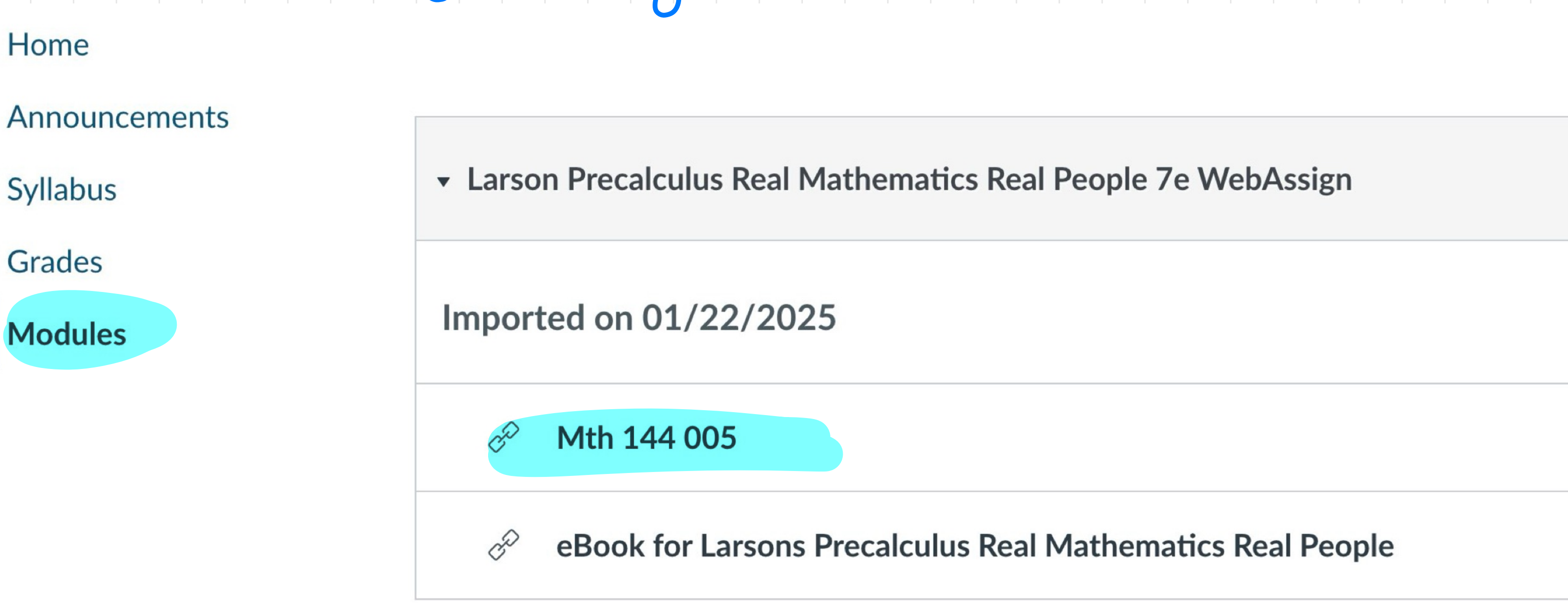
Day 2, Monday Jan 27th

- HW 1 Section 1.2 Functions (WebAssign) due Wed

  Click on Modules tab then MTH 144 005 to register for web Assign.



- Individual Quiz on Functions and their Domains on Wed
- Next Student Hours Wed 10-12pm
- HW 2 will be posted after class today

3. Evaluating a Function

Evaluating a function at a specific input value means plug in that specific value in the formula to get your output.

Example 4. Let $f(x) = 2x^2 + 3x + 1$. Evaluate

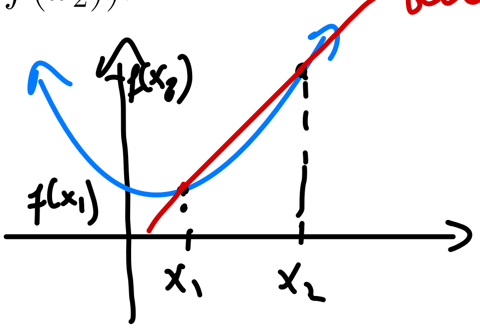
1. $f(2) = 2(2)^2 + 3(2) + 1 = 2 \cdot 4 + 6 + 1 = 15$
2. $f(-3) = 2(-3)^2 + 3(-3) + 1 = 2 \cdot 9 - 9 + 1 = 10$

$f(\star) = 2\star^2 + 3\star + 1$

4. Difference Quotient

The difference quotient represents the average rate of change of a function between two values $x_1 = x$ and $x_2 = x + h$.

