Reinforcing mathematical intuition with the augmented reality serious game *Mathisfactory*

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Abstract. Level curves and directional derivatives are examples of mathematical methods employed in physics to describe and model many real-world phenomena. Unfortunately, they are often taught on a purely theoretical basis, without any visualization, thus failing to spark an intuition about their nature. As a result, students may struggle applying these methods to real-world engineering problems. Augmented reality (AR) has the potential to convey an intuitive understanding of complex mathematical concepts by visualizing abstract mathematical entities overlaid on real-world objects, thus linking real-world phenomena to abstract mathematical entities. We developed *Mathisfactory*, a novel AR serious game that challenges the player to interactively analyze the topographical properties of a physical volcano model, using the notions of level curves and directional derivatives. The game deploys a *planning and simulation* cycle over multiple levels of increasing difficulty. In the planning phase, the player interactively explores the volcano surface, to determine building locations that can stay safe during an eruption. For this, *Mathisfactory* uniquely employs AR by providing players with interactive visualizations of directional derivatives and level curves on the volcano surface. In the simulation phase, a volcanic eruption is simulated, augmenting the corresponding lava flow over the physical model, visually validating the player's building choices. Altogether, these visualizations give players an intuitive understanding of how surface topography interacts with fluid dynamics, allowing them to predict an eruption outcome.

Keywords: augmented reality · educational games · mathematical intuition · level curves · directional derivatives.

1 Introduction

Traditional higher education often focuses on teaching students mathematical concepts on a very theoretical basis. This focus partly stems from the need to teach a large number of students, while providing each one a theoretical basis. However, students often find it challenging to learn abstract concepts, and have difficulty linking mathematical models to real-world applications [2].

Augmented Reality (AR) has the potential to deepen student understanding of educational material, by complementing curriculum knowledge with interactive modalities. Moreover, this technology provides a virtual environment where one can learn in a situated manner, and bridge the gap between knowledge and real-world application. For example, Christian Moro et al. found that AR and VR were valuable for teaching anatomy on tablet devices, as they enhanced student motivation and 3D comprehension of anatomical structures and their functions [13].

In this paper, we explore the use of AR as a tool to promote student intuition towards the abstract mathematical notions of level curves and directional derivatives. The research question we aim to investigate with this paper is: *How can AR effectively help reinforce the mathematical concepts of level curves and directional derivatives in an intuitive and interactive way?* Ultimately, by interactively augmenting mathematical concepts on top of a real-world application, we investigate how the use of AR can enhance knowledge transfer of mathematics to engineering.

Level curves and directional derivatives are abstract mathematical concepts widely used in applied earth sciences and engineering, so we chose a geological context for our application. We created *Mathisfactory*, a serious game that requires players to analyze and predict the flow of lava in a volcanic situation, by applying their knowledge of level curves and directional derivatives. The game makes explicit use of interactive AR mechanics to help the player learn and intuitively understand how the different mathematical concepts relate to one another.

2 Related Work

We briefly, survey previous research related to visualization of mathematical concepts, AR in education and applied games in mathematics education.

2.1 Visualization of mathematical concepts

Visual representations have long been used to supplement the teaching of mathematical concepts to students of all ages [16]. For example, Yilmaz et al. investigated the use of visualizations in geometry teaching, and concluded that visualizations used during the abstraction process significantly aided in the understanding of the concept of congruent figures. Moreover, they conclude that visualizations not only develop mathematical student thought, but also aid in the discovery and reinforcement of relations between abstract mathematical objects [19].

Furthermore, visualization has been proven to have a close connection to mathematical intuition, a cognitive skill which helps students think and rationalize logically about mathematical problems [9]. In particular, the current generation of students have more heavily relied on constant visual stimuli, such as television or computer graphics, as a teaching method to better understand abstract or complicated concepts [3].

