

Instructions of as use the new runVector2d.py program

In Windows operating system:

After the download the [repository](#) access the `runVector2d.py` program and give two clicks and follow the news instructions in display. Too will can run the `runVector2d.py` program using the Command Prompt or Windows Power Shell in **Windows 11** typed the following command:

```
C:\Users\UserName> py runVector2d.py program
```

Or

```
C:\Users\UserName> python runVector2d.py program
```

[Warning]: No type the following command:

```
C:\Users\UserName> python3 runVector2d.py program
```

to run the `runVector2d.py` program in Command Prompt or Windows Power Shell in **Windows 11**.

In Linux operating system:

After the download the [repository](#) access the `runVector2d.py` program file. Exist two options to run the `runVector2d.py` program.

1.) In any Terminal linux type the following command:

```
python3 runVector2d.py after key ENTER
```

2.) Open and run the `runVector2d.py` program file using the **IDLE3** and use the Run ==> Run Module options. After follow the news instructions in display.

*** - - [Warning]*:** When using the `runVector2d.py` program file in Command Prompt or Windows Power Shell in **Windows 11** or in any Terminal linux and this program request any [data enter] type the [components] of the **Vectors** or the [coordinates] of the given points, and after key ENTER to continue, Ok!

Now will use the `runVector2d.py` program in examples below.

Example1 – Given the vectors: $a = \langle 3, 1 \rangle$ and $b = \langle -2, 4 \rangle$, find the Scalar Product $a \cdot b$ of this vectors.

Solution: Use the option: [1] and enter the [components]: $a_1 = 3$, $a_2 = 1$, $b_1 = -2$, and $b_2 = 4$ when requested by `runVector2d.py` program.

```
    **[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
    **[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
                                --[Version: 2.0 -- Stable]--
```

```
    **[INSTRUCTIONS OF USE]**
```

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with he [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]:

A and B key [5]

- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)] To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)] To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 1

[TO FIND THE [SCALAR PRODUCT] OF [TWO VECTORS: A and B] IN THE PLANE]

- Attribute the [Components] of the [1° vectorA]!

- Enter the [coefficient] of the [Component: (i) or (a1)]?
(^{a<a}) Enter the [new] value? 3

- Introduce the [coefficient] of the [Component: (j) or (a2)]?
(^{a<a}) Enter the [new] value? 1

- Provide the [Components] of the [2° vectorB]!

- Enter the [coefficient] of the [Component: (i) or (b1)]?
(^{a<a}) Enter the [new] value? -2

- Introduce the [coefficient] of the [Component: (j) or (b2)]?
(^{a<a}) Enter the [new] value? 4

[Answer]

-- The [vectorA]: vectorA [3.0, 1.0]
-- The [vectorB]: vectorB [-2.0, 4.0]

-- The [Vector] of the [terms of the Scalar Product]: vectorA*B = [-6.0, 4.0]
-- The [Scalar Product(vectorAB)] of the vectors is: -2.00

. . . Key [ENTER] to exit -- Ok! . . .

[Warning]:

When any user run the runVector2d.py program to solve exercises that provide the [coordinates] of the given vertice points: A, B, and C to triangleABC do the folowing:

$(x_A, y_A) = (x_P, y_P) \Rightarrow x_P = x_A$ and $y_P = y_A$ replace the coordinates: x_A and y_A into the coordinates: x_P and y_P of the point: P when the `runVector2d.py` program request to enter the coordinates: x_P and y_P . Of the same manner to the vertexes B and C so:

$(x_B, y_B) = (x_Q, y_Q) \Rightarrow x_Q = x_B$ and $y_Q = y_B$ replace the coordinates: x_B and y_B into the coordinates: x_Q and y_Q of the point: Q when the `runVector2d.py` program request to enter the coordinates: x_Q and y_Q .

$(x_C, y_C) = (x_R, y_R) \Rightarrow x_R = x_C$ and $y_R = y_C$ replace the coordinates: x_C and y_C into the coordinates: x_R and y_R of the point: R when the `runVector2d.py` program request to enter the coordinates: x_R and y_R .

Of the same manner do to the vertex points: A, B, C, and D of the `parallelogram`. Replace your coordinates so:

$(x_A, y_A) = (x_P, y_P) \Rightarrow x_P = x_A$ and $y_P = y_A$
 $(x_B, y_B) = (x_Q, y_Q) \Rightarrow x_Q = x_B$ and $y_Q = y_B$
 $(x_C, y_C) = (x_R, y_R) \Rightarrow x_R = x_C$ and $y_R = y_C$
 $(x_D, y_D) = (x_S, y_S) \Rightarrow x_S = x_D$ and $y_S = y_D$

When the `runVector2d.py` program request enter the [coordinates] of the points: P, Q, R, and S. Will be confused standardize the `letters: A, B, and C to the vectors` and too to the `vertexes points: A, B, and C of the Triangle` into the `code of the classes` developed.

