

Active neutrino self-interaction with NSI

Contents

1	Particle model	2
1.1	Properties of ν_e	2
1.2	Scalar NSI	2
1.3	Vector NSI	3
2	Differences from DW mechanism	3
3	Boltzmann Equation	3
4	Interaction rate	4
4.1	Scalar NSI	4
4.2	Vector NSI	4
5	Thermal potential	4
5.1	Scalar NSI(B.3.1)	4
5.2	Vector NSI(B.3.2)	4
5.3	Scalar Mediator(B.3.3)	5
A	Calculation of Interaction rate	6
A.1	Matrix elements	6
A.1.1	$\nu_e + \nu_e \rightarrow \nu_e + \nu_e$	7
A.1.2	$\nu_e + \bar{\nu}_e \rightarrow \nu_e + \bar{\nu}_e$	7
A.2	Total cross-section	8
A.3	Interaction rate	10
B	Calculation of Thermal potential	11
B.1	Neutrino Dispersion Relation	11
B.2	Effective potential	13
B.3	Self energy calculation	13
B.3.1	Scalar NSI	14
B.3.2	Vector NSI	15
B.3.3	Scalar mediator	16

1 Particle model

We consider an effective scalar and vector NSI between active neutrinos in addition to SM processes.

If the self-interaction is strong, it could keep active neutrinos in thermal equilibrium with themselves longer than the weak interaction, and enhance DW production rate of sterile neutrinos.

General NSI Lagrangian for active neutrino self-interaction,

$$\mathcal{L}_j = -\frac{G_F}{\sqrt{2}} (\epsilon_{j,\nu})^{\alpha\beta\gamma\delta} (\bar{\nu}_\alpha \mathcal{O}_j \nu_\beta) (\bar{\nu}_\gamma \mathcal{O}'_j \nu_\delta) \quad (1.1)$$

Since we want to study testability in KATRIN, we would use electron flavor as the one and only flavor in our model.

Hence, $\alpha = \beta = \gamma = \delta = e$

Then (1.1),

$$\mathcal{L}_j = -\frac{G_F}{\sqrt{2}} (\epsilon_{j,\nu_e})^{eeee} (\bar{\nu}_e \mathcal{O}_j \nu_e) (\bar{\nu}_e \mathcal{O}'_j \nu_e) \quad (1.2)$$

1.1 Properties of ν_e

- The “neutrino” ν_e in the model is a **Dirac fermion**.
what about the point that if ν is Majorana, they can't be sterile?
- This neutrino has two chiral components, $\nu_e = \nu_L + \nu_R$
- ν_L represent LH component of neutrino field and **active neutrino** ν_a .
- ν_R represent RH component of neutrino field and **sterile neutrino** ν_s .
- neutrino oscillation : $\nu_a \leftrightarrow \nu_s (\nu_L \leftrightarrow \nu_R)$

1.2 Scalar NSI

We would introduce a scalar interaction so that we can compare our results with [1]

$$\mathcal{O}_S = (1 - \gamma^5) \quad \mathcal{O}'_S = (1 - \gamma^5)$$

Here $(1 - \gamma^5)$ implies that we are dealing with left-handed active neutrinos.

$$\mathcal{L}_S = -\frac{G_F}{\sqrt{2}} (\epsilon_{S,\nu_e})^{eeee} (\bar{\nu}_e (1 - \gamma^5) \nu_e) (\bar{\nu}_e (1 - \gamma^5) \nu_e) \quad (1.3)$$

We could see this as a low energy EFT for a scalar heavy mediator ϕ ($m_\phi \gg T$) with interaction,

$$\mathcal{L}_s \supset \frac{\lambda_\phi}{2} \nu_e \nu_e \phi + \text{h.c} \quad (1.4)$$

then, coefficients could be,

$$\frac{G_F}{\sqrt{2}} (\epsilon_{j,\nu_e})^{eeee} = \frac{\lambda_\phi^2}{m_\phi^2}$$

and this would correspond to Case A of Fig.2 of [1]

1.3 Vector NSI

We would introduce a vector interaction so that we can compare our results with [2]

$$\begin{aligned}\mathcal{O}_V &= \gamma^\mu(1 - \gamma^5) & \mathcal{O}'_V &= \gamma_\mu(1 - \gamma^5) \\ \mathcal{L}_V &= -\frac{G_F}{\sqrt{2}} (\epsilon_{V,\nu_e})^{eeee} (\bar{\nu}_e \gamma^\mu(1 - \gamma^5) \nu_e) (\bar{\nu}_e \gamma_\mu(1 - \gamma^5) \nu_e)\end{aligned}\quad (1.5)$$

2 Differences from DW mechanism

	DW only	DW + active-active NSI
Interaction rate	Γ_{SM}	$\Gamma_{\text{SM}} + \Gamma_{\text{NSI}}$
Thermal potential	$\mathcal{V}_{\nu_e} + \mathcal{V}_e$	$\mathcal{V}_{\nu_e} + \mathcal{V}_e + \mathcal{V}_{\text{NSI}}$
mechanism	freeze-in	freeze-in
sterile neutrino mass	$\sim \text{keV}$	$\sim \text{keV}$
Temperature range	$\sim \mathcal{O}(100) \text{ MeV}$ to $\sim \mathcal{O}(1) \text{ MeV}$ (neutrino decoupling)	self-interaction could make neutrinos stay longer in the plasma and extend window of production
Sterile neutrino collision terms	$\mathcal{C}_S = 0$	$\mathcal{C}_S = 0$

Are there any other points?

3 Boltzmann Equation

Since we assume NSI can only modify DW mechanism and there are no other channels to produce sterile neutrinos,

$$\frac{\partial}{\partial t} f_s(p, t) - H p \frac{\partial}{\partial p} f_s(p, t) \approx \frac{1}{4} \frac{\Gamma_\alpha(p) \Delta^2(p) \sin^2 2\theta}{\Delta^2(p) \sin^2 2\theta + \left(\frac{\Gamma_\alpha(p)}{2}\right)^2 + [\Delta(p) \cos 2\theta - \mathcal{V}_T(p)]^2} [f_\alpha(p, t) - f_s(p, t)] \quad (3.1)$$

Here,

$$\begin{aligned}\Gamma_\alpha &= \Gamma_{\text{SM}} + \Gamma_{\text{NSI}} \\ \mathcal{V}_T &= \mathcal{V}_{\text{SM}} + \mathcal{V}_{\text{NSI}} \\ \Delta &= \frac{m_s^2}{2p}\end{aligned}$$

We can also set $f_s(p, T) = 0$ if we assume there is no initial abundance of sterile neutrinos before the production.

4 Interaction rate

In the interaction rate calculations, we consider processes involving active neutrinos. We have SM processes with interaction rate[3],

$$\Gamma_a = C_a(p, T) G_F^2 p T^4 \quad (4.1)$$

where $C_a(p, T)$ is a momentum- and temperature dependent coefficient.

In our extension, following processes will get extra contribution from active neutrino self-interaction.

$$\nu_e + \nu_e \rightarrow \nu_e + \nu_e$$

$$\nu_e + \bar{\nu}_e \rightarrow \nu_e + \bar{\nu}_e$$

$$\bar{\nu}_e + \bar{\nu}_e \rightarrow \bar{\nu}_e + \bar{\nu}_e$$

if you check [2], they have only given total cross-section for $\nu_e + \nu_e \rightarrow \nu_e + \nu_e$ and $\nu_e + \bar{\nu}_e \rightarrow \nu_e + \bar{\nu}_e$ (they mention the reason in paragraph between Eq. 3.6 and 3.7

Check Appendix A for detailed calculation

4.1 Scalar NSI

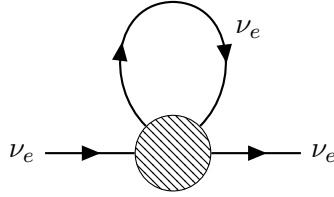
$$\Gamma_S(E, T) = \frac{14\pi G_F^2 \epsilon_S^2}{225} E T^4 \quad (4.2)$$

4.2 Vector NSI

$$\Gamma_V(E, T) = \frac{224\pi G_F^2 \epsilon_V^2}{225} E T^4 \quad (4.3)$$

5 Thermal potential

In addition to SM self-energy diagrams, self-interaction can contribute a new diagram for thermal potential calculation.



