Day 1 Notes
Tuesday Jan 21st
8.1 Sequences and Series

1. Sequences

In this section we are going to introduce Sequences and Series which are fundamental mathematical concepts that are important for understanding patterns and are used in practice to model population growth in biology and analyze algorithms in computer science.

Definition. Roughly speaking, a **sequence** is an infinite list of numbers with some order or pattern

Example 1. A simple example is the sequence:

first term → second term

$$5, \quad 10, \quad 15, \quad 20, \quad 25, 30, \cdots, a_n, \cdots$$

$a_1 \quad a_2 \quad a_3 \quad a_4$

The numbers in the sequence are called terms of the sequence and are often written as $a_1, a_2, a_3, \cdots$. The dots mean that the list continues forever.

Notation:

$a_2 = 10$ (the second term of the sequence)

$a_6 = 30$

a_n is called the general term or the nth term of the sequence

$\{a_n\}$ → represents the entire sequence

$\{a_n\} = \{a_1, a_2, a_3, \cdots\}$

n is a positive intege : 1, 2, 3, 4 . . . .

We can describe the pattern of the sequence displayed above by a formula for the general term: $\boxed{a_n = 5n}$

$a_{99} = 5 \cdot 99 = 495$

$a_{15} = 5 \cdot 15$

Example 2. Find the first three terms and the 100th term of the sequence defined by each formula.

1. $a_n = 2n - 1$

$n=1$ $a_1 = 2\cdot 1 - 1 = 1$

$n=2$ $a_2 = 2\cdot 2 - 1 = 3$

$n=3$ $a_3 = 2\cdot 3 - 1 = 5$

$\vdots$

$n=100$ $a_{100} = 2\cdot 100 - 1 = 199$

$\{a_n\} = \{1, 3, 5, \ldots, 199, \ldots\}$

2. $b_n = n^2 - 1$

$n=1$ $b_1 = 1^2 - 1 = 0$

$n=2$ $b_2 = 2^2 - 1 = 3$

$n=3$ $b_3 = 3^2 - 1 = 8$

$\vdots$

$n=100$ $b_{100} = 100^2 - 1 = 9999$

$\{b_n\} = \{0, 3, 8, \ldots, 9999, \ldots\}$

3. $c_n = \dfrac{n}{n+1}$

$n=1$ $c_1 = \frac{1}{1+1}$

$n=2$ $c_2 = \frac{2}{2+1}$

$n=3$ $c_3 = \frac{3}{3+1}$

$\{c_n\} = \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{100}{101}, \ldots\}$

4. $d_n = \dfrac{(-1)^{n+1}}{n}$

$n=1$ $d_1 = \frac{(-1)^{1+1}}{1} = 1$

$n=2$ $d_2 = \frac{(-1)^{2+1}}{2} = \frac{-1}{2}$

$n=3$ $d_3 = (-1)^{3+1}/3 = 1/3$

$n=4$ $d_4 = (-1)^{4+1}/4 = -1/4$

$\{d_n\} = \{1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \ldots\}$

Example 3. Find the general term of a sequence whose first several terms are given.

(a) $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \cdots$

a_1 a_2 a_3 a_4

$a_n = ?$

$n=1$ $a_1 = \frac{2}{3} = \frac{1+1}{1+2}$

$n=2$ $a_2 = \frac{3}{4} = \frac{2+1}{2+2}$

$\vdots$

$a_n = \frac{n+1}{n+2}$

(b) $-2, 4, -8, 16, -32, \cdots$

$\boxed{a_n = (-2)^n}$ works as well

$a_n = n(-2)$?

$n=1$ $a_1 = -2 = (-1)^1 2^1$

$n=2$ $a_2 = 4 = (-1)^2 2^2$

$n=3$ $a_3 = -8 = (-1)^3 2^3$

$n=4$ $a_4 = 16 = (-1)^4 \cdot 2^4$

$\vdots$

$a_n = (-1)^n 2^n$