UNIVERSITAT DE BARCELONA

>>> IMAGE PROCESSING AND COMPUTATIONAL PHOTOGRAPHY

SESSION 4: ADVANCED FILTERING

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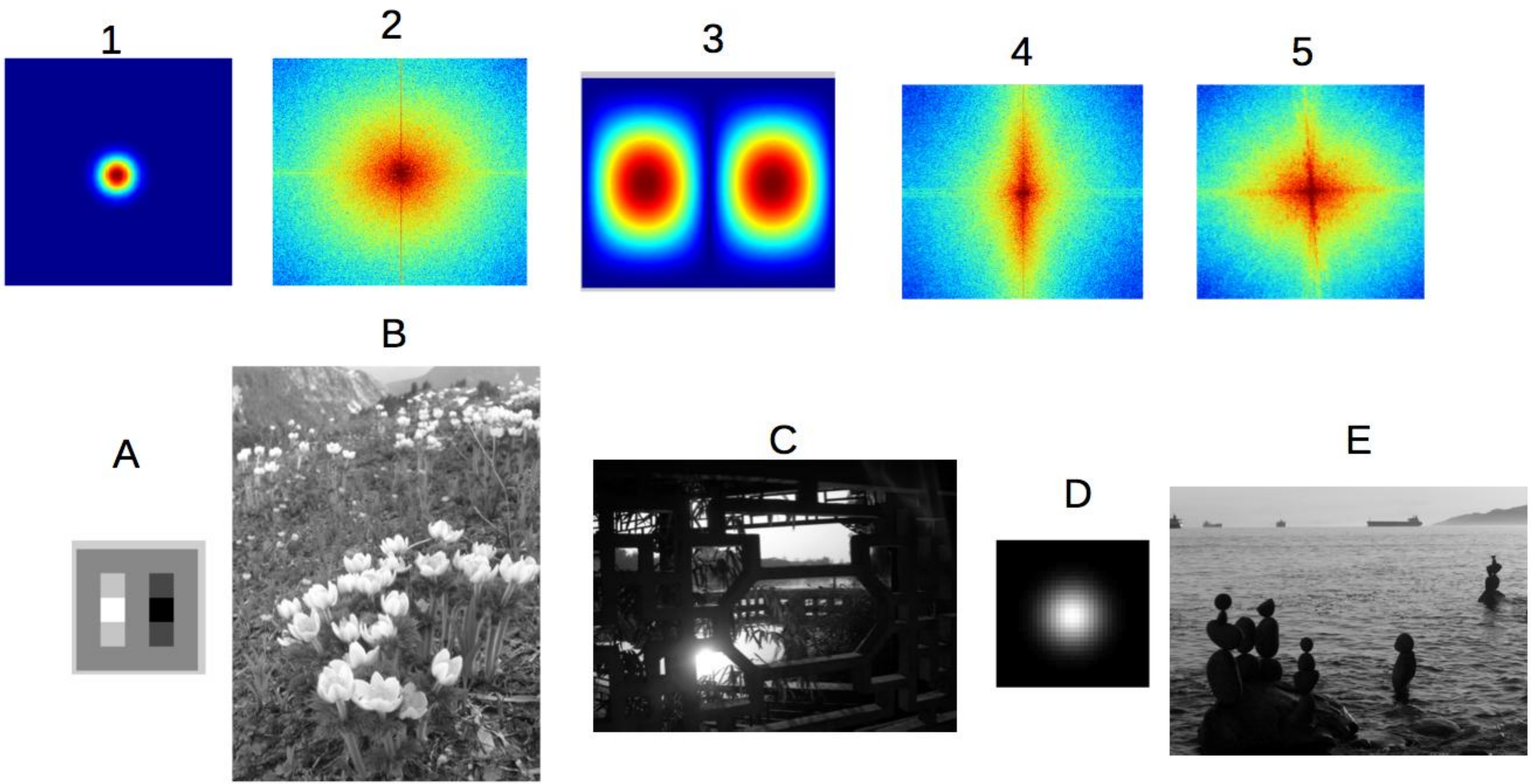
TAKE HOME QUESTIONS

1. Match the spatial domain image to the Fourier magnitude image

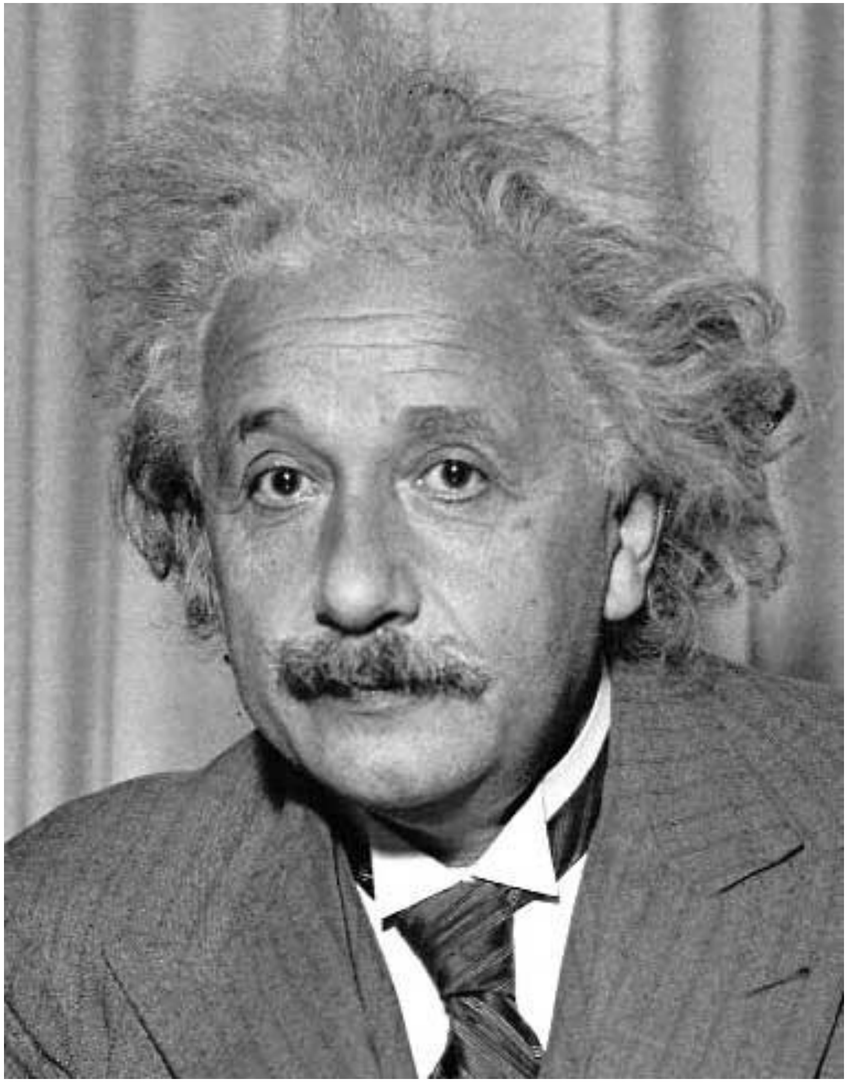


TODAY'S LECTURE

- Template matching
- Coarse-to-fine alignment
- Denoising
- Sharpening
- Anisotropic filtering

TEMPLATE MATCHING

- Goal: find  in image
- Main challenge: What is a good similarity or distance measure between two patches?
  - Correlation
  - Zero-mean correlation
  - Sum Square Difference
  - Normalized Cross Correlation

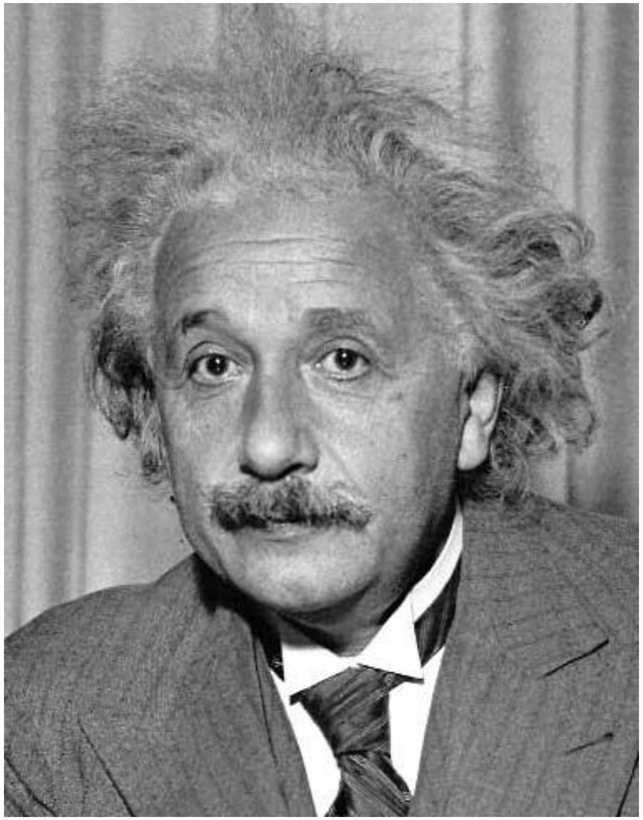


MATCHING WITH FILTERS

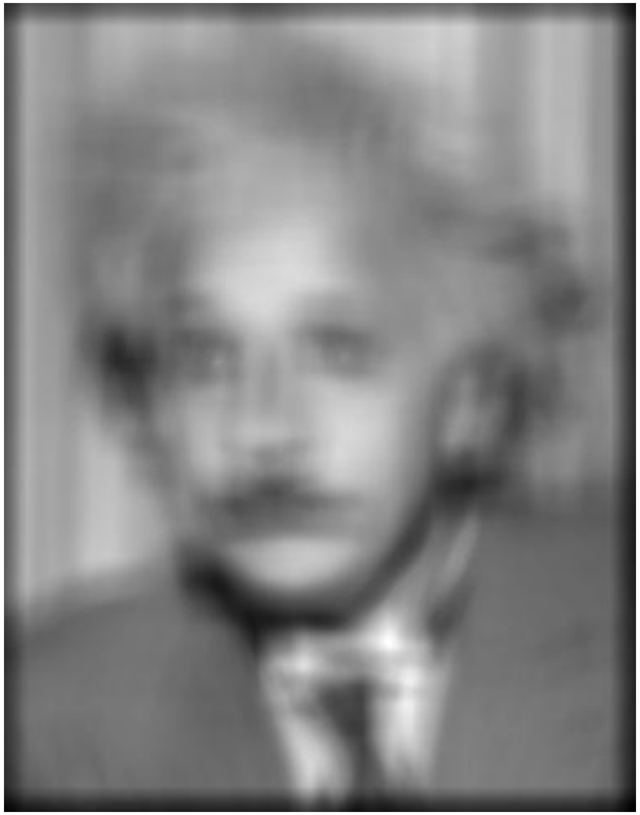
- Goal: find  in image
- Method 0: filter the image with eye patch

$$h(m,n) = \sum_{k,l} g(k,l) f(m+k, n+l)$$

f = image
g = filter



Input



Filtered Image

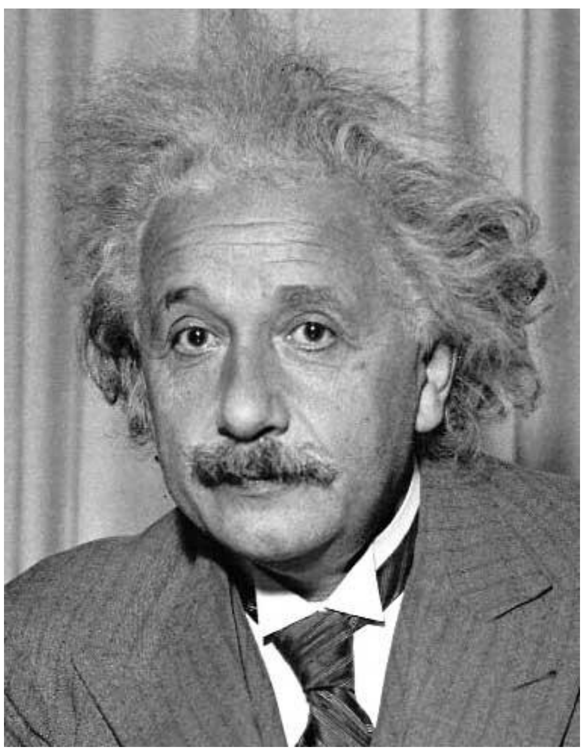
What went wrong?

