University of Bologna
Department of Physics and Astronomy
Master Degree in Science of Climate

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Radiation, Clouds and Climate - Module 2 NOTES

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Student
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Chapter 1

Part One: Atmospheric physical climatology

To investigate the general circulation of the atmosphere we need statistical operations. Consider the general variable $X(\lambda, \phi, z, t)$, the **time average** of x is:

$$\bar{x} = \frac{1}{T}\int_0^T x dt \tag{1.1}$$

1.1 Zonal average

Vertical Integral It is useful to understand the derivation from the mean $x' = x - x_0$ transient and $x^* = x - [x]$ eddy. What happens if I consider the flux of the variable? For example the flux of water vapor qv with q being the speed of the wind and q the concentration of water vapor (specific humidity). So let's consider x and y generic variables and their product xy:

$$\overline{xy} = \overline{(\bar{x}+x')(\bar{y}+y')} = \overline{\bar{x}\bar{y} + x'\bar{y} + \bar{x}y' + x'y'} = \overline{\bar{x}\bar{y}} + \cancel{\overline{x'\bar{y}}} + \cancel{\overline{\bar{x}y'}} + \overline{x'y'} = \underbrace{\bar{x}\bar{y}}_{\text{contribute of the mean circulation}} + \underbrace{\overline{x'y'}}_{\text{contribute of the transient}}$$

Note that

$$[xy] = [x][y] + \underbrace{[x^*y^*]}_{\text{contribute of the eddies}}$$

Statistically speaking, it's a covariance:

- positive $\rightarrow$ x and y covariate
- negative $\rightarrow$ x and y don't covariate
- zero $\rightarrow$ x and y aren't covariated

We have stressed that because there are phenomena that are present in the mean but they disappear if we do the time average or still remain.

1.2 The Observed General Circulation (GC)

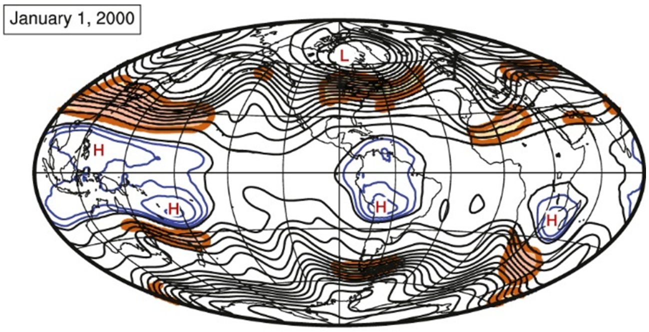


Figure 1.1: Z200 in contrasting season

In the figure 1.1, we see the geopotential height at 200 hPa ($\sim$ 8 km), upper troposphere, that is a proxy for pressure. This picture shows where are the winds: geostrophic approximation is good for large scales (Coriolis balances pressure gradient), and where are the strongest winds: at closed isobars very strong flow ($\sim$ 30 m/s)= jet stream. There are two anticyclonic ridges (H) and one through over Italy (L). If we take the average these structures disappear. Since in this graph it is january, we have summer in the SH $\rightarrow$ high pressure in the upper troposphere. In figure 1.2 we can notice the a southern Asian monsoon (high pressure on tibet), jet stream gets weaker and the transient activity too.

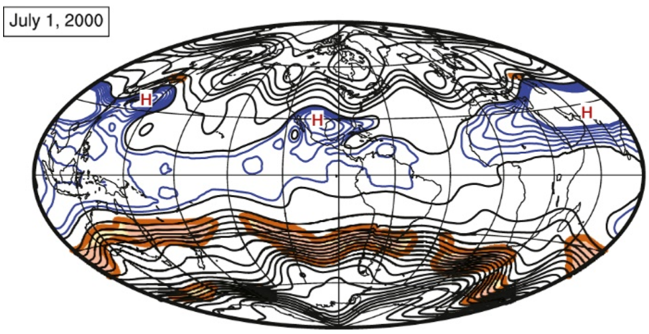
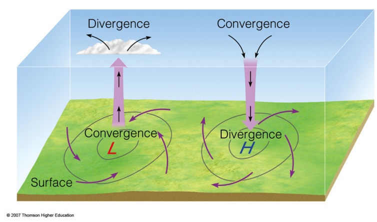


Figure 1.2: Z200 in contrasting season

In 1.3 the transient disappear, we see larger symmetry in the SH and ridging and throughing over America. These meandres are due to the presence of land masses that provide differences between SH and NH in tropics. Tropics have different weather with respect to the rest of the world: here there's a superposition of Rossby and Kelvin waves due to heating associated with land. In 1.4 the difference between location at the same λ (points over land are colder than ocean ones:)$\rightarrow$ different heat capacity.
Fig. 1.5 shows on average annually and seasonally the distribution of sea level pressure and near surface winds. The friction of the surface adds to the pressure force and Coriolis force (H vs L pressure zone). Because of geostrophic balance the friction of the surface acts on the pressure force, curving a bit the isobars;



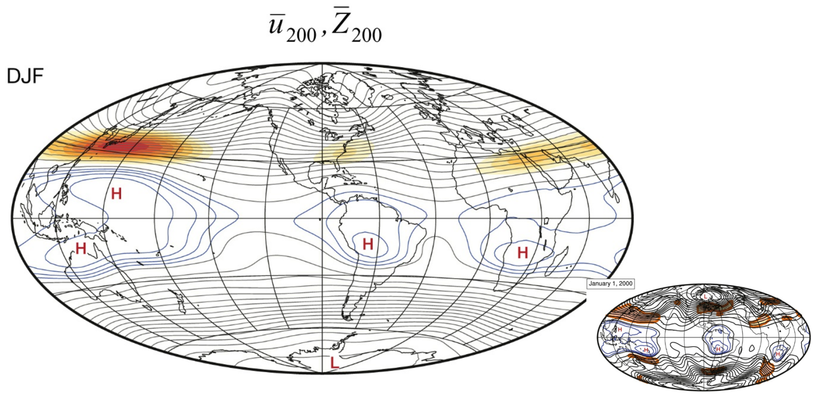


Figure 1.3: Time mean. Transient disappears; larger symmetry in SH; ridging and throughing over America

that's why you have a convergence on the Low pressure zones and a divergence on the Highs, as in fig.1.2.

Annual mean: average over the all months: the upper figure of 1.5: westerly (they come from the west, they go eastward) winds North Atlantic Pacifical High (same heaven subtropical high). In the Northen Hemisphere you have two semipermanent low pressure areas semipermanent Atlantic low, (semipermanent Icelandic Low). Subtropics semipermanent high pressure and semipermanent low pressure at high latitude forces the winds to be westerly. In the tropics winds are easterly: trade winds, in the Equator they tend to converge driven by high pressure systems. Where does the easterly turns into westerly? 30° is this boundary, constant as a consequence of the atmospheric angular momentum. Fig.1.6 is centered in the continental mass of Asia. During winter, winds blow from the continental mass towards the Indian Ocean, in summer they move in the opposite way: Somali jet towards Somalia. This large deviation is due principally to the Asian continental mass. NB: Seasonally, wind reversal means monsoon. The continent plays a very important role in shaping winds. Areas under the high pressures are called subtropical dry zones, where there's very little precipitations, over the ocean there is area with high precipitation, winds converge in the ITCZ (around 7°N) stable seasonally. DJF large amount of rainfall in the South of the Equator, while in the opposite season this area gets completely dry, the rainfall largely shifts northward. The seasonal shift of the tropical Asian monsoon (from Pakistan up to Korea) in the tropics means that you have areas that are dry; over the ocean the ITCZ does not move, in correspondence of the large continental masses, the rain follows the seasons: high in summer, and it shifts largely in terms of degrees up to 10m of rain per year.

If you look at extra tropics (30°N or S), there is another belt where precipitation is high (not high as tropics). Fig.1.7 shows better the continuous belt in extra tropics; this rain is associated with the fact that lots of cyclones form close to USA, meaning there are rainfalls completely different than Indian monsoon, these are **extra tropical cyclones**. These cyclones tend to form over preferential places and move over preferential zones, they are modulated seasonally: for example in winter these cyclones are more intense. The track where preferentially extra tropical cyclones tend to move (the reason why in England it rains a lot) is called the *extra tropical storm track*. Any circulation that is closed is a cyclone, they divided in two kinds:

- *extra tropical cyclones* associated with baroclinic instability, they are responsible for most of the rain and tend to move over extratropical storm tracks.
- *tropical cyclones* (also called hurricanes in North America and typhoons in Asia) associated with very warm SST and heat transfer to the atmosphere.

