Finding Domains of Functions

Overview: How to Find the Domain of a Function

To find the domain of a function, follow these steps:

Step 1. Start with all real numbers as your initial domain

Step 2. Check for restrictions:

- Denominators cannot equal zero
- Even-root expressions must be non-negative
- The expression inside any square root must be non-negative

Step 3. Solve any inequalities to find restricted values

Step 4. Write the domain using interval notation

Practice Problems

Find the domain of each function:

1. $f(x) = \dfrac{x+3}{x-2}$

2. $f(x) = \sqrt[3]{x-4}$

3. $f(x) = \dfrac{x}{x^2-9}$

4. $f(x) = \sqrt{16-x^2}$

5. $f(x) = \dfrac{x+1}{x^2-4}$

6. $f(x) = \dfrac{\sqrt{x+2}}{x-3}$

7. $f(x) = \sqrt[3]{2x+1}$

8. $f(x) = \dfrac{2x+1}{(x+3)(x-1)}$

9. $f(x) = \sqrt{2x+3} + \dfrac{1}{x-4}$

10. $f(x) = \dfrac{\sqrt{x-1}}{x^2-1}$

Answer Key

1. **Domain** $= (-\infty, 2) \cup (2, \infty)$
   - ▷ Denominator cannot equal zero: $x - 2 \neq 0$
   - ▷ Therefore $x \neq 2$

2. **Domain** $= (-\infty, \infty)$
   - ▷ Cube root accepts all real numbers
   - ▷ Therefore x can be any real number

3. **Domain** $= (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$
   - ▷ Denominator cannot equal zero: $x^2 - 9 \neq 0$
   - ▷ Solve: $x^2 - 9 = 0$, $(x + 3)(x - 3) = 0$
   - ▷ Therefore $x \neq -3$ and $x \neq 3$

4. **Domain** $= [-4, 4]$
   - ▷ Inside square root must be non-negative: $16 - x^2 \geq 0$
   - ▷ Solve: $-4 \leq x \leq 4$

5. **Domain** $= (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
   - ▷ Denominator cannot equal zero: $x^2 - 4 \neq 0$
   - ▷ Solve: $(x + 2)(x - 2) = 0$
   - ▷ Therefore $x \neq -2$ and $x \neq 2$

6. **Domain** $= (3, \infty)$
   - ▷ Inside square root must be non-negative: $x + 2 \geq 0$, so $x \geq -2$
   - ▷ Denominator cannot equal zero: $x - 3 \neq 0$, so $x \neq 3$
   - ▷ Combine restrictions: $x > 3$

7. **Domain** $= (-\infty, \infty)$
   - ▷ Cube root accepts all real numbers
   - ▷ Therefore $2x + 1$ can be any real number
   - ▷ Domain is all real numbers

8. **Domain** $= (-\infty, -3) \cup (-3, 1) \cup (1, \infty)$
   - ▷ Denominator cannot equal zero: $(x + 3)(x - 1) \neq 0$
   - ▷ Therefore $x \neq -3$ and $x \neq 1$

9. **Domain** $= (4, \infty)$

   - ▷ Inside square root must be non-negative: $2x + 3 \geq 0$, so $x \geq -\frac{3}{2}$
   - ▷ Denominator cannot equal zero: $x - 4 \neq 0$, so $x \neq 4$
   - ▷ Combine restrictions: $x > 4$

10. **Domain** $= (1, \infty)$

   - ▷ Inside square root must be non-negative: $x - 1 \geq 0$, so $x \geq 1$
   - ▷ Denominator cannot equal zero: $x^2 - 1 \neq 0$
   - ▷ Solve: $(x + 1)(x - 1) = 0$, so $x \neq -1$ and $x \neq 1$
   - ▷ Combine restrictions: $x > 1$