# Day 3 Wednesday Jan 29th

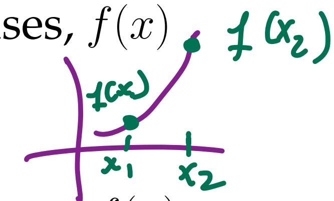
- Finished 1.3 and 1.4
- Group Quiz 1.

## Increasing/decreasing/constant function

Let $f$ be a function.

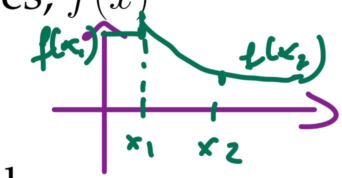
- A function is **increasing** on an interval if as $x$ increases, $f(x)$ increases.

  if $x_1 < x_2$ then $f(x_1) < f(x_2)$



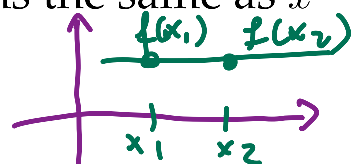

- A function is **decreasing** on an interval if as $x$ increases, $f(x)$ decreases.

  if $x_1 < x_2$ then $f(x_1) > f(x_2)$

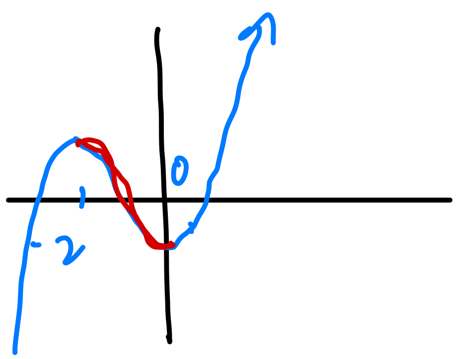



- A function is **constant** on an interval if $f(x)$ remains the same as $x$ changes.

  if $x_1 < x_2$ then $f(x_1) = f(x_2)$



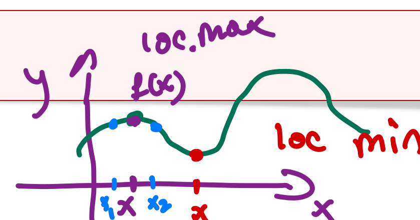

**Example 1.** Graph the function $f(x) = x^3 + 3x^2 - 1$. Then use the graph to describe the increasing and decreasing behavior of the function.




increasing: $(-\infty, -2) \cup (0, \infty)$

decreasing: $(-2, 0)$

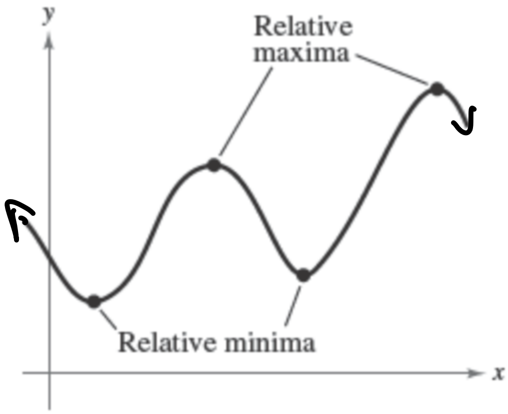
## Relative (local) maxima/minima

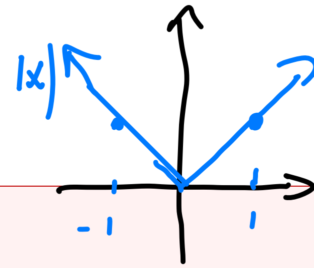


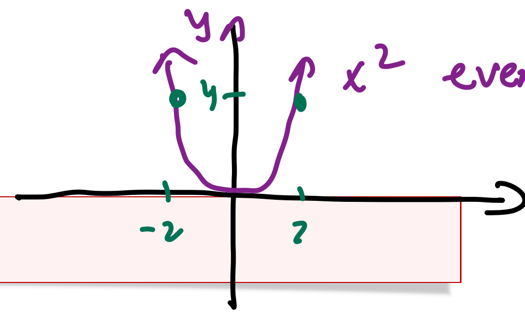

For a function $f$:

- A point is a **local maximum** if $f(x)$ is greater at that point than at nearby points.

