Active neutrino self-interaction

Contents

1 Particle model

We consider an effective scalar and vector NSI between active neutrinos in addition to SM processes.
If the self-interaction is strong, it could keep active neutrinos in thermal equilibrium with themselves longer than the weak interaction, and enhance DW production rate of sterile neutrinos.
General NSI Lagrangian for active neutrino self-interaction,

$$\mathcal{L}_j = -\frac{G_F}{\sqrt{2}} \left(\epsilon_{j,\nu}\right)^{\alpha\beta\gamma\delta} \left(\bar{\nu}_\alpha \mathcal{O}_j \nu_\beta\right) \left(\bar{\nu}_\gamma \mathcal{O}'_j \nu_\delta\right) \tag{1.1}$$

Since we want to study testability in KATRIN, we would use electron flavor as the one and only flavor in our model.
Hence, $\alpha = \beta = \gamma = \delta = e$
Then (1.1),

$$\mathcal{L}_j = -\frac{G_F}{\sqrt{2}} \left(\epsilon_{j,\nu_e}\right)^{eeee} \left(\bar{\nu}_e \mathcal{O}_j \nu_e\right) \left(\bar{\nu}_e \mathcal{O}'_j \nu_e\right) \tag{1.2}$$

1.1 Scalar

We would introduce a scalar interaction so that we can compare our results with [1]

$$\mathcal{O}_S = (1-\gamma^5) \qquad \mathcal{O}'_S = (1-\gamma^5)$$

Here $(1-\gamma^5)$ implies that we are dealing with left-handed active neutrinos.

$$\mathcal{L}_S = -\frac{G_F}{\sqrt{2}}\left(\epsilon_{S,\nu_a}\right)^{eeee}\left(\bar{\nu}_e(1-\gamma^5)\nu_e\right)\left(\bar{\nu}_e(1-\gamma^5)\nu_e\right) \tag{1.3}$$

We could see this as a low energy EFT for a scalar heavy mediator ϕ ($m_\phi \gg T$) with interaction,

$$\mathcal{L}_s \supset \frac{\lambda_\phi}{2}\nu_e\nu_e\phi + \text{h.c} \tag{1.4}$$

then, coefficients could be,

$$\frac{G_F}{\sqrt{2}}\left(\epsilon_{j,\nu_e}\right)^{eeee} = \frac{\lambda_\phi^2}{m_\phi^2}$$

and this would correspond to Case `A` of Fig.2 of [1]

1.2 Vector

We would introduce a vector interaction so that we can compare our results with [2]

$$\mathcal{O}_V = \gamma^\mu(1-\gamma^5) \qquad \mathcal{O}'_V = \gamma_\mu(1-\gamma^5)$$

$$\mathcal{L}_V = -\frac{G_F}{\sqrt{2}}\left(\epsilon_{V,\nu_a}\right)^{eeee}\left(\bar{\nu}_e\gamma^\mu(1-\gamma^5)\nu_e\right)\left(\bar{\nu}_e\gamma_\mu(1-\gamma^5)\nu_e\right) \tag{1.5}$$

2 Differences from DW mechanism

	DW only	DW+NSI
Interaction rate	Γ_{SM}	$\Gamma_{\text{SM}} + \Gamma_{\text{NSI}}$
Thermal potential	$\mathcal{V}_{\nu_e} + \mathcal{V}_e$	$\mathcal{V}_{\nu_e} + \mathcal{V}_e + \mathcal{V}_{\text{NSI}}$
mechanism	freeze-in	freeze-in
sterile neutrino mass	$\sim$ keV	$\sim$ keV
Temperature range	$\sim \mathcal{O}(100)$ MeV to $\sim \mathcal{O}(1)$ MeV(neutrino decoupling)	self-interaction could make neutrinos stay longer in the plasma and extend window of production
Sterile neutrino collision terms	$\mathcal{C}_S = 0$	$\mathcal{C}_S = 0$

Are there any other points?

