

# Code Description for the Technical Challenge

The code is available at <https://github.com/cristianomonteiro/RaizenChallenge>. It was implemented in Python 3.9.7 using Jupyter notebook. The main libraries used were PuLP 2.6, Pandas 1.4.3 and the MIP solver was Coin-OR CBC 2.10.5. The following section summarizes the code structure.

## 1 Code Summary

The code is organized in the following phases:

1. Loading the data into a data frame;
2. Loading dictionaries of containers, boxes and cylinders objects;
3. Building and solving the optimization model;
4. Inserting an additional constraint to force the optimization model not to select again a previous optimal solution;
5. Re-solving the optimization model;
6. Repeating the last two phases until enough different optimal solutions are obtained;
7. Printing a solution selected by the user.

After each optimal solution is obtained, a function is run to check if all rules were properly applied. It was at first implemented to check whether the code has bugs, but I decided to keep it also in this final version. The following section presents the mathematical formulation.

## 2 Mathematical Formulation

The following tables defines the indices, variables and constants used in this formulation.

**Table 1:** Indices of the Technical Challenge

Index	Description
$i \in \mathbb{A}$	Container $i$
$j \in \mathbb{B}_i$	Box $j$ from container $i$
$k \in \mathbb{C}_j \subseteq \mathbb{C}$	Cylinder $k$ from box $j$
$k' \in \mathbb{C}' \subseteq \mathbb{C}$	Cylinder $k$ selected in the previous optimal solution

Equation 1 defines the applied objective function. Since any feasible solution is good enough, all coefficients of the variables are defined as zero. Equations from 2 to 7 defines

**Table 2:** Variables of the Technical Challenge

Variable	Description
$c_i \in \{0, 1\}$	Indicates whether container $i$ is chosen or not
$x_k \in \{0, 1\}$	Indicates whether cylinder $k$ is selected or not
$x_{k'} \in \{0, 1\}$	Indicates the cylinder $k'$ selected in the last optimization run

**Table 3:** Constants of the Technical Challenge

Constant	Description
$v_k$	Volume of the cylinder $k$
$w_k$	Weight of the cylinder $k$

the game's rules. The more complex constraint is the Equation 6 because it uses a Big- $M$  method for assuring that if at least one cylinder is selected, then its container must also be chosen. This constraint is required because the constraints from Equation 5 do not assure, by themselves, that always when a cylinder is selected, its container will be chosen. The number of cylinders in the box was used as Big- $M$  to avoid numbers too big.

$$\arg \min_{c_i, x_k} 0 \quad (1)$$

Subject to:

$$\sum_{i \in \mathbb{A}} c_i = 35 \quad (2)$$

$$\sum_{k \in \mathbb{C}} v_k x_k = 5163.69 \quad (3)$$

$$\sum_{k \in \mathbb{C}} w_k x_k = 18844 \quad (4)$$

$$c_i \leq \sum_{k \in \mathbb{C}_j} x_k \quad \forall i, j \mid i \in \mathbb{A}, j \in \mathbb{B}_i \quad (5)$$

$$|\mathbb{C}_j| c_i \geq \sum_{k \in \mathbb{C}_j} x_k \quad \forall i, j \mid i \in \mathbb{A}, j \in \mathbb{B}_i \quad (6)$$

$$\sum_{k \in \mathbb{C}_j} x_k \leq 1 \quad \forall i, j \mid i \in \mathbb{A}, j \in \mathbb{B}_i \quad (7)$$

Equation 8 is added when a different optimal solution is needed after solving the problem. This constraint forces the solver to find a solution without at least one of the cylinders chosen in the last optimal solution. If multiple different solutions are needed, then this constraint

must be added multiple times, always based on the last optimal solution found. By doing so, every new optimal solution found will be unique.

$$\sum_{k' \in \mathbb{C}'} x_{k'} \leq |\mathbb{C}'| - 1 \quad (8)$$

The following section answers the questions asked in the challenge statement.

### 3 Answering the Questions

The challenge statement has two questions:

#### 3.1 Which containers, boxes and cylinders will you choose?

I will choose the first optimal solution found by the solver (file in Solutions/1.txt). This winner option is presented below. When a container is repeated, its name is not written. By doing so, it is easier to identify containers used more than once.

**Table 4:** Winner Option

Container	Box	Cylinder
AA	LB_1	1
	LB_2	1
AB	LB_1	1
	LB_2	1
AC	LB_1	16
AD	LB_1	1
	LB_2	1
AF	LB_1	14
AG	LB_1	1
	LB_2	1
AI	LB_1	7
	LB_2	1
AJ	LB_1	9
AK	LB_1	4
AL	LB_1	2
	LB_2	1
AM	LB_1	15
	LB_2	2
AN	LB_1	22
	LB_2	2
AO	LB_1	11
	LB_2	1
AQ	LB_1	1
A	LB_1	2

B	LB_1	4
	LB_2	2
C	LB_1	1
	LB_2	2
F	LB_1	8
G	LB_1	1
H	LB_1	2
	LB_2	1
I	LB_1	1
J	LB_1	4
	LB_2	1
K	LB_1	1
L	LB_1	1
M	LB_1	5
	LB_2	1
N	LB_1	13
O	LB_1	2
P	LB_1	4
Q	LB_1	4
R	LB_1	6
S	LB_1	7
T	LB_1	1
	LB_2	4
U	LB_1	4
	LB_2	2
X	LB_1	14
Y	LB_1	1
	LB_2	3

A total of 52 cylinders from 35 different containers were chosen in this winner option.

### 3.2 Is there more than one winner option?

Yes. The folder Solutions has other one hundred possible winner options found. Probably, there are much more different winner options.