Linear Regression

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1. Simple linear regression: A simulated example

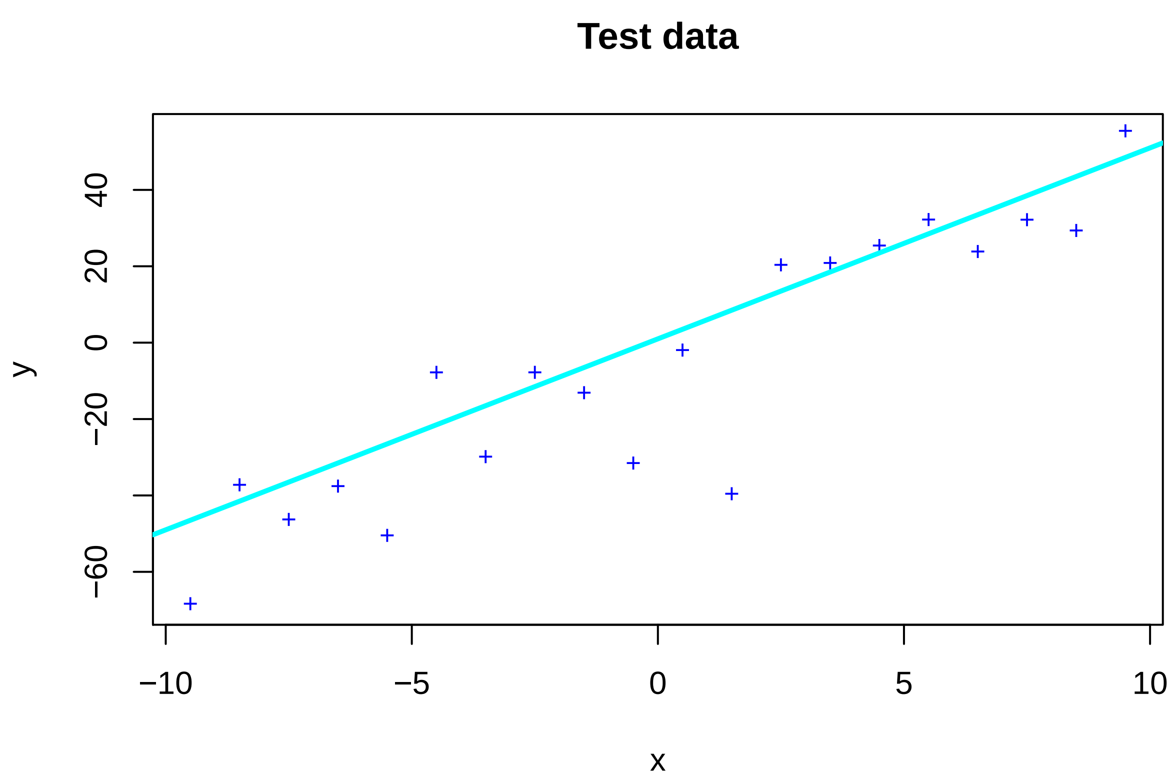
Generate a simulated dataset for simple linear regression

```r
# More or less arbitrary parameters for the data generation

trueA<-1
trueB<-5
trueSd<-15
sampleSize<-20 # n = sampleSize
```

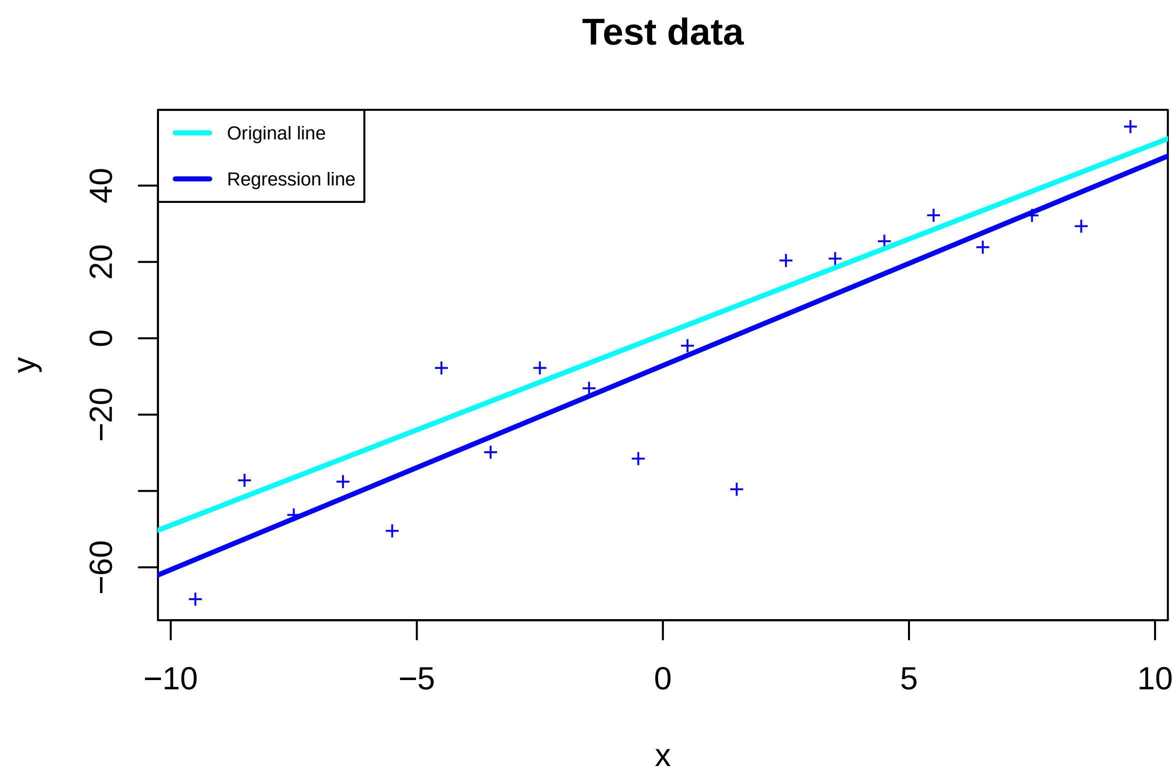
```r
# Uniformly spread x values in the interval [-n/2,n/2]
x<-(-(sampleSize-1)/2):((sampleSize-1)/2)
# y values of the form a + b*x + N(0,trueSd)
y<-trueA+trueB*x+rnorm(n=sampleSize,mean=0,sd=trueSd)
```

```r
# Plot data points plus the line used in the generation
# Note this line is NOT a regression line, which will be computed below
options(repr.plot.width=4, repr.plot.height=4)
plot(x,y,pch='+',cex=0.8,col="blue",main="Test data")
abline(trueA,trueB,lwd=2.5,col="cyan")
```



