> $A := Matrix([[0,-2,0],[1,-2,0],[0,0,-2]]);$

$$A := \begin{bmatrix} 0 & -2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad (1)$$

> $with(Student[LinearAlgebra]):$
> $with(LinearAlgebra): with(linalg):$

> $detA := det(A);$

$$detA := -4 \quad (2)$$

> $invA := A^{-1}$

$$invA := \begin{bmatrix} -1 & 1 & 0 \\ -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \quad (3)$$

> $cp := CharacteristicPolynomial(A, r);$

$$cp := r^3 + 4r^2 + 6r + 4 \quad (4)$$

> $eg := Eigenvectors(A)$

$$eg := \begin{bmatrix} -2 \\ -1+\mathrm{I} \\ -1-\mathrm{I} \end{bmatrix}, \begin{bmatrix} 0 & 1+\mathrm{I} & 1-\mathrm{I} \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad (5)$$

> $ev := Eigenvalues(A)$

$$ev := \begin{bmatrix} -2 \\ -1-\mathrm{I} \\ -1+\mathrm{I} \end{bmatrix} \quad (6)$$

> $u1 := \langle 0, 0, 1 \rangle$

$$u1 := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (7)$$

> $Av := A \cdot u1$

$$Av := \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \quad (8)$$

> $u2 := \langle 1 + I, 1, 0 \rangle$

(9)

$$u2 := \begin{bmatrix} 1+I \\ 1 \\ 0 \end{bmatrix} \quad \textbf{(9)}$$

> $Av2 := A \cdot u2$

$$Av2 := \begin{bmatrix} -2 \\ -1+I \\ 0 \end{bmatrix} \quad \textbf{(10)}$$

> $u3 := \langle 1 - I, 1, 0 \rangle$

$$u3 := \begin{bmatrix} 1-I \\ 1 \\ 0 \end{bmatrix} \quad \textbf{(11)}$$

> $Av3 := A \cdot u3$

$$Av3 := \begin{bmatrix} -2 \\ -1-I \\ 0 \end{bmatrix} \quad \textbf{(12)}$$

> $(-1 - I) \cdot u3$

$$\begin{bmatrix} -2 \\ -1-I \\ 0 \end{bmatrix} \quad \textbf{(13)}$$

> $P := Matrix([\langle u1 \rangle, \langle u2 \rangle, \langle u3 \rangle]);$

$$P := \begin{bmatrix} 0 & 1+I & 1-I \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \textbf{(14)}$$

> $J := Matrix([[-2, 0, 0], [0, -1 + I, 0], [0, 0, -1 - I]]);$

$$J := \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1+I & 0 \\ 0 & 0 & -1-I \end{bmatrix} \quad \textbf{(15)}$$

> $A - P \cdot J \cdot P^{-1}$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \textbf{(16)}$$

>

> $ej := MatrixExponential(t \cdot J)$

**(17)**

$$ej := \begin{bmatrix} e^{-2t} & 0 & 0 \\ 0 & e^{-t}\cos(t) + I\, e^{-t}\sin(t) & 0 \\ 0 & 0 & e^{-t}\cos(t) - I\, e^{-t}\sin(t) \end{bmatrix} \quad (17)$$

> $ea := MatrixExponential(t \cdot A)$

$$ea := \begin{bmatrix} e^{-t}\cos(t) + e^{-t}\sin(t) & -2\, e^{-t}\sin(t) & 0 \\ e^{-t}\sin(t) & e^{-t}\cos(t) - e^{-t}\sin(t) & 0 \\ 0 & 0 & e^{-2t} \end{bmatrix} \quad (18)$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (19)$$

> $Map(limit, ea, t = \text{infinity});$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (20)$$

> $eq := diff(x(t), t) = 1 - x^2(t)$

$$eq := \frac{d}{dt}\, x(t) = 1 - x(t)^2 \quad (21)$$

> $f1 := rhs(\, dsolve(\{eq, x(0) = 1\}, x(t)));$

$$f1 := 1 \quad (22)$$

> $f2 := rhs(\, dsolve(\{eq, x(0) = -1\}, x(t)));$

$$f2 := -1 \quad (23)$$

> $f3 := rhs(\, dsolve(\{eq, x(0) = -2\}, x(t)));$

$$f3 := \coth\left(t - \operatorname{arctanh}\left(\frac{1}{2}\right)\right) \quad (24)$$

