AE 6200 – Advanced Aeroelasticity

Flutter Analysis of General Wing Models

Prof. Cristina Riso, criso@gatech.edu
Daniel Guggenheim School of Aerospace Engineering
Georgia Institute of Technology

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Lecture outline

- Flutter analysis of general wing models
  - Overview
  - Structural model
  - Aerodynamic model
  - Coupling model
  - Analysis process and results

Steps demonstrated using Nastran as an example of a state-of-the-art linear aeroelastic modeling and analysis workflow

Lecture outline

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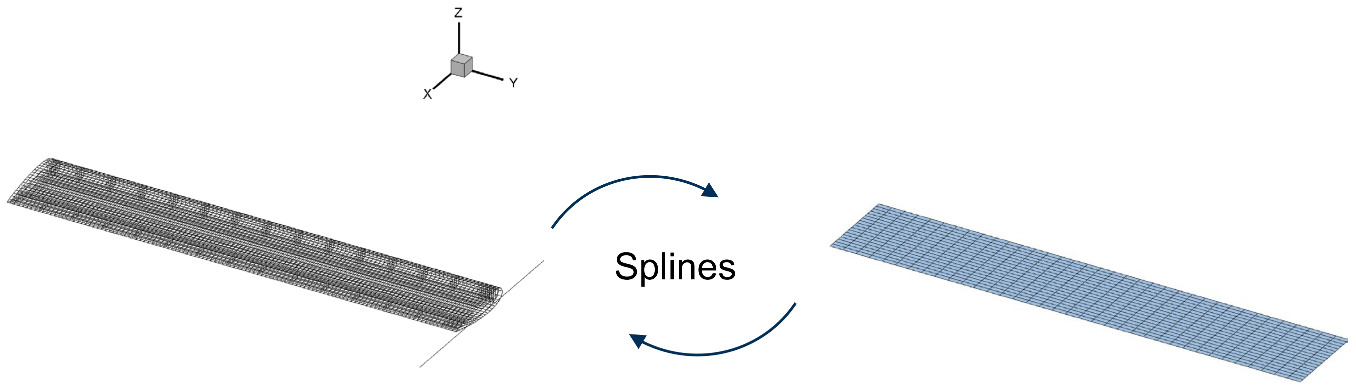
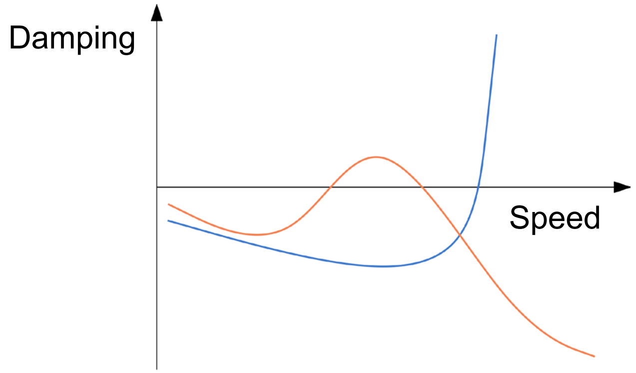
Overview



Structural model based on the finite element method (FEM)

Unsteady aerodynamic model based the doublet-lattice method (DLM)

Flutter analysis using the $p - k$ method



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Structural model: basic theory

- Based on the FEM to handle arbitrary geometries
- **Approach:** approximate the displacement field of a continuous structure in terms of the translations and rotations at discrete points (nodes)

Number of degrees of freedom (DOFs)

Nodal displacements

$$\underbrace{w(x,y,z;t)}_{\text{Displacement field}} \approx \sum_{i=1}^{N} \underbrace{\hat{\phi}_i(x,y,z)}_{\text{Shape functions}} q_i(t) = \widehat{\mathbf{\Phi}}(x,y,z)\mathbf{q}(t)$$

- **Strength:** study the continuous structure as an N-degree-of-freedom system using ordinary differential equations (ODEs) in time

Note: this is generally a large system with sparse matrices

Generalized forces

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{Q}(t)$$

Mass matrix

Stiffness matrix

Generalized coordinates

Structural model: basic theory

- Based on the FEM to handle arbitrary geometries

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{Q}(t)$$

- ODEs more conveniently rewritten using a "special" coordinate change

$$\underbrace{\mathbf{q}(t)}_{\text{Generalized coordinates}} = \underbrace{\mathbf{U}}_{\text{Modal matrix}}\underbrace{\boldsymbol{\eta}(t)}_{\text{Modal coordinates}}$$

$$\downarrow$$

$$\underbrace{\mathbf{U}^T\mathbf{M}\mathbf{U}}_{\text{Modal mass matrix } \bar{\mathbf{M}}}\ddot{\boldsymbol{\eta}}(t) + \underbrace{\mathbf{U}^T\mathbf{K}\mathbf{U}}_{\text{Modal stiffness matrix } \bar{\mathbf{K}}}\boldsymbol{\eta}(t) = \underbrace{\mathbf{U}^T\mathbf{Q}(t)}_{\text{Modal force vector } \mathbf{N}(t)}$$

Note: $\mathbf{U}$ stores $M \leq N$ eigenvectors obtained by solving the generalized eigenvalue problem

$$(\mathbf{K} - \omega^2\mathbf{M})\mathbf{u} = \mathbf{0}$$

Note: all matrices are diagonal

Structural model: basic theory

- Rewritten in **modal form** to *efficiently* handle arbitrary geometries

$$\bar{\mathbf{M}}\ddot{\boldsymbol{\eta}}(t) + \bar{\mathbf{K}}\boldsymbol{\eta}(t) = \mathbf{N}(t)$$

- **Advantages**
  - **Decoupled equations** due to eigenvector orthogonality
  - **Smaller-size system** due to $M \ll N$ thanks to **more effective coordinate choice**
  - **Comparable accuracy** to original discretized model $\underbrace{\text{(if relevant modes are retained)}}_{\text{The relevant modes to be retained depend on the problem}}$

Structural model: basic theory

- Rewritten in **modal form** to *efficiently* handle arbitrary geometries

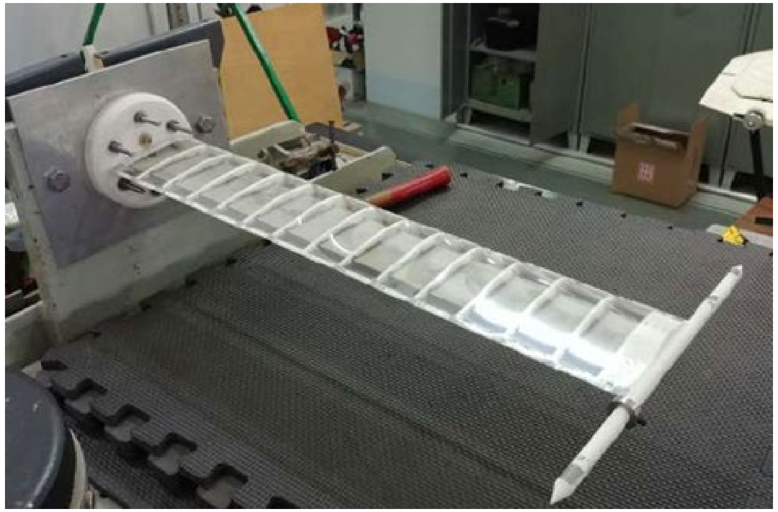
$$\bar{\mathbf{M}}\ddot{\boldsymbol{\eta}}(t) + \bar{\mathbf{K}}\boldsymbol{\eta}(t) = \mathbf{N}(t)$$

- **Standard representation** for practical (linear) flutter analyses
  - The process starts with a modal (eigenvalue) analysis of the undeformed structure
  - Also true for (linear) dynamic aeroelastic response analyses (e.g., to gusts or inputs)
  - Also true for (most linear) dynamic structural response analyses

