Introduction to Word Embeddings
(biased towards machine translation)

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Until now... Words are Words

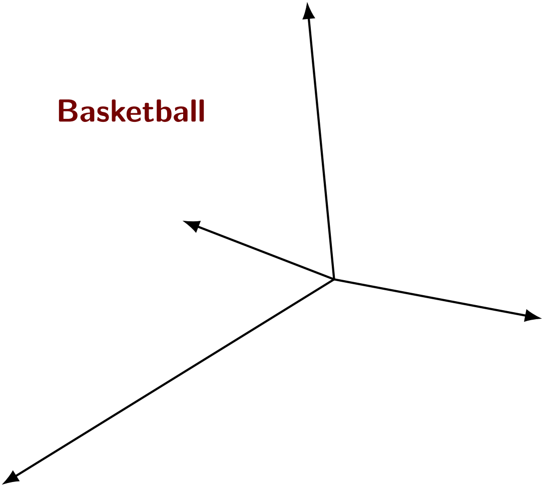
We counted words in corpora and calculated probabilities

Basketball

From now on... Words are Numbers!

We find the numbers that best allow to perform a task

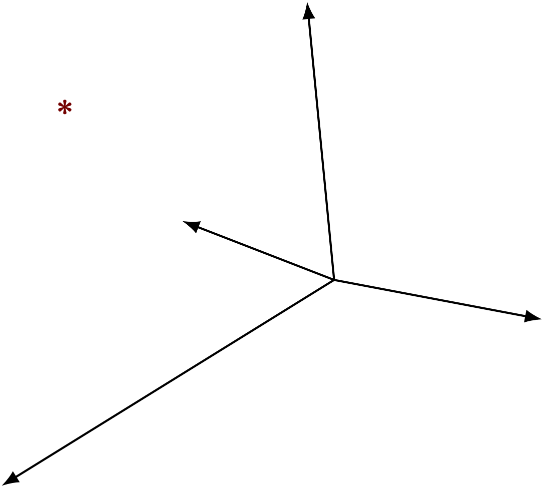
Basketball



***n*-dimensional vector space**

From now on... Words are Numbers!

We find the numbers that best allow to perform a task



***n*-dimensional vector space**

From now on... Words are Numbers!

A word embedding is...

... a numerical representation of a word

- Allow arithmetic operations on text

  Ex: time + flies

From now on... Words are Numbers!

A word embedding is...

... a numerical representation of a word

- Allow arithmetic operations on text
  Ex: time + flies

- Several names
  - Semantic Representations of Words
  - Word Vector Representations
  - Word Embeddings

From now on... Words are Numbers!

A word embedding is...

time = (1.844012, 0.590383, 1.003636, -0.577031, 1.515419, 1.097797, 1.812856, 0.933615, -2.396581, -0.931116, -0.719396, -0.376134, -1.204231, 0.045771, -0.287482, 1.084627, 4.399265, 1.516829, -0.838133, -1.881685, 0.108117, 2.345857, -1.292667, -2.286168, 3.419926, 4.260052, -1.016988, 3.140229, -3.161504, -0.800707, -1.433775, 2.290546, 1.932333, 0.714649, -3.033084, -0.958289, -1.704687, -1.597345, 1.525060, 3.337017, -2.787743, 1.479353, 3.452092, -3.242210, 0.532302, -0.551804, 2.344314, -0.919049, -1.872516, 0.080137, 1.208913, -2.136555, -2.218254, 0.206410, 0.133225, -1.521032, 1.735609, 2.885288, -2.048691, 2.375038, 0.316599, -0.254595, 2.159168, 1.118603, -0.775468, 0.933521, -0.351797, 2.193516, 2.499064, 2.818742, -0.213898, 0.446962, 1.767461, 1.342941, 1.117215, -0.042004, 4.199081, 3.041796, -1.770649, -0.528354, -2.067354, 0.283046, -0.099049, -0.105402, 2.823484, -2.583724, -2.906962, 0.592174, -3.029664, -0.170582, 0.406366, 1.963008, -3.229250, -3.499467, -0.136623, -1.551140, 0.348241, -1.597526, 0.703598, 3.122618, 0.466473, -0.113320, -2.119155, 1.092863, -0.908410, 0.253259, -1.082862, 4.408773, 2.419691, 2.343239, 0.703793, 1.270707, 0.410221, -1.293057, -0.799147, 2.214563, -0.212623, 1.206766, -0.731273, 2.308388, -1.029362, -2.080709, 0.749148, -1.412619, 1.073051, -2.498955, -0.520858, 1.391912, -1.181121, 1.523457, -1.245448, -0.290742, -2.589719, -0.366162, 3.586508, 0.908829, -1.125176, -0.937035, -1.163619, 1.759209, 3.678231, 0.019263, -0.395732, 1.142848, -0.500150, -3.005232, 2.287069, -0.524648, -0.944902, 0.038368, -1.093538, -0.697787, 0.767664, 2.399855, 2.425945, 1.563581, -1.086811, 0.372100, 1.400303, -2.278863, 0.643208, -0.459837, 1.756295, 2.057359, 3.140241, -1.740582, 1.386243, -1.822378, 1.528883, -1.984250, 1.214508, -1.336822, -0.321478, -0.162113, 0.272326, -2.673072, 0.612675, -0.657483, -0.557969, -3.358420, -2.559981, -1.683046, -1.314229, -2.425110, -2.506184, -1.606668, 1.332781, -2.760878, -2.400824, -1.830618, -2.406664,-1.169146, -1.838281, 0.588559, 2.285466, -0.401462, 1.632473, -0.510084, -2.072332, -2.627897, 2.531830, -2.524195, 2.035469, 1.906113, -1.257332, -4.039220, -0.467614, -2.275054, -3.409202, -0.014383, 0.445576, 1.461529, -1.318478, 0.061049, 0.280523, 2.173227, -0.027133, 2.791830, -0.728346, -1.804815, 1.245291, 0.970318, 2.646388, 0.246842, -1.823608, 1.888760, 0.265116, -2.027269, -0.089802, 0.389976, -0.654499, 2.565478, -2.647825, 2.658914, 1.385568, 2.306623, 0.476923, -0.869644, -0.170338, 0.495097, -2.604649, 0.610231, 0.739677, 0.322778, -2.042915, -1.353154, 0.177016, 1.840185, -0.271689, -0.401560, -0.421108, -0.185526, 1.041765, -4.599578, -0.829409, 0.076258, -0.503421, 1.891007, -0.931777, 0.434825, -0.467926, -1.417658, -0.320597, -4.084039, -3.899607, 0.977403, 0.774670, 3.269479, -1.031264, -0.433907, -2.30576 0, 0.811788, 2.347483, -1.254061, -0.861366, 0.080974, -3.666142, -0.363376, -2.384475, -4.290071, -0.924723, 1.257435, 1.223927, 0.276726, 1.541471, 1.274240, 1.883040, -1.987514, -0.809325, 1.252716, 1.812783, -0.511801, -1.657522, 1.196169, 0.804855, -1.861488, -2.113367, 0.429888, -0.920844, 0.377247)

From now on... Words are Numbers!