Check Appendix B for detailed calculation

5.1 Scalar NSI(B.3.1)

$$\mathcal{V}_{\text{scalar}} = \pm \sqrt{2} G_F (\epsilon_{S, \nu_e})^{eeee} (n_\nu - n_{\bar{\nu}}) \quad (5.1)$$

(+ for ν and $(-)$ for $\bar{\nu}$

5.2 Vector NSI(B.3.2)

$$\mathcal{V}_{\text{vector}} = \mp 2\sqrt{2} G_F (\epsilon_{S, \nu_e})^{eeee} (n_\nu - n_{\bar{\nu}}) \quad (5.2)$$

$(-)$ for ν and $(+)$ for $\bar{\nu}$

5.3 Scalar Mediator(B.3.3)

$$\mathcal{V}_\phi = \pm \frac{2\lambda_\phi^2}{m_\phi^2} (n_\nu - n_{\bar{\nu}}) + \frac{7\pi^2\lambda_\phi^2}{45m_\phi^4} ET^4 \quad (5.3)$$

- There is no explicit mediator mass in NSI, hence it is not possible to get a term with $\frac{1}{m^4}$.
(For analogy, if we use 4-Fermi theory to calculate SM thermal potential, we wouldn't be able to get terms $\sim \frac{G_F}{m_{W,Z}^2} ET^4$)
- if the mediator mass is similar to W,Z boson mass, we might miss information on possible resonance(if the sign is opposite) when we use EFT.
- But it is said that NSI is an EFT below EW scale. If it means $m_\phi > 250\text{GeV}$, relevant NSI thermal potential terms would be much smaller than SM thermal potential, and we can possibly neglect them.

A Calculation of Interaction rate

Please check:

- whether co-efficients and signs are correct in the matrix elements
- if any reaction is missing

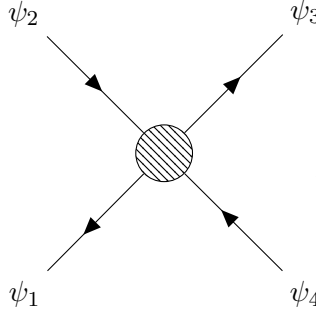
A.1 Matrix elements

EFT vertex

From [4], 4-Fermion EFT vertex for a general Lagrangian

$$\mathcal{L}_{\text{int}} \equiv \sum C^{f_1 f_2 f_3 f_4} \Gamma_{s_1 s_2} \Gamma_{s_3 s_4} \bar{\psi}_{s_1}^{f_1} \psi_{s_2}^{f_2} \bar{\psi}_{s_3}^{f_3} \psi_{s_4}^{f_4} \quad (\text{A.1})$$

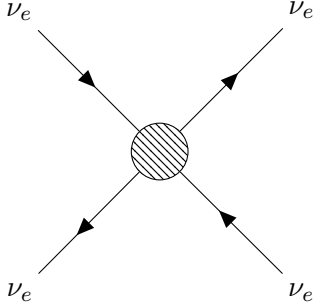
is given by,



$$\longrightarrow i\mathbb{T}_{1234} \equiv 2iC^{f_1 f_2 f_3 f_4} \Gamma_{s_1 s_2} \Gamma_{s_3 s_4} - 2iC^{f_1 f_4 f_3 f_2} \Gamma_{s_1 s_4} \Gamma_{s_3 s_2} \quad (\text{A.2})$$

For NSI Lagrangian,

$$\mathcal{L}_j = -\frac{G_F}{\sqrt{2}} (\epsilon_{j, \nu_e})^{eeee} (\bar{\nu}_e \mathcal{O}_j \nu_e) (\bar{\nu}_e \mathcal{O}_j \nu_e) \quad (\text{A.3})$$

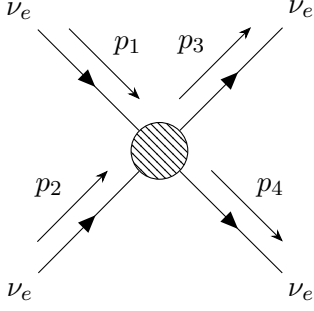


$$\longrightarrow i\mathbb{T}_{1234} \equiv -2i \frac{G_F}{\sqrt{2}} (\epsilon_{j, \nu_e})^{eeee} [\mathcal{O}_{s_1 s_2} \mathcal{O}_{s_3 s_4} - \mathcal{O}_{s_1 s_4} \mathcal{O}_{s_3 s_2}] \quad (\text{A.4})$$

$u(p_i)$ or $\bar{u}(p_i) \equiv$ spinors for neutrinos

$v(p_i)$ or $\bar{v}(p_i) \equiv$ spinors for anti-neutrinos

A.1.1 $\nu_e + \nu_e \rightarrow \nu_e + \nu_e$



$$-i\mathcal{M} = [\bar{u}(p_4)\bar{u}(p_3)] [i\Gamma_{4231}] [u(p_1)u(p_2)] \quad (\text{A.5})$$

$$-i\mathcal{M} = [\bar{u}(p_4)\bar{u}(p_3)] \left(-2i\frac{G_F}{\sqrt{2}} (\epsilon_{j,\nu_e})^{eeee} [\mathcal{O}_{42}\mathcal{O}_{31} - \mathcal{O}_{41}\mathcal{O}_{32}] \right) [u(p_1)u(p_2)] \quad (\text{A.6})$$

$$\mathcal{M} = \sqrt{2}G_F (\epsilon_{j,\nu_e})^{eeee} \left([\bar{u}(p_4)\mathcal{O}u(p_2)\bar{u}(p_3)\mathcal{O}u(p_1)] - [\bar{u}(p_4)\mathcal{O}u(p_1)\bar{u}(p_3)\mathcal{O}u(p_2)] \right) \quad (\text{A.7})$$

$$\mathcal{M}^\dagger = \sqrt{2}G_F (\epsilon_{j,\nu_e})^{eeee} \left([\bar{u}(p_1)\bar{\mathcal{O}}u(p_3)\bar{u}(p_2)\bar{\mathcal{O}}u(p_4)] - [\bar{u}(p_2)\bar{\mathcal{O}}u(p_3)\bar{u}(p_1)\bar{\mathcal{O}}u(p_4)] \right) \quad (\text{A.8})$$

where $\bar{\mathcal{O}} = \gamma^0 \mathcal{O}^\dagger \gamma^0$

Take average over initial spins and sum over final spins:

$$|\overline{\mathcal{M}_j}|^2 = \frac{1}{4} \sum \mathcal{M}_j^\dagger \mathcal{M}_j \quad (\text{A.9})$$

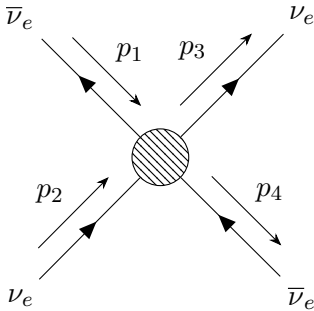
For Scalar NSI:

$$|\overline{\mathcal{M}_S}|^2 = 32 (\epsilon_{S,\nu_e}^{eeee})^2 G_F^2 (p_1 \cdot p_2) (p_3 \cdot p_4) \quad (\text{A.10})$$

For Vector NSI:

$$|\overline{\mathcal{M}_V}|^2 = 512 (\epsilon_{V,\nu_e}^{eeee})^2 G_F^2 (p_1 \cdot p_2) (p_3 \cdot p_4) \quad (\text{A.11})$$