- A point is a **local minimum** if $f(x)$ is less at that point than at nearby points



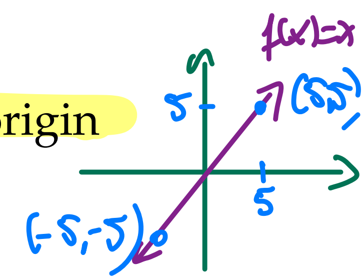

## Even/odd functions

- A function is **even** if its graph is symmetric about the y-axis

$f(1)=1$
$f(-1)=1$

$$f(-x) = f(x) \text{ for all } x \text{ in the domain}$$

$f(-1) = f(1)$




- A function is **odd** if its graph is symmetric about the origin

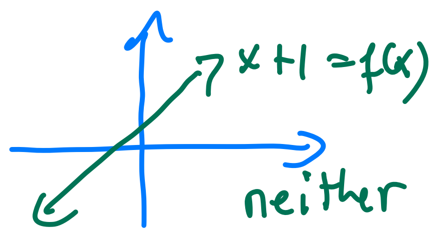
$$f(-x) = -f(x) \text{ for all } x \text{ in the domain}$$

$f(-5) = -f(5)$

$\to -5 = -5$

**Example 2.** Determine whether the function is even, odd or neither.

1. $f(x) = x^3 + 4x^1$
2. $g(x) = 3x - x^2$
3. $f(t) = 5t^{2/3}$




$(-x)^3 = \underbrace{(-x)(-x)}_{x^2}(-x) = x^2(-x)$

1) $f(-x) = (-x)^3 + 4(-x) = -x^3 - 4x = -(x^3+4x) = -f(x)$

odd

2) $g(-x) = 3(-x) - (-x)^2 = \underbrace{-3x - x^2}_{\text{this is not}}$ neither

