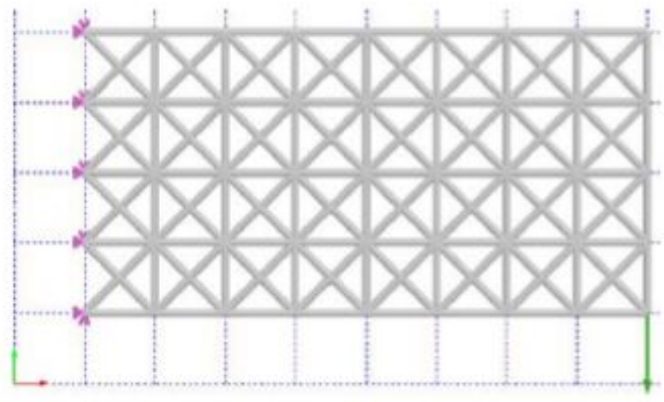
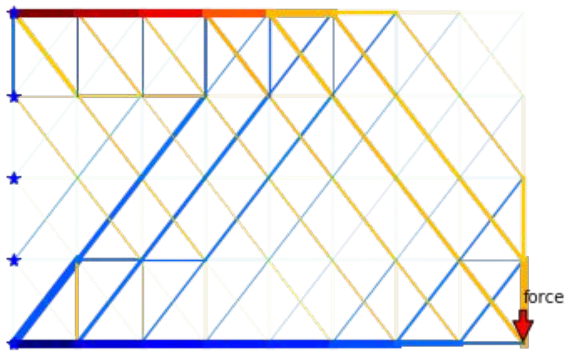
Topology Optimization in Python

Structural Systems I
Maggie Smith

Goal: For a set of members in a 2D truss given fixed node locations, determine the cross-sectional areas that maximize stiffness.





From nikorose87 on GitHub:
https://github.com/nikorose87/2D_topo_opt_truss_structure

The optimization problem

A measure of stiffness called compliance (C) is used.

$$C = F^T u$$

u = node displacements

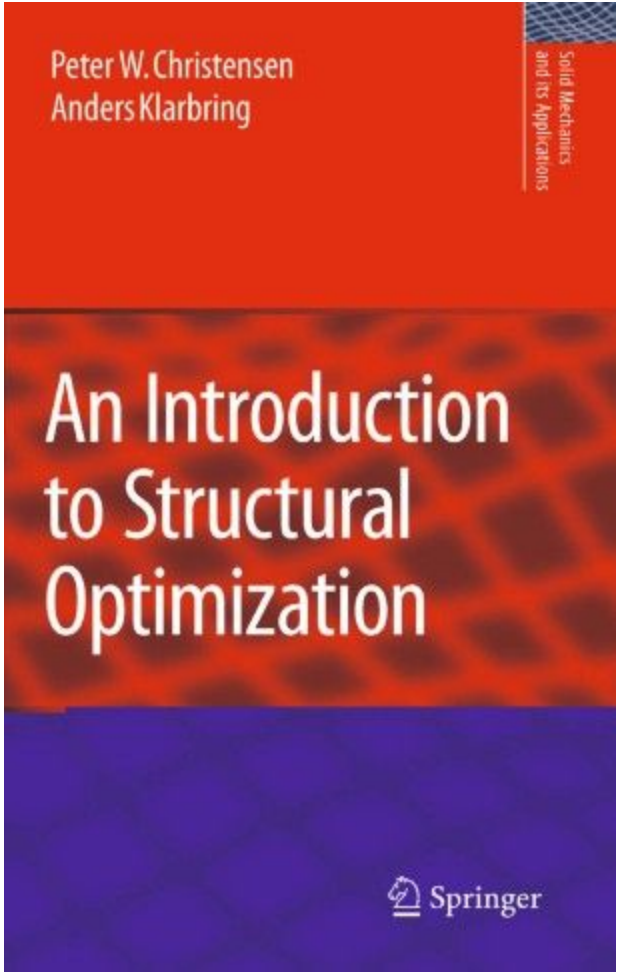
F = forces at nodes

We want to minimize compliance C to maximize stiffness.

An upper bound is placed on the total volume of the members, i.e.

$$\sum_{j=1}^{n} l_j x_j \leq V_{\max}$$

where x_j is the cross-sectional area of bar j and l_j is the length of bar j.