A.1.2 $\nu_e + \bar{\nu}_e \rightarrow \nu_e + \bar{\nu}_e$



$$-i\mathcal{M} = [\bar{u}(p_3)v(p_4)] [i\Gamma_{1234}] [\bar{v}(p_1)u(p_2)] \quad (\text{A.12})$$

$$i\mathcal{M} = [\bar{u}(p_3)v(p_4)] \left(-2i\frac{G_F}{\sqrt{2}} (\epsilon_{j,\nu_e})^{eeee} [\mathcal{O}_{12}\mathcal{O}_{34} - \mathcal{O}_{14}\mathcal{O}_{32}] \right) [\bar{v}(p_1)u(p_2)] \quad (\text{A.13})$$

$$\mathcal{M} = \sqrt{2}G_F (\epsilon_{j,\nu_e})^{eeee} \left([\bar{v}(p_1)\mathcal{O}u(p_2)\bar{u}(p_3)\mathcal{O}v(p_4)] - [\bar{v}(p_1)\mathcal{O}v(p_4)\bar{u}(p_3)\mathcal{O}u(p_2)] \right) \quad (\text{A.14})$$

$$\mathcal{M}^\dagger = \sqrt{2}G_F (\epsilon_{j,\nu_e})^{eeee} \left([\bar{v}(p_4)\bar{\mathcal{O}}u(p_3)\bar{u}(p_2)\bar{\mathcal{O}}v(p_1)] - [\bar{u}(p_2)\bar{\mathcal{O}}u(p_3)\bar{v}(p_4)\bar{\mathcal{O}}v(p_1)] \right) \quad (\text{A.15})$$

where $\bar{\mathcal{O}} = \gamma^0 \mathcal{O}^\dagger \gamma^0$

Take average over initial spins and sum over final spins:

$$\overline{|\mathcal{M}_j|^2} = \frac{1}{4} \sum \mathcal{M}_j^\dagger \mathcal{M}_j \quad (\text{A.16})$$

For Scalar NSI:

$$\overline{|\mathcal{M}_S|^2} = 32 (\epsilon_{S,\nu_e}^{eeee})^2 G_F^2 (p_1 \cdot p_3) (p_2 \cdot p_4) \quad (\text{A.17})$$

For Vector NSI:

$$\overline{|\mathcal{M}_V|^2} = 512 (\epsilon_{V,\nu_e}^{eeee})^2 G_F^2 (p_1 \cdot p_3) (p_2 \cdot p_4) \quad (\text{A.18})$$

A.2 Total cross-section

We will work in the Center of Mass frame.

$$\begin{aligned} p_1 &= (E_1, \mathbf{p}) \\ p_2 &= (E_2, -\mathbf{p}) \\ p_3 &= (E_3, \mathbf{p}^*) \\ p_4 &= (E_4, -\mathbf{p}^*) \\ s &= (p_1 + p_2)^2 = 2p_1 \cdot p_2 = (E_1 + E_2)^2 \\ t &= (p_1 - p_3)^2 = -2p_1 \cdot p_3 = 2(\mathbf{p} \cdot \mathbf{p}^* - E_1 E_3) \end{aligned}$$

angle between \mathbf{p} and \mathbf{p}^* is θ .

For Matrix elements are of the form,

$$\begin{aligned} \overline{|\mathcal{M}|_{\nu\nu-\nu\nu}^2} &= K_1 (p_1 \cdot p_2) (p_3 \cdot p_4) \\ &= K_1 \left(E_1 E_2 E_3 E_4 - E_1 E_2 (\vec{p}_3 \cdot \vec{p}_4) - E_3 E_4 (\vec{p}_1 \cdot \vec{p}_2) + (\vec{p}_1 \cdot \vec{p}_2) (\vec{p}_3 \cdot \vec{p}_4) \right) \\ \overline{|\mathcal{M}|_{\nu\bar{\nu}-\nu\bar{\nu}}^2} &= K_1 (p_1 \cdot p_3) (p_2 \cdot p_4) \\ &= K_1 \left(E_1 E_2 E_3 E_4 - E_1 E_3 (\vec{p}_2 \cdot \vec{p}_4) - E_2 E_4 (\vec{p}_1 \cdot \vec{p}_3) + (\vec{p}_1 \cdot \vec{p}_3) (\vec{p}_2 \cdot \vec{p}_4) \right) \end{aligned} \quad (\text{A.19})$$

In CoM frame,

$$\begin{aligned} \overline{|\mathcal{M}|_{\nu\nu-\nu\nu}^2} &= \frac{K_1}{4} s^2 \\ \overline{|\mathcal{M}|_{\nu\bar{\nu}-\nu\bar{\nu}}^2} &= \frac{K_1}{4} t^2 \end{aligned} \quad (\text{A.20})$$

Starting from,

$$d\sigma = \frac{S\hbar^2}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2}} |\mathcal{M}|^2 d\Pi_{\text{LIPS}} \quad (\text{A.21})$$

For a $2 \rightarrow j$ process, Lorentz Invariant Phase Space,

$$d\Pi_{\text{LIPS}} \equiv \prod_{\text{final states } j} \frac{d^3 p_j}{(2\pi)^3} \frac{1}{2E_{p_j}} (2\pi)^4 \delta^4(\Sigma p) \quad (\text{A.22})$$

Now for a $1 + 2 \rightarrow 3 + 4$ process with massless particles,

$$d\sigma = \frac{S}{2s} |\mathcal{M}|^2 \frac{d^3 p_3}{2E_3(2\pi)^3} \frac{d^3 p_4}{2E_4(2\pi)^3} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \quad (\text{A.23})$$

In CoM frame,

$$\begin{aligned} d\sigma &= \frac{S}{2s} |\mathcal{M}|^2 \frac{d^3 p_3}{2E_3(2\pi)^3} \frac{d^3 p_4}{2E_4(2\pi)^3} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \\ &= \frac{S}{2s} |\mathcal{M}|^2 \frac{d^3 p_3}{2E_3(2\pi)^3} \frac{d^3 p_4}{2E_4(2\pi)^3} (2\pi)^4 \delta(\sqrt{s} - E_3 - E_4) \delta^3(\vec{p}_3 + \vec{p}_4) \end{aligned} \quad (\text{A.24})$$

$$\begin{aligned} \sigma_{\nu\nu-\nu\nu} &= \frac{\frac{1}{2}}{2 \cdot (2\pi)^2 \cdot s} \int \frac{d^3 p_3}{2E_3} \frac{d^3 p_4}{2E_4} \delta(\sqrt{s} - E_3 - E_4) \delta^3(\vec{p}_3 + \vec{p}_4) \cdot \frac{K_1}{4} s^2 \\ &= \frac{S}{2 \cdot (2\pi)^2 \cdot s} |\mathcal{M}|^2 \frac{d^3 p_3}{2E_3} \frac{d^3 p_4}{2E_4} \delta(\sqrt{s} - E_3 - E_4) \delta^3(\vec{p}_3 + \vec{p}_4) \end{aligned} \quad (\text{A.25})$$

integrating out \mathbf{p}_4 with $\delta^3(\vec{p}_3 + \vec{p}_4)$, and we use,

$$\int dx g(x) \delta(f(x)) = g(x_0) \frac{1}{f'(x_0)}$$

where x_0 solves $f(x_0) = 0$.

$$\sigma_{\nu\nu-\nu\nu} = \frac{1}{4 \cdot (2\pi)^2 \cdot s} \cdot \frac{K_1}{4} s^2 \int \frac{d^3 p_3}{(2E_3)^2} \delta(\sqrt{s} - 2E_3) \quad (\text{A.26})$$

$$\begin{aligned} d^3 p_3 &= p_3^2 dp_3 d\Omega_3 \\ E_3 dE_3 &= |\mathbf{p}_3| dp_3 \end{aligned}$$

$$\begin{aligned} \sigma_{\nu\nu-\nu\nu} &= \frac{K_1 s^2}{16 \cdot (2\pi)^2 \cdot s} \int \frac{p_3^2 dp_3 d\Omega_3}{(2E_3)^2} \delta(2E_3 - \sqrt{s}) \\ &= \frac{K_1 s}{16 \cdot (2\pi)^2} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot 4\pi \\ &= \frac{K_1 s}{128\pi} \end{aligned} \quad (\text{A.27})$$

