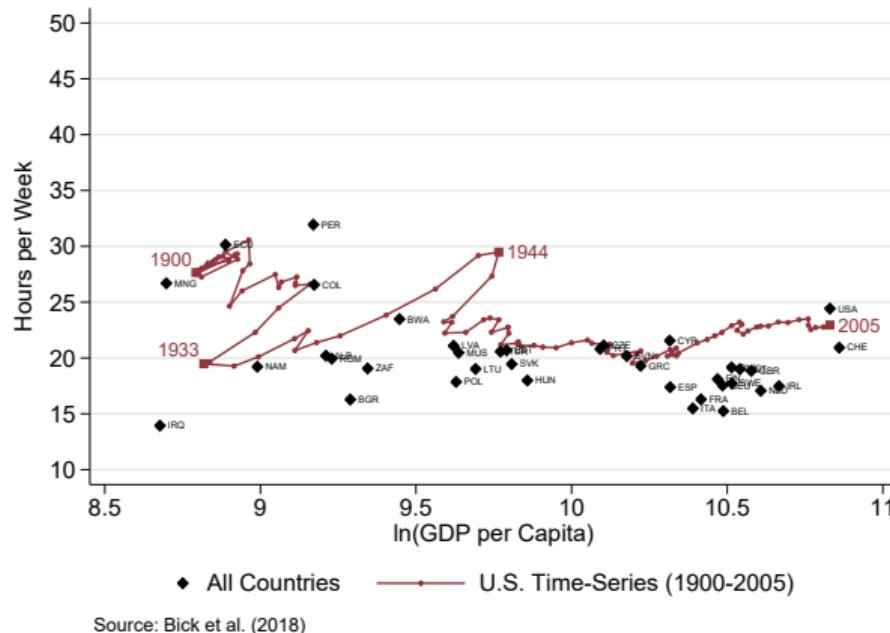


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Hours/capita decline with GDPC in middle income and rich countries.

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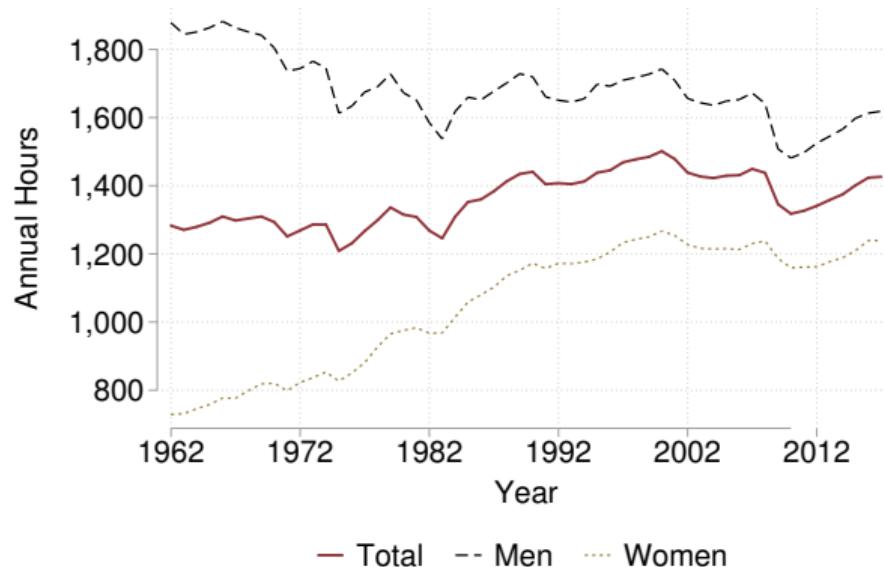
Additional evidence: Comparison with US



Hours/worker decline with GDPC in middle income and rich countries.

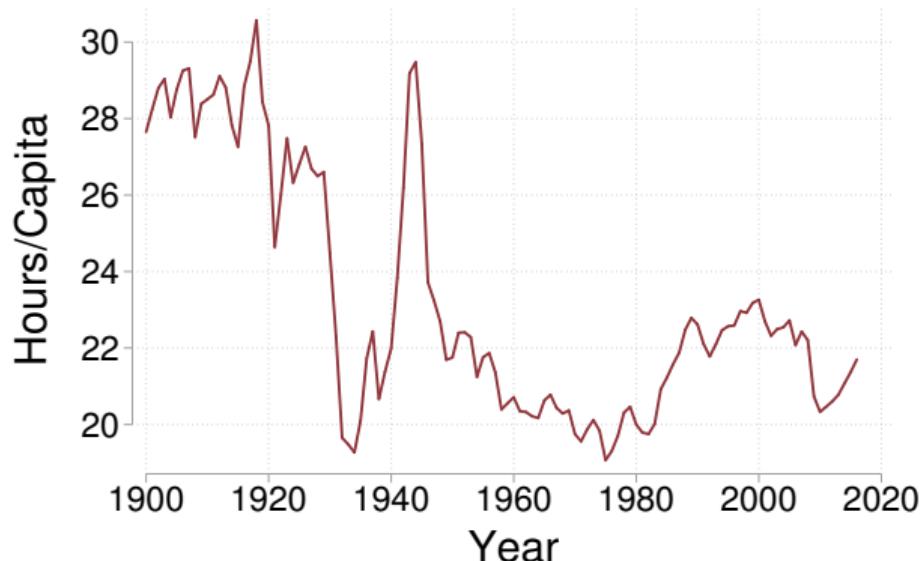
► Back

Additional evidence: Hours by Gender



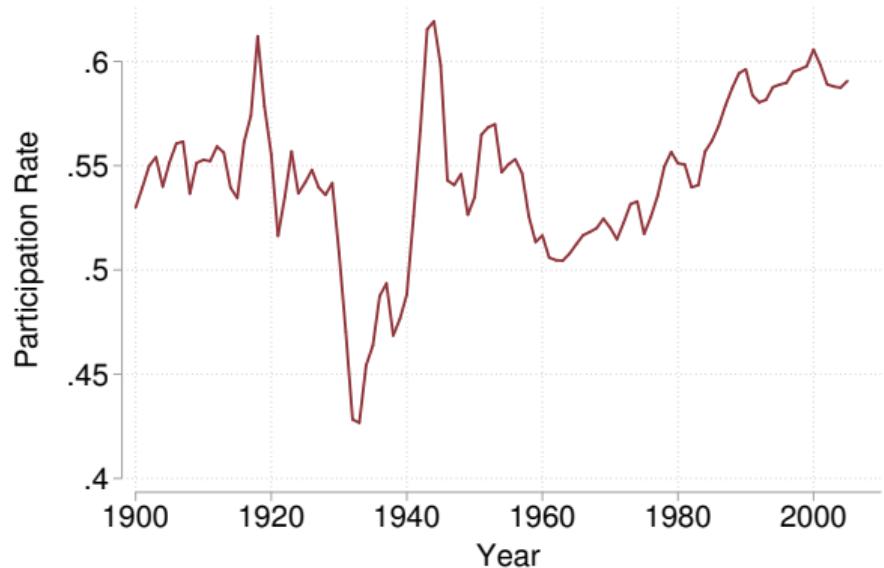
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Additional evidence: Hours per Capita



