Active neutrino self-interaction with NSI (September 14, 2021)

Contents

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1 Aim

- check how DW Mechanism can be improved by NSI.
- work with dim-6 four neutrino operators.

2 Particle model

- We would use electron flavor as the one and only active neutrino flavor($\nu_e \equiv \nu$)(Is electron flavor more constrained than others?).
- N is the sterile neutrino.
- ν & N could be either Dirac or Majorana particles.
- ν & N Dirac $\Rightarrow$ only Vector, Tensor active neutrino self interaction NSIs.
- ν & N Majorana $\Rightarrow$ only Scalar, Pseudoscalar, Axial-vector active neutrino self interaction NSIs.
- We will consider Majorana neutrinos first.
- For Majorana neutrinos,
  - Majorana condition implies $\psi = \psi^c$
  - ν is part of $\mathrm{SU}(2)_L$ doublet $(\nu_L \quad e)^T$.
  - $\nu = \nu_L + \nu_R = \nu_L + (\nu_L)^c = \nu_L + (\nu^c)_R$
  - N is singlet under $SU(2)_L$ and is our sterile neutrino.
  - $N = N_R + N_L = N_R + (N_R)^c = N_R + (N^c)_L$
  - ν and N are independent(N need not to be same flavor as ν).
    They make separate Majorana mass terms,

$$\mathcal{L}_M = m_1 \overline{(\nu_L)^c} \nu_L + m_2 \overline{(N_R)^c} N_R + \text{h.c}$$

2.1 Active neutrino NSI

Most general NSI Lagrangian for active neutrino(ν) interactions with only electron flavor[1],

$$\mathcal{L}_j = -\frac{G_F}{\sqrt{2}} \left(\epsilon_{j,\nu}\right) \left(\bar{\nu}_e \mathcal{O}_j \nu_e\right) \left(\bar{\nu}_e \mathcal{O}'_j \nu_e\right) \tag{2.1}$$

Where $\mathcal{O}_j, \mathcal{O}'_j$ are operators for different kinds of mediators.

2.1.1 Scalar NSI

$\mathcal{O}_S = \mathbb{I}, \mathcal{O}'_S = \mathbb{I}$

$$\begin{aligned}
\mathcal{L}_S &= -\frac{G_F}{\sqrt{2}} \left(\epsilon_{S,\nu}\right) \left(\bar{\nu}\nu\right) \left(\bar{\nu}\nu\right) \\
&= -\frac{G_F}{\sqrt{2}} \left(\epsilon_{S,\nu}\right) \left(\overline{(\nu_L + (\nu_L)^c)}(\nu_L + (\nu_L)^c)\right) \left(\overline{(\nu_L + (\nu_L)^c)}(\nu_L + (\nu_L)^c)\right) \\
&= -\frac{G_F}{\sqrt{2}} \left(\epsilon_{S,\nu}\right) \left(\overline{(\nu_L)^c}\nu_L + \text{h.c}\right) \left(\overline{(\nu_L)^c}\nu_L + \text{h.c}\right) \\
&= -\frac{G_F}{\sqrt{2}} \left(\epsilon_{S,\nu}\right) \left[\left(\overline{(\nu_L)^c}\nu_L\right) \left(\overline{(\nu_L)^c}\nu_L\right) + \left(\overline{(\nu_L)^c}\nu_L\right) \left(\overline{\nu_L}(\nu_L)^c\right) + \text{h.c}\right]
\end{aligned} \tag{2.2}$$

If we had taken $\mathcal{O} = \mathcal{O}'_j = \mathbb{I} - \gamma^5$, we would have missed few terms in the Lagrangian. So it might be better to take $\mathcal{O} = \mathcal{O}'_j = \mathbb{I}$

2.1.2 Vector NSI

Pure vector current would be zero for Majorana neutrinos.

$$\bar{\psi}\gamma^\mu\psi = \overline{\psi^C}\gamma^\mu\psi^C = -\psi^T\mathcal{C}^\dagger\gamma^\mu\mathcal{C}\bar{\psi}^T = \bar{\psi}\mathcal{C}\gamma^{\mu T}\mathcal{C}^\dagger\psi = -\bar{\psi}\gamma^\mu\psi = 0$$

2.1.3 Axial-Vector NSI

An axial-vector interaction with $\mathcal{O}_A = \gamma^\mu\gamma^5, \mathcal{O}'_L = \gamma_\mu\gamma^5$

$$\mathcal{L}_A = -\frac{G_F}{\sqrt{2}} \left(\epsilon_{A,\nu}\right) \left(\bar{\nu}\gamma^\mu\gamma^5\nu\right) \left(\bar{\nu}\gamma_\mu\gamma^5\nu\right) \tag{2.3}$$

2.1.4 Pseudoscalar NSI

$\mathcal{O}_P = \gamma^5, \mathcal{O}'_P = \gamma^5$

$$\mathcal{L}_S = -\frac{G_F}{\sqrt{2}} \left(\epsilon_{P,\nu}\right) \left(\bar{\nu}\gamma^5\nu\right) \left(\bar{\nu}\gamma^5\nu\right) \tag{2.4}$$

2.1.5 Tensor NSI

similar to vector current, pure tensor current would be zero for Majorana neutrinos.

$$\bar{\psi}\sigma^{\mu\nu}\psi = \overline{\psi^C}\sigma^{\mu\nu}\psi^C = -\psi^T\mathcal{C}^\dagger\sigma^{\mu\nu}\mathcal{C}\bar{\psi}^T = \bar{\psi}\mathcal{C}\left(\sigma^{\mu\nu}\right)^T\mathcal{C}^\dagger\psi = -\bar{\psi}\sigma^{\mu\nu}\psi = 0$$

If we consider interaction with $\mathcal{O}_j = \sigma^{\mu\nu}(\mathbb{I} - \gamma^5), \mathcal{O}'_j = \sigma_{\mu\nu}(\mathbb{I} - \gamma^5)$,

$$\mathcal{C}(\sigma^{\mu\nu}P_L)^{\mathrm{T}}\mathcal{C}^{-1} = -P_L\sigma^{\mu\nu} = -\sigma^{\mu\nu}P_L$$

$$\bar{\psi}\sigma^{\mu\nu}(\mathbb{I}-\gamma^5)\psi = \overline{\psi^C}\sigma^{\mu\nu}(\mathbb{I}-\gamma^5)\psi^C = -\psi^T\mathcal{C}^\dagger\sigma^{\mu\nu}(\mathbb{I}-\gamma^5)\mathcal{C}\bar{\psi}^T$$
$$= \bar{\psi}\mathcal{C}\left(\sigma^{\mu\nu}(\mathbb{I}-\gamma^5)\right)^T\mathcal{C}^\dagger\psi = -\bar{\psi}\sigma^{\mu\nu}(\mathbb{I}-\gamma^5)\psi = 0$$

So it is not possible to get a tensor NSI with Majorana neutrinos.(mentioned in [1])

ie, general form of Majorana active neutrino NSI is;

$$\mathcal{L}_j = -\frac{G_F}{\sqrt{2}}\left(\epsilon_{j,\nu}\right)\left(\bar{\nu}_e\mathcal{O}_j\nu_e\right)\left(\bar{\nu}_e\mathcal{O}'_j\nu_e\right) \tag{2.5}$$

with operators

$$\begin{aligned} \text{Scalar:} \quad & \mathcal{O}_S = \mathbb{I} \\ \text{Axial-vector:} \quad & \mathcal{O}_A = \gamma^\mu\gamma^5 \\ \text{Pseudoscalar:} \quad & \mathcal{O}_P = \gamma^5 \end{aligned}$$

3 Differences from DW mechanism

	DW only	DW + active-active NSI
Interaction rate	Γ_{SM}	$\Gamma_{\text{SM}} + \Gamma_{\text{NSI}}$
Thermal potential	$\mathcal{V}_{\nu_e} + \mathcal{V}_e$	$\mathcal{V}_{\nu_e} + \mathcal{V}_e + \mathcal{V}_{\text{NSI}}$
mechanism	freeze-in	freeze-in
sterile neutrino mass	$\sim$ keV	$\sim$ keV
Temperature range	$\sim \mathcal{O}(100)$ MeV to $\sim \mathcal{O}(1)$ MeV(neutrino decoupling)	self-interaction could make neutrinos stay longer in the plasma and extend window of production
Sterile neutrino collision terms	$\mathcal{C}_S = 0$	$\mathcal{C}_S = 0$

4 Boltzmann Equation

Since we assume NSI can only modify DW mechanism and there are no other channels to produce sterile neutrinos,

$$\frac{\partial}{\partial t}f_s(p,t) - Hp\frac{\partial}{\partial p}f_s(p,t) \approx \frac{1}{4}\frac{\Gamma_\alpha(p)\Delta^2(p)\sin^2 2\theta}{\Delta^2(p)\sin^2 2\theta + \left(\frac{\Gamma_\alpha(p)}{2}\right)^2 + \left[\Delta(p)\cos 2\theta - \mathcal{V}_T(p)\right]^2}\left[f_\alpha(p,t) - f_s(p,t)\right] \tag{4.1}$$

Here,

$$\Gamma_\alpha = \Gamma_{SM} + \Gamma_{NSI}$$
$$\mathcal{V}_T = \mathcal{V}_{SM} + \mathcal{V}_{NSI}$$
$$\Delta = \frac{m_s^2}{2p}$$

We can also set $f_s(p,T) = 0$ if we assume there is no initial abundance of sterile neutrinos before the production.