3 Boltzmann Equation

Since we assume NSI can only modify DW mechanism and there are no other channels to produce sterile neutrinos,

$$\frac{\partial}{\partial t} f_s(p,t) - Hp\frac{\partial}{\partial p} f_s(p,t) \approx \frac{1}{4}\frac{\Gamma_\alpha(p)\Delta^2(p)\sin^2 2\theta}{\Delta^2(p)\sin^2 2\theta + \left(\frac{\Gamma_\alpha(p)}{2}\right)^2 + [\Delta(p)\cos 2\theta - \mathcal{V}_T(p)]^2}\left[f_\alpha(p,t) - f_s(p,t)\right] \tag{3.1}$$

Here,

$$\begin{aligned}\Gamma_\alpha &= \Gamma_{SM} + \Gamma_{NSI}\\ \mathcal{V}_T &= \mathcal{V}_{SM} + \mathcal{V}_{NSI}\\ \Delta &= \frac{m_s^2}{2p}\end{aligned}$$

We can also set $f_s(p,T) = 0$ if we assume there is no initial abundance of sterile neutrinos before the production.

4 Interaction rate

In the interaction rate calculations, we consider processes involving active neutrinos.
We have SM processes with interaction rate,

$$\Gamma_a = C_a(p,T)G_F^2 T^4 p \tag{4.1}$$

where $C_a(p,T)$ is a momentum- and temperature dependent coefficient.
In our extension, processes that will get extra contribution from active neutrino self-interaction.

$$\begin{aligned}\nu_e + \nu_e &\to \nu_e + \nu_e\\ \bar{\nu}_e + \bar{\nu}_e &\to \bar{\nu}_e + \bar{\nu}_e\\ \nu_e + \bar{\nu}_e &\to \nu_e + \bar{\nu}_e\end{aligned}$$

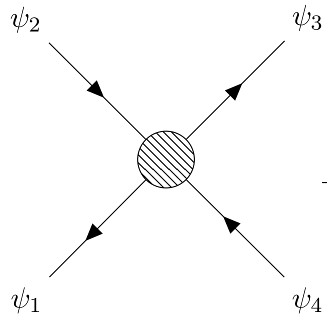
4.1 Matrix elements

EFT vertex

From [3], 4-Fermion EFT vertex for a general Lagrangian

$$\mathcal{L}_{\text{int}} \equiv \sum C^{f_1 f_2 f_3 f_4}\Gamma_{s_1 s_2}\Gamma_{s_3 s_4}\bar{\psi}_{s_1}^{f_1}\psi_{s_2}^{f_2}\bar{\psi}_{s_3}^{f_3}\psi_{s_4}^{f_4} \tag{4.2}$$

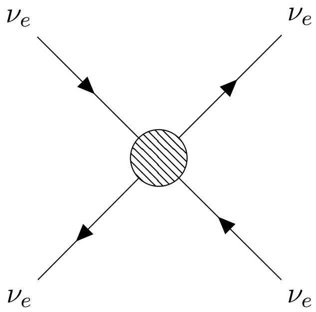
is given by,



$$\longrightarrow i\Gamma_{1234} =\equiv 2iC^{f_1f_2f_3f_4}\Gamma_{s_1s_2}\Gamma_{s_3s_4} - 2iC^{f_1f_4f_3f_2}\Gamma_{s_1s_4}\Gamma_{s_3s_2} \quad (4.3)$$

For NSI Lagrangian,

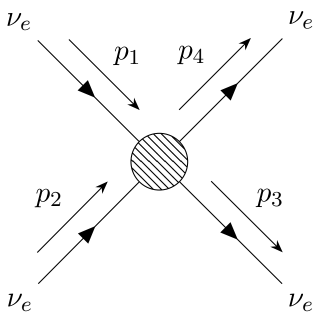
$$\mathcal{L}_j = -\frac{G_F}{\sqrt{2}}\left(\epsilon_{j,\nu_e}\right)^{eeee}\left(\bar{\nu}_e\mathcal{O}_j\nu_e\right)\left(\bar{\nu}_e\mathcal{O}_j\nu_e\right) \tag{4.4}$$



$$\longrightarrow i\Gamma_{1234} =\equiv -2i\frac{G_F}{\sqrt{2}}\left(\epsilon_{j,\nu_e}\right)^{eeee}\left[\mathcal{O}_{s_1s_2}\mathcal{O}_{s_3s_4} - \mathcal{O}_{s_1s_4}\mathcal{O}_{s_3s_2}\right] \quad (4.5)$$

$$u(p_i) \text{ or } \bar{u}(p_i) \equiv \text{spinors for neutrinos}$$
$$v(p_i) \text{ or } \bar{v}(p_i) \equiv \text{spinors for anti-neutrinos}$$