A word embedding is...

flies = (0.101159, 0.550446, 0.543801, -0.973852, -0.680835, 0.417193, -0.247181, 0.209725, -1.136055, -0.059531, -0.401640, 0.171540, 0.925121, -0.143815, 0.781714, -1.482425, 0.347008, -0.112342, 0.442418, -1.020457, -0.071752, 1.873548, -0.222886, -0.729569, -0.830224, -0.868407, 0.203496, 0.469911, -0.191363, 0.565102, 0.687738, 0.480823, 0.842358, -0.173656, -0.265585, 0.685740, 0.488047, -0.359772, -0.576064, -0.802884, 0.081554, 0.046882, -0.861532, -0.461855, 0.613098, -1.534642, -0.884534, 0.207728, 1.396512, -0.242900, -0.383959, 0.570844, -0.703350, -1.368813, -1.008194, 1.534660, 0.171693, 0.640925, -0.233116, 0.324685, 0.483171, 0.337947, -0.963290, -0.400558, 0.830977, 0.913474, 0.251693, -0.589420, -0.299622, 1.047515, -0.266679, -1.247186, 1.087610, -0.549028, 1.600710, -1.538516, -1.703301, -1.393499, -0.894448, 0.717204, 0.105767, -0.189234, -0.615609, -0.658315, 0.051877, 0.014180, -0.791282, 0.150424, 1.343751, -0.464859, 0.871426, 1.542864, -1.202150, -0.767113, -1.734738, 0.073633, -1.012583, 0.747787, 0.476070, -0.454807, 0.642685, -0.854152, -0.071798, 0.233724, 0.712329, -0.097752, -0.531132, 0.323271, -0.447342, 0.657913, 1.199492, -0.107360, -0.154234, -1.131168, 1.354793, 1.721385, -0.240023, 0.655765, -0.217006, -0.801722, 0.553369, 0.213377, 0.323267, -1.516051, 2.106244, -0.134282, 0.742155, 0.426344, 0.197991, -0.806768, 0.372546, -0.160200, -1.552847, -0.286178, -0.707796, 0.527352, -0.259658, 0.230387, 0.105294, -0.194481, 0.301772, -1.022163, 0.557191, 1.096709, 0.058422, -1.036384, 0.353412, -0.623097, -0.689515, 0.091472, 0.783885, 0.184088, -0.367950, 0.952462, 0.183704, 0.677562, 0.293917, -0.214309, -0.487794, 0.934296, 0.311513, 0.286514, -0.085511, 0.777691, 1.232603, -0.309367, -0.225086, 0.005091, -0.099195, -0.293117, 1.305563, 0.595816, 0.950316, 0.568706, -0.561446, 0.911634, -0.383941, 0.758054, -0.197820, 0.506777, -0.290767, -0.356727, 1.229474, -0.156489, -0.782741, -0.210163, -0.029169, 0.602664, 0.418375, 0.148975, -0.761796, 1.322690, -0.173410, 0.204111, -1.344531, 1.081905, -0.660543, -0.225615, -0.444753, -0.929671, 0.054136, 0.052031, -0.164926, 0.159312, -1.316333, 0.837011, -1.290353, 0.958403, 1.247478, 0.442009, 0.455497, -1.856268, -0.358823, -0.230839, -0.206271, 0.227012, -0.454163, 0.747798, -1.252855, 1.436849, -0.427915, -0.810428, -0.628144, -0.288458, 0.087355, 0.356739, 0.153036, 0.516594, -0.504978, 0.814432, 1.052940, 1.094526, -0.219595, 0.722178, 0.267325, -0.087458, -1.270262, -0.039461, 0.991926, -0.112005, -0.009605, 0.149920, 0.164717, 0.280475, 0.966384, 0.327598, 0.189590, -0.208946, 0.838261, 0.051847, -0.277932, -0.788527, -0.768702, -1.688721, 0.388215, 0.170153, -0.555723, -0.529565, -0.528982, -0.659930, 0.588041, -0.368195, -0.850188, -0.004996, 0.925476, 1.046587, -0.731761, 0.519435, 0.193188, -0.709557, 0.123329, -0.454316, 1.885830, -0.201841, -0.728933, -0.953455, -0.205837, -0.724068, 0.120158, 1.765389, -0.192159, 1.062490, -0.002634, 0.125790, -0.846565, 0.548899, -1.062821, -2.146826, 0.134681, 0.570950, 0.851783, 0.436544, 0.688986, 1.229008, 1.435449, 0.118766, -0.132411, 2.527890, 0.778142, 0.269093)

From now on... Words are Numbers!

How can we obtain those numbers?

Outline

1. Introduction
2. Prediction-based Embeddings
3. Cross-lingual Word Embeddings

Introduction

Never ask for the meaning of a word in isolation, but only in the context of a sentence
(Frege, 1884)

Never ask for the meaning of a word in isolation, but only in the context of a sentence
(Frege, 1884)

For a large class of cases... the meaning of a word is its use in the language
(Wittgenstein, 1953)

You shall know a word by the company it keeps
(Firth, 1957)

Words that occur in similar contexts tend to have similar meaning

(Harris, 1954)

Introduction

Similar Meanings...

- ...need for a concept of **distance** to be defined.
- **Geometry** is the branch of mathematics that deals with distances
- **Vector spaces** and linear algebra are our tools

Introduction

Similar Meanings...

Basketball

Sumo

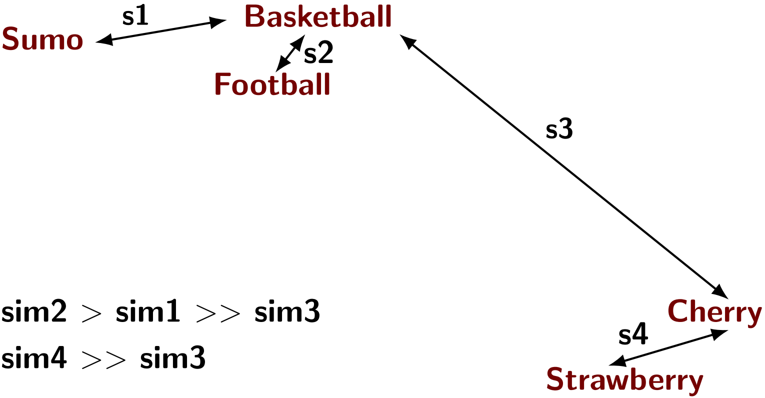
Football

Cherry

Strawberry

Introduction

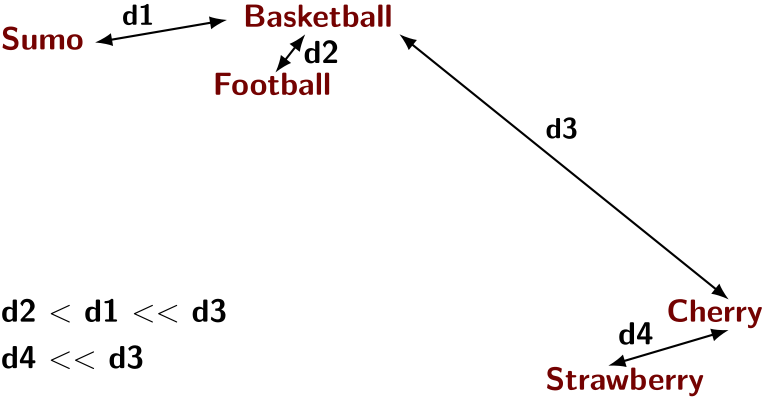
Similar Meanings...



sim2 $>$ sim1 $>>$ sim3

sim4 $>>$ sim3

Similarity vs. Distance

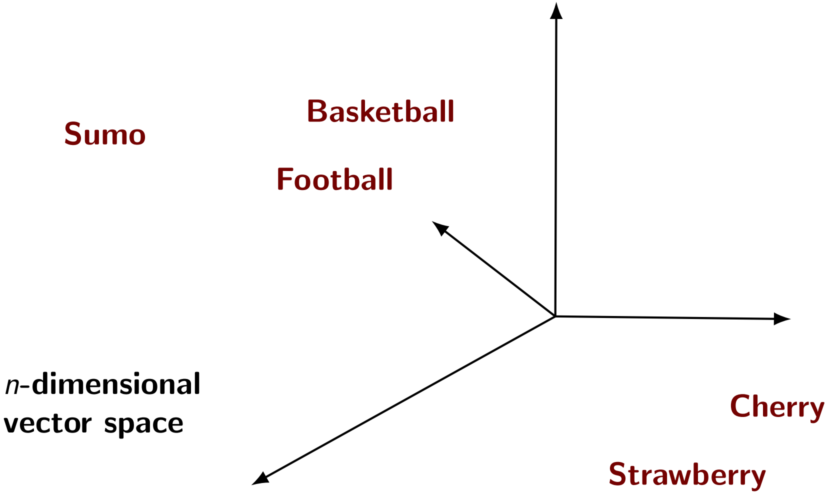


$\mathbf{d2} < \mathbf{d1} << \mathbf{d3}$

$\mathbf{d4} << \mathbf{d3}$

Introduction

Word Vector Space



Frequency-based Embeddings

How to Obtain a Vector for a Word?