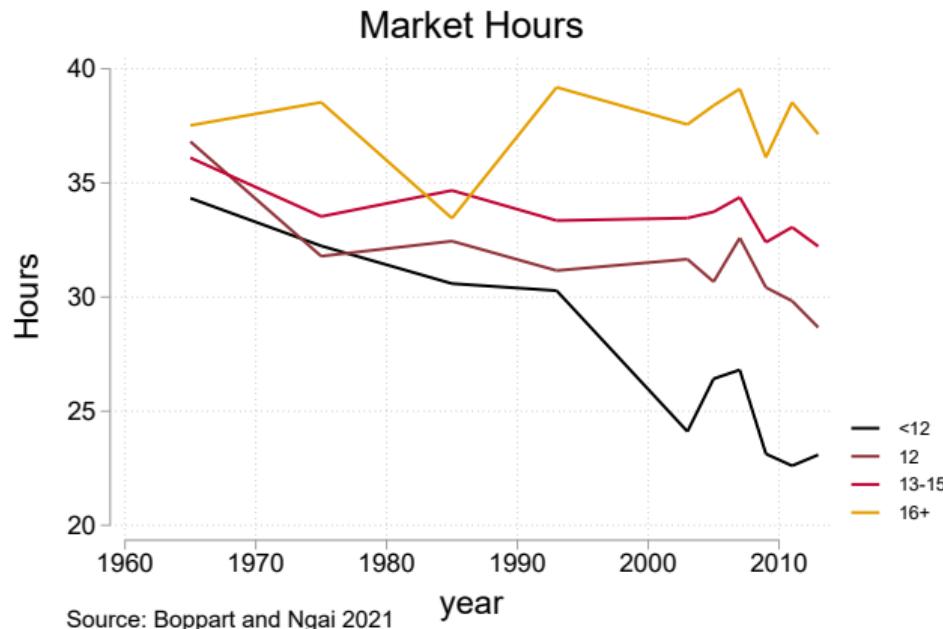
▶ Back

Additional evidence: Extensive Margin



▶ Back

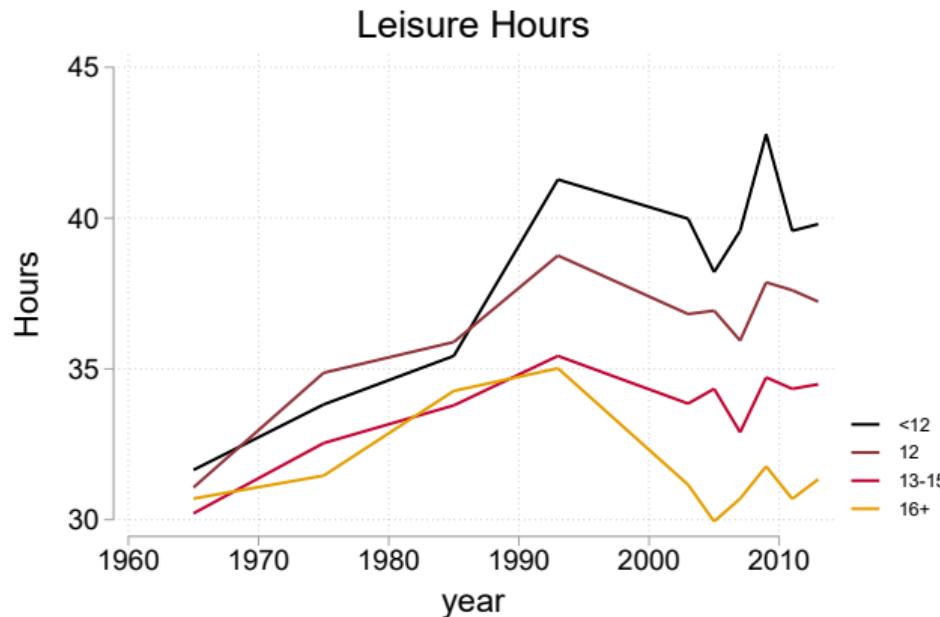
Additional evidence: ATUS data



Skilled workers people work roughly the same; low skilled work less.
Inequality in trends starts from 1980's.

▶ Back

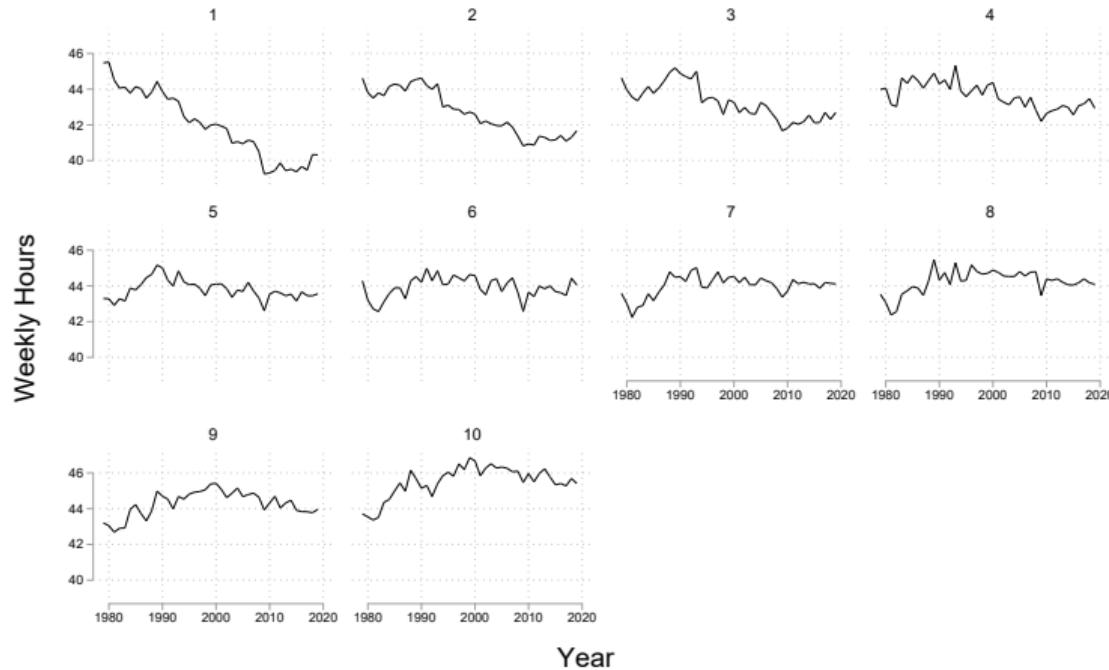
Additional evidence: ATUS data



Skilled workers consume less leisure, but starting from 1990's. Inequality in trends starts from 1980's.

▶ Back

Additional evidence: Hours Worked by Wage P'ctile



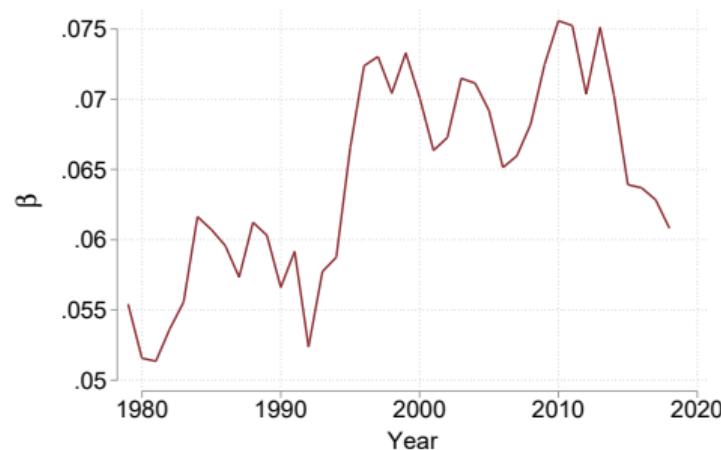
Graphs by hourw2

► Back

Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, both genders):

$$\log(h_i) = \alpha + \beta \log(w_i) + \text{age}_i + \text{age}_i^2 + \epsilon_i$$

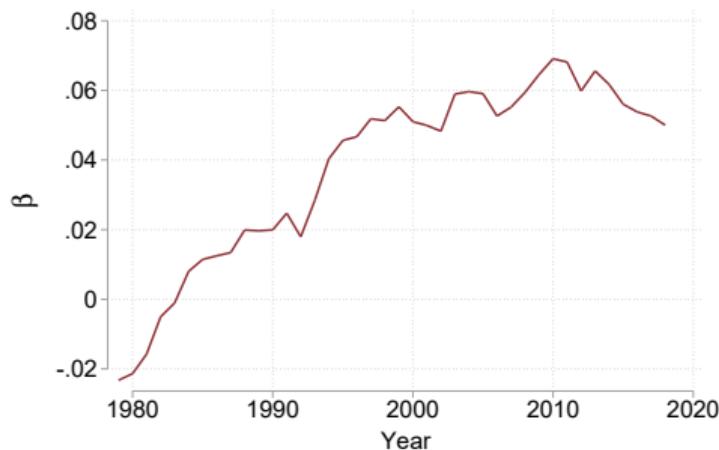


▶ Back

Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, without mult. jobs):

$$\log(h_i) = \alpha + \beta \log(w_i) + \text{age}_i + \text{age}_i^2 + \epsilon_i$$

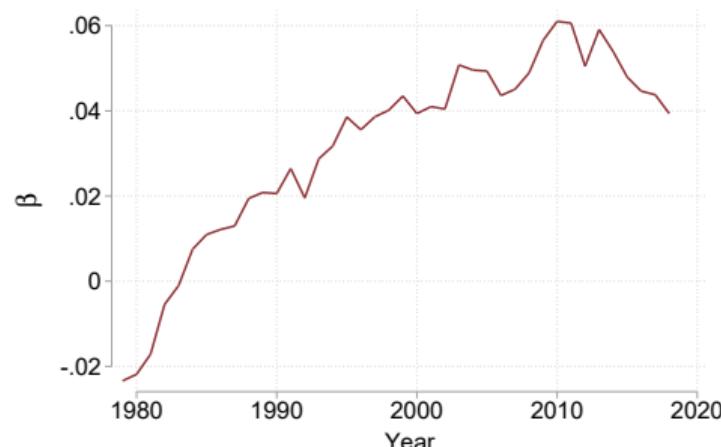


» Back

Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, includes tips/commissions for hourly workers):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$

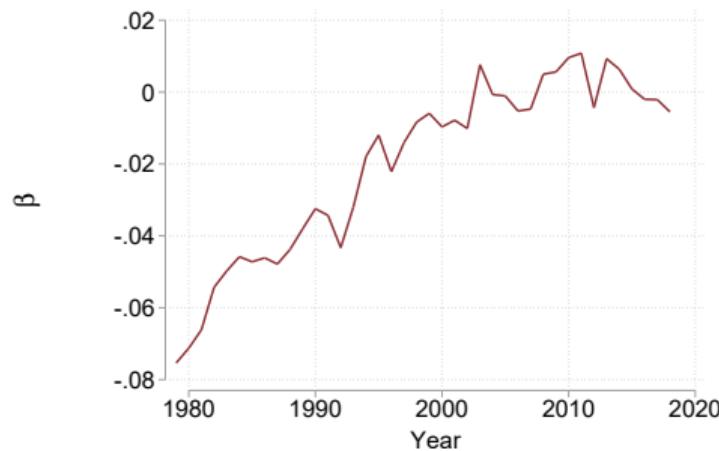


▶ Back

Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, excludes hourly workers):

$$\log(h_i) = \alpha + \beta \log(w_i) + \text{age}_i + \text{age}_i^2 + \epsilon_i$$