MATCHING WITH FILTERS

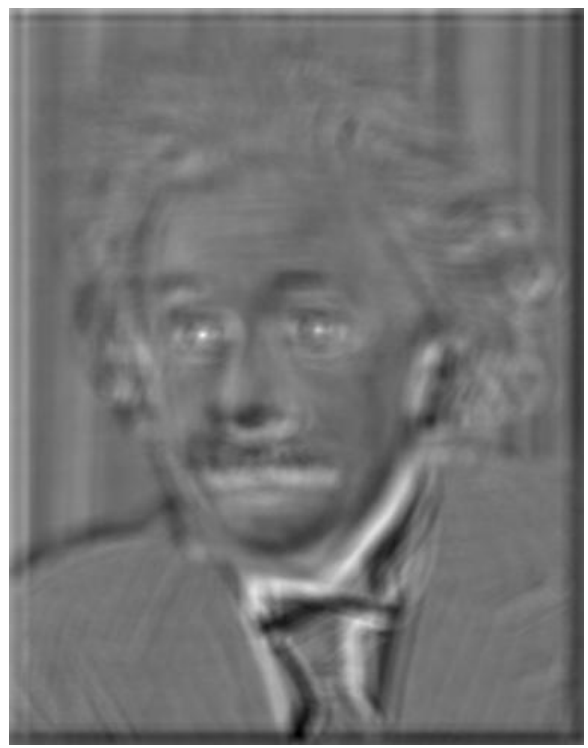
- Goal: find  in image
- Method 1: filter the image with a zero mean filter

$$h(m,n) = \sum_{k,l} (g(k,l) - \bar{g}) f(m+k, n+l)$$

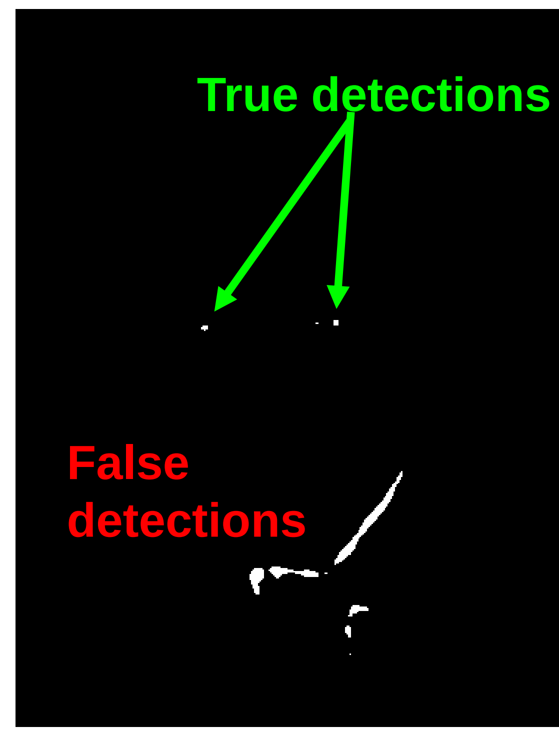
f = image
g = filter



Input



Filtered Image (scaled)



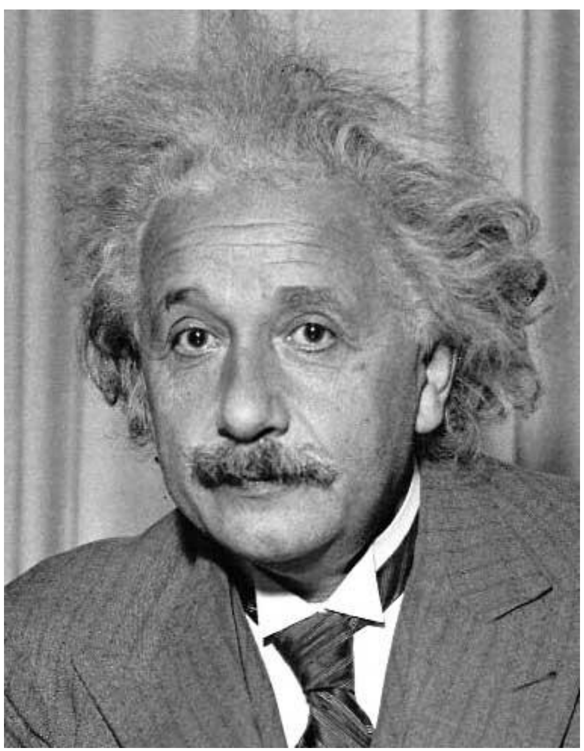
Thresholded Image

MATCHING WITH FILTERS

- Goal: find  in image
- Method 2: sum of squared differences

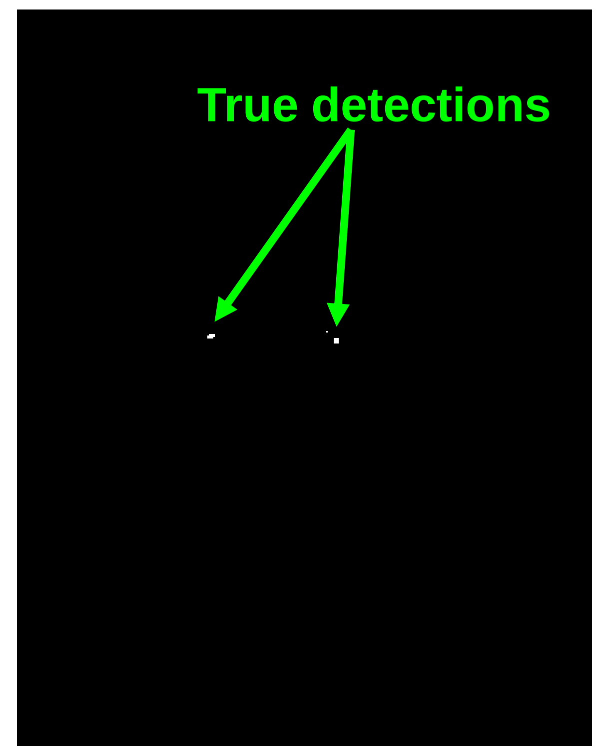
$$h(m,n) = \sum_{k,l} (g(k,l) - f(m+k,n+l))^2$$

f = image
g = filter



Input

1- sqrt(SSD)



Thresholded Image

MATCHING WITH FILTERS

- Question: Can this be implemented using linear filters?

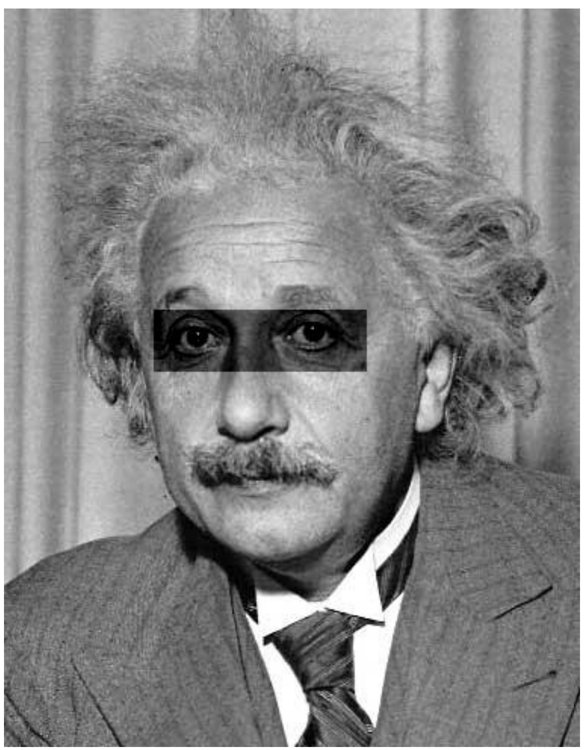
$$h(m,n) = \sum_{k,l} (g(k,l) - f(m+k,n+l))^2$$

MATCHING WITH FILTERS

- Goal: find  in image
- Method 2: sum of squared differences

$$h(m,n) = \sum_{k,l} (g(k,l) - f(m+k, n+l))^2$$

f = image
g = filter



Input

1- sqrt(SSD)

Potential downside !

MATCHING WITH FILTERS

- Goal: find  in image
- Method 3: normalized cross-correlation

f = image
g = filter

Filter mean

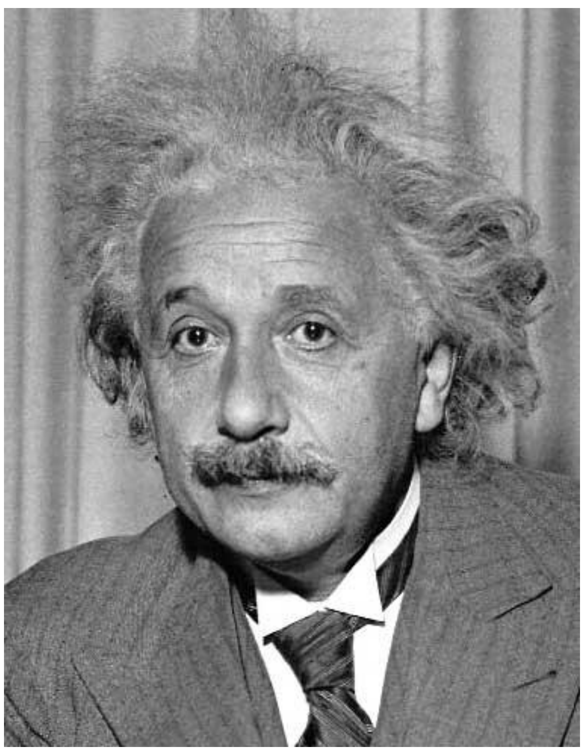
Image patch mean

$$h(m,n) = \frac{\sum_{k,l}(g(k,l) - \bar{g})(f(m+k,n+l) - \bar{f}_{m,n})}{\sqrt{\sum_{k,l}(g(k,l) - \bar{g})^2 \sum_{k,l}(f(m+k,n+l) - \bar{f}_{m,n})^2}}$$