Chapter 5
Sizing Stiffness Optimization of a Truss

In this chapter we will describe in detail how sequential explicit approximations can be used to solve a particular large-scale structural optimization problem, namely that of determining the cross-sectional areas of the bars in a two-dimensional truss with fixed locations of the nodes so that its stiffness is maximized.

5.1 The Simultaneous Formulation of the Problem

In order to maximize the stiffness of a truss, see Fig. 5.1, we naturally need to introduce a suitable measure of stiffness. Here, we will choose to use the compliance C of the truss, i.e. $\boldsymbol{F}^T\boldsymbol{u}$, where $\boldsymbol{u}$ are the displacements of the nonsuppressed nodes of the truss, and $\boldsymbol{F}$ are the given external forces at these nodes. It should be clear that if the compliance is small, the truss will be stiff.

One can easily conceive other measures of stiffness, such as the size of the displacement vector, or $\boldsymbol{u}^T\boldsymbol{u}$. Compliance is a much more popular measure, however. There are at least two reasons for this. First, in a nested formulation, the compliance is a convex function of the design variables, i.e. the cross-sectional areas of the bars, whereas as we have seen in Sect. 2.6, $\boldsymbol{u}^T\boldsymbol{u}$ can be a nonconvex function of these variables. Second, in a truss where the compliance has been minimized for a given amount of material, all bars have the same stress. Intuitively, one has the impression that good use of the available material has been made if all bars have the same stress. Precise formulations and proofs of these statements will be given later.

The optimization problem we are faced with then, may be written as follows in a simultaneous formulation

$$(\mathbb{P})_{\mathrm{sf}} \quad \begin{cases} \min\limits_{\boldsymbol{x},\boldsymbol{u}} \boldsymbol{F}^T\boldsymbol{u} \\ \text{s.t.} \quad \boldsymbol{K}(\boldsymbol{x})\boldsymbol{u} = \boldsymbol{F} \\ \qquad \displaystyle\sum_{j=1}^{n} l_j x_j \le V_{\max} \\ \qquad \boldsymbol{x} \in \mathcal{X} = \{\boldsymbol{x} \in \mathbb{R}^n : x_j^{\min} \le x_j \le x_j^{\max},\ j = 1, \ldots, n\}, \end{cases}$$

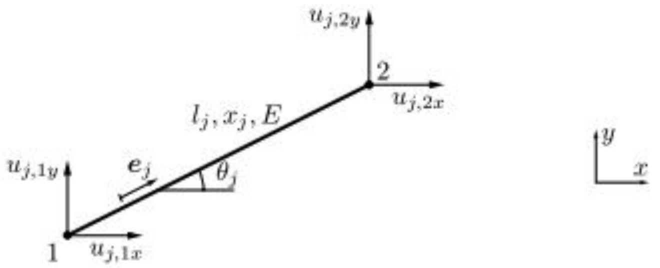


Fig. 5.2 A general bar j in the truss

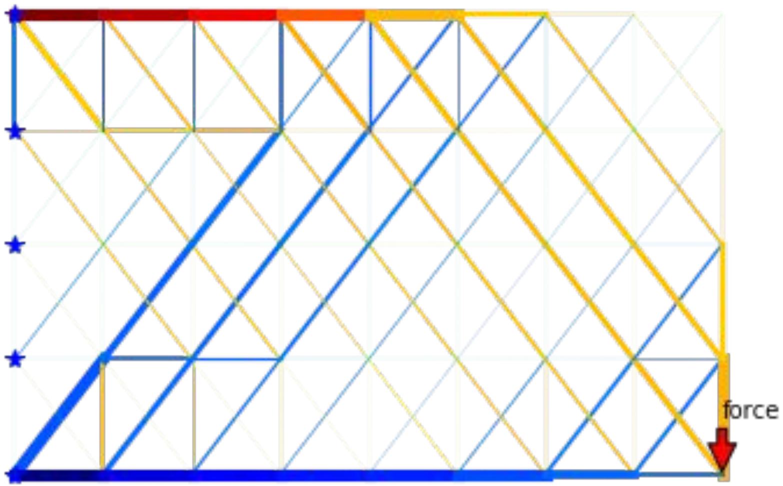
are given lower and upper bounds $x_j^{\min}$ and $x_j^{\max}$ on the design variables. It holds that $x_j^{\min} \geq 0$ and $x_j^{\max}$ is finite. Next, the matrix $\boldsymbol{K}(\boldsymbol{x})$ will be derived. To that end, study a general bar j, and let local node numbers 1 and 2 denote the end points of the bar, cf. Fig. 5.2. A unit vector $\boldsymbol{e}_j$ along the bar is defined so that it points from node 1 to node 2. The orientation of the bar is determined by the angle θ_j, which is the angle from the x-axis to $\boldsymbol{e}_j$, measured anti-clockwise, i.e. around the z-axis. Thus, $\boldsymbol{e}_j$ may be written