Adjust a least squares linear regression model

```
lm.1<-lm(y~x)
options(repr.plot.width=4, repr.plot.height=4)
plot(x,y,pch='+',cex=0.8,col="blue",main="Test data")
abline(trueA,trueB,lwd=2.5,col="cyan")
abline(lm.1,lwd=2.5,col="blue")
legend("topleft",c("Original line","","Regression line"), lwd=2.5,col=c("cyan","white","blue"),cex=0.6)
```



Extract information from the fitted model

```
print(lm.1)
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Coefficients:
## (Intercept)            x
##      -7.135        5.349
```

```
summary(lm.1)
```

```
##
## Call:
## lm(formula = y ~ x)
##
```

```
## Residuals:
##     Min      1Q  Median      3Q     Max
## -41.003  -5.788   1.734   9.846  22.873
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -7.1353     3.3695  -2.118   0.0484 *
## x             5.3490     0.5843   9.154 3.42e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 15.07 on 18 degrees of freedom
## Multiple R-squared:  0.8232, Adjusted R-squared:  0.8133
## F-statistic: 83.79 on 1 and 18 DF,  p-value: 3.419e-08
```

Regression coefficients

```r
Coeff<-coefficients(lm.1)
Coeff
```

```
## (Intercept)           x
##   -7.135320    5.349043
```

```r
a.hat<-Coeff[1]
b.hat<-Coeff[2]
#a.hat<-as.numeric(lm.1$coefficients[1])
#b.hat<-as.numeric(lm.1$coefficients[2])
round(a.hat,3)
```

```
## (Intercept)
##      -7.135
```

```r
round(b.hat,3)
```

```
##     x
## 5.349
```

2. Relevant quantities in a regression

1. Total Sum of Squares

```r
# Total sum of squares
TotalSS<-sum(y^2)
round(TotalSS,3)
```

```
## [1] 24132.67
```

```r
# Centered data
y0<-y-mean(y)
# Centered total sum of squares
TotalSS0<-sum(y0^2)
round(TotalSS0,3)
```

```
## [1] 23114.41
```

```r
# Total number of degrees of freedom
n<-length(y)
Totaldf<-n
```

```r
# Total number of degrees of freedom of the centered y
Totaldf0<-n-1
Totaldf
```

```
## [1] 20
```

```r
Totaldf0
```

```
## [1] 19
```

2. Fitted values and Regression Sum of Squares

```r
# Fitted values
yhat<-fitted.values(lm.1)
#yhat<-as.numeric(lm.1$fitted.values) # Alternative syntax
# Centered fitted values
yhat0<-yhat-mean(yhat)
# Regression Sum of Squares
RegSS<-sum(yhat^2)
round(RegSS,3)
```

```
## [1] 20045.41
```

```r
# Centered Regression Sum if Squares
RegSS0<-sum(yhat0^2)
round(RegSS0,3)
```

```
## [1] 19027.15
```

```r
# Number of degrees of freedom of the regression
Regdf<-2
# Number of degrees of freedom of the regression (centered)
Regdf0<-1

Regdf
```

```
## [1] 2
```

```r
Regdf0
```

```
## [1] 1
```

Check that `mean(y)` coincides with `mean(yhat)`

3. Regression residuals and Residual Sum of Squares

```r
# The regression residuals can be extracted as:
ytilde<-residuals(lm.1)
# ytilde<-as.numeric(lm.11$residuals) # Alternative syntax
# Also:
# ytilde<-y-yhat
# or
# ytilde<-y0-yhat0
#
# Note that, since both y and yhat have the same mean, regression residuals are centered.
ResSS<-sum(ytilde^2)
Resdf<-Totaldf0-Regdf0
round(ResSS,3)
```

```
## [1] 4087.26
```

```
Resdf
```

```
## [1] 18
```

4. [Total Sum of Squares] = [Regression Sum of Squares] + [Residual Sum of Squares]

```
# Both with the non-centered and with the centered version
round(TotalSS-(RegSS+ResSS),10)
```

```
## [1] 0
```

```
round(TotalSS0-(RegSS0+ResSS),10)
```

```
## [1] 0
```

5. Regression Coefficient of Determination (Multiple R-squared)

```
# By definition, with the centered sums of squares
R2<-RegSS0/TotalSS0
round(R2,4)
```

```
## [1] 0.8232
```