Matlab: `normxcorr2(template, im)`

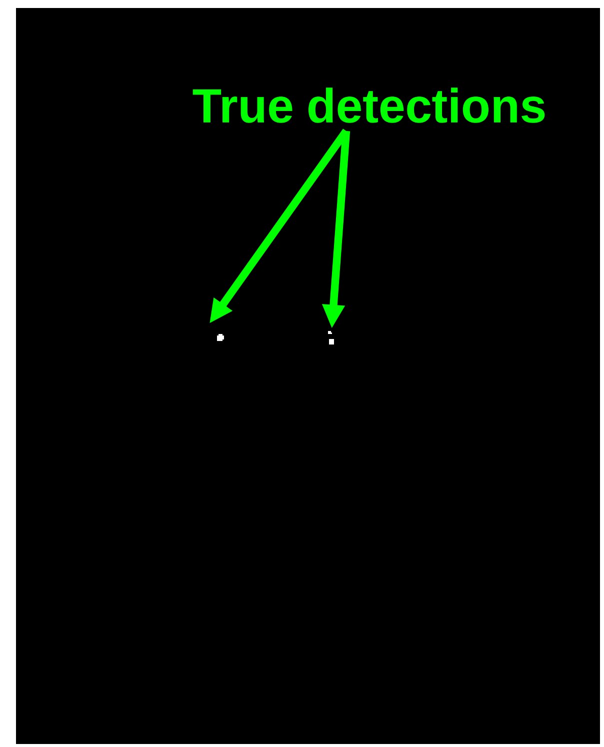
MATCHING WITH FILTERS

- Goal: find  in image
- Method 3: normalized cross-correlation



Input

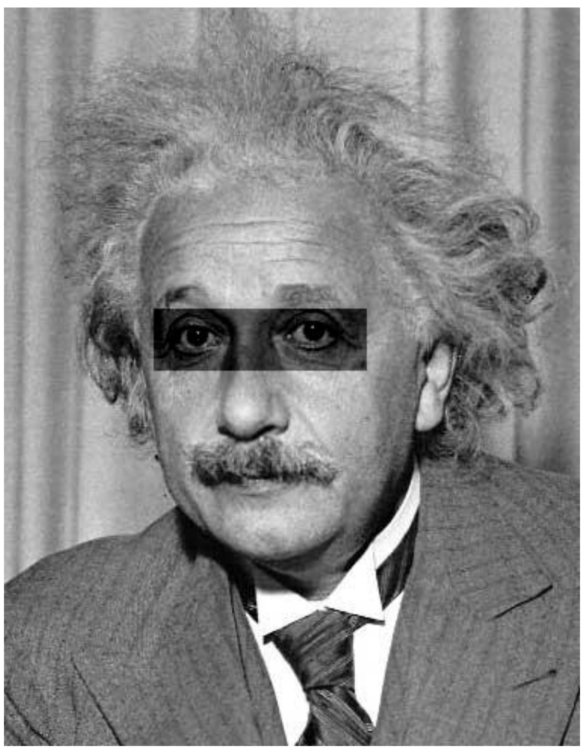
Normalized X-Correlation



Thresholded Image

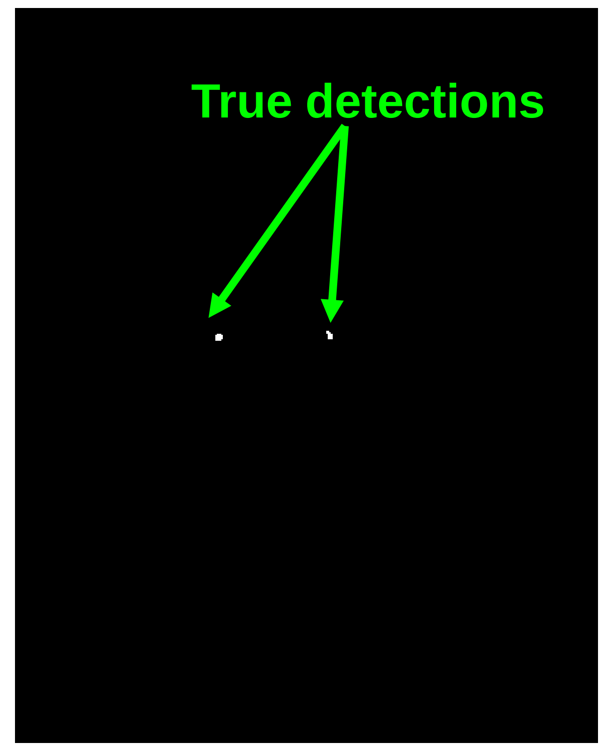
MATCHING WITH FILTERS

- Goal: find  in image
- Method 3: normalized cross-correlation



Input

Normalized X-Correlation



Thresholded Image

MATCHING WITH FILTERS

Q: What is the best method to use?

A: Depends

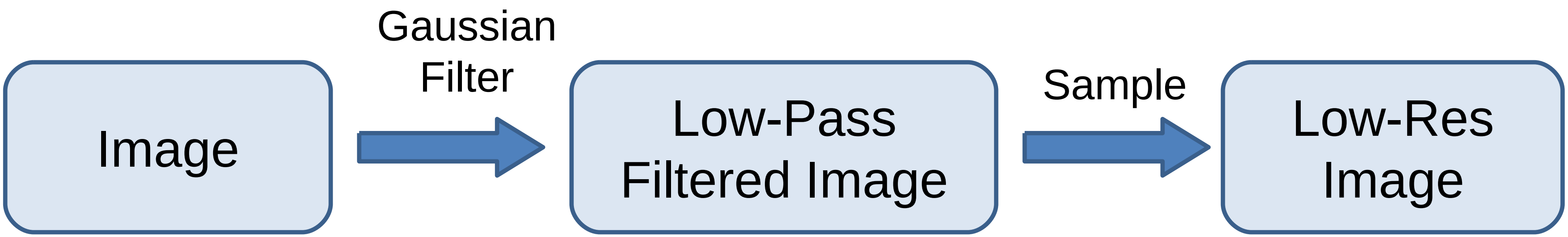
- Zero-mean filter: fastest but not a great matcher
- SSD: next fastest, sensitive to overall intensity
- Normalized cross-correlation: slowest, invariant to local average intensity and contrast

MATCHING WITH FILTERS

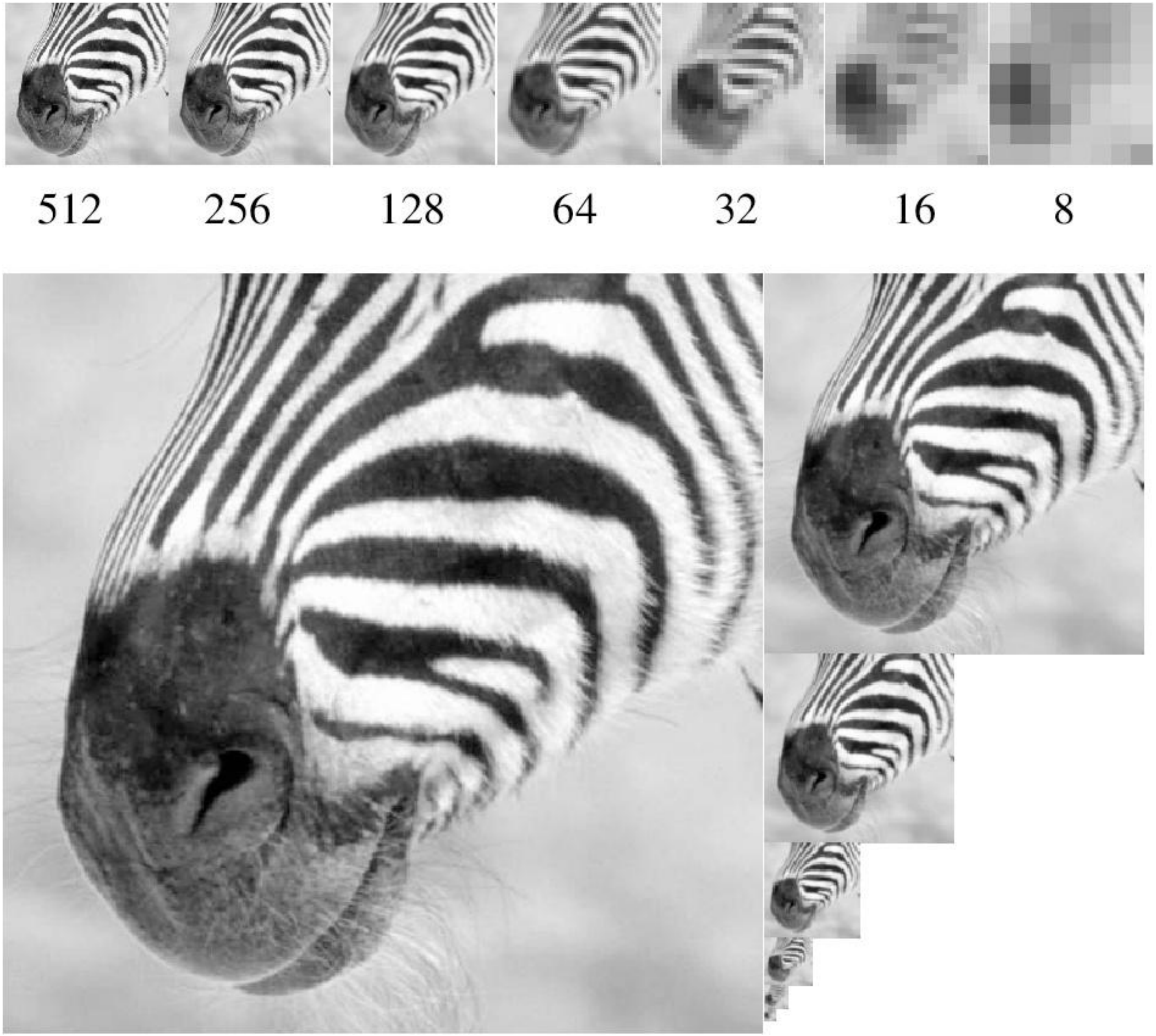
Q: What if we want to find larger or smaller eyes?

