

Kinetic Monte Carlo Modeling with kmos

Mie Andersen, Juan Manuel Lorenzi

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1 Introduction

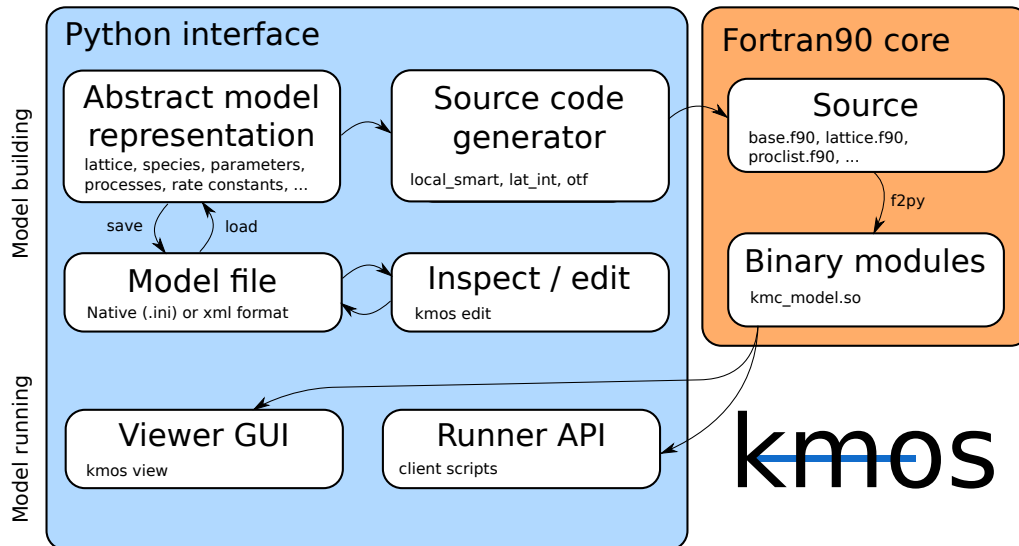


Figure 1: Components of the kmos framework.

For a brief introduction to kinetic Monte Carlo and kmos, see the slides Intro.pdf. For a more thorough introduction to kinetic Monte Carlo, we refer to Ref. [1, 2, 3]. Documentation for kmos can be found at <http://kmos.readthedocs.io>. A detailed description of the algorithms and package structure as well as benchmark data can be found in the kmos paper[4]. The current version of the program can be fetched using *git*

```
git clone http://www.github.com/mhoffman/kmos
```

or directly downloaded from <http://www.github.com/mhoffman/kmos>.

1.1 TASK 1: The Viewer GUI.

Objective: Get used to quickly studying a model's behavior with kmos' viewer GUI

Step-by-step:

- Fire up the viewer GUI by invoking `kmos view` from inside the model folder

```
cd ~/intro2kmos/task_material/CO_oxidation_RuO2_local_smart
kmos view
```

- Use the sliders to reduce oxygen pressure to its lowest value. What do you observe? Slowly increase the temperature. What happens now?

- Set $T \approx 600$ K and $p_{\text{O}_2} \approx 10^{-1}$ bar. Try to tune the value of p_{CO} to get the highest possible value for the turnover frequency (TOF). For which CO partial pressure does this happen? What is the TOF maximum?
Pay special attention to the state of the catalyst (what's adsorbed on it) when the TOF is highest. What happens for other values of p_{CO} ?
- Set $p_{\text{O}_2} \approx 10^{-1}$ bar and $p_{\text{CO}} \approx 2 \times 10^{-1}$ bar and $T \approx 450$ K. Slowly increase T (in steps of ≈ 10 K) up to 600 K, while monitoring the the steady state TOF. How does the surface behave?

2 Quantitative analysis: Using the kmos API

2.1 Scripting, plotting and visualization

The kmos application programming interface (API) uses the Python programming language. Documentation can be found at: <https://docs.python.org/2/>. Some basic usage and examples are provided below.

