Day 3 Notes
Wendesday Jan 29th
1.4 Shifting, Reflecting and Stretching Graphs

1. Transformations of functions

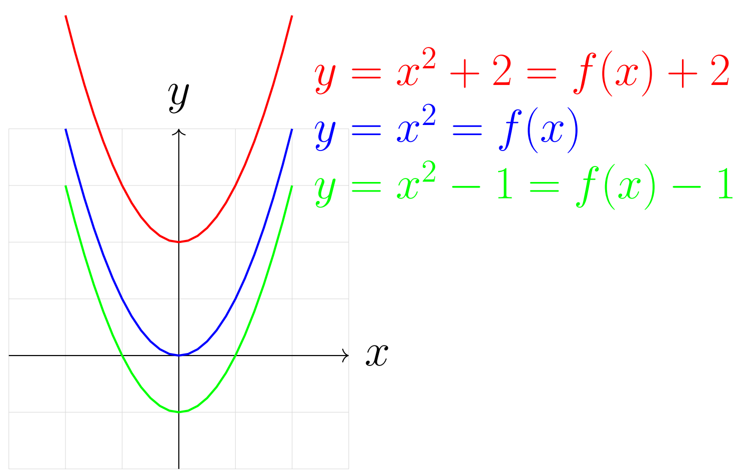
In this section, we'll discuss some ways to graph more complicated popular functions. For example, we will find a quick way to graph the function $f(x) = -(x+2)^2 + 5$ just by knowing that this graph is the graph of x^2 (which we call the **parent** or **basic** function) transformed in some way. These transformation can be rigid or non-rigid.
A **rigid transformation** changes the location of the function in a coordinate plane, but leaves the size and shape of the graph unchanged.
A **non-rigid transformation** changes the size and/or shape of the graph.

1 Shifting

Vertical Shifting (UP or Down)

For any function $f(x)$, adding or subtracting a constant k shifts the graph:

- $f(x) + k$ shifts the graph up k units
- $f(x) - k$ shifts the graph down k units

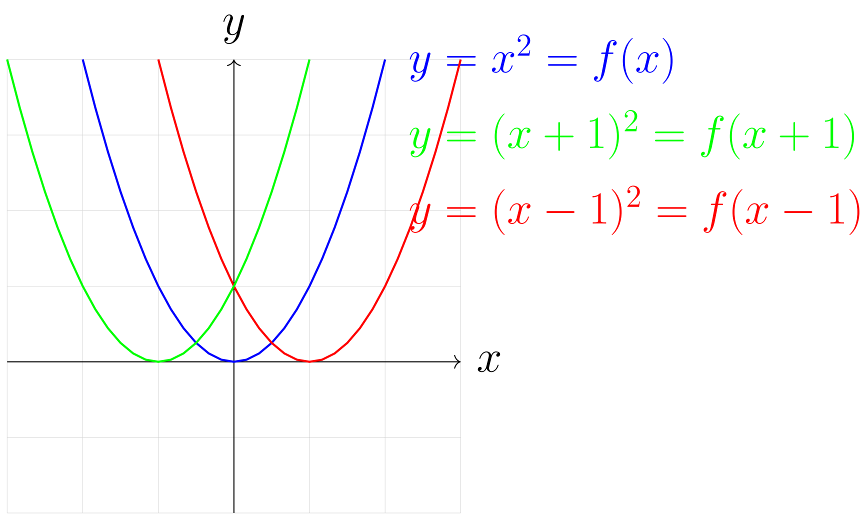


Horizontal Shifting(LEFT or RIGHT)

For any function $f(x)$, replacing x with $(x \pm h)$ shifts the graph:

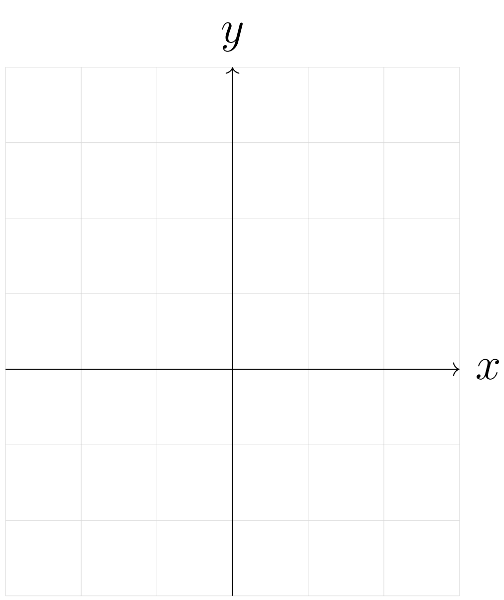
- $f(x-h)$ shifts the graph right h units
- $f(x+h)$ shifts the graph left h units

