Day 5 Wednesday Feb 5th

Comin up:

- WebAssign HW3 due today by 11:59pm
- Exam 1 a week from today
  Wed Feb 12th
- Written HW 1 due Monday 11:59pm
  upload in Canvas

Recall: Combining functions $\pm, \times, \div$

$f \circ g$

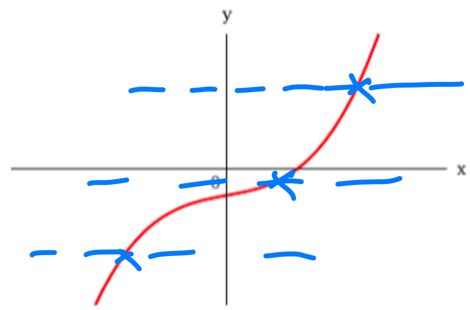
$g \circ f$

, inverses

From previous notes:

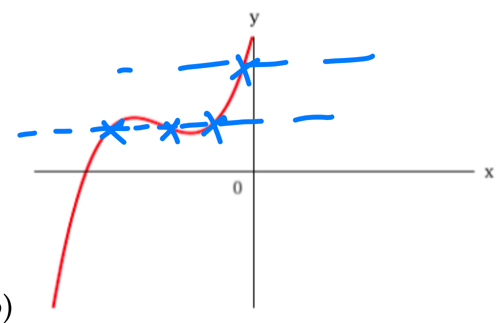
How to determine if a function f is one-to-one - Graphically
Determine if the functions f and g are one-to-one.

we do the Horizontal line test!



(a)

Yes



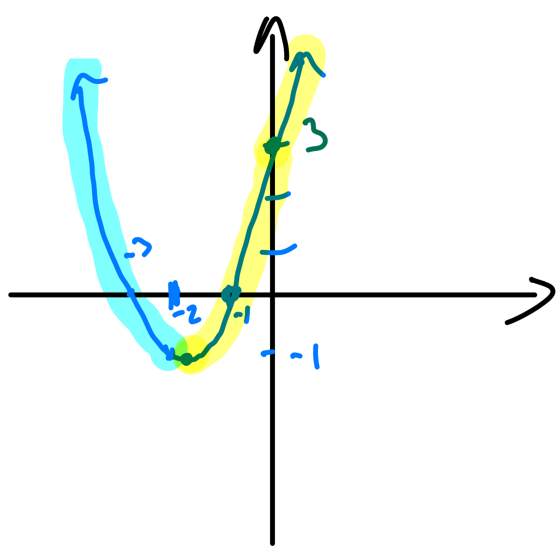
(b)

NOT one-to-one

Practice - Restricted Domain
Graph the function $f(x) = (x+2)^2 - 1$ and decide if it is one-to-one on its entire domain.

y intercept : $x = 0$

$(0+2)^2 - 1 = 3$



x intercepts

$y = (x+2)^2 - 1$

$0 = (x+2)^2 - 1$

$+1 \qquad\qquad +1$

$1 = (x+2)^2$

$\pm\sqrt{1} = \sqrt{(x+2)^2}$

$\pm 1 = x + 2$

$1 = x + 2 \Rightarrow x = -1$

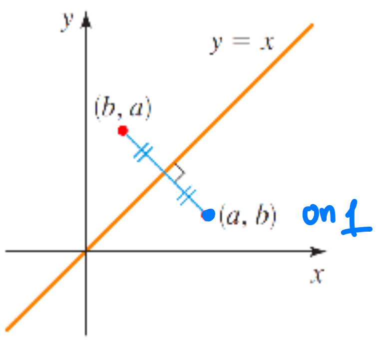
$-1 = x + 2 \Rightarrow x = -3$

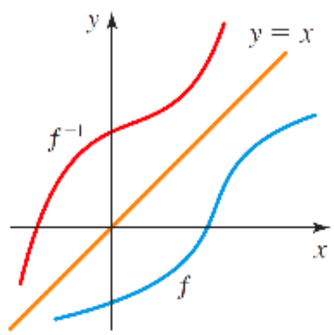
Not one-to-one on its entire domain.
Restrict the domain to $[-2, \infty)$ or $(-\infty, -2]$.
Find inverse of $y = (x+1)^2 - 1$ on $[-2, \infty)$