Example2 - Find the dimensions of the triangleABC given the `vertex points: A(-1,0), B(2,1), and C(1,-2)`

Solution: Do: $A(-1,0) = P(x_P, y_P) \Rightarrow x_P = -1$, and $y_P = 0$
 $B(2,1) = Q(x_Q, y_Q) \Rightarrow x_Q = 2$, and $y_Q = 1$
 $C(1,-2) = R(x_R, y_R) \Rightarrow x_R = 1$, and $y_R = -2$

Begin run the `runVector2d.py` program with the `option[2]` and provide the coordinates: $x_P = -1$, $y_P = 0$, $x_Q = 2$, $y_Q = 1$, $x_R = 1$, and $y_R = -2$, when the `runVector2d.py` program request.

```

**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--

```

****[INSTRUCTIONS OF USE]****

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with he [coordinates] of the given [points: P(x_P, y_P), Q(x_Q, y_Q), and R(x_R, y_R)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(x_P, y_P), Q(x_Q, y_Q), R(x_R, y_R), and S(x_S, y_S)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(x_P, y_P), Q(x_Q, y_Q), and R(x_R, y_R)] key [6]
- To Determine the following [Middle Points]: K(x_K, y_K), L(x_L, y_L), M(x_M, y_M) and N(x_N, y_N) between [points: P(x_P, y_P), Q(x_Q, y_Q), R(x_R, y_R), and S(x_S, y_S)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(x_P, y_P), and Q(x_Q, y_Q)]
- To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(x_P, y_P), and R(x_R, y_R)]

- To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 2

```
**[ TO CALCULATE THE SIDES: SIDE_A, SIDE_B, AND SIDE_C, THE PERIMETER(P),
  THE HEIGHTS: H1,H2, AND H3, AND THE ]**
**[ AREA(A) OF THE [TRIANGLE(PQR)] GIVEN THE [COORDINATES] OF THE
  [ VERTEX POINTS: P, Q, AND R [ IN THE PLANE] ]**
```

--Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].
(^{a<a}) Enter the [new] value? -1

* Enter with the 2°[Coordinate(y)].
(^{a<a}) Enter the [new] value? 0

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1°[Coordinate(x)].
(^{a<a}) Enter the [new] value? 2

* Enter with the 2°[Coordinate(y)].
(^{a<a}) Enter the [new] value? 1

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1°[Coordinate(x)].
(^{a<a}) Enter the [new] value? 1

* Enter with the 2°[Coordinate(y)].
(^{a<a}) Enter the [new] value? -2

- The (Point P): P (-1.0, 0.0)
- The (Point Q): Q (2.0, 1.0)
- The (Point R): R (1.0, -2.0)

[Answer]

```
=====
**[ EXIST THE TRIANGLE(PQR) WITH THE GIVEN POINTS: P, Q, AND R -- OK! ]**
=====
```

- The [vectorA] = vectorQR [-1.0, -3.0]
- The [vectorB] = vectorRP [-2.0, 2.0]
- The [vectorC] = vectorPQ [3.0, 1.0]

-- The [sideA] = $||\text{vectorQR}||$ of the triangle(PQR) is: 3.16
 -- The [sideB] = $||\text{vectorRP}||$ of the triangle(PQR) is: 2.83
 -- The [sideC] = $||\text{vectorPQ}||$ of the triangle(PQR) is: 3.16

-- The [terms] of the [Scalar Product(PQ°QR)] is: [-3.0, -3.0]
 -- The [terms] of the [Scalar Product(QR°RP)] is: [2.0, -6.0]
 -- The [terms] of the [Scalar Product(RP°PQ)] is: [-6.0, 2.0]

-- The [Scalar Product(PQ°QR)] is: -6.00
 -- The [Scalar Product(QR°RP)] is: -4.00
 -- The [Scalar Product(RP°PQ)] is: -4.00

-- The [Perimeter] of the [triangle(PQR)] is 9.15
 -- The [Height(h1) relative as sideQR] is 2.53
 -- The [Height(h2) relative as sideRP] is 2.83
 -- The [Height(h3) relative as sidePQ] is 2.53
 -- The [Area(A)] of the [triangle(PQR)] is 4.0

. . . Key [ENTER] to exit -- Ok! . . .

To calculate all the three Inner angles of the TrianglePQR type the option [6] when requested by runVector2d.py program.

```

**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--

```

[INSTRUCTIONS OF USE]

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM) and N(xN,yN) between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
 To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
 To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]

- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 6

[TO FIND THE THREES SIDES AND [INNER ANGLES] OF THE [TRIANGLE(PQR)]]

[GIVENS THE [COORDINATES] OF THE POINTS: P, Q, AND R IN THE PLANE]

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].

(^a<^a) Enter the [new] value? -1

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? 0

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1°[Coordinate(x)].

(^a<^a) Enter the [new] value? 2

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? 1

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1°[Coordinate(x)].