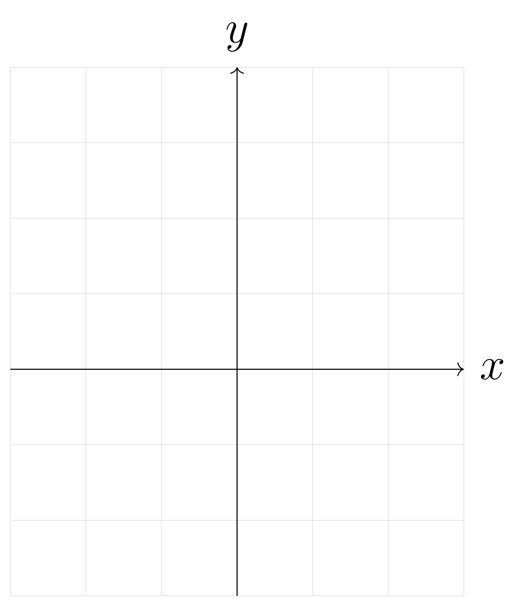
Note: The shift is in the opposite direction of the sign inside the parentheses!



Example 1. Describe the transformation and sketch the graph of each function:

1. $f(x) = |x-2|$
2. $f(x) = \sqrt{x+1} - 2$



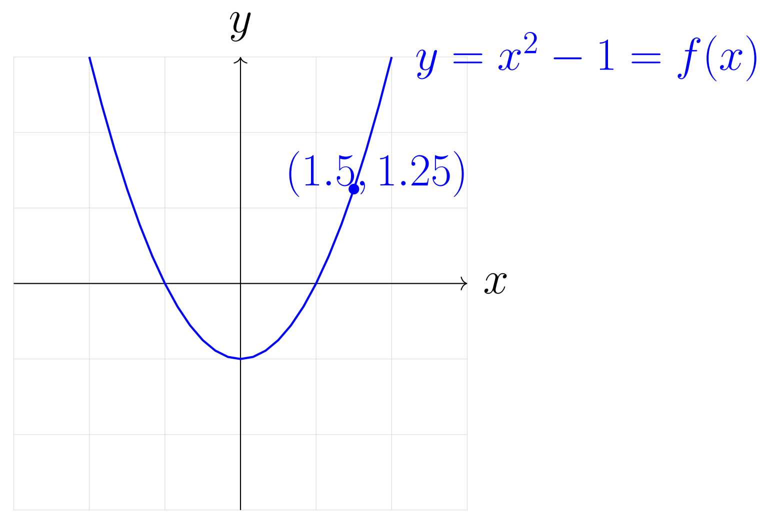


2 Reflecting

Reflecting over x-axis

For any function $f(x)$, multiplying by -1 reflects the graph over the x-axis:

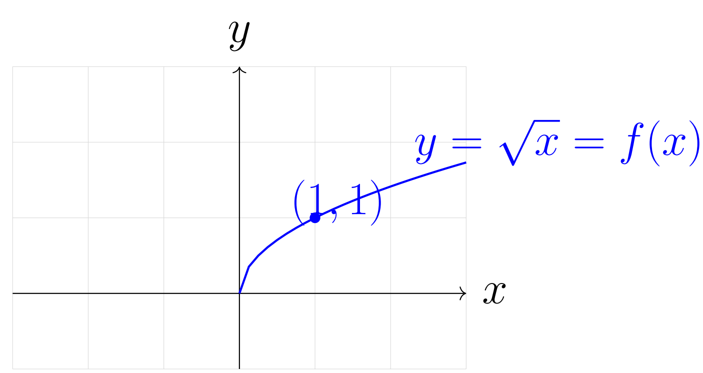
- $y = -f(x)$ reflects the graph of $y = f(x)$ over the x-axis
- Each point (x, y) becomes $(x, -y)$