11. How to find the inverse of a one-to-one function - Graphically

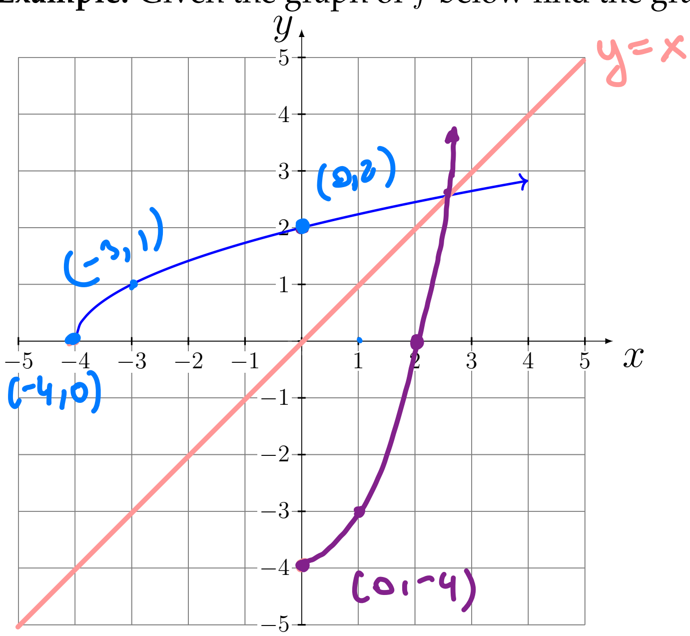
Let f be a one-to one function. Then $f(x) = y$ for any x in the domain of f and f has an inverse f^{-1}. A point (a, b) on the graph of f is going to become (b, a) on the graph of f^{-1} since the input is switched with the output.

The graph of f^{-1} is obtained by reflecting the graph of f in the line $y = x$.





Example: Given the graph of f below find the graph of f^{-1}.



Day 5 Notes
Wednesday Feb 5th
8.1 Sequences and Series

1. Sequences

In this section we are going to introduce Sequences and Series which are fundamental mathematical concepts that are important for understanding patterns and are used in practice to model population growth in biology and analyze algorithms in computer science.

Definition. Roughly speaking, a **sequence** is an infinite list of numbers with some order or pattern.

Example 1. A simple example is the sequence:

first term $n=1$ $n=2$ $n=3$

$$5, \quad 10, \quad 15, \quad 20, \quad \cdots$$

a_1 a_2 a_3 a_4

$a_n = 5n$

$n=1$, $5 \cdot 1 = 5$
$n=2$, $5 \cdot 2 = 10$
$n=3$, $5 \cdot 3 = 15$

The numbers in the sequence are called terms of the sequence and are often written as a_1, a_2, a_3,$\cdots$. The dots mean that the list continues forever.

Formal definition: is a function whose domain is the set of positive integers. The function values are: $a_1, a_2, a_3, a_4, \ldots$

ex 1: Domain: $\{1, 2, 3, 4, \ldots\}$
Range: $\{5, 10, 15, 20, \ldots\}$

$f(3) = 15$
$f(4) = 20$
$f(x) = 5x$

Notation:

a_2 : second term

a_n : general term or the n^{th} term

$\{a_n\}$: represents the whole sequence
$\{a_n\}$ mean $\{a_1, a_2, a_3, a_4 \ldots\}$

We can describe the pattern of the sequence displayed above by a formula for the general term: $a_n = 5n$

Example 2. Find the first three terms and the 100th term of the sequence defined by each formula.