6. Adjusted Coefficient of Determination (Adjusted for the number p of predictors)

$$\bar{R}^2 = 1 - (1 - R^2)\frac{n-1}{n-p-1}$$

The Adjusted Coefficient of Determination is also equal to:

$$\bar{R}^2 = 1 - \frac{SS_{\text{res}}/\text{df}_e}{SS_{\text{tot0}}/\text{df}_{t0}}$$

```
p<-1
R2adj.1<-1-(1-R2)*(n-1)/(n-p-1)
round(R2adj.1,4)
```

```
## [1] 0.8133
```

```
R2adj.2<-1-(ResSS/Resdf)/(TotalSS0/Totaldf0)
round(R2adj.1,4)
```

```
## [1] 0.8133
```

7. Mean squares and regression F statistic

```
TotalMeanS0<-TotalSS0/Totaldf0
RegMeanS0<-RegSS0/Regdf0
ResMeanS<-ResSS/Resdf     # Remember that residuals are centered (hence there is no need of a "0" here)
F<-RegMeanS0/ResMeanS
# The p-value is the probability of obtaining F values larger than the observed one, assuming the null
# hypothesis that there is no regression relationship is true.
p.val<-1-pf(F,df1=Regdf0,df2=Resdf)
round(TotalMeanS0,3)
```

```
## [1] 1216.548
```

```r
round(RegMeanS0,3)
```

```
## [1] 19027.15
```

```r
round(ResMeanS,3)
```

```
## [1] 227.07
```

```r
round(F,3)
```

```
## [1] 83.794
```

```r
round(p.val,10)
```

```
## [1] 3.42e-08
```

The quotient F is a measure of how the Regression Mean Squares exceeds the Residual Mean of Squares.

When the model is a Gauss-Markov normal regression, this quantity follows a Fisher-Snedecor distributions with degrees of freedom Regdf0 and Resdf. The resulting p-value is computed assuming this is true. When it is larger than the standard significance level (p-value > 0.05) we conclude the regression model is non-significant.

8. The `anova()` function

Displays the Sums of Squares and Mean Squares, decomposed by each individual predictor and residuals contribution (here there is a single predictor `x`).

```r
lm.1.anova<-anova(lm.1)
lm.1.anova
```

```
## Analysis of Variance Table
##
## Response: y
##           Df  Sum Sq Mean Sq F value    Pr(>F)
## x          1 19027.2 19027.2  83.794 3.419e-08 ***
## Residuals 18  4087.3   227.1
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# str(lm.1.anova)
```

9. Residual standard error (estimate $\hat{\sigma}$ of the residuals standard deviation)

```r
# Computing from the relevant quantities we have:
ResSE<-sqrt(ResMeanS)
round(ResSE,3)
```

```
## [1] 15.069
```

```r
# Alternatively, from the anova() function output:
sigma2.hat<-lm.1.anova$Sum[2]/lm.1.anova$Df[2]
sigma.hat<-sqrt(sigma2.hat)
round(sigma.hat,3)
```

```
## [1] 15.069
```

10. Estimate of the matrix of variances and covariances of $\hat{a}$ and $\hat{b}$

```r
V<-vcov(lm.1)
round(V,3)
```

```
##             (Intercept)     x
## (Intercept)      11.354 0.000
## x                 0.000 0.341
```

```r
# Null out-of-diagonal entries result from the peculiar way x was constructed for this example.
# In general one would expect non-null entries.
```

11. Estimates of the standard deviations of the regression coefficients

```r
v<-diag(V)
sigma2.a<-as.numeric(v[1])
sigma2.b<-as.numeric(v[2])
sigma.a<-sqrt(sigma2.a)
sigma.b<-sqrt(sigma2.b)
round(sigma.a,4)
```

```
## [1] 3.3695
```

```r
round(sigma.b,4)
```

```
## [1] 0.5843
```

12. Student's t statistics

They are a standardized measure of how each of the regression coefficients, here $\hat{a}$ and $\hat{b}$, differ from zero.

When data are normal and the the regression model is Gauss-Màrkov these quantities follow a Student's distribution $t(n-p-1)$.

For each coefficient, the p-value is, in principle, used to test the hypotheses:

$$\begin{cases} H_0: & \text{The coefficient is null,} \\ H_1: & \text{The coefficient is not zero,} \end{cases}$$

In practice, these p-valors are taken as a mere hint, a first step to a more precise assessment of the importance of each predictor.

```r
t.a<-as.numeric(a.hat/sigma.a)
t.b<-as.numeric(b.hat/sigma.b)
round(t.a,4)
```

```
## [1] -2.1176
```

```r
round(t.b,4)
```

```
## [1] 9.1539
```

```r
p.val.a<-2*pt(-abs(t.a),n-2)
p.val.b<-2*pt(-abs(t.b),n-2)
round(p.val.a,8)
```

```
## [1] 0.048388
```

```r
round(p.val.b,12)
```

```
## [1] 3.4191e-08
```

```r
S<-summary(lm.1)
coefficients(S)
```

```
##              Estimate Std. Error   t value    Pr(>|t|)
## (Intercept) -7.135320  3.3694957 -2.117622 4.83880e-02
## x            5.349043  0.5843447  9.153918 3.41914e-08
```

13. The regression matrix (or model matrix)

Now we see how the least squares regression is actually computed:

```r
X<-model.matrix(lm.1)
str(X)
```

```
##  num [1:20, 1:2] 1 1 1 1 1 1 1 1 1 1 ...
##  - attr(*, "dimnames")=List of 2
##   ..$ : chr [1:20] "1" "2" "3" "4" ...
##   ..$ : chr [1:2] "(Intercept)" "x"
##  - attr(*, "assign")= int [1:2] 0 1
```

```r
X
```

```
##    (Intercept)    x
## 1            1 -9.5
## 2            1 -8.5
## 3            1 -7.5
## 4            1 -6.5
## 5            1 -5.5
## 6            1 -4.5
## 7            1 -3.5
## 8            1 -2.5
## 9            1 -1.5
## 10           1 -0.5
## 11           1  0.5
## 12           1  1.5
## 13           1  2.5
## 14           1  3.5
## 15           1  4.5
## 16           1  5.5
## 17           1  6.5
## 18           1  7.5
## 19           1  8.5
## 20           1  9.5
## attr(,"assign")
## [1] 0 1
```