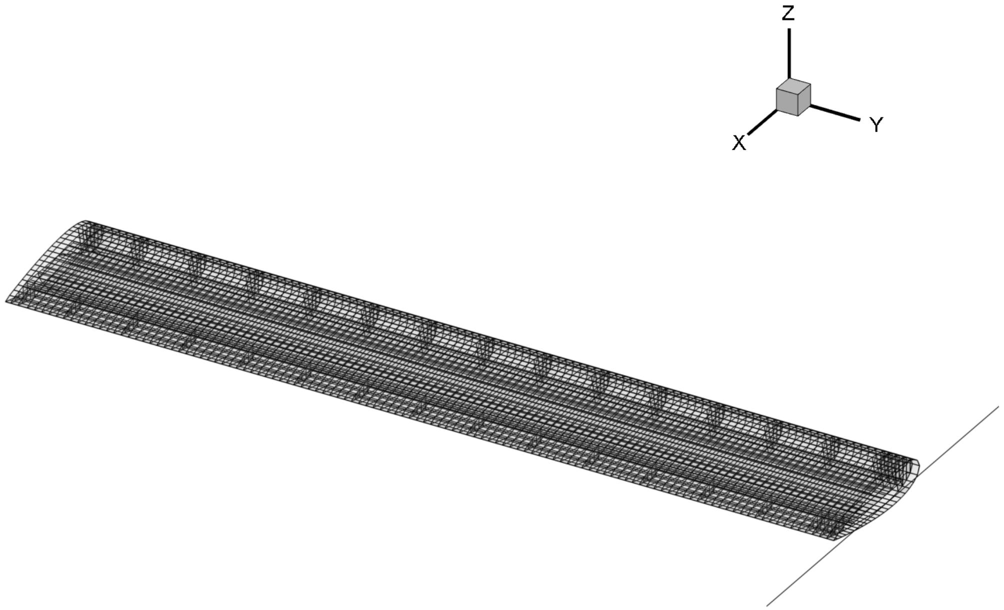
Experimental model figure from: Dr. Daniella Raveh (Technion)
Original modal analysis model with flexible rod and numerical results from: Avin et al., *AIAA J.*, 2022, https://doi.org/10.2514/1.J060621
Modified modal analysis model with rigid rod and numerical results from: Riso and Cesnik, *J. Aircr.*, 2023, https://doi.org/10.2514/1.C036869

Modal analysis: a practical example

- **Wing benchmark model**



Experimental model



Numerical FEM model
(~42k structural DOFs)

Modal analysis: a practical example

- **FEM model** defined in a .bdf file containing commands (cards) for
  - Nodes (GRID)
  - Elements (CBEAM for beam stiffeners and CQUAD4/CTRIA3 for shell panels)
  - Properties (PBEAM for beam stiffeners and PSHELL for shell panels)
  - Material (MAT1 for every elements)
  - Concentrated masses (CONM2)
  - Rigid connections (RBE2)
- **Root-clamped boundary conditions** (BCs) defined in a separate .bdf file
  - Single-point constraint (SPC1)
- **Modal analysis driver** defined in the .dat file
  - Includes the model and BCs
  - Specifies the solution sequence, analysis parameters, and requested outputs
- For complex structures, the model file is large but the analysis driver remains compact

Modal analysis: a practical example

- **Sample structure of an FEM model**

MAT1,51,68.948+9,,0.33,2.814+3	*Isotropic material (Young's Modulus, Poisson ratio, density)*
…	
PROD 701 51 .00129	*Property of rod elements (material ID and area)*
…	
PSHELL 501 51 .00203 51 51	*Property of shell elements (material ID and thickness)*
…	
CROD 301 701 1 5	*Rod element (property ID and nodes)*
…	
CQUAD4 101 501 51 1 5 53	*Shell element (property ID and nodes)*
...	
GRID 1 -.3175 0. -.0508	*Node (coordinates)*
…	

Modal analysis: a practical example

- **Sample structure of an FEM modal analysis**

SOL 103	*Nastran modal analysis solver*
CEND	*End of this file section*
$	
ECHO = NONE	*No model printout*
METHOD = 1000	*Eigenvalue analysis method (EIGRL below)*
SPC = 2000	*Boundary conditions (SPCADD below)*
VECTOR=ALL	*Eigenvector output for all the nodes*
$	
BEGIN BULK	*Begin of bulk section*
$	
PARAM,POST,0	*.xdb output*
PARAM,GRDPNT,0	*Rigid-body mass matrix output*
$	
EIGRL,1000,,,10	*Eigenvalue analysis method (Lanczos) and number of modes*
$	
SPCADD,2000, 1	*Collect boundary conditions*
$	
INCLUDE fem.bdf	*Model include*
INCLUDE bcs.bdf	*Boundary conditions (SPC1,1,123456,grid 1, etc.)*
$	
ENDDATA	*End of file*

Solution sequence

Analysis parameters and requested outputs

Everything else

Modal analysis: a practical example

- **Modal analysis output** given in the .f06 file
  - Default outputs (e.g., model information)
  - Additional requested outputs

- **Main outputs of interest**
  - Natural frequencies (rad/s and Hz)
  - Generalized mass values
  - Generalized stiffness values
  - Mode shapes (discretized displacement fields)

- Additional outputs for postprocessors to visualize the mode shapes

Modal analysis: a practical example

Sample modal parameters output

(length depends on the number of requested structural modes)

R E A L   E I G E N V A L U E S

MODE NO.	EXTRACTION ORDER	EIGENVALUE	RADIANS	CYCLES	GENERALIZED MASS	GENERALIZED STIFFNESS
1	1	6.938675E+02	2.634136E+01	4.192358E+00	1.000000E+00	6.938675E+02
2	2	3.198865E+04	1.788537E+02	2.846545E+01	1.000000E+00	3.198865E+04
3	3	6.553120E+04	2.559906E+02	4.074217E+01	1.000000E+00	6.553120E+04
4	4	2.679516E+05	5.176404E+02	8.238503E+01	1.000000E+00	2.679516E+05
5	5	4.334510E+05	6.583699E+02	1.047828E+02	1.000000E+00	4.334510E+05
6	6	7.044299E+05	8.393032E+02	1.335793E+02	1.000000E+00	7.044299E+05
7	7	8.347316E+05	9.136365E+02	1.454098E+02	1.000000E+00	8.347316E+05
8	8	9.659561E+05	9.828307E+02	1.564224E+02	1.000000E+00	9.659561E+05
9	9	1.162622E+06	1.078250E+03	1.716087E+02	1.000000E+00	1.162622E+06
10	10	1.333351E+06	1.154708E+03	1.837775E+02	1.000000E+00	1.333351E+06