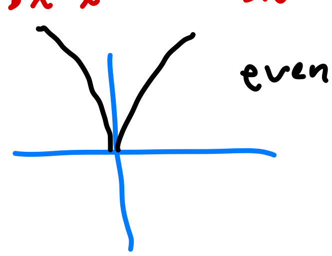
this is not $g(x)$ or $-g(x)$

$3x - x^2$ $\quad$ $-3x + x^2$

3) $f(-t) = 5(-t)^{2/3} = 5\sqrt[3]{(-t)^2} = 5\sqrt[3]{t^2}$

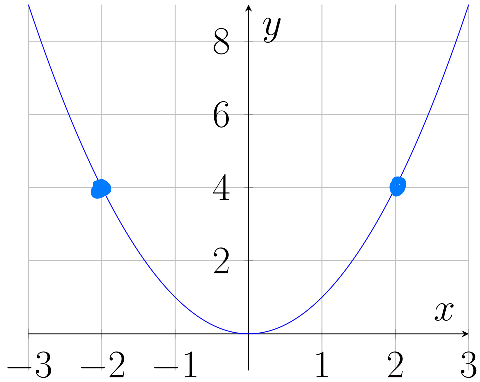
$= 5t^{2/3} = f(t)$

even



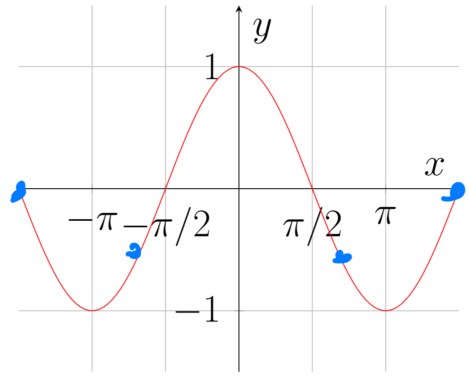
even

Function: $f_1(x) = x^2$



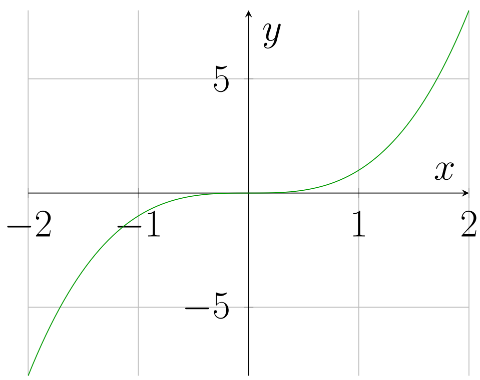

Function: $f_2(x) = \cos(x)$

even



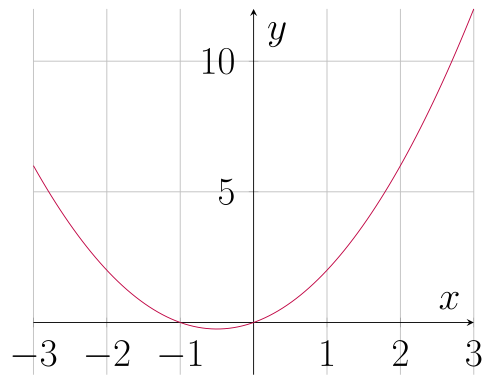

odd

Function: $f_3(x) = x^3$




$f_4(x) = x^2 + x$

neither




## Properties

- For even functions ($f_1$ and $f_2$), $f(x) = f(-x)$ for all $x$ in the domain
- For the odd function ($f_3$), $f(-x) = -f(x)$ for all $x$ in the domain
- For $f_4$, neither property holds: $f_4(-x) \neq f_4(x)$ and $f_4(-x) \neq -f_4(x)$

**Day 3 Notes**
**Wendesday Jan 29th**
*1.4 Shifting, Reflecting and Stretching Graphs*

1. Transformations of functions

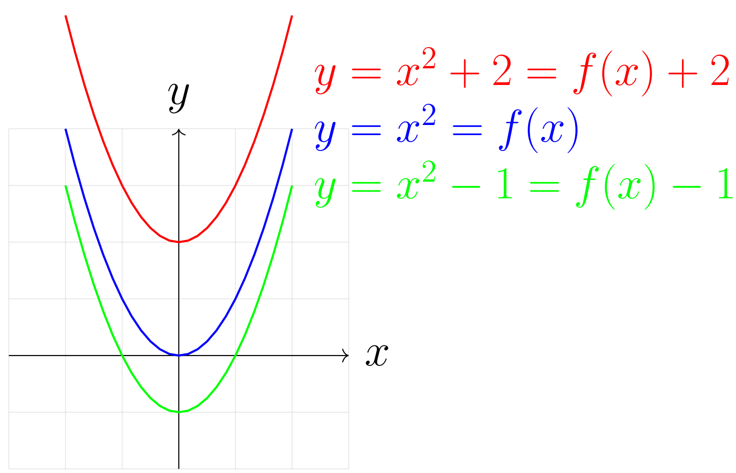
In this section, we'll discuss some ways to graph more complicated popular functions. For example, we will find a quick way to graph the function $f(x) = -(x+2)^2 + 5$ just by knowing that this graph is the graph of $x^2$ (which we call the **parent** or **basic** function) transformed in some way. These transformation can be rigid or non-rigid.
A **rigid transformation** changes the location of the function in a coordinate plane, but leaves the size and shape of the graph unchanged.
A **non-rigid transformation** changes the size and/or shape of the graph.