2.1.1 Elements of Python syntax

1. Hello World is

```
print('Hello_World')
```

or

```
print 'Hello_World'
```

2. Importing modules:

```
import ... as ...
from ... import ...
```

3. (Some of) python variable types:

Basic types:

```
some_text = 'This_is_a_string' # strings
some_integer = 134 # integers
some_float = 0.8; other_float = 600. # floating point numbers
```

Lists:

```
L = ['a', 3, 4.7]
L[0] # 'a'
L[1] # 3
```

Dictionaries:

```
dict = {'Name': 'Zara', 'Age': 7, 'Class': 'First'}
dict['Age'] # 7
```

4. If statements:

```
if (condition):  
    ...  
elif (condition):  
    ...  
else:  
    ...
```

5. Flow control with for loops:

```
for i in range(3):  
    print i
```

0

1

2

```
for i, j in enumerate(['hat', 'cat', 'bat']):  
    print i, j
```

0 hat

1 cat

2 bat

2.1.2 Scientific computing using the numpy package

The numpy package contains many useful things such as N-dimensional array objects, linear algebra and random number capabilities. Documentation can be found at: <http://www.numpy.org/> The package is imported and used as follows:

```
import numpy as np  
A = np.array([0, 1])
```

2.1.3 Plotting using matplotlib

The Python plotting library matplotlib can be used to create publication quality figures in a variety of formats. Documentation can be found at: <http://matplotlib.org/>. Any other plotting software can of course also be used, provided the kmos output data are stored in a convenient format. The advantage of using matplotlib is the easy integration into Python client scripts for running kmos.

2.1.4 Visualizing atomic structures using ASE

The Atomic Simulation Environment (ASE) is a set of tools and Python modules developed for atomistic simulations. It is used in kmos to visualize the current state of a model. Documentation can be found at: <https://wiki.fysik.dtu.dk/ase/>.

2.2 First API steps: Running kmos interactively.

While we can use the GUI viewer to get a general idea of the behavior of the model, quantitative analysis requires the use of the API. To launch the API, we need to launch a terminal, go to the folder containing the compiled kMC model (i.e. the one with the `kmc_model.so` and the `kmc_settings.py` files in it) and run the command:

```
ipython
```

This will initialize `ipython`, an interactive Python interface. Inside this interface we can load our model by writing

```
from kmos.run import KMC_Model
model = KMC_Model()
```

These commands will give some text output (including the model's name and author), and create the `model` object, which we will use to run our simulations.

A useful shortcut is to type:

```
kmos run
```

in the folder containing the compiled kMC model, which directly opens the interactive shell and creates the `model` object.

The most elementary thing we can do with our model is to directly run it:

```
NSTEPS = 1e6
model.do_steps(NSTEPS)
```

which will run `NSTEPS` kMC steps (in this case 1 million). This will take a while, and then we will recover the (`ipython`) command prompt.

Below are listed some other useful commands:

1. Change the value of a parameter (e.g. temperature or pressure)

```
model.parameters.T = 550
model.parameters.p_COgas = 0.5
```

2. To print a list of the current values of all parameters and rate constants in the model, simply type:

```
model
```

3. Print current occupations (as averaged coverages):

```
model.print_coverages()
```

4. Print a list of the number of times each process has been executed since the beginning of the simulation:

```
model.print_procstat()
```

5. Analyze the current state of the model:

```
atoms = model.get_atoms()
```

This command returns an ASE `atoms` object, which can be visualized by typing:

```
from ase.all import view
view(atoms)
```

The atoms object also contains some additional data:

```
atoms.occupation
atoms.tof_data
atoms.kmc_time
atoms.kmc_step
```

The occupations and turn-over-frequencies (TOFs) come in the form of numpy arrays. In order to get the headers for these arrays, type:

```
model.get_occupation_header()
model.get_tof_header()
```

The TOFs that are affiliated with the atoms object are TOFs averaged over the simulated time since the last `model.get_atoms()` call. In contrast, `atoms.occupation` is the current occupation (identical to what is printed using the `model.print_coverages()` command).

6. Sample an average model and return TOFs and coverages in a standardized format:

```
model.get_std_sampled_data(samples, sample_size, tof_method='integ',
output='str')
```

The parameter `samples` is the number of batches to average over. The number of kmc steps in each batch or sample is given by the `sample_size` parameter. In each case check carefully that the desired observable is sampled good enough! The parameter `tof_method` allows to switch between two different methods for evaluating turn-over-frequencies. The default method 'procstat' evaluates the procstat counter, i.e. simply the number of executed events in the simulated time interval. 'integ' will evaluate the number of times the reaction could be evaluated in the simulated time interval based on the local configurations and the rate constant.