2.2 Augmented Reality in education

AR is an emerging technology that has shown success in helping students learn educational material through situated realistic interactions. Moro et al. [13] used AR goggles to teach anatomy to medical students as a substitute for real cadavers. In doing so, they found that visualizing the organs in 3D in combination with the ability to manipulate the organs helped students memorize and associate the organs with their respective functions. In a similar context, Ferrer-Torregrosa et al. [6] investigated the use of AR in the teaching of lower-limb anatomy. They compared two groups studying identical material, with one group being provided additional AR tools that visualize the organs in 3D. They concluded that the AR group comprehended three-dimensional tasks better with higher motivation. In the anatomical domain, AR was not only found to help students better understand the subject matter, but reinforced their spatial insight. As AR provided the students with a cleaner and more flexible experience, students could take the time to consider every aspect of the task at hand, and better relate the organs to their respective purpose and location.

Beyond the field of anatomy, Kirner et al. [11] found that visualizing in 3D gravitational forces between planets using AR helped students better perceive the rules of nature. Fjeld et al. [7] reported a similar improvement in student understanding of the geometry of chemical molecules.

AR is a very flexible technology, providing an environment for students to analyze and learn from their mistakes without serious real-world consequences [10]. Beyond the enthusiastic curiosity towards its ‘game-like technology’, the ability to visualize objects from multiple perspectives and manipulate them has the potential to improve spatial awareness and memory [1]. Moreover, AR provides students the ability to visualize and interact with objects, events or concepts which can hardly be seen in the real world without additional tools and/or precautions, such as a volcanic eruption, gravitational waves, galaxies, or quantum phenomena [4].

2.3 Learning math through games

The idea of using games to improve math teaching is not new. Ramani et al. [15] taught basic arithmetics to children using simple physical games, while Wiersum et al. [17] and Ormsby et al. [14] used board and video games to teach high-school and undergraduate students mathematical concepts such as statistics or calculus. More recently, Xu et al. [18] proposed the procedural generation of arithmetic problems in a gamified math learning context.

In the field of mathematics and engineering, much course material relies extensively on graphical information. Spatial visualization is therefore a key skill to develop. Xiaocen Liu et al. [12] found that video game training led to significant improvements in the mental rotation ability of young children, and Michel Dorval et al. [5] concluded that spatial visualization test scores could be significantly improved by video games in undergraduate students without prior video game experience. Delinda van Garderen et al. [8] found that the use of visual imagery was positively correlated with high mathematical word problem-solving performance, and that schematic imagery was positively correlated with higher spatial visualization performance. These research results together highlight that abstract mathematical problem solving skills and spatial visualization benefit from appropriate visualizations and repetitive training, something that can be achieved through a serious game.

To the best of our knowledge, no AR game has been proposed so far aimed at giving undergraduate students insight into the notions of directional derivatives and level curves. Since AR has the ability to provide 3D visualizations in a situated environment, we believe that using the technology could both improve student's spatial visualization and help them understand and 'experience' how directional derivatives help approach a real-world problem.

3 Game Design

We conceived *Mathisfactory* around a central design idea: reinforcing mathematical concepts by using AR on top of a physical volcano model. This section describes the core aspects of its design, game loop and main game mechanics, all designed with AR in mind.

3.1 Game Synopsis

In a geological setting, you acts as a governmental project manager in charge of choosing safe places for a number of research-oriented buildings on the surface of a volcano. Those locations have to be such that, in the event of an eruption, the damage to the buildings in minimized. Through interactive AR mechanics, you can visualize level curves on the surface of a volcano physical model, as well as probe possible locations for their directional derivatives, before deciding on appropriate placement for the buildings. Subsequently, a volcano eruption is simulated and augmented over the model, making clear which of the placed buildings were impacted by the lava stream. You then receive feedback on your score, and have the opportunity to investigate where and why your prediction may have failed.

3.2 Game mechanics

In order to allow time to reinforce mathematical intuition and understanding of directional derivatives and level curves, *Mathisfactory* is split into a *planning and simulation* cycle, which is repeated over five levels of increasing difficulty (see 1).