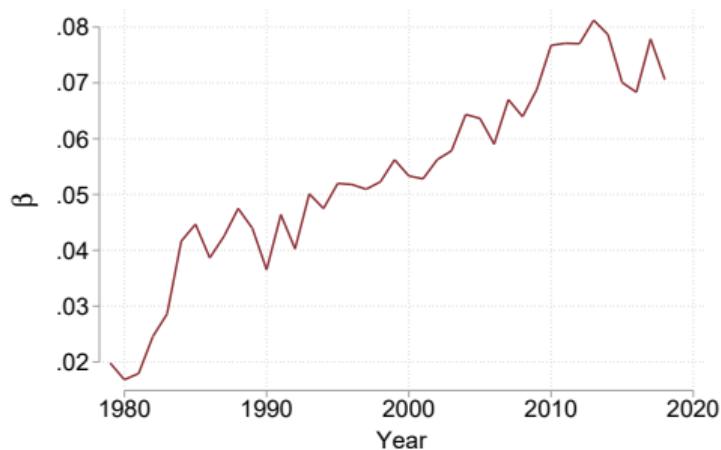


▶ Back

Additional evidence: Regression

- Run the regression for every year 1978-2019 (CPS, males, only hourly workers):

$$\log(h_i) = \alpha + \beta \log(w_i) + \text{age}_i + \text{age}_i^2 + \epsilon_i$$

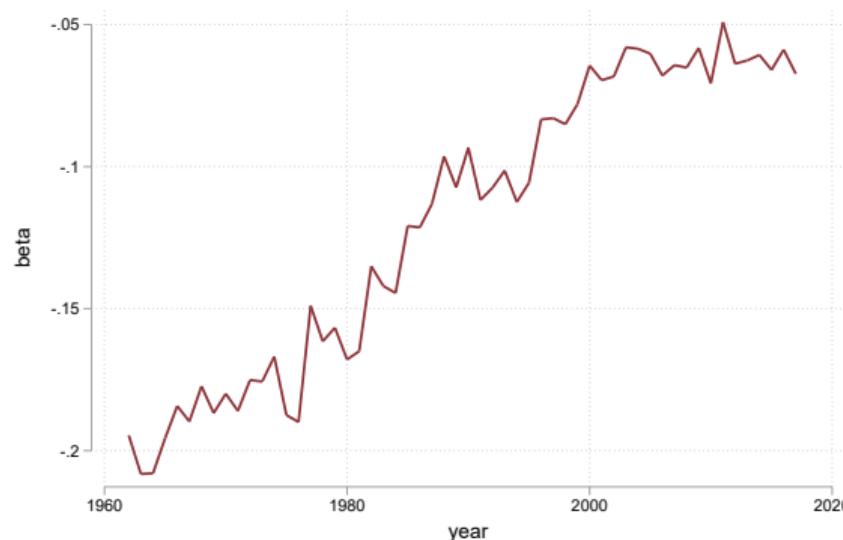


▶ Back

Additional evidence: ASEC

- Run the regression for every year 1962-2019 (ASEC, males, actual hours):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$



► Back

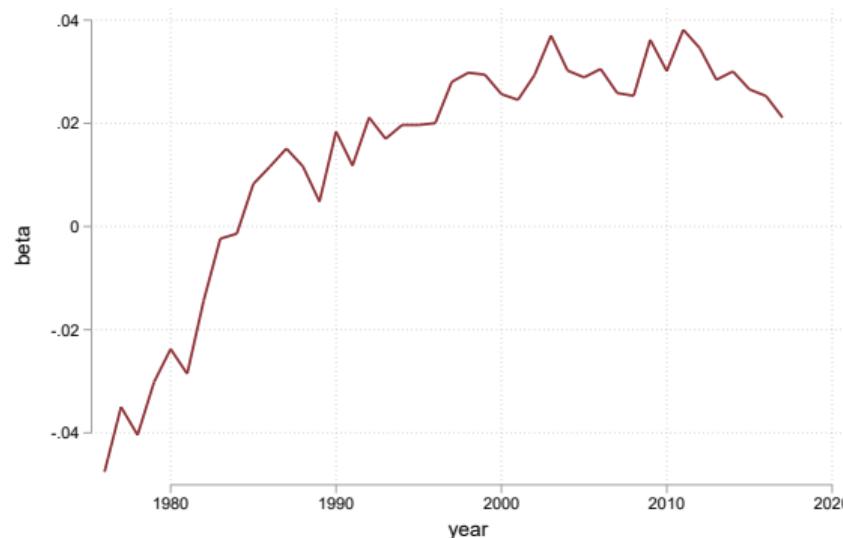
► Robustness: usual hours (M)

► Robustness: usual hours (F)

Additional evidence: ASEC

- Run the regression for every year 1978-2019 (ASEC, males, usual hours):

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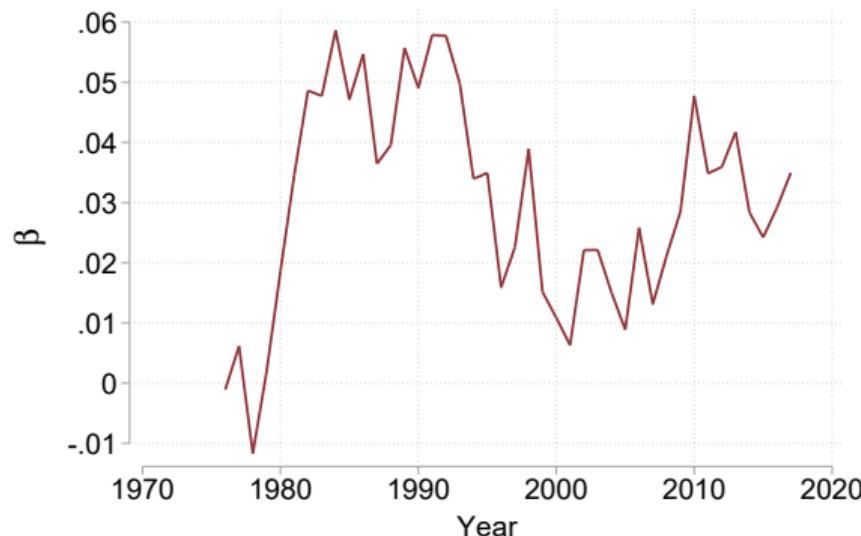
► Back to Main

► Back to ASEC

Additional evidence: ASEC

- Run the regression for every year 1978-2019 (ASEC, females, usual hours):

$$\log(h_i) = \alpha + \beta \log(w_i) + age_i + age_i^2 + \epsilon_i$$

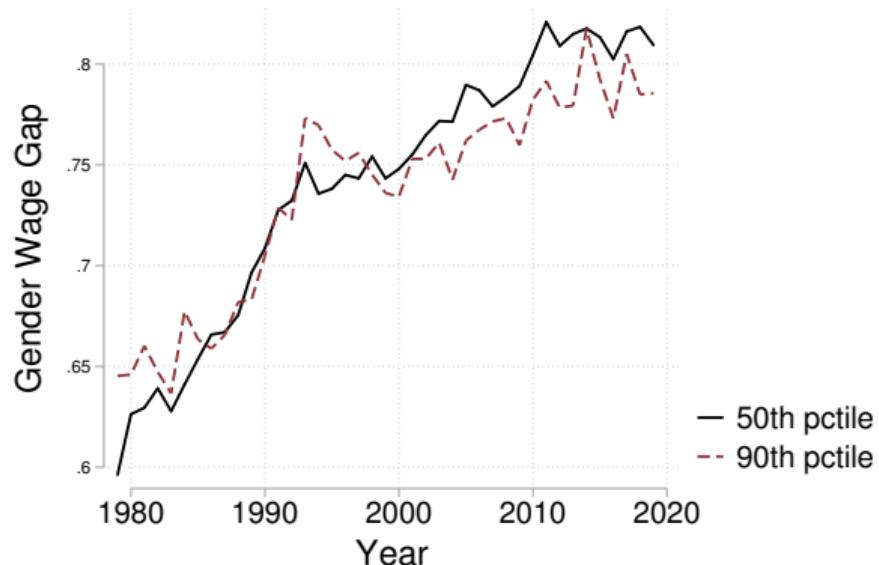


► Back to Main

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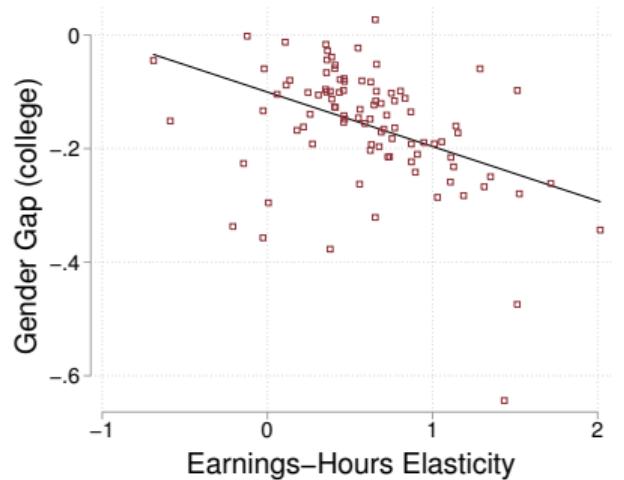
Implications: Hours Constraints and the Gender Gap

Gender gap has been decreasing across the wage distribution.
However, slower decline at the top since 1990's.



Implications: Hours Constraints and the Gender Gap

Tight link between gender gap and how income responds to hours (Goldin, 2014).