A: Image Pyramid

REVIEW OF SAMPLING

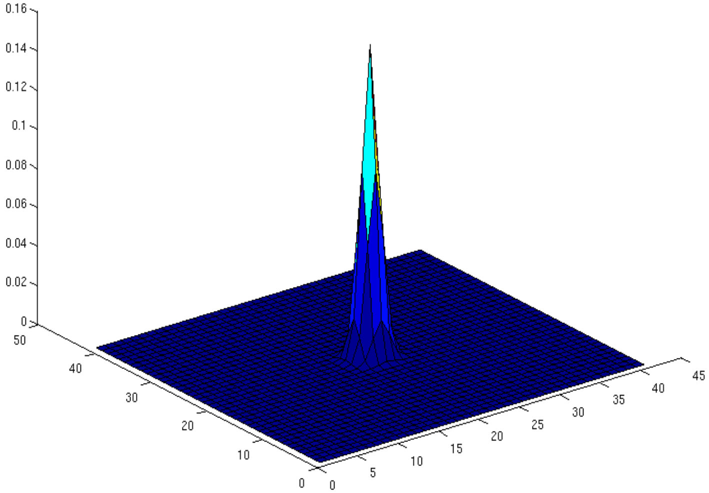


GAUSSIAN PYRAMID



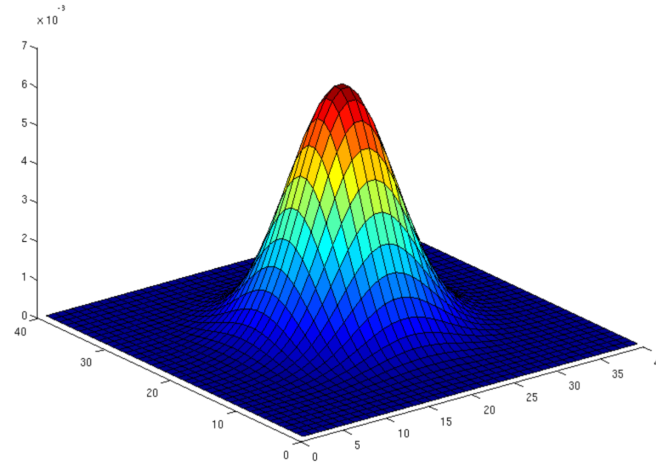
Source: Forsyth

LAPLACIAN FILTER

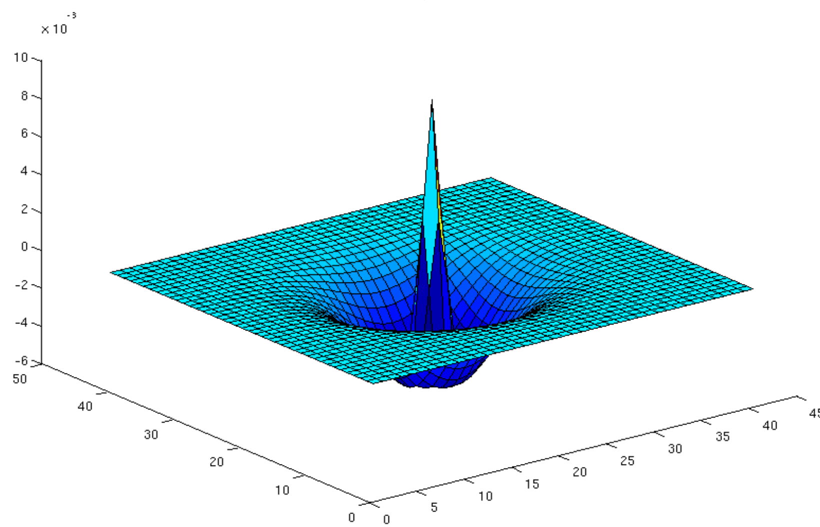


Unit response

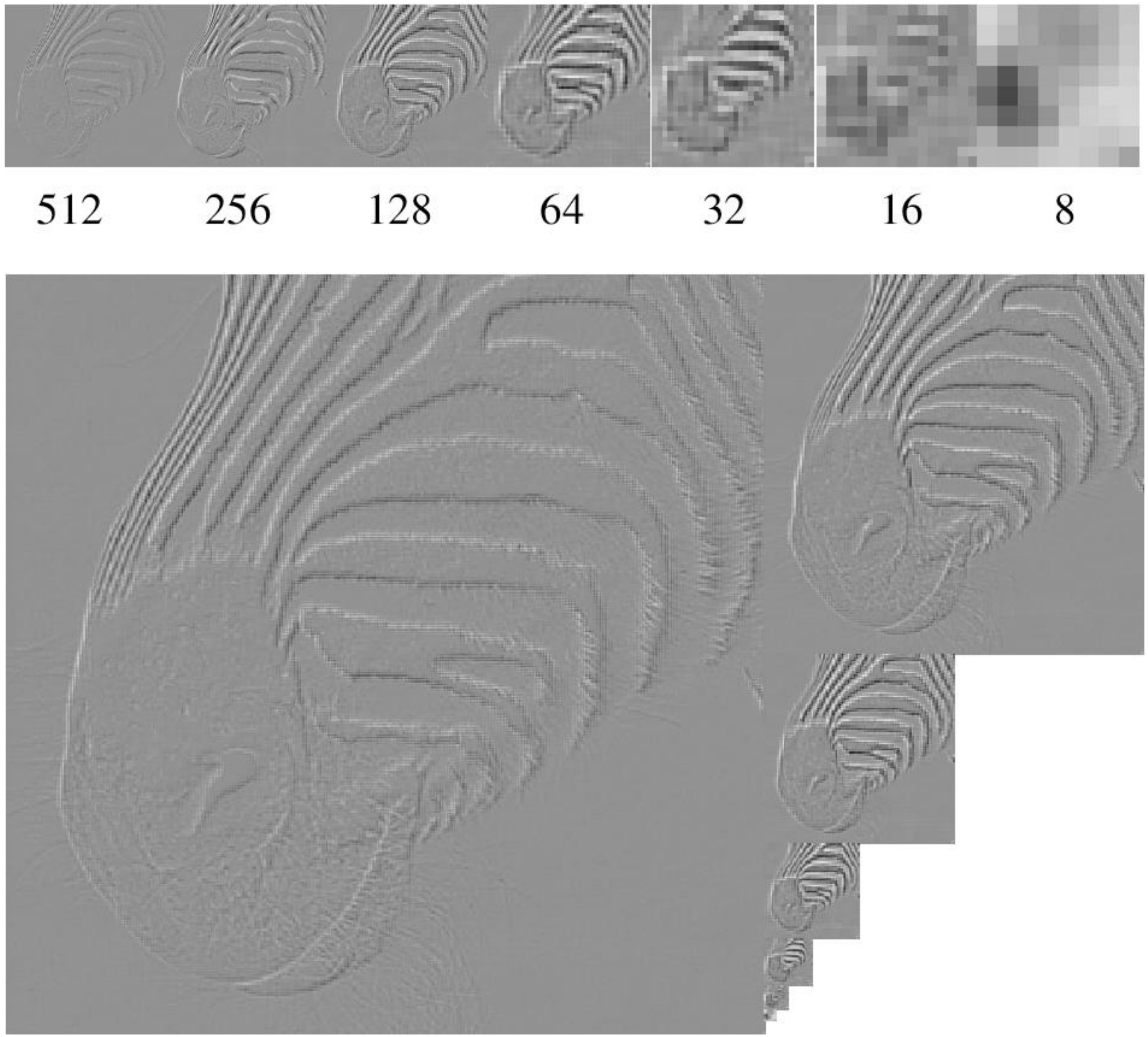
-



Gaussian

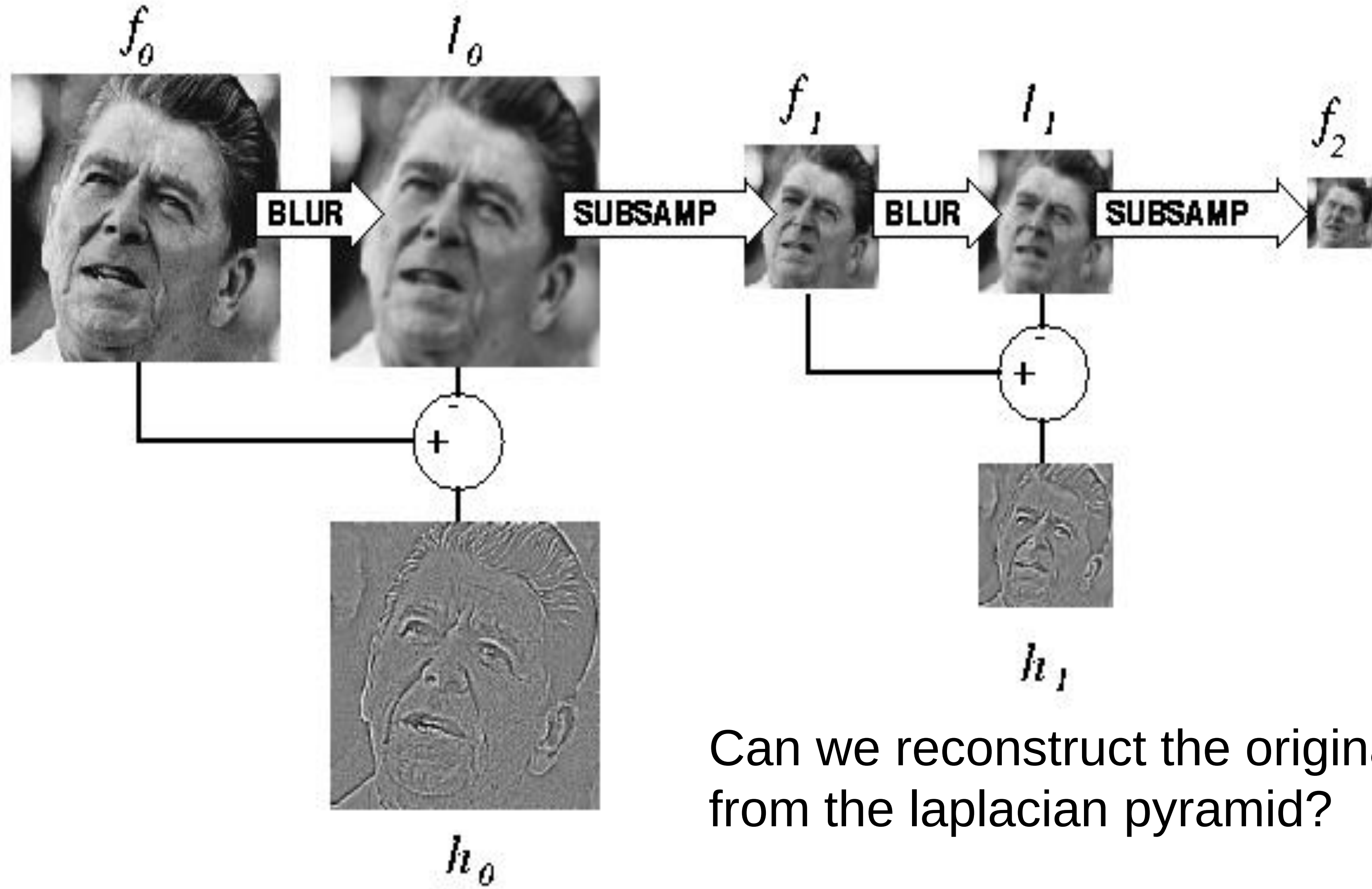


LAPLACIAN PYRAMID



Source: Forsyth

COMPUTING GAUSSIAN/LAPLACIAN PYRAMID

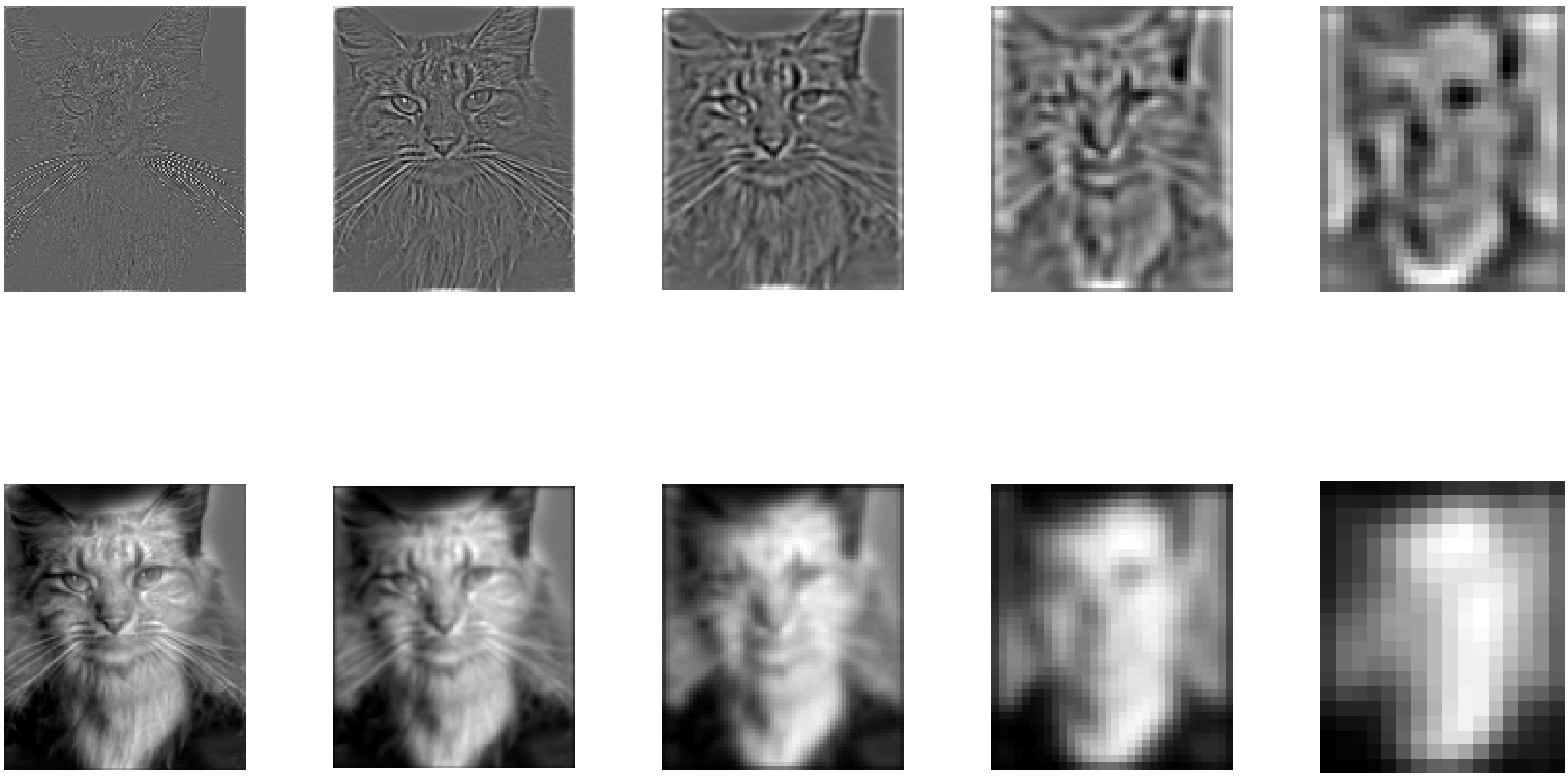


Can we reconstruct the original from the laplacian pyramid?

http://sepwww.stanford.edu/~morgan/texturematch/paper_html/node3.html

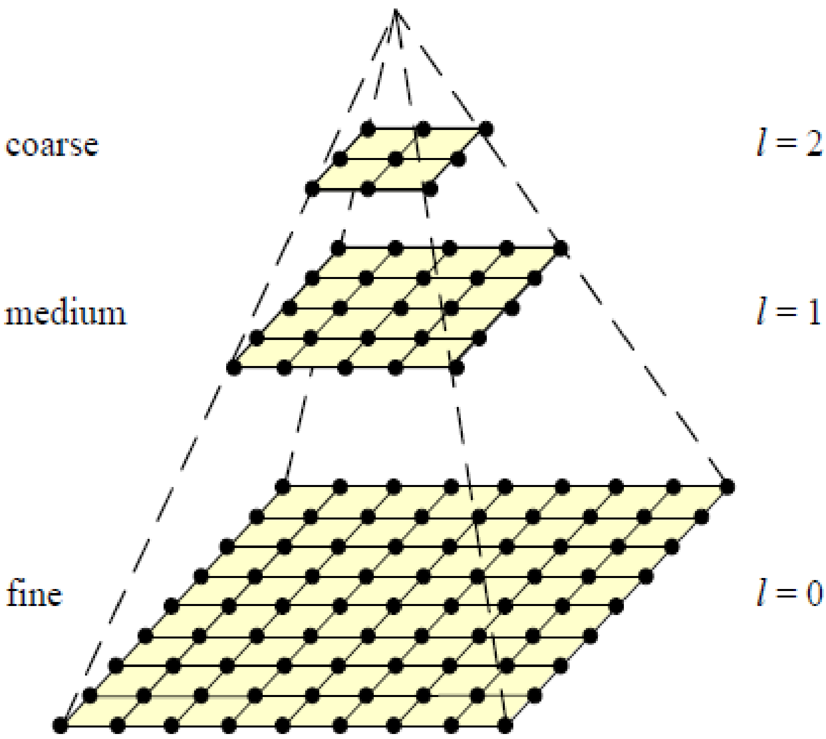
HYBRID IMAGE IN LAPLACIAN PYRAMID

High frequency → Low frequency



COARSE TO FINE IMAGE REGISTRATION

1. Compute Gaussian pyramid
2. Align with coarse pyramid
3. Successively align with finer pyramids
   - Search smaller range



Why is this faster?

