Day 3 notes
Jan 28th
8.2/8.3 Arithmetic and Geometric Sequences

1. Arithmetic Sequences

In this section we are going to introduce Arithmetic Sequences. The simplest way to generate an arithmetic sequence is to start with a number a and add to it a fixed constant d, over and over again.

Definition: An arithmetic sequence is a sequence of the form:

The number a is __________________ and d is ____________________________.

The nth term of an arithmetic sequence is given by:

Example 1: Is this sequence an arithmetic sequence? $13, 7, 1, -5, \cdots$.

If yes, find the common difference, the next three terms, the nth term, and the 300th term of the arithmetic sequence

Example 3: Write the first five terms of the sequence $a_n = -4 + 3n$. Determine whether or not the sequence is arithmetic. If it is, find the common difference.

Example 2: The 11th term of an arithmetic sequence is 52, and the 19th term is 92. Find the 1000th term.

Arithmetic sequence recursive formula:

2. Partial Sums of Arithmetic Sequences

PARTIAL SUMS OF AN ARITHMETIC SEQUENCE

For the arithmetic sequence given by $a_n = a + (n-1)d$, the ***n*th partial sum**

$$S_n = a + (a + d) + (a + 2d) + (a + 3d) + \cdots + [a + (n-1)d]$$

is given by either of the following formulas.

1. $S_n = \frac{n}{2}[2a + (n-1)d]$ **2.** $S_n = n\left(\frac{a + a_n}{2}\right)$

Example 4: a) Find the sum of the first 100 numbers.

b) Find the sum of the first 50 odd numbers.

c) Sum the first 25 terms of the sequence $3, -1, -5, -9, \cdots$

3. Geometric Sequences

Next, we are going to introduce Geometric Sequences. A geometric sequence is generated when we start with a number a and repeatedly multiply by a fixed nonzero constant r.

Definition: A geometric sequence is a sequence of the form:

The number a is ________________ and r is ________________________.

The nth term of a geometric sequence is given by:

Example 5: If $a = 3$ and $r = 2$, then we have the geometric sequence:

Example 6: The sequence $2, -10, 50, -250, 1250, \cdots$ is a geometric sequence with $a = 2$ and $r = -5$. Find a_{10}.

Example 6: Find the common ratio, the first term, the nth term, and the eighth term of the geometric sequence $5, 15, 45, 135, \cdots$

4. Partial Sums of Geometric Sequences

The partial sum s_n of a geometric sequence is given by the formula below.

PARTIAL SUMS OF A GEOMETRIC SEQUENCE

For the geometric sequence defined by $a_n = ar^{n-1}$, the ***n*th partial sum**

$$S_n = a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1} \qquad r \neq 1$$

is given by

$$S_n = a\frac{1 - r^n}{1 - r}$$

Example 7: Find the following partial sum of a geometric sequence: $1 + 4 + 16 + \cdots + 4096$

5. Infinite Geometric Series

An infinite geometric series is a series of the form $a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1} + \cdots$.

SUM OF AN INFINITE GEOMETRIC SERIES

If $|r| < 1$, then the infinite geometric series

$$\sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + ar^3 + \cdots$$

converges and has the sum

$$S = \frac{a}{1 - r}$$

If $|r| \geq 1$, the series diverges.

Example 8: Determine whether the infinite geometric series is convergent or divergent. If it is convergent, find its sum.

(a) $2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \cdots$

(b) $1 + \frac{7}{5} + \frac{49}{25} + \cdots$