5 Interaction rate

In the interaction rate calculations, we consider processes involving active neutrinos.
We have SM processes with interaction rate[2],

$$\Gamma_a = C_a(p,T) G_F^2 p T^4 \tag{5.1}$$

where $C_a(p,T)$ is a momentum- and temperature dependent coefficient.
If we have Majorana neutrinos, we will get extra contribution from ;

$$\nu_e + \nu_e \to \nu_e + \nu_e$$

$\nu_e + \bar{\nu}_e \to \nu_e + \bar{\nu}_e$ would be indistinguishable from $\nu_e + \nu_e \to \nu_e + \nu_e$
("*Obviously, Majorana particles must be genuinely neutral, i.e. they cannot possess any conserved charge-like quantum number that would allow one to discriminate between the particle and its antiparticle*"[3]).
But there would be an additional term in the Matrix element for neutrino scattering which would make Majorana case with one scattering similar to Dirac case with two scatterings.
Check Appendix A for detailed calculation

5.1 Scalar NSI

$$\Gamma_S(E,T) = \frac{7\pi G_F^2 \epsilon_S^2}{180} E T^4 \tag{5.2}$$

5.2 V-A NSI

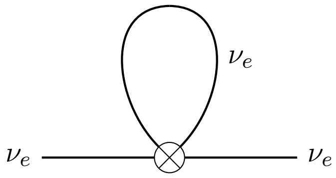
$$\Gamma_A(E,T) = \frac{7\pi G_F^2 \epsilon_A^2}{135} E T^4 \tag{5.3}$$

5.3 Pseudoscalar NSI

$$\Gamma_P(E,T) = \frac{7\pi G_F^2 \epsilon_P^2}{180} E T^4 \tag{5.4}$$

6 Thermal potential

In addition to SM self-energy diagrams, self-interaction can contribute a new diagram for thermal potential calculation.



But to get temperature dependant terms in thermal potential, we have to include higher order term in NSI Lagrangian which would give momentum dependent self-energy.

$$\mathcal{L}_j = -\frac{G_F}{\sqrt{2}} \left(\epsilon_{j,\nu}\right) \left(\left(\bar{\nu}_e \mathcal{O}_j \nu_e\right) \left(\bar{\nu}_e \mathcal{O}'_j \nu_e\right) - \frac{1}{m_\phi^2} \left(\bar{\nu}_e \mathcal{O}_j \nu_e\right) \Box \left(\bar{\nu}_e \mathcal{O}'_j \nu_e\right) \right) \tag{6.1}$$

Check Appendix B for detailed calculation

6.1 Scalar NSI(B.3.1)

$$\mathcal{V}_{\text{scalar}} = -\frac{8\sqrt{2} G_F}{3 m_\phi^2} \left(\epsilon_{S,\nu_e}\right)^{eeee} \cdot \omega \cdot \left[n_\nu \langle E_\nu \rangle + n_{\bar{\nu}} \langle E_{\bar{\nu}} \rangle\right] = -\frac{7\sqrt{2}\pi^2 G_F}{45 m_\phi^2} \left(\epsilon_{S,\nu_e}\right)^{eeee} \cdot \omega T^4 \tag{6.2}$$

6.2 Axial-vector NSI(B.3.2)

$$\mathcal{V}_{\text{V-A}} = -\frac{32\sqrt{2} G_F}{3 m_\phi^2} \left(\epsilon_{A,\nu_e}\right)^{eeee} \cdot \omega \cdot \left[n_\nu \langle E_\nu \rangle + n_{\bar{\nu}} \langle E_{\bar{\nu}} \rangle\right] = -\frac{28\sqrt{2}\pi^2 G_F}{45 m_\phi^2} \left(\epsilon_{A,\nu_e}\right)^{eeee} \cdot \omega T^4 \tag{6.3}$$

6.3 Pseudoscalar NSI(B.3.3)

$$\mathcal{V}_{\text{P}} = -\frac{8\sqrt{2} G_F}{3 m_\phi^2} \left(\epsilon_{P,\nu_e}\right)^{eeee} \cdot \omega \cdot \left[n_\nu \langle E_\nu \rangle + n_{\bar{\nu}} \langle E_{\bar{\nu}} \rangle\right] = -\frac{7\sqrt{2}\pi^2 G_F}{45 m_\phi^2} \left(\epsilon_{P,\nu_e}\right)^{eeee} \cdot \omega T^4 \tag{6.4}$$

A Calculation of Interaction rate

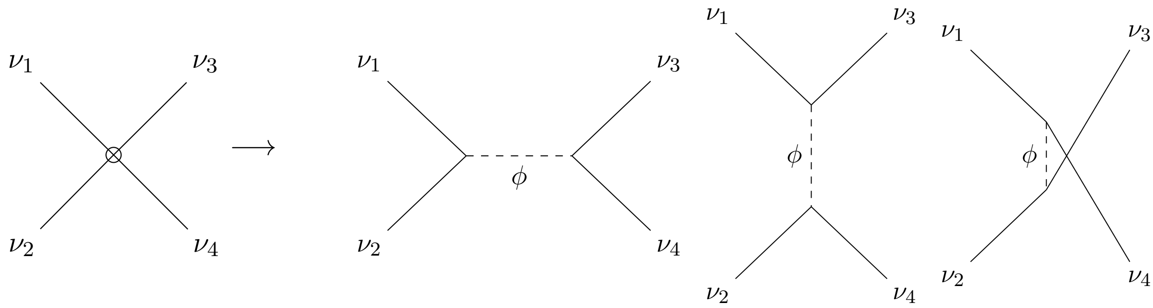
A.1 Matrix elements

EFT vertex

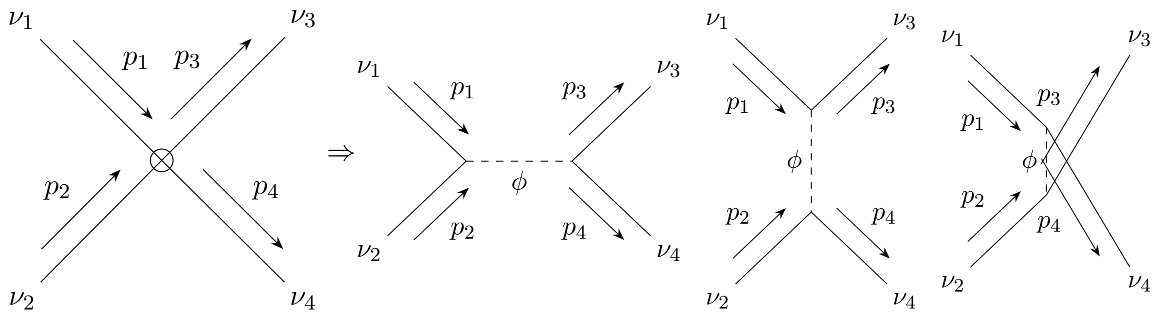
For NSI Lagrangian,

$$\mathcal{L}_j = -\frac{G_F}{\sqrt{2}} \left(\epsilon_{j,\nu_e}\right)^{1234} \left(\bar{\nu}_1 \mathcal{O}_j \nu_2\right) \left(\bar{\nu}_3 \mathcal{O}_j \nu_4\right) \tag{A.1}$$

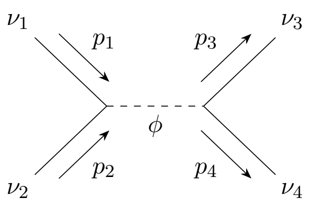
where ν_i is Majorana electron neutrinos, there can be three different topologies[4]. Check Sec VIII.C of [5] for more details on Feynman rules.