The output of `model.get_std_sampled_data()` is determined by the parameter `output`, which can be set to 'str' or 'dict'. The default 'str' returns a text string containing first the values of all adjustable parameters, then the TOF(s) and coverages and finally the total sampled kmc time, the total simulated kmc time (including also the time simulated before the `model.get_std_sampled_data()` call), and the number of sampled kmc steps, all separated by spaces. This text string can be converted to a Python list (L) using the Python `split()` command:

```
data = model.get_std_sampled_data(samples=10, sample_size=1e6)
L = data.split('␣')
```

Alternatively, `output='dict'` returns the above information in the form of a Python dictionary. For example, the TOF for CO oxidation can be retrieved using the command:

```
TOF = output['CO_oxidation']
```

7. Access to the Fortran modules:

The above commands are often sufficient when running and simulating a kmos model, but in certain cases direct access to the Fortran data structures and methods is desirable. The Fortran modules `base`, `lattice`, and `proclist` are attributes of the model instance `kmc_model`.so. This model instance can be explored using `ipython` and `<TAB>`:

```
model.base.<TAB>
model.lattice.<TAB>
model.proclist.<TAB>
```

8. Deallocating memory:

```
model.deallocate()
```

This command is important to call once a simulation has finished, if you want to run a new simulation within the same script, as it frees the memory allocated by the Fortran modules.

9. Reset model:

```
model.reset()
```

This command is called after `model.deallocate()` in order to reset the model to its initial state.

2.3 Client scripts

We have already seen that the kmos GUI is useful to quickly investigate model behavior, and that interactive use of the API allows for finer control and more precise quantitative evaluation. In everyday use however, the most useful way of using kmos is through *client scripts*. With client scripts we can automatize the task we performed using the interactive interface.

2.3.1 Generating an Arrhenius plot.

As a first example of a client script, we will see how to build an Arrhenius plot (i.e. $\log(TOF)$ vs. $1/T$) for the RuO₂ CO oxidation model. The example script `plot_arrhenius.py` can be found in the model folder.

2.3.2 TASK 2: TOF and coverages vs p diagrams.

The objective of this task is to write a client script similar to the one from the previous section. In this case, you need to plot both turnover frequency (TOF) and coverages (discriminated by site type and species) as a function of CO partial pressure, for constant T and oxygen partial pressure. The plot should cover values within $10^{-1} \text{ bar} < p_{\text{CO}} < 10^2 \text{ bar}$ (log-scale should be used)

You can use the previous example as a basis, and either directly use `matplotlib` or another plotting tool you know (if available) by writing to a file.

The strings that label the output important for this task are `'CO_oxidation'`, `'CO_ruo2_bridge'`, `'CO_ruo2_cus'`, `'O_ruo2_bridge'` and `'O_ruo2_cus'`. If writing output to a file, these values will end up in columns 3, 4, 5, 6 and 7, respectively; while `'p_COgas'` is in column 1. All columns are counted starting from 0, according to python standards.

2.3.3 Relaxing the system

We either begin the kmc simulation with a clean surface or prepare the system in some user-specified initial state (see Sec. 2.3.5). In any case, this initial system state might be very different from the steady-state system state. It is therefore necessary to run a number of kmc steps to relax the system before any meaningful information about steady-state TOFs and coverages can be obtained. It should thus always be checked that the system has reached steady-state before calling `model.get_std_sampled_data()`! An example showing how this can be done is provided in the script `relaxation.py`.

2.3.4 Preparing the initial state of the system

Since it can be difficult to estimate if the system has reached steady state or not, it can be useful to test different initial states of the system to check that the same steady-state solution is always found independently of the initial state. The state of the system can be modified at any time during the simulation, but most often one would want to prepare a given initial state before beginning the simulation. The occupation of a site <site> can be changed to the species <species> using the command:

```
model.put(site=[x,y,z,model.lattice.<site>], model.proclist.<species>)
```

where x,y,z are the coordinates of the site. The above command can be quite inefficient if changing many sites at once, since each put() call adjusts the book-keeping database. To circumvent this you can use the _put() method instead:

```
model._put(...)
model._put(...)
...
model._adjust_database()
```

Uncomment lines 16-19 in the script relaxation.py for an example of how to use this.