Modal analysis: a practical example

EIGENVALUE = 6.938675E+02
CYCLES = 4.192358E+00 R E A L E I G E N V E C T O R N O . 1

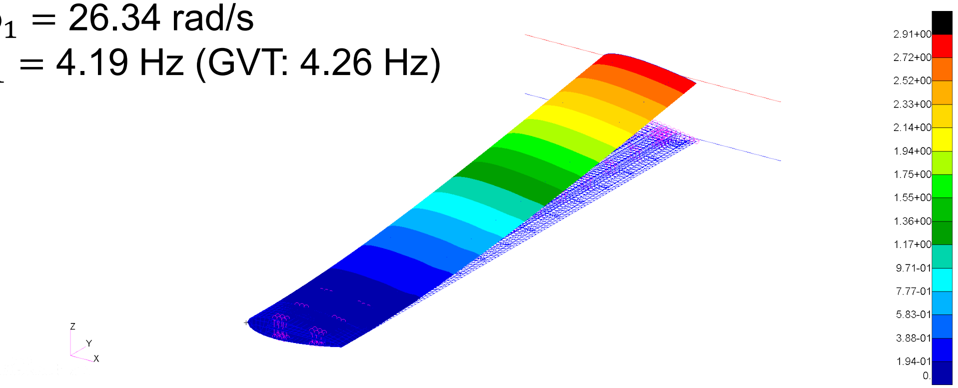
POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	-3.115361E-05	6.304366E-04	1.707131E-01	3.057419E+00	9.165040E-02	-2.682308E-03
2	G	-2.328537E-05	6.605361E-04	1.856150E-01	3.175361E+00	9.487632E-02	-9.830188E-04
3	G	-2.211965E-05	6.889732E-04	2.010742E-01	3.290598E+00	9.807446E-02	1.767459E-04
4	G	-2.523831E-05	7.158479E-04	2.170778E-01	3.403141E+00	1.012425E-01	8.342741E-04
5	G	-3.028496E-05	7.411602E-04	2.336136E-01	3.513075E+00	1.043781E-01	9.939578E-04
6	G	-3.490532E-05	7.649359E-04	2.506685E-01	3.620452E+00	1.074787E-01	6.656336E-04
7	G	-3.680390E-05	7.872008E-04	2.682311E-01	3.725314E+00	1.105418E-01	-1.424485E-04
8	G	-3.368535E-05	8.079564E-04	2.862888E-01	3.827674E+00	1.135647E-01	-1.428804E-03
9	G	-2.332964E-05	8.273089E-04	3.048301E-01	3.927419E+00	1.165449E-01	-3.163544E-03
10	G	1.665615E-04	-5.037961E-03	3.054345E-01	3.924766E+00	1.262005E-01	-8.483466E-03
11	G	2.839866E-04	-8.644536E-03	3.060400E-01	3.923697E+00	1.260462E-01	-9.020258E-03
12	G	3.948272E-04	-1.205693E-02	3.066463E-01	3.923470E+00	1.260389E-01	-9.101017E-03
13	G	4.996775E-04	-1.529093E-02	3.072532E-01	3.922785E+00	1.263228E-01	-5.361345E-03
14	G	5.982988E-04	-1.836456E-02	3.078614E-01	3.923593E+00	1.257813E-01	-1.583862E-03
15	G	5.682366E-04	-2.126797E-02	3.084214E-01	3.922421E+00	1.030313E-01	-3.171666E-04
16	G	4.714925E-04	-2.398113E-02	3.088485E-01	3.921485E+00	7.553189E-02	-1.543231E-04
17	G	3.130816E-04	-2.649292E-02	3.091413E-01	3.921687E+00	4.278090E-02	-2.076919E-04
18	G	1.583838E-04	-2.877774E-02	3.092986E-01	3.921927E+00	1.929198E-02	2.681395E-05
19	G	6.235393E-06	-3.080600E-02	3.093588E-01	3.922183E+00	1.697986E-03	3.647769E-04
20	G	-1.483858E-04	-3.253768E-02	3.093430E-01	3.922434E+00	-1.384600E-02	6.030416E-04
21	G	-3.139654E-04	-3.391881E-02	3.092518E-01	3.922663E+00	-3.181287E-02	5.204130E-04
22	G	-5.014565E-04	-3.488131E-02	3.090620E-01	3.922855E+00	-5.747120E-02	-1.207268E-04
23	G	-6.386704E-04	-3.534318E-02	3.087463E-01	3.923459E+00	-7.196048E-02	-1.253136E-04
24	G	-7.220035E-04	-3.519093E-02	3.083515E-01	3.924091E+00	-8.157552E-02	-1.476060E-04
25	G	-7.036250E-04	-3.426871E-02	3.079526E-01	3.924143E+00	-8.176713E-02	6.983880E-05
26	G	-6.647632E-04	-3.237198E-02	3.075553E-01	3.924700E+00	-8.174886E-02	5.842875E-04
27	G	-5.982253E-04	-2.916351E-02	3.071611E-01	3.925112E+00	-8.181445E-02	6.401077E-04
28	G	-4.916253E-04	-2.403630E-02	3.067758E-01	3.925812E+00	-8.187437E-02	3.955493E-04
29	G	-3.151029E-04	-1.556293E-02	3.064179E-01	3.927322E+00	-8.193000E-02	-2.176099E-04
30	G	9.624760E-06	4.340440E-06	3.062106E-01	3.932123E+00	-1.306891E-02	-9.833179E-05
31	G	9.139550E-06	4.267710E-06	2.876484E-01	3.829473E+00	-1.274609E-02	-1.012194E-04
32	G	8.649972E-06	4.190615E-06	2.695816E-01	3.725078E+00	-1.241796E-02	-1.001872E-04
33	G	8.174703E-06	4.101747E-06	2.520178E-01	3.618942E+00	-1.208485E-02	-9.523553E-05
34	G	7.732503E-06	4.001086E-06	2.349659E-01	3.511065E+00	-1.174710E-02	-8.635898E-05
35	G	7.342077E-06	3.888668E-06	2.184337E-01	3.401442E+00	-1.140499E-02	-7.356698E-05
36	G	7.022193E-06	3.764445E-06	2.024300E-01	3.290075E+00	-1.105887E-02	-5.684357E-05
37	G	6.791646E-06	3.628436E-06	1.869631E-01	3.176957E+00	-1.070900E-02	-3.619271E-05
38	G	6.669122E-06	3.486697E-06	1.720408E-01	3.062090E+00	-1.035568E-02	-1.164045E-05
39	G	-3.714376E-04	-1.211428E-02	1.722835E-01	3.057120E+00	-9.548083E-02	-1.745400E-04
40	G	-5.761598E-04	-1.870833E-02	1.727005E-01	3.055557E+00	-9.526544E-02	4.300348E-04
41	G	-6.991556E-04	-2.269722E-02	1.731486E-01	3.054836E+00	-9.507144E-02	6.734997E-04
42	G	-7.756080E-04	-2.519308E-02	1.736066E-01	3.054412E+00	-9.497267E-02	6.146179E-04
43	G	-8.191061E-04	-2.666834E-02	1.740688E-01	3.053829E+00	-9.498920E-02	9.230403E-05
44	G	-8.371813E-04	-2.738593E-02	1.745318E-01	3.053770E+00	-9.395432E-02	-1.264421E-04
45	G	-7.425245E-04	-2.750369E-02	1.749861E-01	3.053200E+00	-8.302697E-02	-9.787684E-05
46	G	-5.874370E-04	-2.714363E-02	1.753510E-01	3.052658E+00	-6.667247E-02	-9.312495E-05

Sample eigenvector output
(length depends on the number of requested structural modes and DOFs)