4.1.1 $\nu_e + \nu_e \to \nu_e + \nu_e$



$$-i\mathcal{M} = \left[\bar{u}(p_4)\bar{u}(p_3)\right]\left[i\Gamma_{4231}\right]\left[u(p_1)u(p_2)\right] \tag{4.6}$$

$$-i\mathcal{M} = \left[\bar{u}(p_4)\bar{u}(p_3)\right]\left(-2i\frac{G_F}{\sqrt{2}}\left(\epsilon_{j,\nu_e}\right)^{eeee}\left[\mathcal{O}_{42}\mathcal{O}_{31} - \mathcal{O}_{41}\mathcal{O}_{32}\right]\right)\left[u(p_1)u(p_2)\right] \tag{4.7}$$

$$\mathcal{M} = \sqrt{2}G_F\left(\epsilon_{j,\nu_e}\right)^{eeee}\left(\left[\bar{u}(p_4)\mathcal{O}u(p_2)\bar{u}(p_3)\mathcal{O}u(p_1)\right] - \left[\bar{u}(p_4)\mathcal{O}u(p_1)\bar{u}(p_3)\mathcal{O}u(p_2)\right]\right) \tag{4.8}$$

$$\mathcal{M}^\dagger = \sqrt{2}G_F\left(\epsilon_{j,\nu_e}\right)^{eeee}\left(\left[\bar{u}(p_1)\bar{\mathcal{O}}u(p_3)\bar{u}(p_2)\bar{\mathcal{O}}u(p_4)\right] - \left[\bar{u}(p_2)\bar{\mathcal{O}}u(p_3)\bar{u}(p_1)\bar{\mathcal{O}}u(p_4)\right]\right) \tag{4.9}$$

where $\bar{\mathcal{O}} = \gamma^0\mathcal{O}^\dagger\gamma^0$

Take average over initial spins and sum over final spins:

$$\overline{|\mathcal{M}_j|}^2 = \frac{1}{4}\sum \mathcal{M}_j^\dagger \mathcal{M}_j \tag{4.10}$$

For Scalar NSI:

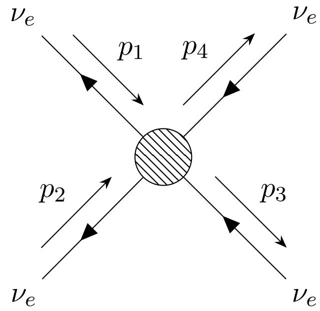
$$\overline{|\mathcal{M}_S|}^2 = 32\left(\epsilon_{S,\nu_e}^{eeee}\right)^2 G_F^2 \left(p_1 \cdot p_2\right)\left(p_3 \cdot p_4\right) \tag{4.11}$$

For Vector NSI:

$$\overline{|\mathcal{M}_V|}^2 = 512\left(\epsilon_{V,\nu_e}^{eeee}\right)^2 G_F^2 \left(p_1 \cdot p_2\right)\left(p_3 \cdot p_4\right) \tag{4.12}$$

4.1.2 $\bar{\nu}_e + \bar{\nu}_e \to \bar{\nu}_e + \bar{\nu}_e$

(Is this really different from the one above?)



$$-i\mathcal{M} = \left[\bar{v}(p_1)\bar{v}(p_2)\right]\left[i\Gamma_{1324}\right]\left[v(p_3)v(p_4)\right] \tag{4.13}$$

$$-i\mathcal{M} = \left[\bar{v}(p_1)\bar{v}(p_2)\right]\left(-2i\frac{G_F}{\sqrt{2}}\left(\epsilon_{j,\nu_e}\right)^{eeee}\left[\mathcal{O}_{13}\mathcal{O}_{24} - \mathcal{O}_{14}\mathcal{O}_{23}\right]\right)\left[v(p_3)v(p_4)\right] \tag{4.14}$$

$$\mathcal{M} = \sqrt{2}G_F\left(\epsilon_{j,\nu_e}\right)^{eeee}\left(\left[\bar{v}(p_1)\mathcal{O}v(p_3)\bar{v}(p_2)\mathcal{O}v(p_4)\right] - \left[\bar{v}(p_1)\mathcal{O}v(p_4)\bar{v}(p_2)\mathcal{O}v(p_3)\right]\right) \tag{4.15}$$

