Day 5 Notes
Wednesday Feb 5th
8.1 Sequences and Series

1. Sequences

In this section we are going to introduce Sequences and Series which are fundamental mathematical concepts that are important for understanding patterns and are used in practice to model population growth in biology and analyze algorithms in computer science.

Definition. Roughly speaking, a **sequence** is an

____

Example 1. A simple example is the sequence:

$$5, \quad 10, \quad 15, \quad 20, \quad \cdots$$

The numbers in the sequence are called ________________________ and are often written as a_1, a_2, a_3,$\cdots$. The dots mean that the list continues forever.

Formal definition:

Notation:

a_2

a_n

$\{a_n\}$

We can describe the pattern of the sequence displayed above by a formula for the general term:

Example 2. Find the first three terms and the 100th term of the sequence defined by each formula.

1. $a_n = 2n - 1$

   $\{a_n\} =$

2. $b_n = n^2 - 1$

   $\{b_n\} =$

3. $c_n = \dfrac{n}{n+1}$

   $\{c_n\} =$

4. $d_n = \dfrac{(-1)^{n+1}}{n}$

   $\{d_n\} =$

Example 3. Find the general term of a sequence whose first several terms are given.

(a) $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \cdots$

(b) $-2, 4, -8, 16, -32, \cdots$

Definition. An **alternating sequence** is a sequence with

__________________________________.

2. Recursively defined sequences

Some sequences do not have simple defining formulas like those of the preceding example.

Definition. When the nth term of a sequence may depend on some or all of the terms preceding it we call the sequence __________________________.

Example 4. Example of recursive sequence:
$a_1 = 4$ and $a_n = 2a_{n-1} + 1$ for $n > 1$.

Example 5. The Fibonacci Sequence:
Find the first 7 terms of the sequence defined recursively by
$a_1 = 1$, $a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$.

If we add the terms of a sequence we obtain a **series**.

Sequence:

ex:

Series:

ex:

Partial sum:

ex:

ex:

Sigma notation: [1]

Example 6. Write the sum using summation notation.

1. $5 + 10 + 15 + 20 + 25 + 30$
2. $\dfrac{\sqrt{3}}{1} + \dfrac{\sqrt{4}}{2} + \dfrac{\sqrt{5}}{3} + \cdots + \dfrac{\sqrt{n+2}}{n}$
3. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$

[1] We use **Sigma notation** for both infinite series and finite series.

Example 7. Write the terms for the sum and evaluate the sum.
$\sum_{n=1}^{5} 2n+3$

Note: In Calculus you will learn how to find what an infinite series adds up to for certain types of series.
Example 8. Consider the series:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$$

Lets investigate the first few partial sums:
$S_1 =$
$S_2 =$
$S_3 =$
$S_4 =$
As we continue adding terms, the sum gets closer and closer to 1.

Example 9. Consider the series:

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$$

Lets investigate the first few partial sums:
$S_1 =$
$S_2 =$
$S_3 =$
$S_4 =$
As we continue adding terms, the sum grows larger and larger without bound. This series does not add up to an actual value.

4. Factorial notation

Factorials form the basis for important series like Taylor series expansions.

Definition. The factorial of a positive integer n, written as $n!$, is

____.

For example, $5! =$

$10! =$

$n! =$

By definition, $0! =$

Example 10. Evaluate the following expressions.

(a) $\dfrac{8!}{3! \cdot 5!}$

(b) $\dfrac{n!}{(n+2)!}$

(c) $\dfrac{(n+1)!}{(n-1)!}$

(d) $\dfrac{(2n)!}{(n)!}$

Example 11. Given the sequence defined by $b_n = \dfrac{n^2}{(n+1)!}$ find b_1 and b_6.