# 1 Shifting

### Vertical Shifting (UP or Down)

For any function $f(x)$, adding or subtracting a constant $k$ shifts the graph:

- $f(x) + k$ shifts the graph up $k$ units
- $f(x) - k$ shifts the graph down $k$ units

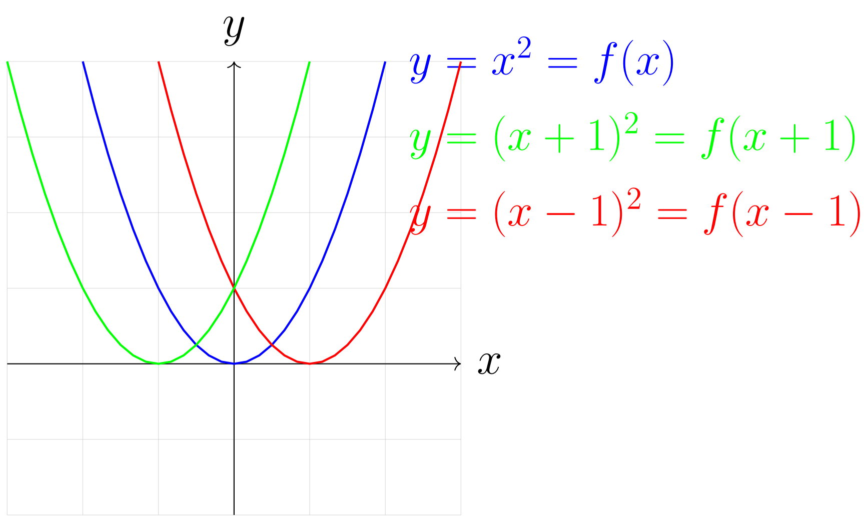


**Horizontal Shifting(LEFT or RIGHT)**

For any function $f(x)$, replacing $x$ with $(x \pm h)$ shifts the graph:

- $f(x-h)$ shifts the graph right $h$ units
- $f(x+h)$ shifts the graph left $h$ units

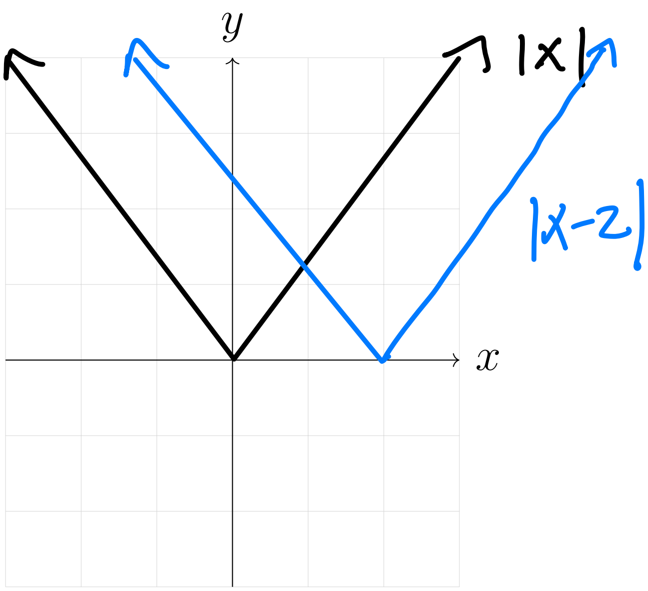
**Note:** The shift is in the opposite direction of the sign inside the parentheses!