$$\mathcal{M}^\dagger = \sqrt{2}G_F\left(\epsilon_{j,\nu_e}\right)^{eeee}\left(\left[\bar{v}(p_4)\bar{\mathcal{O}}v(p_2)\bar{v}(p_3)\bar{\mathcal{O}}v(p_1)\right] - \left[\bar{v}(p_3)\bar{\mathcal{O}}v(p_2)\bar{v}(p_4)\bar{\mathcal{O}}v(p_1)\right]\right) \tag{4.16}$$

where $\bar{\mathcal{O}} = \gamma^0\mathcal{O}^\dagger\gamma^0$

Take average over initial spins and sum over final spins:

$$\overline{|\mathcal{M}_j|}^2 = \frac{1}{4}\sum \mathcal{M}_j^\dagger \mathcal{M}_j \tag{4.17}$$

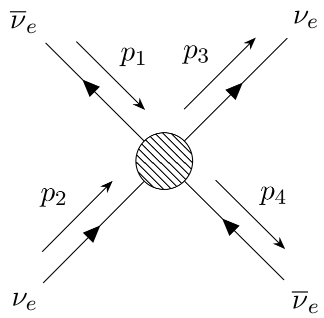
For Scalar NSI:

$$\overline{|\mathcal{M}_S|}^2 = 32\left(\epsilon_{S,\nu_e}^{eeee}\right)^2 G_F^2 \left(p_1 \cdot p_2\right)\left(p_3 \cdot p_4\right) \tag{4.18}$$

For Vector NSI:

$$\overline{|\mathcal{M}_V|}^2 = 512\left(\epsilon_{V,\nu_e}^{eeee}\right)^2 G_F^2 \left(p_1 \cdot p_2\right)\left(p_3 \cdot p_4\right) \tag{4.19}$$

4.1.3 $\nu_e + \bar{\nu}_e \to \nu_e + \bar{\nu}_e$



$$-i\mathcal{M} = [\bar{u}(p_3)v(p_4)]\,[i\Gamma_{1234}]\,[\bar{v}(p_1)u(p_2)] \tag{4.20}$$

$$i\mathcal{M} = [\bar{u}(p_3)v(p_4)]\left(-2i\frac{G_F}{\sqrt{2}}\,(\epsilon_{j,\nu_e})^{eeee}\,[\mathcal{O}_{12}\mathcal{O}_{34} - \mathcal{O}_{14}\mathcal{O}_{32}]\right)[\bar{v}(p_1)u(p_2)] \tag{4.21}$$

$$\mathcal{M} = \sqrt{2}G_F\,(\epsilon_{j,\nu_e})^{eeee}\,\big(\,[\bar{v}(p_1)\mathcal{O}u(p_2)\bar{u}(p_3)\mathcal{O}v(p_4)] - [\bar{v}(p_1)\mathcal{O}v(p_4)\bar{u}(p_3)\mathcal{O}u(p_2)]\,\big) \tag{4.22}$$

$$\mathcal{M}^\dagger = \sqrt{2}G_F\,(\epsilon_{j,\nu_e})^{eeee}\,\big(\,\big[\bar{v}(p_4)\bar{\mathcal{O}}u(p_3)\bar{u}(p_2)\bar{\mathcal{O}}v(p_1)\big] - \big[\bar{u}(p_2)\bar{\mathcal{O}}u(p_3)\bar{v}(p_4)\bar{\mathcal{O}}v(p_1)\big]\,\big) \tag{4.23}$$

where $\bar{\mathcal{O}} = \gamma^0\mathcal{O}^\dagger\gamma^0$

Take average over initial spins and sum over final spins:

$$\overline{|\mathcal{M}_j|}^2 = \frac{1}{4}\sum \mathcal{M}_j^\dagger \mathcal{M}_j \tag{4.24}$$

For Scalar NSI:

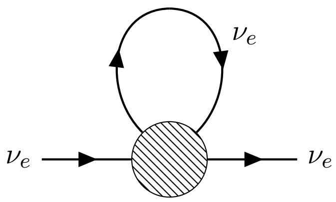
$$\overline{|\mathcal{M}_S|}^2 = 32\left(\epsilon_{S,\nu_e}^{eeee}\right)^2 G_F^2\,(p_1\cdot p_3)\,(p_2\cdot p_4) \tag{4.25}$$

For Vector NSI:

$$\overline{|\mathcal{M}_V|}^2 = 512\left(\epsilon_{V,\nu_e}^{eeee}\right)^2 G_F^2\,(p_1\cdot p_3)\,(p_2\cdot p_4) \tag{4.26}$$

5 Thermal potential

In addition to SM self-energy diagrams, self-interaction can contribute a new diagram for thermal potential calculation.