Are we guaranteed to get the same result?

DENOISING

Typical noise models:

Additive

Gaussian noise

Local variance

Poisson

$$I_f = I + \eta, \quad \eta \in \mathcal{N}(0,1)$$

Shot noise: Often modeled as a Gaussian

Other

Salt and pepper

Speckle

$$I_f = I + \eta \cdot I, \quad \eta \in \mathcal{U}(0,1)$$

DENOISING ADDITIVE GAUSSIAN NOISE


Additive Gaussian noise (0,1)


Gaussian filter (sigma=1)


Gaussian filter (sigma=3)


Gaussian filter (sigma=6)

DENOISING S&P WITH GAUSSIAN FILTER

Salt & Pepper noise (20%)

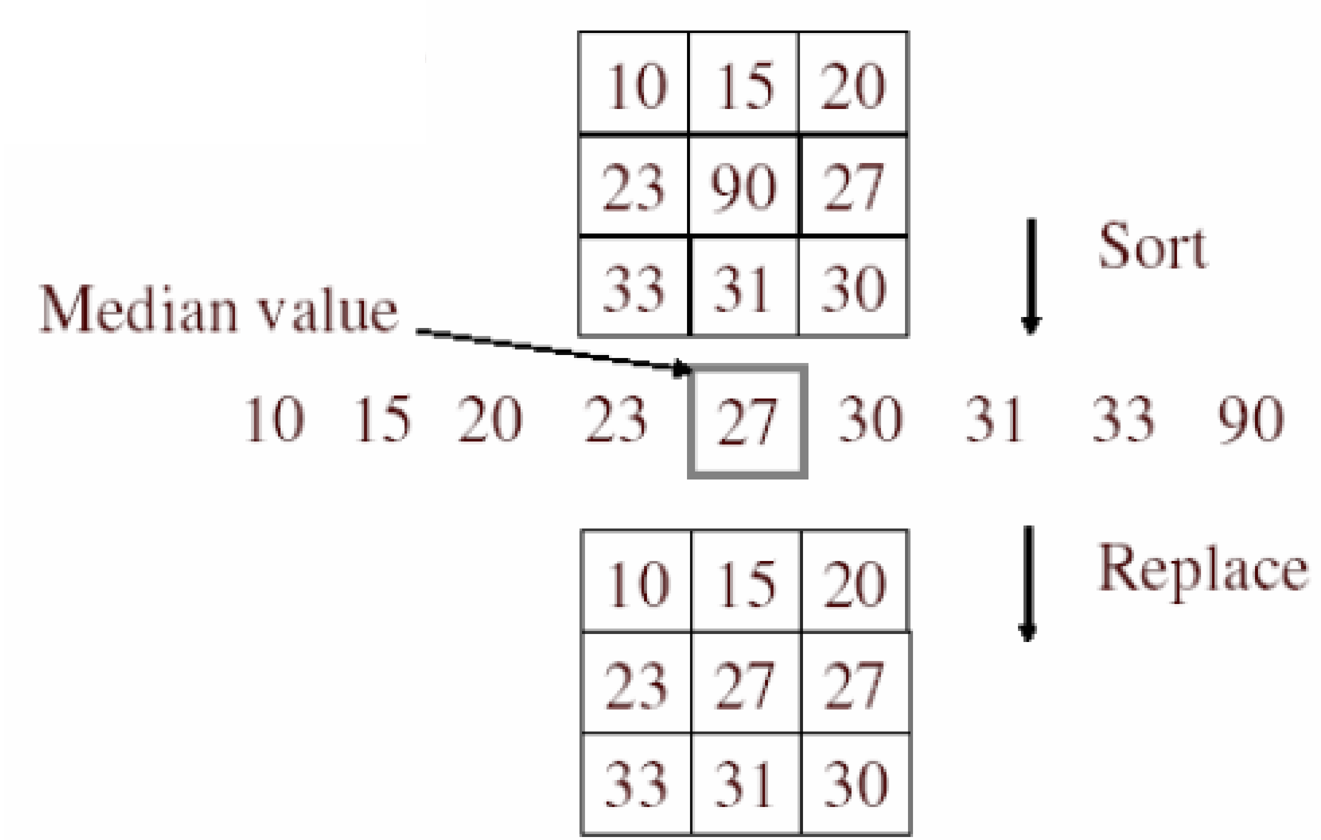
Gaussian filter (sigma=1)

Gaussian filter (sigma=3)

Gaussian filter (sigma=6)

ALTERNATIVE IDEA: MEDIAN FILTER

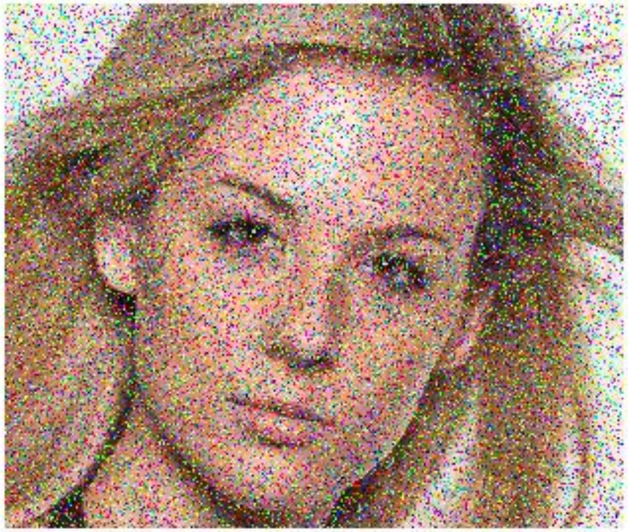
- A **median filter** operates over a window by selecting the median intensity in the window
-
-
-
-
-
-
-



- Is median filtering linear?