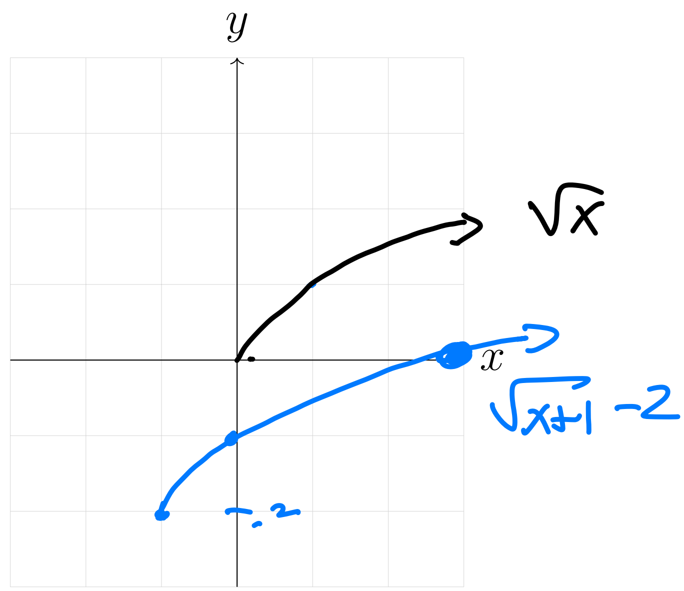



**Example 1.** Describe the transformation and sketch the graph of each function:

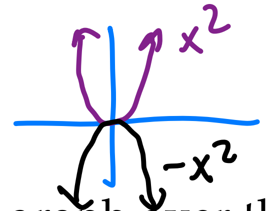
1. $f(x) = |x-2|$
2. $f(x) = \sqrt{x+1} - 2$

$f(x) = |x|$
$f(x) = \sqrt{x}$



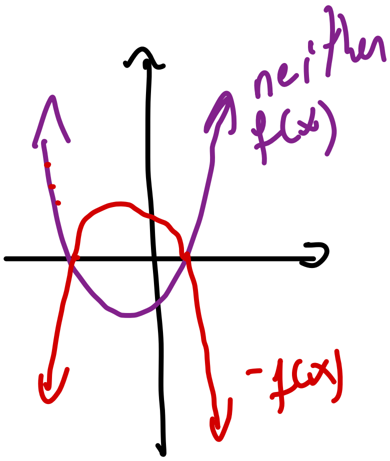
## 2 Reflecting

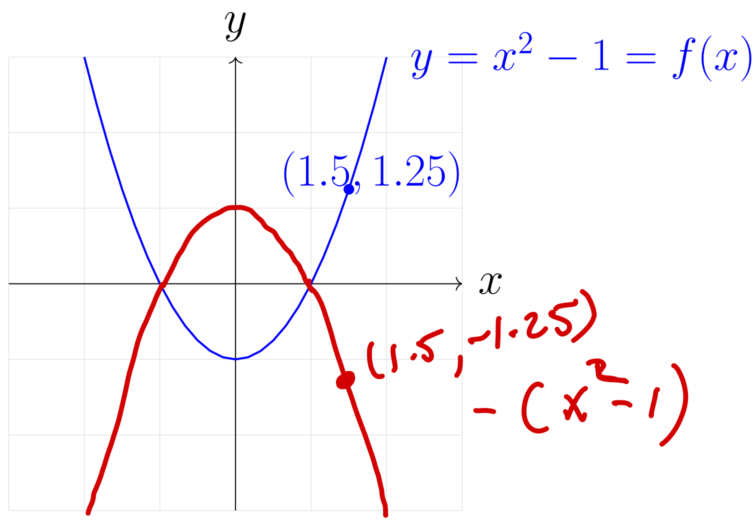


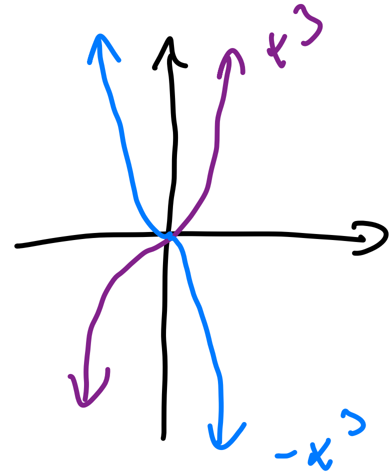

### Reflecting over x-axis

For any function $f(x)$, multiplying by $-1$ reflects the graph over the x-axis:

- $\boxed{y = -f(x)}$ reflects the graph of $y = f(x)$ over the x-axis
- Each point $(x, y)$ becomes $(x, -y)$