Planning phase The goal of the planning phase is to determine the safest locations for placing buildings on the volcano surface. AR is employed in this phase for analyzing the terrain and indicating the player's chosen locations. For this, the AR interactive tools provide topographical insights into the two mathematical concepts of directional derivatives and level curves. With directional derivative information, the player has the ability to visualize the directions of steepest descent at any point selected on the surface. Using this information, the player can infer the direction of the lava flow from that point downwards. With level curve information, the player can derive the steepness variations of the volcano surface. This information can be used to predict the speed of lava down the various regions of the volcano, and can be used in combination with the directional derivatives to fully predict where the lava will flow.

The player has the freedom to explore any possible building location on the volcano surface, as the duration of this phase is not limited. Alternating the focus between the observable surface of the physical volcano, and the augmented

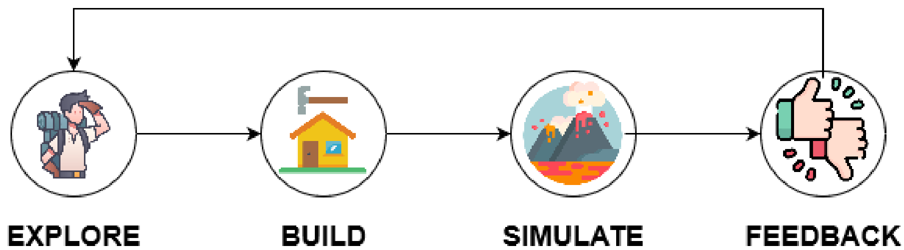


Fig. 1: Diagram of the core game loop. The Planning phase, includes the exploration and building steps; the simulation and feedback steps integrate the Simulation phase. At the end, if enough points have been earned, the player can proceed to the next level.

directional derivative and level curve information aims to reinforce the intuitive connection between the real surface and the mathematical concepts. At the end of this phase, the player should have placed all required buildings on the surface of the volcano.

Simulation Phase During the simulation phase, AR is deployed as an interactive feedback and learning tool, aimed at visually highlight possible disparities between the actual lava flow and its player predicted behavior. During the simulation phase, augmented lava flows down the volcano and collides with any building present in its path. Buildings which come into contact with lava get marked as destroyed, but are not removed, to help the player understand where things went wrong.

To gain better insight, the player has the ability to rewind and pause the flow of lava to focus on particular streams and moments of interest. During the paused time, the player has full access to the interactive AR tools to explore possible causes for any prediction divergence. Without time constraints, the player is encouraged to freely explore the individual streams of lava, (re)playing them to gain a better understanding of how terrain gradient and lava flow relate.

After the simulation phase, a final score is provided to the player, consisting of the score for each building affected by two multipliers: the *height multiplier*, dependent on the height at which the building was placed, and the *building multiplier*, conveying the importance of the specific building. These multipliers are shown to players to motivate them to risk placing buildings on higher ground during a retrial or subsequent level, as this typically requires a more in-depth inspection of the surface topography, and thus a better understanding of the mathematical concepts.

Mathisfactory is played over five levels of increasing difficulty, which repeatedly challenge players, reinforcing their intuition and insight. Each level has a threshold score, to gain access to the next level. This prevents players from placing buildings in ‘too safe’ locations where the lava is likely to never get close, and

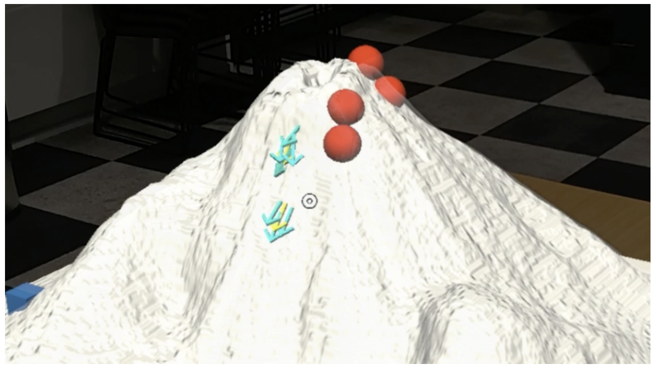


Fig. 2: Interactive visualization of directional derivatives: (yellow arrow) steepest descent at the point selected by the player; (blue arrows) represent the steepest descent around that point on the volcano surface.

forces them to investigate the surface of the volcano. A player who achieves the threshold score is deemed to have gained sufficient understanding and intuition.