Naïve example: **term frequencies** in a corpus

- The basis in our vector space is the vocabulary of the corpus
- Consider the document in which a word occurs its context
- Each word is characterised as the number of times it appears in each document

Frequency-based Embeddings

Example: Toy Corpus

S1: We like to play some sport in the afternoon, I like basketball but John likes sumo more.

S2: Sumo is a sport where a rikishi attempts to force another wrestler out of a circular ring.

S3: Messi scored 4 goals yesterday and kept the ball as a memory of this fantastic sports afternoon!

S4: I ate too many cherries yesterday.

Vocabulary:{*like, play, sport, afternoon, basketball, John, ball, sumo, rikishi, attempt, force, werstler, circular, ring, Messi, score, goal, yesterday, keep, memory, fantastic, eat, cherry*}

Frequency-based Embeddings

Example: Occurrence Matrix

	like	play	sport	afternoon	basketball	John	ball	sumo	rikishi	attempt	force	werstler	circular	ring	Messi	score	goal	yesterday	keep	memory	fantastic	eat	cherry
S1	2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S2	0	0	1	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
S3	0	0	1	1	0	0	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0
S4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1

Frequency-based Embeddings

Example: Occurrence Matrix

	like	play	sport	afternoon	basketball	John	ball	sumo	rikishi	attempt	force	werstler	circular	ring	Messi	score	goal	yesterday	keep	memory	fantastic	eat	cherry
S1	2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S2	0	0	1	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
S3	0	0	1	1	0	0	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0
S4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1

document vector

Frequency-based Embeddings

Example: Occurrence Matrix

	like	play	sport	afternoon	basketball	John	ball	sumo	rikishi	attempt	force	werstler	circular	ring	Messi	score	goal	yesterday	keep	memory	fantastic	eat	cherry
S1	2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S2	0	0	1	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
S3	0	0	1	1	0	0	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0
S4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1

word vector

Frequency-based Embeddings

Example: Text Similarity

Euclidean distance $\mathrm{d}(\overrightarrow{x}, \overrightarrow{y}) = \sqrt{\sum_{\mathrm{i}=1}^{\mathrm{n}} (\mathrm{x_i} - \mathrm{y_i})^2}$

basketball $\rightarrow \{1, 0, 0, 0\}$
sumo $\rightarrow \{1, 1, 0, 0\}$
cherry $\rightarrow \{0, 0, 0, 1\}$

$\mathrm{d}(\text{basketball, sumo}) = \sqrt{(1-1)^2 + (0-1)^2 + (0-0)^2 + (0-0)^2} = 1$
$\mathrm{d}(\text{basketball, cherry}) = \sqrt{(1-0)^2 + (0-1)^2 + (0-0)^2 + (0-0)^2} = \sqrt{2}$
$\mathrm{d}(\text{sumo, cherry}) = \sqrt{(1-0)^2 + (1-0)^2 + (0-0)^2 + (0-1)^2} = \sqrt{3}$

$\mathrm{d}(\text{basketball, sumo}) < \mathrm{d}(\text{basketball, cherry}) < \mathrm{d}(\text{sumo, cherry})$

Frequency-based Embeddings

Example: Text Similarity

Cosine similarity $\mathrm{sim}(\overrightarrow{x}, \overrightarrow{y}) = \frac{\sum_{i=1}^{n} x_i y_i}{|\overrightarrow{x}||\overrightarrow{y}|}$

basketball $\rightarrow$ $\{1, 0, 0, 0\}$
sumo $\rightarrow$ $\{1, 1, 0, 0\}$
cherry $\rightarrow$ $\{0, 0, 0, 1\}$

$\mathrm{sim}(\text{basketball, sumo}){=}1$

$\mathrm{sim}(\text{basketball, cherry}){=}0$

$\mathrm{sim}(\text{sumo, cherry}){=}0$

$\mathrm{sim}(\text{basketball,sumo}) > \mathrm{sim}(\text{basketball,cherry}) = \mathrm{sim}(\text{sumo,cherry})$

Frequency-based Embeddings

Frequency-based Embeddings

Frequency-based Embeddings

- *Term frequency word vectors*
- TF-IDF word vectors
- Co-occurrence word vectors

Frequency-based Embeddings

Term Frequency-Inverse Document Frequency, TF-IDF

Term Frequency

How frequently a term occurs in a document d normalised to account for d length

$$\mathsf{TF}(t,\, d) = \frac{\text{Number of times term } t \text{ appears in a document } d}{\text{Total number of terms in } d}$$

Frequency-based Embeddings

Term Frequency-Inverse Document Frequency, TF-IDF

Term Frequency

How frequently a term occurs in a document d normalised to account for d length

$$\mathsf{TF}(t,\, d) = \frac{\text{Number of times term } t \text{ appears in a document } d}{\text{Total number of terms in } d}$$

Inverse Document Frequency

Measures how important a term is (low weight for stop words)

$$\mathsf{IDF}(t,\, D) = \log_e\left(\frac{\text{Total number of documents } D}{\text{Number of documents with term } t \text{ in it}}\right)$$

Frequency-based Embeddings

Term Frequency-Inverse Document Frequency, TF-IDF

Trivially...

$$\text{TF-IDF}(t, d, D) = \text{TF}(t, d) \times \text{IDF}(t, D)$$

Frequency-based Embeddings

Example: Toy Corpus

d1: We like to play some sport in the afternoon, I like basketball but John likes sumo more.

d2: Sumo is a sport where a rikishi attempts to force another wrestler out of a circular ring.

d3: Messi scored 4 goals yesterday and kept the **ball** as a memory of this fantastic sports afternoon!

d4: I ate too many cherries yesterday.

$$\textbf{TF(ball)} = \left(0, 0, \frac{1}{17}, 0\right);$$

Frequency-based Embeddings

Example: Toy Corpus

d1: We like to play some sport in the afternoon, I like basketball but John likes sumo more.

d2: Sumo is a sport where a rikishi attempts to force another wrestler out of a circular ring.

d3: Messi scored 4 goals yesterday and kept the **ball** as a memory of this fantastic sports afternoon!

d4: I ate too many cherries yesterday.

$$\textbf{TF(ball)} = \left(0, 0, \frac{1}{17}, 0\right); \qquad \textbf{IDF(ball)} = \log_e \left(\frac{4}{1}\right);$$

Frequency-based Embeddings

Example: Toy Corpus

d1: We like to play some sport in the afternoon, I like basketball but John likes sumo more.

d2: Sumo is a sport where a rikishi attempts to force another wrestler out of a circular ring.

d3: Messi scored 4 goals yesterday and kept the **ball** as a memory of this fantastic sports afternoon!

d4: I ate too many cherries yesterday.

$$\textbf{TF(ball)} = \left(0, 0, \frac{1}{17}, 0\right); \qquad \textbf{IDF(ball)} = \log_e \left(\frac{4}{1}\right);$$

$$\textbf{TF-IDF(ball)}_3 = \frac{1}{17} \times \log_e (4) = 0.08$$

Frequency-based Embeddings

Example: Toy Corpus

d1: We like to play some sport in the afternoon, I like basketball but John likes sumo more.

d2: Sumo is **a** sport where **a** rikishi attempts to force another wrestler out of **a** circular ring.

d3: Messi scored 4 goals yesterday and kept the ball as **a** memory of this fantastic sports afternoon!

d4: I ate too many cherries yesterday.

$$\textbf{TF(a)} = \left(0, \frac{3}{17}, \frac{1}{17}, 0\right);$$

Frequency-based Embeddings

Example: Toy Corpus

d1: We like to play some sport in the afternoon, **I** like basketball but John likes sumo more.

d2: Sumo is **a** sport where **a** rikishi attempts to force another wrestler out of **a** circular ring.

d3: Messi scored 4 goals yesterday and kept the ball as **a** memory of this fantastic sports afternoon!

d4: **I** ate too many cherries yesterday.

$$\mathbf{TF(a)} = \left(0, \frac{3}{17}, \frac{1}{17}, 0\right); \qquad \mathbf{IDF(a)} = \log_e \left(\frac{4}{2}\right);$$

Frequency-based Embeddings

Example: Toy Corpus

d1: We like to play some sport in the afternoon, **I** like basketball but John likes sumo more.

d2: Sumo is **a** sport where **a** rikishi attempts to force another wrestler out of **a** circular ring.

d3: Messi scored 4 goals yesterday and kept the ball as **a** memory of this fantastic sports afternoon!

d4: **I** ate too many cherries yesterday.

$$\textbf{TF(a)} = \left(0, \frac{3}{17}, \frac{1}{17}, 0\right); \qquad \textbf{IDF(a)} = \log_e \left(\frac{4}{2}\right);$$

$$\textbf{TF-IDF(a)}_2 = \frac{3}{17} \times \log_e (2) = 0.12; \quad \textbf{TF-IDF(a)}_3 = 0.04$$

Frequency-based Embeddings

Term Frequency-Inverse Document Frequency, TF-IDF

- Word vectors of D dimensions
- Distances between words as before:
  - Euclidean distance
  - Cosine similarity
  - ...