14 . Regression coefficients (estimates of)

The regression coefficients vector $\hat{\beta} = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix}$ is computed as:

$$\hat{\beta} = (X' \cdot X)^{-1} \cdot X' \cdot y$$

```r
# Matrix to invert
Q<-t(X) %*% X
round(Q,3)
```

```
##             (Intercept)   x
## (Intercept)          20   0
## x                     0 665
```

```
# Compute regression coefficients:
Q1<-solve(Q)   # inverse matrix
beta.hat<-Q1 %*%t(X) %*% y
round(beta.hat,4)
```

```
##                [,1]
## (Intercept) -7.1353
## x            5.3490
```

15. The hat matrix and the fitted $\hat{y}$ values

The hat matrix for a regression with model matrix X is:

$$H = X \cdot (X' \cdot X)^{-1} \cdot X'.$$

It satisfies that:

$$H \cdot y = X \cdot (X' \cdot X)^{-1} \cdot X' \cdot y = X \cdot (X' \cdot X)^{-1} \cdot X' \cdot y = X \cdot \hat{\beta} = \hat{y}.$$

```
# The hat matrix is the [n,n] matrix:
H<-X %*% solve(Q)%*%t(X)
# H is the operator that "puts a hat" on y, giving yhat.
```

```
# Check that indeed this yhat coincides with the one obtained above
yhat.1<-H %*% y
round(max(abs(yhat-yhat.1)),15)
```

```
## [1] 2.1e-14
```

Properties of H

```
# The sum of diagonal entries in H (the trace of H) is equal to the rank of X
sum(diag(H))
```

```
## [1] 2
```

```
# H is an idempotent matrix. The square of H is equal to H.
round(max(abs(H %*% H-H)),15)
```

```
## [1] 0
```

```
#round(H,3)
```

16. Covariances of regression coefficients estimates

The matrix of variances and covariances of the coefficients vector:

$$\hat{\beta} = (X' \cdot X)^{-1} \cdot X' \cdot y$$

is computed by:

$$\mathrm{Var}(\hat{\beta}) = (X' \cdot X)^{-1} \cdot X' \cdot \mathrm{Var}(y) \cdot X \cdot (X' \cdot X)^{-1}$$

If the Gauss-Markov condition holds, then $\operatorname{Var}(y) = \sigma^2 \, I$ and the expression above is simplified:

$$\operatorname{Var}(\hat{\beta}) = \sigma^2 \, (X' \cdot X)^{-1}$$

```
# Compare this V1 with the one obtained above with the vcov() function
V1<-Q1*sigma2.hat
V1
```

```
##             (Intercept)         x
## (Intercept)     11.3535 0.0000000
## x                0.0000 0.3414587
```

3. The `Advertising` dataset

Download `Advertising.csv`, from the textbook web page An Introduction to Statistical Learning with Applications in R (ISLR). This dataset is used both in lesson 3 in this course and in Chapter 3 of the textbook. In this section we follow the treatment of the `Advertising` dataset from his chapter.

Set the directory where you saved the dataset file as the RStudio working directory.

Read dataset.

Omit the first column, variable `X`, the index of each sample. Anyway it can be recovered by: `row.names(Advertising)`.

```
Advertising<-read.csv("Advertising.csv")
Advertising<-Advertising[,-1]
```

Pairwise simple linear regressions

Following ISLR, Chapter 3:

Compute simple linear regressions, one for each of the predictors, `TV`, `Radio`, `Newspapers`.

Plot all three scatterplots, superimposing on them the regression line.

What can be said about goodness-of-fit of these models?

Which variable, of the possible predictors, `TV`, `Radio`, `Newspapers`, is a better predictor of `Sales`?

Compare the goodness of fit of these linear regressions with the k-NN regressions from last lesson.

```
#
# Insert your code here
#
```

