Day 3 notes Jan 28th

8.2/8.3 Arithmetic and Geometric Sequences

1. Arithmetic Sequences

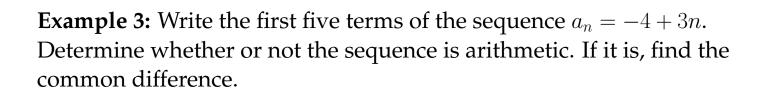
In this section we are going to introduce Arithmetic Sequences. The simplest way to generate an arithmetic sequence is to start with a number a and add to it a fixed constant d, over and over again.

Definition: An arithmetic sequence is a sequence of the form:

The number a is _____ and d is _____. The nth term of an arithmetic sequence is given by:

Example 1: Is this sequence an arithmetic sequence? $13, 7, 1, -5, \cdots$.

If yes, find the common difference, the next three terms, the nth term, and the 300th term of the arithmetic sequence



Example 2: The 11th term of an arithmetic sequence is 52, and the 19th term is 92. Find the 1000th term.

Arithmetic sequence recursive formula:

2. Partial Sums of Arithmetic Sequences

PARTIAL SUMS OF AN ARITHMETIC SEQUENCE

For the arithmetic sequence given by $a_n = a + (n-1)d$, the **nth partial sum**

$$S_n = a + (a + d) + (a + 2d) + (a + 3d) + \cdots + [a + (n - 1)d]$$

is given by either of the following formulas.

1.
$$S_n = \frac{n}{2} [2a + (n-1)d]$$
 2. $S_n = n \left(\frac{a+a_n}{2} \right)$

Example 4: a) Find the sum of the first 100 numbers.

b) Find the sum of the first 50 odd numbers.

c) Sum the first 25 terms of the sequence $3, -1, -5, -9, \cdots$

3. Geometric Sequences

Next, we are going to introduce Geometric Sequences. A geometric sequence is generated when we start with a number a and repeatedly multiply by a fixed nonzero constant r.

Definition: A geometric sequence is a sequence of the form:

The number a is _____ and r is

The nth term of a geometric sequence is given by:

Example 5: If a = 3 and r = 2, then we have the geometric sequence:

Example 6: The sequence

 $2,-10,50,-250,1250,\cdots$ is a geometric sequence with a=2 and r=-5. Find a_{10} .

Example 6: Find the common ratio, the first term, the nth term, and the eighth term of the geometric sequence $5, 15, 45, 135, \cdots$

4. Partial Sums of Geometric Sequences

The partial sum s_n of a geometric sequence is given by the formula below.

PARTIAL SUMS OF A GEOMETRIC SEQUENCE

For the geometric sequence defined by $a_n = ar^{n-1}$, the **nth partial sum**

$$S_n = a + ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1}$$
 $r \neq 1$

is given by

$$S_n = a \frac{1 - r^n}{1 - r}$$

Example 7: Find the following partial sum of a geometric sequence:

$$1 + 4 + 16 + \dots + 4096$$

5. Infinite Geometric Series

An infinite geometric series is a series of the form

$$a + ar + ar^{2} + ar^{3} + ar^{4} + \dots + ar^{n-1} + \dots$$

SUM OF AN INFINITE GEOMETRIC SERIES

If |r| < 1, then the infinite geometric series

$$\sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + ar^3 + \cdots$$

converges and has the sum

$$S = \frac{a}{1 - r}$$

5

If $|r| \ge 1$, the series diverges.

Example 8: Determine whether the infinite geometric series is convergent or divergent. If it is convergent, find its sum.

- (a) $2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \cdots$
- (b) $1 + \frac{7}{5} + \frac{49}{25} + \cdots$