Day 4

- Today Group Quiz ((at the end)

Arithmetic sequences

$$a, a+d, a+2d, a+3d, \ldots\ldots, a+(n-1)d, \ldots$$

$a = ?$
$d = ?$

a_n

S_n = the sum of the first n term of an arithmetic seq

$$S_n = n\left(\frac{a_1+a_n}{2}\right)$$

Office hours 12:30 - 2:30 pm

3. Geometric Sequences

Next, we are going to introduce Geometric Sequences. A geometric sequence is generated when we start with a number a and repeatedly multiply by a fixed nonzero constant r.

Definition: A geometric sequence is a sequence of the form:

$a, ar, ar^2, ar^3, \ldots, ar^{n-1}, \ldots$

$a_1 = a$

$a_2 = ar$

$a_3 = (ar)r = ar^2$

$a_4 = ar^3$

$\vdots$

The number a is first term and r is common ratio.

$\frac{a_2}{a_1} = \frac{ar}{a} = r$; $\frac{a_4}{a_3} = \frac{ar^3}{ar^2} = r$

The nth term of a geometric sequence is given by:

$$\boxed{a_n = ar^{n-1}}$$

Example 5: If $a = 3$ and $r = 2$, then we have the geometric sequence: $a_{10} = ?$

$3, 6, 12, 24, \ldots$

$n = 10$

$a_{10} = ar^{10-1}$
$= 3 \cdot 2^9$
$= 3 \cdot 512$
$= 1536$

Example 6: The sequence $2, -10, 50, -250, 1250, \cdots$ is a geometric sequence with $a = 2$ and $r = -5$. Find a_10.

$a_{10} = ?$ $a_{10} = ar^{10-1} = 2(-5)^9 = -3906250$

Example 6: Find the common ratio, the first term, the nth term, and the eighth term of the geometric sequence $5, 15, 45, 135, \cdots$

$\frac{15}{5} = 3$ $\frac{45}{15} = 3$ $\frac{135}{45} = 3$

$r = 3$
$a = 5$
$a_n = 5 \cdot 3^{n-1}$

$a_8 = 5 \cdot 3^7$
$= 10935$

4. Partial Sums of Geometric Sequences

The partial sum s_n of a geometric sequence is given by the formula below.

PARTIAL SUMS OF A GEOMETRIC SEQUENCE

For the geometric sequence defined by $a_n = ar^{n-1}$, the ***n*th partial sum**

$S_n = a_1 + a_2 + a_3 + a_4 + a_5 + \cdots a_n$

$$S_n = a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1} \qquad r \neq 1$$

is given by

$S_{10} = a\dfrac{1-r^{10}}{1-r}$

$$S_n = a\frac{1-r^n}{1-r}$$

$S_6 = a\dfrac{1-r^6}{1-r}$

Example 7: Find the following partial sum of a geometric sequence:

$1 + 4 + 16 + \cdots + 4096$

a_n, a_7

$S_n = a\dfrac{1-r^n}{1-r}$

$a = 1$

$r = 4$

$n = ?$

$\boxed{S_7 = 1\dfrac{1-4^7}{1-4} = 5461}$

calc: $(1-4\wedge 7)/(1-4)$

$a_n = 4096$

$a_n = ar^{n-1}$

$\boxed{4096 = 1 \cdot 4^{n-1}}$

$4096 = 4^{n-1}$

$4^6 = 4^{n-1}$

$6 = n-1$

$n = 7$

5. Infinite Geometric Series

An infinite geometric series is a series of the form $a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1} + \cdots$.

SUM OF AN INFINITE GEOMETRIC SERIES

If $|r| < 1$, then the infinite geometric series

$$\sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + ar^3 + \cdots$$

converges and has the sum

$$* \quad \boxed{S = \frac{a}{1-r}} \quad *$$

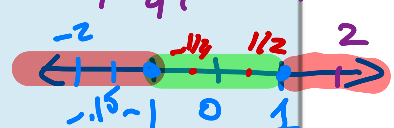
If $|r| \geq 1$, the series diverges. *

$|r| < 1$

$|-3| = 3$

$|1/2| = 1/2$

$|-\frac{1}{4}| = 1/4$



$\sum_{n=1}^{\infty} ar^{n-1}$

Example 8: Determine whether the infinite geometric series is convergent or divergent. If it is convergent, find its sum.