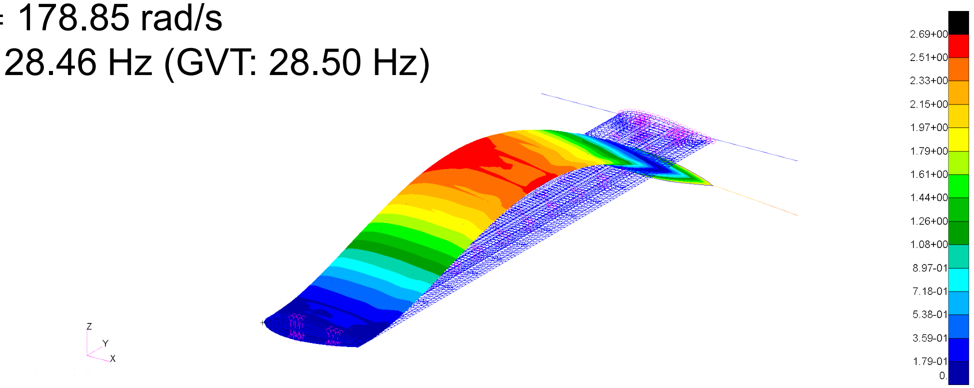
Modal analysis: a practical example

- **Sample mode shape visualization output**

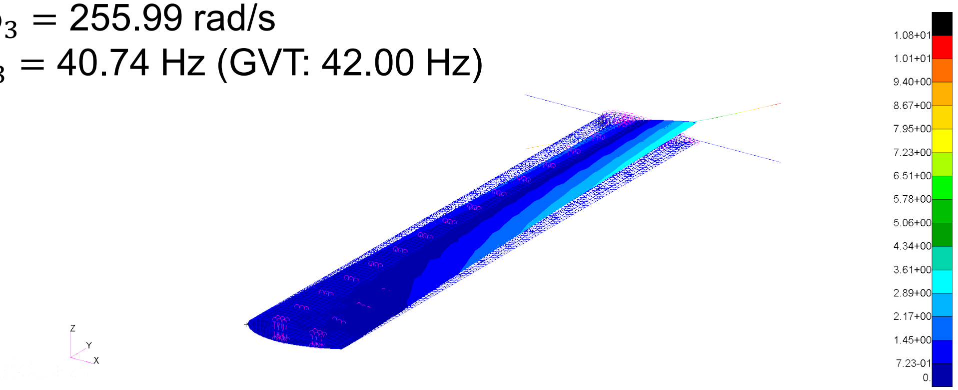
$\omega_1 = 26.34$ rad/s
$f_1 = 4.19$ Hz (GVT: 4.26 Hz)



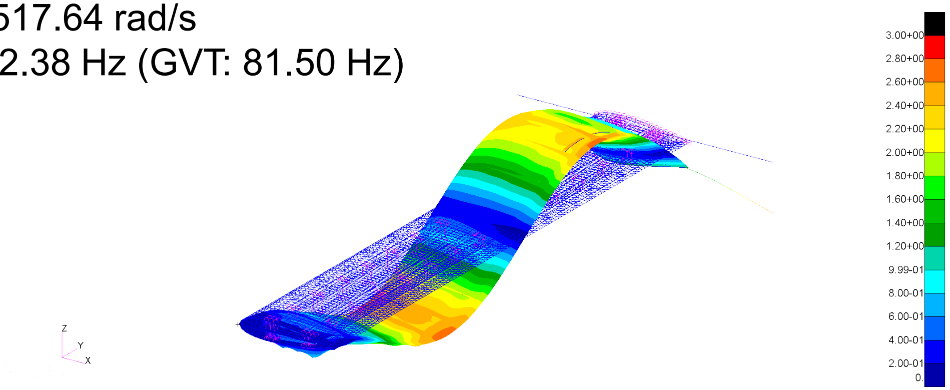
$\omega_2 = 178.85$ rad/s
$f_2 = 28.46$ Hz (GVT: 28.50 Hz)



$\omega_3 = 255.99$ rad/s
$f_3 = 40.74$ Hz (GVT: 42.00 Hz)



$\omega_4 = 517.64$ rad/s
$f_4 = 82.38$ Hz (GVT: 81.50 Hz)



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Aerodynamic model: basic theory

- Based on the DLM to handle three-dimensional unsteady aerodynamics
- **Approach:** approximate the unsteady aerodynamic loads on a lifting surface by discretizing it into panels
- **Assumptions**
  - No thickness and camber
  - Small-amplitude harmonic motion
  - Potential flow subsonic aerodynamics
  - Constant pressure difference over a panel
- **Features**
  - Fully unsteady, three-dimensional aerodynamics
  - Interactional aerodynamics effects among lifting surfaces
  - No wake discretization (unlike the unsteady vortex-lattice method)

Doublet-lattice method reference: Albano and Rodden, *AIAA J.*, 1969, https://doi.org/10.2514/3.5086
See also Demasi, *Introduction to Unsteady Aerodynamics and Dynamic Aeroelasticity*, Springer, 2024

Note: equations written in the frequency domain

Aerodynamic model: basic theory

- Based on the DLM to handle three-dimensional unsteady aerodynamics
- **Approach:** approximate the unsteady aerodynamic loads on a lifting surface by discretizing the wetted area into panels

Modal forces

Modal coordinates

$$\underbrace{(s^2\bar{\mathbf{M}} + \bar{\mathbf{K}})\bar{\boldsymbol{\eta}}}_{\text{Structural dynamics terms}} = \bar{\mathbf{N}} = \frac{\rho U^2}{2}\overbrace{\mathbf{U}^T\mathbf{G}^T\mathbf{S}\underbrace{\mathbf{A}(k,M)}_{\text{Aerodynamic influence coefficients (AIC) matrix}}\mathbf{D}(k)\mathbf{G}\mathbf{U}}^{\text{Generalized aerodynamic force (GAF) matrix (complex-valued function of } k \text{ and } M)}\bar{\boldsymbol{\eta}}$$

Modal matrix

Displacement interpolation matrix

Downwash matrix

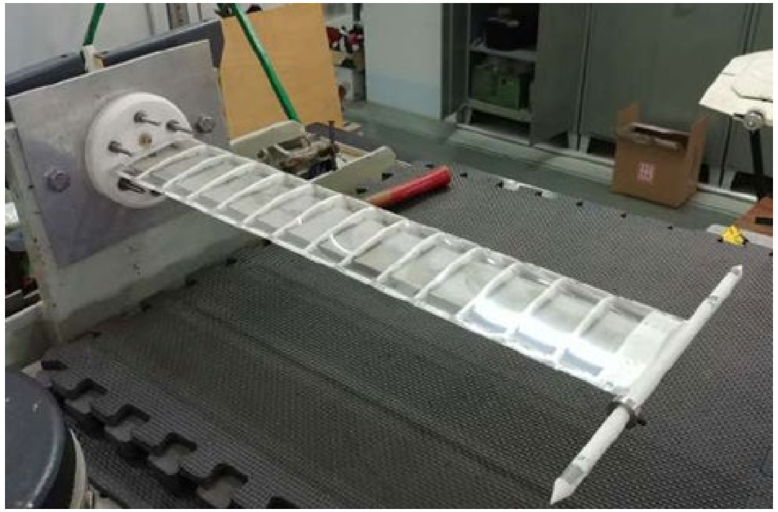
Pressure difference integration matrix

Load transfer matrix

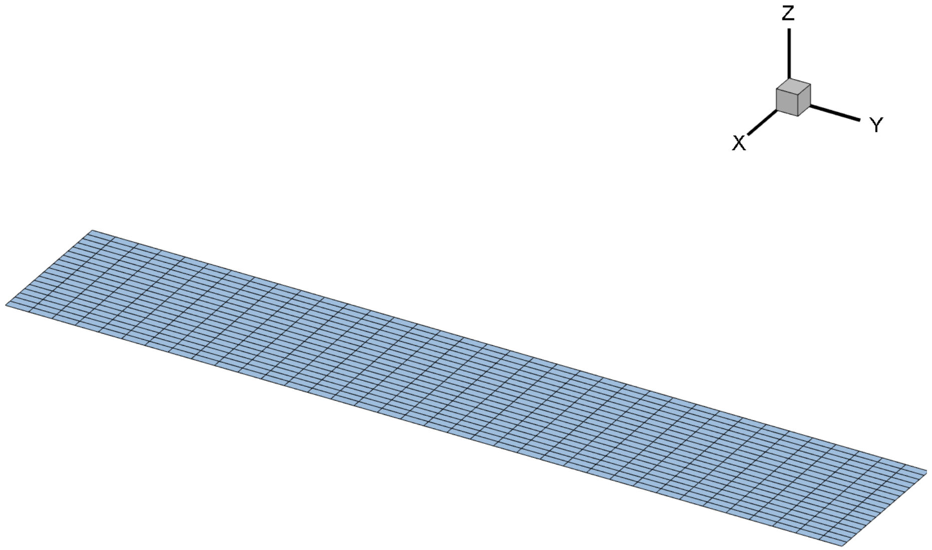
Modal projection matrix

Aerodynamic model: a practical example

- **Wing benchmark model**



Experimental model



Numerical DLM model
(~650 aerodynamic panels)