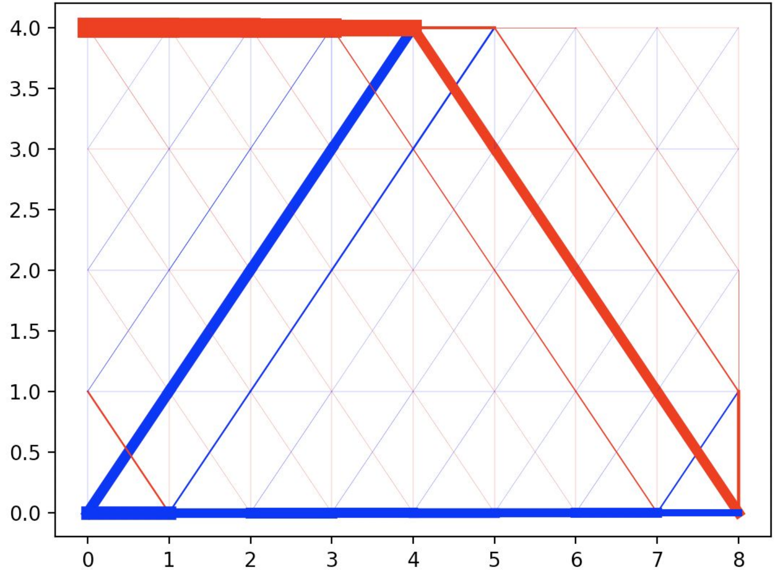
$$\boldsymbol{e}_j = \begin{bmatrix} \cos\theta_j \\ \sin\theta_j \end{bmatrix}.$$

The displacements of the end points of bar j are collected in a vector

$$\boldsymbol{u}_j = \begin{bmatrix} \boldsymbol{u}_{j,1} \\ \boldsymbol{u}_{j,2} \end{bmatrix}, \quad \text{where} \quad \boldsymbol{u}_{j,1} = \begin{bmatrix} u_{j,1x} \\ u_{j,1y} \end{bmatrix} \quad \text{and} \quad \boldsymbol{u}_{j,2} = \begin{bmatrix} u_{j,2x} \\ u_{j,2y} \end{bmatrix}.$$