Frequency-based Embeddings

Co-Occurence Matrix, Count Vectors

- Words co-occurrence statistics describes how words occur together

- Counts how two or more words occur together in a given corpus

Frequency-based Embeddings

Example: Toy Corpus

d1: We like to play some sport in the afternoon, I like basketball but John likes sumo more.

d2: Sumo is a sport where a rikishi attempts to force another wrestler out of a circular ring.

d3: Messi scored 4 goals yesterday and kept the ball as a memory of this fantastic sports afternoon!

d4: I ate too many cherries yesterday.

Frequency-based Embeddings

Example: Toy Corpus

d1: We **like** to play some sport in the afternoon, **I like** basketball but John likes sumo more.

d2: Sumo is **a** sport where **a** rikishi **attempts** to force another wrestler out of **a** circular ring.

d3: **Messi** scored 4 goals yesterday and kept the **ball** as **a** memory of this fantastic sports afternoon!

d4: **I** ate too many cherries yesterday.

Frequency-based Embeddings

Example: Co-Occurrence Matrix

	like	to	sport	afternoon	basketball	John	ball	a	rikishi	attempt	force	werstler	circular	as	Messi	score	goal	yesterday	keep	memory
like	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
attempt	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Messi	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
a	0	0	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1
ball	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
...																				

Frequency-based Embeddings

Co-occurence matrix, count vectors

- Simple bigram frequencies of all possible word-pairs need a size $N \times N$ matrix to represent N words in a corpus
- Real models use context windows, not only bigrams
- Counts are converted into probabilities
- In general, one has sparse matrices
- Dimensionality reduction (SVD, Extra Slides)

Frequency-based Embeddings

Aside Comment: One-Hot Encodings

	like	to	sport	afternoon	basketball	John	ball	a	rikishi	attempt	force	werstler	circular	as	Messi	score	goal	yesterday	keep	memory
like	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
attempt	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
Messi	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
a	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
...																				

One-hot encoding for *like* with this vocabulary:
like = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)

Frequency-based Embeddings

Aside Comment: One-Hot Encodings

	like	to	sport	afternoon	basketball	John	ball	a	rikishi	attempt	force	werstler	circular	as	Messi	score	goal	yesterday	keep	memory
like	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
attempt	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Messi	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
a	0	0	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1
...																				

Co-occurence word vector for *like* with this vocabulary in the previous corpus:

like $= (0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$

Prediction-based Embeddings

Prediction-based Embeddings

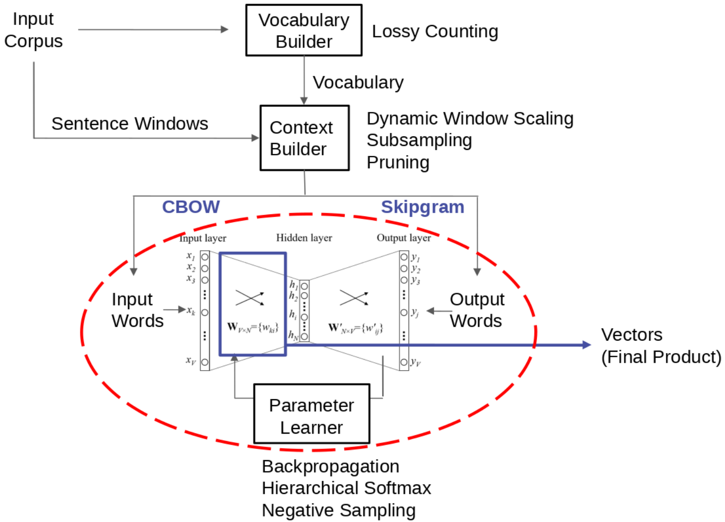
Word Embeddings (word2vec example)

Word vectors learned by a neural network in two tasks:

1. predict the probability of a **word given a context** (CBoW)
2. predict the **context given a word** (skip-gram)

Prediction-based Embeddings

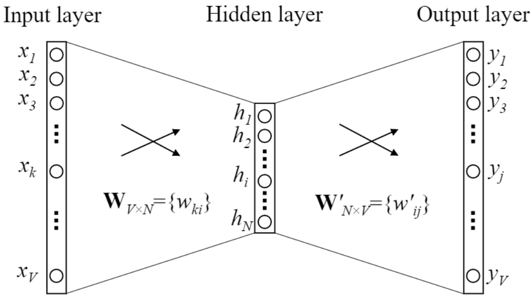
Word Embeddings (word2vec example)



Credits: Xin Rong

Prediction-based Embeddings

Word Embeddings (word2vec example)



Look at the network: simple feed-forward network learned by backpropagation with cross-entropy loss

No deep learning at all!

Prediction-based Embeddings

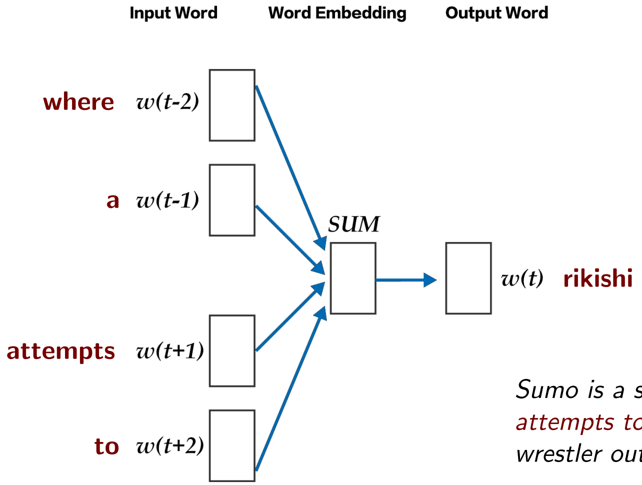
Word Embeddings (word2vec example)

Comments:

- A hidden layer in a NN interprets the input in his own way to optimise his work in the concrete task
- The size of the hidden layer gives you the dimension of the word embeddings
- Too few neurons could not have enough capacity to learn everything needed
- Too many neurons would need a very large corpus to be meaningful

Prediction-based Embeddings

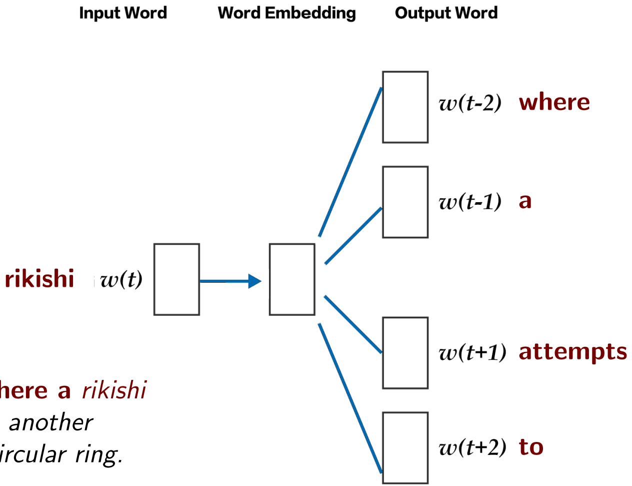
Continuous Bag of Words, CBoW



Sumo is a sport where a ***rikishi*** *attempts to force another wrestler out of a circular ring.*