Reflection over the y-axis

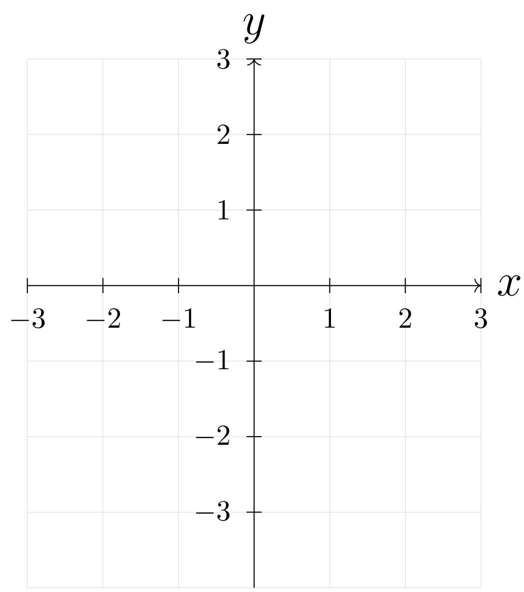
For any function $f(x)$, replacing x with $-x$ reflects the graph over the y-axis:

- $y = f(-x)$ reflects the graph of $y = f(x)$ over the y-axis
- Each point (x, y) becomes $(-x, y)$

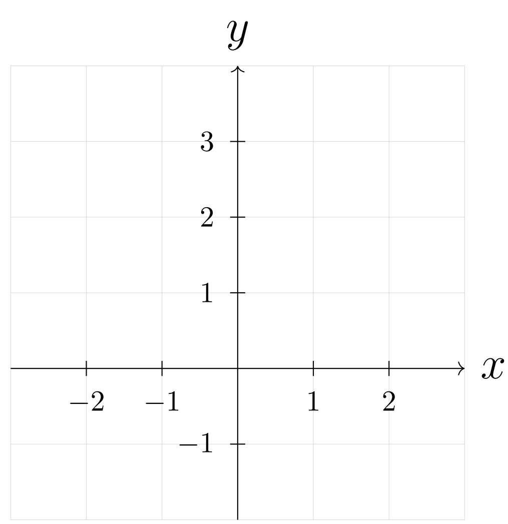


Example 2. Find the equation and sketch the graph of each reflection:

1. Reflect $y = x^2 + 2$ over the x-axis



2. Reflect $y = \sqrt{x - 1}$ over the y-axis



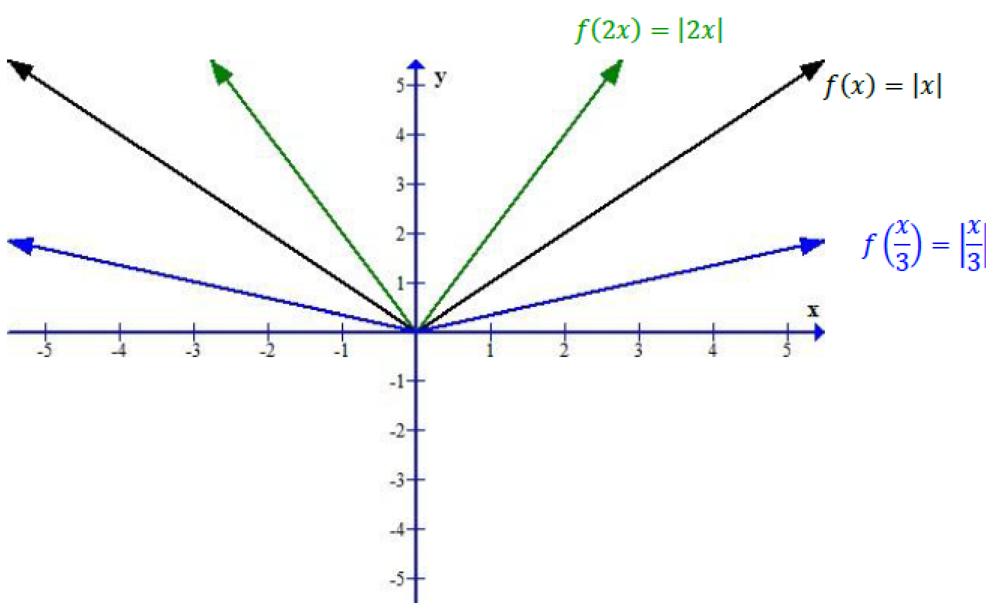
3 Stretching (non-rigid transformation)

Horizontal stretch/shrink

For any function $f(x)$, if c is multiplied to the variable of the function then the graph of the function will undergo a horizontal stretching or compression.