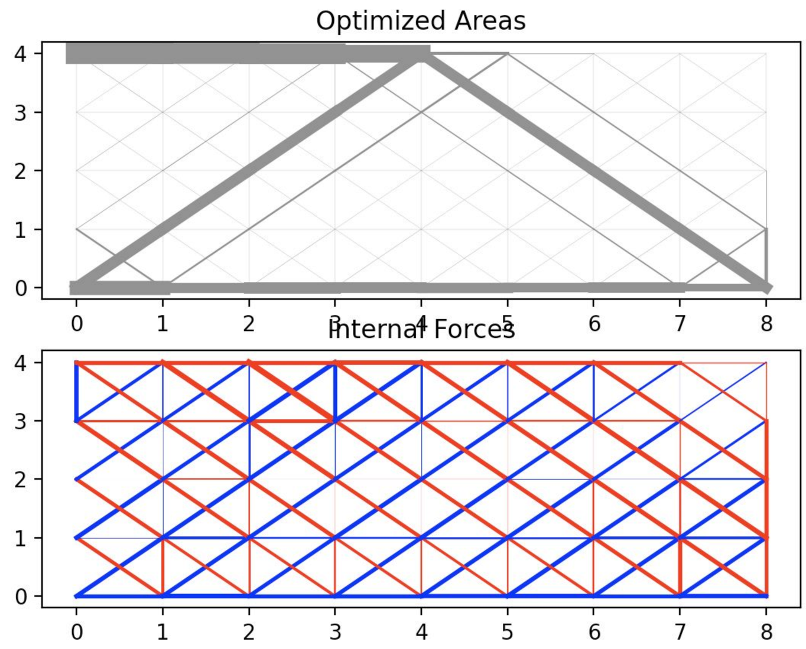
The original problem & my solution



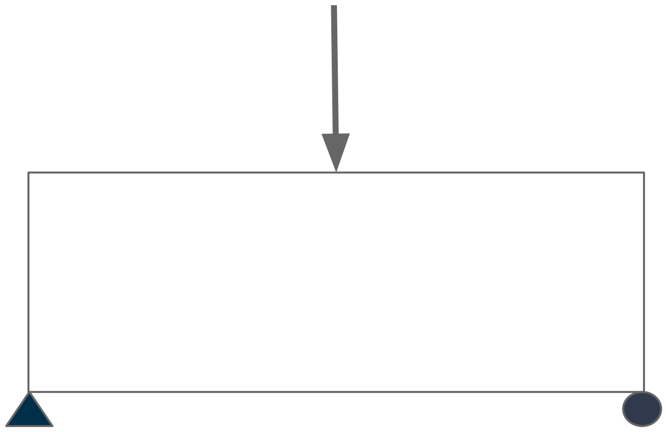


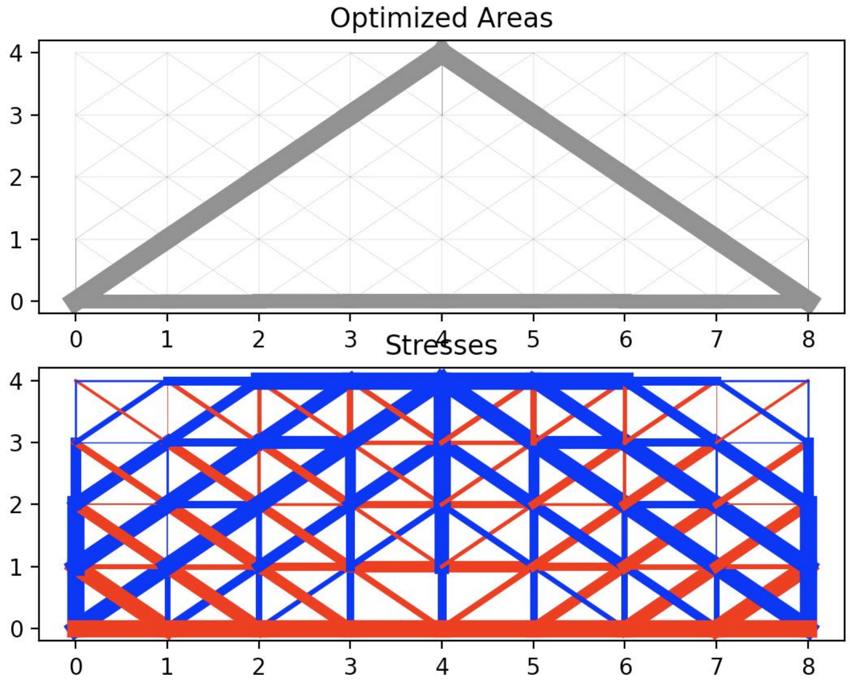
“in a truss where the compliance has been minimized for a given amount of material, all bars have the same stress. Intuitively, one has the impression that good use of the available material has been made if all bars have the same stress.”