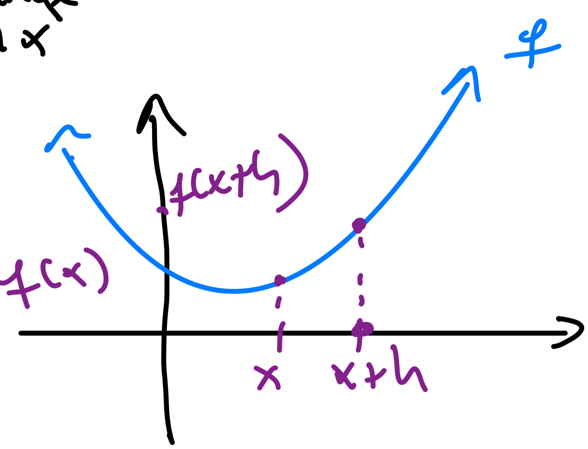
Recall that the Average rate of change of a function f between two points x_1 and x_2 is just the slope of the line going through the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$.



Average rate of change = slope of secant line

$$\frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Formula:



$$\text{Difference quotient} = \frac{f(x+h) - f(x)}{\cancel{x} + h - \cancel{x}}$$

$$\text{difference quotient} = \frac{f(x+h) - f(x)}{h}, \quad h \neq 0$$

$h \neq 0$ (small positive number)

Example 5. Find the difference quotient for $\boxed{f(x) = 2x^2 + 3x + 1.}$

goal: cancel h

$$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2+3(x+h)+1-(2x^2+3x+1)}{h}$$

$$= \frac{2(x^2+2xh+h^2)+3x+3h+1-2x^2-3x-1}{h}$$

$$= \frac{\cancel{2x^2}+4xh+2h^2\cancel{+3x}+3h\cancel{+1}\cancel{-2x^2}\cancel{-3x}\cancel{-1}}{h}$$

$$= \frac{4xh+2h^2+3h}{h} = \frac{\cancel{h}(4x+2h+3)}{\cancel{h}} = 4x+3+2h$$

$(a-b)^2 \neq a^2-b^2$

$(a-b)(a-b)$
$= a^2-2ab+b^2$

Example 6. Find the difference quotient for $\boxed{f(x) = \frac{1}{x+1}.}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{(x+h+1)}\frac{(x+1)}{(x+1)} - \frac{1}{(x+1)}\frac{(x+h+1)}{(x+h+1)}}{h} = \frac{\frac{\cancel{x}\cancel{+1}\cancel{-x}-h\cancel{-1}}{(x+h+1)(x+1)}}{h}$$

$$= \frac{\frac{-h}{(x+h+1)(x+1)}}{\frac{h}{1}}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b}\cdot\frac{d}{c}$$

$$= \frac{-\cancel{h}}{(x+h+1)(x+1)}\cdot\frac{1}{\cancel{h}} = \frac{-1}{(x+h+1)(x+1)}$$

! $-\frac{1}{(x+1)^2}$

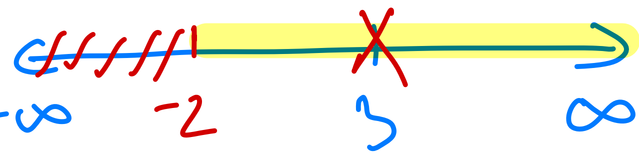
Practice 1: Find the domain of functions:

$\sqrt{-8} = -2$

$(-2)^3 = (-2)(-2)(-2)$
$= -8$

1. $f(x) = \dfrac{\sqrt{2+x}}{3-x}$

Domain:



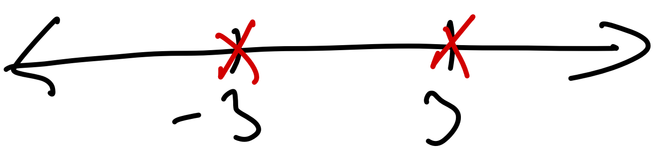
$[-2, 3) \cup (3, \infty)$

2. $g(x) = \sqrt[3]{x-1}$

$(-\infty, \infty)$

3. $h(x) = \dfrac{x^4}{(x-3)(x+3)}$

Factoring

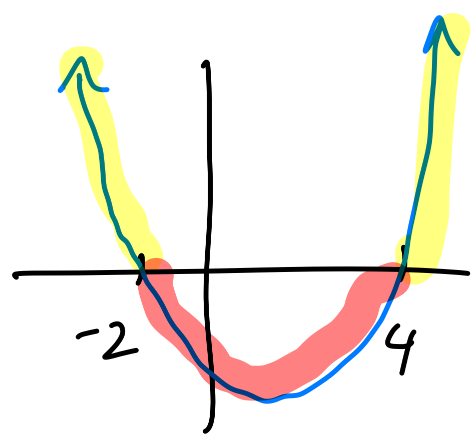


Ans: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

4. $f(t) = \sqrt{t^2 - 2t - 8}$

$t^2 - 2t - 8 = (t-4)(t+2)$

Domain $(-\infty, -2] \cup [4, \infty)$



Practice 2: Evaluate (if possible) the function $g(x) = \dfrac{1-x}{1+x}$ at:

1. $g(2) = \dfrac{1-2}{1+2} = -1/3$

2. $g(-1) = \dfrac{1-(-1)}{1+(-1)}$ 0

not possible

3. $g(a-1) = \dfrac{1-(a-1)}{1+a-1} = \dfrac{2-a}{a}$

4. $g(x^2-1) = \dfrac{1-(x^2-1)}{1+x^2-1} = \dfrac{2-x^2}{x^2}$

Practice 3: Find the difference quotient for $f(x) = x^2 + 2x - 3$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+2(x+h)-3-(x^2+2x-3)}{h}$$

$$= \frac{\cancel{x^2}+2xh+h^2+\cancel{2x}+2h\cancel{-3}\cancel{-x^2}\cancel{-2x}\cancel{+3}}{h}$$

$$= \frac{2xh+h^2+2h}{h} = \frac{\cancel{h}(2x+h+2)}{\cancel{h}} = \boxed{2x+2+h}$$

Practice 4: Evaluate the piecewise defined function at the indicated values:

$$f(x) = \begin{cases} x^2+2x & x \le -1 \\ x & -1 < x \le 1 \\ -1 & 1 < x \end{cases}$$

1. $f(-4) = (-4)^2+2(-4) = 8$

2. $f(25) = -1$

3. $f(-1) = (-1)^2+2(-1) = 1-2 = -1$

4. $f(0) = 0$

Day 2 Notes
Monday Jan 27th
1.3 Graphs of Functions

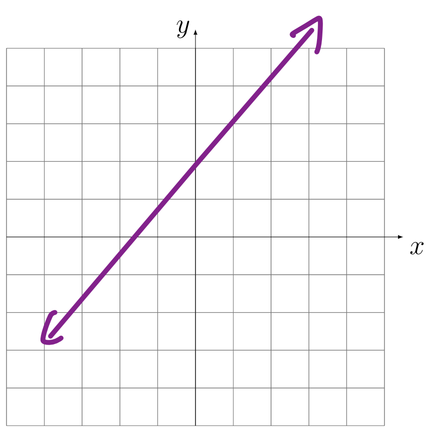
1. Graphs of Functions: Introduction

The most important way to visualize a function is through its graph. In this section we investigate in more detail the concept of graphing functions.