Prediction-based Embeddings

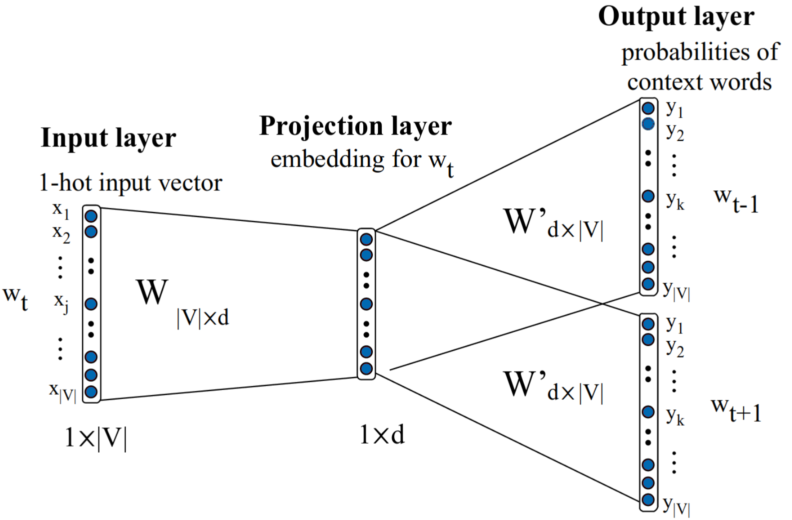
Skip-Gram Model



Sumo is a sport **where a** *rikishi* **attempts to** *force another wrestler out of a circular ring.*

Prediction-based Embeddings

More Detailed Architecture (skip-gram)



Credits: Xin Rong

Prediction-based Embeddings

More Detailed Architecture (schematic matrix visualisation)

$$\begin{array}{ccccc} \begin{pmatrix} \\ \\ V \\ \\ \\ \end{pmatrix} & \begin{pmatrix} & & & \\ & & & \\ & V \times d & & \\ & & & \\ & & & \end{pmatrix} & \begin{pmatrix} d \end{pmatrix} & \begin{pmatrix} & d \times V & \end{pmatrix} & \begin{pmatrix} \\ \\ V \\ \\ \\ \end{pmatrix} \\ \mathbf{x} & \mathbf{W} & \mathbf{h} & \mathbf{W}' & \mathbf{y} \end{array}$$

Input Embedding

The row *i* of the input matrix W is the $1 \times \mathrm{d}$ for word *i* in the vocabulary

Prediction-based Embeddings

More Detailed Architecture (schematic matrix visualisation)

$$\underset{\mathbf{x}}{\begin{pmatrix} \\ \\ V \\ \\ \\ \end{pmatrix}} \underset{\mathbf{W}}{\begin{pmatrix} \\ \\ \quad V \times d \quad \\ \\ \\ \end{pmatrix}} \underset{\mathbf{h}}{\begin{pmatrix} \\ d \\ \\ \end{pmatrix}} \underset{\mathbf{W'}}{\begin{pmatrix} \\ \quad d \times V \quad \\ \\ \end{pmatrix}} \underset{\mathbf{y}}{\begin{pmatrix} \\ \\ V \\ \\ \\ \end{pmatrix}}$$

Output Embedding

The column j of the output matrix W' is the $\mathrm{d} \times 1$ for word j in the vocabulary

Prediction-based Embeddings

Let's Play!

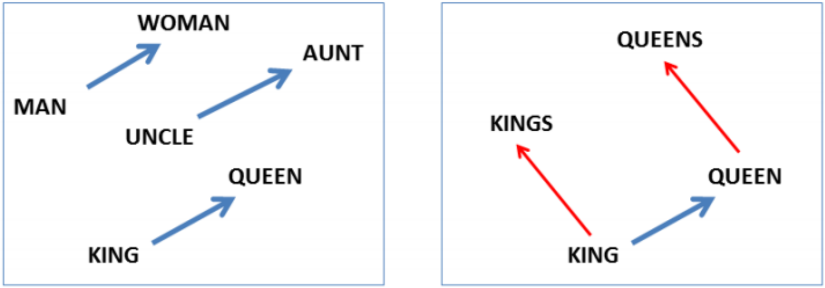
Word Embedding Visual Inspector, wevi

`https://ronxin.github.io/wevi/`

Prediction-based Embeddings

Nice Properties

King - Man + Woman = Queen

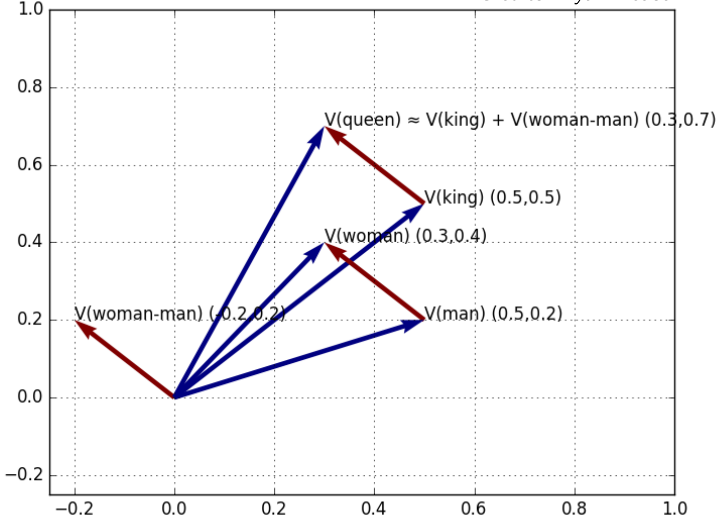


(Mikolov et al., NAACL HLT, 2013)

Prediction-based Embeddings

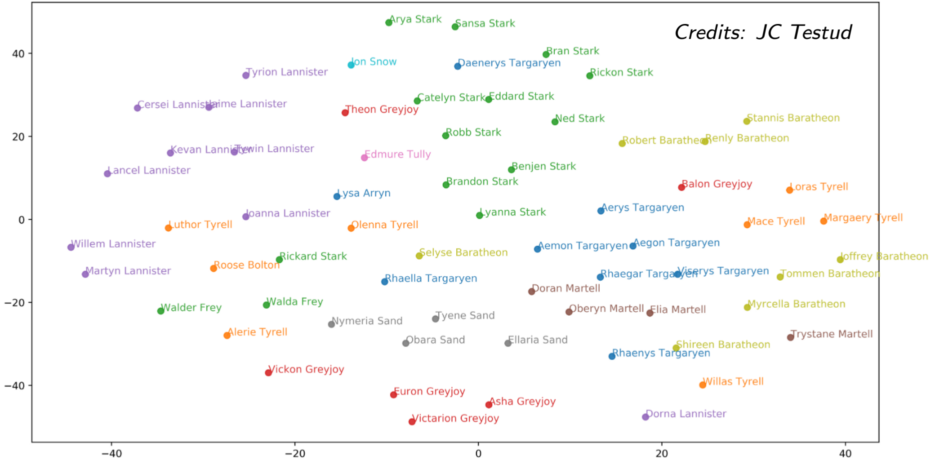
Nice Properties

Credits: Ryan Heuser



Prediction-based Embeddings

Nice Properties

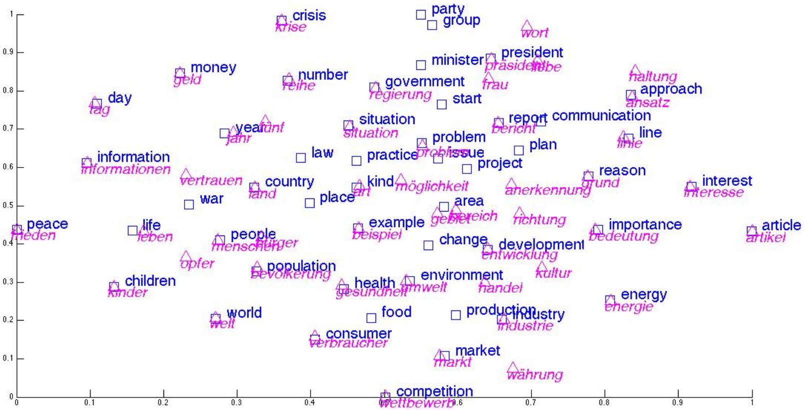


2D tSNE projection of the main characters of Game of thrones colored by House

Prediction-based Embeddings

Nice Properties

(Luong, Pham & Manning, NAACL, 2015)



Barnes-Hut-SNE visualisation of bilingual embeddings German/English

Prediction-based Embeddings

Let's Explore!