3.3 AR-based analysis in a geological context

A core aspect of *Mathisfactory* is the interactive analysis of the terrain relief information through the use of AR. In this way, by allowing a hands-on exploration of directional derivatives and level curves, the game reinforces the player's understanding and mathematical intuition of these concepts [9].

AR for interactions Directional derivative and level curve information can be obtained via physical interactions with the 3D printed model. When touching an area on the volcano surface, vectors indicating derivative information appear around the selected area. These vectors individually point along the surface in the direction of the steepest descent, and remain there for a few seconds;

Fig. 3: Visualization of the level curves (top) with and (bottom) without the virtual volcano overlay. By moving around the 3D model, the player can gain an intuition on how the 2-dimensional level curves represent the 3-dimensional surface of the volcano.

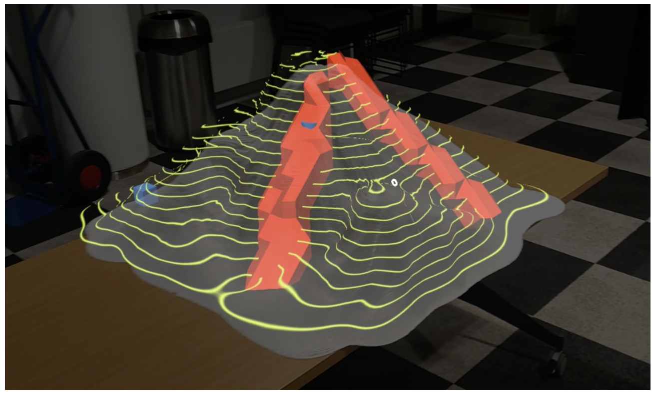


Fig. 4: Augmenting the flow of lava down the surface of the volcano.

see Figure 2. Multiple areas on the volcano can be selected at the same time, each individually disappearing independently. Additionally, level curves can be toggled to be augmented on the volcano surface, with the inter-ring height adjustable in real-time through the use of a slider; see Figure 3. The combination of these two AR interactions aim to reinforce the player's understanding that the shape of the volcano can be modelled using a derivative-based mathematical model. In tandem with the gameplay mechanics discussed in the previous section, these interactions lead the player to associate the different concepts together and reinforce the player's spatial visualization and mathematical intuition.

AR for Feedback Beyond providing interactions to help the player understand how the surface of a volcano can be mathematically modelled, AR is also used as a feedback mechanism. Using a simple gradient-based model, the flow of lava down the volcano is simulated and displayed using AR on the 3D model; see Figure 4. As the augmented lava flow is simulated, the player is led to intuitively better understand how the flow of a fluid relates to the surface it is flowing on.

4 Evaluation

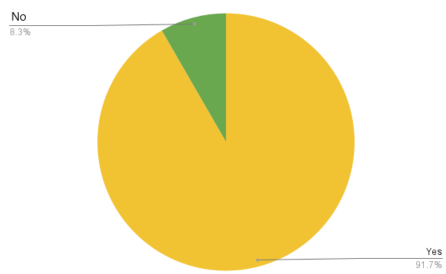
4.1 Method

In order to evaluate the extent to which the use of Augmented Reality was useful in enhancing the player's mathematical intuition, multiple play sessions were held with 12 freshman college students. Prior to each session, students were given a short explanation to get familiar with the concepts and functionality of the game.

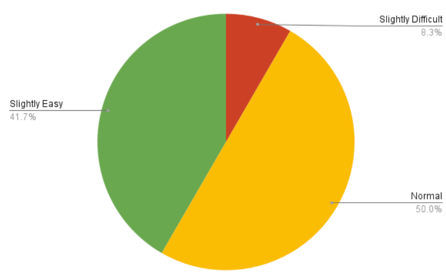
After playing the game, each student was filled in a survey, to debrief their play experience and feedback. The form consisted of both open and rating questions, aimed at assessing players prior experience with the Microsoft HoloLens, and determining whether AR had helped players better understand the mathematical notions involved.