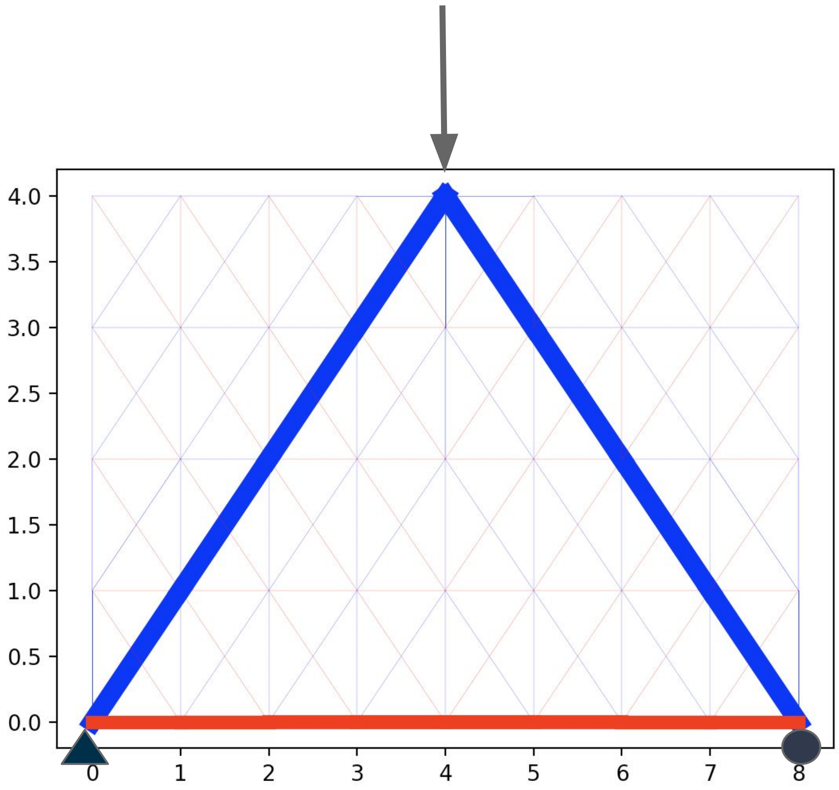
Christensen & Klabring, An introduction to structural optimization, p. 77



Beam: 4x8







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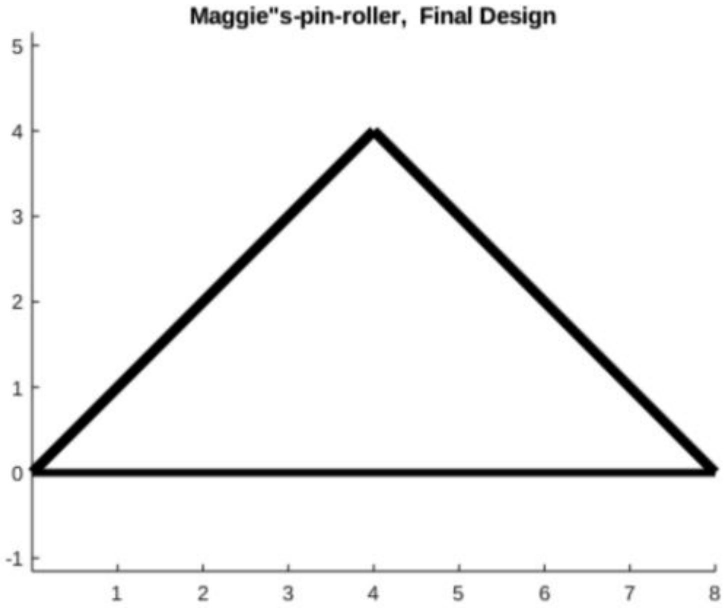

Maggie"s-pin-roller, Final Design
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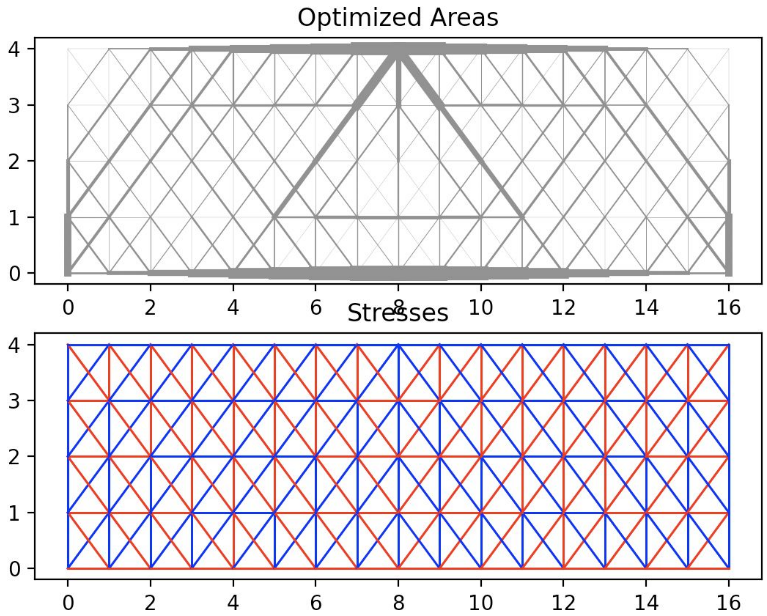
Beam: 4x16