A.1.1 $\nu_e + \nu_e \to \nu_e + \nu_e$

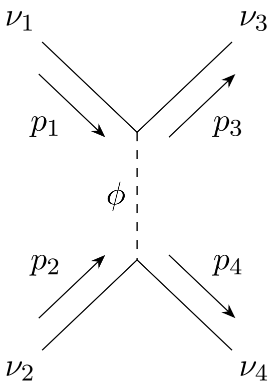


Following [5] and [6] for Feynman rules with Majorana neutrinos



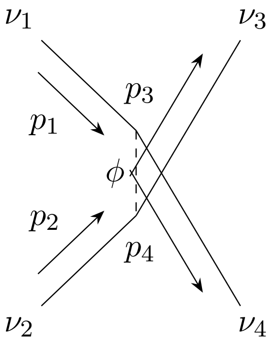
$$\begin{aligned} -i\mathcal{M}_s = -i\frac{G_F}{\sqrt{2}} \left(\epsilon_{j,\nu_e}\right)^{eeee} \big[&(\bar{v}(p_1)\mathcal{O}u(p_2) - \bar{v}(p_2)\mathcal{O}u(p_1)) \\ &\times(\bar{u}(p_3)\mathcal{O}v(p_4) - \bar{u}(p_4)\mathcal{O}v(p_3))\big] \end{aligned} \tag{A.2}$$

$$\begin{aligned} -i\mathcal{M}_s = -i\frac{G_F}{\sqrt{2}} \left(\epsilon_{j,\nu_e}\right)^{eeee} \Big[&(\bar{v}(p_1)\big[\mathcal{O} + C\mathcal{O}^{\mathrm{T}}C^{-1}\big]u(p_2)) \\ &\times(\bar{u}(p_3)\big[\mathcal{O} + C\mathcal{O}^{\mathrm{T}}C^{-1}\big]v(p_4))\Big] \end{aligned} \tag{A.3}$$



$$-i\mathcal{M}_t = -i\frac{G_F}{\sqrt{2}}\left(\epsilon_{j,\nu_e}\right)^{eeee}\left[\left(\bar{v}(p_1)\mathcal{O}v(p_3) - \bar{u}(p_3)\mathcal{O}u(p_1)\right)\right.$$
$$\left.\times\left(\bar{u}(p_4)\mathcal{O}u(p_2)\right) - \bar{v}(p_2)\mathcal{O}v(p_4)\right] \tag{A.4}$$

$$-i\mathcal{M}_t = -i\frac{G_F}{\sqrt{2}}\left(\epsilon_{j,\nu_e}\right)^{eeee}\Bigg[\left(\bar{v}(p_1)\left[\mathcal{O} + C\mathcal{O}^{\mathrm{T}}C^{-1}\right]v(p_3)\right)$$
$$\times\left(\bar{u}(p_4)\left[\mathcal{O} + C\mathcal{O}^{\mathrm{T}}C^{-1}\right]u(p_2)\right)\Bigg] \tag{A.5}$$



$$-i\mathcal{M}_u = -i\frac{G_F}{\sqrt{2}}\left(\epsilon_{j,\nu_e}\right)^{eeee}\left[\left(\bar{v}(p_1)\mathcal{O}v(p_4) - \bar{u}(p_4)\mathcal{O}u(p_1)\right)\right.$$
$$\left.\times\left(\bar{u}(p_3)\mathcal{O}u(p_2)\right) - \bar{v}(p_2)\mathcal{O}v(p_3)\right] \tag{A.6}$$

$$-i\mathcal{M}_u = -i\frac{G_F}{\sqrt{2}}\left(\epsilon_{j,\nu_e}\right)^{eeee}\Bigg[\left(\bar{v}(p_1)\left[\mathcal{O} + C\mathcal{O}^{\mathrm{T}}C^{-1}\right]v(p_4)\right)$$
$$\times\left(\bar{u}(p_3)\left[\mathcal{O} + C\mathcal{O}^{\mathrm{T}}C^{-1}\right]u(p_2)\right)\Bigg] \tag{A.7}$$

See Sec. VIII.C of [5] to see where additional $C\mathcal{O}^{\mathrm{T}}C^{-1}$ come from.

$$\begin{aligned}
C\mathcal{O}^{\mathrm{T}}C^{-1} &= \mathcal{O} \quad \text{for } \mathcal{O} = \mathbb{I}, i\gamma^5, \gamma^\mu\gamma^5 \\
C\mathcal{O}^{\mathrm{T}}C^{-1} &= -\mathcal{O} \quad \text{for } \mathcal{O} = \gamma^\mu, \sigma^{\mu\nu} \\
C(\gamma^\mu P_L)^{\mathrm{T}}C^{-1} &= -P_L\gamma^\mu = -\gamma^\mu P_R \\
C(\sigma^{\mu\nu} P_L)^{\mathrm{T}}C^{-1} &= -P_L\sigma^{\mu\nu} = -\sigma^{\mu\nu} P_L
\end{aligned} \tag{A.8}$$

$$-i\mathcal{M}_{\text{total}} = -i(\mathcal{M}_s - \mathcal{M}_t - \mathcal{M}_u) \tag{A.9}$$

Check A.2 of [4] to clarify about signs
Take average over initial spins and sum over final spins:

$$\overline{|\mathcal{M}_{\text{tot}}|}^2 = \frac{1}{4}\sum \mathcal{M}_{\text{tot}}^\dagger \mathcal{M}_{\text{tot}} \tag{A.10}$$

For Scalar NSI:

$$\overline{|\mathcal{M}_S|}^2 = 4\left(\epsilon_{S,\nu_e}^{eeee}\right)^2 G_F^2\left(s^2 + 5t^2 + u^2\right) \tag{A.11}$$

For Axial-vector NSI:

$$\overline{|\mathcal{M}_A|}^2 = 64\left(\epsilon_{A,\nu_e}^{eeee}\right)^2 G_F^2\left(t^2\right) \tag{A.12}$$

For Pseudoscalar NSI:

$$\overline{|\mathcal{M}_P|}^2 = 4\left(\epsilon_{P,\nu_e}^{eeee}\right)^2 G_F^2\left(s^2 + 5t^2 + u^2\right) \tag{A.13}$$

A.2 Total cross-section

We will work in the Center of Mass frame.

$$
\begin{aligned}
p_1 &= (E_1, \mathbf{p}) \\
p_2 &= (E_2, -\mathbf{p}) \\
p_3 &= (E_3, \mathbf{p}^*) \\
p_4 &= (E_4, -\mathbf{p}^*) \\
s &= (p_1 + p_2)^2 \\
t &= (p_1 - p_3)^2 \\
u &= (p_1 - p_4)^2
\end{aligned}
$$

angle between $\mathbf{p}(\vec{p}_1)$ and $\mathbf{p}^*(\vec{p}_3)$ is θ and s, t, u are Mandelstam variables
Matrix elements are of the form,

$$\overline{|\mathcal{M}|}^2 = K_s s^2 + K_t t^2 + K_u u^2 \tag{A.14}$$

Starting from,

$$d\sigma = \frac{S\hbar^2}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2}} |\mathcal{M}|^2 d\Pi_{\text{LIPS}} \tag{A.15}$$

For a $2 \to j$ process, Lorentz Invariant Phase Space,

$$d\Pi_{\text{LIPS}} \equiv \prod_{\text{final states } j} \frac{d^3 p_j}{(2\pi)^3} \frac{1}{2E_{p_j}} (2\pi)^4 \delta^4(\Sigma p) \tag{A.16}$$

Now for a $1 + 2 \to 3 + 4$ process with massless particles,

$$d\sigma = \frac{S}{2s} |\mathcal{M}|^2 \frac{d^3 p_3}{2E_3 (2\pi)^3} \frac{d^3 p_4}{2E_4 (2\pi)^3} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \tag{A.17}$$

In CoM frame,

$$
\begin{aligned}
d\sigma &= \frac{S}{2s} |\mathcal{M}|^2 \frac{d^3 p_3}{2E_3 (2\pi)^3} \frac{d^3 p_4}{2E_4 (2\pi)^3} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \\
&= \frac{S}{2s} \mathcal{M}|^2 \frac{d^3 p_3}{2E_3 (2\pi)^3} \frac{d^3 p_4}{2E_4 (2\pi)^3} (2\pi)^4 \delta(\sqrt{s} - E_3 - E_4) \delta^3(\vec{p}_3 + \vec{p}_4) \\
&= \frac{S}{2 \cdot (2\pi)^2 \cdot s} |\mathcal{M}|^2 \frac{d^3 p_3}{2E_3} \frac{d^3 p_4}{2E_4} \delta(\sqrt{s} - E_3 - E_4) \delta^3(\vec{p}_3 + \vec{p}_4)
\end{aligned} \tag{A.18}
$$

symmetry factor for neutrino - neutrino scattering, $S = \frac{1}{2}$.