Aerodynamic model: a practical example

- **DLM model** defined in a .bdf file containing commands (cards) for
  - Aerodynamic group (PAERO1)
  - Macro DLM aerodynamic surface (CAERO1)

Panels along the span

Panels along the chord

Aerodynamic surface ID

Interference group ID (dummy for an isolated wing)

```
PAERO1,9999

CAERO1,100001,9999,,36,18,,,1,
,0.0,2.15e-3,0.0,0.0989,0.0,5.52e-3,0.0,0.098
```

Root leading-edge coordinates

Root chord

Tip leading-edge coordinates

Tip chord

Best aerodynamic discretization practices:
- Panel chordwise length $< 0.08\, U_{min}/f_{max}$
- At least four panels along the chord
- Refine around expected gradients
- Panel aspect ratio from 0.3 to 3

- For complex configurations, one may have multiple aerodynamic groups and surfaces

Lecture outline

- Flutter analysis of general wing models
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Coupling model: basic theory

- **Issue:** structural and aerodynamic models used in flutter analyses do not typically have consistent discretizations
- **Solution:** establish schemes to
  - Interpolate displacements (structure → aerodynamics)
  - Transfer loads (aerodynamics → structure)

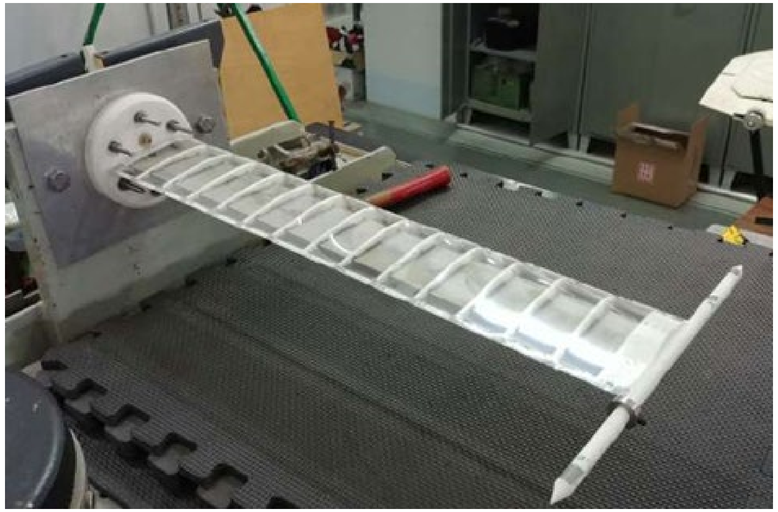
$$\bar{\mathbf{N}} = \frac{\rho U^2}{2} \mathbf{U}^T \mathbf{G}^T \mathbf{S}\mathbf{A}(k, M)\mathbf{D}(k)\mathbf{G}\mathbf{U}\bar{\boldsymbol{\eta}}$$

- If different schemes are used, $\mathbf{G}^T$ is replaced by a different matrix

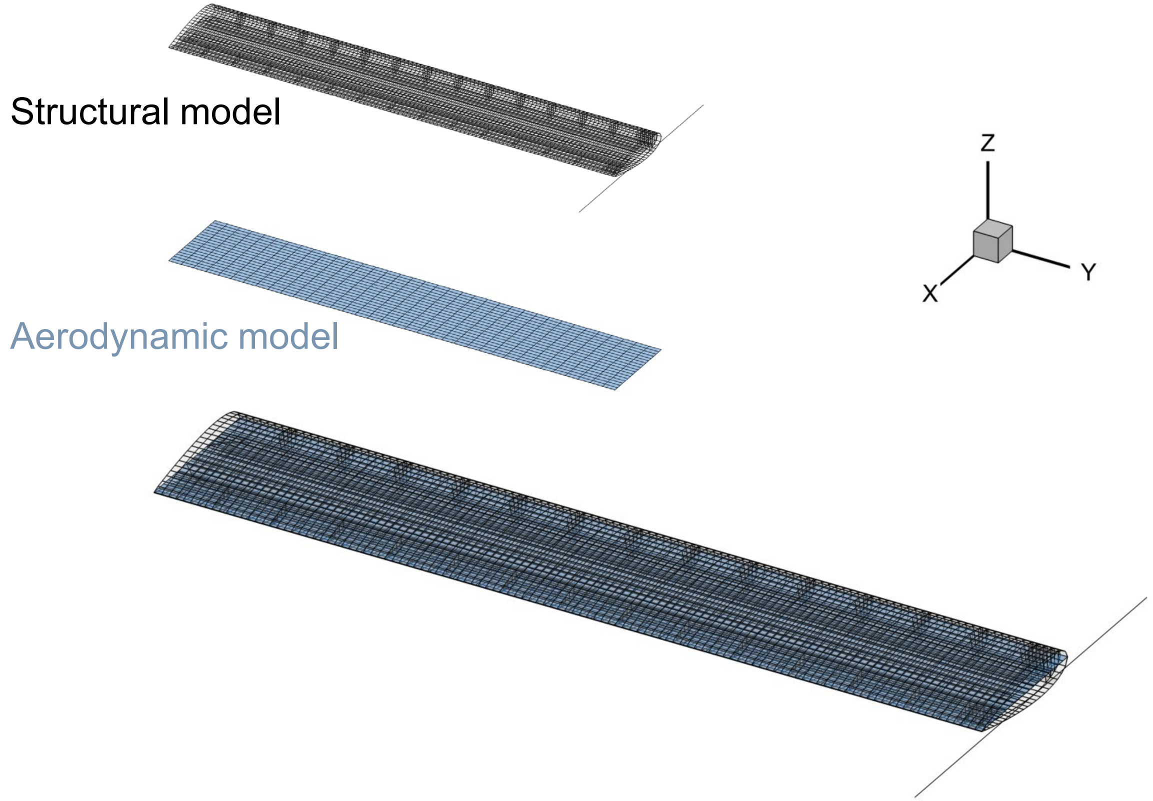
- The structural-aerodynamic coupling can significantly affect the results

Coupling model: a practical example

- **Wing benchmark model**



Experimental model



Numerical FEM-DLM model
(~42k DOFs and ~650 panels)

Coupling model: a practical example

- **Coupling model** defined in a .bdf file containing coupling commands

Aerodynamic surface ID

Spline ID

First and last panels to be coupled

Coordinate system ID

SPLINE2,1000,100001,100001,100648,100,0.0,1.0,111
,0.0,0.0

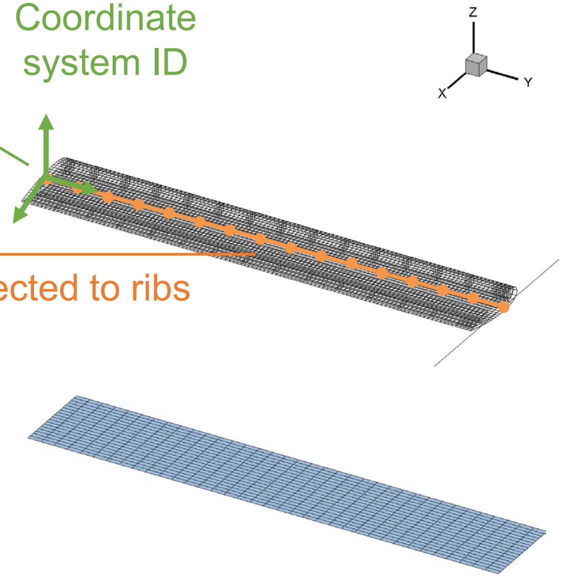
Node set ID

Beam spline (suitable for slender lifting surfaces)