Optimized Areas
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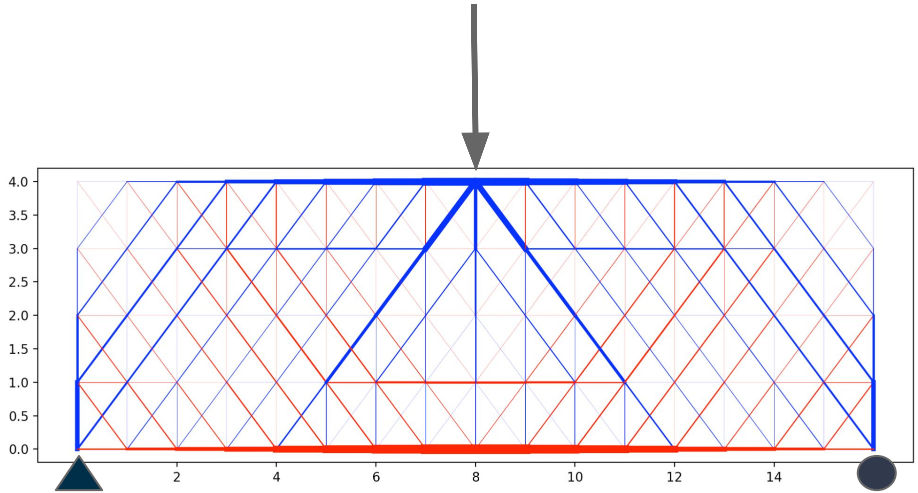
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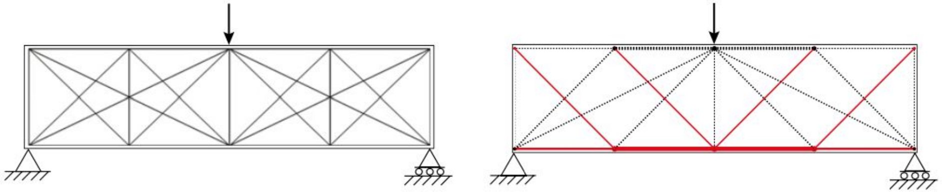
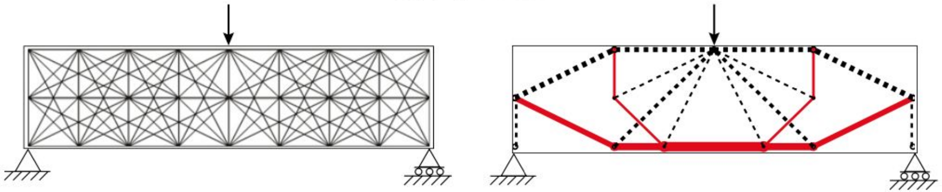
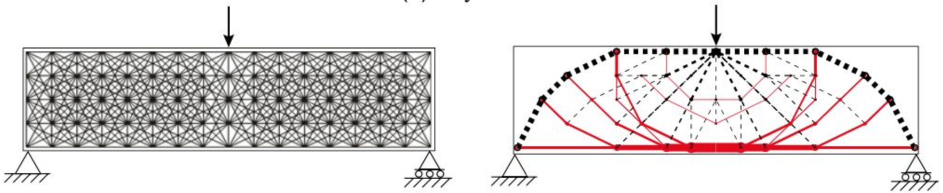