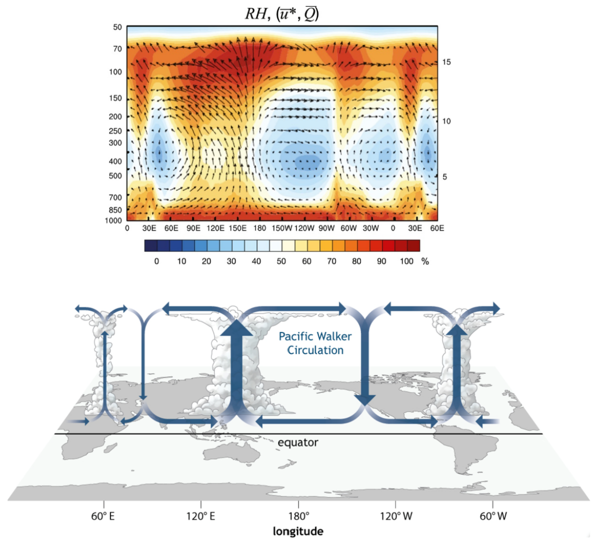


Figure 1.4: Skin Temperature surface

Due to extra tropical cyclone movement, you have a strong maximum associated with ITCZ in the equator and another maximum associated with extra tropical tracks as in Fig.1.7. Winds are fundamental in understanding the movement of water masses, oceans are the source of energy that keep hurricanes.

ITCZ intertropical convergence zone. Most people indicate it to be over the oceans and monsoon just over land because they are different: ITCZ stays constant, and monsoon changes over continuous lands shifting a lot between seasons. In fig. 1.9 we are in the middle of the troposphere. We see an up-welling and down-welling motion. Negative p coordinate velocity (ω) means an upwelling motion (upward motion) because height grows while the pressure decrease; positive omega down-welling motion makes sense as you're carring moisture away. We can say the mean vertical velocity in pressure coordinates ω is a proxy for precipitations as it correlates quite well with fig.1.7 in the tropics (areas of large precipitation related to negative ω) but not in the mid latitudes. This is because tropical rainfall are associated with a mean circulation that is persistent (all days uprising motion); in storm tracks we have cyclones continually reforming and replaced by anticyclones (downward motion), meaning that on average they cancel out (they are transient): the rainfall in the extra tropics is associated with a transient phenomena: averaging it goes to zero. Fundamental difference: in tropics precipitation is associated with mean circulation, in extra tropic precipitation is associated with transient: you don't see them in mean, you have to take quadratic terms like stand deviation in order to see them.

Until now we considered time means fixing some height (surface, 500 hPa, ...) without really focusing on how things vary with height. To analyse this we should look at 3D, a bit complicated, an easy way to look how things depend on height is to look zonally. Remember that zonal mean implies averaging along circles having the same latitude, by doing that you compress the longitudinal dimension and in a 3D field you get to work only with latitude and height. As we said the main properties of the atmospheric circulation at first order depend on latitude, they also depend on longitude but higher order.

Fig.1.10 shows the zonal mean of the time mean zonal wind (component along the parallels). Winds increase with height in the extra tropics and has a maximum value around the same 10 km: the two cores in the S and N hemisphere jet streams. In the stratosphere it is different: you don't have two westerly jets (u positive means it grows from West towards East), there is a stronge easterly jet in the polar *polar night jet* observed. Close to the surface the transition from westerly to easterly is preatty stable across the season and it is 30°. The jet in

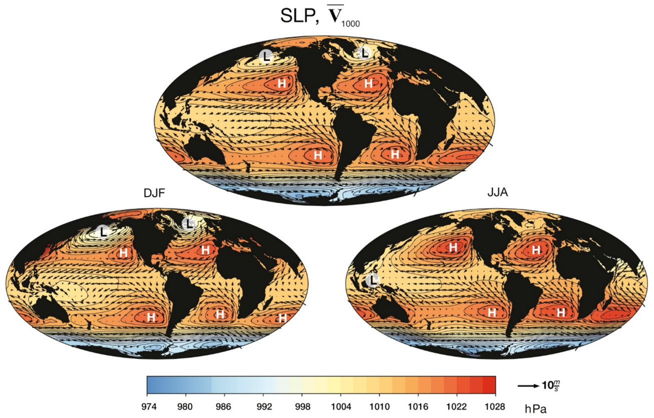


Figure 1.5: Distribution of sea level pressure.

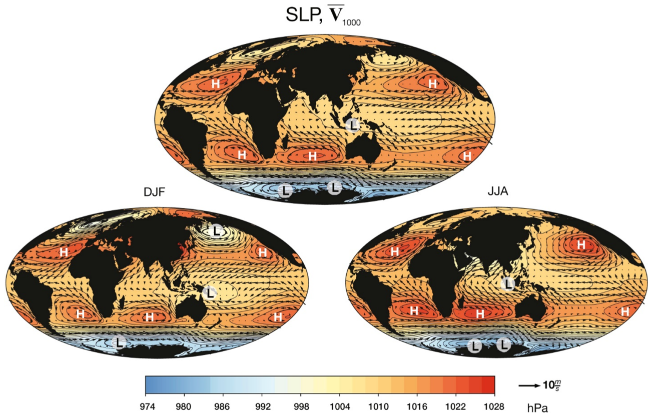


Figure 1.6: SLP centered in Asia

the winter is stronger in the NH, due to the thermal wind balance that relates the gradient of T at surface with the shear of the wind (large in winter than in summer).

Fig.1.11 shows the thermal structure of the atmosphere. You have average everywhere worldwide. In troposphere T decreases: this rate is the vertical lapse rate $\Gamma = -dT/dz$, about 6.5 K km^{-1}: similar to what the moist (or saturated) lapse rate predicts (dry lapse rate is steeper $\sim$ 10 K km^{-1}), because of water vapor (condenses when it rains) the real lapse rate is less. This means that on average the lapse rate in the global troposphere is controlled by the saturated lapse rate, in the tropics especially: lots of rainfall.

From fig.1.12, we see that temperature overall decreases when you go from the surface up to the tropopause and it decreases when you move from the tropical regions poleward. In the tropics temperature doesn't vary much, while in the extra tropics you have a large gradient $\leftrightarrow$ tropospheric jet streams are placed here: here there's the largest meridional temperature gradient and because of the thermal wind balance this is consistent with a vertical wind shear (wind grows very rapidly in the jet core where there's large thermal gradient).

Now let us look at where water vapor is in the atmosphere. The concentration of water vapor is very high near the surface and near the Tropics, it drops moving upward. The zonal mean of specific humidity closely mirrors the distribution of temperature. Where is warmer you have the highest concentration of water

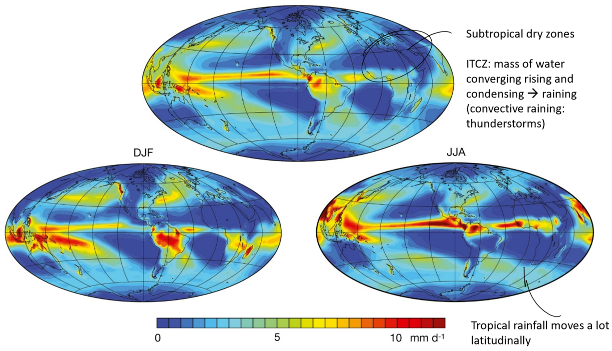


Figure 1.7: Annual mean of precipitation. How much precipitation falls on average on a year (or season) for every grid point.