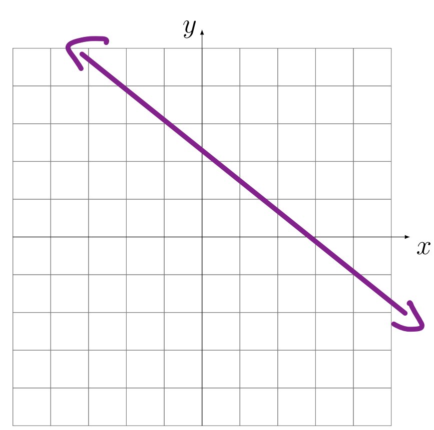
To graph of a function f, we plot the points (x, y) in a coordinate plane where the x coordinate represents an input and the y coordinate is the corresponding output of the function, $y = f(x)$.

Examples of some functions and their graphs.

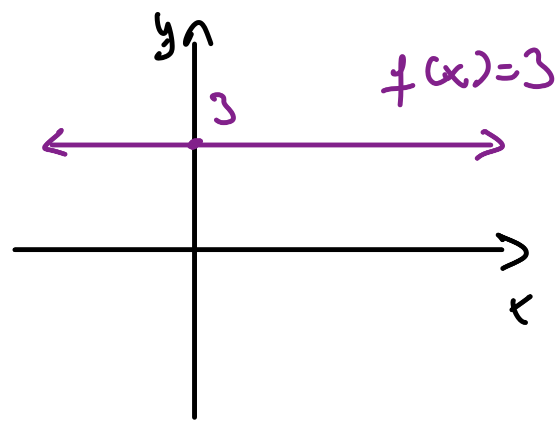
Linear Functions $f(x) = mx + b$, m = slope , b : y intercept