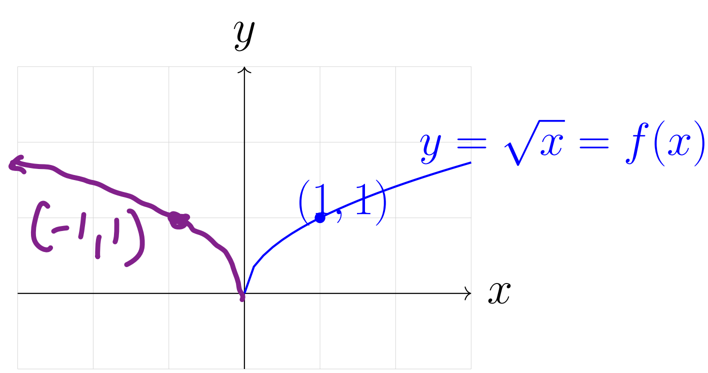




### Reflection over the y-axis

For any function $f(x)$, replacing $x$ with $-x$ reflects the graph over the y-axis:

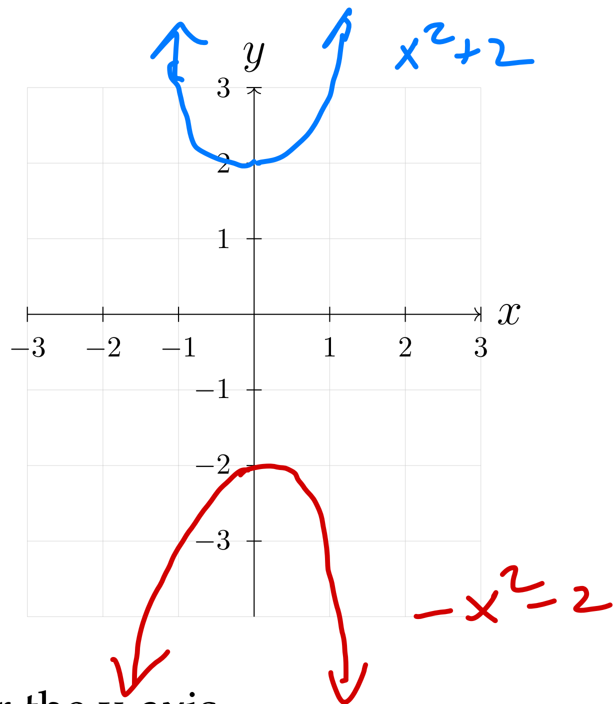
- $\boxed{y = f(-x)}$ reflects the graph of $y = f(x)$ over the y-axis
- Each point $(x, y)$ becomes $(-x, y)$

$f(-x) = \sqrt{-x}$



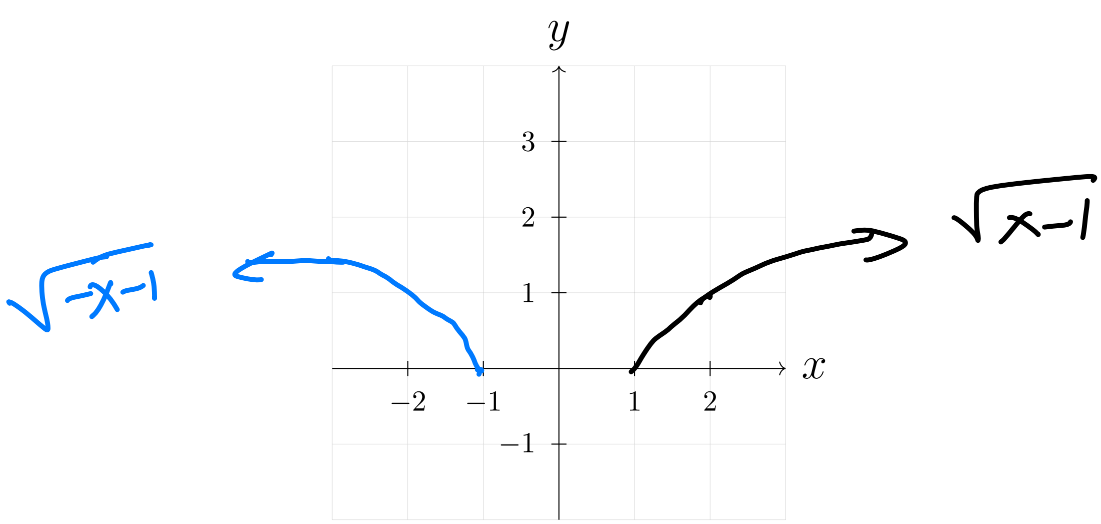
**Example 2.** Find the equation and sketch the graph of each reflection:

