Meta-Learning methods

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Outline

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Multiclassifiers, Meta-learners, Ensemble Learners

- Combining several *weak learners* to give a *strong learner*
- A kind of *multiclassifier* systems and *meta-learners*
- *Ensemble* typically applied to a single type of weak learner
  - All built by same algorithm, with different data or parameters
- Lots of what I say applies to multiclassifier systems in general

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   - And we can incorporate domain knowledge into different learners

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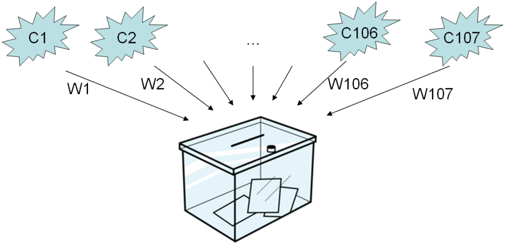
2. Combine strengths of different classifier builders
   - And we can incorporate domain knowledge into different learners
3. May help avoiding overfitting
   - This is paradoxical because more expressive than weak learners!

Combining weak learners

- Voting
  - Each weak learner votes, and votes are combined
- Experts that abstain
  - A weak learner only counts when it's expert on this kind of instances
  - Otherwise it abstains (or goes to sleep)

Outline

Voting



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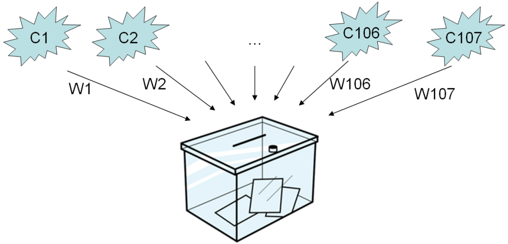

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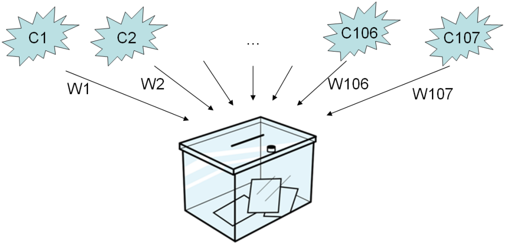
Voting



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- Simple *majority vote*
- Weights depend on *errors* ($1 - e_i$? $1/e_i$? $\exp(-e_i)$? $\ldots$)

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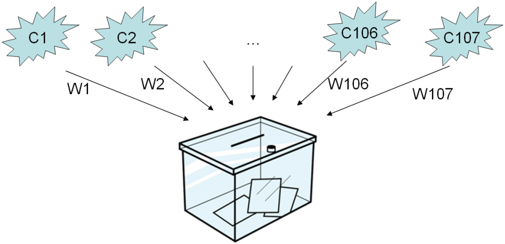

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- Weights depend on *errors* ($1 - e_i$? $1/e_i$? $\exp(-e_i)$? ...)
- Weights depend on *confidences*
- Maximizing *diversity*

Outline

Stacking (Wolpert 92)

A meta-learner that learns to weight its weak learner

- Dataset with instances `(x,y)`
- Transform dataset to have instances `(x,c1(x),...cN(x),y)`
- Train metaclassifier M with enriched dataset

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Often, `x` not given to M, just the votes
Often, just linear classifier
Can simulate most other voting schemes

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- Weights depend exponentially on error
- At least as good as best weak learner in time $O(\log N)$
- Often much better; more when classifiers are uncorrelated
- Good for online prediction and when many classifiers
- E.g. when 1 classifier = 1 feature

Outline

Introduction

- Definition

Voting schemes

- Stacking
- Weighted majority
- Bagging and Random Forests
- Boosting

Bagging (Breiman 96)

1. Get a dataset S of N labeled examples on A attributes;
2. Build N bagging replicas of S: $S_1, \ldots, S_N$;
   - S_i = draw N samples from S *with replacement*;
3. Use the N replicas to build N weak learners $C_1, \ldots, C_N$;
4. Predict using majority vote of the C_i's

Bagging I

Example of building training sets:

Original:	1	2	3	4	5	6	7	8
Training Set1:	2	7	8	3	7	6	3	1
Training Set2:	7	8	5	6	4	2	7	1
Training Set3:	3	6	2	7	5	6	2	2
Training Set4:	4	5	1	4	6	4	3	8

Specially useful in Decision trees and Perceptrons (high variance or sensibility to training data set)

Bagging II

Way to do majority voting when small samples or only one classifier builder

Improves unstable weak learners (= with high variance)

