Day 6, Monday Feb 10th

- Exam 1 on Wednesday
- Written HW 1 due today by 11:59 pm
- Practice Problems for exam 1 posted in WebAssign
  (material on exam - everything but arithmetic sequences)

Example 7. Write the terms for the sum and evaluate the sum.
$\sum_{n=1}^{5} 2n+3$ $= 5 + (2\cdot 2+3) + (2\cdot 3+3) + (2\cdot 4+3) + (2\cdot 5+3)$
$= 5 + 7 + 9 + 11 + 13 = 45$

Note: In Calculus you will learn how to find what an infinite series adds up to for certain types of series.

Example 8. Consider the series:

$$\sum_{i=1}^{\infty} \frac{1}{2^i} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

convergent series to 1

Lets investigate the first few partial sums:

$S_1 =$ $1/2 = 0.5$
$S_2 =$ $1/2 + 1/4 = 3/4 = 0.75$
$S_3 =$ $S_2 + a_3 = 3/4 + 1/8 = 0.875$
$S_4 =$ 0.9375

$S_5 = 0.96875$
$\vdots$

As we continue adding terms, the sum gets closer and closer to 1.

Example 9. Consider the series:

$$\sum_{i=1}^{\infty} \frac{1}{i} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

divergent!

Lets investigate the first few partial sums:

$S_1 =$ 1
$S_2 =$ $1 + 1/2 = 1.5$
$S_3 =$ 1.8333
$S_4 =$ $2.0833\ldots$

As we continue adding terms, the sum grows larger and larger without bound. This series does not add up to an actual value.

4. Factorial notation

Factorials form the basis for important series like Taylor series expansions.

Definition. The factorial of a positive integer n, written as $n!$, is the product of all positive integers. less than or equal to n.

For example, $5! = 1\cdot 2\cdot 3\cdot 4\cdot 5 = 120$ $\qquad 5! = 5\cdot 4\cdot 3\cdot 2\cdot 1$

$10! = 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8\cdot 9\cdot 10 = 3628800$

$n! = 1\cdot 2\cdot 3\cdot 4\cdot\cdots\cdot(n-2)\cdot(n-1)\cdot n$

By definition, $0! = 1$

Example 10. Evaluate the following expressions.

Write the bigger factorial in terms of smaller factorial.

(a) $\dfrac{8!}{3!\cdot 5!}$

$$\frac{\cancel{1\cdot 2\cdot 3\cdot 4\cdot 5}\cdot\cancel{6}\cdot 7\cdot 8}{\cancel{3!}\ \cancel{5!}} = 56$$

$8! = 5!\ 6\cdot 7\cdot 8$

(b) $\dfrac{n!}{(n+2)!} = \dfrac{\cancel{n!}}{\cancel{n!}\,(n+1)(n+2)} = \dfrac{1}{(n+1)(n+2)}$

(c) $\dfrac{(n+1)!}{(n-1)!} = \dfrac{\cancel{1\cdot 2\cdot 3\cdots(n-1)}\,n\,(n+1)}{\cancel{(n-1)!}}$

$\dfrac{n+1}{n-1}$, $n(n+1)$, $n!$

(d) $\dfrac{(2n)!}{(n)!} = \dfrac{\cancel{1\cdot 2\cdot 3\cdots(n)}\,(2n-3)(2n-2)(2n-1)\,(2n)}{\cancel{n!}}$

$= (n+1)(n+2)(n+3)\cdots(2n)$

$(n+2)! = \underbrace{1\cdot 2\cdot 3\cdot\ \cdots\ \cdot n}\,(n+1)\,(n+2) = n!\,(n+1)(n+2)$

Example 11. Given the sequence defined by $b_n = \dfrac{n^2}{(n+1)!}$ find b_1 and b_6.

$$b_1 = \frac{1^2}{(1+1)!} = \frac{1}{2!} = \frac{1}{2}$$

$$b_6 = \frac{6^2}{(6+1)!} = \frac{36}{7!} = \frac{\cancel{36}}{\cancel{1\cdot 2\cdot 3}\cdot 4\cdot 5\cdot\cancel{6}\cdot 7} = \frac{1}{140}$$

Day 6 notes
Feb 10th
8.2/8.3 Arithmetic and Geometric Sequences

1. Arithmetic Sequences

In this section we are going to introduce Arithmetic Sequences. The simplest way to generate an arithmetic sequence is to start with a number a and add to it a fixed constant d, over and over again.
can be negative

Definition: An arithmetic sequence is a sequence of the form:

$a, a+d, a+2d, a+3d, \dots$
a_1 a_2 a_3 a_4

The number a is the first term and d is called the common difference.

$a_2 - a_1 = d$
$a_4 - a_3 = d$

The nth term of an arithmetic sequence is given by:

$$\boxed{a_n = a + (n-1)d}$$

$a_1 = a$
$a_2 = a + 1d$
$a_3 = a + 2d$

Example 1: Is this sequence an arithmetic sequence? $13, 7, 1, -5, \cdots$.

$-6 \quad -6 \quad -6$

$7 - 13 = -6$
$1 - 7 = -6$
$-5 - 1 = -6$

If yes, find the common difference, the next three terms, the nth term, and the 300th term of the arithmetic sequence

$d = -6$

$a = 13$

$a_5 = -5 - 6 = -11$
$a_6 = -11 - 6 = -17$
$a_7 = -17 - 6 = -23$

$$\boxed{a_n = a + (n-1)d}$$

$$\boxed{a_n = 13 + (n-1)(-6)}$$

$a_{300} = 13 + (300-1)(-6) = -1781$ ✓

Example 3: Write the first five terms of the sequence $a_n = -4 + 3n$. Determine whether or not the sequence is arithmetic. If it is, find the common difference.

$a_1 = -4+3 = -1$
$a_2 = 2$ $+3$
$a_3 = 5$ $+3$
$a_4 = 8$ $+3$
$a_5 = 11$ $+3$

If yes

$\boxed{d = 3}$

$\boxed{a = -1}$

Example 2: The 11th term of an arithmetic sequence is 52, and the 19th term is 92. Find the 1000th term.

$a_{11} = 52$

$a_{19} = 92$

$a_{1000} = ?$

$52 = a + 10d$
$92 = a + 18d$ subtract

$40 = (\cancel{a} + 18d) - (\cancel{a} + 10d)$

$40 = 8d$

$d = 5$

$52 = a + 10(5)$

$52 = a + 50$

$a = 2$

$a_{1000} = 4997$

$a_n = a + (n-1)d$

$a_{11} = a + (11-1)d$

$a_{11} = a + 10d$

$a_{19} = a + 18d$

$a_{1000} = 2 + (999)5 = 4997$

Arithmetic sequence recursive formula:

$a_1 = a$

$a_n = a_{n-1} + d$

$$\boxed{\begin{aligned} a_1 &= a \\ a_n &= a + (n-1)d \end{aligned}}$$

$a_n = 2a_{n-1} + 5$

$a_n =$