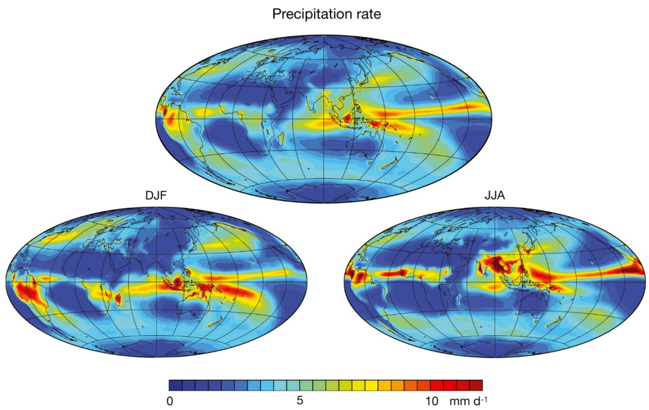


Figure 1.8: Annual mean of precipitation centered in Asia

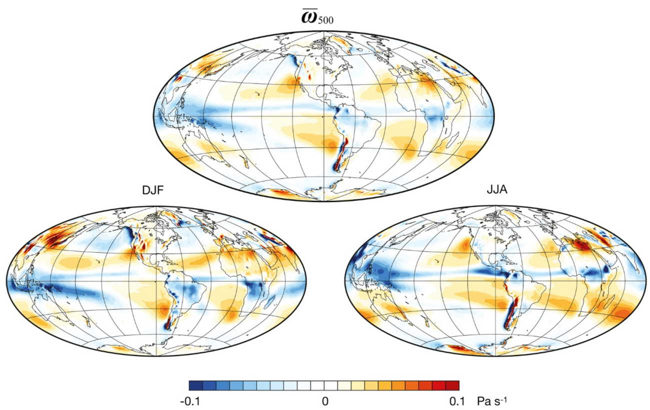


Figure 1.9: Vertical pressure level at 500 hPa: ω is the vertical velocity in p-coordinates.

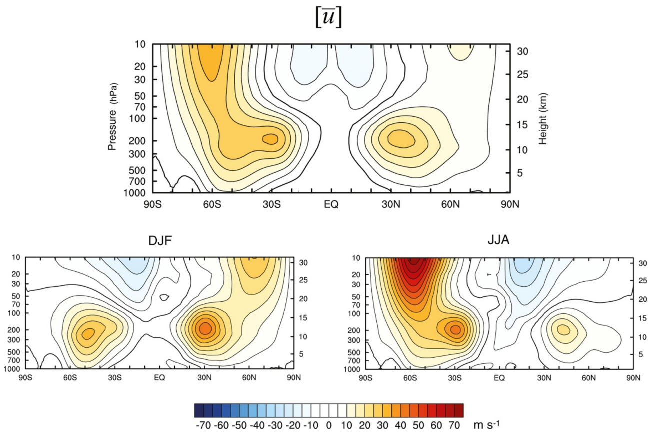


Figure 1.10: Zonal mean of the time mean zonal wind.

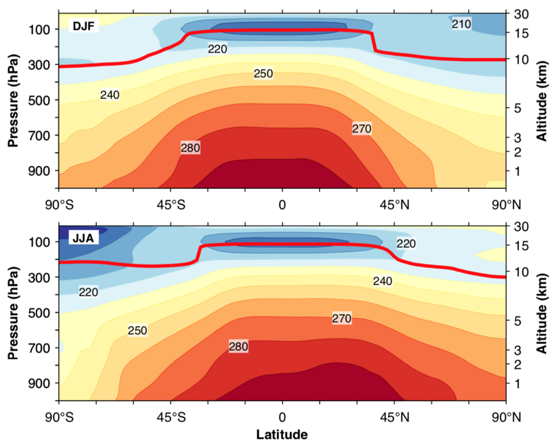


Figure 1.11: Thermal structure of the atmosphere

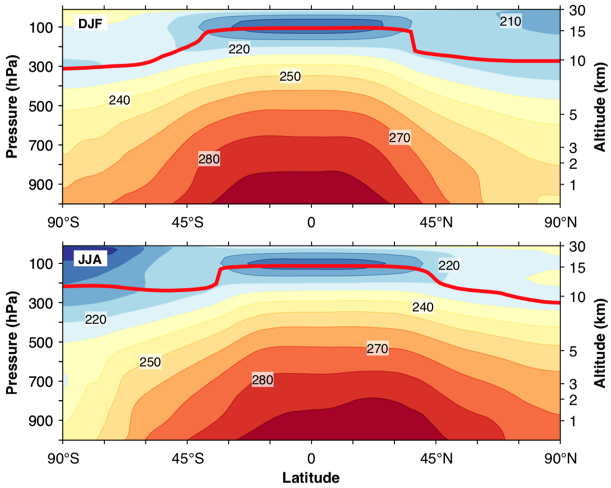


Figure 1.12: Vertical structure of temperatures in the zonal mean.

vapor. The relationship between temperature and humidity is expressed by the Clausius-Clapeyron relation,

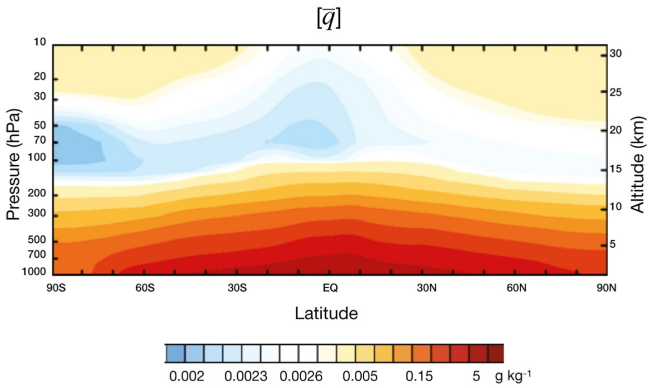


Figure 1.13: Zonal mean of specific humidity: where water vapor is in the atmosphere.

which expresses for each degree of warming how much water vapor we will have. Related to an increase in precipitation extremes. The concentration of water vapor drops as you move upward and upward. Clausius-Clapeyron relation relates the concentration of water vapor to temperature:

$$\frac{dq_s}{q_s} \simeq \left(\frac{L}{R_v T^2}\right) dT \tag{1.2}$$

where on the left there's the relative change of specific humidity, the fraction on the right is the relative latent heat proportional to a variation in temperature and to R_v gas constant for water vapor, with $L \simeq 2.5 \quad 10^6$ J/K, $R_v \simeq 461$ J/(K kg) and $T \simeq 300$ K $\simeq 20$ °C, we obtain a proportional factor $\alpha \simeq 7\%$, meaning that for every 1°C warming, the relative humidity increases of 7%.

This plot shows zonal mean time mean meridional velocity. Around the Eq there's an area with large ascent. Two meridional circulations associated with Hadley cells. There's a better way to see: the meridional stream function.

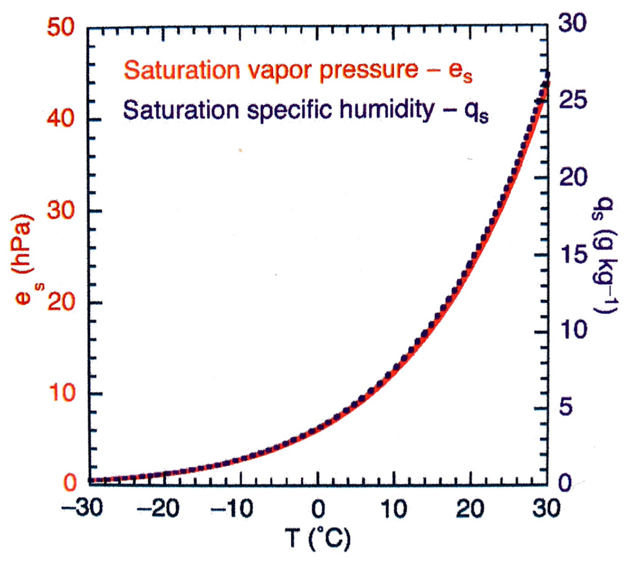


Figure 1.14: Exact solution of the Clausius-Clapeyron equation. NB in very cold conditions the atmosphere is dry.