> $f4 := rhs(\, dsolve(\{eq, x(0) = 0\}, x(t)));$

$$f4 := \tanh(t) \quad (25)$$

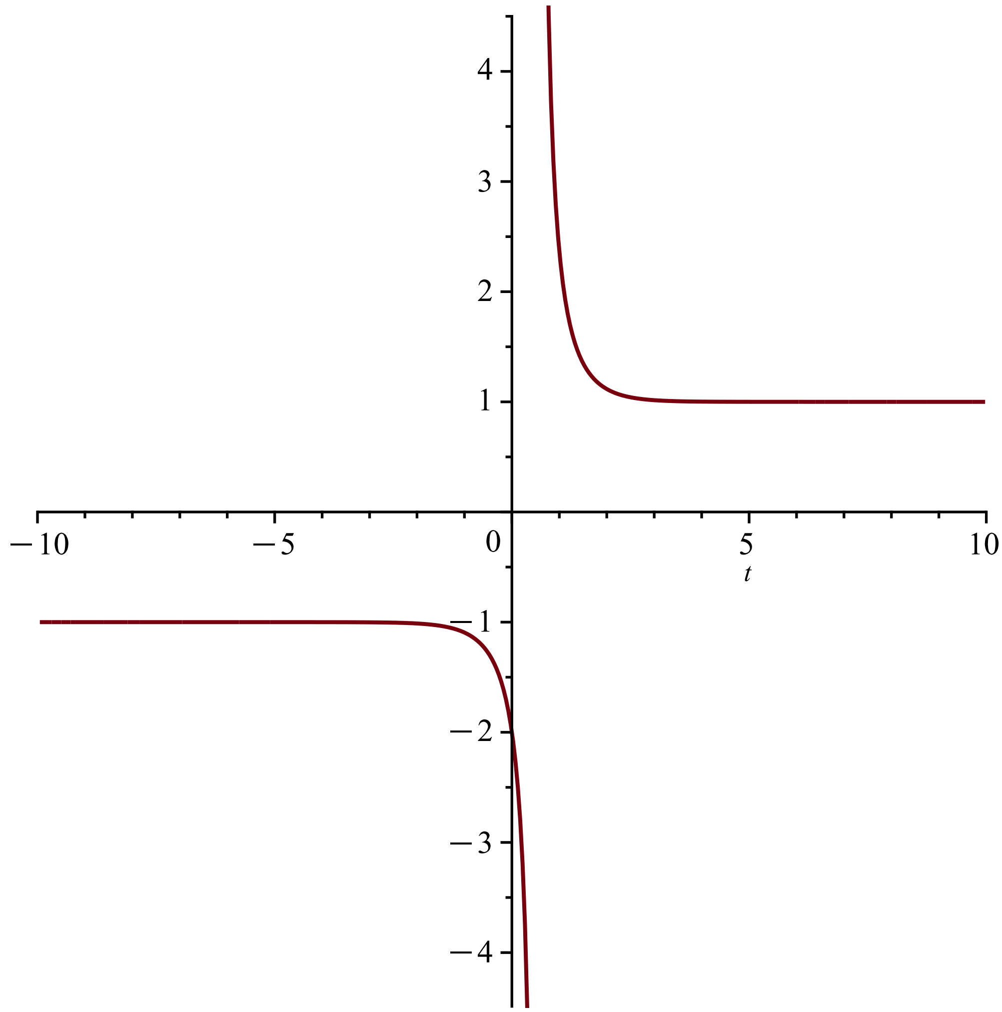
> $f5 := rhs(\, dsolve(\{eq, x(0) = 2\}, x(t)));$

$$f5 := \coth\left(\operatorname{arctanh}\left(\frac{1}{2}\right) + t\right) \quad (26)$$

