

Key

Day 1 Group Activity
Tuesday Jan 21st
8.1 Sequences and Series

1. Group Instructions:

During the activity, make sure everyone has a chance to participate and contribute. Listen to each other's ideas and encourage quieter team members to share their thoughts. Show respect for your teammates by not interrupting when others are speaking and considering all suggestions thoughtfully, even if you disagree.

2. Concept Matching

Draw lines to match each concept on the left with its corresponding example on the right.

1. Writing terms from nth term

2. Writing recursive sequences

3. Factorial notation

4. Summation notation

a) $\sum_{i=1}^4 i = 1+2+3+4 = 10$

b) $4! = 4 \times 3 \times 2 \times 1 = 24$

c) $a_n = 2n + 1$ gives $\{3, 5, 7, 9, \dots\}$

d) $a_1 = 2, a_n = a_{n-1} + 3$

3. Practice

Write down complete solutions for the following exercises.

1. Given $a_n = 3n - 2$, write the first four terms.

$n=1: a_1 = 3(1) - 2 = 1$

$n=2: a_2 = 3(2) - 2 = 4$

$n=3: a_3 = 3(3) - 2 = 7$

$n=4: a_4 = 3(4) - 2 = 10$

$\{a_n\} = \{1, 4, 7, 10, \dots\}$

2. Given $a_1 = 5$ and $a_n = 3a_{n-1} - 4$, write the first five terms.

$$n=2: a_2 = 3a_1 - 4 = 3(5) - 4 = 11$$

$$n=3: a_3 = 3a_2 - 4 = 3(11) - 4 = 29$$

$$n=4: a_4 = 3a_3 - 4 = 3(29) - 4 = 83$$

$$n=5: a_5 = 3a_4 - 4 = 245$$

$$\{a_n\} = \{5, 11, 29, 83, 245, \dots\}$$

$$a_n = \frac{n!}{3^n}$$

3. Find the general term, a_n , for the following sequences.

(a) $-5, 6, -7, 8, \dots$

(b) $\frac{1}{5}, -\frac{4}{25}, \frac{9}{125}, -\frac{16}{625}, \dots$

(c) $\frac{1}{3}, \frac{1 \cdot 2}{9}, \frac{1 \cdot 2 \cdot 3}{27}, \frac{1 \cdot 2 \cdot 3 \cdot 4}{81}, \dots$

a) sequence starts with negative so use $(-1)^n$

$$n=1 \quad a_1 = (-1)^1 (1+4)$$

$$n=2 \quad a_2 = (-1)^2 (2+4)$$

$$n=3 \quad a_3 = (-1)^3 (3+4)$$

$$\text{so } a_n = (-1)^n (n+4)$$

b) sequence starts with positive so use $(-1)^{n+1}$

$$n=1 \quad a_1 = (-1)^{1+1} \frac{1^2}{5^1}$$

$$n=2 \quad a_2 = (-1)^{2+1} \frac{2^2}{5^2}$$

$$\text{so } a_n = (-1)^{n+1} \frac{n^2}{5^n}$$

4. Evaluate $\frac{(n-1)!}{n!} = \frac{1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1)}{1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n} = \frac{1}{n}$

5. Write the sum using summation notation.

$$\frac{2}{1} - \frac{3}{4} + \frac{4}{9} - \frac{5}{16} + \frac{6}{25} = \sum_{i=1}^5 (-1)^{i+1} \frac{n+1}{n^2}$$

6. Write the terms for the ~~series~~ ^{sum} and evaluate the sum.

$$\sum_{n=2}^5 (-1)^n \frac{1}{2^n} = (-1)^2 \frac{1}{2^2} + (-1)^3 \frac{1}{2^3} + (-1)^4 \frac{1}{2^4} + (-1)^5 \frac{1}{2^5}$$

$$= \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} = 0.15625$$