Day 5, Tuesday Feb 4th

- Exam 1 next Tuesday
- Written HW 1 posted and due Tuesday
- Tacos & Tutoring
  L135 Wed Feb 5th
  11am - 2pm

Day 5 Notes
Tuesday February 4th
Review 1.4 Factoring Quadratics and 2.2 Circles

1. Review Quadratics

In this section we will review how to factor quadratics.

Definition. Let a, b, and c be real numbers with $a \neq 0$. A **quadratic equation** in the variable x is an equation of the form

$$ax^2 + bx + c = 0$$

To solve a quadratic means to find the x value(s) that satisfy the equation.

$(x-1)(x-1) = 0$

$x = 1$ is only solution

Examples:

- $4x^2 - 10x + 12 = 0$
  trinomial $\Rightarrow$ 3 terms
- $7x^2 + 8x = 0$
  binomial $\Rightarrow$ 2 terms
- $8x^2$
  monomial $\Rightarrow$ 1 term

$9 = 3^2$, $5 = ?$, $16 = 4^2$ — perfect squares

Solving using the zero product property and factoring

(1) $x^2 - 8x = 0$

$x(x-8) = 0$ solutions $x = 0$

$x - 8 = 0$

$x = 8$

ex: $(x+1)(x+1) = 0$

$(x+1)^2 = 0$

binomial squared

$x^2 + 2x + 1 = (x+1)^2$

trinomial — binomial squared

Example 1. Solve the following quadratic equations.

1. $x^2 - 8x = 0$
2. $2x(2x - 7) = 0$
3. $x^2 - 5x + 6 = 0$
4. $x^2 + 4x - 5 = 0$
5. $x^2 + 10x + 24 = 0$

2) $2x(2x-7)=0$

$2x=0$ or $2x-7=0 \Rightarrow x=7/2$

$2(0)(2\cdot 0-7)=2\cdot 0\cdot(-7)=0$

$2\cdot\frac{7}{2}\cdot(2\cdot\frac{7}{2}-7)=2(\frac{7}{2})(0)=0$

2B) $(x+2)(x-1)=0$

$x+2=0$ or $x-1=0$

$x=-2$ $\quad x=1$

3) $x^2-5x+6=0$

$(x+\square)(x+\square)=0$

$(x-3)(x-2)=0$?

$x^2-2x-3x+6=0$

$(x-1)(x-6)=0$?

what 2 numbers multiplied together give me 6

what 2 numbers added together give -5

Solving quadratics using the Square Root Property

When we can bring the equation in the form $x^2 = k$, we can solve by square rooting both sides.

Example 2. Solve using the square root property.

1. $x^2 = 64$
2. $2y^2 - 36 = 0$
3. $(w + 3)^2 = 8$

$x=?$

$x^2=64$ / square root both sides

don't forget $\pm$!

$\sqrt{x^2}=\pm\sqrt{64}$

$x=\pm 8$ $\quad \begin{cases} x=8 \\ x=-8 \end{cases}$

$y=?$

$+36$

$2y^2=36$ / $\div 2$

$y^2=18$

$y=\pm\sqrt{18}$, $y=\pm\sqrt{9\cdot 2}=\pm 3\sqrt{2}$

5) $x^2 + 10x + 24 = 0$

$(x+4)(x+6) = 0$

$x = -4 \quad x = -6$

24: (1)(24) ✗

(2)(12) ✗

3. 8 ✗

4. 6 ✓

Solving quadratics by completing the square

We can manipulate the quadratic equation $ax^2 + bx + c = 0$ with $a \neq 0$ to write as the square of a binomial equal to a constant.

rewrite $ax^2+bx+c=0$ as

$(x + \text{something})^2 = \text{constant}$

or $(x - \text{something})^2 = \text{constant}$

Example 3. Solve by completing the square.

steps

1) Rewrite $ax^2+bx+c=0$ as $ax^2+bx=-c$

2) Take half of coefficient of x, square it, and add to both sides $(b/2)^2$ added to both sides

3) Factor the left hand side

1. $x^2 - 3 = -10x$

(1) $x^2+10x = 3$

$x^2+10x+\left(\frac{10}{2}\right)^2 = 3+\left(\frac{10}{2}\right)^2$

$x^2+10x+25 = 3+25$

$(x+5)(x+5) = 28$

$(x+5)^2 = 28$ square root $x+5 = \pm\sqrt{28}$

$x = -5 \pm 2\sqrt{7}$

2. $x^2 - 8x - 2 = 0$

3. $-2x^2 - 3x - 5 = 0$

Solving quadratics by using the quadratic formula

$ax^2+bx+c=0$, $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Example 4. Use quadratic formula to solve $x^2 - 6x = 3$

$x^2-6x-3=0$, $a=1, b=-6, c=-3$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2-4(1)(-3)}}{2\cdot 1} = \frac{6 \pm \sqrt{36+12}}{2} = \frac{6 \pm \sqrt{48}}{2}$$

$$= \frac{6 \pm 4\sqrt{3}}{2} = 3 \pm 2\sqrt{3}$$

(3) $-2x^2 - 3x - 5 = 0$ $\div(-2)$

solve by completing the square

$x^2 + \frac{3}{2}x + \frac{5}{2} = 0$

Step 0

make the coefficient of x^2, 1 by dividing by -2

Step 1: $x^2 + \frac{3}{2}x = -\frac{5}{2}$

Step 2: $\left(\frac{b}{2}\right)^2 = \left(\frac{\frac{3}{2}}{2}\right)^2$

$= \left(\frac{3}{4}\right)^2$

$= \frac{9}{16}$

$\frac{\frac{3}{2}}{2} = \frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4}$

$x^2 + \frac{3}{2}x + \frac{9}{16} = -\frac{5}{2} \cdot \frac{8}{8} + \frac{9}{16}$

Step 3: Factor LHS

$\left(x + \frac{3}{4}\right)\left(x + \frac{3}{4}\right) = \frac{-40+9}{16}$

$\left(x + \frac{3}{4}\right)^2 = \frac{-31}{16}$

check

$\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$

$\frac{3}{4} + \frac{3}{4} = \frac{6}{4}$

$= \frac{3}{2}$

2) $x^2 - 8x - 2 = 0$ Solve by completing the square

1) $x^2 - 8x = 2$

2) $x^2 - 8x + \left(-\frac{8}{2}\right)^2 = 2 + \left(-\frac{8}{2}\right)^2$

$x^2 - 8x + 16 = 2 + 16$

$(x-4)(x-4) = 18$

$(x-4)^2 = 18$ / Square root both sides ±!

$x - 4 = \pm\sqrt{18}$

$x - 4 = \pm\sqrt{2 \cdot 9}$

$x - 4 = \pm 3\sqrt{2}$

$x = 4 \pm 3\sqrt{2}$