1.2.1 Stream function

On fig.1.16 lines of constant meridional stream function. In gen the flow is along lines on contant streamfunction. A streamfunction is the best way to describe meridional mean. In general you can introduce a streamfunction in a 2D flow if the divergence of the flow is zero.

$$\text{if } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Leftrightarrow \exists \psi(x,y) \text{ such that } u = \frac{\partial \psi}{\partial y} \quad v = \frac{\partial \psi}{\partial x}$$

note that the condition that the divergence of the flow is zero corresponds to the incompressible flow approximation. Hence, the streamfunction simplifies the flow and it gives the direction of the flow. The latter is because as $\nabla\psi \cdot (u,v) = 0$ then the gradient is perpendicular to the wind, meaning that on the lines of constant ψ the wind will always be perpendicular to the flow $\leftrightarrow$ the flow is always along the lines of constant ψ. Notice that $\psi_0 - \psi_1$ describes the mass flux of air in that region:

$$\int_{y_0}^{y_1} \rho u dy = \rho \int_{y_0}^{y_1} \frac{\partial \psi}{\partial y} dy = \rho(\psi_1 - \psi_0)$$

meaning that the largest ψ the largest is the mass transport. In these considerations we're taking the zonal mean: we consider only 2D without the vertical component; we cannot define the streamfunction for the atmosphere but we can for the zonal mean of the atmosphere. Meridional ψ is useful to describe the three cells circulation:

$$\nabla_h \cdot \vec{V}_h + \frac{\partial \omega}{\partial p} = 0$$

that is the continuity equation in p-coordinates, in spherical coordinates it can be translated into:

$$\frac{1}{R_E \cos\varphi} \left[\frac{\partial u}{\partial \lambda}\right] + \frac{1}{R_E \cos\varphi} \frac{\partial}{\partial \varphi} [v \cos\varphi] + \frac{\partial [\omega]}{\partial p} = 0$$

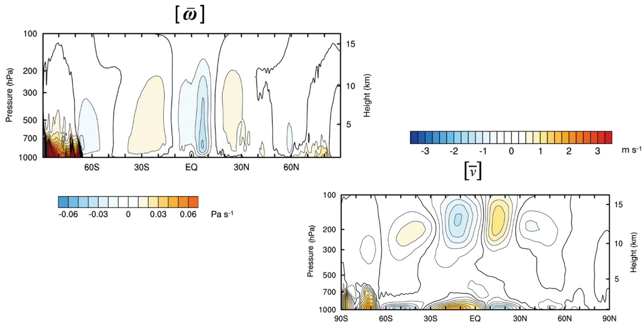


Figure 1.15: Mean meridional circulation

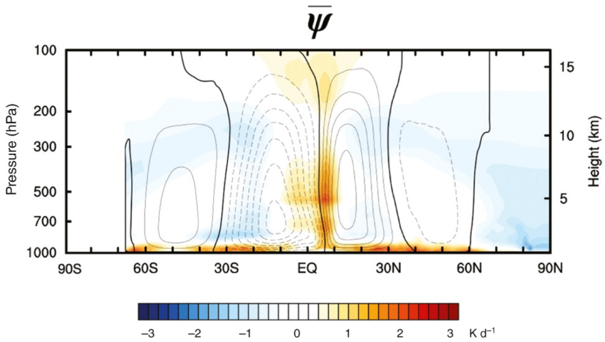


Figure 1.16: Meridional mass streamfunction (contours) and diabatic heating rate (shading)

where we took the zonal mean, recall:

$$[\quad] = \frac{1}{2\pi}\int_0^{2\pi} d\lambda$$

since $\frac{1}{2\pi} u\big|_0^{2\pi} = 0$ (over a circle),

$$\frac{\partial}{\partial \varphi}\cos\varphi[v] + \frac{\partial}{\partial p}\left(R_E\cos\varphi[\omega]\right) = 0 \Leftrightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

so that we can introduce a streamfunction such that

$$\cos\varphi[v] = \frac{\partial \psi_M}{\partial p}$$
$$R_E\cos\varphi[\omega] = -\frac{\partial \psi_M}{\partial \phi}$$

that is called the *meridional streamfunction*

$$\psi_M(\phi, p) = \int_{p_s}^{p} \cos\phi[v]dp \tag{1.3}$$

Tropics associated with ITCZ. We described the mean meridional circulation with the meridional streamfunction.

$$\psi_M(\phi, p, t)$$

is formed like a 2D divergence equation. The streamfunction is useful as it defines the stream of the flow (how wide the Hadley cell is) and also the strength of the overturning meridional circulation. In the scientific literature it is used to define the edge of the Hadley cell in the middle of the troposphere (500m). You find the latitude of the limit of the cell where the $\psi_M = 0$: people defined this latitude as the edge of the Hadley cell. This is important because the weather and climate in the Tropics (tropical circulation) is always below the Hadley cell.

Research question. You have to find out if the Hadley cell is moving foreward: under the effect of global warming the cell expands. Overturning tropical circulation is getting weaker due to global warming: you can take the difference of the streamfunction $\rightarrow$ as it's streamflow over two points you could take the difference between the two points. The maximum of the Hadley cell equals the mass flow transported: how strong or weak it is. If the Hadley cell expands the Tropics get closer to the poles. Where there's the downwelling branch of the Hadley cell we have the dry zones (while upwelling we have precipitations). If the Hadley expands, the dry zones expand poleward. Mean meridional circulation, function of (ϕ, p, t) often we look it as annual mean but we can look it as seasonal mean. Longitudinally the mean meridional circulation can have different shape.

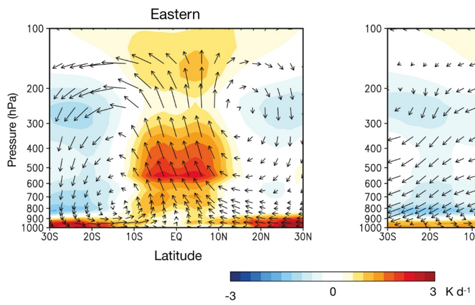


Figure 1.17: Contributions of the eastern and western hemispheres to the annual mean pattern of 1.18. The streamfunction cannot be computed for partial zonal averages, so the overturning circulation is represented by vectors representing the average of $\bar{v}$ and $\bar{\omega}$ over the respective hemispheres. The narrow chimney of ascent in the western hemisphere corresponds to the ITCZ.

The annual mean Hadley cell is quite symmetric seasonally due to one cell larger than the other (smaller and weaker).

Seasonal evolution of the Hadley cell There's one cell much stronger that goes as uprising in the SH and upwelling in the NH.

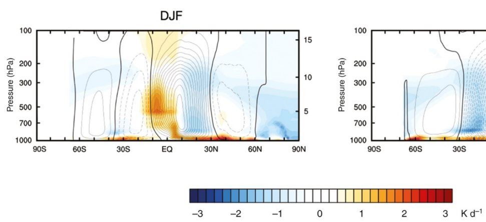


Figure 1.18: Meridional mass stream function (contours) and diabatic heating rate (shading), seasonal variation

- **Winter Hadley cell** is the strongest one. It has part of it in the northen hemisphere. The upward branch is NH in the summer (rising in the winter); down-welling motion in the winters. It dominates over the two. It's the only one of the two (in fig.1.18) that has a component in the Northen Hemisphere. Uprising branch always in the summer hemisphere.

- The weaker one has uprising and downward branch in the summer. It gets weaker and narrower.

Seasonally these two change a lot. These variations are associated with monsoons. They start when there is a strong rearragement of the Hadley cell (April, May). These variations are associated with monsoons in the Northen Hemisphere in summer (we have uprising and it rains) with down-welling in the other hemisphere. From 20 N to 10 S, Hadley cell shifts and the winter cell comes forward: understanding this shift is crucial to understand monsoons.

Looking at the annual mean 1.16 the monsoons start when there's strong rearragement: representing a shift northward. Fig.1.16 shows tjhe climatological mean meridional mass streamfunction and diabatic heating rate per day (colored shading). The zero contour is thickened. The circulation is clockwise around the maxima and counterclockwise around the minima in ψ field. The Hadley cells flank the band of tropical ascent, marked by the narrow maximum in Q. They are flanked by much weaker Ferrel cells centered in midlatitudes. The mean meridional circulation is associated with monsoons, rainfalls and position of the dry zones.

Research question: understanding how MMC changes with global warming is crucial. Climate models project on expansion of Hadley cell. Wakening of the overturning meridional circulation also affects other circulation, like Walker circulation (that is East-West), where you have down-welling in the Pacific.

Recall that condensation heats the atmosphere, while drop evaporating cool the atmosphere. Fig.1.18 shows that a large amount of heat is release because there's condensation. Because of convection the atmosphere is heated due to thunderstorms. Where the atmosphere is cold there's emission of IR radiation. You can conclude that the atmosphere is heated up in the middle of the troposphere and tropical rainfalls where you have monsoons in tropical convergence zone and near the surface (fluxes of sensitive heat and latent heat).

1.2.2 Eddies and Transients

How do we identify where the transient acts? There are pars of the world where circulation is zero. If we look at a single day we'll see a different structure evolving that has both vertical and meridional component $\leftrightarrow \quad \bar{v} = 0 \quad v' \neq 0$. We can look at the variance $\overline{v'^2}$ or the standard deviation $\sigma(v') = \sqrt{\overline{v'^2}}$ that can be different from zero even in the case of $\overline{v'} = 0$. This is a good proxy to identify where the transient is acting and how strong.

Time mean of geopotential height sigma figure. $\sigma(v'_{200})$ meridional wind at 200 hPa in a single day: there are two regions in the extra tropics where it is higher: stormtracks. At tropics circulation is more stable in time (a lot of transients = very large $\sigma \rightarrow v' \neq 0$).