(a) $2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \cdots = \sum_{i=1}^{\infty} 2\left(\frac{1}{5}\right)^{i-1}$

$a = 2$

$r = \frac{2/5}{2} = \frac{1}{5}$

(b) $1 + \frac{7}{5} + \frac{49}{25} + \cdots = \sum_{i=1}^{\infty} ar^{i-1} = \sum_{i=1}^{\infty} 1 \cdot \left(\frac{7}{5}\right)^{i-1}$

a) $r = \frac{1}{5}$; $|r| = \left|\frac{1}{5}\right| < 1$ yes so series converges to $\frac{a}{1-r} = \frac{2}{\left(1-\frac{1}{5}\right)} = \frac{2}{\frac{5}{5}-\frac{1}{5}}$

$= \frac{2}{\frac{4}{5}} = \cancel{2} \cdot \frac{5}{\cancel{4}\,2} = \frac{5}{2} = 2.5$

b) $r = \frac{7}{5} = 1.4$

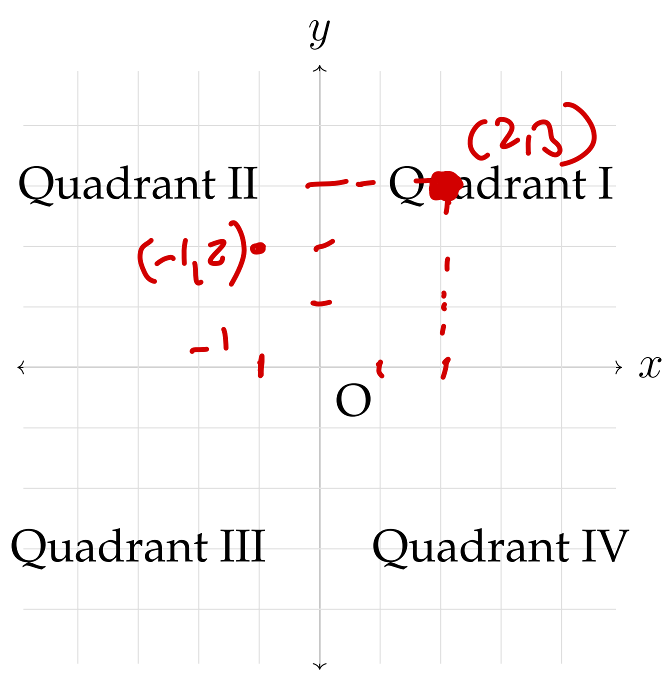
$\left|\frac{7}{5}\right| > 1$ so series diverges

Day 4 Notes
Thursday Jan 30th
2.1 The coordinate system

1. Introduction to the Coordinate System

The rectangular coordinate system (also called the Cartesian plane) consists of two perpendicular number lines:

- The horizontal line is called the x axis
- The vertical line is called the y axis
- The point where these lines intersect is called the origin



point
(2,3)
(-1,2)

Quadrants:

- Quadrant I: Both x and y are positive
- Quadrant II: x is negative and y is positive
- Quadrant III: Both x and y are negative
- Quadrant IV: x is positive and y is negative

Plotting Points: Any point in the plane can be written as an ordered pair (x, y) where:

- x represents the horizontal distance from the origin (left/right)
- y represents the vertical distance from the origin (up/down)