A Calculation of Interaction rate

Following [4] and [5],
For a reaction $\psi+a+b+\cdots \rightarrow i+j+\ldots$ Collision term in the RHS of Boltzmann equation,

$$\begin{aligned} C_{\psi}^{(f)}= & -\frac{1}{2 E_{\psi}} \int d \Pi_{a} d \Pi_{b} \cdots d \Pi_{i} d \Pi_{j} \cdots(2 \pi)^{4} \delta^{4}\left(p_{\psi}+p_{a}+p_{b}+\cdots-p_{i}-p_{j}-\cdots\right) \\ & \times S\left[|\mathcal{M}|_{\psi+a+b+\cdots \rightarrow i+j+\cdots}^{2} f_{\psi} f_{a} f_{b} \cdots\left(1 \pm f_{i}\right)\left(1 \pm f_{j}\right) \cdots\right. \\ & \left.-|\mathcal{M}|_{i+j+\cdots \rightarrow \psi+a+b+\cdots}^{2} f_{i} f_{j} \cdots\left(1 \pm f_{\psi}\right)\left(1 \pm f_{a}\right)\left(1 \pm f_{b}\right) \cdots\right] \end{aligned} \tag{A.1}$$

where $d\Pi_x \equiv \frac{g_x}{(2\pi)^3}\frac{d^3p_x}{2E_x}, x \in \{\psi, a, b, \cdots i, j, \cdots\}$
For $1+2 \rightarrow 3+4$ reaction,

$$\begin{aligned} C_{1}^{(f)}= & -\frac{1}{2 E_{1}} \int d \Pi_{2} d \Pi_{3} d \Pi_{4}(2 \pi)^{4} \delta^{4}\left(p_{1}+p_{2}-p_{3}-p_{4}\right) \\ & \times S\left[|\mathcal{M}|_{1+2 \rightarrow 3+4}^{2} f_{1} f_{2}\left(1 \pm f_{3}\right)\left(1 \pm f_{4}\right)\right. \\ & \left.-|\mathcal{M}|_{3+4 \rightarrow 1+2}^{2} f_{3} f_{4}\left(1 \pm f_{1}\right)\left(1 \pm f_{2}\right)\right] \end{aligned} \tag{A.2}$$

From A.2, we can extract interaction rate (From Eq. 3 in https://arxiv.org/pdf/astro-ph/9506015.pdf to Eq. B7 in https://arxiv.org/pdf/1507.06655.pdf)

can you check if there is a minus sign? also it is not exactly clear to me how they get rid of second term with $f_3 f_4 (1 \pm f_1)(1 \pm f_2)$ in 1507.06655

$$\Gamma_{1}=\frac{1}{2 E_{1}} \int d \Pi_{2} d \Pi_{3} d \Pi_{4}(2 \pi)^{4} \delta^{4}\left(p_{1}+p_{2}-p_{3}-p_{4}\right) \times \mathcal{S} \sum|\mathcal{M}|^{2} f_{2}\left(1 \pm f_{3}\right)\left(1 \pm f_{4}\right) \tag{A.3}$$

$$\begin{aligned} \delta^{4}\left(p_{1}+p_{2}-p_{3}-p_{4}\right) &= \delta\left(E_{1}+E_{2}-E_{3}-E_{4}\right) \delta^{3}\left(\vec{p}_{1}+\vec{p}_{2}-\vec{p}_{3}-\vec{p}_{4}\right) \\ \delta^{3}\left(\vec{p}_{1}+\vec{p}_{2}-\vec{p}_{3}-\vec{p}_{4}\right) &= \int \frac{d^{3} \lambda}{2 \pi^{3}} e^{\left(\vec{p}_{1}+\vec{p}_{2}-\vec{p}_{3}-\vec{p}_{4}\right) \cdot \lambda} \\ d^{3} p_{i} &= p_{i}^{2} d p_{i} d\left(\cos \theta_{i}\right) d \phi_{i} \equiv p_{i}^{2} d p_{i} d \Omega_{i} \end{aligned}$$

