Mth 122: Group Quiz 1 Group Names & group #: Key

- The following exercises must be completed as a group and turned in at the end of class. Everyone has to write the answers and turn in their quiz. I will randomly select one of the quizzes to grade, and everyone in the group gets the same grade.

1. Find the general term a_n for each sequence:

   (a) $2, 6, 18, 54, 162, \ldots$

   a_1

   $n=1 \quad a_1 = 2\cdot 3^0 \qquad a_3 = 2\cdot 3^2$

   $n=2 \quad a_2 = 2\cdot 3^1 \qquad a_4 = 2\cdot 3^3$

   so $\boxed{a_n = 2\cdot 3^{n-1}}$

   (b) $\frac{5}{4}, \frac{10}{9}, \frac{15}{16}, \frac{20}{25}, \ldots$

   $\frac{5\cdot 1}{2^2}, \frac{5\cdot 2}{3^2}, \frac{5\cdot 3}{4^2}, \frac{5\cdot 4}{5^2}, \ldots$

   $n=1 \quad n=2 \quad n=3 \quad n=4$

   $a_n = \frac{5n}{(n+1)^2}$

2. Simplify the following factorial expressions:

   (a) $\frac{7!}{5!} = \frac{5!\,6\cdot 7}{5!} = 42$

   (b) $\frac{(n+2)!}{n!} = \frac{\cancel{n!}(n+1)(n+2)}{\cancel{n!}} = (n+1)(n+2)$

3. Find the general term a_n for the sequence:

   use $a_n = a + (n-1)d$

   $3, 6, 9, 12, 15, \ldots$

   arithmetic $a=3$, $d=3$

   $\boxed{a_n = 3 + (n-1)3}$

4. Write a recursive formula for each sequence:

   (a) $2, 4, 8, 16, 32, \ldots$

   $a_1 = 2$, $a_2 = a_1\cdot 2$, $a_3 = a_2\cdot 2$

   $a_n = a_{n-1}\cdot 2$

   (b) $-3, -5, -11, -29, -83, \ldots$

Bonus:

$a_1 = -3$

$\boxed{a_n = 3a_{n-1} + 4}$

check $a_2 = 3(-3) + 4$

✓ $a_2 = -9 + 4 = -5$

✓ $a_3 = 3a_2 + 4 = 3(-5) + 4 = -11$

5. Determine if each sequence is arithmetic. If it is, find the common difference.

(a) $-2, 1, 4, 7, 10, \ldots$ yes $d=3$

$+3$

(b) $2, 6, 12, 20, 30, \ldots$ NO

4 6 8

6. Find the sum of each arithmetic sequence:

(a) Find the sum of the first 20 terms of an arithmetic sequence where $a_1 = 3$ and $a_{20} = 79$

wants $S_{20} = ?$

use $S_n = n\left(\frac{a_1 + a_n}{2}\right)$

$S_{20} = n\left(\frac{3+79}{2}\right) = 20\left(\frac{82}{2}\right) = 820$

(b) Find the sum of an arithmetic sequence where $a_1 = 5$, $d = 3$, and $n = 25$

$S_{25} = n\left(\frac{a_1 + a_{25}}{2}\right) = 25\left(\frac{5 + a_{25}}{2}\right) = 25\left(\frac{5+77}{2}\right)$

$a_{25} = a + 24d = 5 + 24(3) = 77$

$= 1025$

(c) Find the sum of the arithmetic sequence $-1 + 4 + 9 + \ldots + 49$

$+5$

Find n first: $a_n = 49$

$49 = a + (n-1)d = -1 + (n-1)5$

$49 = -1 + 5n - 5 \Rightarrow n = 11$

Now $S_{11} = 11\left(\frac{-1+49}{2}\right) = 264$

7. Express using sigma notation and find the sum:

(a) $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81}$ $= \frac{1}{3^1} - \frac{1}{3^2} + \frac{1}{3^3} - \frac{1}{3^4} = \sum_{i=1}^{4} (-1)^{i+1} \frac{1}{3^n}$

(b) $\sum_{j=1}^{6} (j+1)(j-1)$ $= \underbrace{(1+1)(1-1)}_{j=1} + \underbrace{(2+1)(2-1)}_{j=2} + \underbrace{(3+1)(3-1)}_{j=3} +$

$\underbrace{(4+1)(4-1)}_{j=4} + \underbrace{(5+1)(5-1)}_{j=5} + \underbrace{(6+1)(6-1)}_{j=6} = 0 + 3 + 8 + 15 + 24 + 35 = 85$