Stationary waves Fig.1.19 provide a global survey of the amplitude of the DJF stationary waves in the climatological mean meridional wind components on latitude circles. There's a maximum in $\sigma(\bar{v}^*)$ at 52 °N. Fig.1.19 provide a global survey of the amplitude of the DJF stationary waves in the climatological mean meridional wind component $\bar{v^*}$ on latitude circles.

Transient Zonally Symmetric Variability Seasonal variations in the climatology of the zonal ...

Large nonseasonal variability is observed. Fig.libro shows the standard deviations of $[u]'$ and $[v]'$ about their respective seasonally varying climatological means. They are computed on the basis of daily data and thus include variability on timescales ranging from days to years, but since the reference state is seasonally varying, it specifically excludes the variance associated with seasonality. In general, $\sigma([u]') >> \sigma([v]')$: just as the kinetic energy of the climatological mean zonally symmetric circulation is dominated by the zonal wind component, the same is true of the variations about that climatology. That of $\sigma([v]')$ is so small reflects the inhibition of the meridional motion of zonally symmetric rings of air on a rotating planet, a constraint imposed

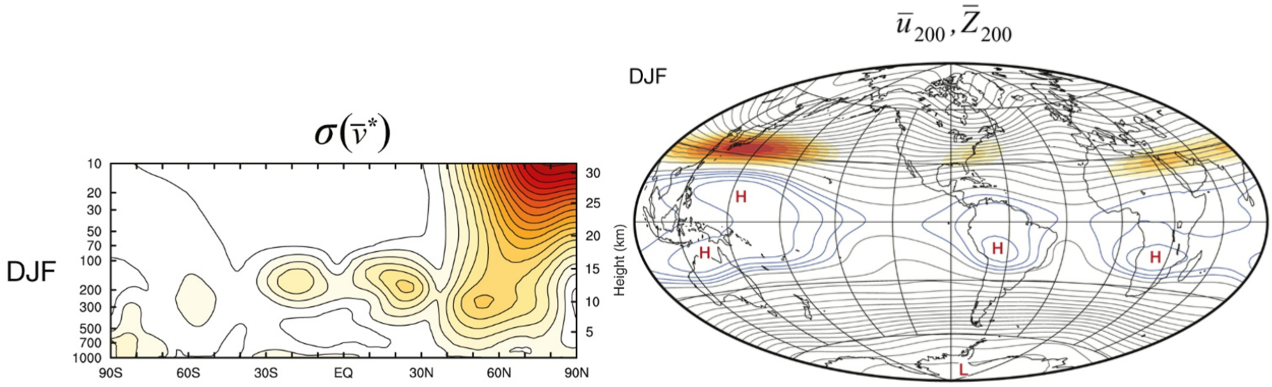


Figure 1.19: $\sigma(\bar{v}^*)$, climatological mean longitudinal standard deviation averaged around latitude circles in the stationary waves. The zonally average reference field for computing the standard deviation in each panel is the seasolan mean, not the annual mean.

by the conservation of angular momentum. The only notable feature in the distribution of $\sigma([v'])$ is the pair of equatorial maxima, one in the boundary layer and the other just above the 200 hPa level. Anomalies in $[v]$ at these levels tend to occur out-of-phase with one another (not shown). These features are associated with nonseasonal meridional shifts in the Hadley cell that mirror the seasonal shifts in fig.1.18, but occur on a wide range of timescales.

Transient Eddies After the variance associated with the stationary waves and the nonseasonal zonally symmetric variability is accounted for, there remains the variability associated with the transient eddies, which are neither anchored in place longitudinally nor zonally symmetric. It is the only kind of eddy-related variability that appears on a rotating planet with zonally symmetric, temporally invariant boundary conditions and external forcing. The distinction between all transients and transient eddies is subtle and has often been ignored in the literature. For example in extra tropical latitudes the transient and transient eddy components of the v field are virtually indistinguishable. Accordingly, we will denote the transient eddy component of v simply as v' rather that v'^*. Notable exemples are the extratropical cyclones and the so-called baroclinic waves in which they are embedded. The meridional cross section of $\sigma(v')$ shown in fig.1.20, is dominated by broad maxima centered ad $\sim 50°$N/S, just above the 300 hPa level. The transient eddies are stronger that the stationary waves fig.1.19 by about a factor of two. At higher levels in the stratosphere, the variability peaks at higher latitudes, around the periphery of the polar cap regions, where it is roughly comparable to that associated with the stationary waves at the same level. The corresponding distribution of zonally varying $\sigma(v')$ at the 200 hPa level fig.1.21 confirm that the upper tropospheric, midlatitude maximum in transient eddy amplitude is a robust feature that is present at almost all longitudes. We will refer to this feature as the midlatitude "*storm track*".

Remember: if I have a through there will be a cyclone developing under it.

For each single day you can work out the deviation and square time mean and variance up to 200 hPa; you'll see that there are two regions on the extra tropics where the variance is larger: they corresponds to storm tracks. Circulation in the tropics is much more stable. These waves that form and move around mean a lot of transient, reflected in very large standard deviation in the mean meridional wind almost close to zero (isobars are almost symmetric).

Research question: how storm tracks change under global warming. Climate models suggestions show that extra tropical storm tracks will shift poleward with global warming. How do people identify this? Working out standard deviation of meridional wind, you can also other values such as vertical wind: that's a quantity that if you average over time in extra tropics goes to zero.On days with high pressure, the vertical wind is downward,

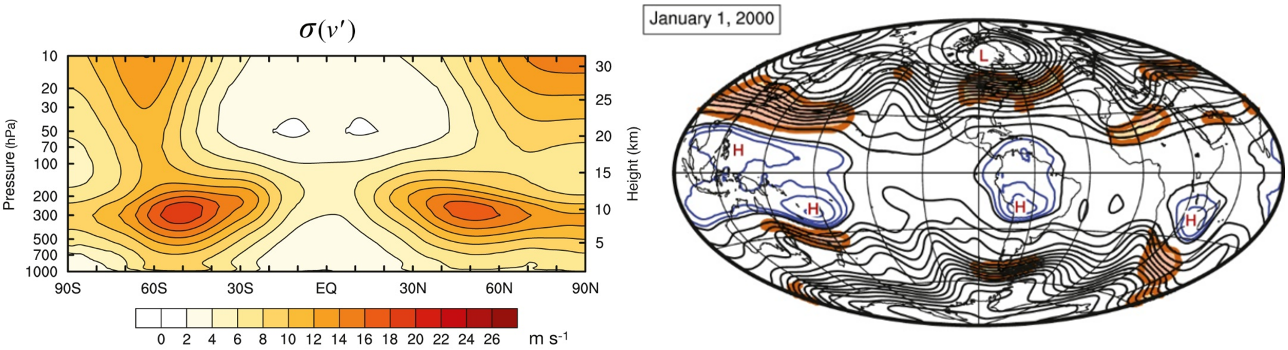


Figure 1.20: Zonally averaged temporal standard deviation of the meridional wind component v based on daily data, referred to as the *transient*, for all calendar months, computed at each grid point and level. The temporal standard deviations about the seasonally varying climatological means $\sigma(v^*)$ and $\sigma(v'^*)$ are virtually indistinguishable, from which it follows that these patterns can be interpreted as representative of the transient eddies. The maxima in the variance at temperate latitudes are associated with the *storm tracks*.

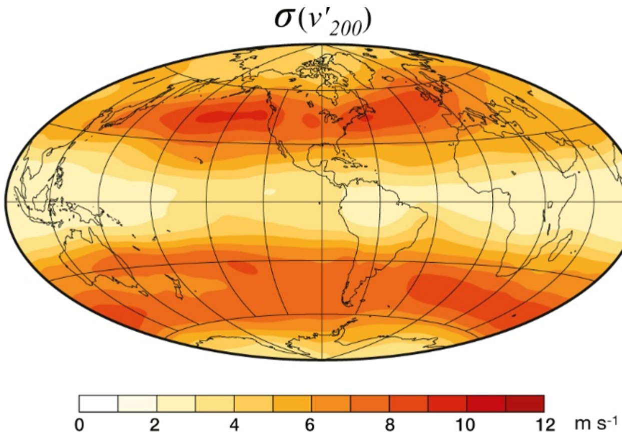


Figure 1.21: Temporal standard deviation of v' at the 200 hPa level based on year-round data.

whereas, in the presence of a cyclone (which brings rain), the vertical wind is upward. In regions with both high and low pressure, the average vertical velocity tends to zero. To capture the transient events, you can use the standard deviation of ω, which is one of the best proxies for identifying when it rains (see Fig. 1.22). Rain primarily forms in the first few kilometers of the atmosphere. Transients in the vertical are particularly useful for capturing what occurs during tropical cyclones. The $\sigma(w'_{700})$ represents the vertical velocity at 700 hPa, approximately 3 km above the surface, where condensation occurs. In this figure, we clearly observe the North Atlantic and North Pacific storm tracks.