May degrade stable weak learners (= with low variance)

Random Forests (Breiman 01, Ho 98)

1. Parameters k and a;
2. Get a dataset S of N labeled examples on A attributes;
3. Build k bagging replicas of S: $S_1, \ldots, S_k$;
4. Use the k replicas to build k *random trees* $T_1, \ldots, T_k$;
   - At each node split, randomly select $a \leq A$ attributes, and choose best of these a;
   - Grow each tree as deep as possible: not pruning!!
5. Predict using majority vote of the T_i's

Random Forests II

Weak learner strength vs. weak learner variance

- More attributes a increases strength, overfits more
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Can be shown to be similar to weighted k-NN
Top performer in many tasks

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Boosting I

- Bagging tries to reduce variance of base classifiers by building different bootstrapping datasets
- Boosting tries to actively improve accuracy of weak classifiers
- How? By training a sequence of specialized classified based on previous errors

Boosting I (Schapire 92)

Adaptively, sequentially, creating classifiers

Classifiers *and instances* have varying weights

Increase weight of incorrectly classified instances

Boosting II

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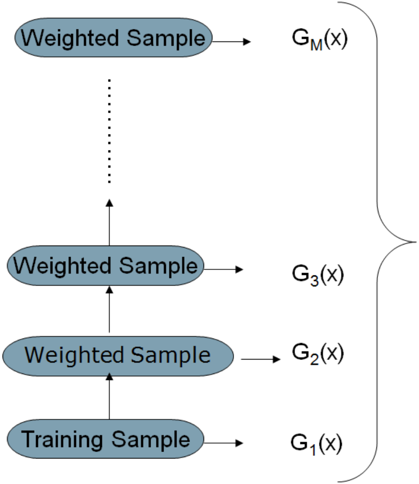
- Increase weight of incorrectly classified instances
- Final label as weighting voting of sequence of classifiers

Preliminaries

- Only two classes
- Output: $y \in \{-1, 1\}$
- Examples: X
- Weak Classifier: G(X)
- Error de training (err_{train})