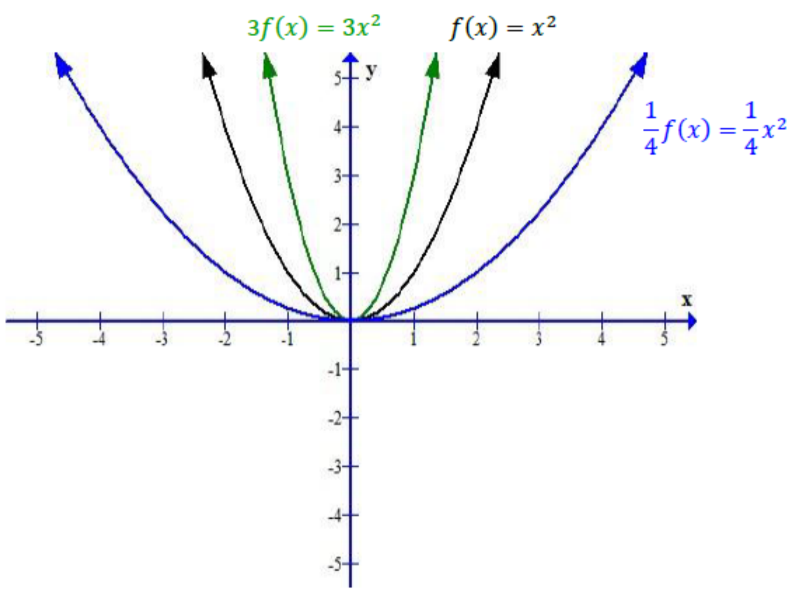
- When the function becomes $y = f(cx)$ and $0 < c < 1$, a horizontal stretching of the graph of will occur.
- Graphically, a horizontal stretching pulls the graph of $y = f(x)$ away from the y-axis.
- When $c > 1$ in the function $y = f(cx)$, a horizontal shrinking of the graph of $y = f(x)$ will occur.
- A horizontal shrinking pushes the graph of toward the y-axis.
- In general, a horizontal stretching or shrinking means that every point (x, y) on the graph of is transformed to $\left(\frac{x}{c}, y\right)$ on the graph of $y = f(cx)$.

Example 3.
Horizontal Stretch and shrink



Vertical stretching and shrinking: non-rigid

- If c is multiplied to the function then the graph of the function will undergo a vertical stretching or compression.
- So when the function becomes $y = cf(x)$ and $0 < c < 1$, a vertical shrinking of the graph of will occur.
- Graphically, a vertical shrinking pulls the graph of $y = f(x)$ toward the x-axis.
- When $c > 1$ in the function $y = cf(x)$, a vertical stretching of the graph of $y = f(x)$ will occur.
- A vertical stretching pushes the graph of $y = f(x)$ away from the x-axis.
- In general, a vertical stretching or shrinking means that every point (x, y) on the graph of $f(x)$ is transformed to (x, cy) on the graph of $y = cf(x)$.



4 Using transformations to graph functions

Transformations can be combined within the same function so that one graph can be shifted, stretched, and reflected. If a function contains more than one transformation, **perform the transformations in the following order** to graph the function:

1. Horizontal translation
2. Stretching or shrinking
3. Reflecting
4. Vertical translation

Summary

Transformations of the graphs of functions	
$f(x)+c$	shift $f(x)$ up c units
$f(x)-c$	shift $f(x)$ down c units
$f(x+c)$	shift $f(x)$ left c units
$f(x-c)$	shift $f(x)$ right c units
$f(-x)$	reflect $f(x)$ about the y-axis
$-f(x)$	reflect $f(x)$ about the x-axis
$cf(x)$	When $0 < c < 1$ – vertical shrinking of $f(x)$ When $c > 1$ – vertical stretching of $f(x)$ **Multiply the y values by c**
$f(cx)$	When $0 < c < 1$ – horizontal stretching of $f(x)$ When $c > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by c**

Practice: Graph the parent function $\sqrt{x}$ and the function
$f(x) = -2\sqrt{x+3} + 1$.
Where are the points $(0, 0)$, $(1, 1)$, and $(4, 2)$ on the graph of $\sqrt{x}$ transformed to on the graph of $-2\sqrt{x+3} + 1$?