▶ Back

Distributions: From the Data

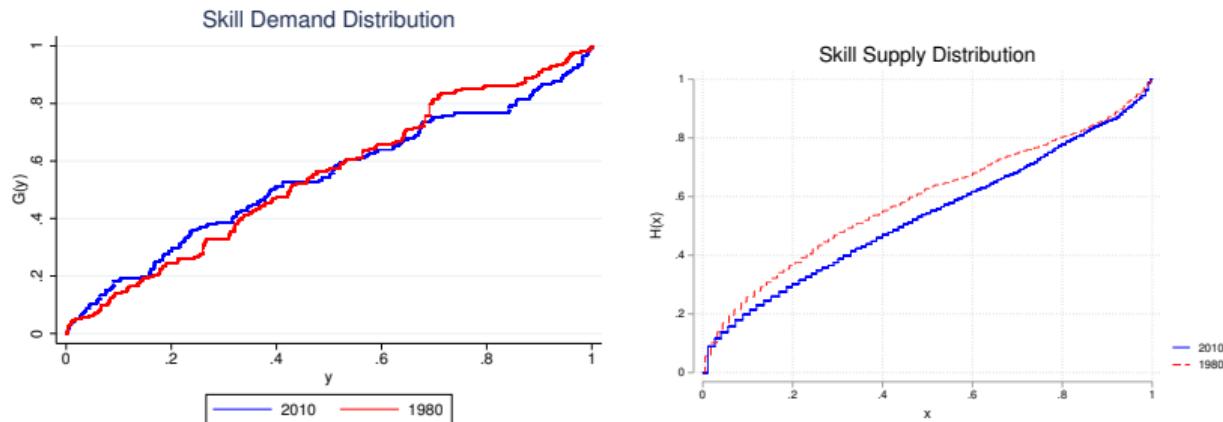


Figure: Estimated skill demand (left) and supply (right) distributions, for 1980 and 2010.

► Back

Robustness

On preferences:

- Income effects (different σ) [» Results](#)
- Idiosyncratic tastes for leisure [» Results](#)

On distributions:

- Take distributions from the data [» Distributions](#) [» Results 1](#) [» Results 2](#)
- Assume Beta distributions (Lise, Meghir, Robin 2016) [» Results](#)
- Estimate only σ_x [» Fit](#) [» Estimates](#)

On technology/estimation:

- Different nest of CES function [» Nest 2: Simulation](#) [» Nest 2: Results](#) [» Nest 3: Results](#)
[» Nests: Comparison](#)
- Simulate increase in tax progressivity (in progress)
- Standard errors + Sensitivity matrix (Shapiro et al, 2017 QJE) ✓

Estimates: Fitted Distributions

Parameter	1980	2015	Meaning
β	0.91 (0.02)	0.83 (0.02)	weight of (x, y) in prod.
α	0.24 (0.04)	0.79 (0.08)	weight of skills in prod.
γ	0.22 (0.02)	-0.39 (0.02)	compl. $(h, (x, y))$
ρ	-1.58 (0.27)	-4.24 (1.0)	compl. (x, y)
A	183.1 (27)	127.1 (1.0)	TFP

Notes: Standard errors in parentheses.

Key parameter changes:

- $\alpha \uparrow$ and $\rho \downarrow$: x, y more complementary
- $\gamma \downarrow$: hours/skills are more complementary ($f_{xh} \uparrow$)

▶ Back

Results: Fixed Hours Model

What is the role of **endogenous** hours response?

$$f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)h^\gamma \right)^{\frac{1}{\gamma}}$$

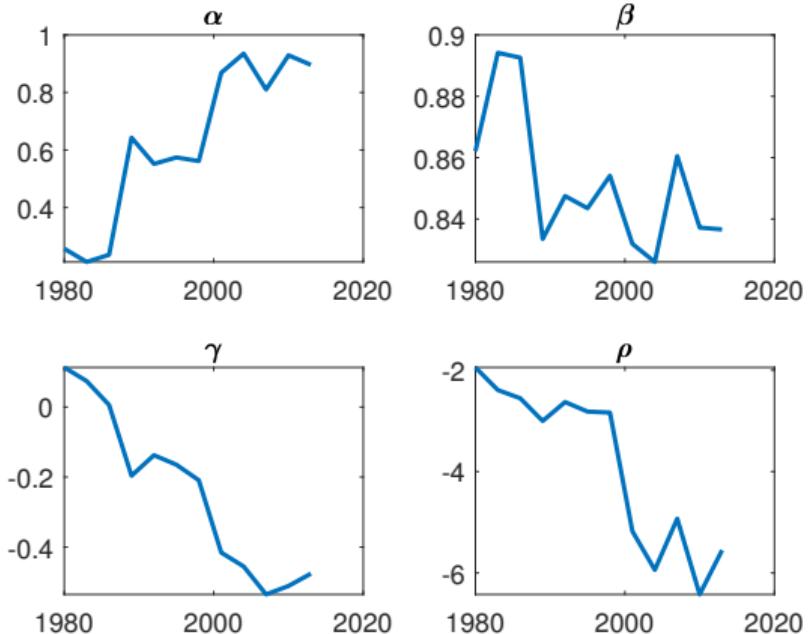
Estimated Change 1980-2015		
Parameter	Baseline	Fixed-hours
β	-60%	+2%
α	+199%	+102%
γ	-202%	-88%
ρ	+294%	+1200%

Fixed-h model **overstates** magnitude of SBTC (α, ρ) and **understates** HBTC ($\gamma \downarrow$)

» Back

Estimates: Fitted Distributions

Parameters Estimated Over Time



▶ Back

Estimates: Lower Income Effects

Parameter	1980	2015	Meaning
β	0.99	0.98	weight of (x, y) in prod.
α	0.05	0.13	weight of skills in prod.
γ	0.51	0.09	compl. $(h, (x, y))$
ρ	0.98	-0.57	compl. (x, y)
A	28,5179	26,677	TFP

Key parameter changes very similar to baseline specification:

- $\alpha \uparrow$ and $\rho \downarrow$: x, y more complementary
- $\gamma \downarrow$: hours/skills are more complementary ($f_{xh} \uparrow$)

▶ Back

Estimates: Idiosyncratic Tastes for Leisure

Introduce idiosyncratic tastes for leisure ψ_i in the utility function to match dispersion in hours.

Captures preferences, amenities, health shocks..

Parameter	1980	2015	Meaning
β	0.81	0.60	weight of (x, y) in prod.
α	0.23	0.67	weight of skills in prod.
γ	0.02	-0.13	compl. $(h, (x, y))$
ρ	0.98	-0.57	compl. (x, y)
A	6,5220	7,1405	TFP

Key parameter changes very similar to baseline specification.

▶ Back

Estimates: Assume Beta Distributions

Parameter	1980	2015	Meaning
β	0.94	0.91	weight of (x, y) in prod.
α	0.08	0.14	weight of skills in prod.
γ	0.30	-0.07	compl. $(h, (x, y))$
ρ	0.93	-2.48	compl. (x, y)
A	16,5103	16,5703	TFP

Key parameter changes very similar to baseline specification.

▶ Back

Transformed Production

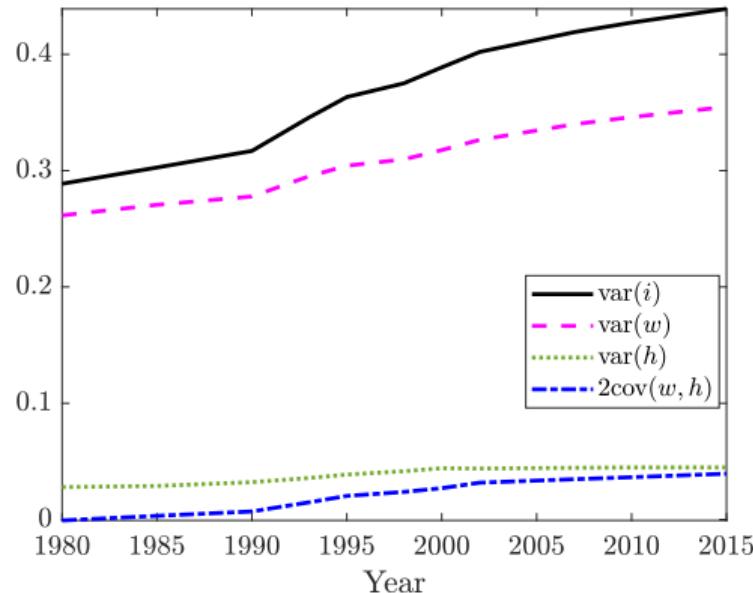
- Note: $f(x, y, h)$ is equivalent to

$$f(x, y, h) = \left((A_x x^\rho + A_y y^\rho)^{\frac{\gamma}{\rho}} + A_h h^\gamma \right)^{\frac{1}{\gamma}}$$

where A_x is the weight of skill in the transformed production, and similarly for y and h .

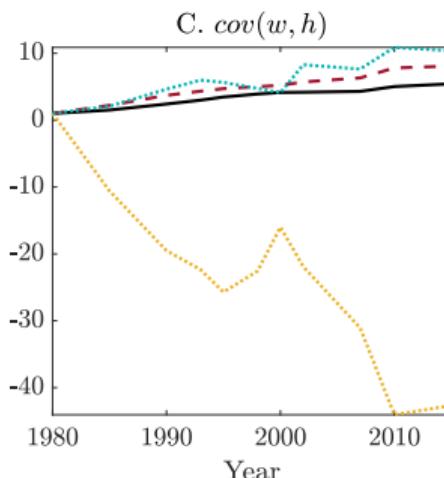
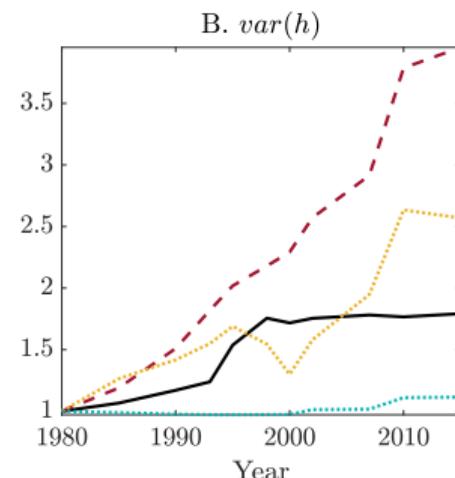
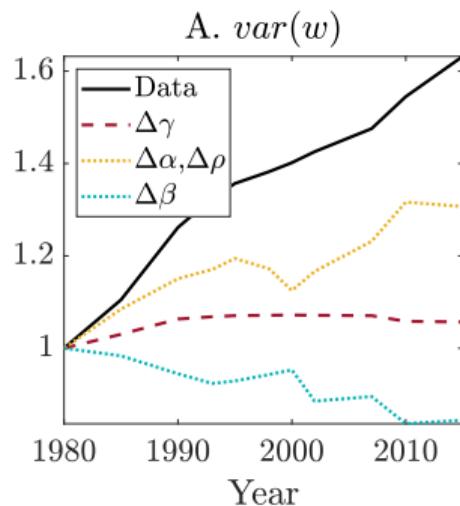
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Income Inequality: Levels