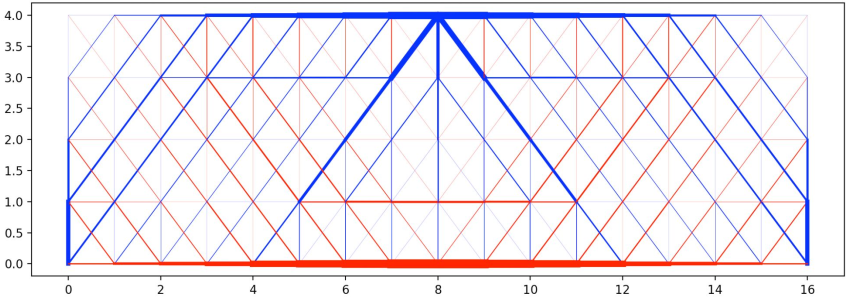
(a) 5 by 2 mesh



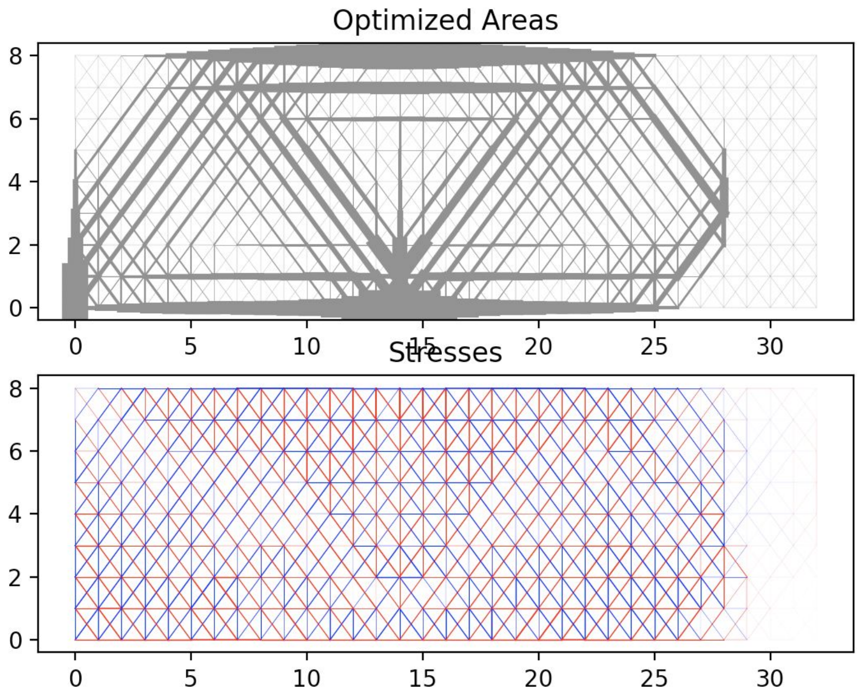
(b) 9 by 3 mesh



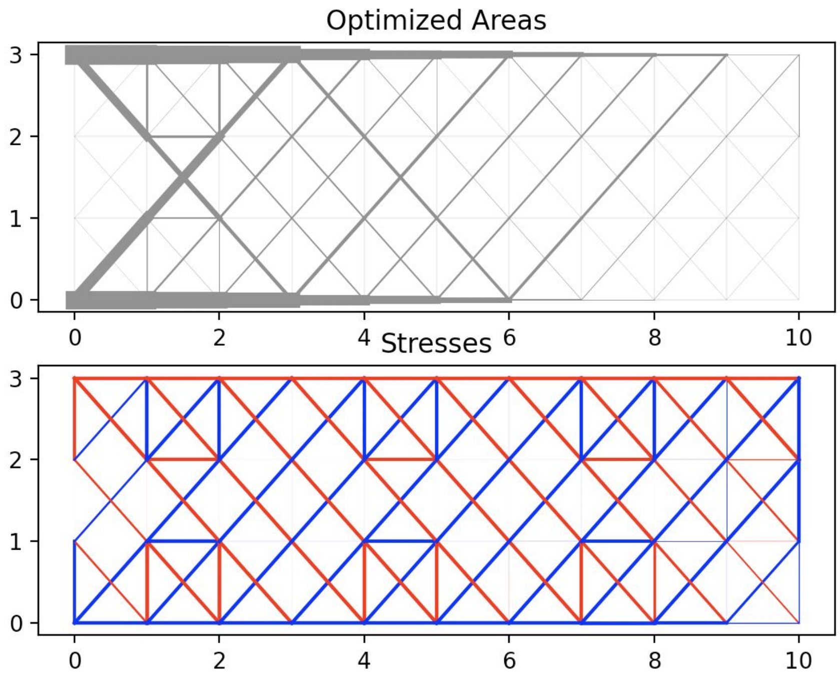
(c) 17 by 5 mesh