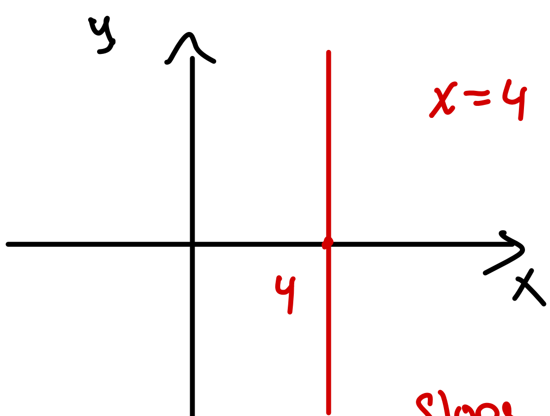
$m > 0$



$m < 0$



$m = 0$



vertical line not a linear function

slope undefined

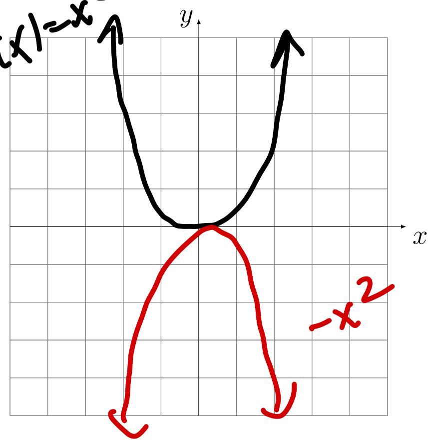
Power Functions - Positive Exponents

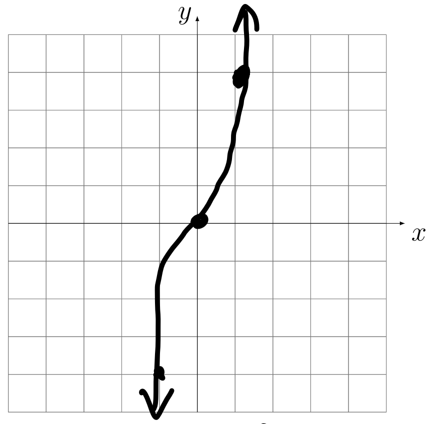
integer

$f(x) = k\,x^p$

p - positive integer

$f(x) = -x^2$





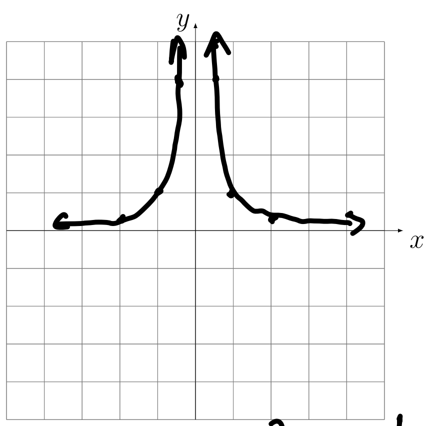
x	$g(x)$
-1	-4
0	0
1	4

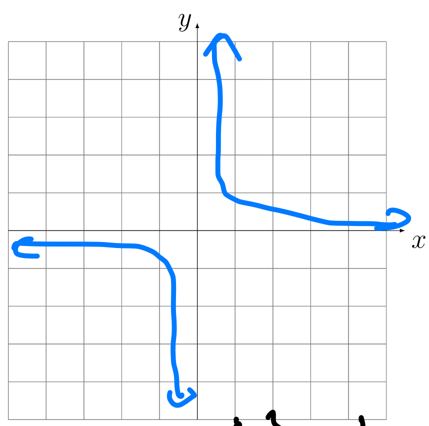
$f(x) = x^2$
$= 1 \cdot x^2$

$g(x) = 4x^3$

Power Functions - Negative Exponents

$f(x) = k\,x^p$



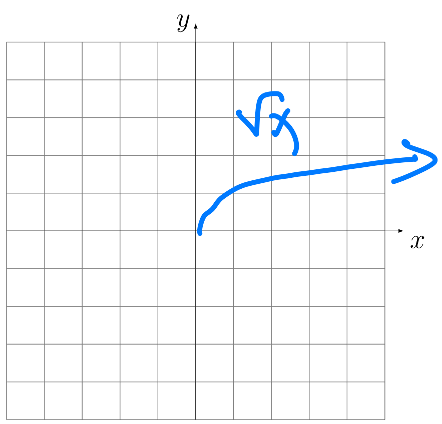


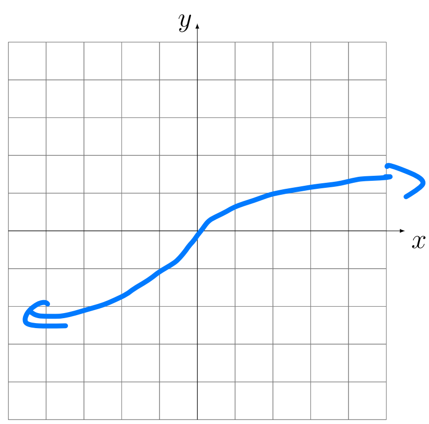
$f(x) = x^{-2} = \frac{1}{x^2}$

$g(x) = x^{-3} = \frac{1}{x^3}$

x	$f(x)$
-2	$(-2)^{-2} = \frac{1}{(-2)^2} = \frac{1}{4}$
-1	$\frac{1}{(-1)^2} = 1$
0	10
1	1
2	$\frac{1}{4}$
$-1/2$	$\frac{1}{\left(\frac{-1}{2}\right)^2} = \frac{1}{1/4} = 4$
$1/2$	4

Root Functions





$\sqrt[3]{x}$

2. Graphing Piecewise defined functions

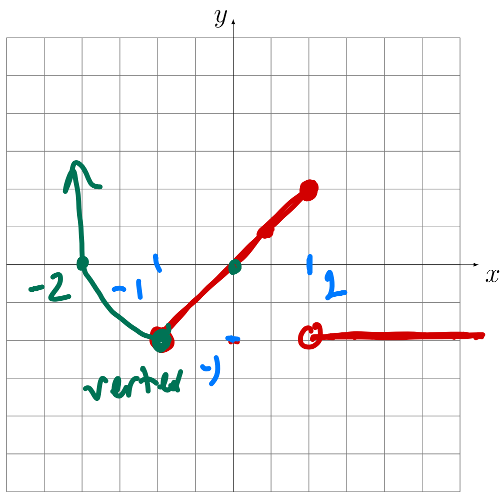
Graph the function

$$f(x) = \begin{cases} x^2 + 2x & x \leq -1 \\ x & -1 < x \leq 1 \\ -1 & 1 < x \end{cases}$$

parabola

line $m=1, b=0$

hor. line $m=0$, $b=-1$



Steps to graph a piecewise function:

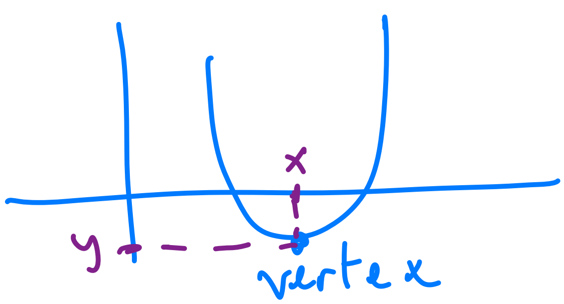
1. Graph each piece on its given domain
2. Use solid dots (•) for included endpoints
3. Use open dots (○) for excluded endpoints
4. Check that the pieces connect as specified

Vertex $(-1,-1)$

parabola up : $x^2+2x = x(x+2)$

$x=0$ $\quad$ $x=-2$

vertex :



x coordinate $x = -\frac{b}{2a} = \frac{-2}{2(1)}$ $\quad$ ax^2+bx+c

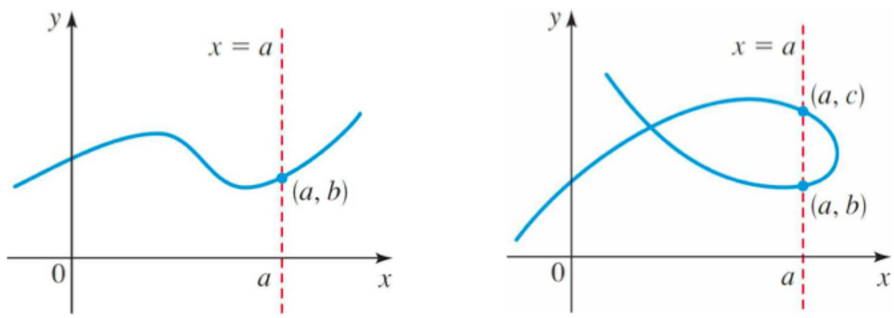
x^2+2x

$a=1, \ b=2, \ c=0$

y coord $f(-1)$ $x=-1$,

3. Function or not a function?

Given a graph, one can decide if the graph represents a function if the graph passes the **vertical line test**: A curve in the xy-plane represents a function if and only if no vertical intersects the curve more than once.



Graph of a function

Not a graph of a function

Vertical Line Test

Given the equation of the relationship between x and y, one can decide if y represents a function of x by solving for y and seeing if each x value has exactly one y value assigned to it.

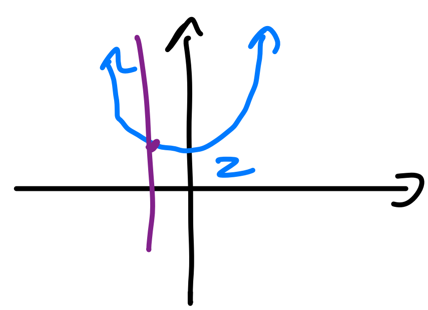
Examples: Does the equation define y as a function of x?

1. $y – x^2 = 2$

$y = 2 + x^2$

$x = -4$, $y = 2 + (-4)^2 = 18$

$x = 4$, $y = 2 + 4^2 = 18$



2. $x^2 + y^2 = 4$

$-x^2 \quad -x^2$

$y^2 = 4 - x^2$ | square root both sides

$\cancel{\sqrt{y^2}} = \pm\sqrt{4 - x^2}$

$y = \pm\sqrt{4 - x^2}$

NO!