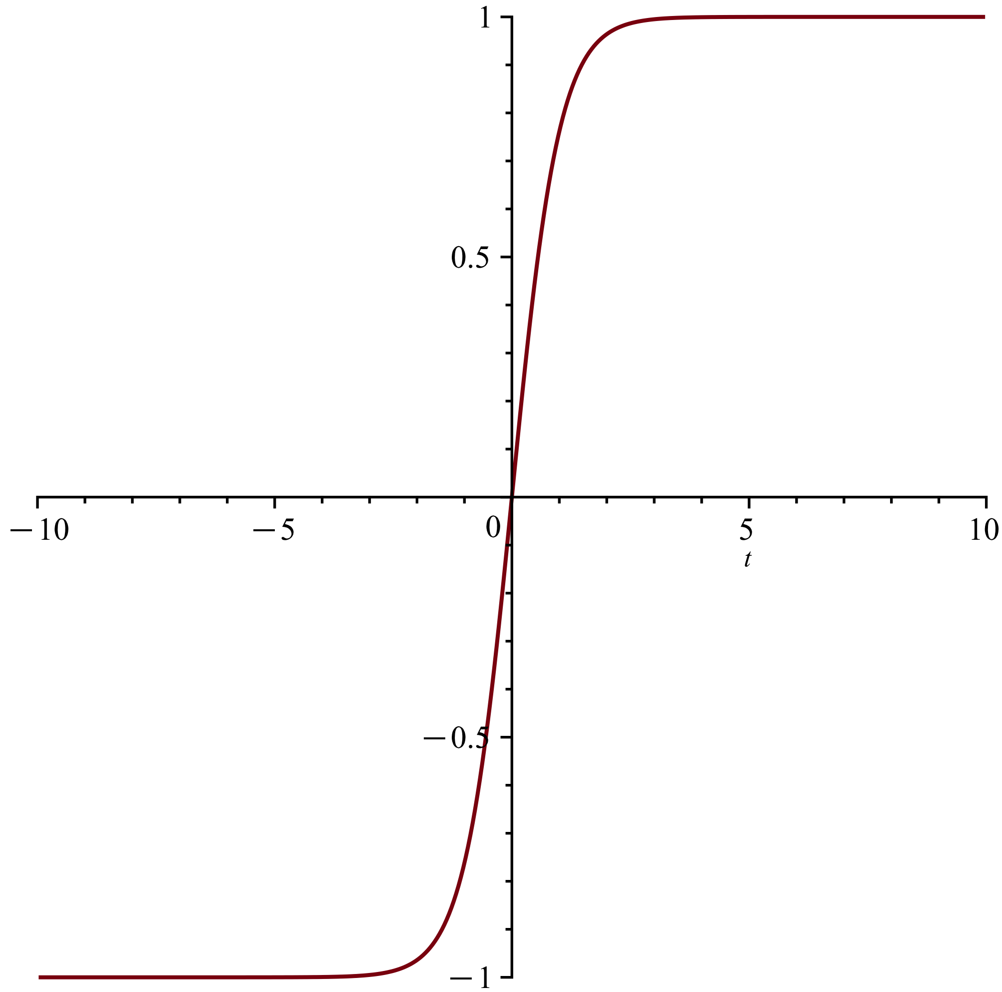
> $convert(convert(f5, \exp), \exp);$

$$\frac{3\, e^{2t} + 1}{3\, e^{2t} - 1} \quad (27)$$

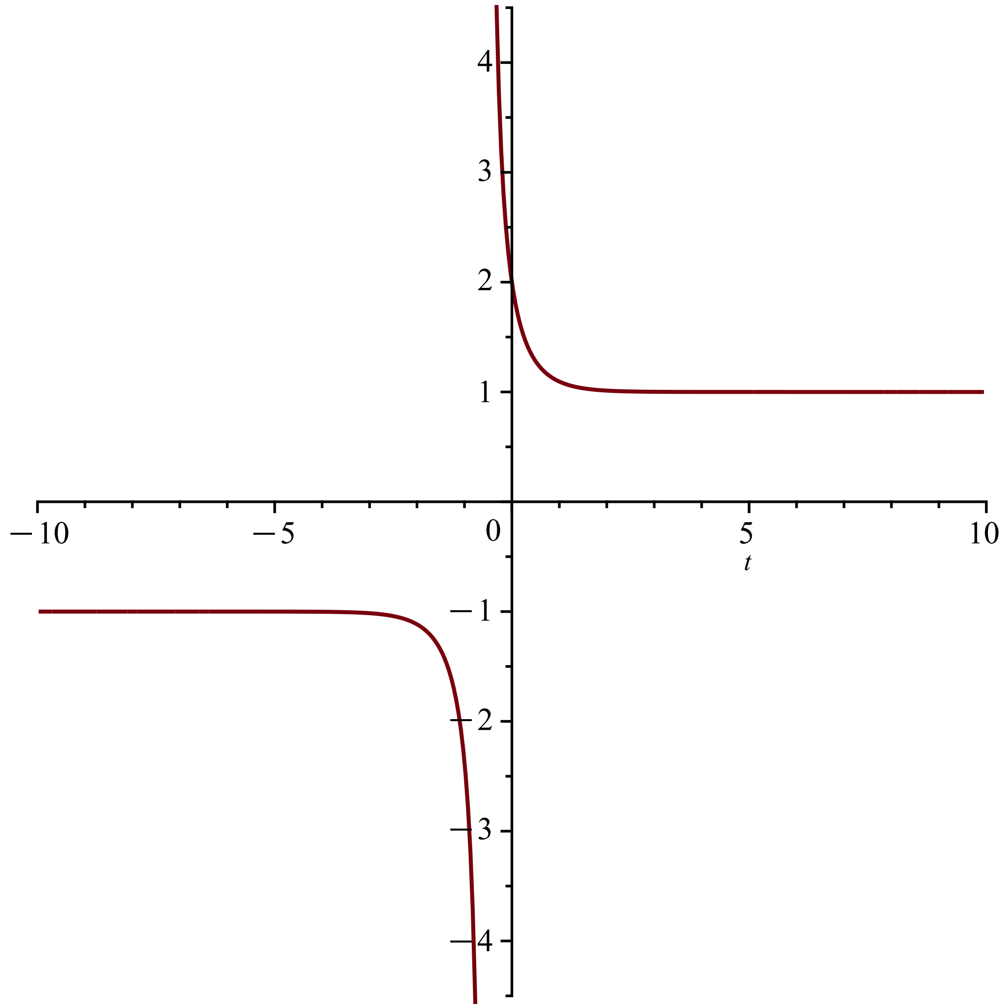
> $plot(f3)$




> $plot(f4)$




> $plot(f5)$




> $l3 := limit(f3, t = \text{infinity});$

$$l3 := 1 \tag{28}$$

> $l4 := limit(f4, t = \text{infinity});$

$$l4 := 1 \tag{29}$$

> $l5 := limit(f5, t = \text{infinity});$

$$l5 := 1 \tag{30}$$

>