Day 1 Notes
Wednesday Jan 22nd
1.2 Functions and Relations

1. Functions and Relations

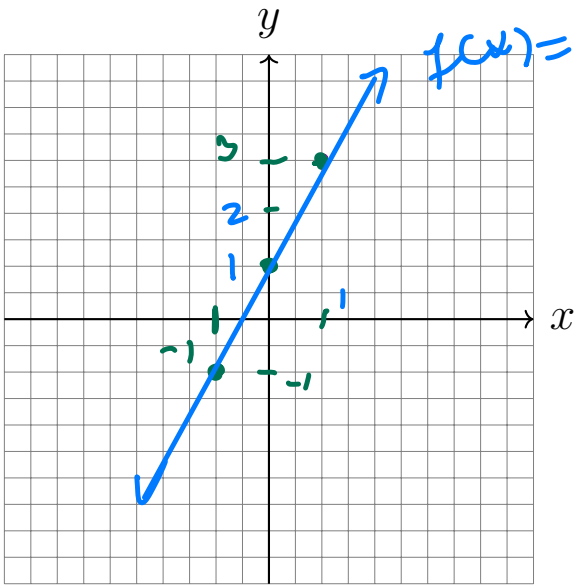
In nearly every physical phenomenon, we observe that one quantity depends on another. For example, the cost of tuition depends on the number of credits you take, or the area of a circle depends on its radius.

We use the term **function** to describe this dependence of one quantity on another.
We will use letters such as $f, g, h \cdots$ to represent functions.
We can represent a function in **four different ways**. For example, we can use the letter f to represent a rule that doubles the input and then adds 1.

1. **Verbally:** f is the rule that doubles the input x and then adds 1.
2. **Algebraically**: $y = 2x+1$ or function notation $f(x) = 2x+1$
   x → independent variable input and y output → dependent variable
3. **Numerically:** a table

x	-1	0	1	2
y	-1	1	3	5

4. **Graphically**:



Formal definition of a function:

A function f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B. $y = f(x)$

Set A is called the domain of the function and it consists of valid inputs.

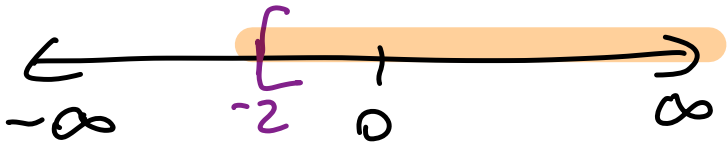
Set B is called the range of the function and it consists of all possible $f(x)$, as x varies throughout the domain.

Ex 0 $f(x) = \sqrt{x+2}$

Domain : $[-2, \infty)$

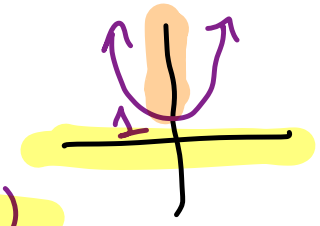
Range : $[0, \infty)$

use interval notation



Example 1. Identify if the following relations are functions. If yes, identify the domain and range.

1. $y = x^2 + 1$



yes

Domain: all Real numbers $(-\infty, \infty)$

Range : $[1, \infty)$

2.

x	y
1	2
1	3
2	4

No

3.

x	y
1	5
2	7
3	5

yes

Domain : $\{1, 2, 3\}$

Range : $\{5, 7\}$

2. Piecewise defined functions

Sometimes a function has different rules for different portions of the domain and when that happens we call those **piecewise defined functions**.

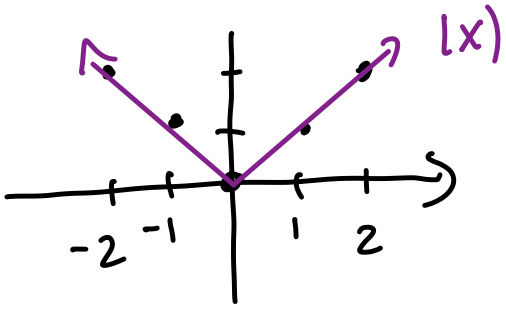
$$f(x) = \begin{cases} \text{formula 1}, & x \geq 0 \\ \text{formula 2}, & x < 0 \end{cases} \qquad f(x) = \begin{cases} 2x+1, & x \geq 0 \\ x^2+1, & x < 0 \end{cases}$$

Example 2. So, for example, if a cell phone company charges \$40 a month if you use 500 minutes or less, but for every minute beyond the allowed 500 minutes you get charged a quarter a minute, the function that calculates the cost of the phone plan will be a piecewise defined function.
Find a formula for the cost of this phone plan, $C(x)$ where x is the number of minutes used in a month period.

$$C(x) = \begin{cases} 40, & x \leq 500 \\ 40 + 0.25(x-500), & x > 500 \end{cases}$$

Example 3. Recall the absolute value function $f(x) = |x|$.

1. Graph f.



$f(2) = |2| = 2$
$f(-2) = |-2| = 2$
$f(0) = 0$

2. Rewrite f as a piecewise defined function.

$$f(x) = \begin{cases} -x, & x \leq 0 \\ x, & x > 0 \end{cases}$$

MTH 144 Introductions Please take a few moments to complete this form. (2)