2.3.5 TASK 3: The effect of the initial state

Try relaxing the model from different initial states, e.g. clean, CO@br, O@cus, etc., by doing the corresponding modifications to the script relaxation.py. Also try to vary the number of kMC steps taken in each sample (sample_size) and the number of samples (Nsamples).

2.3.6 TASK 4: Random initial state from guess coverages

In some cases one might have a good guess of the steady-state system state in terms of (averaged) coverages. This could for example be obtained by solving the corresponding model in the mean-field approximation (MFA) using rate equations. Using a good guess for the final coverages can substantially speed up the time required to relax the system. Since the MFA does not take into account lattice inhomogeneity, the best way to convert the MF coverages to a kMC lattice occupation is to assume a random occupation of the lattice sites. In order to do this, the guess coverages must be converted to the number of sites occupied by each species. For each species, the corresponding number of sites is then chosen randomly among the total number of available sites in the system using the Python random.sample() method. Take a look at the script relaxation_random_initialization.py and try relaxing the model from different random initial states.

2.3.7 TASK 5: Sensitivity analysis

In a catalytic system, the macroscopic TOF is often controlled by only one or a few microscopic rate constants. This can be quantified using the so-called Degree of Rate Control (DRC) [5]. Several definitions of this concept exists, but here we will be concerned with quantifying how much the change of the rate constant k of a single microscopic process i (+ for forward process, – for reverse process) affects the TOF according to the formula:

$$x_i^+ = \frac{k_i^+}{\text{TOF}} \left. \frac{\partial \text{TOF}}{\partial k_i^+} \right|_{k_{j \neq i}^+, K_{j \neq i}, k_i^-} \quad (1)$$

Take a look at the script `DRC.py`, which calculates the DRC for CO adsorption on the *cus* site. The derivative is calculated using finite differences, changing the rate constant by plus and minus 2% (the delta value). Try to play around with the delta value and the number of sample steps used to get an averaged TOF. Hint: finite differences are extremely sensitive to numerical noise and therefore require very long sampling times to sufficiently converge. Does your result match the result for the appropriate process and CO pressure in the lower panel of Fig. 4 in Ref. [6]?

2.3.8 TASK 6: ModelRunner

For some kMC applications you simply require a large number of data points across a set of external parameters (phase diagrams, microkinetic models). For this case there is a convenient class `ModelRunner` to work with. For example:

```
from kmos.run import ModelRunner, PressureParameter, TemperatureParameter

class ScanKinetics(ModelRunner):
    p_O2gas = PressureParameter(1)
    T = TemperatureParameter(600)
    p_COgas = PressureParameter(min=1e-1, max=1e2, steps=20)

ScanKinetics().run(init_steps=1e6, sample_steps=1e7, cores=4)
```

This script generates data points over the specified range. Using the `PressureParameter` or `TemperatureParameter` assures that the corresponding parameters will be sampled in a log- or reciprocal-scale, respectively. The script above runs several kmos jobs synchronously (as many as indicated with the `cores` argument), and generates an output file with results (in this case called `ScanKinetics.dat`).

TASK: Try to reproduce the Arrhenius plot and the plots from Task 2 using the `ModelRunner` class. Compare the time spent by the original scripts with this one. Use as many cores as your workstation has.

Hint: `ModelRunner` uses a file to keep track of which calculations took place. If you need to restart the calculations from scratch, you have to remove the file ending in `.lock`.

3 Building a kmos model

Now that we know how to run an existing kmos model, we will learn how to generate a new model. While doing this we will learn about the way models are abstractly represented. We will also discuss how the models are saved and how the source code necessary for running the simulations is generated and compiled.

