DAY 3, Tuesday Jan 28th

Recall:

- Sequence: $a_1, a_2, a_3, \dots$
- Series: $a_1 + a_2 + a_3 + \dots$
- Sigma notation $\sum_{i=1}^{\infty} a_i$
- Partial sum $a_1 + a_2 + \dots + a_{10} = \sum_{i=1}^{10} a_i$
- Factorial notation $n! = 1 \cdot 2 \cdot 3 \cdots n$

$$(n+2)! = 1 \cdot 2 \cdot 3 \cdots n(n+1)(n+2)$$

$$(n+2)! = n!\,(n+1)(n+2)$$

$$(n+2)! = (n+1)!\,(n+2)$$

ex

$$\frac{(n-1)!}{(n+3)!} = \frac{\cancel{(n-1)!}}{\cancel{1 \cdot 2 \cdot 3 \cdots (n-1)}\, n \cdot (n+1)(n+2)(n+3)}$$

$$= \frac{1}{n(n+1)(n+2)(n+3)}$$

Day 3 notes
Jan 28th
8.2/8.3 Arithmetic and Geometric Sequences

1. Arithmetic Sequences

In this section we are going to introduce Arithmetic Sequences. The simplest way to generate an arithmetic sequence is to start with a number a and add to it a fixed constant d, over and over again.

positive or negative

Definition: An arithmetic sequence is a sequence of the form:

$$\underset{a_1}{a}, \underset{a_2}{a+d}, \underset{a_3}{a+2d}, \underset{a_4}{a+3d}, \ldots, \underset{a_n}{a+(n-1)d}, \ldots$$

The number a is <u>the first term</u> and d is called <u>the common difference</u>.

$a_2 - a_1 = \cancel{a} + d - \cancel{a} = d$
$a_3 - a_2 = a + 2d - a - d$
$= d$

The nth term of an arithmetic sequence is given by:

$$\boxed{a_n = a + (n-1)d}$$

Example 1: Is this sequence an arithmetic sequence? $\underset{a_1}{13}, \underset{a_2}{7}, \underset{a_3}{1}, \underset{a_4}{-5}, -11, -17, -23, \ldots$

$+(-6)$ $+(-6)$

$a_2 - a_1 = 7 - 13 = -6$, $a_3 - a_2 = 1 - 7 = -6$, $a_4 - a_3 = -5 - 1 = -6$

If yes, find the common difference, the next three terms, the nth term, and the 300th term of the arithmetic sequence

$a_5 = ?$ $a_6 = ?$, $a_7 = ?$

yes. $d = -6$

$a = 13$

$a_n = a + (n-1)d$ $\xrightarrow{n=300}$ $a_{300} = 13 + (300-1)(-6) = -1781$

$= 13 + 299(-6) = -1781$

$a_5 = 13 + (4-1)(-6) = -11$

$a_6 = 13 + (6-1)(-6) = -17$

$a_7 = 13 + (7-1)(-6) = -23$

Example 3: Write the first five terms of the sequence $a_n = -4 + 3n$. Determine whether or not the sequence is arithmetic. If it is, find the common difference.

$a_1 = -4 + 3(1) = -1$
$a_2 = -4 + 3(2) = 2$
$a_3 = 5$
$a_4 = 8$
$a_5 = 11$

+3 +3 +3

yes $d = 3$
$a = -1$

Example 2: The 11th term of an arithmetic sequence is 52, and the 19th term is 92. Find the 1000th term.

$a_{11} = 52$

$a_{19} = 92$

$a + 10d = 52$
$a + 18d = 92$ (subtract first equation from it)

$a + 18d - (a + 10d) = 92 - 52$
$\cancel{a} + 18d \cancel{-a} - 10d = 40$
$8d = 40 \Rightarrow d = 5$

$a_{1000} = ?$
$a_{1000} = a + (1000 - 1)d$
$a_{1000} = 2 + 999(5)$
$a_{1000} = 4997$

$a_n = a + (n-1)d$
$a_{11} = a + (11-1)d$
$\boxed{a_{11} = a + 10d}$
$a_{19} = a + (19-1)d$
$a_{19} = a + 18d$

$a = 2$ from $a + 10d = 52$
$a + 10 \cdot 5 = 52$

Arithmetic sequence recursive formula:

$$\boxed{\begin{aligned} a_1 &= a \\ a_n &= a_{n-1} + d \end{aligned}}$$

$$\boxed{\begin{aligned} a_1 &= a \\ a_n &= a + (n-1)d \end{aligned}}$$

$a_1 = a$
$a_2 = a_1 + d$
$a_3 =$

$a_1 = a$
$a_2 = a + d$
$a_3 = a + 2d = (a + d) + d$
$= a_2 + d$

2. Partial Sums of Arithmetic Sequences

PARTIAL SUMS OF AN ARITHMETIC SEQUENCE $S_n = a_1 + a_2 + \cdots + a_n$

For the arithmetic sequence given by $a_n = a + (n-1)d$, the ***n*th partial sum**

$$S_n = \underbrace{a}_{a_1} + \underbrace{(a+d)}_{a_2} + \underbrace{(a+2d)}_{a_3} + \underbrace{(a+3d)}_{a_4} + \cdots + \underbrace{[a+(n-1)d]}_{a_n}$$

is given by either of the following formulas.

1. $S_n = \frac{n}{2}[2a + (n-1)d]$ **2.** $S_n = n\left(\frac{a + a_n}{2}\right)$ $S_n = n\left(\frac{a_1 + a_n}{2}\right)$

Example 4: a) Find the sum of the first 100 numbers.

$1 + 2 + 3 + \cdots + 100 = ?$

$1, 2, 3, \ldots, 100, \ldots$

$a = 1$

$d = 1$

$S_{100} = ?$

$S_{100} = 100\left(\frac{1+100}{2}\right) = \frac{100 \cdot 101}{2} = 5050$

b) Find the sum of the first 50 odd numbers.

$\underset{a_1}{1} + \underset{a_2}{3} + \underset{a_3}{5} + \underset{a_4}{7} + \cdots + \underset{a_{50}}{99} = ?$

$1, 3, 5, 7, \ldots$

$a = 1$

$d = 2$

First find $a_{50} = a + (50-1)d$ $\quad a_n = a + (n-1)d$

$a_{50} = 1 + 49(2) = 99$

used $S_n = n\left(\frac{a_1 + a_n}{2}\right)$

Ans: $S_{50} = 1 + 3 + 5 + \cdots + 99 = 50\left(\frac{1+99}{2}\right) = 2500$

c) Sum the first 25 terms of the sequence $3, -1, -5, -9, \cdots$

$-1 - 3 = -4$

$a = 3$

$d = -4$

$a_{25} = 3 + (25-1)(-4)$

$= -93$

$\underset{a_1}{3}, \underset{a_2}{-1}, \underset{a_3}{-5}, \underset{a_4}{-9}, \ldots, -93$

$a_{25} = ?$

$S_{25} = 3 + (-1) + (-5) + (-9) + \cdots + (-93)$

$= 25\left(\frac{3 + (-93)}{2}\right) = -1125$