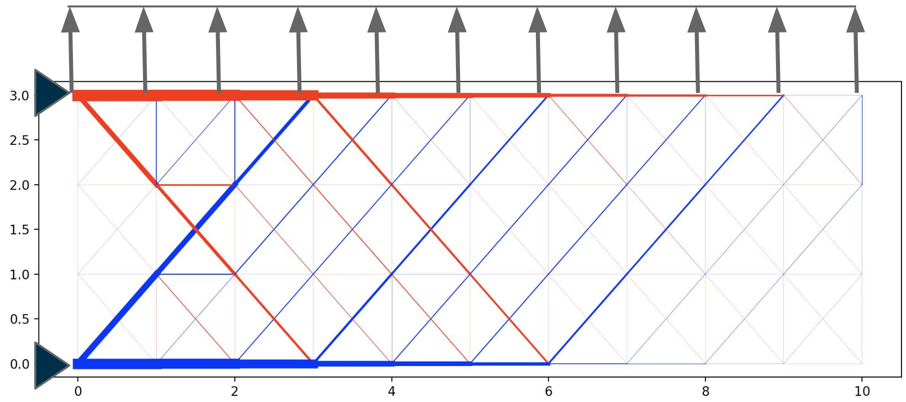


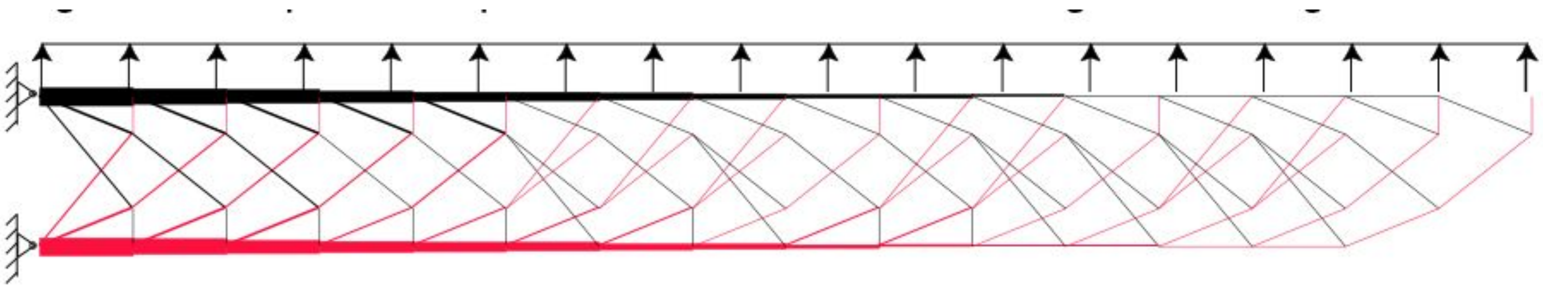
Beam: 8x32

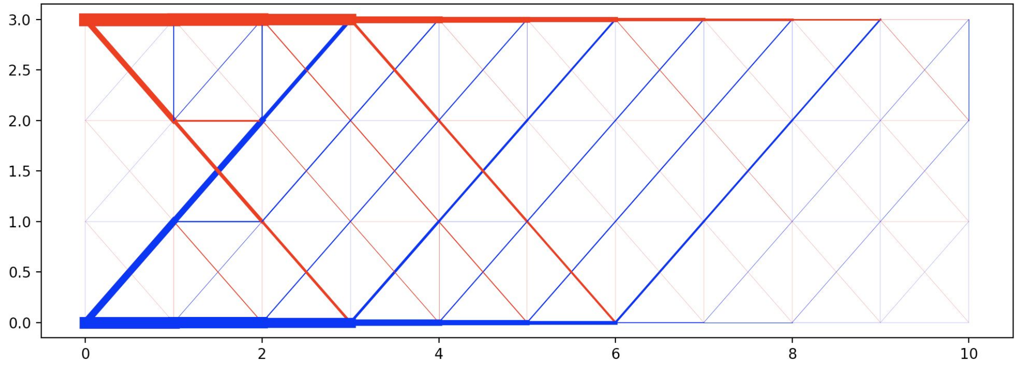


Plane Wing: 3x10









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