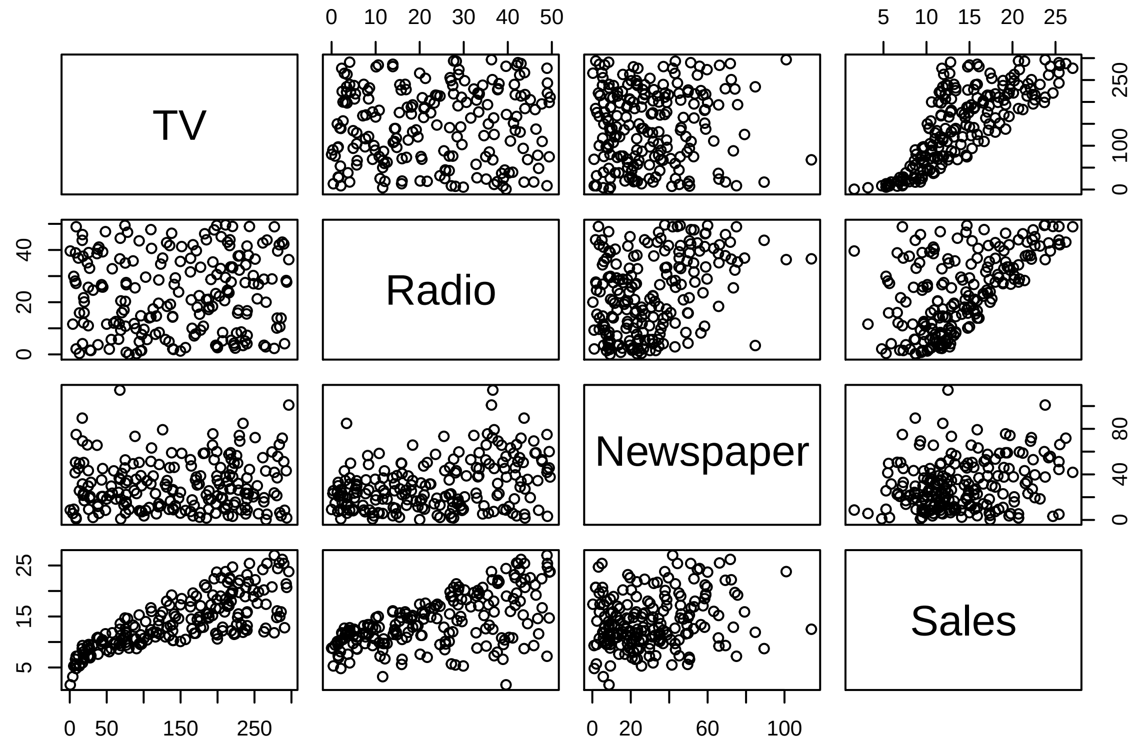
4. Multiple regression

```
Advertising<-read.csv("Advertising.csv")
Advertising<-Advertising[,-1]
```

```
str(Advertising)
```

```
## 'data.frame':    200 obs. of  4 variables:
##  $ TV       : num  230.1 44.5 17.2 151.5 180.8 ...
##  $ Radio    : num  37.8 39.3 45.9 41.3 10.8 48.9 32.8 19.6 2.1 2.6 ...
##  $ Newspaper: num  69.2 45.1 69.3 58.5 58.4 75 23.5 11.6 1 21.2 ...
##  $ Sales    : num  22.1 10.4 9.3 18.5 12.9 7.2 11.8 13.2 4.8 10.6 ...
```

```r
plot(Advertising)
```




```r
cor(Advertising)
```

```
##                   TV      Radio  Newspaper     Sales
## TV        1.00000000 0.05480866 0.05664787 0.7822244
## Radio     0.05480866 1.00000000 0.35410375 0.5762226
## Newspaper 0.05664787 0.35410375 1.00000000 0.2282990
## Sales     0.78222442 0.57622257 0.22829903 1.0000000
```

```r
round(cor(Advertising),2)
```

```
##             TV Radio Newspaper Sales
## TV        1.00  0.05      0.06  0.78
## Radio     0.05  1.00      0.35  0.58
## Newspaper 0.06  0.35      1.00  0.23
## Sales     0.78  0.58      0.23  1.00
```

```r
round(cor(Advertising),1)
```

```
##            TV Radio Newspaper Sales
## TV        1.0   0.1       0.1   0.8
## Radio     0.1   1.0       0.4   0.6
## Newspaper 0.1   0.4       1.0   0.2
## Sales     0.8   0.6       0.2   1.0
```

Multiple linear regression of Sales on all three predictors

```r
lm.Advertising.01<-lm(Sales~TV+Radio+Newspaper,data=Advertising)
summary(lm.Advertising.01)
```

```
##
## Call:
## lm(formula = Sales ~ TV + Radio + Newspaper, data = Advertising)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.8277 -0.8908  0.2418  1.1893  2.8292
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.938889   0.311908   9.422   <2e-16 ***
## TV           0.045765   0.001395  32.809   <2e-16 ***
## Radio        0.188530   0.008611  21.893   <2e-16 ***
## Newspaper   -0.001037   0.005871  -0.177     0.86
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.686 on 196 degrees of freedom
## Multiple R-squared:  0.8972, Adjusted R-squared:  0.8956
## F-statistic: 570.3 on 3 and 196 DF,  p-value: < 2.2e-16
```

```r
anova(lm.Advertising.01)
```

```
## Analysis of Variance Table
##
## Response: Sales
##            Df Sum Sq Mean Sq   F value Pr(>F)
## TV          1 3314.6  3314.6 1166.7308 <2e-16 ***
## Radio       1 1545.6  1545.6  544.0501 <2e-16 ***
## Newspaper   1    0.1     0.1    0.0312 0.8599
## Residuals 196  556.8     2.8
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# Equivalent, alternative, notation
lm.Advertising.02<-lm(Sales~.,data=Advertising)
summary(lm.Advertising.02)
```

```
##
## Call:
## lm(formula = Sales ~ ., data = Advertising)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.8277 -0.8908  0.2418  1.1893  2.8292
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.938889   0.311908   9.422   <2e-16 ***
## TV           0.045765   0.001395  32.809   <2e-16 ***
## Radio        0.188530   0.008611  21.893   <2e-16 ***
## Newspaper   -0.001037   0.005871  -0.177     0.86
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.686 on 196 degrees of freedom
## Multiple R-squared:  0.8972, Adjusted R-squared:  0.8956
## F-statistic: 570.3 on 3 and 196 DF,  p-value: < 2.2e-16
```

```
anova(lm.Advertising.02)
```

```
## Analysis of Variance Table
##
## Response: Sales
##            Df Sum Sq Mean Sq   F value Pr(>F)
## TV          1 3314.6  3314.6 1166.7308 <2e-16 ***
## Radio       1 1545.6  1545.6  544.0501 <2e-16 ***
## Newspaper   1    0.1     0.1    0.0312 0.8599
## Residuals 196  556.8     2.8
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

5. The *Credit* dataset