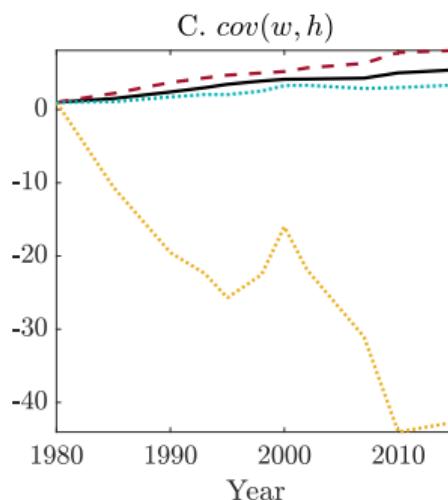
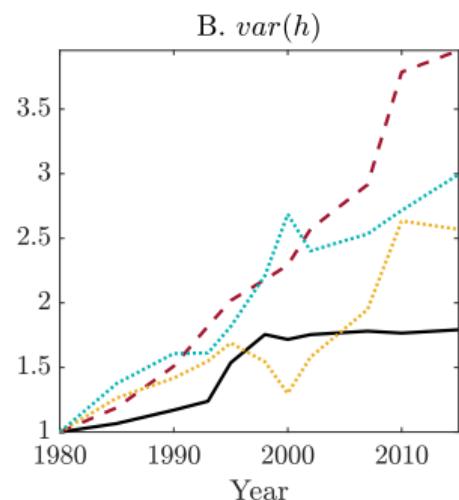
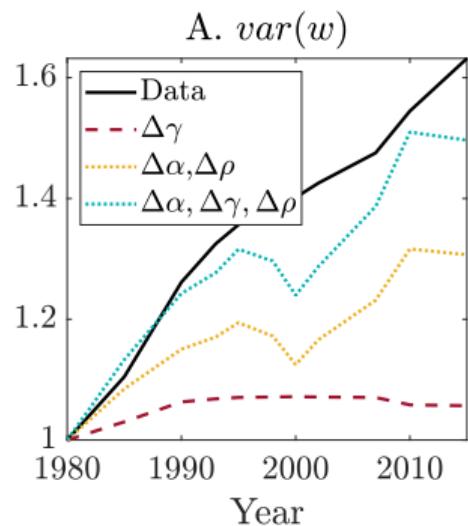
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Counterfactuals: Drivers of Inequality (One parm. at a time) 1/2



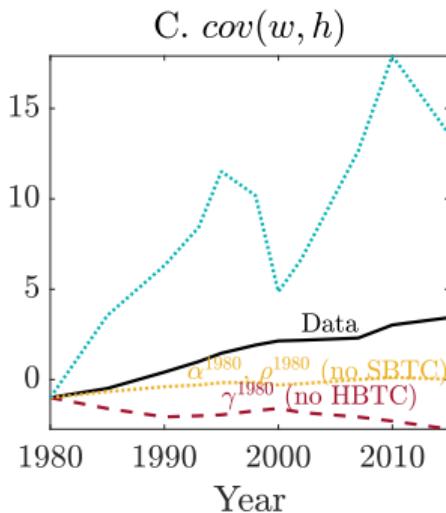
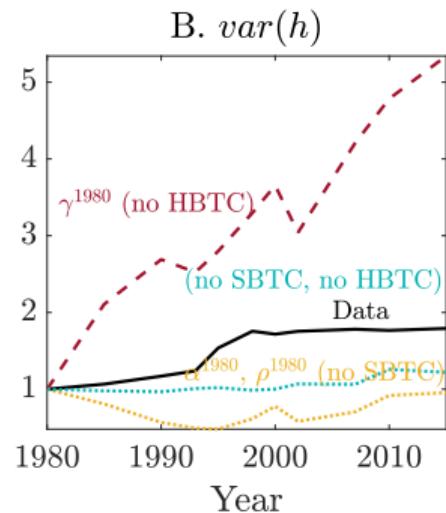
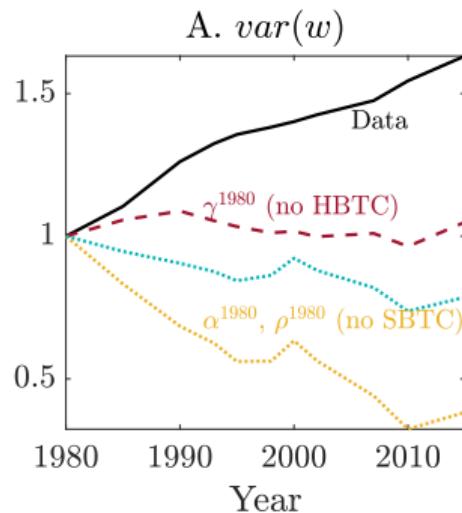
▶ Back

Counterfactuals: Drivers of Inequality (One parm. at a time) 2/2



▶ Back

Counterfactuals: Drivers of Inequality (Neither HBTC nor SBTC)



▶ Back

Additional evidence: progressivity across countries

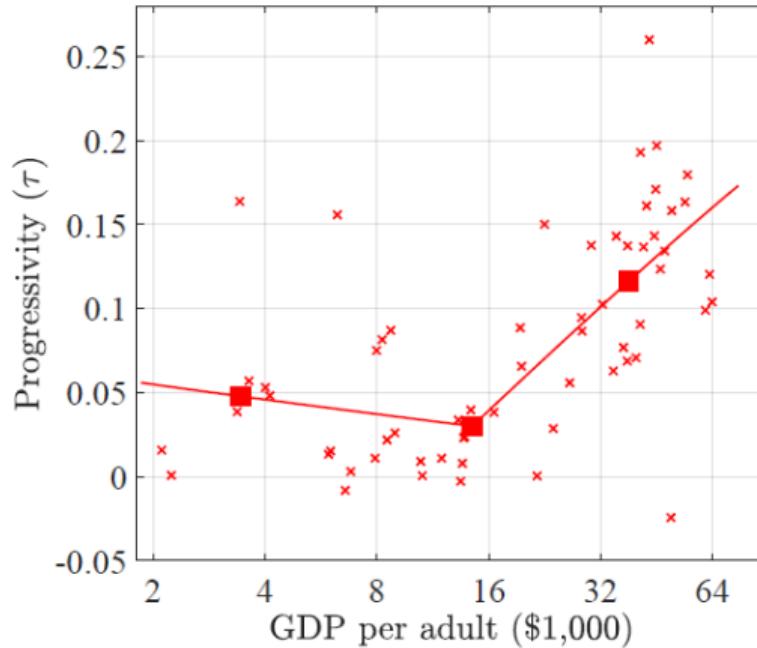


Figure: Fiscal progressivity across countries (source: Bick et al, 2021)

Market Clearing

Complete Definition

- Under Positive Assortative Matching (PAM), market clearing condition requires:

$$\int_{\mu(x)}^{\bar{y}} g^y(s)ds = \int_x^{\bar{x}} g^x(s)ds ,$$

where g^x and g^y are the densities of workers and jobs, respectively.

- Starting at the top, the highest type \bar{x} matches with the highest type \bar{y} .
- The type $x < \bar{x}$ matches with $y = \mu(x)$ if the measure of workers above x is equal to the measure of jobs above $\mu(x)$.
- This ensures that matching is measure-preserving.

Equilibrium definition

Market Clearing & Equilibrium

- Both households and firms take hedonic income function $e()$ as given.
- Market clearing determines matching function $\mu()$.
Under Positive Assortative Matching (PAM):

$$\int_{\mu(x)}^{\bar{y}} g^y(s)ds = \int_x^{\bar{x}} g^x(s)ds \quad (1)$$

Definition 1: Competitive equilibrium.

