E.7.1. PENDULUM CASE: NUTS (MCMC) PERFORMANCE

Aiming to check how the performance of the MCMC method is affected by the time-length (number of data points) and the size of the state, we explore the pendulum tracking with EKF presented in (Särkkä & Svensson, 2023).

The main task is estimate the EKF parameters based on the simulated data of pendulum using BAT approach and check the NUTS performance using the computation duration of NUT as metric. Therefore, we executed this task through three tests with different setups, see Table 21. The setups include: size of state variables, number of estimated parameters, number of data points (measured data used to generate innovation errors in EKF) and number of iterations for NUTS. It important to point out that to be able to check the impact of size of state variable vector, we compare the test 2 with test 4 that is the task 1 of Bioprocess monitoring problem described in Section 5.1. The Table 20 shows the results obtained by sample the BAT posterior of EKF parameters give the simulated data. All estimation are closed to the ground truth. However, in the Table 21, we can see the difference in the compute duration of NUTS obtained for each test. Comparing test 2 with test 4, we can see that increasing the size of state variables vector, cause a significant increasing in the computation duration of ≈ 3247 sec. However, comparing test 2 with test 1, we can see that increasing the number of data points, cause a increasing in the computation duration of only ≈ 301 sec. Therefore, we conclude that increasing in the state variable vector can cause more impact on the computation duration than increase the number of data points.

The details about the dynamic model of pendulum and simulated data can be seen in (Särkkä & Svensson, 2023). Furthermore, the code of this example is available in our anonymous repository.

It important to point out that the ACP plots of all testes presented faster convergence, see Figures 12, 13 and 14. Convergence in this context means that the chain has reached a stationary distribution that adequately represents the posterior. This is crucial for ensuring that the samples drawn from the chain can be used for reliable statistical inference.

Table 20. Results of sampling the BAT for EKF with simulated data of pendulum.

		TEST1		TEST 2		TEST 3	
UKF COMPONENTS	GROUND TRUTH	MEAN	STD	MEAN	STD	MEAN	STD
$Q_{1,1}$	3.3E-9	3.2345E-9	9.726E-11	3.1345E-9	9.226E-11	3.3345E-9	9.2345E-11
$Q_{2,2}$	5.0E-7	5.1011E-7	1.630E-8	5.0011E-7	1.330E-8	5.555E-7	1.243E-8
$Q_{3,3}$	5.0E-7	5.0306E-7	1.501E-8	5.0006E-7	1.2201E-8	5.0456E-7	1.3425E-8
$Q_{4,4}$	0.0001	9.9708E-5	9.6978E-6	9.0008E-5	9.08E-6	9.5656E-5	9.3244E-6
R	0.1	0.0997	0.0102	0.1004	0.0101	0.10006	0.0102
g	9.81	9.8111	0.0193	9.8103	0.0200	9.8101	0.0203
$P_{1,1}$	1	1.029	0.5124	0.9962	0.4802	0.9963	0.4876
$P_{1,2}$	0	-	-	-	-	0.9966	0.4903
$P_{2,1}$	0	-	-	-	-	0.4036	0.3060
$P_{2,2}$	1	0.9791	0.5046	0.9964	0.4991	0.9966	0.3010
$\mathrm{M}_1(0)$	0	-	-	-	-	0.3946	0.3066
$\mathrm{M}_2(0)$	0	-	-	-	-	0.3958	0.2953

Table 21. NUTS (MCMC) performance scale with time-length (number of data points) and the size of the state.

	Pendulum problem with EKF			Bioprocess monitoring problem (Region 2 in Task1)
	Test 1	Test 2	Test 3	Test 4
Size of state variables	2	2	2	7
Number of estimated parameters	8	8	12	8
Number of data points	500	53	53	53
Number of iterations for NUTS	3000	3000	3000	3000
Compute Durations of NUTS	$\approx$354 sec	$\approx$53 sec	$\approx$123 sec	$\approx$ 3300 sec

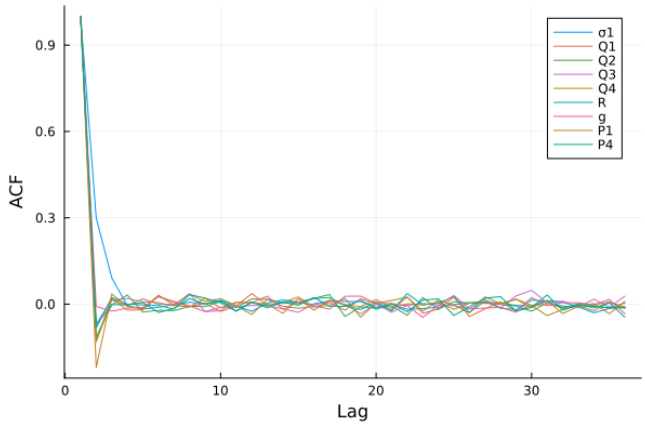

Figure 12. ACF plots of BAT samples computed up to a lag of 30 during test 1.

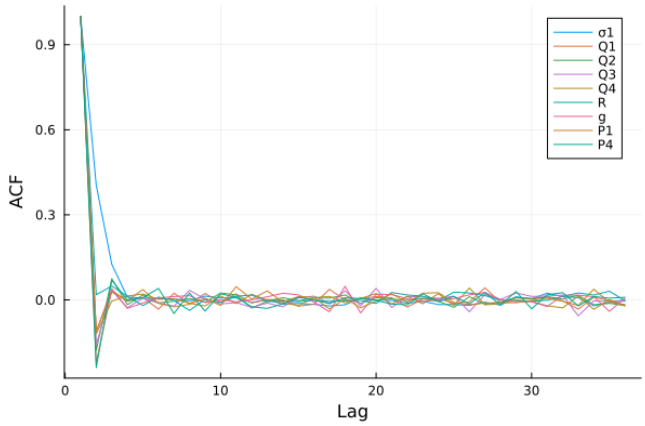

Figure 13. ACF plots of BAT samples computed up to a lag of 30 during test 2.

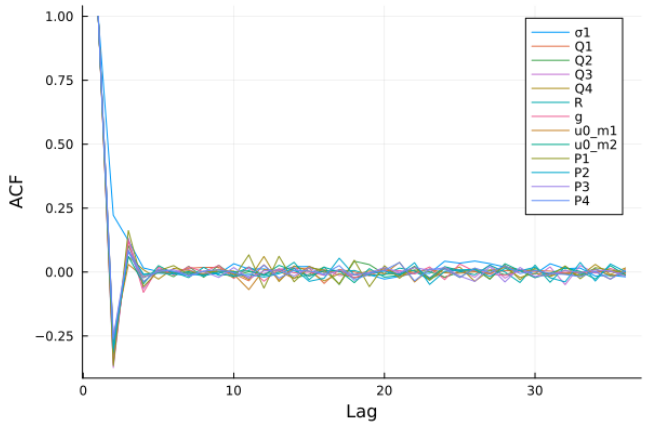

Figure 14. ACF plots of BAT samples computed up to a lag of 30 during test 3.