For $\nu_e + \bar{\nu}_e \rightarrow \nu_e + \bar{\nu}_e$,

$$\sigma_{\nu\bar{\nu}-\nu\bar{\nu}} = \frac{1}{2 \cdot (2\pi)^2 \cdot s} \int \frac{d^3 p_3}{2E_3} \frac{d^3 p_4}{2E_4} \delta(\sqrt{s} - E_3 - E_4) \delta^3(\vec{p}_3 + \vec{p}_4) \cdot \frac{K_1}{4} t^2 \quad (\text{A.28})$$

$$t = (p_1 - p_3)^2 = -2p_1 \cdot p_3 = 2(\vec{p}_1 \cdot \vec{p}_3 - E_1 E_3) = 2|p_1||p_3|(\cos\theta - 1)t^2 = 4|p_1|^2|p_3|^2(1 - \cos\theta)^2$$

$$\sigma_{\nu\bar{\nu}-\nu\bar{\nu}} = \frac{1}{2 \cdot (2\pi)^2 \cdot s} \int \frac{d^3 p_3}{2E_3} \frac{d^3 p_4}{2E_4} \delta(\sqrt{s} - E_3 - E_4) \delta^3(\vec{p}_3 + \vec{p}_4) \cdot \frac{K_1}{4} 4|p_1|^2|p_3|^2(1 - \cos\theta)^2 \quad (\text{A.29})$$

integrating out \mathbf{p}_4 with $\delta^3(\vec{p}_3 + \vec{p}_4)$,

$$\sigma_{\nu\bar{\nu}-\nu\bar{\nu}} = \frac{K_1 |p_1|^2}{8\pi^2 s} \int \frac{d^3 p_3}{4} \delta(\sqrt{s} - 2E_3) (1 - \cos\theta)^2 \quad (\text{A.30})$$

$$d^3p_3 = p_3^2 dp_3 d\Omega_3 = p_3^2 dp_3 d(\cos \theta) d\phi$$

$$E_3 dE_3 = |\mathbf{p}_3| dp_3$$

$$\begin{aligned} \sigma_{\nu\bar{\nu}-\nu\bar{\nu}} &= \frac{K_1 |p_1|^2}{32\pi^2 s} \int p_3^2 dp_3 d(\cos \theta) d\phi \delta(2p_3 - \sqrt{s})(1 - \cos \theta)^2 \\ &= \frac{K_1 \frac{s}{4}}{32\pi^2 s} \int p_3^2 \delta(2p_3 - \sqrt{s}) dp_3 \int_{-1}^1 d(\cos \theta) (1 - \cos \theta)^2 \int_0^{2\pi} d\phi \\ &= \frac{K_1}{128\pi^2} \cdot \frac{1}{2} \left(\frac{\sqrt{s}}{2} \right)^2 \frac{8}{3} \cdot 2\pi = \frac{K_1 s}{3 \cdot 128\pi} \end{aligned} \quad (\text{A.31})$$

$$\sigma_{tot} = \sigma_{\nu\nu-\nu\nu} + \sigma_{\nu\bar{\nu}-\nu\bar{\nu}} = \frac{K_1 s}{128\pi} \left(1 + \frac{1}{3} \right) = \frac{K_1 s}{96\pi} \quad (\text{A.32})$$

A.3 Interaction rate

Now Interaction rate from Eq. 3.6 of [2] for a neutrino with fixed energy E , by averaging over the phase space of the other neutrino (or anti-neutrino) it scatters with,

$$\Gamma_V(E, T) = 2 \int \frac{d^3\vec{p}_2}{(2\pi)^3} f_\nu(E_2, T) \sigma_{tot}(\vec{p}_1, \vec{p}_2) v_{rel} \quad (\text{A.33})$$

where $f_\nu(E_2, T) = 1/[1 + \exp(E_2/T)]$ is the Fermi-Dirac distribution for active neutrino or antineutrino, σ_{tot} is the sum of the two cross-sections. The relative velocity is equal to $\sqrt{2(1 - \cos \theta)}$ for ultra-relativistic neutrinos [2]. The prefactor of 2 captures both helicity states of the active neutrinos and antineutrinos. Hence, Γ_V also includes the reaction rate of antineutrinos

Now for $p_1 = (E, \vec{p}), p_2 = (E', \vec{p}')$

$$s = 2p_1 \cdot p_2 = 2(E E' - \vec{p} \cdot \vec{p}') = 2|p||p'|(1 - \cos \theta)$$

$$\begin{aligned} \Gamma(E, T) &= 2 \int \frac{d^3\vec{p}_2}{(2\pi)^3} f_\nu(E_2, T) \sigma_{tot}(\vec{p}_1, \vec{p}_2) v_{rel} \\ &= 2 \int \frac{|p'|^2 dp' d\Omega}{(2\pi)^3} \frac{1}{1 + \exp(E'/T)} \frac{K_1}{96\pi} \cdot 2|p||p'|(1 - \cos \theta) \cdot \sqrt{2(1 - \cos \theta)} \\ &= \frac{4K_1 E}{96\pi} \int \frac{E'^2 dE' d(\cos \theta) d\phi}{(2\pi)^3} \frac{1}{1 + \exp(E'/T)} E' \sqrt{2}(1 - \cos \theta)^{3/2} \\ &= \frac{K_1 E}{24\pi} \int \frac{E'^3 dE'}{(2\pi)^3} \frac{1}{1 + \exp(E'/T)} \int_{-1}^1 d(\cos \theta) \sqrt{2}(1 - \cos \theta)^{3/2} \int_0^{2\pi} d\phi \quad (\text{A.34}) \\ &= \frac{K_1 E}{24\pi \cdot (2\pi)^2} \int \frac{E'^3 dE'}{1 + \exp(E'/T)} \int_{-1}^1 d(\cos \theta) \sqrt{2}(1 - \cos \theta)^{3/2} \\ &= \frac{K_1 E}{24\pi \cdot (2\pi)^2} \frac{7\pi^4 T^4}{120} \cdot \frac{16}{5} \\ \Gamma(E, T) &= \frac{7K_1 \pi}{3600} E T^4 \end{aligned}$$

(E' integration in Mathematica.)

If we use MB distribution, we get $6T^4$ instead of $\frac{7\pi^4 T^4}{120} \approx 5.68T^4$)

$K_S = 32G_F^2\epsilon_S^2$ for scalar NSI and $K_V = 512G_F^2\epsilon_V^2$ for Vector NSI
ie,

$$\begin{aligned}\Gamma_S(E, T) &= \frac{14\pi G_F^2\epsilon_S^2}{225}ET^4 \\ \Gamma_V(E, T) &= \frac{224\pi G_F^2\epsilon_V^2}{225}ET^4\end{aligned}\tag{A.35}$$

B Calculation of Thermal potential

B.1 Neutrino Dispersion Relation

The properties of a neutrino that propagates through a medium are determined from the Dirac equation, which in momentum space is

$$(\not{k} - \Sigma_{\text{eff}}) \psi = 0\tag{B.1}$$

Here k_μ is the neutrino momentum and Σ_{eff} is the neutrino self-energy, which includes the effects of the background. The chiral nature of the neutrino interactions implies that the self-energy of a (left-handed) neutrino is of the form