Embedding Projector

`http://projector.tensorflow.org/`

First Short Recap

Assuming Contextuality Holds...

- A word can be represented by a vector that describes it with respect to the context it is usually used
- This vector can be estimated by **counts** in a corpus
- This vector can be **learned** by examples in a corpus
- In fact, skipgram and co-occurence matrix factorisation are equivalent under certain conditions
- They are very useful to characterise text, but we haven't talked about **ambiguity** problems for instance

First Short Recap

Assuming Compositionality Holds... (backup slides for probe)

- A sentence can be decomposed into the vectors of its constituents
- Simple operations such as **sum** and **product** work surprisingly well
- Sentence/Document vectors can also be **learned** in more general tasks such as translation
- They are very useful to characterise text, but we haven't talked about **idioms** or **negation** problems for instance

First Short Recap

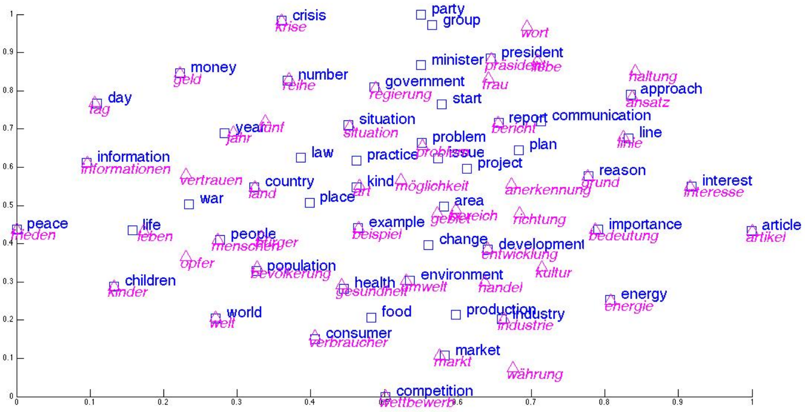
Wait!

Cross-lingual Word Embeddings

Cross-lingual Word Embeddings

Nice Properties, is this Translation?

(Luong, Pham & Manning, NAACL, 2015)



Barnes-Hut-SNE visualisation of bilingual embeddings German/English

How do we achieve this bilingualism?

Cross-lingual embeddings,
bilingual embeddings,
multi-lingual embeddings

Cross-lingual Word Embeddings

Taxonomy

1. **Supervised**
   - Joint learning
     - Regularization term in the loss function
     - Creating pseudo-bilingual corpora
   - Mapping (post-hoc alignment)
2. **Unsupervised**
   - Mapping with self-learning
   - Mapping with adversarial training

Why cross-lingual embeddings?

- Multilingual modeling of meaning
- Support for cross-lingual NLP, and will help MT!

Cross-lingual Word Embeddings

Whys

Why cross-lingual embeddings?

- Multilingual modeling of meaning
- Support for cross-lingual NLP, and will help MT!

Why supervised cross-lingual embeddings?

- Simplicity
- Supervision mostly possible
  (small dictionaries, common words...)

Why unsupervised cross-lingual embeddings?

- Sometimes outperformed supervised ones!
- Cases without dictionaries

Cross-lingual Word Embeddings

Summary of Approaches

- The summary is not comprehensive at all
- Selection biased towards understanding unsupervised NMT at the end of the course
- Lot of info in Sebastian Ruder's blogs and tutorials

Supervised Cross-lingual Embeddings

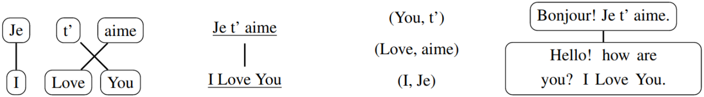
Let's Start with Supervised Methods

1. **Supervised**
   - Joint learning
     - Regularization term in the loss function
     - Creating pseudo-bilingual corpora
   - Mapping (post-hoc alignment)
2. **Unsupervised**
   - Mapping with self-learning
   - Mapping with adversarial training

Supervised Cross-lingual Embeddings

Form of Cross-lingual Supervision

- **Word level**: bilingual dictionaries, word alignments
- **Sentence level**: parallel corpora, sentence aligments
- **Document level**: comparable corpora, document alignments



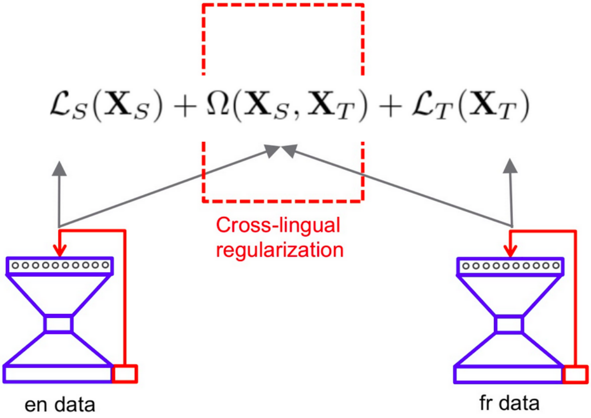
(a) **BiSkip** (b) **BiCVM** (c) **BiCCA** (d) **BiVCD**

Figure 2: Forms of supervision required by the four models compared in this paper. From left to right, the cost of the supervision required varies from expensive (BiSkip) to cheap (BiVCD). BiSkip requires a parallel corpus annotated with word alignments (Fig. 2a), BiCVM requires a sentence-aligned corpus (Fig. 2b), BiCCA only requires a bilingual lexicon (Fig. 2c) and BiVCD requires comparable documents (Fig. 2d).

(Upadhyay, Faruqui, Dyer & Roth, ACL, 2016)

Supervised Cross-lingual Embeddings

Joint Learning Approaches

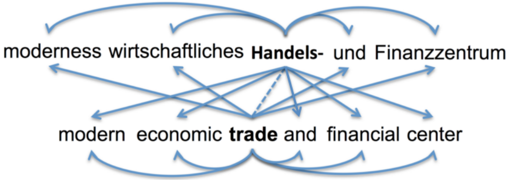


https://tinyurl.com/xlingual

Supervised Cross-lingual Embeddings

Joint Learning Approaches: Bilingual Skipgram

Luong et al., 2015: Bilingual skipgram, direct but expensive

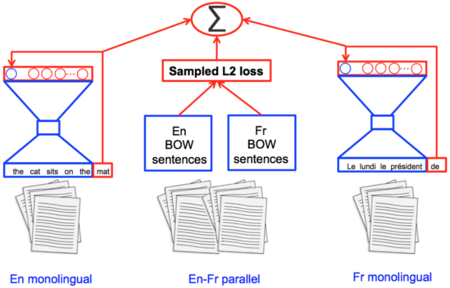


- predict words in the source language **and** predict aligned words in the target language
- parallel corpora + (learned) word aligments

Supervised Cross-lingual Embeddings

Joint Learning Approaches: Bilingual BilBOWA

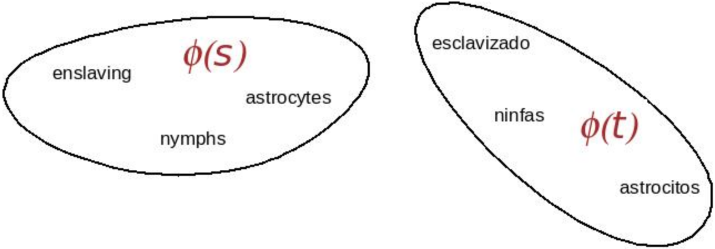
Guows et al., 2015: Bilingual Bag-of-Words without Word Alignments (**Coulmance et al., 2015**: Trans-gram)



- monolingual skipgram loss
- every word in Source is uniformly aligned to every word in Target
- BilBOWA: minimise distance between the means of the words in the aligned sentences
- Trans-gram: every word in Target as context of every word in Source