$$err_{train} = \frac{1}{N} \sum_{i=1}^{N} I(y_i \neq G(x_i))$$

Preliminaries



$$G(x) = \text{sign}\left(\sum_{m=1}^{M} \alpha_m G_m(x) \right)$$

Adaboost algorithm

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Return classifier:
```

$$G(x) = \operatorname{sign}\left(\sum_{t=1}^{L} \alpha_t \, G_t(x)\right)$$

Adaboost algorithm

$$\alpha_m = \frac{1}{2} \ln \left(\frac{1 - err_m}{err_m} \right) > 0$$

$$w_i \leftarrow \frac{w_i}{Z_m} \cdot e^{-[\alpha_m \cdot y_i \cdot G(x_i)]}$$

$$G(x) = \operatorname{sign}\left(\sum_{m=1}^{L} \alpha_m G_m(x) \right)$$

Adaboost algorithm

$$\alpha_m = \frac{1}{2} \ln \left(\frac{1 - err_m}{err_m} \right) > 0$$

$$w_i \leftarrow \frac{w_i}{Z_m} \cdot \begin{cases} e^{-\alpha_t} & \text{if } y_i = G_t(x_i) \\ e^{\alpha_t} & \text{if } y_i \neq G_t(x_i) \end{cases}$$

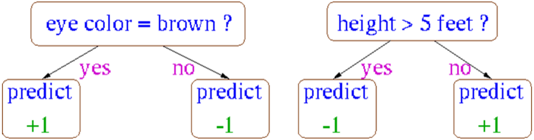
$$G(x) = \operatorname{sign} \left(\sum_{m=1}^{L} \alpha_m G_m(x) \right)$$

Simple example

We will use *Decision stumps* as the weak learner

Decision stumps are decision trees pruned to only one level. Good candidates to weak learners: above 0.5 accuracy and high variance.

Two examples of decision stumps.



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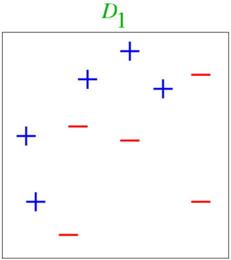

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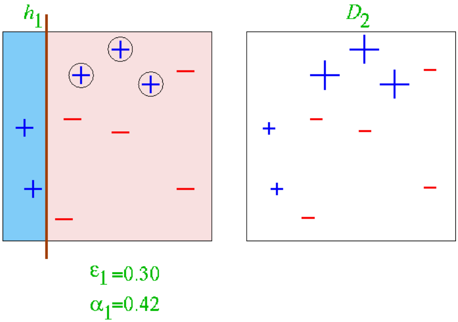

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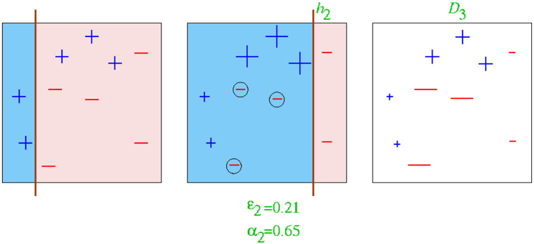

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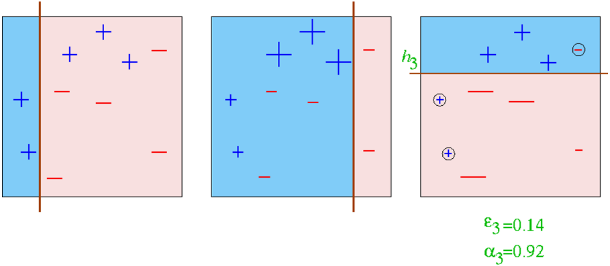

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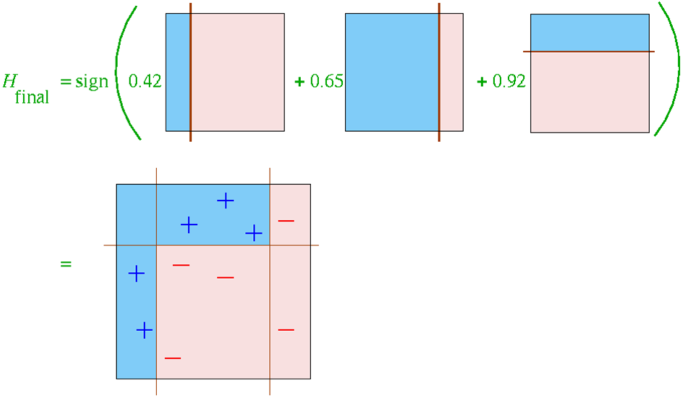

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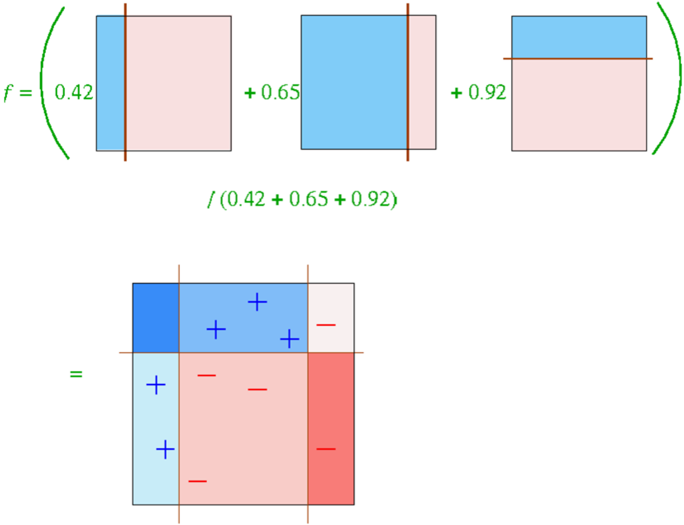

AdaBoost III

Theorem. *Suppose that the error of classifier* h_t *is* $1/2 - \gamma_t$, $t = 1..T$. *Then the error of the combination* H *of* $h_1, \ldots h_T$ *is at most*

$$\exp\left(-\sum_{t=1}^{T} \gamma_t^2\right)$$

Note: It tends to 0 if we can guarantee $\gamma_i \geq \gamma$ for fixed γ

Boosting vs. Bagging

- Fruitful investigation on how and why they differ
- On average, Boosting provides a larger increase in accuracy than Bagging
- But Boosting fails sometimes (particularly in noisy data)
- while bagging consistently gives an improvement

Possible reasons why this works

1. Statistical reasons: We do not rely on one classifier, so we reduce variance
2. Computational reasons: A weak classifier can be stuck in local minima. When starting from different training data sets, we can find better solution
3. Representational reasons: Combination of classifiers return solutions outside the initial set of hypothesis, so they adapt better to the problem

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In practice they work very well, sometimes better that SVMs.