3.1 The elements of a kmos project

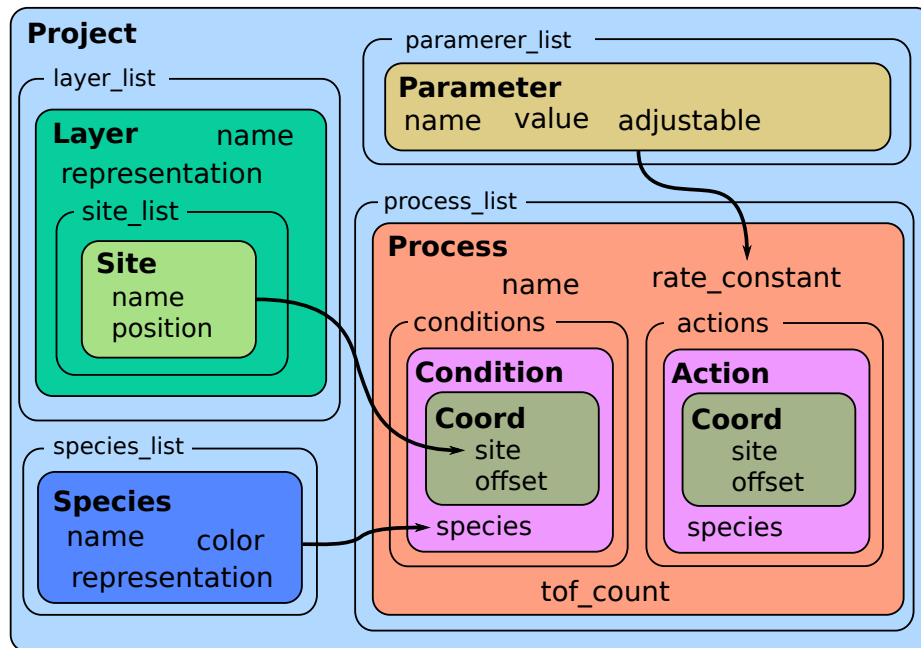


Figure 2: Structure of a kmos model.

A kmos model is built by putting together several building blocks. Those building blocks are found within the `kmos.types`.

3.1.1 Project

The `Project` is the structure that contains all other elements.

3.1.2 Meta

`Meta` contains the meta-data of the project. This includes the model's name, the author's name and email and, importantly, the dimensionality (1D, 2D or 3D) of the system.

3.1.3 Layer

A kmos `Project` contains a list of `Layer` objects. In this tutorial we will only consider models with a single layer, but more could be used. Each `Layer` has an unique *name*, an optional *representation* and a list of *sites*.

Site Each site has an unique *name* and a *position* in the unit cell.

3.1.4 Species

A `Project` also contains a list of `Species`. Each species needs to have an unique *name*. A *color* and a *representation* can additionally be added; these are useful while using the Editor and the GUI Viewer, respectively.

3.1.5 Parameter

A list of *parameters* is included in a *Project*. Each parameter needs an unique *name* and a default *value*. Parameters are used in the definition of the rate constants, and their values can be modified at run-time using the API or (if defined as *adjustable*) the Viewer GUI.

3.1.6 Process

The processes are perhaps the most complex structure in a kmos model, and they are built using the elements previously discussed. Apart from an unique *name*, processes are composed of a list of *conditions*, a list of *actions* and a *rate constant expression*. Optionally, a process can also contain a *tof_count* attribute, if it contributes to some of the turnover frequencies.

Condition A condition is defined by selecting a single *species* and a single *coordinate*. A coordinate is a site, but containing additionally information about its *offset*, i.e. the relative position of this coordinate with respect to a reference central coordinate. The list of conditions of a process encodes the necessary (local) occupation pattern necessary for the process to occur.

Action Like conditions, actions are also composed of a *species* and a *coordinate*. The list of actions of a process determines what is changed in the system state if the process is actually executed.

Rate constant Each process needs to be assigned a rate constant expression. These are given as strings that can contain common mathematical expression, any of the user-defined parameters as well as some other predefined parameters and constants. All rate constants are evaluated when the model is first loaded and whenever the value of a parameter is changed.

3.2 A model step-by-step: O₂ adsorption / desorption

We will learn about the components of a kmos models through a simple example: A model for oxygen adsorption and desorption into a fcc(100) (square) lattice.