SET1,100,80000001,THRU, 80000016

Extra nodes connected to ribs

CORD2R,111,,4.3589e-2,2.15e-3,0.0,4.3589e-2,2.15e-
3,1.0,
,1.0,2.15e-3,0.0



- For complex configurations, one may have multiple splines per surface

Lecture outline

- Flutter analysis of general wing models
  - Overview
  - Structural model
  - Aerodynamic model
  - Coupling model
  - Analysis process and results

Flutter analysis: basic theory

- **Approach:** solve the $p - k$ flutter determinant

$$(s^2\bar{\mathbf{M}} + \bar{\mathbf{K}})\bar{\boldsymbol{\eta}} = \bar{\mathbf{N}} = \frac{\rho U^2}{2}\mathbf{U}^T\mathbf{G}^T\mathbf{S}\mathbf{A}(k, M)\mathbf{D}(k)\mathbf{G}\mathbf{U}\bar{\boldsymbol{\eta}}$$

$$\downarrow$$

$$\left[s^2\bar{\mathbf{M}} + \bar{\mathbf{K}} - \frac{\rho U^2}{2}\mathbf{U}^T\mathbf{G}^T\mathbf{S}\mathbf{A}(k, M)\mathbf{D}(k)\mathbf{G}\mathbf{U}\right]\bar{\boldsymbol{\eta}} = \mathbf{0}$$

- Recall that s and k are related
- Flutter determinant solved for combinations of ρ, M, U
- Matching conditions not necessarily enforced automatically

Flutter analysis: a practical example

- **Sample structure of an FEM-DLM flutter FEM-DLM analysis**

SOL 145	*Nastran flutter analysis solver*	**Solution sequence**
CEND	*End of this file section*	
$		
ECHO = NONE	*No model echo*	**Analysis parameters and requested outputs**
METHOD = 1000	*Eigenvalue analysis method (EIGRL below)*	
FMETHOD = 2000	*Flutter analysis method (FLUTTER on next page)*	
SPC = 3000	*Boundary conditions (SPCADD below)*	
$		
BEGIN BULK	*Begin of bulk section*	**Everything else (continues on the next slide…)**
$		
PARAM,GRDPNT,0	*Rigid-body mass matrix output*	
PARAM,POST,0	*.xdb output*	
$		
PARAM,LMODES,10	*Number of modes retained in flutter analysis*	
$		
INCLUDE fem.bdf	*Model include*	
INCLUDE bcs.bdf	*Boundary conditions (SPC1,1,123456,grid 1, etc.)*	
INCLUDE reference_axis_grids.bdf	*Spline nodes along the reference axis*	
INCLUDE reference_axis_rbe3s.bdf	*Interpolation elements to connect reference axis nodes to ribs*	
$		
SPCADD,3000,1	*Collect boundary conditions (SPC1,1,123456,grid 1, etc.)*	
$		
EIGRL,1000,,,20	*Eigenvalue analysis method (Lanczos) and number of modes*	

Flutter analysis: a practical example

- **Sample structure of an FEM-DLM flutter FEM-DLM analysis**

```
AERO,,1.0,0.0989,1.225,1                    Aerodynamic parameters
$
INCLUDE dlm.bdf                             DLM model include
INCLUDE spl.bdf                             Spline model include
$
FLUTTER,2000,PK,2001,2002,2003,,,0.0001     Flutter analysis method and ranges of variables
$
FLFACT,2001,1.0000                          Density multiplication factor
FLFACT,2002,0.0010                          Mach number
FLFACT,2003,1.0,THRU,121.0,121              Velocity range (negative values to output eigenvectors)
$
INCLUDE mkaero.bdf                          GAF matrix database for various (M, k) pairs
$
ENDDATA                                     End of file

MKAERO1,0.001,,,,,,,                                            Mach number
,0.0100,0.0500,0.1000,0.1500,0.2000,0.2500,0.3000,0.3500        Reduced frequencies
MKAERO1,0.001,,,,,,,
,0.4000,0.4500,0.5000,0.5500,0.6000,0.6500,0.7000,0.7500
…continues….
```

Everything else (continues from the previous slide…)

Flutter analysis: a practical example

- **Flutter analysis output** given in the .f06 file
  - Default outputs (e.g., model information)
  - Additional requested outputs

- **Main outputs of interest**
  - Modal analysis outputs (see the previous slides)
  - Evolution of selected aeroelastic eigenvalues (not mode tracked)
  - Selected aeroelastic mode shapes (participation of structural modal coordinates)

- Additional outputs for postprocessors (e.g., to visualize the mode shapes)

Flutter analysis: a practical example

FLUTTER SUMMARY

CONFIGURATION = AEROSG2D XY-SYMMETRY = ASYMMETRIC XZ-SYMMETRY = SYMMETRIC

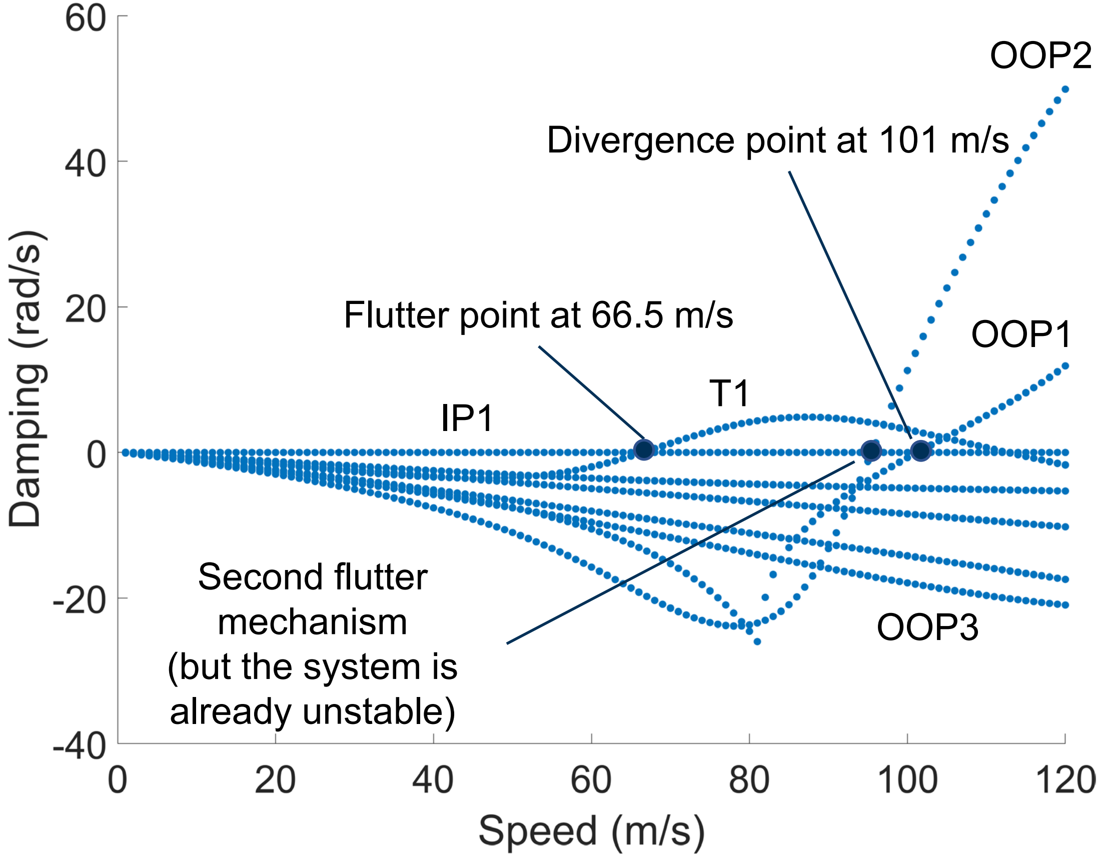
POINT = 1 MACH NUMBER = 0.0010 DENSITY RATIO = 1.0000E+00 METHOD = PK