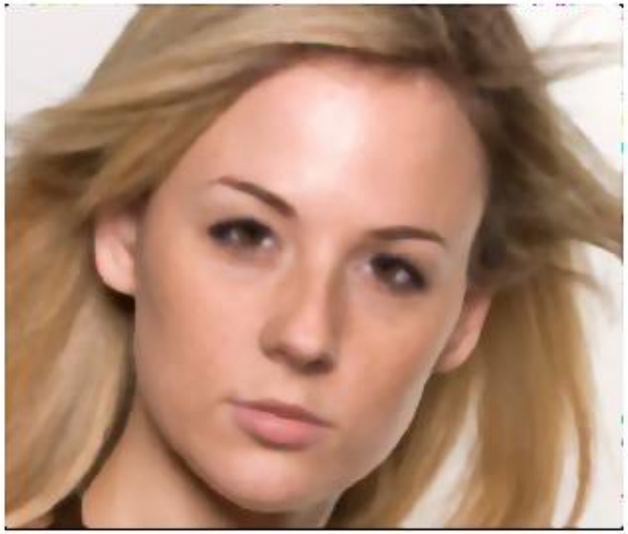
Source: K. Grauman

DENOISING S&P WITH MEDIAN FILTER

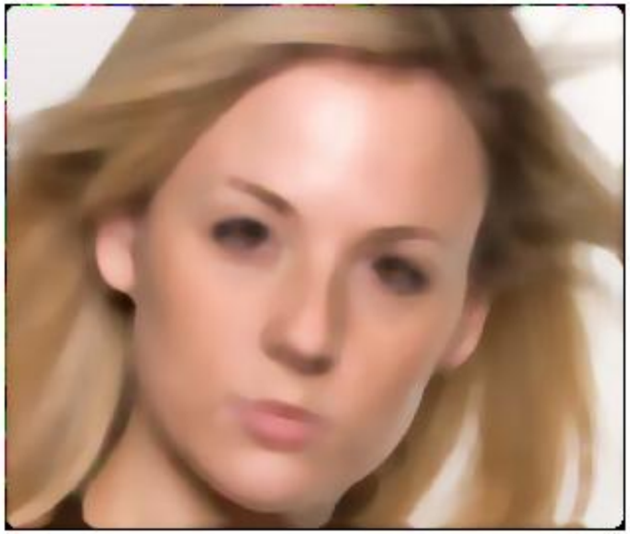


Salt & Pepper noise (20%)

Median filter (w=5)



Median filter (w=10)



Median filter (w=20)

DENOISING S&P WITH MEDIAN FILTER

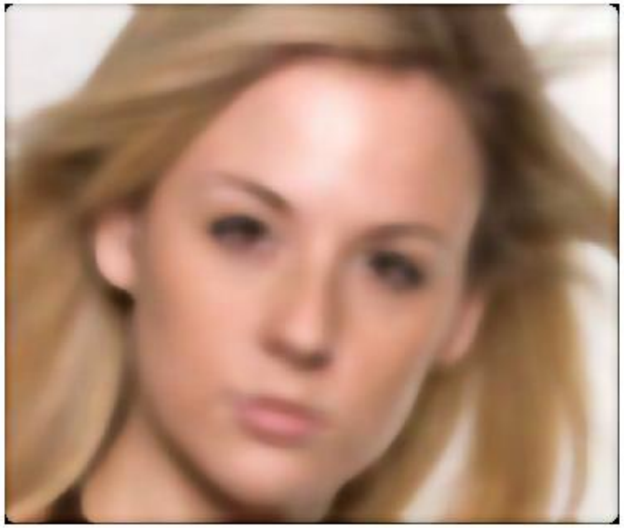
CLOSE UP

DENOISING G-N WITH MEDIAN FILTER

Additive Gaussian noise

Median filter (w=5)

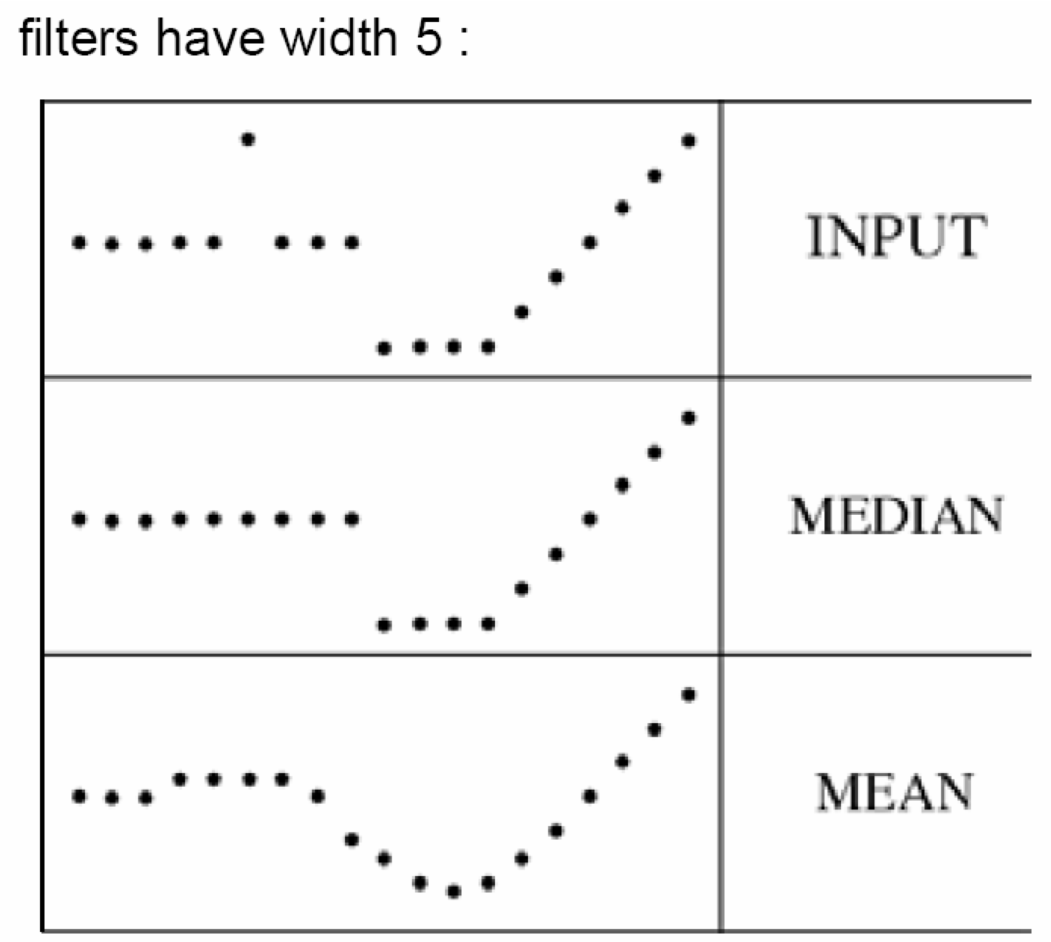
Median filter (w=10)



Median filter (w=20)

MEDIAN FILTER

- What advantage does median filtering have over Gaussian filtering?
  - Robustness to outliers



Source: K. Grauman

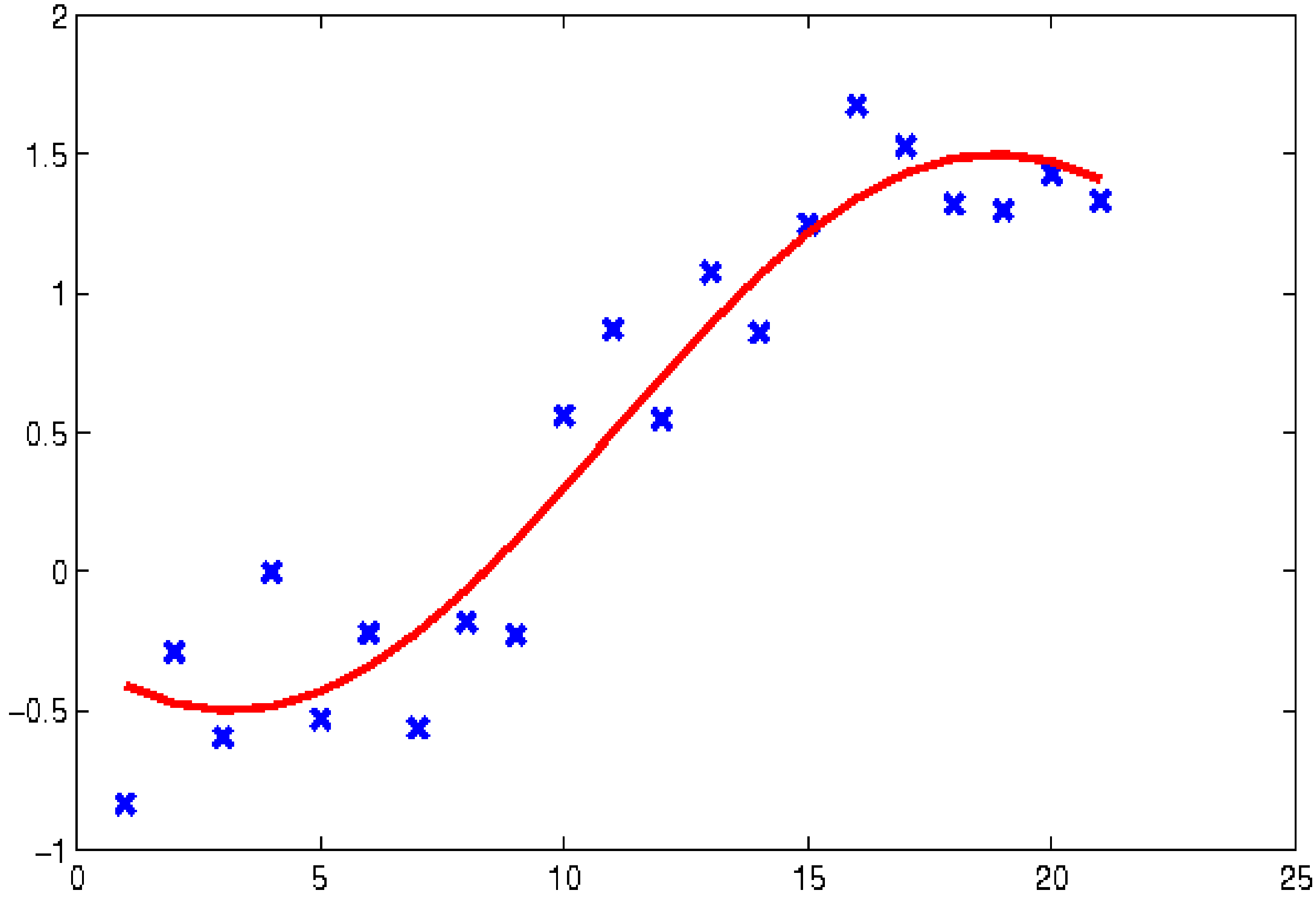
ADVANCED FILTERING

Gaussian and median filtering tend to destroy details.

Would not it be nice to have a filter that smooths but preserve structures?

RECONSTRUCTION GENERALITIES

The problem of signal reconstruction from observations involves problems such as denoising, deblurring, interpolation, etc.

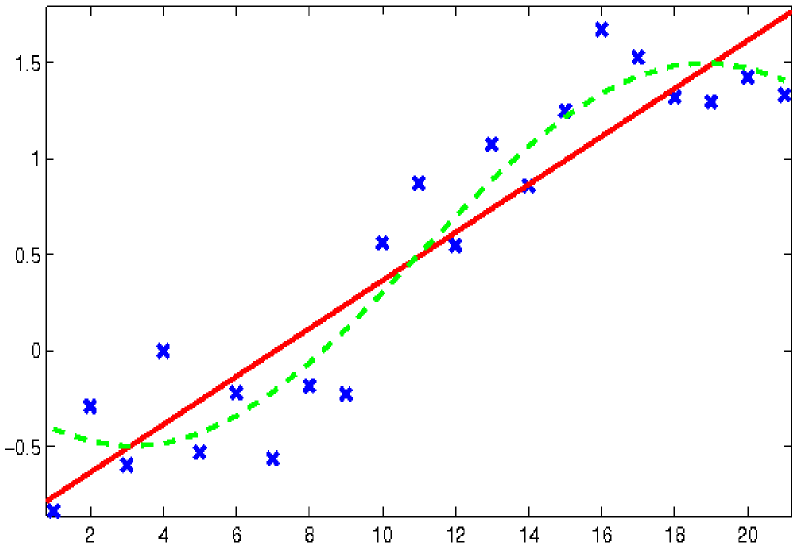


Original signal and noisy sampling.