Now,

$$\begin{aligned} \Gamma_{1}= & \frac{1}{2 E_{1}(2 \pi)^{3}} \int \prod_{i=2}^{4} p_{i}^{2} d p_{i} d \Omega_{i} \times(2 \pi) \delta\left(E_{1}+E_{2}-E_{3}-E_{4}\right) \\ & \int d^{3} \lambda e^{\left(\vec{p}_{1}+\vec{p}_{2}-\vec{p}_{3}-\vec{p}_{4}\right) \cdot \lambda} \times \mathcal{S} \sum|\mathcal{M}|^{2} f_{2}\left(1 \pm f_{3}\right)\left(1 \pm f_{4}\right) \end{aligned} \tag{A.4}$$

$$\Gamma_{1}=\frac{1}{64 \pi^{3} E_{1} p_{1}} \int \frac{p_{2} d p_{2}}{E_{2}} \frac{p_{3} d p_{3}}{E_{3}} \frac{p_{4} d p_{4}}{E_{4}} \times(2 \pi) \delta\left(E_{1}+E_{2}-E_{3}-E_{4}\right) F(f) D\left(p_{1}, p_{2}, p_{3}, p_{4}\right) \tag{A.5}$$

Where

$$F(f)=f_{2}\left(1 \pm f_{3}\right)\left(1 \pm f_{4}\right), \quad f_{i}=\frac{1}{\exp \left(\frac{\mathrm{E_i}}{\mathrm{T_i}}\right)+1}$$

When $E_i \gg T_i$, $F_{FD} \to F_{MB} = \exp\left(\frac{-\mathrm{E_i}}{\mathrm{T_i}}\right)$

Matrix elements are of the form,

$$\begin{aligned} \mathcal{S}\overline{|\mathcal{M}|}^2 &= K_1 \left(p_1 \cdot p_2\right)\left(p_3 \cdot p_4\right) \\ &= K_1 \Big(E_1 E_2 E_3 E_4 - E_1 E_2 \left(\vec{p}_3 \cdot \vec{p}_4\right) - E_3 E_4 \left(\vec{p}_1 \cdot \vec{p}_2\right) + \left(\vec{p}_1 \cdot \vec{p}_2\right)\left(\vec{p}_3 \cdot \vec{p}_4\right) \Big) \end{aligned} \tag{A.6}$$

$$D(p_1, p_2, p_3, p_4) = \frac{p_1 p_2 p_3 p_4}{64\pi^5} \int_0^\infty \lambda^2 d\lambda d\Omega_\lambda \prod_{i=2}^{4} d\Omega_i e^{-i(\mathbf{p_1}+\mathbf{p_2}-\mathbf{p_3}-\mathbf{p_4})\cdot\lambda} S\overline{|\mathcal{M}|}^2 \tag{A.7}$$

$$D(p_1, p_2, p_3, p_4) = \frac{p_1 p_2 p_3 p_4}{64\pi^5} \int_0^\infty \lambda^2 d\lambda \int e^{-i\vec{p_1}\cdot\lambda} d\Omega_\lambda \int e^{-i\vec{p_2}\cdot\lambda} d\Omega_2 \int e^{+i\vec{p_3}\cdot\lambda} d\Omega_3 \int e^{+i\vec{p_4}\cdot\lambda} d\Omega_4 \times S\overline{|\mathcal{M}|}^2 \tag{A.8}$$

$$D(p_1, p_2, p_3, p_4) = K_1 \Big(E_1 E_2 E_3 E_4 D_{SS} - E_1 E_2 p_3 p_4 D_{SC} - E_3 E_4 p_1 p_2 D_{CS} + p_1 p_2 p_3 p_4 D_{CC} \Big) \tag{A.9}$$

where,

$$D_{SS} = \frac{p_1 p_2 p_3 p_4}{64\pi^5} \int_0^\infty \lambda^2 d\lambda \frac{4\pi \cdot \sin(p_1\lambda)}{p_1\lambda} \frac{4\pi \cdot \sin(p_2\lambda)}{p_2\lambda} \frac{4\pi \cdot \sin(p_3\lambda)}{p_3\lambda} \frac{4\pi \cdot \sin(p_4\lambda)}{p_4\lambda} \tag{A.10}$$