1. $a_n = 2n - 1$

$a_1 = 2(1) - 1 = 1$

$a_2 = 2 \cdot 2 - 1 = 3$

$a_3 = 2 \cdot 3 - 1 = 5$

$a_{100} = 2 \cdot 100 - 1 = 199$

$\{a_n\} = \{1, 3, 5, \ldots, 199, \ldots\}$

2. $b_n = n^2 - 1$

$\{b_n\} = \{0, 3, 8, 15, \ldots\}$

3. $c_n = \dfrac{n}{n+1}$

$\{c_n\} = \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\}$

4. $d_n = \dfrac{(-1)^{n+1}}{n}$

$n=1: \ d_1 = \dfrac{(-1)^{1+1}}{1} = 1$

$n=2: \ d_2 = \dfrac{(-1)^{2+1}}{2} = -1/2$

$n=3: \ d_3 = \dfrac{(-1)^{3+1}}{3} = 1/3$

$n=4: \ d_4 = (-1)^{4+1}/4 = -1/4$

$\{d_n\} = \{1, -1/2, 1/3, -1/4, \ldots$

Example 3. Find the general term of a sequence whose first several terms are given.

(a) $\dfrac{2}{3}, \dfrac{3}{4}, \dfrac{4}{5}, \dfrac{5}{6}, \cdots$

$a_n = \dfrac{n+1}{n+2}$

(b) $-2, 4, -8, 16, -32, \cdots$

$n=1, \ a_1 = -2 = -(2^1)$

$n=2, \ a_2 = 4 = (2^2)$

$n=3, \ a_3 = -8 = -(2^3)$

$n=4, \ a_4 = 16 = 2^4$

$\boxed{a_n = (-1)^n 2^n}$

we start w/ negative

Definition. An **alternating sequence** is a sequence with alternating
sign $+, -, +, -, +, -, \ldots$. $\rightarrow$ use $(-1)^{n+1}$

$-, +, -, +, -, +, \ldots \rightarrow$ use $(-1)^n$

2. Recursively defined sequences

Some sequences do not have simple defining formulas like those of the preceding example.

Definition. When the nth term of a sequence may depend on some or all of the terms preceding it we call the sequence recursive.

Example 4. Example of recursive sequence:
$a_1 = 4$ and $\underbrace{a_n = 2a_{n-1} + 1}$ for $n > 1$.

$\downarrow$ the next term is twice the previous term plus 1.

$n=1 \quad a_1 = 4$

$n=2 \quad a_2 = 2a_{2-1} + 1 = 2a_1 + 1 = 2(4) + 1 = 9$

$n=3 \quad a_3 = 2a_2 + 1 = 2(9) + 1 = 19$

$4, 9, 19, 39, \ldots$

Example 5. The Fibonacci Sequence:
Find the first 7 terms of the sequence defined recursively by
$a_1 = 1$, $a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$.

$\{1, 1, 2, 3, 5, 8, \ldots\}$

If we add the terms of a sequence we obtain a **series**.

Sequence: $\{a_1, a_2, a_3, \ldots\}$

ex: $\{5, 10, 15, 20, \ldots\}$

Series: $a_1 + a_2 + a_3 + \ldots$ = infinite sum

ex: $5 + 10 + 15 + 20 + \ldots$

Partial sum: is a finite series or a finite sum of numbers.

ex: $a_1 + a_2 + a_3 = S_3$ → the third partial sum

ex: $5 + 10 + 15 = 30 = S_3$

$S_n = a_1 + a_2 + a_3 + \ldots + a_n$ = the nth partial sum.

Sigma notation: [1] is a compact way of writing sums.

It uses the greek letter Σ ↳ read sigma to represent sum.

$$a_1 + a_2 + a_3 + \ldots + a_n = \sum_{i=1}^{n} a_i$$

$$a_1 + a_2 + a_3 + \ldots = \sum_{i=1}^{\infty} a_i$$

Example 6. Write the sum using summation notation.

1. $5 + 10 + 15 + 20 + 25 + 30 = \sum_{n=1}^{6} 5n$
2. $\frac{\sqrt{3}}{1} + \frac{\sqrt{4}}{2} + \frac{\sqrt{5}}{3} + \cdots + \frac{\sqrt{n+2}}{n} = \sum_{i=1}^{n} \frac{\sqrt{i+2}}{i}$
3. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = \sum_{j=1}^{\infty} \frac{1}{2^j}$

[1]We use **Sigma notation** for both infinite series and finite series.