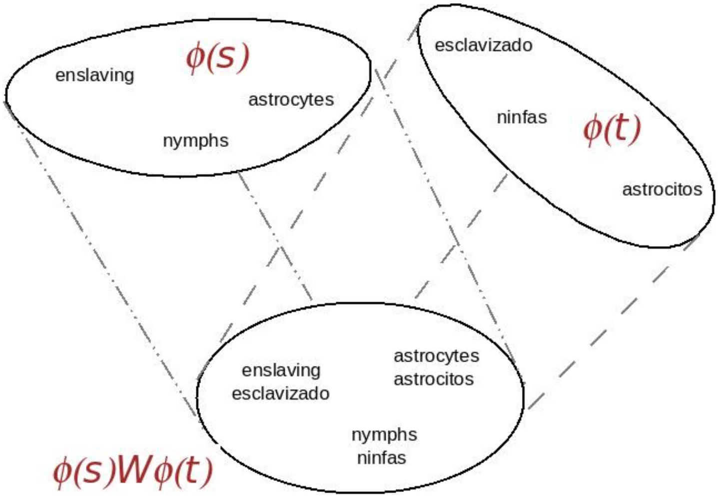
Supervised Cross-lingual Embeddings

Mapping Approaches



Supervised Cross-lingual Embeddings

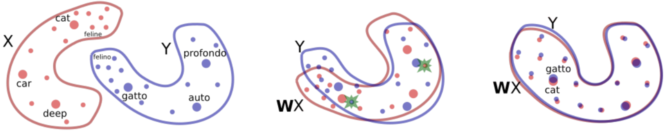
Mapping Approaches (aka finding W)



Supervised Cross-lingual Embeddings

Mapping Approaches: Isomorphism (and Other!) Assumption

Spaces should be isomorphic for (linear) mappings to be effective

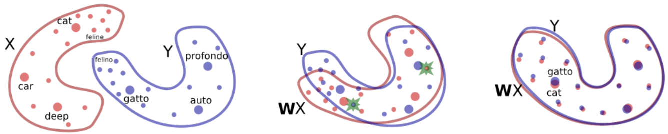


(Figure from *Conneau et al., 2017*)

Supervised Cross-lingual Embeddings

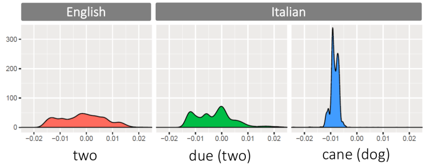
Mapping Approaches: Isomorphism (and Other!) Assumption

Spaces should be isomorphic for (linear) mappings to be effective



(Figure from *Conneau et al., 2017)*

Similarly, similar intra-lingual similarity would be expected



(Figure from *Artetxe et al., 2018)*

Supervised Cross-lingual Embeddings

Mapping Approaches, a bit of History (towards UnsupMT!)

Mikolov et al., 2013: Minimise Euclidean distance

$W^* = \arg\min_W \parallel Wx_i - y_i \parallel^2, \quad (x_i, y_i)$ pairs in a dictionary

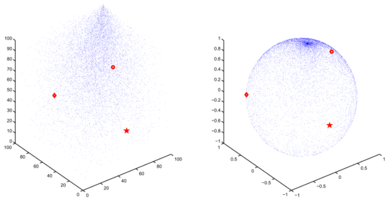
Supervised Cross-lingual Embeddings

Mapping Approaches, a bit of History (towards UnsupMT!)

Mikolov et al., 2013: Minimise Euclidean distance

$$W^* = \arg\min_W \parallel Wx_i - y_i \parallel^2, \qquad (x_i, y_i) \text{ pairs in a dictionary}$$

Xing et al., 2015: Minimise Cosine distance



Mismatch between the initial objective function, the distance measure, and the test distance measure

$$W^* = \arg\max_W \cos(Wx_i, y_i)$$

Second Short Recap

Cross-lingual Embeddings

- One can obtain semantic representation of words in the form of word embeddings using monolingual data
- Joint learning methods allow to obtain meaningful bilingual embeddings in a similar way
  - Methods differ in the amount (type) of supervision
- Mapping methods depart from the monolingual embeddings
- Next (not today!) we'll learn about methods that can do the mapping without supervision

Is this Machine Translation?

Remember SMT:

$$\mathbf{T(f)} = \hat{\mathbf{e}} = \mathrm{argmax_e}\ \mathbf{P(e)}\,\mathbf{P(f|e)}$$

- $P(e)$ monolingual data, $P(f|e)$ parallel data
- $P(f|e)$ was our translation table:
  barbe noire ||| black beard ||| 0.324064 0.188708 0.388876 0.445356
- Can we get something similar to $P(f|e)$ from (unsupervised) bilingual embeddings?
- If so, we can train MT with only monolingual data!

Is this Machine Translation?

What Next with Embeddings?

- Learn more on how neural nets (NN) work
  - cbow and skip-gram are learnt with a feed forward NN $\rightarrow$ how??
- Describe (with equations) NNs used for machine translation
  - They have embeddings everywhere: word embeddings, sentence embedings...
- (Pre-trained) embeddings can be used to initialise MT NNs, as key concept for unsupervised MT, etc.

Is this Machine Translation?

Wait!

Introduction to Word Embeddings
(biased towards machine translation)

Cristina España-Bonet
DFKI GmbH

Summer Semester 2023

15th May 2023

Extra Slides

A few Clarifications

The meaning of a compound expression is a function of the meanings of its parts and of the way they are syntactically combined.

(Partee, 1984)

Beyond *Word* Embeddings

Is Language Compositional?

Computer Scientist-like Background Yes!

$$\texttt{meaning}(\text{Eat the icecream}) =$$
$$\texttt{meaning}(\text{Eat}) + \texttt{meaning}(\text{the}) + \texttt{meaning}(\text{icecream})$$

Is Language Compositional?

Computer Scientist-like Background Yes!

$$\texttt{meaning}(\text{Eat the icecream}) = \texttt{meaning}(\text{Eat}) + \texttt{meaning}(\text{the}) + \texttt{meaning}(\text{icecream})$$

Linguist-like Background No!

$$\texttt{meaning}(\text{Break the ice}) \neq \texttt{meaning}(\text{Break}) + \texttt{meaning}(\text{the}) + \texttt{meaning}(\text{ice})$$

Beyond *Word* Embeddings

Is Language Compositional?

Computer Scientist-like Background Yes!

$$\texttt{meaning}(\text{Eat the icecream}) = \texttt{meaning}(\text{Eat}) + \texttt{meaning}(\text{the}) + \texttt{meaning}(\text{icecream})$$

Linguist-like Background No!

$$\texttt{meaning}(\text{Break the ice}) \neq \texttt{meaning}(\text{Break}) + \texttt{meaning}(\text{the}) + \texttt{meaning}(\text{ice})$$

Beyond *Word* Embeddings

Representations for Phrases, Sentences or Paragraphs

- **Composition** of word embeddings using operations ($+$,$\times$) on vectors and matrices
- **Latent paragraph vectors** in word2vec-like NNs
- Internal representations in **seq2seq** architectures or auto-encoders (NMT context vectors, skip-thought vectors...)

Composition I

(Mitchell & Lapata, 2010)

Table 5
Composition functions considered in our experiments

Model	Function
Additive	$p_i = u_i + v_i$
Kintsch	$p_i = u_i + v_i + n_i$
Multiplicative	$p_i = u_i \cdot v_i$
Tensor product	$p_{i,j} = u_i \cdot v_j$
Circular convolution	$p_i = \sum_j u_j \cdot v_{i-j}$
Weighted additive	$p_i = \alpha v_i + \beta u_i$
Dilation	$p_i = v_i \sum_j u_j u_j + (\lambda - 1) u_i \sum_j u_j v_j$
Head only	$p_i = v_i$
Target unit	$p_i = v_i(t_1 t_2)$

Beyond *Word* Embeddings

Composition II

(Mitchell & Lapata, 2010)

Table 6
Correlation coefficients of model predictions with subject similarity ratings (Spearman's ρ) using a simple semantic space

Model	Adjective–Noun	Noun–Noun	Verb–Object
Additive	.36	.39	.30
Kintsch	.32	.22	.29
Multiplicative	.46	.49	.37
Tensor product	.41	.36	.33
Convolution	.09	.05	.10
Weighted additive	.44	.41	.34
Dilation	.44	.41	.38
Target unit	.43	.34	.29
Head only	.43	.17	.24
Humans	.52	.49	.55