KFREQ	1./KFREQ	VELOCITY	DAMPING	FREQUENCY	COMPLEX	EIGENVALUE
1.2974	7.7079486E-01	1.0000000E+00	-6.5038454E-03	4.1755629E+00	-8.5316908E-02	2.6235835E+01
0.6491	1.5406481E+00	2.0000000E+00	-1.3993750E-02	4.1781150E+00	-1.8368105E-01	2.6251871E+01
0.4330	2.3096253E+00	3.0000000E+00	-2.2113120E-02	4.1805513E+00	-2.9042464E-01	2.6267178E+01
0.3249	3.0780229E+00	4.0000000E+00	-3.0596143E-02	4.1825581E+00	-4.0203007E-01	2.6279788E+01
0.2600	3.8460778E+00	5.0000000E+00	-3.9296055E-02	4.1841358E+00	-5.1654075E-01	2.6289701E+01
0.2167	4.6139808E+00	6.0000000E+00	-4.8135482E-02	4.1853262E+00	-6.3291372E-01	2.6297180E+01
0.1858	5.3818732E+00	7.0000000E+00	-5.7074299E-02	4.1861849E+00	-7.5060053E-01	2.6302576E+01
0.1626	6.1498655E+00	8.0000000E+00	-6.6093524E-02	4.1867613E+00	-8.6933464E-01	2.6306197E+01
0.1445	6.9180709E+00	9.0000000E+00	-7.5180939E-02	4.1870807E+00	-9.8893774E-01	2.6308204E+01
0.1301	7.6865712E+00	1.0000000E+01	-8.4334646E-02	4.1871757E+00	-1.1093717E+00	2.6308801E+01
0.1183	8.4554513E+00	1.1000000E+01	-9.3550377E-02	4.1870652E+00	-1.2305667E+00	2.6308107E+01
0.1084	9.2247888E+00	1.2000000E+01	-1.0282974E-01	4.1867656E+00	-1.3525311E+00	2.6306224E+01
0.1001	9.9946638E+00	1.3000000E+01	-1.1217674E-01	4.1862870E+00	-1.4753046E+00	2.6303217E+01
0.0929	1.0765156E+01	1.4000000E+01	-1.2159653E-01	4.1856369E+00	-1.5989416E+00	2.6299132E+01
0.0867	1.1536334E+01	1.5000000E+01	-1.3109426E-01	4.1848246E+00	-1.7234981E+00	2.6294028E+01
0.0812	1.2308267E+01	1.6000000E+01	-1.4067586E-01	4.1838578E+00	-1.8490403E+00	2.6287954E+01
0.0764	1.3081027E+01	1.7000000E+01	-1.5034805E-01	4.1827403E+00	-1.9756434E+00	2.6280933E+01
0.0722	1.3854689E+01	1.8000000E+01	-1.6011800E-01	4.1814756E+00	-2.1033890E+00	2.6272986E+01
0.0684	1.4629333E+01	1.9000000E+01	-1.6999342E-01	4.1800638E+00	-2.2323635E+00	2.6264116E+01
0.0649	1.5405041E+01	2.0000000E+01	-1.7998234E-01	4.1785055E+00	-2.3626574E+00	2.6254324E+01
0.0618	1.6181905E+01	2.1000000E+01	-1.9009323E-01	4.1767980E+00	-2.4943650E+00	2.6243596E+01
0.0590	1.6960020E+01	2.2000000E+01	-2.0033492E-01	4.1749392E+00	-2.6275845E+00	2.6231917E+01
0.0564	1.7739482E+01	2.3000000E+01	-2.1071658E-01	4.1729266E+00	-2.7624176E+00	2.6219271E+01
0.0540	1.8520397E+01	2.4000000E+01	-2.2124784E-01	4.1707560E+00	-2.8989699E+00	2.6205633E+01
0.0518	1.9302886E+01	2.5000000E+01	-2.3193889E-01	4.1684212E+00	-3.0373514E+00	2.6190963E+01
0.0498	2.0087065E+01	2.6000000E+01	-2.4280023E-01	4.1659179E+00	-3.1776765E+00	2.6175234E+01
0.0479	2.0873061E+01	2.7000000E+01	-2.5384286E-01	4.1632401E+00	-3.3200627E+00	2.6158409E+01
0.0462	2.1661016E+01	2.8000000E+01	-2.6507812E-01	4.1603805E+00	-3.4646296E+00	2.6140442E+01
0.0445	2.2451075E+01	2.9000000E+01	-2.7651798E-01	4.1573319E+00	-3.6115027E+00	2.6121287E+01
0.0430	2.3243390E+01	3.0000000E+01	-2.8817509E-01	4.1540873E+00	-3.7608146E+00	2.6100900E+01
0.0416	2.4038136E+01	3.1000000E+01	-3.0006305E-01	4.1506369E+00	-3.9127052E+00	2.6079221E+01
0.0403	2.4835494E+01	3.2000000E+01	-3.1219624E-01	4.1469711E+00	-4.0673220E+00	2.6056188E+01
0.0390	2.5635669E+01	3.3000000E+01	-3.2459012E-01	4.1430780E+00	-4.2248206E+00	2.6031727E+01
0.0378	2.6438875E+01	3.4000000E+01	-3.3726102E-01	4.1389463E+00	-4.3853656E+00	2.6005766E+01

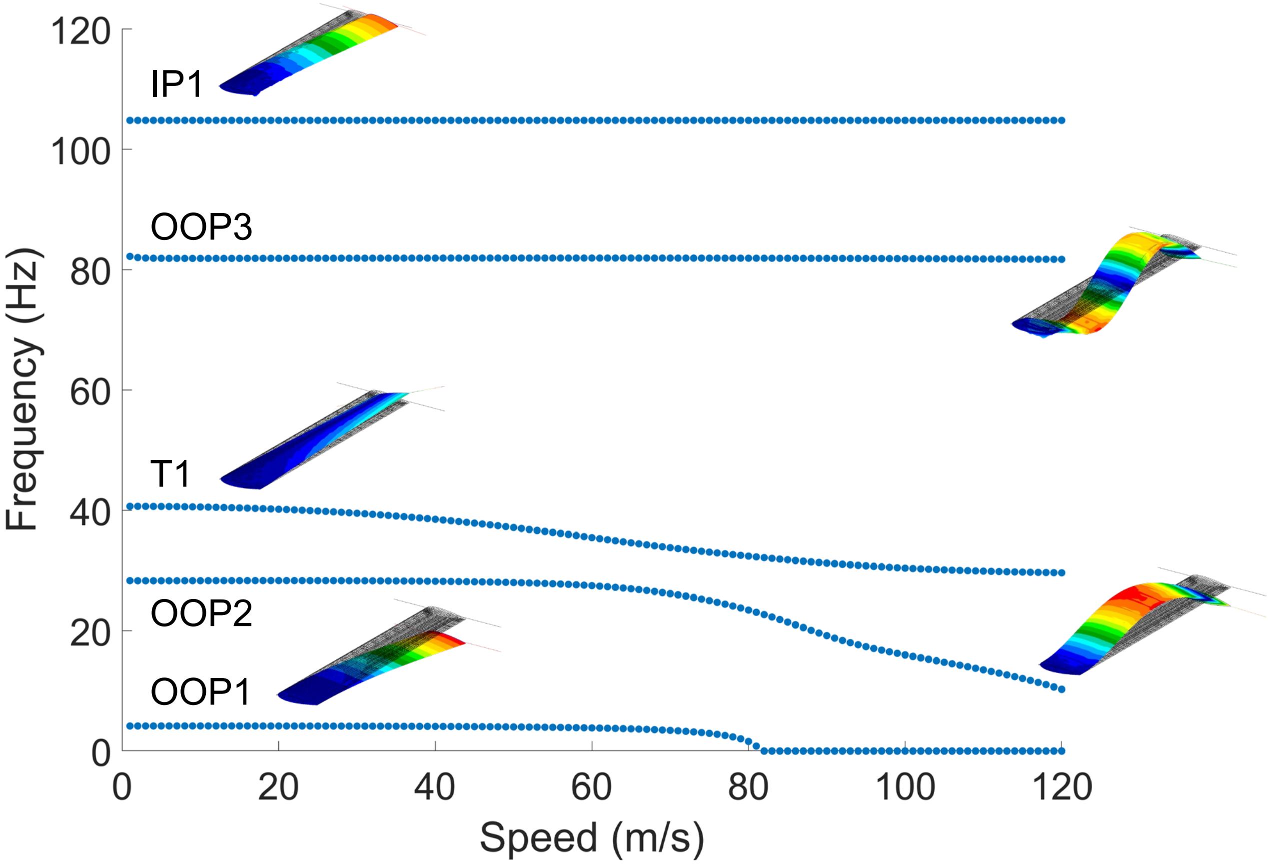
Sample eigenvalues output
(length depends on the number of requested aeroelastic modes and operating conditions)