```
#install.packages("ISLR", repos="https://cloud.r-project.org/")
require(ISLR)
```

```
## Loading required package: ISLR
```

```
data(Credit)
str(Credit)
```

```
## 'data.frame':    400 obs. of  12 variables:
##  $ ID       : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ Income   : num  14.9 106 104.6 148.9 55.9 ...
##  $ Limit    : int  3606 6645 7075 9504 4897 8047 3388 7114 3300 6819 ...
##  $ Rating   : int  283 483 514 681 357 569 259 512 266 491 ...
##  $ Cards    : int  2 3 4 3 2 4 2 2 5 3 ...
##  $ Age      : int  34 82 71 36 68 77 37 87 66 41 ...
##  $ Education: int  11 15 11 11 16 10 12 9 13 19 ...
##  $ Gender   : Factor w/ 2 levels " Male","Female": 1 2 1 2 1 1 2 1 2 2 ...
##  $ Student  : Factor w/ 2 levels "No","Yes": 1 2 1 1 1 1 1 1 1 2 ...
##  $ Married  : Factor w/ 2 levels "No","Yes": 2 2 1 1 2 1 1 1 1 2 ...
##  $ Ethnicity: Factor w/ 3 levels "African American",..: 3 2 2 2 3 3 1 2 3 1 ...
##  $ Balance  : int  333 903 580 964 331 1151 203 872 279 1350 ...
```

```
# See the Credit dataset help file
# From the information we see there, we should:
# 1. Remove the ID from the dataset
# 2. Check the qualitative predictors are indeed coded as factors
Credit<-Credit[,-1]
str(Credit)
```

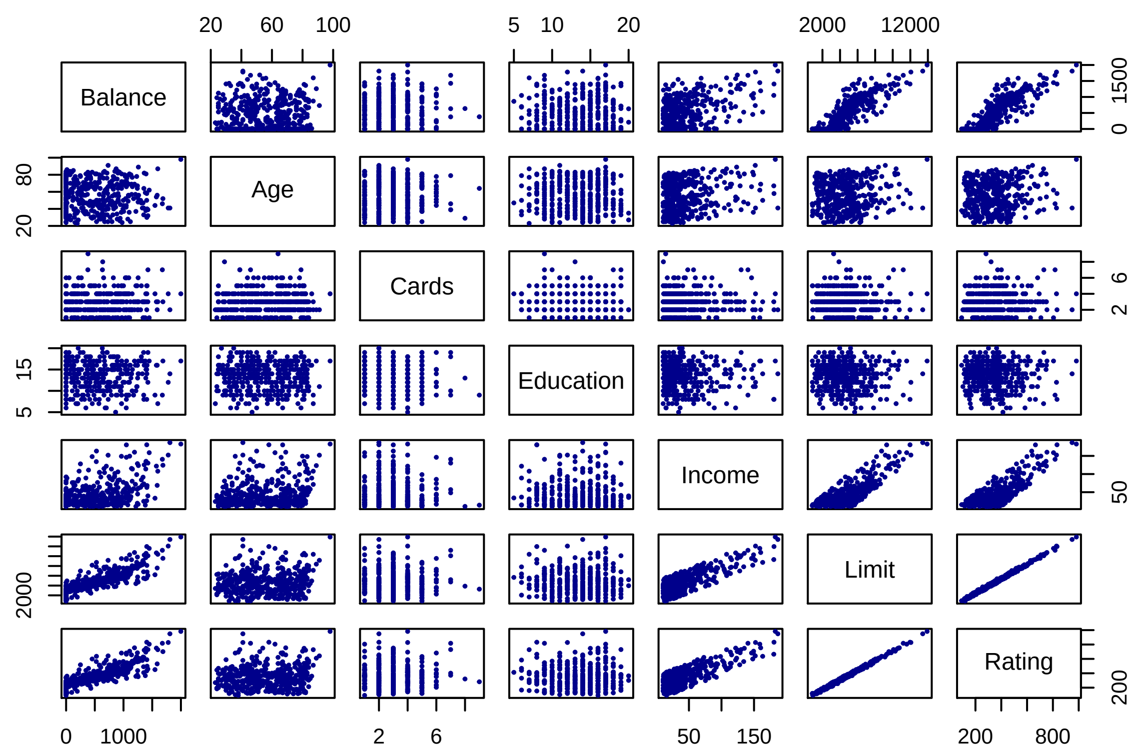
```
## 'data.frame':    400 obs. of  11 variables:
##  $ Income   : num  14.9 106 104.6 148.9 55.9 ...
##  $ Limit    : int  3606 6645 7075 9504 4897 8047 3388 7114 3300 6819 ...
##  $ Rating   : int  283 483 514 681 357 569 259 512 266 491 ...
##  $ Cards    : int  2 3 4 3 2 4 2 2 5 3 ...
```

```
##  $ Age      : int  34 82 71 36 68 77 37 87 66 41 ...
##  $ Education: int  11 15 11 11 16 10 12 9 13 19 ...
##  $ Gender   : Factor w/ 2 levels " Male","Female": 1 2 1 2 1 1 2 1 2 2 ...
##  $ Student  : Factor w/ 2 levels "No","Yes": 1 2 1 1 1 1 1 1 1 2 ...
##  $ Married  : Factor w/ 2 levels "No","Yes": 2 2 1 1 2 1 1 1 1 2 ...
##  $ Ethnicity: Factor w/ 3 levels "African American",..: 3 2 2 2 3 3 1 2 3 1 ...
##  $ Balance  : int  333 903 580 964 331 1151 203 872 279 1350 ...
```

```r
# Isolate the quantitative variables from Credit (as in Figure 3.6)
with(Credit,Credit.Quant<<-data.frame(Balance,Age,Cards,Education, Income,Limit,Rating))
str(Credit.Quant)
```