4.2 Results and Discussion

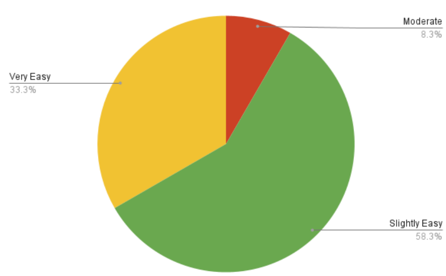
The first questions of the survey inquired about each players's familiarity with the Microsoft HoloLens. Out of the 12 participants, 11 reported having never interacted with it before; see Figure 5a. Also, the majority of the players found the HoloLens relatively easy to use, with only 1 individual finding it slightly difficult to use; see Figure 5b. During the debriefings, it was observed that most of the players found the HoloLens to be initially overwhelming, though all eventually got used to its unique experience, and found the game mechanics to be easy to use, as shown in Figure 5c.



(a) was this your first time using the HoloLens 2?

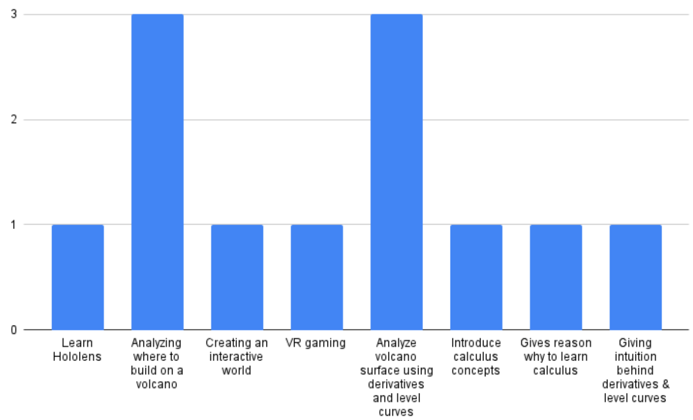


(b) How easy did you find using the HoloLens 2?

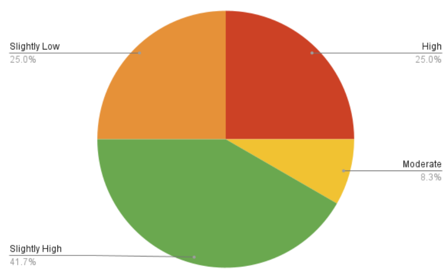


(c) How easy was it to pick up the mechanics of the game?

Fig. 5: Survey outcome about the HoloLens 2 deployment.



(a) What do you think this serious game is trying to achieve?

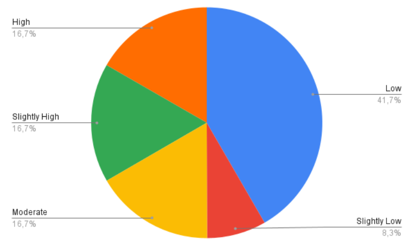


(b) To which extent did AR enhance the learning aspects of the game?

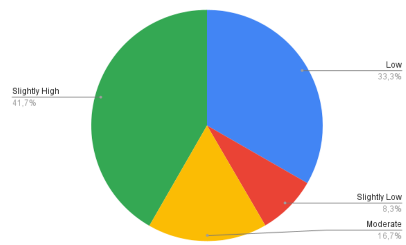
Fig. 6: Survey outcome about the game purpose.

Regarding the game purpose, we first asked players what they thought the game was about, before inquiring about the possible benefits of AR for that goal; see Figure 6. Student answers included using/learning directional derivatives and level curves, analyzing the surface of the volcano and learning calculus. Similarly, 66% of the students reported that AR (somehow) enhanced the learning aspects of the game, with only 3 students failing to recognize that.