$$\sigma_{\nu\nu-\nu\nu} = \frac{\frac{1}{2}}{2 \cdot (2\pi)^2 \cdot s} \int \frac{d^3 p_3}{2E_3} \frac{d^3 p_4}{2E_4} \delta(\sqrt{s} - E_3 - E_4) \delta^3(\vec{p}_3 + \vec{p}_4) \cdot \left(K_s s^2 + K_t t^2 + K_u u^2\right) \tag{A.19}$$

$$\sigma_{\nu\nu-\nu\nu} = \sigma_s + \sigma_t + \sigma_u \tag{A.20}$$

Where

$$\sigma_s = \frac{\frac{1}{2}}{2\cdot(2\pi)^2\cdot s}\int \frac{d^3p_3}{2E_3}\frac{d^3p_4}{2E_4}\delta(\sqrt{s}-E_3-E_4)\delta^3(\vec{p}_3+\vec{p}_4)\cdot K_s s^2$$
$$\sigma_t = \frac{\frac{1}{2}}{2\cdot(2\pi)^2\cdot s}\int \frac{d^3p_3}{2E_3}\frac{d^3p_4}{2E_4}\delta(\sqrt{s}-E_3-E_4)\delta^3(\vec{p}_3+\vec{p}_4)\cdot K_t t^2 \tag{A.21}$$
$$\sigma_u = \frac{\frac{1}{2}}{2\cdot(2\pi)^2\cdot s}\int \frac{d^3p_3}{2E_3}\frac{d^3p_4}{2E_4}\delta(\sqrt{s}-E_3-E_4)\delta^3(\vec{p}_3+\vec{p}_4)\cdot K_u u^2$$

First for σ_s, integrating out $\mathbf{p}_4$ with $\delta^3(\vec{p}_3+\vec{p}_4)$ with,

$$\int dx\, g(x)\,\delta(f(x)) = g(x_0)\,\frac{1}{f'(x_0)} \qquad \text{where } x_0 \text{ solves } f(x_0)=0.$$

$$\sigma_{\nu\nu-\nu\nu} = \frac{1}{4\cdot(2\pi)^2\cdot s}\cdot K_s s^2\int\frac{d^3p_3}{(2E_3)^2}\delta(\sqrt{s}-2E_3) \tag{A.22}$$
$$d^3p_3 = p_3^2dp_3d\Omega_3$$
$$E_3dE_3 = |\mathbf{p}_3|dp_3$$

$$\begin{aligned}\sigma_s &= \frac{K_s s^2}{4\cdot(2\pi)^2\cdot s}\int\frac{p_3^2dp_3d\Omega_3}{(2E_3)^2}\delta(2E_3-\sqrt{s})\\ &= \frac{K_s s}{4\cdot(2\pi)^2}\cdot\frac{1}{2}\cdot\frac{1}{4}\cdot 4\pi\\ &= \frac{K_s s}{32\pi}\end{aligned} \tag{A.23}$$

For σ_t,

$$\sigma_t = \frac{\frac{1}{2}}{2\cdot(2\pi)^2\cdot s}\int \frac{d^3p_3}{2E_3}\frac{d^3p_4}{2E_4}\delta(\sqrt{s}-E_3-E_4)\delta^3(\vec{p}_3+\vec{p}_4)\cdot K_t t^2 \tag{A.24}$$

$t=(p_1-p_3)^2 = -2p_1\cdot p_3 = 2(\vec{p}_1\cdot\vec{p}_3 - E_1E_3) = 2|p_1||p_3|(\cos\theta-1)t^2 = 4|p_1|^2|p_3|^2(1-\cos\theta)^2$

$$\sigma_{\nu\bar{\nu}-\nu\bar{\nu}} = \frac{1}{4\cdot(2\pi)^2\cdot s}\int\frac{d^3p_3}{2E_3}\frac{d^3p_4}{2E_4}\delta(\sqrt{s}-E_3-E_4)\delta^3(\vec{p}_3+\vec{p}_4)\cdot K_t\cdot 4|p_1|^2|p_3|^2(1-\cos\theta)^2 \tag{A.25}$$

integrating out $\mathbf{p}_4$ with $\delta^3(\vec{p}_3+\vec{p}_4)$,

$$\sigma_{\nu\bar{\nu}-\nu\bar{\nu}} = \frac{K_t|p_1|^2}{4\pi^2 s}\int\frac{d^3p_3}{4}\delta(\sqrt{s}-2E_3)(1-\cos\theta)^2 \tag{A.26}$$
$$d^3p_3 = p_3^2dp_3d\Omega_3 = p_3^2dp_3d(\cos\theta)d\phi$$
$$E_3dE_3 = |\mathbf{p}_3|dp_3$$

$$\begin{aligned}\sigma_t &= \frac{K_t|p_1|^2}{16\pi^2 s}\int p_3^2dp_3d(\cos\theta)d\phi\delta(2p_3-\sqrt{s})(1-\cos\theta)^2\\ &= \frac{K_t\frac{s}{4}}{16\pi^2 s}\int p_3^2\delta(2p_3-\sqrt{s})dp_3\int_{-1}^{1}d(\cos\theta)(1-\cos\theta)^2\int_0^{2\pi}d\phi\\ &= \frac{K_t}{64\pi^2}\cdot\frac{1}{2}\left(\frac{\sqrt{s}}{2}\right)^2\frac{8}{3}\cdot 2\pi = \frac{K_t s}{3\cdot 64\pi}\end{aligned} \tag{A.27}$$

For σ_u,

$$\sigma_u = \frac{\frac{1}{2}}{2 \cdot (2\pi)^2 \cdot s} \int \frac{d^3 p_3}{2E_3} \frac{d^3 p_4}{2E_4} \delta(\sqrt{s} - E_3 - E_4) \delta^3(\vec{p}_3 + \vec{p}_4) \cdot K_u u^2 \tag{A.28}$$

$$u = (p_1 - p_4)^2 = -2p_1 \cdot p_4 = 2\,(\vec{p}_1 \cdot \vec{p}_4 - E_1 E_4) = 2|p_1||p_4|(\cos(\pi - \theta) - 1)$$

$$u^2 = 4|p_1|^2 |p_4|^2 (1 + \cos\theta)^2$$

$$\sigma_u = \frac{1}{4 \cdot (2\pi)^2 \cdot s} \int \frac{d^3 p_3}{2E_3} \frac{d^3 p_4}{2E_4} \delta(\sqrt{s} - E_3 - E_4) \delta^3(\vec{p}_3 + \vec{p}_4) \cdot K_u \cdot 4|p_1|^2 |p_4|^2 (1 + \cos\theta)^2 \tag{A.29}$$

integrating out $\mathbf{p}_3$ with $\delta^3(\vec{p}_3 + \vec{p}_4)$,

$$\sigma_u = \frac{K_u |p_1|^2}{4\pi^2 s} \int \frac{d^3 p_4}{4} \delta(\sqrt{s} - 2E_4)(1 + \cos\theta)^2 \tag{A.30}$$

$$d^3 p_4 = p_4^2 dp_4 d\Omega_4 = p_4^2 dp_4 d(\cos\theta) d\phi$$

$$E_4 dE_4 = |\mathbf{p}_4| dp_4$$

$$\begin{aligned} \sigma_u &= \frac{K_u |p_1|^2}{16\pi^2 s} \int p_4^2 dp_4 d(\cos\theta) d\phi \delta(2p_4 - \sqrt{s})(1 + \cos\theta)^2 \\ &= \frac{K_u \frac{s}{4}}{16\pi^2 s} \int p_4^2 \delta(2p_4 - \sqrt{s}) dp_4 \int_{-1}^{1} d(\cos\theta)(1 + \cos\theta)^2 \int_0^{2\pi} d\phi \\ &= \frac{K_u}{64\pi^2} \cdot \frac{1}{2} \left(\frac{\sqrt{s}}{2}\right)^2 \frac{8}{3} \cdot 2\pi = \frac{K_u s}{3 \cdot 64\pi} \end{aligned} \tag{A.31}$$

$$\sigma_{tot} = \sigma_s + \sigma_t + \sigma_u = \frac{s}{64\pi} \left(2K_s + \frac{K_t + K_u}{3}\right) \tag{A.32}$$

A.3 Interaction rate

Now Interaction rate from Eq. 3.6 of [7] for a neutrino with fixed energy E, by averaging over the phase space of the other neutrino (or anti-neutrino) it scatters with,

$$\Gamma(E, T) = 2 \int \frac{d^3 \vec{p}_2}{(2\pi)^3} f_\nu\left(E_2, T\right) \sigma_{\text{tot}}\left(\vec{p}_1, \vec{p}_2\right) v_{\text{Møller}} \tag{A.33}$$

where $f_\nu\left(E_2, T\right) = 1/\left[1 + \exp\left(E_2/T\right)\right]$ is the Fermi-Dirac distribution for active neutrino or antineutrino, σ_{tot} is the sum of the two cross-sections. The Møller velocity is equal to $(1 - \cos\theta)$ for ultra-relativistic neutrinos [7]. The prefactor of 2 captures both helicity states of the active neutrinos and antineutrinos(Is it same for Majorana neutrinos?).

