

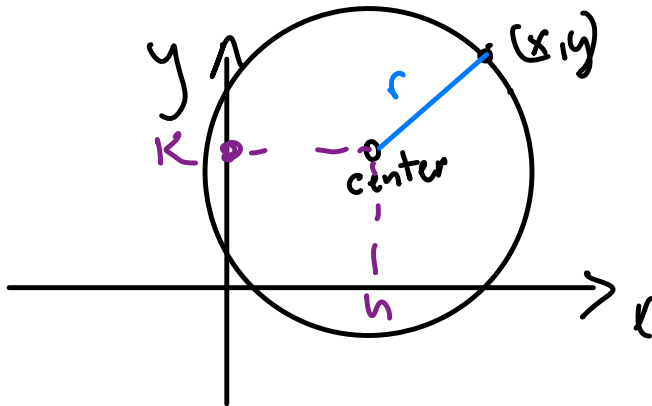
Day 6

- Exam 1 on Tuesday
- HWH 1 posted

2. Circles

Next, we will start graphing specific categories of equations such as circles, ellipses, hyperbolas and parabola. We begin with circles.

Definition. A circle is the set of all points in a plane that are **equidistant** from a fixed point called center. The fixed distance from any point on the circle to the center is called the radius.



r - radius
 center: (h, k)
 \downarrow \downarrow
 x coord. of center y coord. of center

Standard form of an equation of a circle

Given a circle centered at (h, k) with radius r , the standard form is

$$(x - h)^2 + (y - k)^2 = r^2$$

Example 5.

- $(x - 4)^2 + (y + 3)^2 = 25$ \rightarrow center: $(h, k) = (4, -3)$; $r = 5$
 $(x - 4)^2 + (y - (-3))^2 = 5^2$
- $x^2 + (y - 1/2)^2 = 12$ \rightarrow center: $(h, k) = (0, 1/2)$; $r = \sqrt{12} = \sqrt{4 \cdot 3}$
 $(x - 0)^2 + (y - 1/2)^2 = (\sqrt{12})^2$
 $r = \sqrt{4 \cdot 3}$
 $r = 2\sqrt{3}$
- $x^2 + y^2 = 7$ \rightarrow center: $(h, k) = (0, 0)$; $r = \sqrt{7}$
 $(x - 0)^2 + (y - 0)^2 = (\sqrt{7})^2$

Example 6. Write the standard form of an equation of a circle with center $(-4, 6)$ and radius 2. Graph the circle.

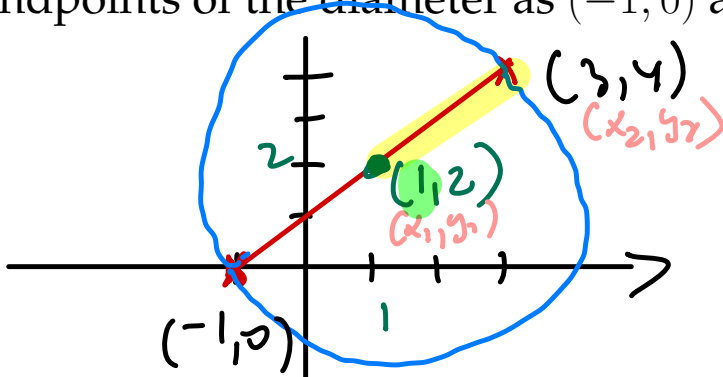
$$(h, k) = (-4, 6)$$

$$r = 2$$

$$(x - (-4))^2 + (y - 6)^2 = 4$$

$$(x + 4)^2 + (y - 6)^2 = 4$$

Example 7. Write the standard form of an equation of a circle with the endpoints of the diameter as $(-1, 0)$ and $(3, 4)$. Graph the circle.



center = midpoint

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-1 + 3}{2}, \frac{0 + 4}{2} \right) = (1, 2) \quad (h, k)$$

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{(3 - (-1))^2 + (4 - 0)^2} = \sqrt{8} = 2\sqrt{2}$$

$$(x - 1)^2 + (y - 2)^2 = 8$$

General form of an Equation of a circle

An equation of a circle written in the form $x^2 + y^2 + Ax + By + C = 0$ is called the general form of an equation of a circle.

Example 8. Write the equation of the circle in standard form.

$$x^2 + y^2 + 10x - 6y + 25 = 0$$

Steps

$$x^2 + 10x + y^2 - 6y = -25$$

1) Rewrite the equation by grouping the x terms and y terms.

$$(x^2 + 10x + 25) + (y^2 - 6y + 9) = -25 + 25 + 9$$

2) Complete the square for binomial in x and for binomial in y

$$(x + 5)^2 + (y - 3)^2 = 9$$

center $(-5, 3)$

$$r = 3$$

Exam 1 Material

8.1 Sequences and series

- Given $a_n = 2n-1$

$$a_1, a_2, a_3 = ? \quad a_{70} = ?$$

- Given a_1, a_2, a_3, \dots

$$a_n = ?$$

$n=1 \rightarrow a_1 = \dots *$
 $n=2 \rightarrow a_2 = \dots *$

- Recursive sequences

$$a_1 = 2, \quad a_n = 2a_{n-1} + 5$$

- Factorial notation

$$\frac{2n!}{(2n+1)!} = \frac{\cancel{(2n)!}}{\cancel{1 \cdot 2 \cdot \dots \cdot (2n)} (2n+1)}$$

- Sigma notation $\sum_{i=1}^n i^2$

- Series $a_1 + a_2 + a_3 + \dots$
Sequence a_1, a_2, a_3, \dots

8.2 & 8.3 Arithmetic & Geom.

$$a_n =$$

$$S_n =$$

d

r

2.1 Midpoint & Distance Formulas

1.4 Quadratic Equation

- factoring

- sq root

- quadratic formula

- completing square

2.2 Circles