3.3 TASK 7: The ZGB Model

The ZGB model [7] is a classical example of a kinetic Monte Carlo model of CO oxidation. It includes dissociative oxygen adsorption, molecular CO adsorption and a CO-O reaction to form gaseous CO₂. The only parameter in the model is the fraction of CO in the gas mixture y_{CO} . The CO adsorption rate per unit cell on empty sites is equal to y_{CO} . The Oxygen adsorption rate per unit cell is $(1 - y_{\text{CO}})$. Desorption of the reactants (CO and O₂) is neglected. To avoid deadlocks, i.e. states in which no process can happen, desorption process with very small rate (i.e. 10^{-10}s^{-1}) should be included. CO₂ formation has “infinite” rate constant in the model, which can be modeled by using a very large value (i.e. 10^{10}s^{-1}).

Hint: Beware of double counting when implementing the oxygen adsorption process.

3.4 Modeling (lattice) diffusion

3.4.1 A simple ion diffusion model

Another type of system that can be modeled with kmos is that of a set of particles diffusing on a lattice. Here we will consider a simple example of particles diffusing on a 2D square lattice. The example render script `render_LGD.py` can be found in the `task_material` folder.

A similar simple model in a realistic context (Li diffusion in graphene) can be found in [8].

3.4.2 Preparing the system

If used as given, trying to run the model will not work. This is because at start-up the system will be completely empty, a state in which no process can occur. To solve this problem there are two possibilities, depending on our objectives:

- One option is to prepare the system so that it contains a certain number of ions at the start, as was done in section 2.3.4. This is useful if one ones to study the relaxation of the system from the selected configuration, or to study equilibrium properties.
- Alternative, it is possible to impose special boundary conditions to the system. In this way it is also possible to study steady-state behavior under non-equilibrium conditions.

In this course we will focus only on the second case.

3.4.3 TASK 8: Implement the boundary conditions

The objective of this task is to set up the boundary conditions for the diffusion model. This can be achieved by including a pair of auxiliary species ('source' and 'drain') and the corresponding entry and exit processes (as shown in the slides). Additionally, the `kmc_settings.py` file needs to be modified in two ways:

- First, the default system size should be changed from the default (20×20) to something that is longer in the direction of prominent diffusion, e.g. (50×20)
- Second, the `setup_model` function should be modified to place the *sources* and *drains* in the correct positions.

If you manage to set this up correctly, the model can be visualized using `kmos view`.

3.4.4 TASK 9: Extending and testing the lattice diffusion model

Extend the diffusion model to include the effects of an external electric field. For this, introduce a new parameter that measures the strength of the field, and modify the diffusion processes accordingly. Once this is set in place, test how the effects of field strength and particle concentration in the current (details in the slides).

3.5 Lateral interactions in kmos

Up to now we have only considered models with a relatively small number of processes. In some situations, however, it is possible that the rate constants depend not only on the *class* of process being executed, but also on the local environment around the adsorbates. For example, the particles in our diffusion problem could interact repulsively, changing the rates of diffusion. In these cases we say that there are *lateral interactions* in the system.

The standard way to treat this in kmos is to explicitly incorporate all different processes arising from the interactions. In order to do this, we can use the `itertools` python module, to programmatically explore possible local states. An example of this is presented in the slides.

3.6 TASK 10: Solid-on-solid crystal growth model

Another problem that can be studied with kinetic Monte Carlo is that of crystal growth. We will consider a very simple model of crystal growth: the Solid-on-Solid (SOS) model. The model we will consider only includes adsorption and desorption processes, neglecting diffusion. The adsorption rate is constant, but the desorption rate is affected by both the system temperature and by lateral interactions (details in the slides).

Interesting examples of use of kinetic Monte Carlo in studies of growth processes can be found in [9, 10].

TASK: Implement the SOS model in kmos. Simulate crystal growth for two different temperatures (350 K and 450 K), and observe the resulting structures in both cases.

3.7 TASK 11: Diffusion in the SOS model

Implement diffusion processes in the SOS growth model. Compare growth patterns in the model with and without diffusion. Consider also a model without desorption (only adsorption and diffusion).

3.8 TASK 12: Lateral interactions in the diffusion model

Recalculate the plots obtained in TASK 9 for the diffusion model that includes lateral interaction. Do this for different values of the lateral interaction strength.

3.9 TASK 13: Defects in the diffusion model

Extend the 2D diffusion model by adding a 'defect' species, that blocks diffusion. Generate initialization script that prepares the system with defects randomly located. Test the effect of the presence of defects in the current. Do this for different defect concentrations.

References

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