- The mean vertical velocity at 500 hPa ($\overline{w}_{500}$) correlates well with precipitation, but it does not capture transient events.
- In high latitudes, rainfall is associated with transients.

Those studying monsoons tend to focus on the mean flow, while those studying transients focus on variations in the mean flow.

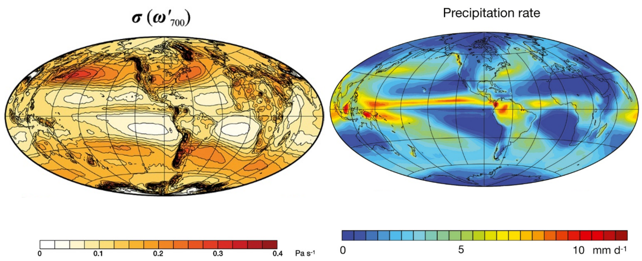


Figure 1.22: Correlation between sigma and precipitations

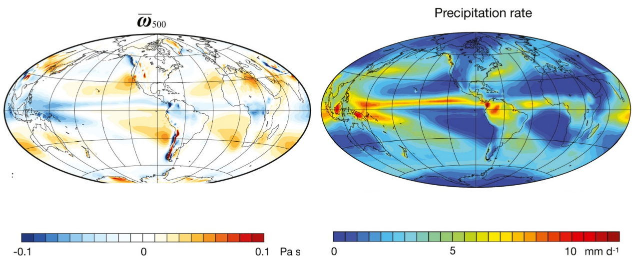


Figure 1.23: Correlation between ω and precipitation

When examining the mean precipitation, we observe that the spatial distribution of vertical velocity aligns well with the overall structure of precipitation, except for the ITCZ. Precipitation in high latitudes is not linked to the mean circulation but rather to transient events. When it rains in high latitudes, it is typically due to the presence of a cyclone, not because of high pressure.

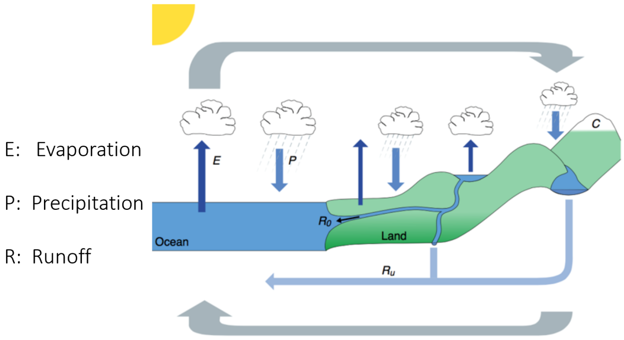
When we examined the mean vertical velocity, we observed its correlation with tropical rainfall, although it does not reveal storm tracks. We can conclude that tropical rainfall in the ITCZ or monsoons is primarily linked to the mean meridional circulation (which shows a strong correlation) and the Hadley cells. In contrast, precipitation in the extratropics (storm track region) is not associated with the mean circulation. Instead, it is tied to transient phenomena, as rainfall in the extratropics is linked to transient events rather than persistent patterns.

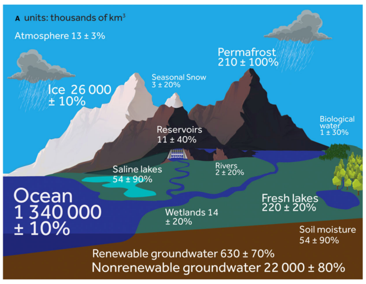
1.3 The water cycle

Transport is influenced by the atmospheric general circulation. Evaporation (or evapotranspiration) is a source and precipitation is a sink. Surface runoff (rivers and channel) or underground runoff (groundwater). Atmospheric branch and land branch of the water cycle connected by processes of precipitation and evaporation. Atmosphere contains only 0.001% of water: it's the smaller reservoir but the most dynamic in terms of transferring water from one place to another. Atmosphere is moving water vapor around: the fluxes related to it are big. Important variables:

$$\text{specific humidity:} \quad q \quad [\text{g/kg}]$$

Mostly concentrated in the few km of the atmosphere and it's higher is in the Tropic: the warmer atmosphere, the more water vapor it can contain. Clouds are made of droplets or ice in the pole. Water is also contained in clouds: $q + q_l + q_i$ specific concentration of water vapor, liquid water and solid water, we will just consider





Reservoir	Percent (%)
Oceans	97.39
Cryosphere	2.010
Underground water, Soil moisture	0.580
Lakes and rivers	0.020
Atmosphere	0.001
water (total)	100.0
drinking water	2.60

Allan et al., Ann. N.Y. Acad. Sci., 1472 (2020)

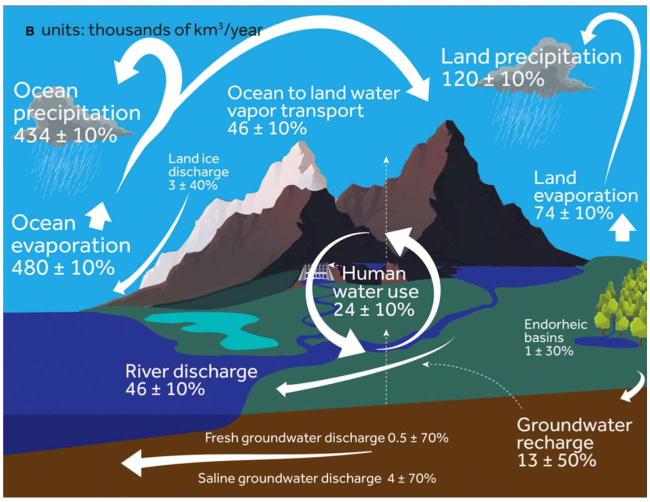


Figure 1.24: Water fluxes

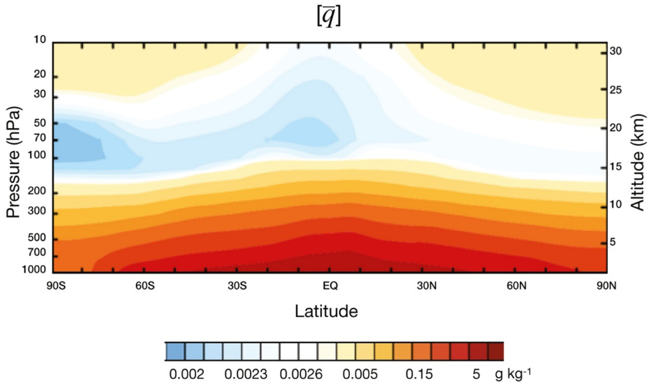


Figure 1.25: Specific humidity: concentration of water vapor in the atmosphere.

q: we can do it because the gas component accounts for most of all water in the atmosphere. Clouds contain either q_l or q_i, if you consider $\frac{\text{mass of clouds}}{\text{mass of all water on the atmosphere}} = \frac{1}{300}$ clouds are important for radiative balance but in terms of mass of water that they contain they are very negligible.

Precipitable water/total column water vapor (TCWV) is just the vertical mass integral of q:

$$W = \int_{p_s}^{0} -\frac{dp}{g} q \quad \text{kg/m}^2 \tag{1.4}$$

it says how much water vapor you have inside an atmospheric column of section 1 m^2, it gives an indication locally on how much water vapor you have (and could be precipitation, usually it precipitates less).