Chiral nature of neutrino interactions imply [5],

$$\Sigma_{\text{eff}} = P_R \Sigma P_L\tag{B.2}$$

Where $P_{R/L} = \frac{1 \pm \gamma_5}{2}$

In vacuum, the only term self energy can depend on is the neutrino four-momentum k_μ .
i.e.,

$$\Sigma = a\not{k}\tag{B.3}$$

But in a medium with velocity u_μ (we would assume rest frame where $u_\mu = (1, 0, 0, 0)$),

$$\Sigma = a\not{k} + b\not{u} + c[\not{k}, \not{u}]\tag{B.4}$$

From [6], we can see that at one-loop level, $c = 0$. Then,

$$\Sigma = a\not{k} + b\not{u}\tag{B.5}$$

a, b depend on invariant quantities $\omega = k \cdot u$ and $\kappa = \sqrt{\omega^2 - k^2}$.
Now Dirac Equation,

$$(\not{k} - \Sigma_{\text{eff}}) \psi = 0\tag{B.6}$$

$$(\not{k} - P_R(a\not{k} + b\not{u})P_L) \psi = 0\tag{B.7}$$

for active neutrinos, $\psi = \psi_L = P_L \psi$ also $P_L^2 = P_L$

$$\begin{aligned}(\not{k} - P_R(a\not{k} + b\not{u})P_L) P_L \psi &= 0 \\ (\not{k} P_L - P_R(a\not{k} + b\not{u})P_L) \psi &= 0 \\ (\not{k} P_L^2 - P_R(a\not{k} + b\not{u})P_L) \psi &= 0 \\ (P_R \not{k} P_L - P_R(a\not{k} + b\not{u})P_L) \psi &= 0\end{aligned}\tag{B.8}$$

$$P_R [(1-a) \not{k} - b \not{u}] P_L \psi = 0 \Rightarrow \not{V} \psi = 0 \quad (\text{B.9})$$

where $V_\mu = (1-a) k_\mu - b u_\mu$ For non-trivial solutions of $\not{V} \psi = 0$, $V^2 = 0$

$$\begin{aligned} [(1-a) k_\mu - b u_\mu] [(1-a) k^\mu - b u^\mu] &= 0 \\ (1-a)^2 k_\mu k^\mu + b^2 u_\mu u^\mu - 2(1-a)b(k \cdot u) &= 0 \\ (1-a)^2 k^2 + b^2 u^2 - 2b(1-a)\omega &= 0 \\ (1-a)^2 (\omega^2 - \kappa^2) + b^2 - 2b(1-a)\omega &= 0 \\ [(1-a)(\omega - \kappa) - b] [(1-a)(\omega + \kappa) - b] &= 0 \\ f(\omega) \times \bar{f}(\omega) &= 0 \end{aligned} \quad (\text{B.10})$$

So, $V^2 = 0 \Rightarrow f(\omega) \bar{f}(\omega) = 0$

This equation has the solution:

$$f(\omega_\kappa) = 0 \quad (\text{B.11})$$

or,

$$\bar{f}(-\bar{\omega}_\kappa) = 0 \quad (\text{B.12})$$

Since we can separate self-energy into gauge dependent and independent parts, we can separate f as,

$$f = f_0 + f_\xi = [(1-a_0)(\omega - \kappa) - b_0] + [(1-a_\xi)(\omega - \kappa) - b_\xi] \quad (\text{B.13})$$

Dispersion relation is independent of gauge parameter. f_0 and f_ξ must vanish separately at $\omega = \omega_\kappa$

$$\begin{aligned} f_0(\omega_\kappa) &\equiv (1-a_0)(\omega_\kappa - \kappa) - b_0 = 0 \\ \Rightarrow b_0 &= (1-a_0)(\omega_\kappa - \kappa) \Rightarrow \omega_\kappa - \kappa = b_0 + a_0(\omega_\kappa - \kappa) \\ \omega_\kappa &= \kappa + b_0 + a_0(\omega_\kappa - \kappa) \end{aligned} \quad (\text{B.14})$$

In zeroth order, $\omega_\kappa = \kappa$. Substituting this in (B.14)

$$\omega_\kappa = \kappa + b_0(\omega_\kappa = \kappa)$$

So for neutrinos,

$$\omega_\kappa = \kappa + b_0(\kappa) + \text{h.o} \quad (\text{B.15})$$

Similarly, for anti-neutrinos,

$$\bar{f}(-\bar{\omega}_\kappa) = 0 \quad (\text{B.16})$$

$$\begin{aligned} \bar{f}_0(-\bar{\omega}_\kappa) &\equiv (1-a_0)(-\bar{\omega}_\kappa + \kappa) - b_0 = 0 \\ \Rightarrow b_0 &= (1-a_0)(-\bar{\omega}_\kappa + \kappa) \Rightarrow \omega_\kappa = \kappa - b_0 - a_0(-\bar{\omega}_\kappa + \kappa) \end{aligned} \quad (\text{B.17})$$

Keeping terms in the first order,

$$\omega_\kappa = \kappa - b_0(\omega_\kappa = -\kappa)$$

So for anti-neutrinos,

$$\bar{\omega}_\kappa = \kappa - b_0 (-\kappa) + \text{h.o} \quad (\text{B.18})$$

B.2 Effective potential

We can see the effective potential as a contribution to the energy of the particle by background medium. ie,

Energy of Neutrino propagating through a medium =

. Energy of neutrino in vacuum + Effective potential in the medium

$$\omega_\kappa = \kappa + \mathcal{V}_l \quad (\text{B.19})$$

We are only considering the lowest order, hence \mathcal{V}_l coincide with b_0 , but in general it depends on a_0 too.

Now our problem has reduced to finding b_0 in $\Sigma = (a_0 + a_\xi)\not{k} + (b_0 + b_\xi)\not{p}$

B.3 Self energy calculation

- Please check if there are any minus signs missing

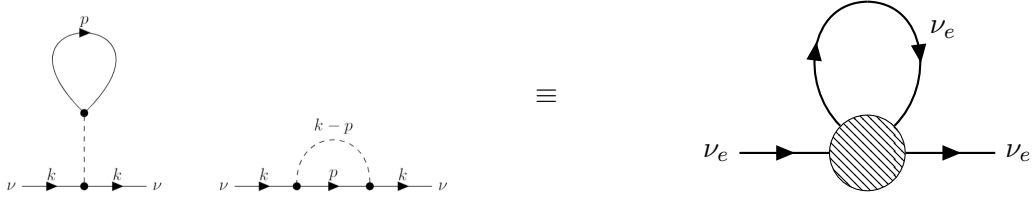


Figure 1: Self energy diagrams in NSI

From [4] (or we can infer from Fig.1) we can write Feynman rules for 4-fermion vertex Matrix elements for an NSI Lagrangian,

$$\mathcal{L}_j = -\frac{G_F}{\sqrt{2}} (\epsilon_{j,\nu_e})^{eeee} (\bar{\nu}_e \mathcal{O}_j \nu_e) (\bar{\nu}_e \mathcal{O}'_j \nu_e) \quad (\text{B.20})$$

$$\nu_e \rightarrow \text{[shaded circle]} \rightarrow \nu_e \quad \equiv \quad i\mathcal{M} = \frac{i^2 G_F}{\sqrt{2}} (\epsilon_{j,\nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^4} (\bar{u} S_F(p) \mathcal{O}_j \mathcal{O}'_j u - \bar{u} \mathcal{O}_j u \text{Tr} [\mathcal{O}'_j S_F(p)]) \quad (\text{B.21})$$

where $S_F(p)$ is the fermion propagator at finite temperature,

$$S_F(p) = (\not{p} + m_l) \left[\frac{1}{p^2 - m_l^2} + 2\pi i \delta(p^2 - m_l^2) \eta(p \cdot u) \right] \quad (\text{B.22})$$

with,

$$\eta(p \cdot u) = \frac{\theta(p \cdot u)}{e^x + 1} + \frac{\theta(-p \cdot u)}{e^{-x} + 1}, x = \frac{(p \cdot u - \mu)}{T} \quad (\text{B.23})$$

for neutrinos,

$$S_F(p) = \not{p} \left[\frac{1}{p^2} + 2\pi i \delta(p^2 - m_\nu^2) \eta(p \cdot u) \right] \quad (\text{B.24})$$