A competitive equilibrium of this economy is a tuple of functions (e, μ, h) such that:

- e and h maximize utility and profits (*optimality*)
- equation (1) holds (*market clearing*)

Derivation: Benchmark Model

- FOC:

$$f_x(x, y) - \frac{\partial w(x)}{\partial x} = 0$$

- Market clearing: $\mu()$ (measure preserving). PAM is defined as $\mu'(x) > 0$
- If $\mu'(x) > 0$, then we can write: $\Gamma(x) = \Phi(\mu(x)) \implies \mu(x) = \Phi^{-1}\Gamma(x)$.
- SOC:

$$f_{xx}(x, y) - w_{xx}(x) < 0$$

- To get rid of w_{xx} , we evaluate the FOC at $y = \mu(x)$:

$$f_{xx}(x, \mu(x)) + f_{xy}(x, \mu(x)) \cdot \frac{d\mu(x)}{dx} = w_{xx}(x)$$

- Substituting back, we get:

$$f_{xy}(x, \mu(x)) \cdot \frac{d\mu(x)}{dx} > 0$$

- Hence the sign of $\mu(x)$ is determined by the sign of f_{xy} (provided differentiability of $\mu()$)

Derivation: This Model

- Here we have two optimization problems (firm + hh). Hence FOC is a system of equations:

$$f_{\tilde{x}}(\tilde{x}, y) - \frac{\partial w(\tilde{x})}{\partial \tilde{x}} = 0 \quad (\text{firm})$$
$$u'_w \cdot w'_h + u'_h = 0 \quad (\text{hh})$$

- We repeat the previous derivation to understand properties of \mathbf{u} and \mathbf{f} that gives us PAM. The Hessian (SOC) is:

$$\begin{pmatrix} f_{xx}(x, y) - w_{xx}(x) & f_{xh}(x, y) - w_{xh}(x) \\ \frac{\partial u'_w \cdot w'_h}{\partial x} & \frac{\partial u'_w \cdot w'_h + u'_h}{\partial h} \end{pmatrix}$$

- Optimality requires $|H| \geq 0$. As before, we evaluate the FOC at $y = \mu(x)$ and substitute.
- Solving for $|H| \geq 0$ after some algebra we have that:

$$|H| \geq 0 \implies \text{the main condition.}$$

▶ Back

▶ Standard model

Hours choice: Derivation

The FOC for hours reads:

$$u_c f_h + u_h = 0$$

By monotone comparative statics, h_x increases in x if:

$$\frac{\partial (u_c f_h + u_h)}{\partial x} > 0$$

From which one gets:

$$\left(\underbrace{u_{cc} f_h f_x}_{\text{income effects}} + \underbrace{u_c f_{hx}}_{\text{subs. effects}} \right) > 0$$

Hours choice

Special Case: CRRA

- Hours choice:

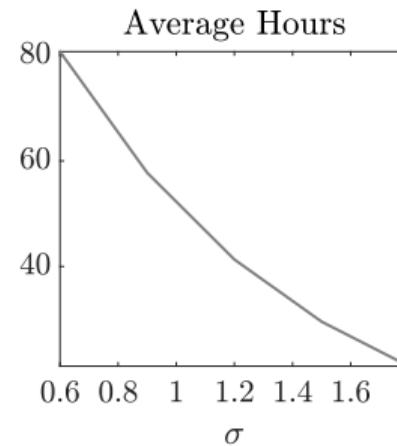
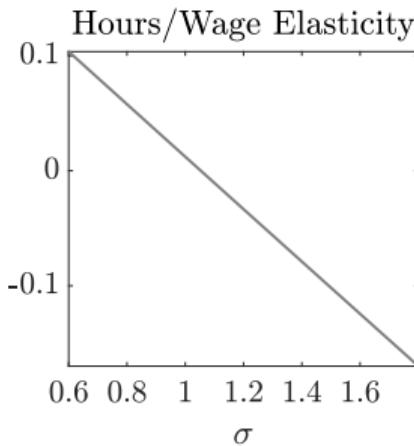
$$\left(\underbrace{u_{cc} f_h f_x}_{\text{income effects}} + \underbrace{u_c f_{hx}}_{\text{subs. effects}} \right) > 0$$

- **Sufficient condition:** relative risk aversion coefficient $\sigma = -c \frac{u_{cc}}{u_c} < 1$
 - σ governs strength of income vs substitution effects in macro models (King, Plosser, Rebelo 1988).
 - $\sigma > 1$ needed to match aggregate evidence (Boppart and Krusell, 2020)

▶ Back

Comparative Statics

Preferences



An increase in σ :

- Decreases hours wage elasticity (higher income effects) and average hours worked
- Elasticity turns negative for lower values of σ , if low complementarities.

▶ back

Assortative Matching

- Pair x and y solve the problem:

$$U(x, y, V) = \max_h u(c, h) \quad \text{s.t.} \quad f(x, y, h) - c \geq V$$

where $U(x, y, .)$ traces a **Pareto frontier**.

- Hence we maximize U w.r.t. to job y to get the FOC:

$$U_y + U_\pi \pi_y = 0$$

- The equilibrium matching μ is optimal provided that the SOC < 0 , hence:

$$U_{yy} + 2U_{y\pi}\pi' + U_{\pi\pi}\pi'^2 + U_\pi\pi'' < 0$$

Assortative Matching

- Differentiate FOC along the candidate equilibrium $y = \mu(x)$:

$$U_{yy} + U_{xy}\mu' + U_{x\pi}\pi' + U_{x\pi}\pi' + U_{y\pi}\mu'\pi' + U_{\pi\pi}\pi'^2 + U_\pi\pi'' = 0$$

- Combining this condition with $SOC < 0$ gives:

$$\mu' \left[U_{xy} - \frac{U_y}{U_\pi} U_{x\pi} \right] \geq 0$$

- Hence PAM ($\mu' > 0$) obtains iff

$$U_{xy} - \frac{U_y}{U_\pi} U_{x\pi} \geq 0$$

Which completes the proof.

Assortative Matching

Alternative condition for PAM

- Corollary: A condition for PAM in terms of primitives is:

$$(f_{yx}(\underbrace{-u_{cc}f_h f_h - u_c f_{hh} - u_{hh}}_{>0}) + f_{hy}(u_{cc}f_x f_h + u_c f_{hx})) > 0$$

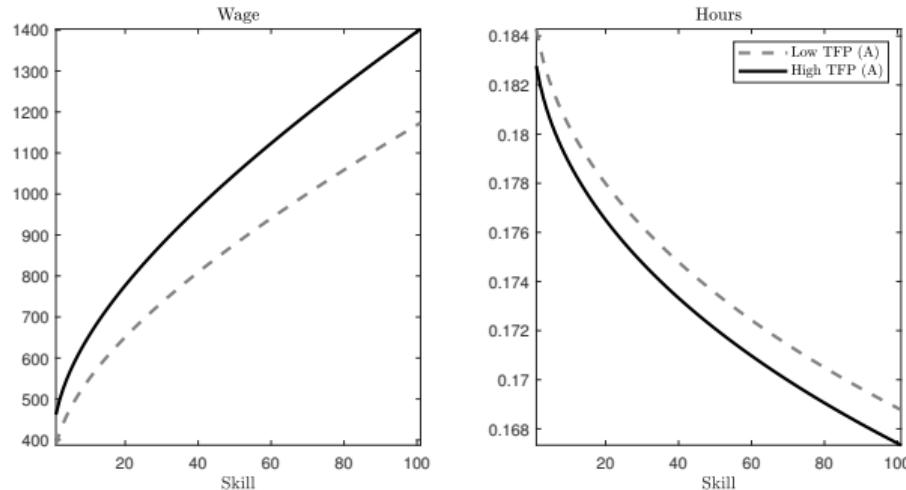
- Intuition:
 - f_{xy} : skill-job complementarity (Becker 1973)
 - f_{xh}, f_{yh} : **hours** complementarities (better workers produce more with more time)
 - $u_{cc} < 0$: income effects push towards NAM

▶ Proof

▶ Back

Prod. function: comparative statics

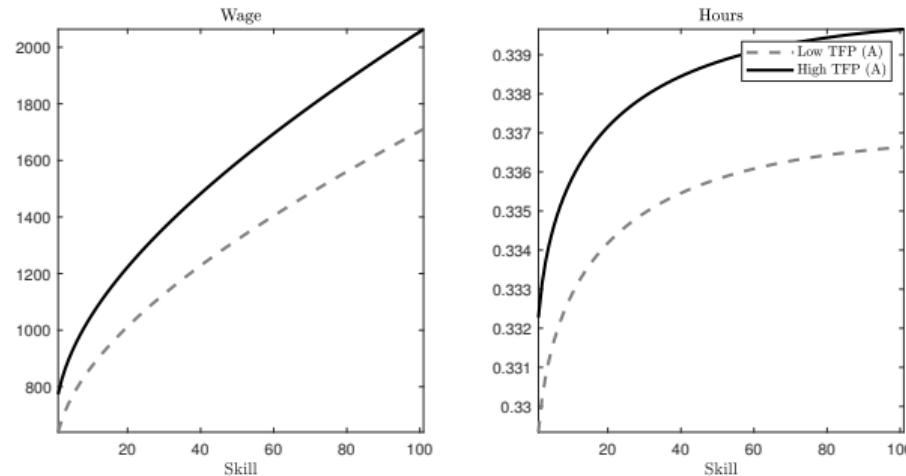
TFP: A, baseline



Increasing TFP has a level shift on wages (positive) and hours (negative, $\sigma > 1$).

Prod. function: comparative statics

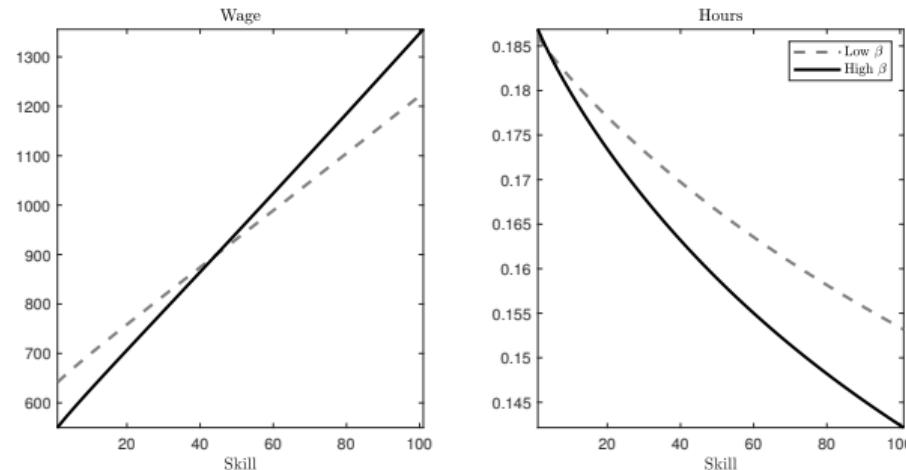
TFP: A, low σ



Increasing TFP has a level shift on wages (positive) and hours (positive, $\sigma < 1$).