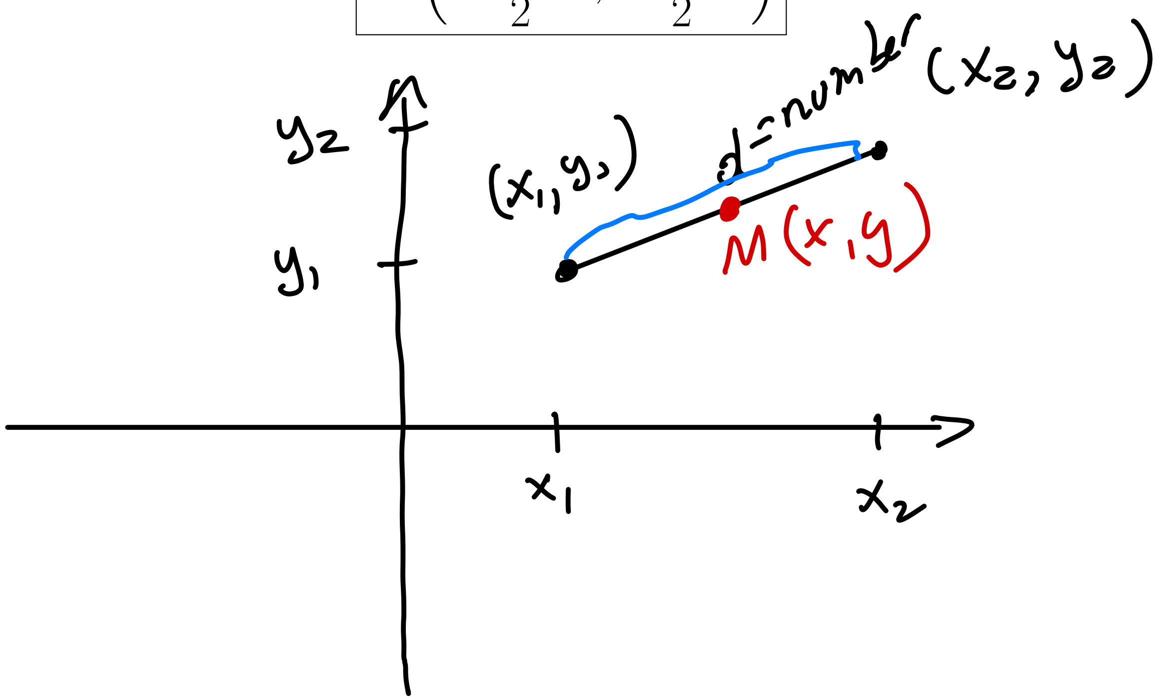
2. Distance and Midpoint Formulas

Distance Formula: The distance d between two points (x_1, y_1) and (x_2, y_2) is:

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

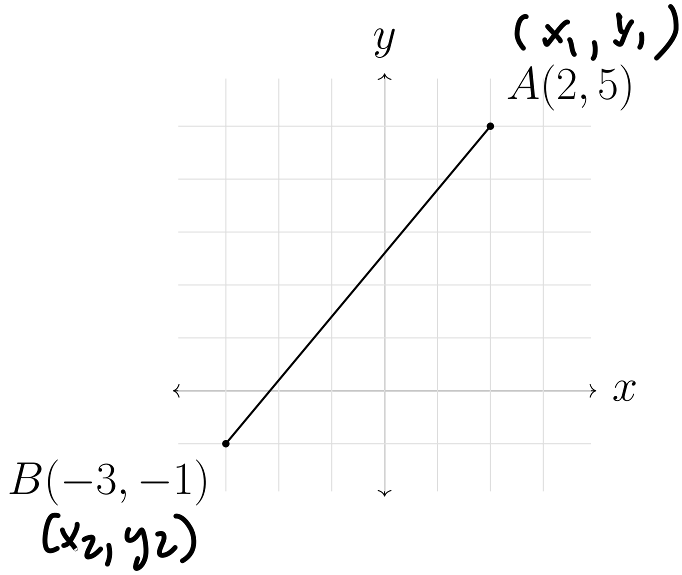
Midpoint Formula: The midpoint $M(x, y)$ of a line segment with endpoints (x_1, y_1) and (x_2, y_2) is:

$$\boxed{M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$



3. Examples

Example 1: Find the distance between points $A(2,5)$ and $B(-3,-1)$

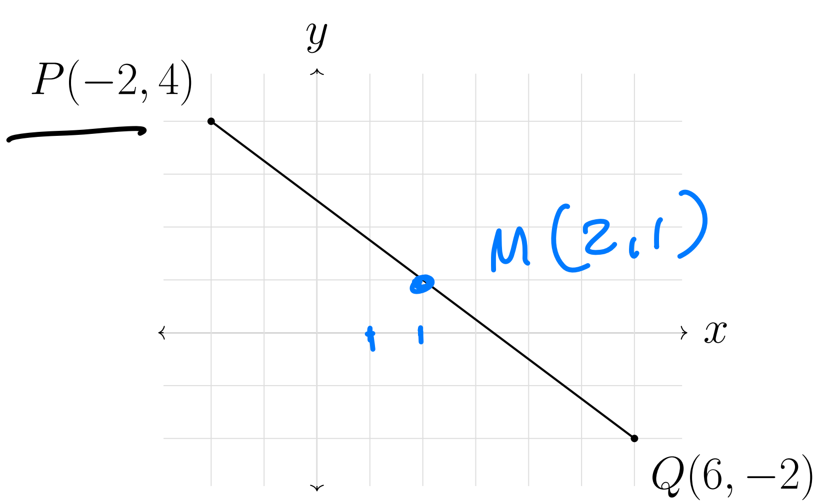


Solution:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-3-2)^2 + (-1-5)^2} = \sqrt{25+36}$$
$$= \sqrt{61}$$

Example 2: Find the midpoint of the line segment with endpoints $P(-2,4)$ and $Q(6,-2)$

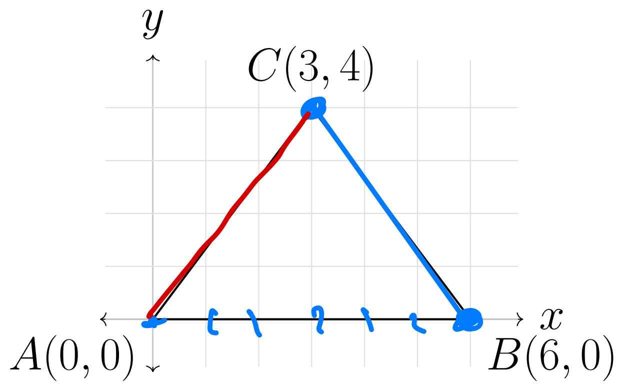


$M(2,1)$

Solution: M have $x = \frac{x_1 + x_2}{2} = \frac{-2+6}{2} = 2$

$y = \frac{y_1 + y_2}{2} = \frac{4 + (-2)}{2} = \frac{2}{2} = 1$

Example 3: Determine if triangle ABC with vertices $A(0,0)$, $B(6,0)$, and $C(3,4)$ is a right triangle.



Solution:

$AB = 6$

$BC = \sqrt{(6-3)^2 + (0-4)^2} = \sqrt{25} = 5$

$AC = \sqrt{(3-0)^2 + (4-0)^2} = 5$

Does it check Pythagorean! $a^2 + b^2 = c^2$

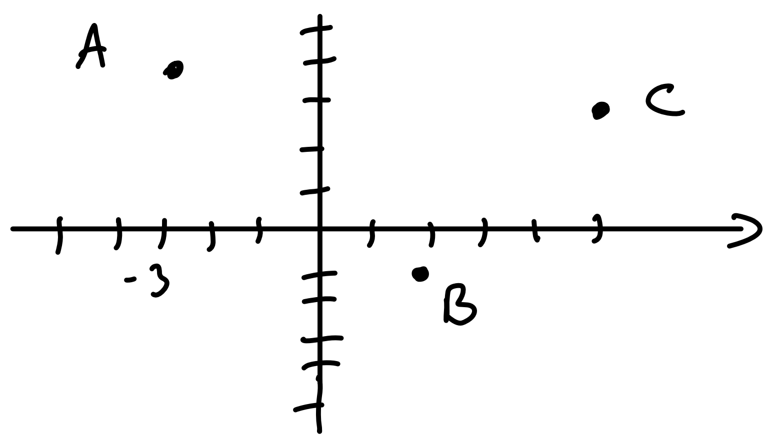
$5^2 + 5^2 \stackrel{?}{=} 6^2$ NO

Practice Problems:

II IV I

1. Plot the points $A(-3,4)$, $B(2,-1)$, and $C(5,3)$. Label which quadrant each point is in.

Check your answer:



2. Find the distance between points $(1,7)$ and $(4,-2)$.

$$d = \sqrt{(1-4)^2 + (7-(-2))^2} = \sqrt{(-3)^2 + 9^2} = \sqrt{9+81}$$
$$= \sqrt{90}$$
$$= 3\sqrt{10}$$