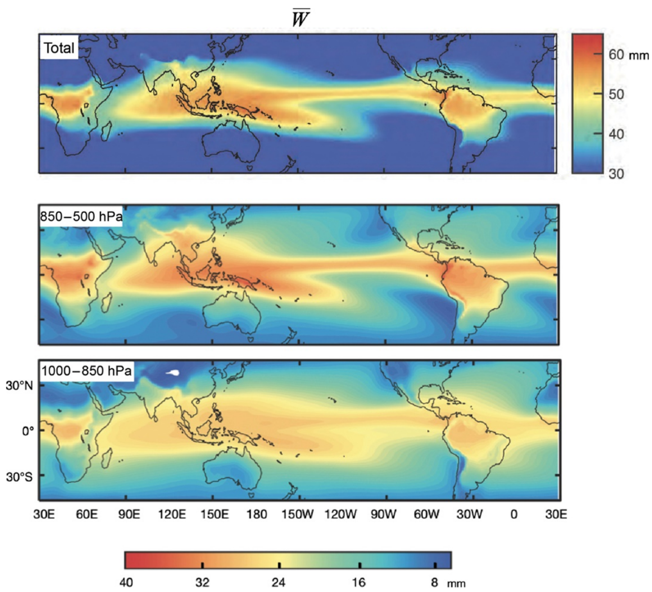


Figure 1.26: Most of the water in atmosphere is concentrated in the Tropics

Vertically integrated vapor flux (IVF) A water vapor flux is defined as the movement of water vapor carried by the wind: $q\mathbf{v} = (qu, qv, qw)$ specific humidity times the wind, in general it is a 3D quantity: how water vapor is transferred zonally, meridionally and vertically. The IVF is:

$$\text{IVF} = \int_{p_s}^{0} -\frac{dp}{g} q\mathbf{v} \tag{1.5}$$

that is a 2D quantity, meaning that the amount of water vapor transported is moved horizontally through the atmosphere. You integrate over the vertical: we're interested in what happens horizontally. In fig.1.27 Vaia

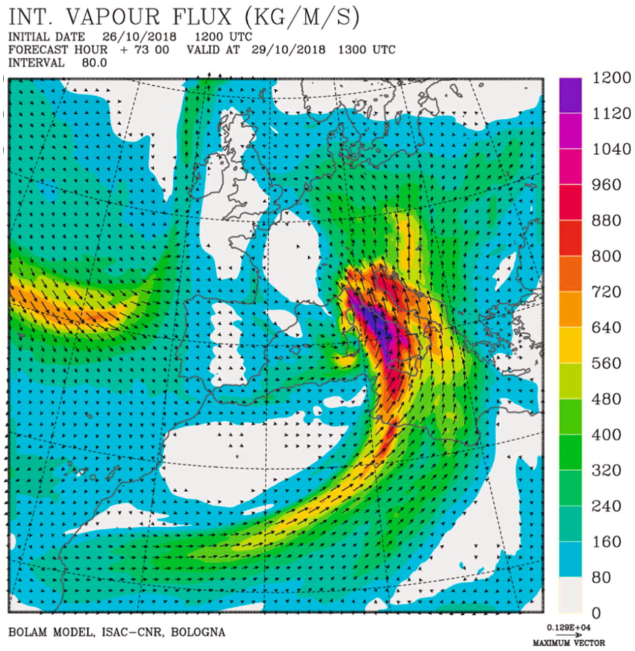


Figure 1.27: IWVF during Vaia's high precipitation event

was a strorm in extra tropical cyclone: strong wind and large precipitation. The arrows show the IWVF, where you have a lot of water vapor in the atmosphere, there was a river of water vapor transported over the Atlantic converging. The atmospheric circulation on that day was so that a lot of water vapor was transported. IWVP is a very important diagnostic tool for weather forecasting. *Atmospheric rivers* are large transport of water vapor from the Tropics to the extra-tropics, they stretch seen as interactions between extra-tropical systems, and they typically impact the West Coast of the USA.

In order to see where it's raining, you take the divergence $\nabla \cdot (\text{IVF})$ because divergence tells you where you have a sink or a source: negative value means it's raining. In situations in where the flow is converging somewhat, the inside area will have a negative divergence (convergence); while positive divergence is what divergence literally means. When you take IVF, you work out the divergence: piling up water vapor, it cannot disappear due to conservation of mass: you'll have large precipitation. Also, where the field is decreasing you'll have a convergence (negative derivative) you bring in more than you bring out. In a 3D field, divergence of the hor component where it's negative upwelling, positive down-welling, in this case we integrated over the vertical hence it's a 2D field, the only way to get away is by rain.

$$W = \int_{p_s}^{0} -\frac{dp}{g} q$$

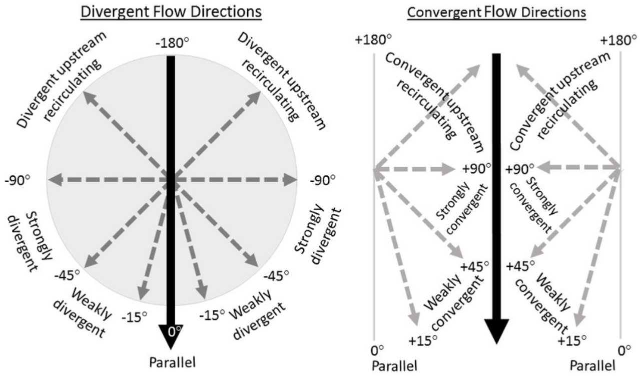


Figure 1.28: Convergence and divergence

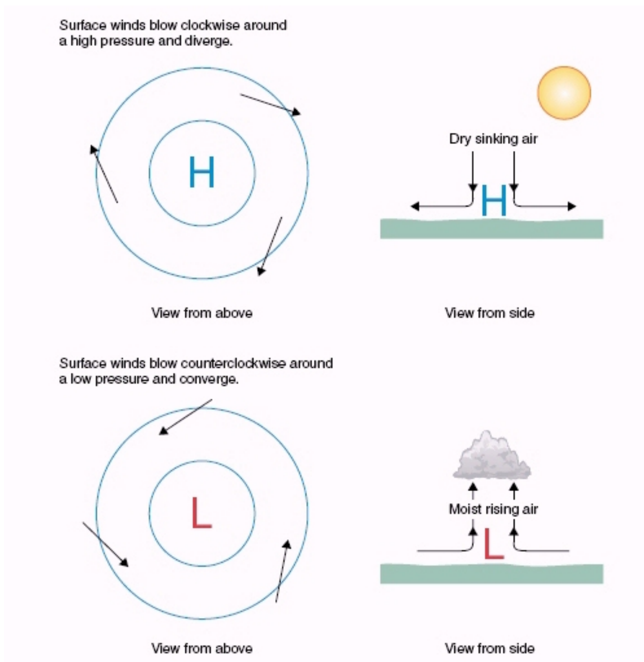


Figure 1.29: Convergence recall

water vapor in the atmosphere. Vertical integrated water vapor flux:

$$\mathbf{Q} = \int_{ps}^{0} -\frac{dp}{g} d\mathbf{v}$$

how water vapor is moving overall

P how much water is going out from the atmosphere, water is like a sink for the atmosphere: when it rains atm is losing water vapor E evapo-transiparion (over land evapor- as it's mediated by land). I is a source of water vapor for the atmosphere.

We will demostrate balance equation for the vertical integrated water vapor balance eq in the atmosphere

$$\frac{\partial W}{\partial t} + \nabla_h \cdot \mathbf{Q} = E - P \tag{1.6}$$

where the divergence is a 2D divergence. The time derivative f the tot amount of water vapor in the water column is give by three terms:

$$\frac{\partial W}{\partial t} = \underbrace{(E-P)}_{\text{sources/sinks}} \underbrace{-\nabla_h \cdot \mathbf{Q}}_{\text{transport of } q \text{ associated with } \mathbf{v}}$$

differential eq: everything is true at any point.

You may now understand why E is a source and P is a sink. The sign of divergence depends on the specific vertically integrated water vapor flux. In general, when

- $\boldsymbol{\nabla} \cdot F > 0$ it means something is leaving my field
- $\boldsymbol{\nabla} \cdot F < 0$ you have convergence to my field.

applying this principle to Q, in the horizontal plane, regions where the horizontal divergence of Q is positive, the flow (Q is associated with v) tends to carry water vapor away to certain regions. Carrying water vapor away you're emptying that region: contributing negatively in the balance: decreasing water vapor in that column. Whereas, converging water vapor in a specific region, in the balance it contributes positively: increase of water vapor in a given column in time. W can change in time either because locally it's raining (P) or evaporation (E) or because the flow is carrying water vapor in or out.