```
## 'data.frame':    400 obs. of  7 variables:
##  $ Balance  : int  333 903 580 964 331 1151 203 872 279 1350 ...
##  $ Age      : int  34 82 71 36 68 77 37 87 66 41 ...
##  $ Cards    : int  2 3 4 3 2 4 2 2 5 3 ...
##  $ Education: int  11 15 11 11 16 10 12 9 13 19 ...
##  $ Income   : num  14.9 106 104.6 148.9 55.9 ...
##  $ Limit    : int  3606 6645 7075 9504 4897 8047 3388 7114 3300 6819 ...
##  $ Rating   : int  283 483 514 681 357 569 259 512 266 491 ...
```

```r
plot(Credit.Quant,pch=19,col="DarkBlue",cex=0.3)
```




```r
round(cor(Credit.Quant),2)
```

```
##           Balance  Age Cards Education Income Limit Rating
## Balance      1.00 0.00  0.09     -0.01   0.46  0.86   0.86
## Age          0.00 1.00  0.04      0.00   0.18  0.10   0.10
```

```
## Cards          0.09 0.04  1.00      -0.05  -0.02  0.01   0.05
## Education     -0.01 0.00 -0.05       1.00  -0.03 -0.02  -0.03
## Income         0.46 0.18 -0.02      -0.03   1.00  0.79   0.79
## Limit          0.86 0.10  0.01      -0.02   0.79  1.00   1.00
## Rating         0.86 0.10  0.05      -0.03   0.79  1.00   1.00
```

6. Regression with qualitative predictors

Predicting `Balance` from `Gender`

```r
lm.Credit.Gender<-lm(Balance~Gender,data=Credit)
summary(lm.Credit.Gender)
```

```
##
## Call:
## lm(formula = Balance ~ Gender, data = Credit)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -529.54 -455.35  -60.17  334.71 1489.20
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    509.80      33.13  15.389   <2e-16 ***
## GenderFemale    19.73      46.05   0.429    0.669
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 460.2 on 398 degrees of freedom
## Multiple R-squared:  0.0004611,  Adjusted R-squared:  -0.00205
## F-statistic: 0.1836 on 1 and 398 DF,  p-value: 0.6685
```

```r
anova(lm.Credit.Gender)
```

```
## Analysis of Variance Table
##
## Response: Balance
##            Df   Sum Sq Mean Sq F value Pr(>F)
## Gender      1    38892   38892  0.1836 0.6685
## Residuals 398 84301020  211812
```

```r
X1<-model.matrix(lm.Credit.Gender)
```

By default, `lm()` prepares a model matrix for a qualitative predictor entered as a `factor` with g levels, adding $g-1$ columns constructed as follows: it takes one level as the base level and the remaining ones as "treatments" coded with dummy variables, where a 1 in a given column means that the corresponding treatment is present.

The `contr.*()` functions are internally used to generate these columns. In the above example for the `Credit` data, the two-levels version was used.

```r
contr.treatment(2)
```

```
##   2
## 1 0
## 2 1
```

```r
contr.treatment(3)
```

```
##   2 3
## 1 0 0
## 2 1 0
## 3 0 1
```

```r
contr.treatment(4)
```

```
##   2 3 4
## 1 0 0 0
## 2 1 0 0
## 3 0 1 0
## 4 0 0 1
```

Other codings are possible. For instance, if one does not want to distinguish one level as base:

```r
contr.sum(2)
```

```
##   [,1]
## 1    1
## 2   -1
```

```r
contr.sum(3)
```

```
##   [,1] [,2]
## 1    1    0
## 2    0    1
## 3   -1   -1
```

```r
contr.sum(4)
```

```
##   [,1] [,2] [,3]
## 1    1    0    0
## 2    0    1    0
## 3    0    0    1
## 4   -1   -1   -1
```

```r
lm.Credit.Ethnicity<-lm(Balance~Ethnicity,data=Credit)
summary(lm.Credit.Ethnicity)
```

```
## 
## Call:
## lm(formula = Balance ~ Ethnicity, data = Credit)
## 
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -531.00 -457.08  -63.25  339.25 1480.50 
## 
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)    
## (Intercept)           531.00      46.32  11.464   <2e-16 ***
## EthnicityAsian        -18.69      65.02  -0.287    0.774    
## EthnicityCaucasian    -12.50      56.68  -0.221    0.826    
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 460.9 on 397 degrees of freedom
```

```
## Multiple R-squared:  0.0002188,  Adjusted R-squared:  -0.004818
## F-statistic: 0.04344 on 2 and 397 DF,  p-value: 0.9575
```

```r
anova(lm.Credit.Ethnicity)
```

```
## Analysis of Variance Table
##
## Response: Balance
##            Df   Sum Sq Mean Sq F value Pr(>F)
## Ethnicity   2    18454    9227  0.0434 0.9575
## Residuals 397 84321458  212397
```

Interaction terms

```r
lm.Advertising.Radio.TV.1<-lm(Sales~Radio+TV,data=Advertising)
summary(lm.Advertising.Radio.TV.1)
```

```
##
## Call:
## lm(formula = Sales ~ Radio + TV, data = Advertising)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.7977 -0.8752  0.2422  1.1708  2.8328
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.92110    0.29449   9.919   <2e-16 ***
## Radio        0.18799    0.00804  23.382   <2e-16 ***
## TV           0.04575    0.00139  32.909   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.681 on 197 degrees of freedom
## Multiple R-squared:  0.8972, Adjusted R-squared:  0.8962
## F-statistic: 859.6 on 2 and 197 DF,  p-value: < 2.2e-16
```