Flutter analysis: a practical example

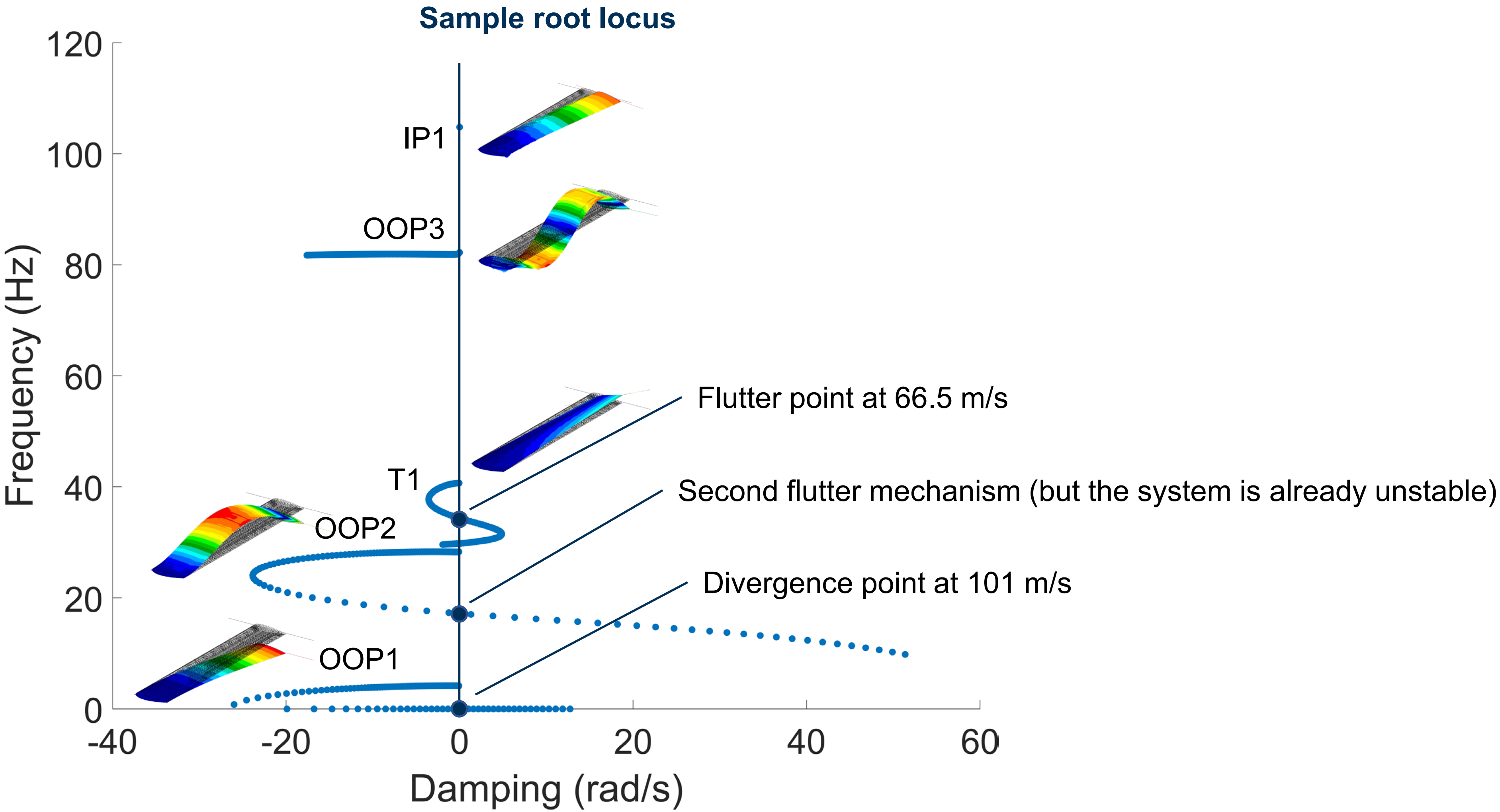
Sample damping diagram



Sample frequency diagram



Flutter analysis: a practical example



Flutter analysis: a practical example

```
                                             EIGENVECTOR FROM THE     PK METHOD
EIGENVALUE =    -8.59429E-03    2.15887E+02        VELOCITY =      6.65000E+01

EIGENVECTOR
                 3.27007E-01   -3.06581E-02
                 1.00000E+00    0.00000E+00
                 4.80899E-01   -2.03772E-01
                 8.50485E-03   -2.99258E-03
                -1.53305E-05    5.57343E-06
                -1.55238E-02    4.93248E-03
                -1.01110E-02    2.92536E-03
                 2.05729E-06   -1.71169E-07
                 2.67930E-03   -9.65530E-04
                -3.63384E-05   -1.08279E-05
```

Sample eigenvector output
(modal participations as real and imaginary parts, length depends on the number of retained structural modes)

- **Flutter mechanism**
  - First torsional mode
  - Second out-of-plane bending mode
- Out-of-plane bending and torsion **deformations are not in phase**
- Other modes contribute slightly

References

- This lecture is partially based on a lecture by Prof. Riso in the AE 544 – Aeroelasticity course taught by Prof. Cesnik at the University of Michigan in Fall 2021

- Additional details on modal analysis of MDOF systems
  - Meirovitch, *Fundamentals of Vibrations*, Waveland Press Inc., 2010
  - Inman, *Engineering Vibrations*, Pearson, 2013
  - Friedmann, Lesieutre, and Huang, *Structural Dynamics*, Cambridge, 2023

- Additional details on unsteady aerodynamic theories
  - Demasi, *Introduction to Unsteady Aerodynamics and Dynamic Aeroelasticity*, Springer, 2024
  - Dimitriadis, *Unsteady Aerodynamics: Potential and Vortex Methods*, Wiley, 2023
  - Consider taking AE 6030 if you have not

References

- Additional details on the wing test case and numerical models used in this lecture

  - Avin et al., “Experimental Aeroelastic Benchmark of a Very Flexible Wing,” *AIAA Journal*, 2022. https://doi.org/10.2514/1.J060621
  - Riso and Cesnik, “Impact of Low-Order Modeling on Aeroelastic Predictions for Very Flexible Wings,” *Journal of Aircraft*, 2023. https://doi.org/10.2514/1.C036869
  - Ritter et al., “Collaborative Pazy Wing Analyses for the Third Aeroelastic Prediction Workshop,” *AIAA SciTech Forum*, 2024. https://doi.org/10.2514/6.2024-0419