

# Linear Algebra Structure for Word Senses

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# Motivation

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- Polysemous words yield a huge headache for word embeddings. Essentially, different senses of a polysemy should predict different neighbours.

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e.g. 'Tie' could mean either 'a piece of clothes' or 'an undecided match'.

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How elegant would it be if we could decompose word senses linearly. Say,

$$\mathbf{v}_{tie} = \alpha \mathbf{v}_{tie1} + \beta \mathbf{v}_{tie2}$$

# Are There Really Linear Structures

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Do the word senses really yield a linear structure?

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Do the word senses really yield a linear structure? Let's have some fun ;).

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1. Take Mister Johnson's favourite pre-trained word embeddings.

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Do the word senses really yield a linear structure? Let's have some fun ;).

1. Take Mister Johnson's favourite pre-trained word embeddings.
2. Randomly pick two words  $w_1, w_2$  and create a new word  $w$  to represent a polysemy with two senses  $w_1, w_2$ .



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2. Randomly pick two words  $w_1, w_2$  and create a new word  $w$  to represent a polysemy with two senses  $w_1, w_2$ .
3. Delete embeddings  $v_{w_1}, v_{w_2}$  and train an embedding  $v_w$ .

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3. Delete embeddings  $v_{w1}, v_{w2}$  and train an embedding  $v_w$ .

Now the fun part. We have a look at the cosine distance between  $v_w$  and the span of  $\{v_{w1}, v_{w2}\}$ . The average is 0.97, with an SD of 0.02. Wow.

# A Closer Look at the Linear Structure

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Now we can confidently assume that for polysemous  $w$ ,

$$v_w = \alpha v_{w1} + \beta v_{w2}$$

but what are these coefficients \*?

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$$v_w = \alpha v_{w1} + \beta v_{w2}$$

but what are these coefficients \*? Statistics show that the Pearson correlation

$$\rho(\beta, \text{freq}(w_1)/\text{fre1}(w_2)) = 0.67$$

and we have

$$\beta \approx 1 - \text{clg}(\text{freq}(w_1)/\text{fre1}(w_2))$$

# A Closer Look at the Linear Structure II

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Now we have

$$v_w = \alpha v_{w1} + \beta v_{w2}$$

and a plausible interpretation of the coefficients. If we are faced with a potentially polysemous word  $w$ , how to obtain the senses  $v_{w1}, v_{w2}$ ? What are they after all\* ?

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- In the context of word embeddings, senses are distinguished by their behaviours of predicting context words, regardless of their frequencies.

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- In the context of word embeddings, senses are distinguished by their behaviours of predicting context words, regardless of their frequencies.
- These senses are considered as vectors on the **unit sphere**, i.e. directions in the embedding space, termed *discourses*.



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- In the context of word embeddings, senses are distinguished by their behaviours of predicting context words, regardless of their frequencies.
- These senses are considered as vectors on the **unit sphere**, i.e. directions in the embedding space, termed *discourses*.
- Behind the scene there is a corpus generation model. Given a discourse  $c$ , the probability of generating a word  $w$

$$pr[w|c] \propto \exp(c \cdot v_w)$$

# Model

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We now have

$$v_w = \sum_{j \leq m} \alpha_{w,j} A_j + \eta_w$$

where  $A_j$ s are the discourse atoms and  $\eta_w$  is a noise vector, s.t.  
at most  $k$  of the coefficients are non-zero.

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The model yields two hyper-parameters  $k$  (max number of senses per word) and  $m$  (number of different discourses). This is a standard sparse coding task. Best performance is achieved with  $k = 5$  and  $m = 2000$ .

# Hierarchy

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Using  $m = 2$ ,  $k = 200$  yields a less fine-grained decomposition.  
Which is interesting.

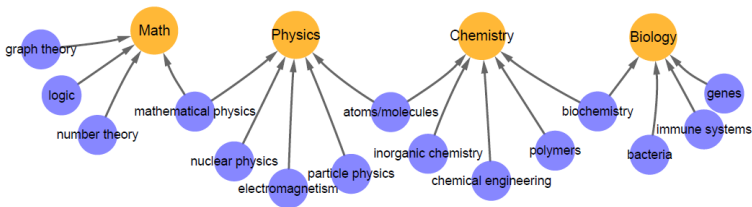
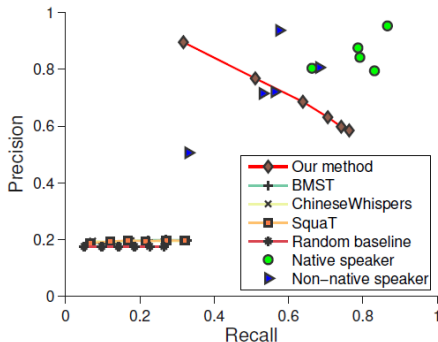


Figure: Discourses in different hierarchies.

# Word Sense Disambiguation



**Figure:** Performance on word sense disambiguation. The algorithm is comparable with non-native speakers (those living in the states and have been learning English for at least 10 years).

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- We see an approach of decomposing word embedding vectors into a linear combination of their senses.

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- The decomposition result supports a word sense decomposition algorithm whose performance is comparable to that of non-native speakers.

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- The senses are represented as vectors on the unit sphere.
- The decomposition result supports a word sense decomposition algorithm whose performance is comparable to that of non-native speakers.
- Unless this lame for-fun presentation, the paper elegantly and convincingly illustrated the procedure of the research. Would make an excellent read.