1. Reflect $y = x^2 + 2$ over the x-axis $\quad -(x^2+2) = -x^2 - 2$




2. Reflect $y = \sqrt{x-1}$ over the y-axis

$\sqrt{-x-1}$



## 3 Stretching (non-rigid transformation)

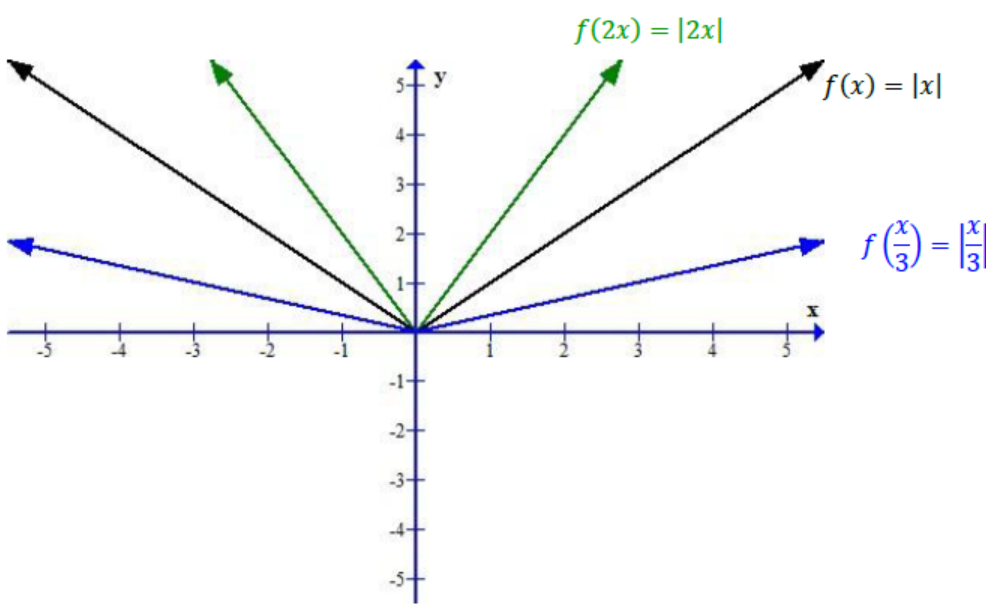
### Horizontal stretch/shrink

For any function $f(x)$, if $c$ is multiplied to the variable of the function then the graph of the function will undergo a horizontal stretching or compression.