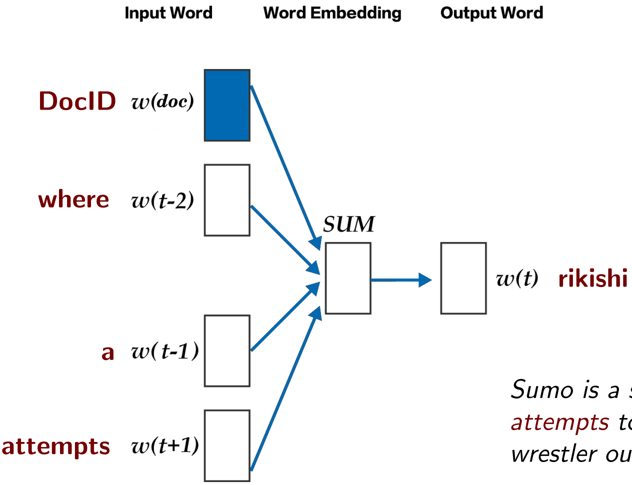
Beyond *Word* Embeddings

Distributed Representations of Sentences and Documents

- word2vec-like architecture where a **document vector** is added to word vectors and learned simultaneously (PV-DM)
- **Generation** of words from a document from its document vector (skip-gram-like architecture!) (PV-DBoW)

Beyond *Word* Embeddings

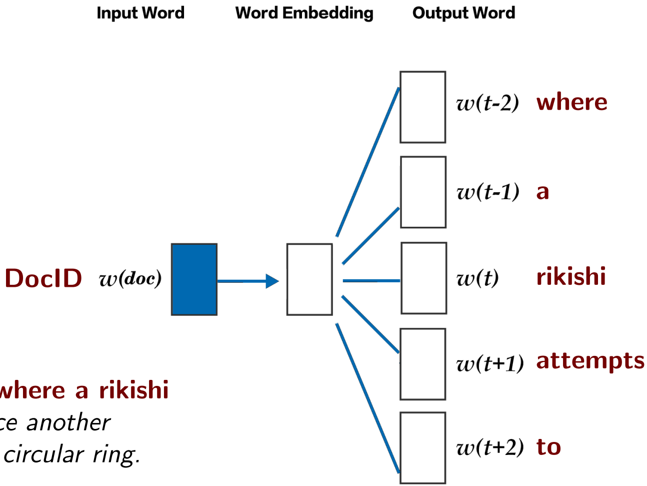
Distributed Memory Model of Paragraph Vectors, PV-DM



Sumo is a sport where a ***rikishi*** *attempts to force another wrestler out of a circular ring.*

Beyond *Word* Embeddings

Distributed Bag of Words version of Paragraph Vector



Sumo is a sport **where a rikishi attempts to** *force another wrestler out of a circular ring.*

Some Maths: SVD

Singular-Value Decomposition, SVD

- Linear algebra

Some Maths: SVD

Singular-Value Decomposition, SVD

- Linear algebra
- **Factorisation** of a matrix $\mathbf{M}$ as $\mathbf{M} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathsf{T}}$

Some Maths: SVD

Singular-Value Decomposition, SVD

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- **Factorisation** of a matrix $\mathbf{M}$ as $\mathbf{M} = \mathbf{U\Sigma V^T}$
  - ✓ $\mathbf{U}$ is an $m \times m$ orthogonal matrix,

Some Maths: SVD

Singular-Value Decomposition, SVD

- Linear algebra
- **Factorisation** of a matrix $\mathbf{M}$ as $\mathbf{M} = \mathbf{U\Sigma V^T}$
  - ✓ $\mathbf{U}$ is an $m \times m$ **orthogonal matrix**,
    - $\mathbf{U^T U} = \mathbf{U U^T} = \mathbf{I}$
    - or, equivalently, $\mathbf{U^T} = \mathbf{U}^{-1}$

Some Maths: SVD

Singular-Value Decomposition, SVD

- Linear algebra
- **Factorisation** of a matrix $\mathbf{M}$ as $\mathbf{M} = \mathbf{U\Sigma V^T}$
  - ✓ $\mathbf{U}$ is an $m \times m$ orthogonal matrix,
  - ✓ $\mathbf{\Sigma}$ is a diagonal $m \times n$ matrix with non-negative real numbers,

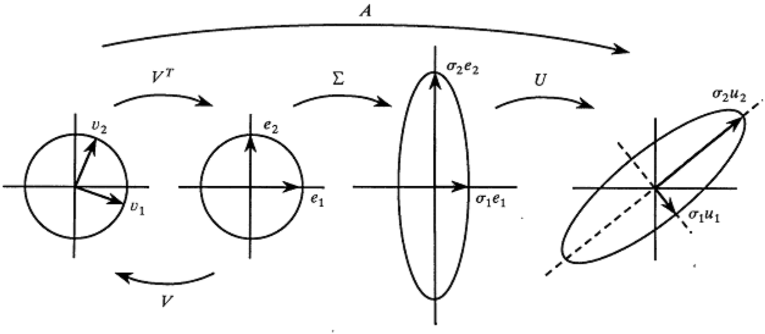
Some Maths: SVD

Singular-Value Decomposition, SVD

- Linear algebra
- **Factorisation** of a matrix $\mathbf{M}$ as $\mathbf{M} = \mathbf{U\Sigma V^T}$
  - ✓ $\mathbf{U}$ is an $m \times m$ orthogonal matrix,
  - ✓ $\mathbf{\Sigma}$ is a diagonal $m \times n$ matrix with non-negative real numbers,
  - ✓ $\mathbf{V^T}$ is the conjugate transpose of an $n \times n$ orthogonal matrix

Some Maths: SVD

SVD: 2×2 Geometric Interpretation



a linear transformation is a rotation or reflection, followed by a scaling, followed by another rotation or reflection

`https://blogs.sas.com`

Some Maths: SVD

Singular-Value Decomposition, SVD

$$\begin{pmatrix} \\ \\ m \times n \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \quad\quad m \times m \quad\quad \\ \\ \\ \end{pmatrix} \begin{pmatrix} \\ \\ m \times n \\ \\ \\ \end{pmatrix} \begin{pmatrix} n \times n \end{pmatrix}$$

$$\mathbf{M} = \mathbf{U} \quad \boldsymbol{\Sigma} \quad \mathbf{V}^{\mathbf{T}}$$

Some Maths: SVD

SVD: Singular Values

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_1 & & & \\ & \ddots & \text{\Huge 0} & \\ \text{\Huge 0} & & \sigma_r & \\ & & & 0 \end{pmatrix};$$

σ_1 ... σ_r, singular values of $\mathbf{M}$ (in decreasing order)

r, rank of $\mathbf{M}$

Some Maths: SVD

SVD: Singular Values

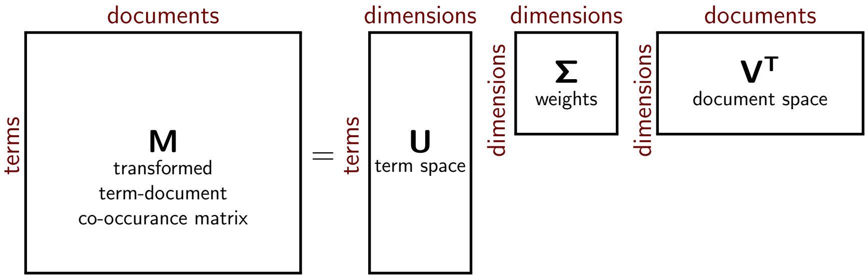
$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1 & & & & \\ & \ddots & & \mathbf{0} & \\ & \mathbf{0} & \ddots & & \\ & & & \sigma_r & \\ & & & & 0 \end{pmatrix}; \qquad \mathbf{M_r} = \sum_{i=1}^{r} \sigma_i \overrightarrow{u}_i \overrightarrow{v}_i^T$$

σ_1 ... σ_r, singular values of **M** (in decreasing order)

r, rank of **M**

Some Maths: SVD

SVD: Application, Latent Semantic Analysis



$$\Sigma_r \Longrightarrow \mathbf{M}_r$$

Some Maths: SVD

SVD: Learn & Practice

`https://nlp.stanford.edu/IR-book/pdf/18lsi.pdf`

Christopher D. Manning, Prabhakar Raghavan, and Hinrich Schütze. 2008. **Introduction to Information Retrieval**. Cambridge University Press, New York, NY, USA.