Now for $p_1 = (E, \vec{p}), p_2 = (E', \vec{p'})$

$$s = 2p_1 \cdot p_2 = 2(EE' - \vec{p} \cdot \vec{p'}) = 2|p||p'|(1 - \cos\theta)$$

$$
\begin{aligned}
\Gamma(E,T) &= 2\int \frac{d^3\vec{p}_2}{(2\pi)^3} f_\nu\left(E_2,T\right)\sigma_{\text{tot}}\left(\vec{p}_1,\vec{p}_2\right) v_{\text{Møller}} \\
&= 2\int \frac{|p'|^2 dp' d\Omega}{(2\pi)^3}\frac{1}{1+\exp\left(E'/T\right)}\left(2K_s+\frac{K_t+K_u}{3}\right)\cdot\frac{1}{64\pi}\cdot 2|p||p'|(1-\cos\theta)\cdot(1-\cos\theta) \\
&= \frac{4E}{64\pi}\left(2K_s+\frac{K_t+K_u}{3}\right)\int\frac{E'^2 dE' d(\cos\theta) d\phi}{(2\pi)^3}\frac{1}{1+\exp\left(E'/T\right)}E'(1-\cos\theta)^2 \\
&= \frac{E}{16\pi}\left(2K_s+\frac{K_t+K_u}{3}\right)\int\frac{E'^3 dE'}{(2\pi)^3}\frac{1}{1+\exp\left(E'/T\right)}\int_{-1}^{1} d(\cos\theta)(1-\cos\theta)^2\int_0^{2\pi} d\phi \\
&= \frac{E}{16\pi\cdot(2\pi)^2}\left(2K_s+\frac{K_t+K_u}{3}\right)\int\frac{E'^3 dE'}{1+\exp\left(E'/T\right)}\int_{-1}^{1} d(\cos\theta)(1-\cos\theta)^2 \\
&= \frac{E}{16\pi\cdot(2\pi)^2}\left(2K_s+\frac{K_t+K_u}{3}\right)\cdot\frac{7\pi^4T^4}{120}\cdot\frac{8}{3}
\end{aligned}
\tag{A.34}
$$

With MB distribution, we get $6T^4$ instead of $\frac{7\pi^4T^4}{120}\approx 5.68T^4$

$$
\Gamma(E,T)=\left(2K_s+\frac{K_t+K_u}{3}\right)\frac{7\pi}{2880}ET^4 \tag{A.35}
$$

For Scalar NSI, $K_s=4\left(\epsilon^{eeee}_{S,\nu_e}\right)^2G_F^2, K_t=20\left(\epsilon^{eeee}_{S,\nu_e}\right)^2G_F^2, K_u=4\left(\epsilon^{eeee}_{S,\nu_e}\right)^2G_F^2$

$$
\Gamma_S(E,T)=\frac{7\pi G_F^2\epsilon_S^2}{180}ET^4 \tag{A.36}
$$

For Axial-Vector NSI, $K_s=0, K_t=64\left(\epsilon^{eeee}_{A,\nu_e}\right)^2G_F^2, K_u=0$

$$
\Gamma_A(E,T)=\frac{7\pi G_F^2\epsilon_A^2}{135}ET^4 \tag{A.37}
$$

For Pseudoscalar NSI, $K_s=4\left(\epsilon^{eeee}_{P,\nu_e}\right)^2G_F^2, K_t=20\left(\epsilon^{eeee}_{P,\nu_e}\right)^2G_F^2, K_u=4\left(\epsilon^{eeee}_{P,\nu_e}\right)^2G_F^2$

$$
\Gamma_P(E,T)=\frac{7\pi G_F^2\epsilon_P^2}{180}ET^4 \tag{A.38}
$$

B Calculation of Thermal potential

We follow the method presented in *"Finite temperature corrections to the effective potential of neutrinos in a medium"*, (J. C. D'Olivo, J. F. Nieves, M. Torres)[8] for SM thermal potential

B.1 Neutrino Dispersion Relation

The properties of a neutrino that propagates through a medium are determined from the Dirac equation, which in momentum space is

$$(\not{k} - \Sigma_{\text{eff}})\, \psi = 0 \tag{B.1}$$

Here k_μ is the neutrino momentum and Σ_{eff} is the neutrino self-energy, which includes the effects of the background. The chiral nature of the neutrino interactions implies that the self-energy of a (left-handed) neutrino in SM is of the form[9],

$$\Sigma_{\text{eff}} = P_R \Sigma P_L \tag{B.2}$$

Where $P_{R/L} = \frac{1 \pm \gamma_5}{2}$
Since we are considering Non-standard neutrino interactions, Σ doesn't have to follow chiral nature.

$$\Sigma_{\text{eff}} = \Sigma \tag{B.3}$$

In vacuum, the only term self energy can depend on is the neutrino four-momentum k_μ. i.e.,

$$\Sigma = a\not{k} \tag{B.4}$$

But in a medium with velocity u_μ(we would assume rest frame where $u_\mu = (1, 0, 0, 0)$),

$$\Sigma = a\not{k} + b\not{u} + c\,[\not{k}, \not{u}] \tag{B.5}$$

From [10], we can see that at one-loop level, $c = 0$. Then,

$$\Sigma = a\not{k} + b\not{u} \tag{B.6}$$

a,b depend on invariant quantities $\omega = k \cdot u$ and $\kappa = \sqrt{\omega^2 - k^2}$.
Now Dirac Equation,

$$(\not{k} - \Sigma_{\text{eff}})\, \psi = 0 \tag{B.7}$$

$$(\not{k} - (a\not{k} + b\not{u}))\, \psi = 0 \tag{B.8}$$

$$[(1 - a)\,\not{k} - b\not{u}]\, \psi = 0 \Rightarrow \not{V}\psi = 0 \tag{B.9}$$

where $V_\mu = (1-a)\,k_\mu - bu_\mu$ For non-trivial solutions of $\not{V}\psi$=0, $V^2 = 0$

$$\begin{aligned}
\left[(1-a)\,k_\mu - bu_\mu)\right]\left[(1-a)\,k^\mu - b\mu^\mu\right] &= 0\\
(1-a)^2\,k_\mu k^\mu + b^2 u_\mu u^\mu - 2\,(1-a)\,b\,(k\cdot u) &= 0\\
(1-a)^2\,k^2 + b^2 u^2 - 2b\,(1-a)\,\omega &= 0\\
(1-a)^2\left(\omega^2-\kappa^2\right) + b^2 - 2b\,(1-a)\,\omega &= 0\\
\left[(1-a)\,(\omega-\kappa) - b\right]\left[(1-a)\,(\omega+\kappa) - b\right] &= 0\\
f\,(\omega)\times\overline{f}\,(\omega) &= 0
\end{aligned} \tag{B.10}$$

So, $V^2 = 0 \Rightarrow f\,(\omega)\,\overline{f}\,(\omega) = 0$
This equation has the solution:

$$f\,(\omega_\kappa) = 0 \tag{B.11}$$

or,

$$\overline{f}\,(-\overline{\omega}_\kappa) = 0 \tag{B.12}$$

Since we can separate self-energy into gauge dependent and independent parts, we can separate f as,

$$f = f_0 + f_\xi = \left[(1-a_0)\,(\omega-\kappa) - b_0\right] + \left[(1-a_\xi)\,(\omega-\kappa) - b_\xi\right] \tag{B.13}$$

Dispersion relation is independent of gauge parameter. f_0 and f_ξ must vanish separately at $\omega = \omega_\kappa$

$$\begin{aligned}
f_0\,(\omega_\kappa) \equiv (1-a_0)\,(\omega_\kappa-\kappa) - b_0 = 0\\
\Rightarrow b_0 = (1-a_0)\,(\omega_\kappa-\kappa) \Rightarrow \omega_\kappa - \kappa = b_0 + a_0\,(\omega_\kappa-\kappa)\\
\omega_\kappa = \kappa + b_0 + a_0\,(\omega_\kappa-\kappa)
\end{aligned} \tag{B.14}$$

In zeroth order, $\omega_\kappa = \kappa$. Substituting this in (B.14)

$$\omega_\kappa = \kappa + b_0\,(\omega_\kappa = \kappa)$$

So for neutrinos,

$$\omega_\kappa = \kappa + b_0\,(\kappa) + \mathrm{h.o} \tag{B.15}$$

Similarly, for anti-neutrinos,

$$\overline{f}\,(-\overline{\omega}_\kappa) = 0 \tag{B.16}$$

$$\begin{aligned}
\overline{f}_0\,(-\overline{\omega}_\kappa) \equiv (1-a_0)\,(-\overline{\omega}_\kappa+\kappa) - b_0 = 0\\
\Rightarrow b_0 = (1-a_0)\,(-\overline{\omega}_\kappa+\kappa) \Rightarrow \omega_\kappa = \kappa - b_0 - a_0\,(-\overline{\omega}_\kappa+\kappa)
\end{aligned} \tag{B.17}$$