$(2x)^2$ $|1/3x|$

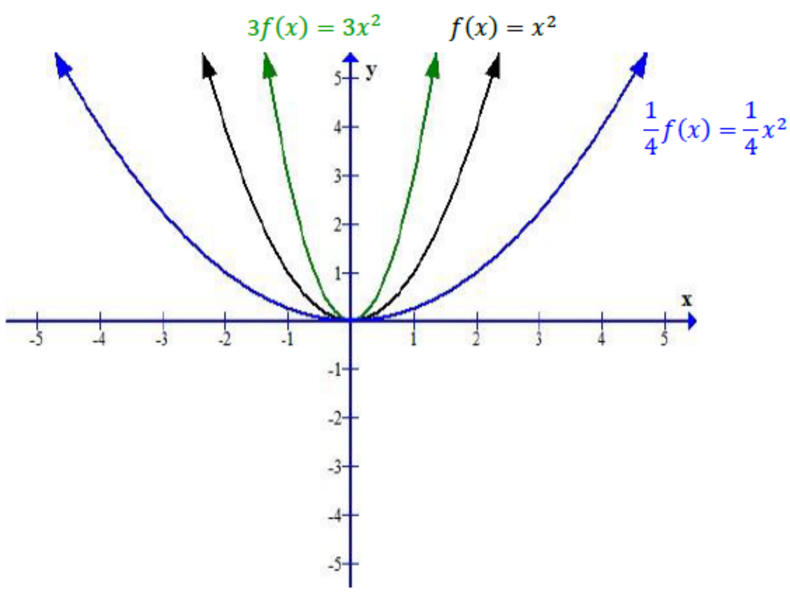
- When the function becomes $y = f(cx)$ and $0 < c < 1$, a horizontal stretching of the graph of will occur.
- Graphically, a horizontal stretching pulls the graph of $y = f(x)$ away from the $y$-axis.
- When $c > 1$ in the function $y = f(cx)$, a horizontal shrinking of the graph of $y = f(x)$ will occur.
- A horizontal shrinking pushes the graph of toward the $y$-axis.
- In general, a horizontal stretching or shrinking means that every point $(x, y)$ on the graph of is transformed to $\left(\frac{x}{c}, y\right)$ on the graph of $y = f(cx)$.

**Example 3.**
**Horizontal Stretch and shrink**



**Vertical stretching and shrinking: non-rigid**

- If $c$ is multiplied to the function then the graph of the function will undergo a vertical stretching or compression.
- So when the function becomes $y = cf(x)$ and $0 < c < 1$, a vertical shrinking of the graph of will occur.
- Graphically, a vertical shrinking pulls the graph of $y = f(x)$ toward the $x$-axis.
- When $c > 1$ in the function $y = cf(x)$, a vertical stretching of the graph of $y = f(x)$ will occur.
- A vertical stretching pushes the graph of $y = f(x)$ away from the $x$-axis.
- In general, a vertical stretching or shrinking means that every point $(x, y)$ on the graph of $f(x)$ is transformed to $(x, cy)$ on the graph of $y = cf(x)$.



## 4 Using transformations to graph functions

Transformations can be combined within the same function so that one graph can be shifted, stretched, and reflected. If a function contains more than one transformation, **perform the transformations in the following order** to graph the function:

1. Horizontal translation
2. Stretching or shrinking
3. Reflecting
4. Vertical translation

### Summary

| **Transformations of the graphs of functions** | |
|---|---|
| $f(x)+c$ | shift $f(x)$ up c units |
| $f(x)-c$ | shift $f(x)$ down c units |
| $f(x+c)$ | shift $f(x)$ left c units |
| $f(x-c)$ | shift $f(x)$ right c units |
| $f(-x)$ | reflect $f(x)$ about the y-axis |
| $-f(x)$ | reflect $f(x)$ about the x-axis |
| $cf(x)$ | When $0<c<1$ – vertical shrinking of $f(x)$<br>When $c>1$ – vertical stretching of $f(x)$<br>**Multiply the y values by c** |
| $f(cx)$ | When $0<c<1$ – horizontal stretching of $f(x)$<br>When $c>1$ – horizontal shrinking of $f(x)$<br>**Divide the x values by c** |

**Practice:** Graph the parent function $\sqrt{x}$ and the function $f(x) = -2\sqrt{x+3} + 1$.
Where are the points $(0, 0)$, $(1, 1)$, and $(4, 2)$ on the graph of $\sqrt{x}$ transformed to on the graph of $-2\sqrt{x+3} + 1$?

Warm up next time