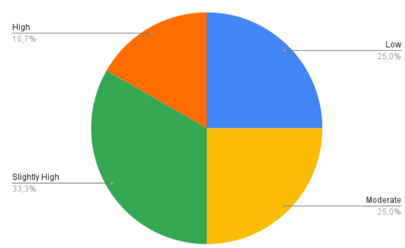
In terms of learning experience, 75% students found the game helpful in understanding level curves and their relation to a 3D volume (see Figure 7c). Around 50% of the students found the game to provide no substantial benefit towards understanding directional derivatives (see Figure 7a), and a mix of students found the game useful towards understanding fluid flow and its relation to the surface slope (see Figure 7b). However, a large majority of students (75%) found the use of the build, simulation feedback loop to be useful for focusing on the key mathematical concepts, and reported the interactivity to be engaging and helpful.



(a) To which extent did your understanding about directional derivatives improve?



(b) To which extent did your understanding about fluid flow improve?



(c) To which extent did your understanding about level curves improve?

Fig. 7: Survey outcome about perceived learning.

4.3 Limitations

Here we acknowledge some limitations present both in the conducted evaluation, and in the current game design. Firstly, the limited playtesting sample size may have an impact on the game evaluation. Moreover, due to time restrictions, this evaluation was exclusively focused on freshman students of the Earth Sciences faculty. A broader evaluation effort is recommended, in order to better identify both the specific topics the game is applicable to, and the age group that most benefits from its AR-based mechanics.

Regarding specifically the AR interaction, various students reported that the arrows representing the directional derivatives were difficult to visualize, and provided valuable improvement suggestions. It was also observed that the game menus were often lost when a player moved around the physical model, causing a momentary disconnect from the game. To prevent this, one could try to make all menus follow the player without needing to drag them to where you are standing. Some of these improvements, however, are difficult to implement on the current HoloLens 2 due to its technical limitations (e.g. range of gestures, field of view). We believe that as the technology behind the HoloLens further develops, a much better experience will be provided to the players of *Mathisfactory*.

5 Conclusion

Many students have trouble applying the mathematical notions of level curves and directional derivatives in real-world problems. We designed and developed *Mathisfactory*, a novel serious game that makes use of Augmented Reality (AR) to help students better understand these concepts. The game is based on exploratory gameplay, augmented on a 3D physical model of a volcano. The game cycle combines a *planning phase*, in which the player analyses the volcano surface to choose safe building locations, with a *simulation phase*, in which a volcanic eruption is simulated, augmenting the lava flow over the physical model.

In both phases, AR interactive tools help the player intuitively understand how the topography of a volcano can be represented and interpreted using mathematical concepts. By pausing and rewinding the lava flow during the simulation phase, the player is encouraged to explore and understand how their prediction deviated from the actual flow of lava.

Evaluation of the game has shown that this game loop is useful in keeping students engaged and focused on the mathematical methods of interest. However, evaluation has also indicated that *Mathisfactory* had a limited pedagogical benefit on the current playtest students: while the visualization and utility of level curves was found to be clear and intuitive, the perception and interactivity of directional derivatives was found to be less beneficial.

We believe that AR has the potential to help students develop their mathematical intuition on abstract math concepts other than directional derivatives and level curves. It would therefore be interesting to extend *Mathisfactory* to other real-world scenarios and dilemmas, in which different mathematical concepts could benefit from its augmented visualization and interactivity facilities.

References

1. Band, R., Lips, M., Prawira, J., Schagen, J.v., Tulling, S., Zhang, Y., Benaiss, A.A., Ham, I.J.v.d., Bueno, M., Bidarra, R.: Training and assessing perspective taking through a hole new perspective. In: Proceedings of CoG 2022 - IEEE Conference on Games. IEEE (Jun 2022)
2. Crouch, R., Haines, C.: Mathematical modelling: transitions between the real world and the mathematical model. International Journal of Mathematical Education in Science and Technology **35**(2), 197–206 (2004). https://doi.org/10.1080/00207390310001638322
3. Dianne Hansford, G.E.F.: Volume visualization. Mathematical Principles for Scientific Computing and Visualization p. 223–246 (2008). https://doi.org/10.1201/b10694-17
4. Dorland, B., van Hal, L., Lageweg, S., Mulder, J., Schreuder, R., Zaidi, A., Alderliesten, J.W.D., Bidarra, R.: Quantum physics vs. classical physics: introducing the basics with a virtual reality game. In: Proceedings of the Games and Learning