Background-independent part of the fermion propagator only renormalizes the wave function and does not contribute to the dispersion relation in the lowest order[7]. To simplify the calculations, we will only consider background-dependent terms.

$$S_F^T(p) = 2\pi i \delta(p^2 - m_l^2) \eta(p \cdot u) (\not{p} + m_l) \quad (\text{B.25})$$

Now,

$$i\mathcal{M} = \bar{u} [i\Sigma_{\text{eff}}] u = \bar{u} \left[\frac{i^2 G_F}{\sqrt{2}} (\epsilon_{j,\nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^4} (S_F(p) \mathcal{O}_j \mathcal{O}'_j - \mathcal{O}_j \text{Tr} [\mathcal{O}'_j S_F(p)]) \right] u \quad (\text{B.26})$$

$$i\Sigma_{\text{eff}} = \frac{-G_F}{\sqrt{2}} (\epsilon_{j,\nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^4} (S_F^T(p) \mathcal{O}_j \mathcal{O}'_j - \mathcal{O}_j \text{Tr} [\mathcal{O}'_j S_F^T(p)]) \quad (\text{B.27})$$

Does the order of operators matter? which one is correct, $S_F^T(p) \mathcal{O}_j \mathcal{O}'_j$ or $\mathcal{O}_j S_F^T(p) \mathcal{O}'_j$?

B.3.1 Scalar NSI

For scalar NSI, $\mathcal{O}_S = (1 - \gamma^5)$ and $\mathcal{O}'_S = (1 - \gamma^5)$

$$\begin{aligned} i\Sigma_{\text{eff}} &= \frac{-G_F}{\sqrt{2}} (\epsilon_{S,\nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^4} (S_F^T(p) \mathcal{O}_S \mathcal{O}'_S - \mathcal{O}_S \text{Tr} [\mathcal{O}'_S S_F^T(p)]) \\ &= -\frac{G_F}{\sqrt{2}} (\epsilon_{S,\nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^4} (S_F^T(p) (1 - \gamma^5) (1 - \gamma^5) - (1 - \gamma^5) \text{Tr} [(1 - \gamma^5) S_F^T(p)]) \end{aligned} \quad (\text{B.28})$$

$$i\Sigma_{\text{eff}} = -\frac{G_F}{\sqrt{2}} (\epsilon_{S,\nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^4} (2 \cdot S_F^T(p) - \text{Tr} [(1 - \gamma^5) S_F^T(p)]) (1 - \gamma^5) \quad (\text{B.29})$$

$$\text{Tr} [(1 - \gamma^5) S_F^T(p)] = 2\pi i \delta(p^2 - m_\nu^2) \eta(p \cdot u) \text{Tr} [(1 - \gamma^5) \not{p}] = 0 \quad (m_\nu \approx 0)$$

$$\begin{aligned} i\Sigma_{\text{eff}} &= -\frac{G_F}{\sqrt{2}} (\epsilon_{S,\nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^4} (2 \cdot 2\pi i \delta(p^2 - m_\nu^2) \eta(p \cdot u) \gamma_\mu p^\mu) (1 - \gamma^5) \\ &= -\frac{2iG_F}{\sqrt{2}} (\epsilon_{S,\nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_\nu^2) \eta(p \cdot u) \gamma_\mu p^\mu \underbrace{u_\mu u^\mu}_{=1(\text{B.1})} (1 - \gamma^5) \end{aligned} \quad (\text{B.30})$$

$$\begin{aligned} &= -\sqrt{2}iG_F (\epsilon_{S,\nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_\nu^2) \eta(p \cdot u) (p \cdot u) \not{p} (1 - \gamma^5) \\ &= -\sqrt{2}iG_F (\epsilon_{S,\nu_e})^{eeee} J_1^{(\nu)} \not{p} (1 - \gamma^5) \end{aligned}$$

$$\Sigma_{\text{eff}} = -2\sqrt{2}G_F (\epsilon_{S,\nu_e})^{eeee} J_1^{(\nu)} \not{p} P_L \quad (\text{B.31})$$

where (check [B.3.3](#)),

$$J_n^{(f)} = \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_f^2) \eta(p \cdot u)(p \cdot u)^n \quad (\text{B.32})$$

Now comparing (B.31) with $\Sigma_{\text{eff}} = P_R \Sigma P_L = P_R(b_0 \not{u}) P_L = -b_0 \not{u} P_L$,

$$b_0 = 2\sqrt{2} G_F (\epsilon_{S, \nu_e})^{eeee} J_1^{(\nu)} = \sqrt{2} G_F (\epsilon_{S, \nu_e})^{eeee} (n_\nu - n_{\bar{\nu}}) \quad (\text{B.33})$$

From (B.2) and (B.1), Effective potential $\mathcal{V}_l = b_0(\omega_k = \kappa)$ for ν and $\mathcal{V}_l = -b_0(\omega_k = -\kappa)$ for $\bar{\nu}$

ie,

$$\mathcal{V}_{\text{scalar}} = \pm \sqrt{2} G_F (\epsilon_{S, \nu_e})^{eeee} (n_\nu - n_{\bar{\nu}}) \quad (\text{B.34})$$

(+) for ν and (-) for $\bar{\nu}$

B.3.2 Vector NSI

For Vector NSI, $\mathcal{O}_V = \gamma_\mu(1 - \gamma^5)$ and $\mathcal{O}'_V = \gamma^\mu(1 - \gamma^5)$

$$\begin{aligned} i\Sigma_{\text{eff}} &= \frac{-G_F}{\sqrt{2}} (\epsilon_{V, \nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^4} (\mathcal{O}_V S_F^T(p) \mathcal{O}'_V - \mathcal{O}_V \text{Tr} [\mathcal{O}'_V S_F^T(p)]) \\ &= -\frac{G_F}{\sqrt{2}} (\epsilon_{V, \nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^4} (\gamma_\mu(1 - \gamma^5) S_F^T(p) \gamma^\mu(1 - \gamma^5) - \gamma_\mu(1 - \gamma^5) \text{Tr} [\gamma^\mu(1 - \gamma^5) S_F^T(p)]) \end{aligned} \quad (\text{B.35})$$

$$i\Sigma_{\text{eff}} = -\frac{G_F}{\sqrt{2}} (\epsilon_{V, \nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^4} (\gamma_\mu(1 - \gamma^5) S_F^T(p) - \text{Tr} [\gamma_\mu(1 - \gamma^5) S_F^T(p)]) \gamma^\mu(1 - \gamma^5) \quad (\text{B.36})$$

$$\begin{aligned} \text{Tr} [\gamma^\mu(1 - \gamma^5) S_F^T(p)] &= 2\pi i \delta(p^2 - m_\nu^2) \eta(p \cdot u) \text{Tr} [\gamma^\mu(1 - \gamma^5) \not{p}] \\ &= 2\pi i \delta(p^2 - m_\nu^2) \eta(p \cdot u) \text{Tr} [\gamma^\mu \not{p}] = 2\pi i \delta(p^2 - m_\nu^2) \eta(p \cdot u) \cdot 4p^\mu \end{aligned}$$

$$\gamma_\mu(1 - \gamma^5) \not{p} \gamma^\mu(1 - \gamma^5) = -\not{p} \gamma_\mu(1 + \gamma^5) \gamma^\mu(1 - \gamma^5) = -\not{p} \gamma_\mu \gamma^\mu (1 - \gamma^5)^2 = -\not{p} (-4I_4) \cdot 2 \cdot (1 - \gamma^5) = 8\not{p}(1 - \gamma^5)$$

$$\begin{aligned} \Sigma_{\text{eff}} &= -\frac{G_F}{\sqrt{2}} (\epsilon_{V, \nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_\nu^2) \eta(p \cdot u) (\gamma_\mu(1 - \gamma^5) \not{p} - 4p_\mu) \gamma^\mu(1 - \gamma^5) \\ &= -\frac{G_F}{\sqrt{2}} (\epsilon_{V, \nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_\nu^2) \eta(p \cdot u) (8\not{p} - 4\not{p}) (1 - \gamma^5) \\ &= \frac{4G_F}{\sqrt{2}} (\epsilon_{V, \nu_e})^{eeee} \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_\nu^2) \eta(p \cdot u) \not{p}(1 - \gamma^5) \quad \text{similar to (B.30)} \\ &= \frac{4G_F}{\sqrt{2}} (\epsilon_{V, \nu_e})^{eeee} J_1^{(\nu)} \not{u}(1 - \gamma^5) \end{aligned} \quad (\text{B.37})$$

$$\Sigma_{\text{eff}} = 4\sqrt{2}G_F (\epsilon_{V,\nu_e})^{eeee} J_1^{(\nu)} \not{p} P_L \quad (\text{B.38})$$