```r
anova(lm.Advertising.Radio.TV.1)
```

```
## Analysis of Variance Table
##
## Response: Sales
##            Df  Sum Sq Mean Sq F value    Pr(>F)
## Radio       1 1798.67 1798.67  636.25 < 2.2e-16 ***
## TV          1 3061.57 3061.57 1082.98 < 2.2e-16 ***
## Residuals 197  556.91    2.83
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
lm.Advertising.Radio.TV.2<-lm(Sales~Radio+TV+Radio:TV,,data=Advertising)
summary(lm.Advertising.Radio.TV.2)
```

```
##
## Call:
## lm(formula = Sales ~ Radio + TV + Radio:TV, data = Advertising)
##
```

```
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.3366 -0.4028  0.1831  0.5948  1.5246
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 6.750e+00  2.479e-01  27.233   <2e-16 ***
## Radio       2.886e-02  8.905e-03   3.241   0.0014 **
## TV          1.910e-02  1.504e-03  12.699   <2e-16 ***
## Radio:TV    1.086e-03  5.242e-05  20.727   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9435 on 196 degrees of freedom
## Multiple R-squared:  0.9678, Adjusted R-squared:  0.9673
## F-statistic:  1963 on 3 and 196 DF,  p-value: < 2.2e-16
```

```
anova(lm.Advertising.Radio.TV.2)
```

```
## Analysis of Variance Table
##
## Response: Sales
##            Df  Sum Sq Mean Sq F value    Pr(>F)
## Radio       1 1798.67 1798.67 2020.47 < 2.2e-16 ***
## TV          1 3061.57 3061.57 3439.11 < 2.2e-16 ***
## Radio:TV    1  382.43  382.43  429.59 < 2.2e-16 ***
## Residuals 196  174.48    0.89
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# Alternative syntax
lm.Advertising.Radio.TV.3<-lm(Sales~Radio*TV,data=Advertising)
summary(lm.Advertising.Radio.TV.3)
```

```
##
## Call:
## lm(formula = Sales ~ Radio * TV, data = Advertising)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.3366 -0.4028  0.1831  0.5948  1.5246
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 6.750e+00  2.479e-01  27.233   <2e-16 ***
## Radio       2.886e-02  8.905e-03   3.241   0.0014 **
## TV          1.910e-02  1.504e-03  12.699   <2e-16 ***
## Radio:TV    1.086e-03  5.242e-05  20.727   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9435 on 196 degrees of freedom
## Multiple R-squared:  0.9678, Adjusted R-squared:  0.9673
## F-statistic:  1963 on 3 and 196 DF,  p-value: < 2.2e-16
```

```r
anova(lm.Advertising.Radio.TV.3)
```

```
## Analysis of Variance Table
##
## Response: Sales
##            Df  Sum Sq Mean Sq F value    Pr(>F)
## Radio       1 1798.67 1798.67 2020.47 < 2.2e-16 ***
## TV          1 3061.57 3061.57 3439.11 < 2.2e-16 ***
## Radio:TV    1  382.43  382.43  429.59 < 2.2e-16 ***
## Residuals 196  174.48    0.89
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Interaction of a quantitative predictor and a qualitative predictor in the `Credit` dataset

```r
lm.Credit.Income.Student.1<-lm(Balance~Income+Student,data=Credit)
summary(lm.Credit.Income.Student.1)
```

```
##
## Call:
## lm(formula = Balance ~ Income + Student, data = Credit)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -762.37 -331.38  -45.04  323.60  818.28
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 211.1430    32.4572   6.505 2.34e-10 ***
## Income        5.9843     0.5566  10.751  < 2e-16 ***
## StudentYes  382.6705    65.3108   5.859 9.78e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 391.8 on 397 degrees of freedom
## Multiple R-squared:  0.2775, Adjusted R-squared:  0.2738
## F-statistic: 76.22 on 2 and 397 DF,  p-value: < 2.2e-16
```

```r
anova(lm.Credit.Income.Student.1)
```

```
## Analysis of Variance Table
##
## Response: Balance
##            Df   Sum Sq  Mean Sq F value    Pr(>F)
## Income      1 18131167 18131167 118.119 < 2.2e-16 ***
## Student     1  5269691  5269691  34.331 9.776e-09 ***
## Residuals 397 60939054   153499
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
lm.Credit.Income.Student.2<-lm(Balance~Income*Student,data=Credit)
summary(lm.Credit.Income.Student.2)
```

```
##
## Call:
## lm(formula = Balance ~ Income * Student, data = Credit)
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -773.39 -325.70  -41.13  321.65  814.04
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)       200.6232    33.6984   5.953 5.79e-09 ***
## Income              6.2182     0.5921  10.502  < 2e-16 ***
## StudentYes        476.6758   104.3512   4.568 6.59e-06 ***
## Income:StudentYes  -1.9992     1.7313  -1.155    0.249
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 391.6 on 396 degrees of freedom
## Multiple R-squared:  0.2799, Adjusted R-squared:  0.2744
## F-statistic:  51.3 on 3 and 396 DF,  p-value: < 2.2e-16
```

```
anova(lm.Credit.Income.Student.2)
```

```
## Analysis of Variance Table
##
## Response: Balance
##                 Df   Sum Sq  Mean Sq  F value    Pr(>F)
## Income           1 18131167 18131167 118.2184 < 2.2e-16 ***
## Student          1  5269691  5269691  34.3593 9.661e-09 ***
## Income:Student   1   204509   204509   1.3334    0.2489
## Residuals      396 60734545   153370
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

7. The ***House prices*** dataset

From Kaggle House Prices: Advanced Regression Techniques