Keeping terms in the first order,

$$\omega_\kappa = \kappa - b_0\,(\omega_\kappa = -\kappa)$$

So for anti-neutrinos,

$$\overline{\omega}_\kappa = \kappa - b_0\,(-\kappa) + \mathrm{h.o} \tag{B.18}$$

B.2 Effective potential

We can see the effective potential as a contribution to the energy of the particle by background medium.ie,
Energy of Neutrino propagating through a medium =
. Energy of neutrino in vacuum + Effective potential in the medium

$$\omega_\kappa = \kappa + \mathcal{V}_l \tag{B.19}$$

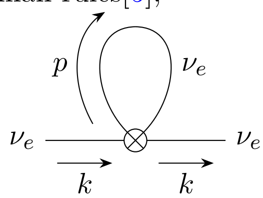
We are only considering the lowest order, hence $\mathcal{V}_l$ coincide with b_0, but in general it depends on a_0 too.
Now our problem has reduced to finding b_0 in $\Sigma = (a_0 + a_\xi)\not{k} + (b_0 + b_\xi)\not{u}$

B.3 Self energy calculation

Figure 1: Self energy diagrams in NSI

We can conveniently ignore tadpole diagram in Fig.1 from calculations as it would only give potential related to lepton asymmetry. We can safely neglect it for DW mechanism relevant matrix element for Lagrangian using Majorana neutrino Feynman rules[5],

$$\mathcal{L}_j = -\frac{G_F}{\sqrt{2}}\,(\epsilon_{j,\nu})\left(\,(\bar{\nu}_e\mathcal{O}_j\nu_e)\,(\bar{\nu}_e\mathcal{O}'_j\nu_e) - \frac{1}{m_\phi^2}\,(\bar{\nu}_e\mathcal{O}_j\nu_e)\,\Box\,(\bar{\nu}_e\mathcal{O}'_j\nu_e)\right) \tag{B.20}$$



$$-i\mathcal{M} = \frac{i^4 G_F \epsilon_{j,\nu_e}}{\sqrt{2}}\int\frac{d^4p}{(2\pi)^4}\left(\bar{u}(\mathcal{O}_j + C\mathcal{O}_j^{\mathrm{T}}C^{-1})S_F(p)(\mathcal{O}'_j + C{\mathcal{O}'_j}^{\mathrm{T}}C^{-1})u\right)[1 + \frac{q_\mu q^\mu}{m_\phi^2}] \tag{B.21}$$

$$q = k - p$$
$$p^2 = k^2 = m_\nu^2 \approx 0$$

$$-i\mathcal{M} = \frac{i^4 G_F \epsilon_{j,\nu_e}}{\sqrt{2}}\int\frac{d^4p}{(2\pi)^4}\left(\bar{u}(\mathcal{O}_j + C\mathcal{O}_j^{\mathrm{T}}C^{-1})S_F(p)(\mathcal{O}'_j + C{\mathcal{O}'_j}^{\mathrm{T}}C^{-1})u\right)\left(1 - \frac{2p\cdot k}{m_\phi^2}\right) \tag{B.22}$$

where $S_F(p)$ is the fermion propagator at finite temperature,

$$S_F(p) = (\not{p} + m_l)\left[\frac{1}{p^2 - m_l^2} + 2\pi i\delta\left(p^2 - m_l^2\right)\eta(p\cdot u)\right] \tag{B.23}$$

with,

$$\eta(p\cdot u) = \frac{\theta(p\cdot u)}{e^x + 1} + \frac{\theta(-p\cdot u)}{e^{-x} + 1}, x = \frac{(p\cdot u - \mu)}{T} \tag{B.24}$$

for neutrinos,

$$S_F(p) = \not{p}\left[\frac{1}{p^2 + i\eta} + 2\pi i\delta\left(p^2 - m_\nu^2\right)\eta(p\cdot u)\right] \tag{B.25}$$

Background-independent part of the fermion propagator only renormalizes the wave function and does not contribute to the dispersion relation in the lowest order[8]. To simplify the calculations, we will only consider background-dependent terms.

$$S_F^T(p) = 2\pi i\delta\left(p^2 - m_l^2\right)\eta(p\cdot u)\left(\not{p} + m_l\right) \tag{B.26}$$

Now,

$$-i\mathcal{M} = \bar{u}\left[-i\Sigma_{\text{eff}}\right]u = \bar{u}\left[\frac{G_F\epsilon_{j,\nu_e}}{\sqrt{2}}\int\frac{d^4p}{(2\pi)^4}(\mathcal{O}_j + C\mathcal{O}_j^{\mathrm{T}}C^{-1})S_F(p)(\mathcal{O}'_j + C{\mathcal{O}'_j}^{\mathrm{T}}C^{-1})\left(1 - \frac{2p\cdot k}{m_\phi^2}\right)\right]u \tag{B.27}$$

$$i\Sigma_{\text{eff}} = -\frac{G_F\epsilon_{j,\nu_e}}{\sqrt{2}}\int\frac{d^4p}{(2\pi)^4}(\mathcal{O}_j + C\mathcal{O}_j^{\mathrm{T}}C^{-1})S_F(p)(\mathcal{O}'_j + C{\mathcal{O}'_j}^{\mathrm{T}}C^{-1})\left(1 - \frac{2p\cdot k}{m_\phi^2}\right) \tag{B.28}$$

There will be an extra minus sign for scalar mediator which comes from the difference between scalar propagator $\Delta_\phi = \frac{i}{q^2 - m_\phi^2}$ and vector propagator $\Delta_V = \frac{-ig^{\mu\nu}}{q^2 - m_V^2}$

B.3.1 Scalar NSI

For scalar NSI,

$$\mathcal{O}_S = \mathbb{I}, \quad C\mathcal{O}_S^{\mathrm{T}}C^{-1} = \mathbb{I}, \quad \mathcal{O}'_S = \mathbb{I}, \quad C{\mathcal{O}'_S}^{\mathrm{T}}C^{-1} = \mathbb{I}$$

$$\begin{aligned} i\Sigma_{\text{eff}} &= \frac{G_F}{\sqrt{2}}\left(\epsilon_{S,\nu_e}\right)\int\frac{d^4p}{(2\pi)^4}(\mathcal{O}_S + C{\mathcal{O}_S}^{\mathrm{T}}C^{-1})S_F(p)(\mathcal{O}'_S + C{\mathcal{O}'_S}^{\mathrm{T}}C^{-1})\left(1 - \frac{2p\cdot k}{m_\phi^2}\right) \\ &= \frac{4G_F}{\sqrt{2}}\left(\epsilon_{S,\nu_e}\right)^{eeee}\int\frac{d^4p}{(2\pi)^4}S_F^T(p)\left(1 - \frac{2p\cdot k}{m_\phi^2}\right) \end{aligned} \tag{B.29}$$

$$\begin{aligned} i\Sigma_{\text{eff}} &= \frac{4G_F}{\sqrt{2}}\left(\epsilon_{S,\nu_e}\right)^{eeee}\int\frac{d^4p}{(2\pi)^4}\left(2\pi i\delta\left(p^2 - m_\nu^2\right)\eta(p\cdot u)\gamma_\mu p^\mu\right)\left(1 - \frac{2p\cdot k}{m_\phi^2}\right) \\ &= \frac{4iG_F}{\sqrt{2}}\left(\epsilon_{S,\nu_e}\right)^{eeee}\int\frac{d^4p}{(2\pi)^3}\delta\left(p^2 - m_\nu^2\right)\eta(p\cdot u)\gamma_\mu p^\mu\left(1 - \frac{2p\cdot k}{m_\phi^2}\right) \\ &= \frac{4iG_F}{\sqrt{2}}\left(\epsilon_{S,\nu_e}\right)^{eeee}\gamma^\mu\left[I_\mu - \frac{2k^\nu}{m_\phi^2}I_{\mu\nu}\right] \end{aligned} \tag{B.30}$$

Where

$$
\begin{aligned}
I_\mu &= \int \frac{d^4p}{(2\pi)^3}\delta\left(p^2-m_l^2\right)\eta(p\cdot u)p_\mu \\
I_{\mu\nu} &= \int \frac{d^4p}{(2\pi)^3}\delta\left(p^2-m_l^2\right)\eta(p\cdot u)p_\mu p_\nu
\end{aligned}
\tag{B.31}
$$

I_μ is manifestly covariant and depends only on the vector u_μ.