Let's consider a finite region $\mathcal{S}$ (instead of infinitesimal point) and we integrate over that region. $\partial\mathcal{S}$ defines the border of the region:

$$\int_{\mathcal{S}} \frac{\partial W}{\partial t} = -\int_{\mathcal{S}} \nabla_h \cdot \mathbf{Q} d\mathcal{S} + \int_{\mathcal{S}} (E-P) d\mathcal{S}$$

$$\frac{\partial}{\partial t} \int_{\mathcal{S}} W d\mathcal{S} = -\int_{\mathcal{S}} \nabla_h \cdot \mathbf{Q} d\mathcal{S} + \int_{\mathcal{S}} (E-P) d\mathcal{S}$$

because the region doesn't vary in time

the first one is the time derivative of the total amount of water vapor in that region (W is define per meter squared). The last term is just what's the tot amount of evaporation from land or ocean into that region and the amount of precipitation. Let's analyze the meaning of

$$-\int_{\mathcal{S}} \nabla_h \cdot \mathbf{Q} d\mathcal{S}$$

using Gauss theorem (from a 2D integral to a 1D integral):

$$\int\int_{\mathcal{S}} \boldsymbol{\nabla} \cdot \mathcal{Q}\mathcal{S} = \oint_{\mathcal{S}} \mathcal{Q} \cdot \hat{n} dl \tag{1.7}$$

Int od divergence is equal to the flux. The scalar product gives how much goes out (Q and $\hat{n}$ are in the same direction or not). Hence,

$$\frac{\partial}{\partial t}\int_{\mathcal{S}} W d\mathcal{S} = -\oint_{\mathcal{S}} \mathcal{Q} \cdot \hat{n} dl + \int_{\mathcal{S}} (E-P) d\mathcal{S}$$

how much water vapor is transported inside this region. divergence associated with a flux inside or outside the flow.

Note that in large scale dynamics we don't consider it. More important is the balance eq of q. We want to determine $\frac{Dq}{Dt}$ (Lagrangian derivative). It tells you how that something changes following the parcel, sitting over the parcel at each instant you can check how much q stays in the parcel, this rate of change is what we call lagrangian. Whatever can make q change in the motion:

- Condensation. Droplets or rain (sink term) that can happen inside the parcel.
- droplet falling in the parcel, a part of it can evaporate (source). Part of the rain re-evaporates before it can reach the ground.
- The parcel goes very close to a liquid surface. Physically there will be diffusion of molecule of water vapor. Some molecules will just diffuse upwards, meaning that close to liquid surfaces diffusion of water vapor occurs. You can schematize it with a flux $\mathbf{J}$ found near the surface, usually vertical (one component). We're talking about molecular diffusion. NB eddy diffusion is the consequence of turbulence, lots of vortexes. The net effect of all eddies is acting like molecular diffusion: still transfer of water vapor above, that explains latent and sensible heat we found at surface, they are determined by turbulent diff: surf warm atm cold and turbulence kicks off. Transpor prop from surface to the atm. $\mathbf{J}$ means a diffudive water vapor flux, the nature of it can be either molecular or turbulent. In the first few mm you have molecula, then in the first 100m you have turbulent diffusion. $\mathbf{J}$ is again a flux with a divergence, it can change the content of water vapor: $\boldsymbol{\nabla} \cdot \mathbf{J}$: "convergence of diffusive water vapor".

$$\frac{Dq}{Dt} = S(q) \tag{1.8}$$

$$S(q) = e - c - \frac{1}{\rho}\boldsymbol{\nabla} \cdot \mathbf{J} \tag{1.9}$$

This is one of the starting equations. A passive tracer is $S(q) = 0$ substance can be traced only flow, it doesn't undergo phase transitions. However, diff water vapor fluxes, condensation, and evaporation can act on the change of water vapor. We now want to go from (1.8) to (1.6). That is, you have to move a Lagrangian derivative. In general, when you have a Lagrangian equation and you couple it with a continuity equation (in p-coordinates so we can split vertical and horizontal):

$$\begin{cases} \frac{DA}{Dt} = F \\ \nabla_h \cdot \mathbf{v}_h + \frac{\partial \omega}{\partial p} = 0 \end{cases} \Leftrightarrow \frac{\partial A}{\partial t} + \nabla \cdot (\mathbf{v}A) = F \tag{1.10}$$

Therefore, with $A = q$:

$$\frac{Dq}{Dt} = \frac{\partial q}{\partial t} + \underbrace{\mathbf{v}_h \cdot \nabla_h q + \omega\frac{\partial q}{\partial p}}_{\nabla_h \cdot (\mathbf{v}_h q) - q\nabla_h \cdot \mathbf{v}_h + \frac{\partial}{\partial p}(\omega p) - q\frac{\partial \omega}{\partial p}} = \frac{\partial q}{\partial t} + \nabla_h \cdot (\mathbf{v}_h q) + \frac{\partial}{\partial p}(\omega q) - q\cancel{\left(\nabla_h \mathbf{v}_h + \frac{\partial \omega}{\partial p}\right)}$$

hence,

$$\frac{\partial q}{\partial t} + \nabla_h \cdot (\mathbf{v}_h q) + \frac{\partial}{\partial p}(\omega q) = S(q)$$

we demonstrated that via cont equation we have the form of Eulerian derivative that equals the Lagrangian. To arrive at (1.6), we need to take the vertical integral. When you want to study the global scale, we don't care about the vertical, we want to understand meridionally or horizontally how the water vapor is transported: we integrate vertically. Note that in this form, you can see the fluxes of water vapor. We said that $\mathbf{J}$ is mostly vertical (from the surface up): $\mathbf{J} \simeq (0, 0, J)$ so

$$\boldsymbol{\nabla} \cdot \mathbf{J} \simeq \frac{\partial J}{\partial z} = \frac{\partial J}{\partial p}\frac{\partial p}{\partial z} = -\frac{\partial J}{\partial p}\rho g$$

Meaning

$$-\frac{1}{\rho} \approx g\frac{\partial J}{\partial p}$$

$$\frac{\partial q}{\partial t} + \nabla_h \cdot (\mathbf{v}_h q) + \frac{\partial}{\partial p}(\omega q) = e - c + g\frac{\partial J}{\partial p}$$

Recall that the vertical integral: $\int_{p_s}^{0} () - \frac{dp}{g} = \{\}$ then

$$\{\frac{\partial q}{\partial t}\} + \{\nabla_h \cdot (\mathbf{v}_h q)\} + \{\frac{\partial}{\partial p}(\omega q)\} = \{e - c\} + \{g\frac{\partial J}{\partial p}\}$$

$$\{\frac{\partial q}{\partial t}\} = \frac{\partial}{\partial t}\{q\} = \frac{\partial W}{\partial t}$$

$$\{\nabla_h \cdot (\mathbf{v}_h q)\} = \nabla_h \cdot \{\mathbf{v}_h q\} = \nabla_h \cdot \mathbf{Q}$$

$$\left\{\frac{\partial}{\partial p}(\omega q)\right\} = -\int_{p_s}^{0} \frac{\partial}{\partial p}(\omega q)\frac{dp}{g} = \frac{1}{g}\int_{0}^{p_s} \frac{\partial}{\partial p}(\omega q) dp = \frac{1}{g}(\cancel{\omega q\big|_{surface}} - \cancel{\omega q\big|_{top}})$$

as vertical velocity at the surface is zero and q at the top is zero. Then notice that integrating vertically evaporation and condensation you get all the $e - c$ is the precipitation that actually hits the ground. In gen c is always larger than e, the net precipitation term that reaches the ground. Precipitation is the net amount of water that reaches the ground, it is what I measure, not what condenses.

$$\{e - c\} = P$$

That leaves one last term:

$$\int_{0}^{p_s} dp\frac{\partial J}{\partial p} = J\big|_{surface} - \cancel{J\big|_{top}} = E$$

evaporation is nothing but the diffusion of water vapor in the atmosphere, it is the vertical diffusive flux of water vapor from the surface up in the atmosphere:

$$E = \lim_{p \to p_s} J(p)$$

hence,

$$\frac{\partial W}{\partial t} + \nabla_h \cdot \mathbf{Q} = E - P$$

this eq is true at any time-steps, you don't need any approximation as long as you know E and P.

We are interested in what happens in the atm over periods that are long (at least 30 years). As long as you take one or more years $\frac{\partial W}{\partial t}$ goes to zero → residence time of water vapor in the atmosphere.The climatological equation of water vapor balance remains. wxcharts.com

1.4 Balance requirements for the general circulation

1.4.1 Balance of total energy

1.4.2 The angular momentum balance

Chapter 2

Part 2: Climate Variability

2.1 Climate variability: basic tools (EOF analysis) and concepts (teleconnections)

2.2 Stationary Rossby waves

2.3 Low frequency climate variability in the extratropical atmosphere