Now comparing (B.38) with $\Sigma_{\text{eff}} = P_R \Sigma P_L = P_R (b_0 \not{p}) P_L = -b_0 \not{p} P_L$,

$$b_0 = -4\sqrt{2}G_F (\epsilon_{V,\nu_e})^{eeee} J_1^{(\nu)} = -2\sqrt{2}G_F (\epsilon_{S,\nu_e})^{eeee} (n_\nu - n_{\bar{\nu}}) \quad (\text{B.39})$$

From (B.2) and (B.1), Effective potential $\mathcal{V}_l = b_0(\omega_k = \kappa)$ for ν and $\mathcal{V}_l = -b_0(\omega_k = -\kappa)$ for $\bar{\nu}$

ie,

$$\mathcal{V}_{\text{vector}} = \mp 2\sqrt{2}G_F (\epsilon_{S,\nu_e})^{eeee} (n_\nu - n_{\bar{\nu}}) \quad (\text{B.40})$$

(-) for ν and (+) for $\bar{\nu}$

B.3.3 Scalar mediator

Let's assume we have a scalar mediator with mass m_ϕ as in [1].

$$\mathcal{L}_s \supset \frac{\lambda_\phi}{2} \nu_e \nu_e \phi + \text{h.c} \quad (\text{B.41})$$

There will be two diagrams(Fig 1),

$$\begin{aligned} i\Sigma_{\text{eff}} &= \lambda_\phi^2 \int \frac{d^4 p}{(2\pi)^4} S_F^T(p) (1 - \gamma^5) \frac{-1}{(k-p)^2 - m_\phi^2} (1 - \gamma^5) - \\ &\quad (1 - \gamma^5) \frac{-1}{(k-p)^2 - m_\phi^2} \text{Tr} [(1 - \gamma^5) S_F^T(p)] \\ &= 2\lambda_\phi^2 \int \frac{d^4 p}{(2\pi)^4} S_F^T \left(\frac{-1}{(k-p)^2 - m_\phi^2} \right) (1 - \gamma^5) \\ &= -2\lambda_\phi^2 \int \frac{d^4 p}{(2\pi)^4} \frac{2\pi i \delta(p^2 - m_\nu^2) \eta(p \cdot u) \gamma_\mu p^\mu}{(k-p)^2 - m_\phi^2} (1 - \gamma^5) \end{aligned} \quad (\text{B.42})$$

ie,

$$\Sigma_{\text{eff}} = -4\lambda_\phi^2 \gamma_\mu \mathcal{P}^\mu(1) P_L \quad (\text{B.43})$$

where,

$$\mathcal{P}_\mu(1) = \int \frac{d^4 p}{(2\pi)^3} \frac{\delta(p^2 - m_l^2) \eta(p \cdot u)}{(p-k)^2 - m_\phi^2} p_\mu \quad (\text{B.44})$$

Now, with $\mathcal{M}^2 = m_l^2 + k^2 - m_\phi^2$

$$\mathcal{P}_\mu(1) = \int \frac{d^4 p}{(2\pi)^3} \frac{\delta(p^2 - m_l^2) \eta(p \cdot u)}{p^2 + k^2 - 2 \cdot k - m_\phi^2} p_\mu \quad (\text{B.45})$$

$$\mathcal{P}_\mu(1) = \int \frac{d^4 p}{(2\pi)^3} \frac{\delta(p^2 - m_l^2) \eta(p \cdot u)}{\mathcal{M}^2 - 2p \cdot k} p_\mu \quad (\text{B.46})$$

$$\mathcal{P}_\mu(1) = \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_l^2) \eta(p \cdot u) \frac{p_\mu}{\mathcal{M}^2} \left(1 - \frac{2p \cdot k}{\mathcal{M}^2} \right)^{-1} \quad (\text{B.47})$$

Only keeping terms up to \mathcal{M}^{-4} ,

$$\mathcal{P}_\mu(1) = \frac{1}{\mathcal{M}^2} \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_l^2) \eta(p \cdot u) p_\mu \left(1 + \frac{2p \cdot k}{\mathcal{M}^2} + O(\mathcal{M}^{-4}) \right) \quad (\text{B.48})$$

$$\mathcal{P}_\mu(1) = \frac{1}{\mathcal{M}^2} \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_l^2) \eta(p \cdot u) p_\mu \left(1 + \frac{2p_\nu k^\nu}{\mathcal{M}^2} + O(\mathcal{M}^{-4}) \right) \quad (\text{B.49})$$

$$\mathcal{P}_\mu(1) = \frac{1}{\mathcal{M}^2} \left[I_\mu + \frac{2k^\nu}{\mathcal{M}^2} I_{\mu\nu} \right] \quad (\text{B.50})$$

Where

$$\begin{aligned} I_\mu &= \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_l^2) \eta(p \cdot u) p_\mu \\ I_{\mu\nu} &= \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_l^2) \eta(p \cdot u) p_\mu p_\nu \end{aligned} \quad (\text{B.51})$$

I_μ is manifestly covariant and depends only on the vector u_μ .

$$I_\mu = A u_\mu \quad (\text{B.52})$$

Therefore, contracting I_μ with u_μ ,

$$I_\mu u^\mu = A u_\mu u^\mu = A \quad (\text{B.53})$$

$$I_\mu u^\mu = \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_l^2) \eta(p \cdot u) p_\mu u^\mu = J_1^{(l)} \quad (\text{B.54})$$

ie,

$$I_\mu = J_1^{(l)} u_\mu \quad (\text{B.55})$$

Similarly, $I_{\mu\nu} = \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_l^2) \eta(p \cdot u) p_\mu p_\nu$ depends only on u

$$I_{\mu\nu} = A g_{\mu\nu} + B u_\mu u_\nu \quad (\text{B.56})$$

By contracting this expression with $u_\mu u_\nu$ and $g_{\mu\nu}$, we obtain two equations for A and B ,

$$g^{\mu\nu} I_{\mu\nu} = \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_l^2) \eta(p \cdot u) p_\mu g^{\mu\nu} p_\nu = m_l^2 J_0^{(l)} = 4A + B \quad (\text{B.57})$$

$$u^\mu u^\nu I_{\mu\nu} = \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_l^2) \eta(p \cdot u) p_\mu u^\mu u^\nu p_\nu = J_2^{(l)} = A + B \quad (\text{B.58})$$

$$I_\mu^\mu = m_l^2 J_0^{(l)} \quad (\text{B.59})$$

Solving for A and B , these are then determined as

$$\begin{aligned} A &= \frac{1}{3} \left(m_l^2 J_0^{(l)} - J_2^{(l)} \right) \\ B &= \frac{1}{3} \left(4J_2^{(l)} - m_l^2 J_0^{(l)} \right) \end{aligned}$$