Alliance conference (GaLA 2019), Athens, Greece. pp. 383–393. No. LNCS 11899, Springer (2019)
5. Dorval, M., Pépin, M.: Effect of playing a video game on a measure of spatial visualization. Perceptual and Motor Skills **62**(1), 159–162 (1986). https://doi.org/10.2466/pms.1986.62.1.159, pMID: 3960656
6. Ferrer-Torregrosa, J., Torralba, J., Jimenez, M.A., García, S., Barcia, J.M.: Arbook: Development and assessment of a tool based on augmented reality for anatomy. Journal of Science Education and Technology **24**(1), 119–124 (2014). https://doi.org/10.1007/s10956-014-9526-4
7. Fjeld, M., Voegtli, B.M.: Augmented chemistry: An interactive educational workbench. In: Proceedings. International Symposium on Mixed and Augmented Reality. pp. 259–321. IEEE (2002)
8. van Garderen, D.: Spatial visualization, visual imagery, and mathematical problem solving of students with varying abilities. Journal of Learning Disabilities **39**(6), 496–506 (2006). https://doi.org/10.1177/00222194060390060201, pMID: 17165617
9. Giardino, V.: Intuition and visualization in mathematical problem solving. Topoi **29**(1), 29–39 (2010)
10. Kegeleers, M., Miglani, S., Reichert, G.M.W., Salamon, N.Z., Balint, J.T., Lukosch, S.G., Bidarra, R.: Star: Superhuman training in augmented reality. In: Proceedings of the First Superhuman Sports Design Challenge: First International Symposium on Amplifying Capabilities and Competing in Mixed Realities. pp. 7:1–7:6. No. 7 in SHS 18, ACM (jul 2018)
11. Kirner, T.G., Reis, F.M.V., Kirner, C.: Development of an interactive book with augmented reality for teaching and learning geometric shapes. In: 7th Iberian Conference on Information Systems and Technologies (CISTI 2012). pp. 1–6 (2012)
12. Liu, X., Huang, H., Yu, K., Dou, D.: Can video game training improve the two-dimensional mental rotation ability of young children? In: Fang, X. (ed.) HCI in Games. pp. 305–317. Springer International Publishing, Cham (2020)
13. Moro, C., Štromberga, Z., Raikos, A., Stirling, A.: The effectiveness of virtual and augmented reality in health sciences and medical anatomy. Anatomical Sciences Education **10**(6), 549–559 (2017). https://doi.org/https://doi.org/10.1002/ase.1696
14. Ormsby, R., Daniel, R., Ormsby, M.: Preparing for the future with games for learning: Using video games and simulations to engage students in science, technology, engineering, and math. Astropolitics **9**(2-3), 150–164 (2011). https://doi.org/10.1080/14777622.2011.625924
15. Ramani, G.B., Eason, S.H.: It all adds up: Learning early math through play and games. Phi Delta Kappan **96**(8), 27–32 (2015). https://doi.org/10.1177/0031721715583959
16. Shabiralyani, G., Hasan, K., Hamad, N., Iqbal, N.: Impact of visual aids in enhancing the learning process case research: District dera ghazi khan. Journal of Education and Practice **6**, 226–233 (2015)
17. Wiersum, E.G.: Teaching and learning mathematics through games and activities. Acta Electrotechnica et Informatica **12**(3), 23–26 (2012)
18. Xu, Y., Smeets, R., Bidarra, R.: Procedural generation of problems for elementary math education. International Journal of Serious Games **8**(2), 49 – 65 (jun 2021). https://doi.org/10.17083/ijsg.v8i2.396, `http://graphics.tudelft.nl/Publications-new/2021/XSB21`, https://doi.org/10.17083/ijsg.v8i2.396
19. Yilmaz, R., Argun, Z.: Role of visualization in mathematical abstraction: The case of congruence concept. International Journal of Education in Mathematics, Science and Technology **6**, 1–1 (07 2017). https://doi.org/10.18404/ijemst.328337