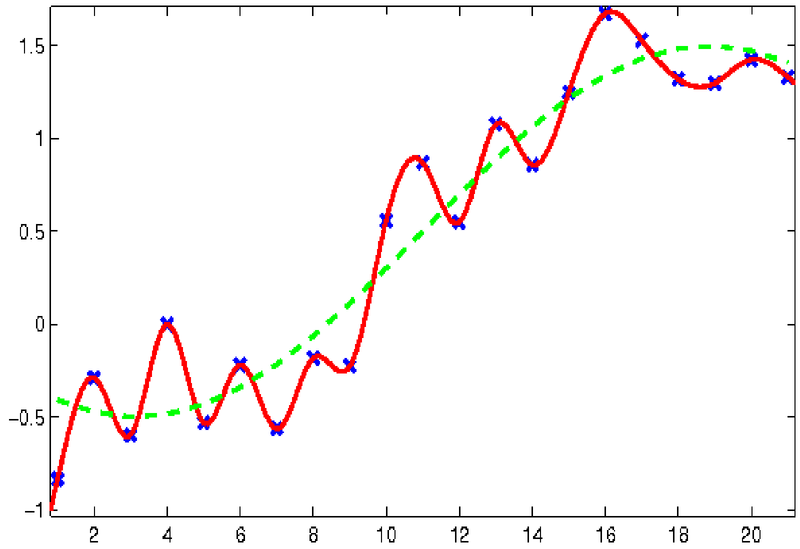
Goal: To reconstruct the red signal from the blue samples.

RECONSTRUCTION GENERALITIES

Goal: To reconstruct the green signal from the blue samples.



Reconstruction 1



Reconstruction 2

How can we measure the quality of the reconstruction?

RECONSTRUCTION GENERALITIES

How can we measure the quality of the reconstruction?

Creating a function representing our dislike or unacceptability about how well the reconstructed function approximates the ideal one.

$$\text{minimize} \quad \phi(y - z)$$

The most well known quality function is:
Least euclidean norm (aka Least Squares or Sum of Least Squares)

$$\text{minimize} \quad \|y - z\|_2^2$$

$$= (y - z)^T (y - z) = \sum_i (y_i - z_i)^2$$

A GENERAL FRAMEWORK FOR NON-LINEAR FILTERING

We can easily extend the measure by adding a weight that depends on the position in the image (p) and the value of the image pixel (y)

$$\hat{z}(p_j) = \arg\min_{z(p_j)} \sum_{i=1}^{n} (y_i - z(p_j))^2 K(p_i, p_j, y_i, y_j)$$

Measures the similarity between two pixels

Observations: for each pixel j we use the weighted value of all the image

WEIGHTED LEAST SQUARES PROBLEM

$$\hat{z}(p_j) = \arg\min_{z(p_j)} (y - z(p_j)1_n)^T K_j (y - z(p_j)1_n)$$

where

$$y = \begin{pmatrix} y_1 \\ y_2 \\ . \\ y_n \end{pmatrix} \; 1_n = \begin{pmatrix} 1 \\ 1 \\ . \\ 1 \end{pmatrix} \; K_j = \mathrm{diag}\begin{pmatrix} K(p_1, p_j, y_1, y_j) \\ K(p_2, p_j, y_2, y_j) \\ . \\ K(p_n, p_j, y_n, y_j) \end{pmatrix}$$

WEIGHTED LEAST SQUARES PROBLEM

$$\hat{z}(p_j) = \arg\min_{z(p_j)} (y - z(p_j)1_n)^T K_j (y - z(p_j)1_n)$$

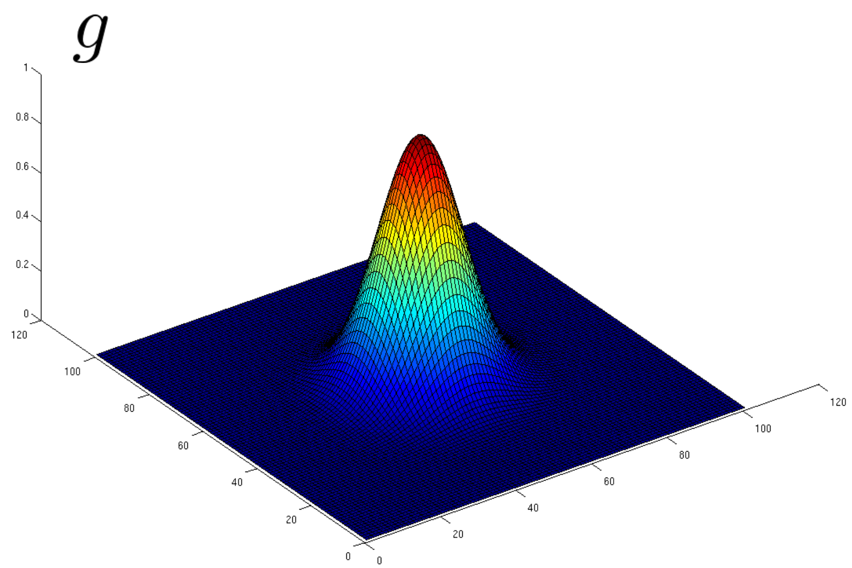
$$\Downarrow$$

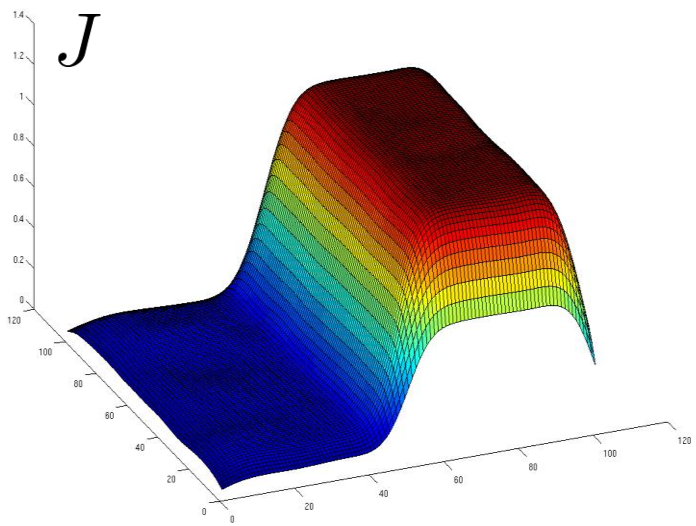
$$\begin{aligned}
\hat{z}(p_j) &= (1_n^T K_j 1_n)^{-1} 1_n^T K_j y \\
&= \sum_i \frac{K(p_i, p_j, y_i, y_j)}{\sum_i K(p_i, p_j, y_i, y_j)} y_i \\
&\boxed{= \sum_i W_{i,j} y_i} \\
&= w_j^T y
\end{aligned}$$