Prod. function: comparative statics

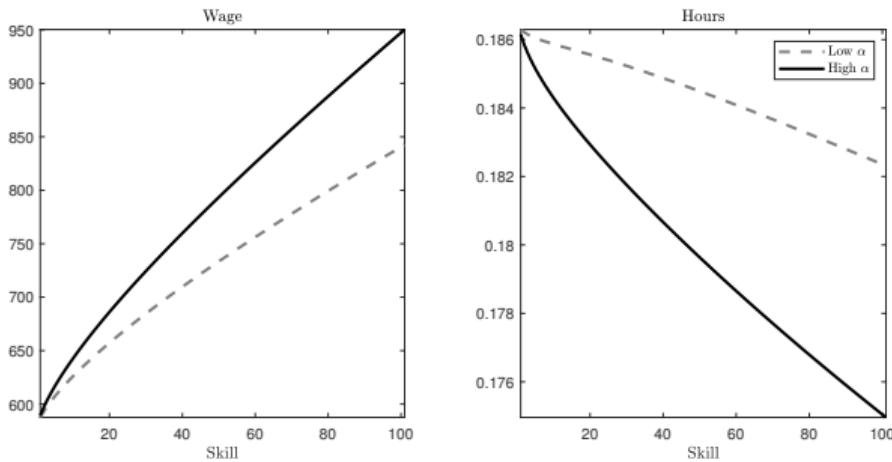
Hours/Match share: β



Increasing β decreases importance of hours (hence hours choice \downarrow).

Prod. function: comparative statics

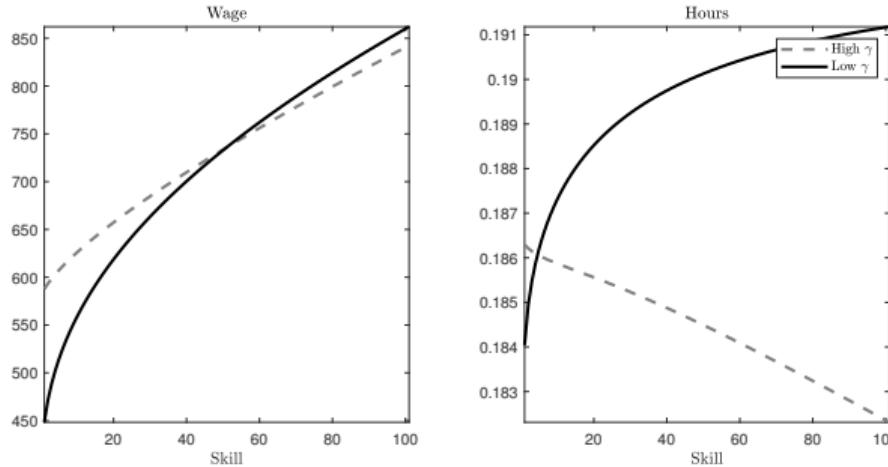
Skill importance: α



Increasing α makes wage steeper in skills; through income effects, hours less steep (hence $\partial h / \partial x \downarrow$).

Prod. function: comparative statics

Match/hours complementarity: γ



Increasing γ pushes towards $\partial h / \partial x \uparrow$. Also, it increases inequality. Key parameter to inform positive/increasing hours/wage elasticity! Missing in the previous specification ($\gamma = 0$).

Estimation

Jobs distribution

- Need distribution of worker and job types, $H(x)$ and $G(y)$, for 1980 and 2010
- Solution: estimate directly from the data
- $G(y) \approx \text{cognitive skill requirement}$
 - 1980: DOT (Dictionary of occupational titles); $y \approx$ mathematical reasoning
 - 2010: O*NET: PCA on cognitive/math variables
- Final step: merge estimated y with CPS for both periods to get $G(y)$
- Same approach as in Lindenlaub(2014), Lindenlaub and Chade(2022)

Estimation

Skill/Ability Distributions

- $H(x)$ test scores
 - 1980: NLSY79
 - 2010: NLSY97
- I rely on Altonji, Bharadwaj, and Lange (2009) criterion to make test scores comparable across the two periods¹
- (adjusted) distribution of test scores gives our measure of $H(x)$

¹e.g. test respondents were administered the test at different ages in the two cohorts

Estimation

Skill/Ability Distributions

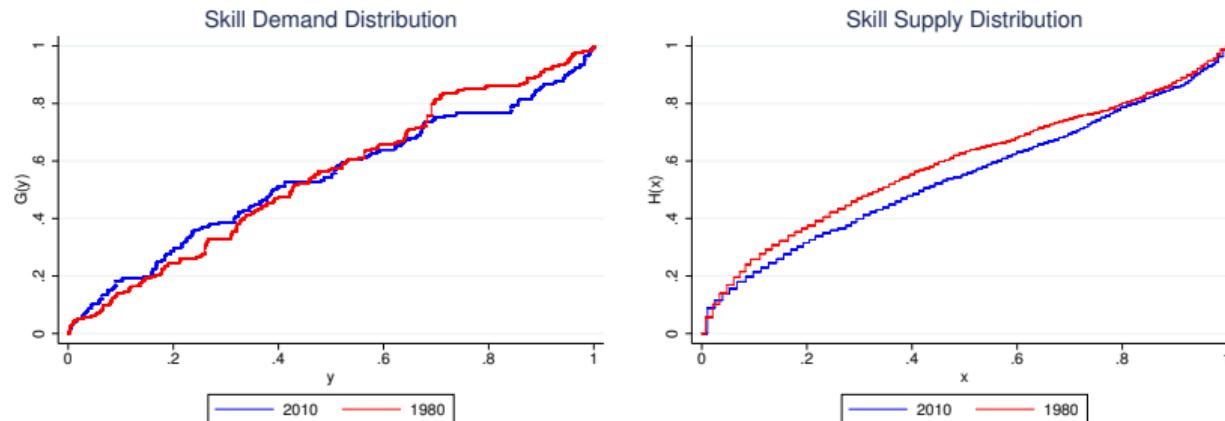


Figure: Estimated skill demand and supply distributions, for 1980 and 2015.

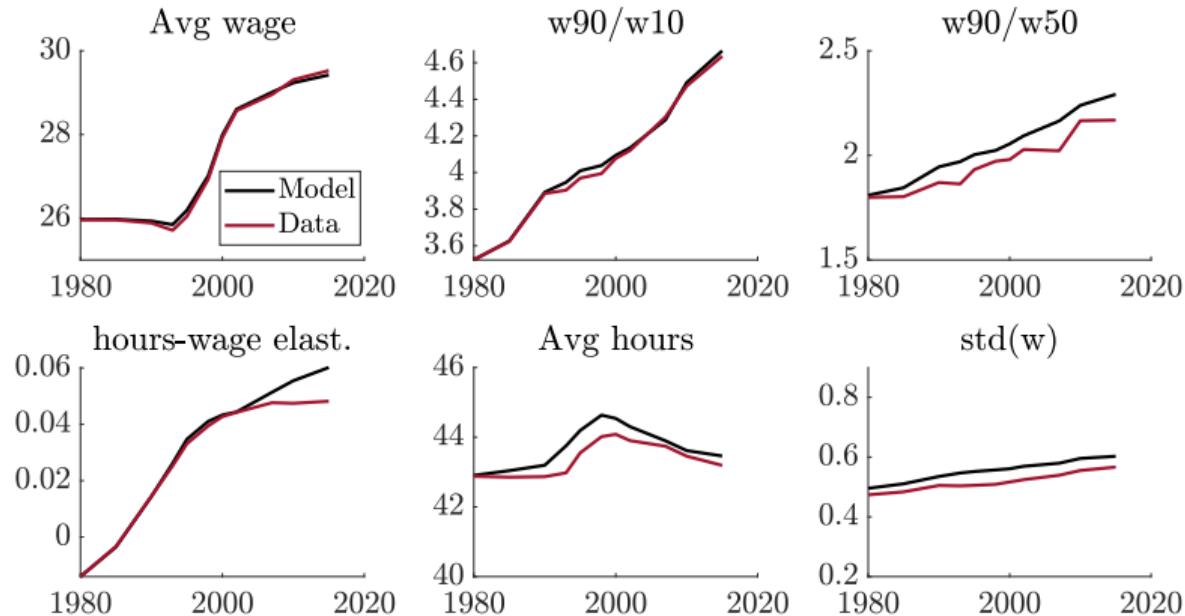
- Jobs (left): slight change in dispersion
- Workers (right): 'better' distribution of skills

Targets

Results						
Moment	Data 1980	Data 2015	Model 1980	Model 2015	Data Δ	Model Δ
E(w)	19.6	23.9	19.6	23.9	21%	21%
E(h)	37.9	38.4	37.9	38.4	+1%	+1%
W75/W25	1.86	2.29	1.86	2.29	23%	23%
W50/W10	1.92	2.03	1.92	2.03	6%	6%
h/w elast.	-0.012	0.062	-0.012	0.062	7%	7%