Substituting the expressions for I_μ and $I_{\mu\nu}$ into (B.50),

$$\mathcal{P}_\mu(1) = \frac{1}{\mathcal{M}^2} \left\{ \left[J_1^{(l)} + \frac{2\omega}{3\mathcal{M}^2} \left(4J_2^{(l)} - m_l^2 J_0^{(l)} \right) \right] u_\mu + \frac{2k_\mu}{3\mathcal{M}^2} \left(m_l^2 J_0^{(l)} - J_2^{(l)} \right) \right\} \quad (\text{B.60})$$

Now,

$$\Sigma_{\text{eff}} = -4\lambda_\phi^2 \gamma_\mu \mathcal{P}^\mu(1) P_L \quad (\text{B.61})$$

$$m_l = m_\nu \approx 0$$

$$\Sigma_{\text{eff}} = -\frac{4\lambda_\phi^2}{\mathcal{M}^2} \gamma_\mu \left\{ \left[J_1^{(\nu)} + \frac{8\omega}{3\mathcal{M}^2} J_2^{(\nu)} \right] u^\mu - \frac{2J_2^{(\nu)}}{3\mathcal{M}^2} k^\mu \right\} P_L \quad (\text{B.62})$$

$$\Sigma_{\text{eff}} = -\frac{4\lambda_\phi^2}{\mathcal{M}^2} \left\{ \left[J_1^{(\nu)} + \frac{8\omega}{3\mathcal{M}^2} J_2^{(\nu)} \right] \not{u} - \frac{2J_2^{(\nu)}}{3\mathcal{M}^2} \not{k} \right\} P_L \quad (\text{B.63})$$

Comparing with $\Sigma_{\text{eff}} = P_R \Sigma P_L = P_R (a_0 \not{k} + b_0 \not{u}) P_L = -(a_0 \not{k} + b_0 \not{u}) P_L$,

$$b_0 = \frac{4\lambda_\phi^2}{\mathcal{M}^2} \left[J_1^{(\nu)} + \frac{8\omega}{3\mathcal{M}^2} J_2^{(\nu)} \right] \quad (\text{B.64})$$

$$\mathcal{M}^2 \approx m_\phi^2$$

$$b_0 = \frac{4\lambda_\phi^2}{m_\phi^2} \left[\frac{1}{2} (n_\nu - n_{\bar{\nu}}) + \frac{8\omega}{3m_\phi^2} \cdot \frac{1}{2} (n_\nu \langle E_\nu \rangle + n_{\bar{\nu}} \langle E_{\bar{\nu}} \rangle) \right] \quad (\text{B.65})$$

Effective potential $\mathcal{V}_l = b_0(\omega_k = \kappa)$ for ν and $\mathcal{V}_l = -b_0(\omega_k = -\kappa)$ for $\bar{\nu}$

$$\mathcal{V}_\phi = \pm \frac{2\lambda_\phi^2}{m_\phi^2} (n_\nu - n_{\bar{\nu}}) + \frac{16\lambda_\phi^2}{3m_\phi^4} \cdot \omega \cdot \frac{7\pi^2 T^4}{240} \quad (\text{B.66})$$

$$\mathcal{V}_\phi = \pm \frac{2\lambda_\phi^2}{m_\phi^2} (n_\nu - n_{\bar{\nu}}) + \frac{7\pi^2 \lambda_\phi^2}{45m_\phi^4} E T^4 \quad (\text{B.67})$$

- probably a $\frac{1}{2}$ missed in the start
- sign is different from [1], probably because a minus sign is misplaced

Evaluating $J_n^{(f)}$

$$J_n^{(f)} = \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_f^2) \eta(p \cdot u) (p \cdot u)^n \quad (\text{B.68})$$

δ -function,

$$\begin{aligned} \delta(p^2 - m_f^2) &= \delta((p - m_f)(p + m_f)) \\ &= \delta(p_0^2 - \vec{p}^2 - m_f^2) \\ &= \delta(p_0^2 - \omega_p^2) \\ &= \delta((p_0 - \omega_p)(p_0 + \omega_p)) \\ &= \frac{1}{2\omega_p} [\delta(p_0 - \omega_p) + \delta(p_0 + \omega_p)] \end{aligned} \quad (\text{B.69})$$

where $\omega_p = \sqrt{p^2 + m_f^2} = E_p$.

$$\eta(p \cdot u) = \frac{\theta(p \cdot u)}{e^{\frac{(p \cdot u - \mu)}{T}} + 1} + \frac{\theta(-p \cdot u)}{e^{-\frac{(p \cdot u - \mu)}{T}} + 1} \quad (\text{B.70})$$

In the rest frame of the medium, $u_\mu = (1, 0, 0, 0)$,

$$\eta(p \cdot u) = \frac{\theta(p_0)}{e^{\frac{(p_0 - \mu)}{T}} + 1} + \frac{\theta(-p_0)}{e^{-\frac{(p_0 - \mu)}{T}} + 1} = \theta(p_0) f_f(p_0) + \theta(-p_0) f_{\bar{f}}(-p_0) \quad (\text{B.71})$$

where, we have introduced the particle and antiparticle momentum distributions,

$$f_{f,\bar{f}}(E) = \frac{1}{e^{\beta(E \mp \mu)} + 1}$$

number densities are given by

$$n_{f,\bar{f}} = g_f \int \frac{d^3 \mathbf{p}}{(2\pi)^3} f_{f,\bar{f}}$$

and the thermal average of \mathcal{E}^n ,

$$\langle \mathcal{E}_{f,\bar{f}}^n \rangle \equiv \frac{g_f}{n_{f,\bar{f}}} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \mathcal{E}^n f_{f,\bar{f}}$$

Now,

$$\begin{aligned} J_n^{(f)} &= \int \frac{d^4 p}{(2\pi)^3} \delta(p^2 - m_f^2) \eta(p \cdot u) (p \cdot u)^n \\ &= \int \frac{d^3 \mathbf{p} dp_0}{(2\pi)^3} \frac{1}{2\omega_p} [\delta(p_0 - \omega_p) + \delta(p_0 + \omega_p)] [\theta(p_0) f_f(p_0) + \theta(-p_0) f_{\bar{f}}(-p_0)] (p_0)^n \\ &= \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left[\frac{\omega_p^n}{2\omega_p} f_f(\omega_p) + \frac{(-\omega_p)^n}{2\omega_p} f_{\bar{f}}(\omega_p) \right] \\ &= \frac{1}{2} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left[E_f^{n-1} f_f(E_p) + (-1)^n E_{\bar{f}}^{n-1} f_{\bar{f}}(E_p) \right] \\ &= \frac{1}{2} \left[\frac{n_f}{g_f} \langle E_f^{n-1} \rangle + (-1)^n \frac{n_{\bar{f}}}{g_{\bar{f}}} \langle E_{\bar{f}}^{n-1} \rangle \right] \end{aligned} \quad (\text{B.72})$$

For neutrinos,

$$g_\nu = g_{\bar{\nu}} = 1$$

$$J_0^{(\nu)} = \frac{1}{2} \left[n_\nu \left\langle \frac{1}{E_\nu} \right\rangle + n_{\bar{\nu}} \left\langle \frac{1}{E_{\bar{\nu}}} \right\rangle \right] \quad (\text{B.73})$$

$$J_1^{(\nu)} = \frac{1}{2} \left[\frac{n_\nu}{g_\nu} - \frac{n_{\bar{\nu}}}{g_{\bar{\nu}}} \right] = \frac{1}{2} (n_\nu - n_{\bar{\nu}}) \quad (\text{B.74})$$

$$J_2^{(\nu)} = \frac{1}{2} [n_\nu \langle E_\nu \rangle + n_{\bar{\nu}} \langle E_{\bar{\nu}} \rangle] = \frac{7\pi^2 T^4}{240} \quad (\text{B.75})$$

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