ex: $x = 2$

$y = \pm\sqrt{4 - 2^2} = \pm\sqrt{0} = 0$

$x = 1$

$y = \pm\sqrt{4 - 1^2} = \pm\sqrt{3}$

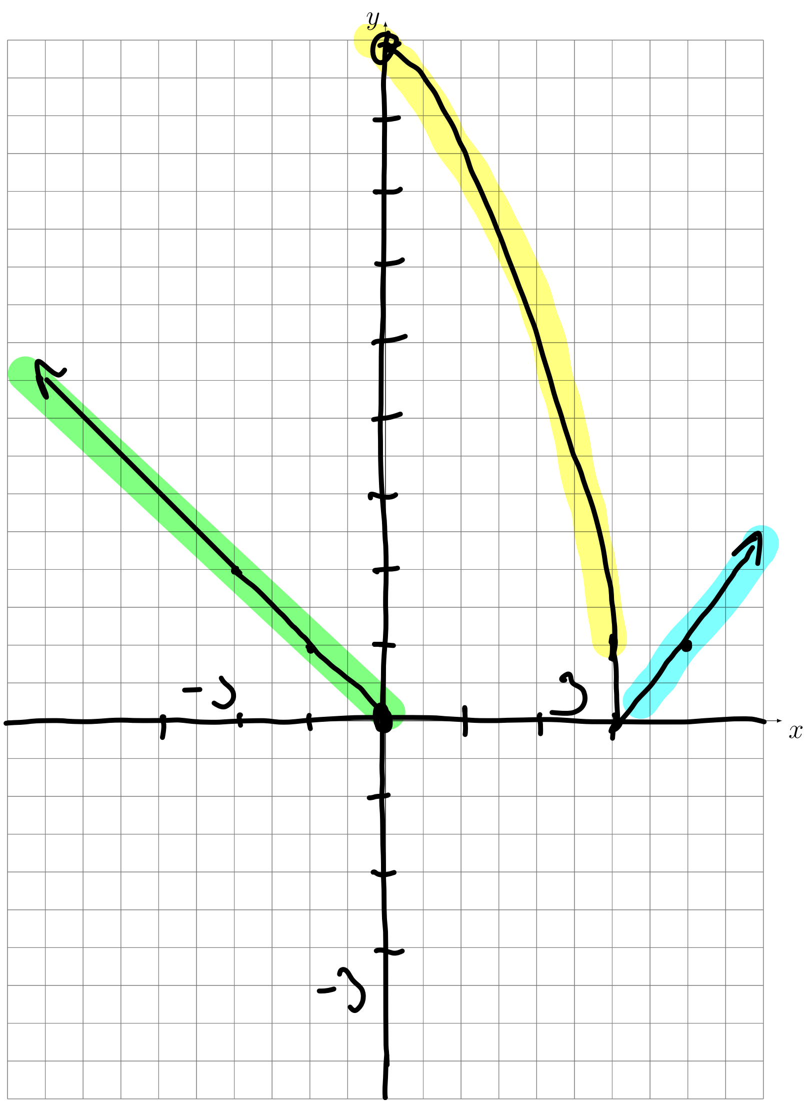
Practice: Graph the following function by plotting points.

$$f(x) = \begin{cases} -x & x \leq 0 \\ 9 - x^2 & 0 < x \leq 3 \\ x - 3 & 3 < x \end{cases}$$

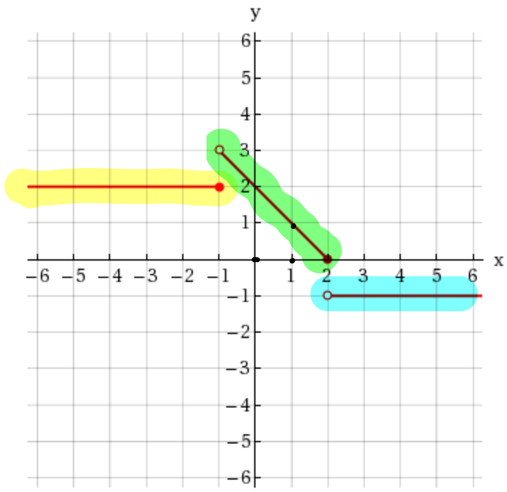
line : m = -1, b = 0

parabola down

line : m = 1, b = -3



Practice: Find a formula for the function and state its domain and range.



$$f(x) = \begin{cases} 2, & x \le -1 \\ -x+2, & -1 < x \le 2 \\ -1, & x > 2 \end{cases}$$