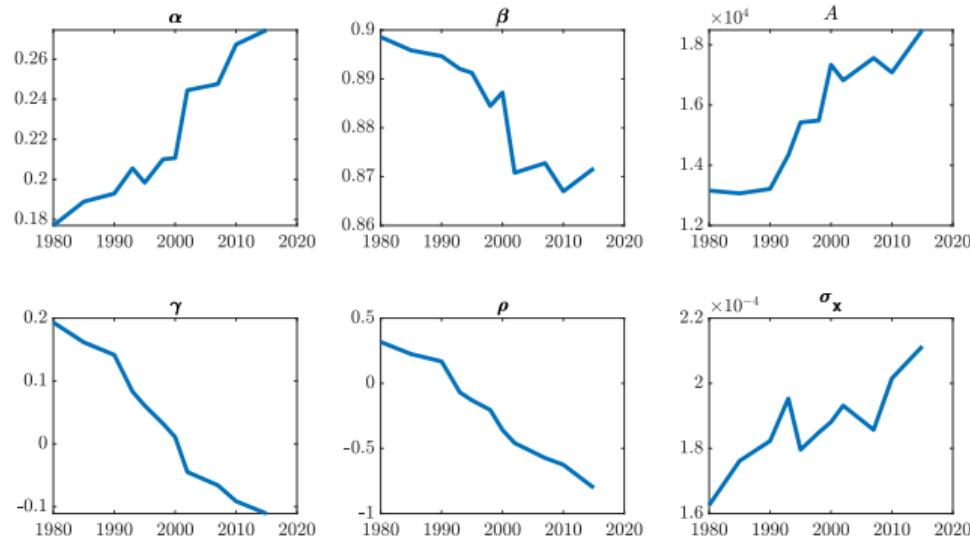
► back

Model Fit



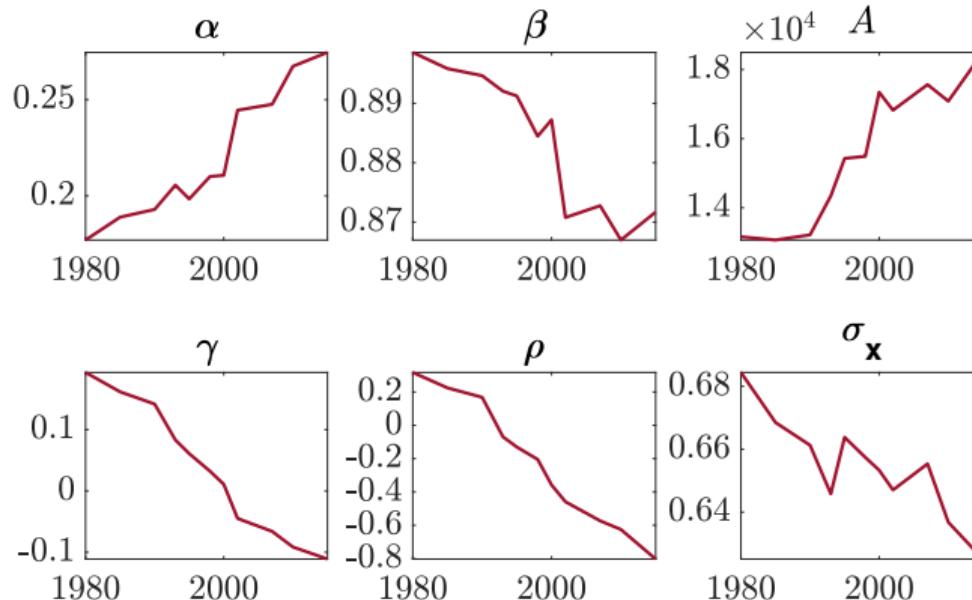
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Estimates



▶ Back

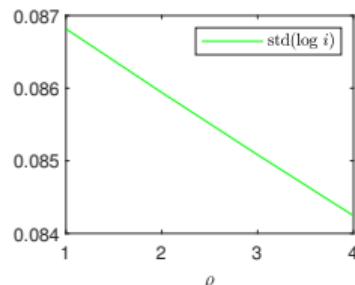
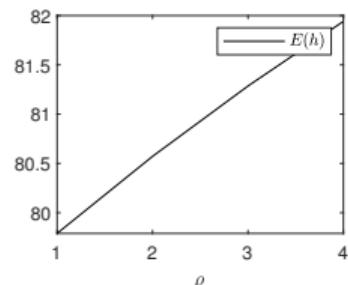
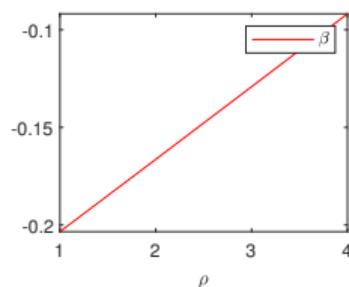
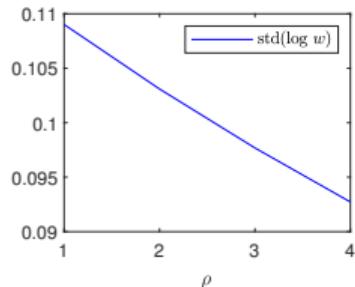
Estimates



▶ Back

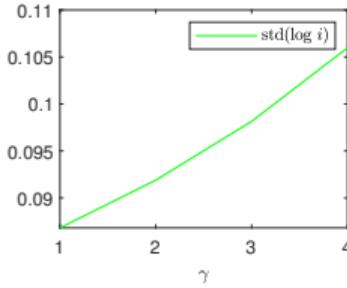
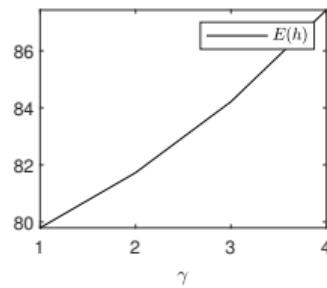
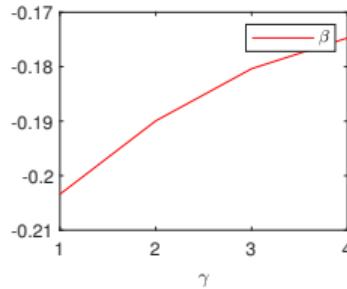
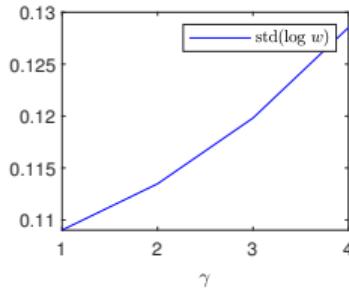
Different CES nesting

$$f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)y^\gamma \right)^{\frac{1}{\gamma}}$$



Different CES nesting

$$f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)y^\gamma \right)^{\frac{1}{\gamma}}$$



▶ Back

Different CES nesting

Assume a different nest for the CES: workers and hours form a 'bundle',
 $\tilde{x} = \tilde{x}(x, h)$.

$$f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)y^\gamma \right)^{\frac{1}{\gamma}}$$

Parameter	1980	2015	Meaning
β	0.09	0.17	weight of (x, h) in prod.
α	0.80	0.85	weight of skills in prod.
γ	1.01	-0.34	compl. (\tilde{x}, y)
ρ	0.17	0.11	compl. (h, x)
A	32,209	27,276	TFP

► Back

Different CES nesting

Assume a different nest for the CES: firms/jobs and hours nested as $CES((y, h), x)$.

$$f(x, y, h) = A \left(\beta(\alpha y^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)x^\gamma \right)^{\frac{1}{\gamma}}$$

Parameter	1980	2015	Meaning
β	0.40	0.90	weight of (y, h) in prod.
α	0.73	0.97	weight of jobs y in prod.
γ	-3.3	-4.5	compl. $(x, y/h)$
ρ	0.56	0.04	compl. (h, y)
A	4,489	27,597	TFP

► Back

Different CES nesting

$$\text{Nest1} \quad f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)y^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)h^\gamma \right)^{\frac{1}{\gamma}}$$

$$\text{Nest2} \quad f(x, y, h) = A \left(\beta(\alpha x^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)y^\gamma \right)^{\frac{1}{\gamma}}$$

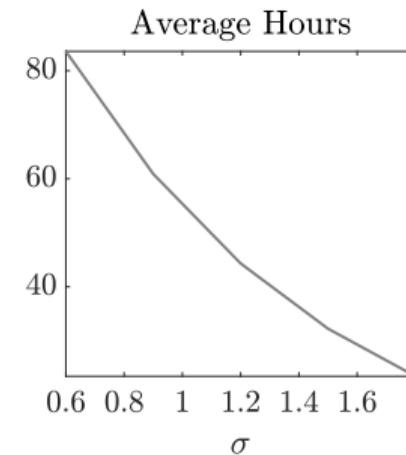
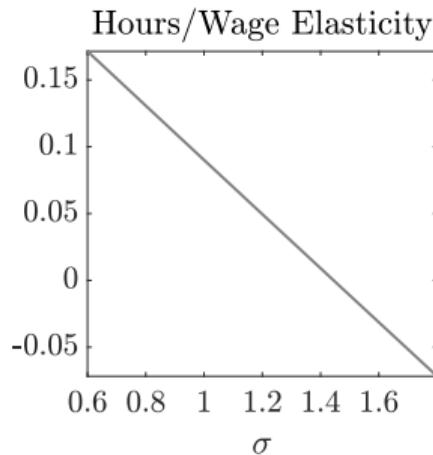
$$\text{Nest3} \quad f(x, y, h) = A \left(\beta(\alpha y^\rho + (1 - \alpha)h^\rho)^{\frac{\gamma}{\rho}} + (1 - \beta)x^\gamma \right)^{\frac{1}{\gamma}}$$

	Nest 1		Nest 2		Nest 3	
Parameter	1980	2015	1980	2015	1980	2015
β	0.90	0.87	0.09	0.17	0.40	0.90
α	0.13	0.21	0.80	0.85	0.73	0.97
γ	0.32	-0.07	0.98	-0.34	-3.3	-4.5
ρ	0.22	-1.20	0.17	0.11	0.56	0.04
A	12,871	22,218	32,209	27,276	4,489	27,597

► Back

Comparative Statics

Preferences



An increase in σ :

- Decreases hours wage elasticity (higher income effects) and average hours worked

▶ Low compl.

▶ Back