$$I_\mu = Au_\mu \tag{B.32}$$

Therefore, contracting I_μ with u_μ,

$$I_\mu u^\mu = Au_\mu u^\mu = A \tag{B.33}$$

$$I_\mu u^\mu = \int \frac{d^4p}{(2\pi)^3}\delta\left(p^2-m_l^2\right)\eta(p\cdot u)p_\mu u^\mu = J_1^{(l)} \tag{B.34}$$

ie,

$$I_\mu = J_1^{(l)}u_\mu \tag{B.35}$$

Similarly, $I_{\mu\nu} = \int \frac{d^4p}{(2\pi)^3}\delta\left(p^2-m_l^2\right)\eta(p\cdot u)p_\mu p_\nu$ depends only on u

$$I_{\mu\nu} = Ag_{\mu\nu} + Bu_\mu u_\nu \tag{B.36}$$

By contracting this expression with $u_\mu u_v$ and $g_{\mu v}$, we obtain two equations for A and B ,

$$g^{\mu\nu}I_{\mu\nu} = \int \frac{d^4p}{(2\pi)^3}\delta\left(p^2-m_l^2\right)\eta(p\cdot u)p_\mu g^{\mu\nu}p_\nu = m_l^2 J_0^{(l)} = 4A+B \tag{B.37}$$

$$u^\mu u^\nu I_{\mu\nu} = \int \frac{d^4p}{(2\pi)^3}\delta\left(p^2-m_l^2\right)\eta(p\cdot u)p_\mu u^\mu u^\nu p_\nu = J_2^{(l)} = A+B \tag{B.38}$$

$$I_\mu^\mu = m_l^2 J_0^{(l)} \tag{B.39}$$

Solving for A and B, these are then determined as

$$
\begin{aligned}
A &= \tfrac{1}{3}\left(m_l^2J_0^{(l)} - J_2^{(l)}\right) \\
B &= \tfrac{1}{3}\left(4J_2^{(l)} - m_l^2J_0^{(l)}\right)
\end{aligned}
$$

Substituting the expressions for I_μ and $I_{\mu v}$ into (B.30),

$$i\Sigma_{\text{eff}} = \frac{4iG_F}{\sqrt{2}}\left(\epsilon_{S,\nu_e}\right)^{eeee}\gamma^\mu\left[I_\mu - \frac{2k^\nu}{m_\phi^2}I_{\mu\nu}\right] \tag{B.40}$$

$$
\begin{aligned}
i\Sigma_{\text{eff}} &= \frac{4iG_F}{\sqrt{2}}\left(\epsilon_{S,\nu_e}\right)^{eeee}\left[J_1^{(l)}\not{u} - \frac{2\gamma^\mu k^\nu}{3m_\phi^2}\left(m_l^2J_0^{(l)} - J_2^{(l)}\right)g_{\mu\nu} - \frac{2\gamma^\mu k^\nu}{3m_\phi^2}\left(4J_2^{(l)} - m_l^2J_0^{(l)}\right)u_\mu u_\nu\right] \\
&= \frac{4iG_F}{\sqrt{2}}\left(\epsilon_{S,\nu_e}\right)^{eeee}\left[J_1^{(l)}\not{u} - \frac{2}{3m_\phi^2}\left(m_l^2J_0^{(l)} - J_2^{(l)}\right)\not{k} - \frac{2\omega}{3m_\phi^2}\left(4J_2^{(l)} - m_l^2J_0^{(l)}\right)\not{u}\right]
\end{aligned}
\tag{B.41}
$$

for neutrinos, $m_\nu \approx 0$,

$$i\Sigma_{\text{eff}} = \frac{4iG_F}{\sqrt{2}} \left(\epsilon_{S,\nu_e}\right)^{eeee} \left[-\frac{2J_2^{(\nu)}}{3m_\phi^2}\not{k} + \left(J_1^{(\nu)} - \frac{8\omega}{3m_\phi^2}J_2^{(\nu)}\right)\not{u}\right] \tag{B.42}$$

where(check B.3.3),

$$J_n^{(f)} = \int \frac{d^4p}{(2\pi)^3}\delta\left(p^2 - m_f^2\right)\eta(p\cdot u)(p\cdot u)^n \tag{B.43}$$

Now comparing (B.42) with $\Sigma_{\text{eff}} = a_0\not{k} + b_0\not{u}$,

$$b_0 = \frac{4G_F}{\sqrt{2}} \left(\epsilon_{S,\nu_e}\right)^{eeee} \left(J_1^{(\nu)} - \frac{8\omega}{3m_\phi^2}J_2^{(\nu)}\right) \tag{B.44}$$

$$b_0 = \frac{4G_F}{\sqrt{2}} \left(\epsilon_{S,\nu_e}\right)^{eeee} \left[\frac{1}{2}\left(n_\nu - n_{\bar{\nu}}\right) - \frac{8\omega}{3m_\phi^2}\cdot\frac{1}{2}\left(n_\nu\left\langle E_\nu\right\rangle + n_{\bar{\nu}}\left\langle E_{\bar{\nu}}\right\rangle\right)\right] \tag{B.45}$$

From (B.2) and (B.1), Effective potential $\mathcal{V}_l = b_0(\omega_k = \kappa)$ for ν and $\mathcal{V}_l = -b_0(\omega_k = -\kappa)$ for $\bar{\nu}$

We can also ignore $J_1^{(\nu)}$ which only contributes to lepton asymmetry potential.

i.e. in the lowest order of T,

$$\mathcal{V}_{\text{scalar}} = -\frac{16\sqrt{2}G_F}{3m_\phi^2}\left(\epsilon_{S,\nu_e}\right)^{eeee}\cdot\omega\cdot\frac{7\pi^2T^4}{240} \tag{B.46}$$

B.3.2 Axial-Vector NSI

For Axial-vector NSI,

$$\mathcal{O}_A = \gamma_\mu\gamma^5, \quad C\mathcal{O}_A^{\mathrm{T}}C^{-1} = \gamma_\mu\gamma^5, \quad \mathcal{O}'_A = \gamma^\mu\gamma^5, \quad C\mathcal{O}'^{\,\mathrm{T}}_A C^{-1} = \gamma^\mu\gamma^5$$

$$\mathcal{O}_L + C\mathcal{O}_L^{\mathrm{T}}C^{-1} = 2\gamma_\mu\gamma^5, \quad \mathcal{O}'_L + C\mathcal{O}'^{\,\mathrm{T}}_L C^{-1} = 2\gamma^\mu\gamma^5$$

$$\begin{aligned}
i\Sigma_{\text{eff}} &= \frac{-G_F}{\sqrt{2}}\left(\epsilon_{L,\nu_e}\right)^{eeee}\int\frac{d^4p}{(2\pi)^4}(\mathcal{O}_L + C{\mathcal{O}_L}^{\mathrm{T}}C^{-1})S_F(p)(\mathcal{O}'_L + C{\mathcal{O}'_L}^{\mathrm{T}}C^{-1})\left(1 - \frac{2p\cdot k}{m_\phi^2}\right)\\
&= \frac{-4G_F}{\sqrt{2}}\left(\epsilon_{L,\nu_e}\right)^{eeee}\int\frac{d^4p}{(2\pi)^4}\gamma_\mu\gamma^5S_F^T(p)\gamma^\mu\gamma^5\left(1 - \frac{2p\cdot k}{m_\phi^2}\right)\\
&= \frac{4G_F}{\sqrt{2}}\left(\epsilon_{L,\nu_e}\right)^{eeee}\int\frac{d^4p}{(2\pi)^4}\gamma_\mu\gamma^\mu S_F^T(p)\left(1 - \frac{2p\cdot k}{m_\phi^2}\right)
\end{aligned} \tag{B.47}$$

$$i\Sigma_{\text{eff}} = \frac{16G_F}{\sqrt{2}}\left(\epsilon_{L,\nu_e}\right)^{eeee}\int\frac{d^4p}{(2\pi)^4}S_F^T(p)\left(1 - \frac{2p\cdot k}{m_\phi^2}\right) \tag{B.48}$$

This is similar to (B.29). ie, from (B.42)

$$i\Sigma_{\text{eff}} = \frac{16iG_F}{\sqrt{2}} \left(\epsilon_{L,\nu_e}\right)^{eeee} \left[-\frac{2J_2^{(\nu)}}{3m_\phi^2}\not{k} + \left(J_1^{(\nu)} - \frac{8\omega}{3m_\phi^2}J_2^{(\nu)}\right)\not{u}\right] \tag{B.49}$$

Comparing with $\Sigma_{\text{eff}} = a_0 \not{k} + b_0 \not{u}$,

$$b_0 = 8\sqrt{2} G_F \left(\epsilon_{L,\nu_e}\right)^{eeee} \left(J_1^{(\nu)} - \frac{8\omega}{3m_\phi^2} J_2^{(\nu)} \right) \tag{B.50}$$

$$b_0 = 8\sqrt{2} G_F \left(\epsilon_{L,\nu_e}\right)^{eeee} \left[\frac{1}{2}\left(n_\nu - n_{\bar{\nu}}\right) - \frac{8\omega}{3m_\phi^2} \cdot \frac{1}{2}\left(n_\nu \langle E_\nu \rangle + n_{\bar{\nu}} \langle E_{\bar{\nu}} \rangle\right) \right] \tag{B.51}$$

From (B.2) and (B.1), Effective potential $\mathcal{V}_l = b_0(\omega_k = \kappa)$ for ν and $\mathcal{V}_l = -b_0(\omega_k = -\kappa)$ for $\bar{\nu}$
We can also ignore $J_1^{(\nu)}$ which contributes to lepton asymmetry potential.
i.e. in the lowest order,

$$\mathcal{V}_{\text{A}} = -\frac{64\sqrt{2} G_F}{3m_\phi^2} \left(\epsilon_{A,\nu_e}\right)^{eeee} \cdot \omega \cdot \frac{7\pi^2 T^4}{240} \tag{B.52}$$