Convex combination
of all the data

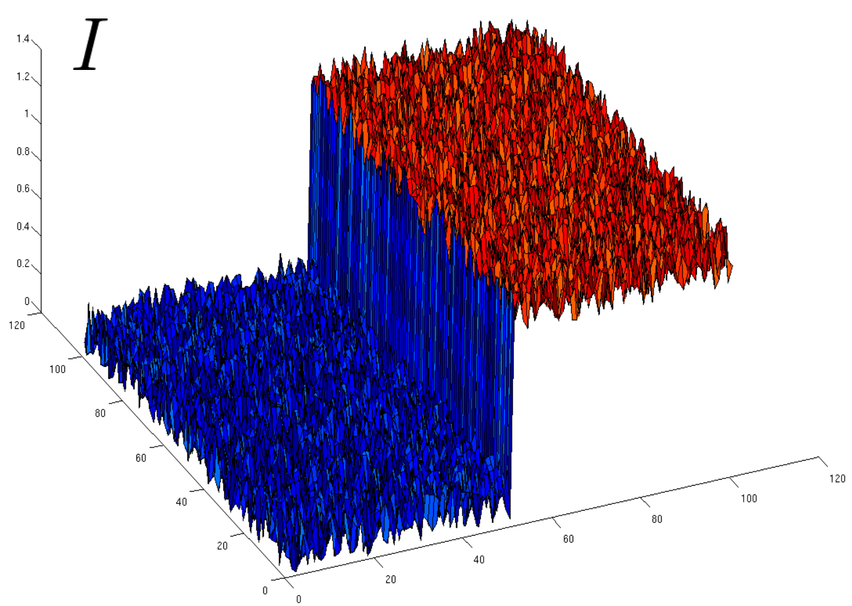
BILATERAL FILTERING

Recall a basic Gaussian filter

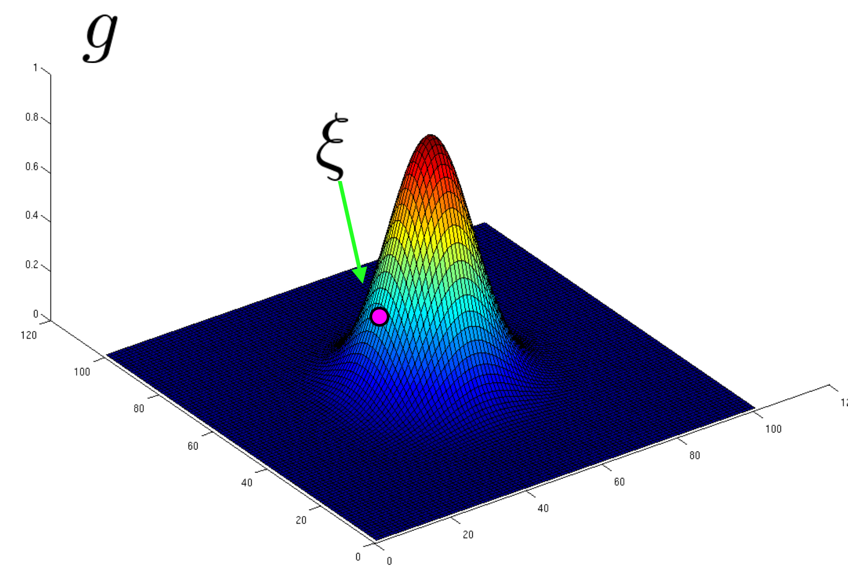


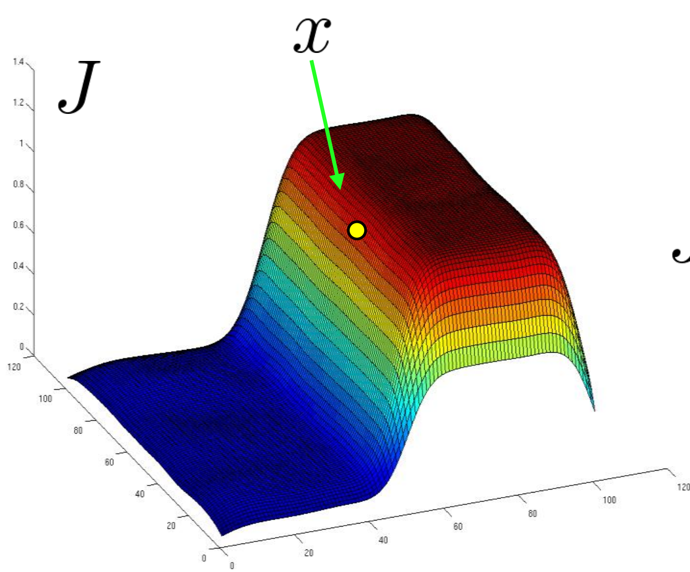


$$J = I \otimes g$$

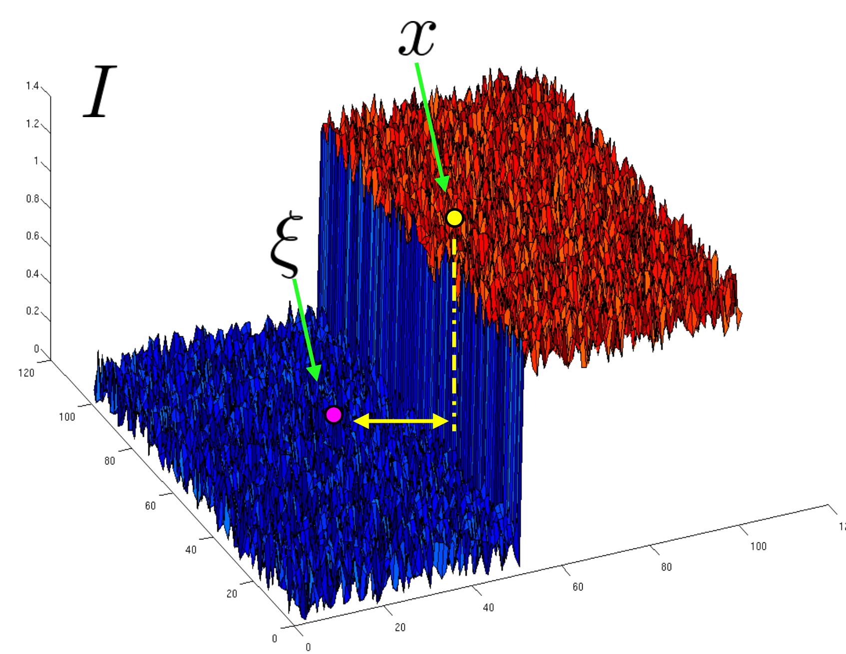


BILATERAL FILTERING





$$J(x) = \sum_{\xi} g(\xi, x) I(\xi)$$



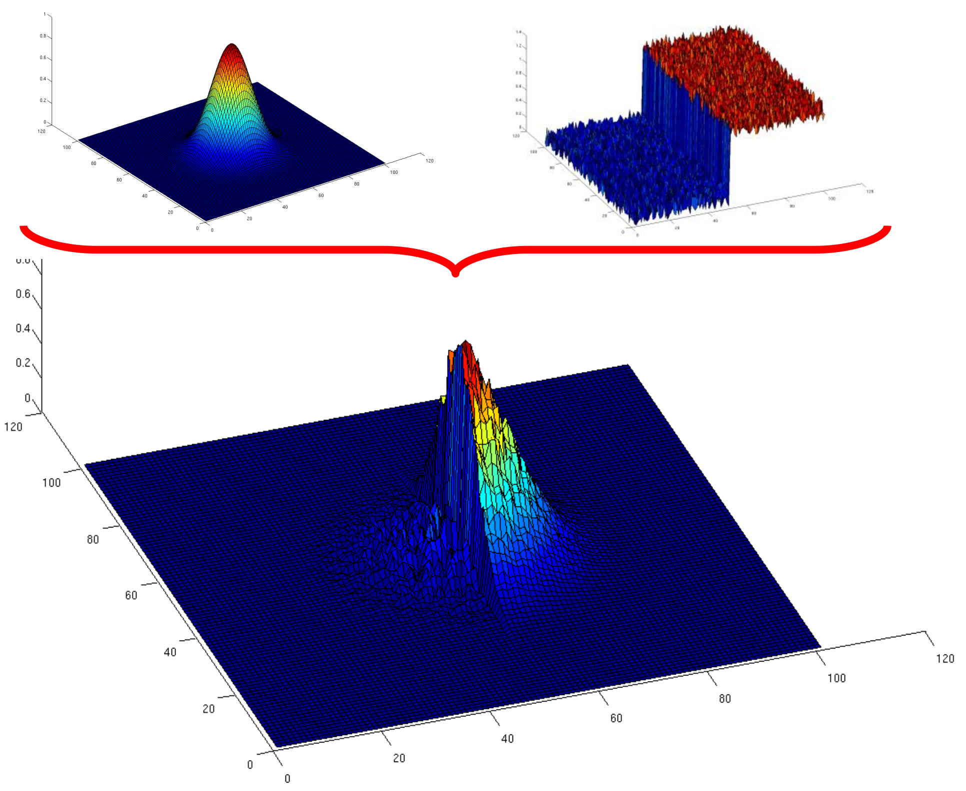
Recall a basic Gaussian filter. Observe that it can be seen as a weighted average in which g encodes the notion of distance.

But $I(\xi)$ pollutes the estimate … it is so different to $I(x)$!!!

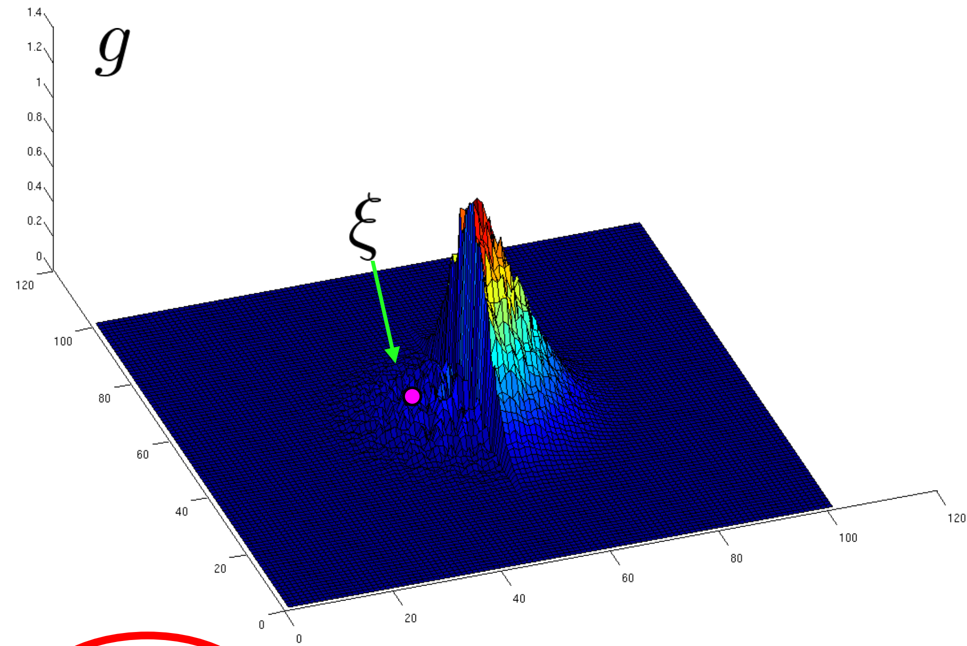
BILATERAL FILTERING

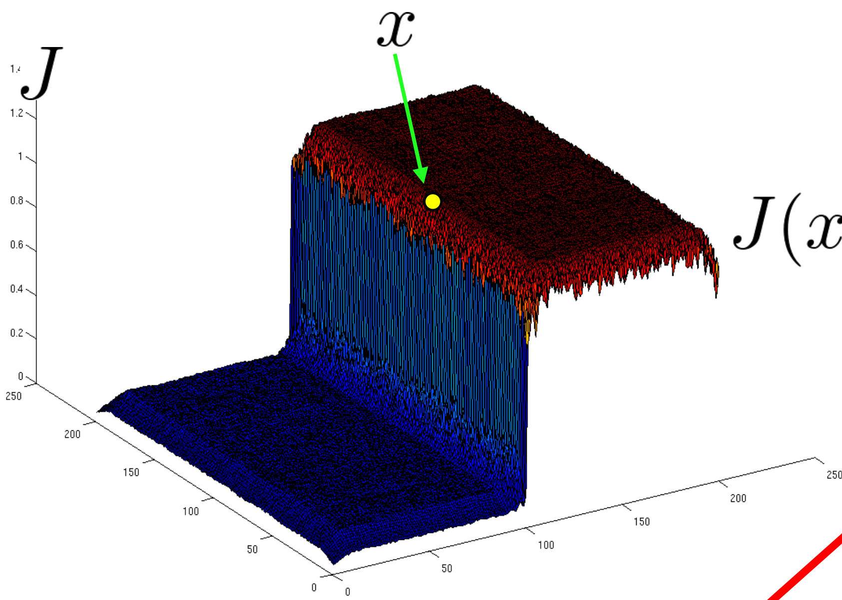
So, why don't we penalize intensity differences?

$$J(x) = \frac{1}{k(x)} \sum_{\xi} g_a(\xi, x) g_b(I(\xi) - I(x)) I(\xi)$$

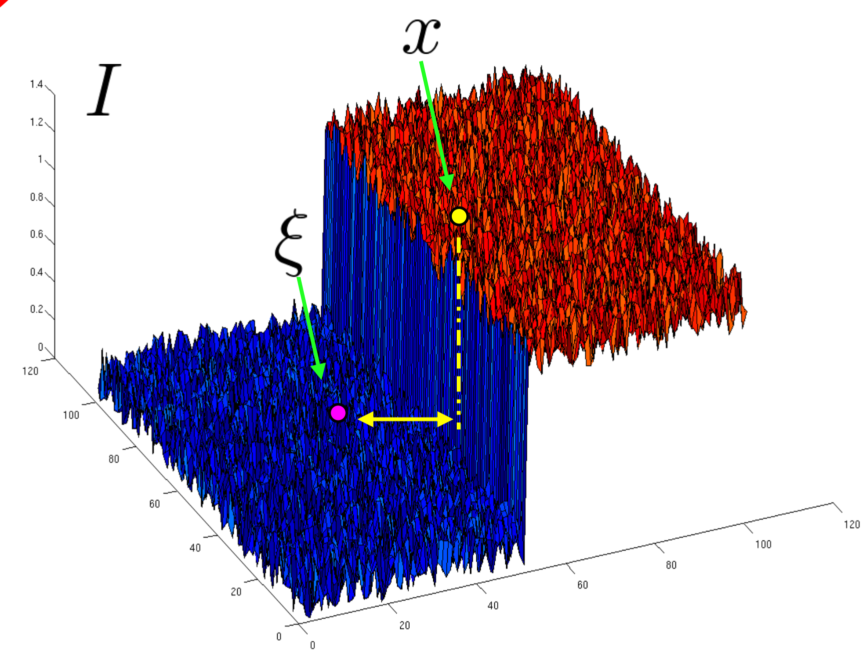


BILATERAL FILTERING





$$J(x) = \sum_{\xi} g(\xi, x) I(\xi)$$



But this filter changes for each output value!

BILATERAL FILTERING

Formulation:

$$J(x) = \frac{1}{k(x)} \sum_{\xi} g_a(\xi, x) g_b(I(\xi) - I(x)) I(\xi)$$

$$k(x) = \sum_{\xi} g_a(\xi, x) g_b(I(\xi) - I(x))$$

BILATERAL FILTERING

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