$$\begin{aligned} D_{SC} = \frac{-4}{\pi} \int \frac{d\lambda}{\lambda^4} \cdot \sin(p_1\lambda) \cdot \sin(p_2\lambda) \cdot \left(\sin\left(p_3\lambda\right) - p_3\lambda \cos\left(\left(p_3\lambda\right)\right)\right. \\ \times \left(\sin\left(p_4\lambda\right) - p_4\lambda \cos\left(\left(p_4\lambda\right)\right)\right) \end{aligned} \tag{A.11}$$

$$\begin{aligned} D_{CS} = \frac{-4}{\pi} \int \frac{d\lambda}{\lambda^4} \cdot \left(\sin\left(p_1\lambda\right) - p_1\lambda \cos\left(\left(p_1\lambda\right)\right) \cdot \left(\sin\left(p_2\lambda\right) - p_2\lambda \cos\left(\left(p_2\lambda\right)\right)\right)\right. \\ \times \sin(p_3\lambda) \cdot \sin(p_4\lambda) \end{aligned} \tag{A.12}$$

and,

$$\begin{aligned} D_{CC} = \frac{4}{\pi} \int \frac{d\lambda}{\lambda^6} \cdot \left(\sin\left(p_1\lambda\right) - p_1\lambda \cos\left(\left(p_1\lambda\right)\right) \cdot \left(\sin\left(p_2\lambda\right) - p_2\lambda \cos\left(\left(p_2\lambda\right)\right)\right)\right. \\ \times \left(\sin\left(p_3\lambda\right) - p_3\lambda \cos\left(\left(p_3\lambda\right)\right) \cdot \left(\sin\left(p_4\lambda\right) - p_4\lambda \cos\left(\left(p_4\lambda\right)\right)\right)\right. \end{aligned} \tag{A.13}$$

These can be evaluated with Mathematica and have 4 different possibilities for different conditions for momentum:

1. $p_1 + p_2 > p_3 + p_4$ and $p_1 + p_4 > p_2 + p_3$,
2. $p_1 + p_2 > p_3 + p_4$ and $p_1 + p_4 < p_2 + p_3$,
3. $p_1 + p_2 < p_3 + p_4$ and $p_1 + p_4 < p_2 + p_3$,
4. $p_1 + p_2 < p_3 + p_4$ and $p_1 + p_4 > p_2 + p_3$,

Results are calculated in Mathematica and can be compared with Eq. C20-24 of [4]
Taking the massless assumption $E_i = p_i$,

$$D(p_1, p_2, p_3, p_4) = K_1 \Big(E_1E_2E_3E_4D_{SS} - E_1E_2E_3E_4D_{SC} - E_3E_4E_1E_2D_{CS} + E_1E_2E_3E_4D_{CC} \Big) \tag{A.14}$$

$$\Gamma_1 = \frac{K_1}{32\pi^2 p_1} \int p_2 dp_2 p_3 dp_3 p_4 dp_4 \delta\left(p_1 + p_2 - p_3 - p_4\right) F\left(f\right) \left(D_{SS} - D_{CS} - D_{SC} + D_{CC}\right) \tag{A.15}$$

Similar to Eq. C29 in [4]. We can numerically integrate this using Mathematica.

References

[1] A. de Gouvẩa, M. Sen, W. Tangarife and Y. Zhang, *Dodelson-widrow mechanism in the presence of self-interacting neutrinos*, *Physical Review Letters* **124** (2020) .

[2] K.J. Kelly, M. Sen, W. Tangarife and Y. Zhang, *Origin of sterile neutrino dark matter via secret neutrino interactions with vector bosons*, *Physical Review D* **101** (2020) .

[3] M. Paraskevas, *Dirac and Majorana Feynman Rules with four-fermions*, 1802.02657.

[4] X. Luo, W. Rodejohann and X.-J. Xu, *Dirac neutrinos and* $N_{\rm eff}$, *JCAP* **06** (2020) 058 [2005.01629].

[5] A. Dolgov, S. Hansen and D. Semikoz, *Non-equilibrium corrections to the spectra of massless neutrinos in the early universe*, *Nuclear Physics B* **503** (1997) 426 .