B.3.3 Pseudoscalar NSI

For pseudoscalar NSI,

$$\mathcal{O}_P = \gamma^5, \quad C\mathcal{O}_P^{\text{T}} C^{-1} = \gamma^5, \quad \mathcal{O}'_P = \gamma^5, \quad C{\mathcal{O}'_P}^{\text{T}} C^{-1} = \gamma^5$$

$$\mathcal{O}_P + C\mathcal{O}_P^{\text{T}} C^{-1} = 2\gamma^5, \quad \mathcal{O}'_P + C{\mathcal{O}'_P}^{\text{T}} C^{-1} = 2\gamma^5$$

$$\begin{aligned} i\Sigma_{\text{eff}} &= \frac{-G_F}{\sqrt{2}} \left(\epsilon_{P,\nu_e}\right)^{eeee} \int \frac{d^4p}{(2\pi)^4} (\mathcal{O}_P + C{\mathcal{O}_P}^{\text{T}} C^{-1}) S_F(p) (\mathcal{O}'_P + C{\mathcal{O}'_P}^{\text{T}} C^{-1}) \left(1 - \frac{2p \cdot k}{m_\phi^2}\right) \\ &= \frac{-4G_F}{\sqrt{2}} \left(\epsilon_{P,\nu_e}\right)^{eeee} \int \frac{d^4p}{(2\pi)^4} \gamma^5 S_F^T(p) \gamma^5 \left(1 - \frac{2p \cdot k}{m_\phi^2}\right) \\ &= \frac{4G_F}{\sqrt{2}} \left(\epsilon_{P,\nu_e}\right)^{eeee} \int \frac{d^4p}{(2\pi)^4} S_F^T(p) \left(1 - \frac{2p \cdot k}{m_\phi^2}\right) \end{aligned} \tag{B.53}$$

$$i\Sigma_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \left(\epsilon_{P,\nu_e}\right)^{eeee} \int \frac{d^4p}{(2\pi)^4} S_F^T(p) \left(1 - \frac{2p \cdot k}{m_\phi^2}\right) \tag{B.54}$$

This is similar to (B.29). ie, from (B.42)

$$i\Sigma_{\text{eff}} = \frac{4iG_F}{\sqrt{2}} \left(\epsilon_{P,\nu_e}\right)^{eeee} \left[-\frac{2J_2^{(\nu)}}{3m_\phi^2} \not{k} + \left(J_1^{(\nu)} - \frac{8\omega}{3m_\phi^2} J_2^{(\nu)} \right) \not{u} \right] \tag{B.55}$$

Comparing with $\Sigma_{\text{eff}} = a_0 \not{k} + b_0 \not{u}$,

$$b_0 = 2\sqrt{2} G_F \left(\epsilon_{P,\nu_e}\right)^{eeee} \left(J_1^{(\nu)} - \frac{8\omega}{3m_\phi^2} J_2^{(\nu)} \right) \tag{B.56}$$

$$b_0 = 2\sqrt{2} G_F \left(\epsilon_{P,\nu_e}\right)^{eeee} \left[\frac{1}{2}\left(n_\nu - n_{\bar{\nu}}\right) - \frac{8\omega}{3m_\phi^2} \cdot \frac{1}{2}\left(n_\nu \langle E_\nu \rangle + n_{\bar{\nu}} \langle E_{\bar{\nu}} \rangle\right) \right] \tag{B.57}$$

From (B.2) and (B.1), Effective potential $\mathcal{V}_l = b_0(\omega_k = \kappa)$ for ν and $\mathcal{V}_l = -b_0(\omega_k = -\kappa)$ for $\bar{\nu}$

We can also ignore $J_1^{(\nu)}$ which contributes to lepton asymmetry potential.

i.e. in the lowest order,

$$\mathcal{V}_{\mathrm{P}} = -\frac{16\sqrt{2}G_F}{3m_\phi^2}\left(\epsilon_{P,\nu_e}\right)^{eeee} \cdot \omega \cdot \frac{7\pi^2 T^4}{240} \tag{B.58}$$

Evaluating $J_n^{(f)}$

$$J_n^{(f)} = \int \frac{d^4p}{(2\pi)^3}\delta\left(p^2 - m_f^2\right)\eta(p\cdot u)(p\cdot u)^n \tag{B.59}$$

δ-function,

$$\begin{aligned}
\delta\left(p^2 - m_f^2\right) &= \delta\left(\left(p - m_f\right)\left(p + m_f\right)\right) \\
&= \delta\left(p_0^2 - \vec{p}^2 - m_f^2\right) \\
&= \delta\left(p_0^2 - \omega_p^2\right) \\
&= \delta\left(\left(p_0 - \omega_p\right)\left(p_0 + w_p\right)\right) \\
&= \frac{1}{2\omega_p}\left[\delta\left(p_0 - w_p\right) + \delta\left(p_0 + w_p\right)\right]
\end{aligned} \tag{B.60}$$

where $\omega_p = \sqrt{p^2 + m_f^2} = E_p$.

$$\eta\left(p\cdot u\right) = \frac{\theta\left(p\cdot u\right)}{e^{\frac{(p\cdot u - \mu)}{T}} + 1} + \frac{\theta\left(-p\cdot u\right)}{e^{-\frac{(p\cdot u - \mu)}{T}} + 1} \tag{B.61}$$

In the rest frame of the medium, $u_\mu = (1, 0, 0, 0)$,

$$\eta\left(p\cdot u\right) = \frac{\theta\left(p_0\right)}{e^{\frac{(p_0 - \mu)}{T}} + 1} + \frac{\theta\left(-p_0\right)}{e^{-\frac{(p_0 - \mu)}{T}} + 1} = \theta\left(p_0\right) f_f(p_0) + \theta\left(-p_0\right) f_{\bar{f}}(-p_0) \tag{B.62}$$

where, we have introduced the particle and antiparticle momentum distributions,

$$f_{f,\bar{f}}(E) = \frac{1}{e^{\beta(E \mp \mu)} + 1}$$

number densities are given by

$$n_{f,\bar{f}} = g_f \int \frac{d^3\mathbf{p}}{(2\pi)^3} f_{f,\bar{f}}$$

and the thermal average of $\mathcal{E}^n$,

$$\left\langle \mathcal{E}^n_{f,\bar{f}} \right\rangle \equiv \frac{g_f}{n_{f,\bar{f}}} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \mathcal{E}^n f_{f,\bar{f}}$$

Now,

$$
\begin{aligned}
J_n^{(f)} &= \int \frac{d^4p}{(2\pi)^3}\delta\left(p^2 - m_f^2\right)\eta(p\cdot u)(p\cdot u)^n \\
&= \int \frac{d^3\mathbf{p}dp_0}{(2\pi)^3}\frac{1}{2\omega_p}\left[\delta\left(p_0-\omega_p\right)+\delta\left(p_0+\omega_p\right)\right]\left[\theta\left(p_0\right)f_f(p_0)+\theta\left(-p_0\right)f_{\bar{f}}(-p_0)\right](p_0)^n \\
&= \int \frac{d^3\mathbf{p}}{(2\pi)^3}\left[\frac{\omega_p^n}{2w_p}f_f\left(\omega_p\right)+\frac{\left(-\omega_p\right)^n}{2\omega_p}f_{\bar{f}}\left(\omega_p\right)\right] \\
&= \frac{1}{2}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\left[E_f^{n-1}f_f(E_p)+(-1)^n E_f^{n-1}f_{\bar{f}}(E_p)\right] \\
&= \frac{1}{2}\left[\frac{n_f}{g_f}\left\langle E_f^{n-1}\right\rangle+(-1)^n\frac{n_{\bar{f}}}{g_{\bar{f}}}\left\langle E_{\bar{f}}^{n-1}\right\rangle\right]
\end{aligned}
\tag{B.63}
$$

For neutrinos,
$g_\nu = g_{\bar{\nu}} = 1$

$$J_0^{(\nu)} = \frac{1}{2}\left[n_\nu\left\langle\frac{1}{E_\nu}\right\rangle+n_{\bar{\nu}}\left\langle\frac{1}{E_{\bar{\nu}}}\right\rangle\right] \tag{B.64}$$

$$J_1^{(\nu)} = \frac{1}{2}\left[\frac{n_\nu}{g_\nu}-\frac{n_{\bar{\nu}}}{g_{\bar{\nu}}}\right] = \frac{1}{2}\left(n_\nu - n_{\bar{\nu}}\right) \tag{B.65}$$

$$J_2^{(\nu)} = \frac{1}{2}\left[n_\nu\left\langle E_\nu\right\rangle+n_{\bar{\nu}}\left\langle E_{\bar{\nu}}\right\rangle\right] = \frac{7\pi^2T^4}{240} \tag{B.66}$$

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