(^a<^a) Enter the [new] value? 1

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? -2

- The (Point P): P (-1.0, 0.0)

- The (Point Q): Q (2.0, 1.0)

- The (Point R): R (1.0, -2.0)

[Answer]

- The [vectorB]=vectorPR [2.0, -2.0]

- The [vectorC]=vectorPQ [3.0, 1.0]

- The [vectorA]=vectorQR [-1.0, -3.0]

- The [vectorC]=vectorQP [-3.0, -1.0]

- The [vectorA]=vectorRQ [1.0, 3.0]

- The [vectorB]=vectorRP [-2.0, 2.0]

=====
[EXIST THE TRIANGLE(PQR) WITH THE GIVEN POINTS: P, Q, AND R -- OK!]
=====

-- The [sideA]=|vectorA| of the triangle(PQR) is: 3.16

-- The [terms] of the [Scalar Profuct(AC°AB)] is the vector[AC°AB]: [6.0, -2.0]

-- The [value] of the [Scalar Profuct(AC°AB)] is: 4.00

- The value of the [ALPHA ANGLE] was calculate is: 63.43

-- The [sideB]=|vectorB| of the triangle(PQR) is: 2.83

-- The [terms] of the [Scalar Product($BC^\circ BA$)] is the vector[$BC^\circ BA$]: [3.0, 3.0]

-- The [value] of the [Scalar Product($BC^\circ BA$)] is: 6.00

- The value of the [BETA ANGLE] was calculate is: 53.13

-- The [sideC]=|vectorC| of the triangle(PQR)] is: 3.16

-- The [terms] of the [Scalar Product($CB^\circ CA$)] is the vector[$CB^\circ CA$]: [-2.0, 6.0]

-- The [value] of the [Scalar Product($CB^\circ CA$)] is: 4.00

- The value of the [GAMA ANGLE] was calculate is: 63.43

-- The triangle(PQR) is [Isosceles]!

-- The [Add] of the [Inner Angles: ALPHA, BETA, GAMA] is 180.0

- THE [ADD] OF THE [INNER ANGLES] OF THE [TRIANGLE-PQR] is 180.00

--[END CALCULUS-OK!]

. . . Key [ENTER] to exit -- Ok! . . .

New Example2.1 - Now determine the measures of the sides: sideA, sideB, and sideC, perimeter(P), height(h1,h2,h3), and Area(A) of the TrianglePQR whose vertexes are P(-5,7), Q(1,-3), and R(3, 1).

Solution: To find the sides: sideA, sideB, and sideC as too the perimeter(P), and Area(A) key in option: [2] and after ENTER. Wait the runVector2d.py program request enter the [coordinates] of the vertice points: P, Q, and R according viewed below.

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--
```

[INSTRUCTIONS OF USE]

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with he [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)] To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)] To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]

- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[>] Provide the [new] value? 2

```

**[ TO CALCULATE THE SIDES: SIDE_A, SIDE_B, AND SIDE_C, THE PERIMETER(P),
    THE HEIGHTS: H1,H2, AND H3, AND THE ]**
**[ AREA(A) OF THE [TRIANGLE(PQR)] GIVEN THE [COORDINATES] OF THE
    [ VERTEX POINTS: P, Q, AND R [ IN THE PLANE] ]**

```

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? -5

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? 7

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? 1

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? -3

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? 3

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? 1

- The (Point P): P (-5.0, 7.0)

- The (Point Q): Q (1.0, -3.0)

- The (Point R): R (3.0, 1.0)

[Answer]

```

=====
**[ EXIST THE TRIANGLE(PQR) WITH THE GIVEN POINTS: P, Q, AND R -- OK! ]**
=====

```

-- The [vectorA] = vectorQR [2.0, 4.0]

-- The [vectorB] = vectorRP [-8.0, 6.0]

-- The [vectorC] = vectorPQ [6.0, -10.0]

-- The [sideA] = ||vectorQR|| of the triangle(PQR) is: 4.47

-- The [sideB] = ||vectorRP|| of the triangle(PQR) is: 10.00

-- The [sideC] = ||vectorPQ|| of the triangle(PQR) is: 11.66

-- The [terms] of the [Scalar Product(PQ°QR)] is: [12.0, -40.0]
 -- The [terms] of the [Scalar Product(QR°RP)] is: [-16.0, 24.0]
 -- The [terms] of the [Scalar Product(RP°PQ)] is: [-48.0, -60.0]

-- The [Scalar Product(PQ°QR)] is: -28.00
 -- The [Scalar Product(QR°RP)] is: 8.00
 -- The [Scalar Product(RP°PQ)] is: -108.00

-- The [Perimeter] of the [triangle(PQR)] is 26.13
 -- The [Height(h1) relative as sideQR] is 9.84
 -- The [Height(h2) relative as sideRP] is 4.40
 -- The [Height(h3) relative as sidePQ] is 3.77
 -- The [Area(A)] of the [triangle(PQR)] is 22.00

... Key [ENTER] to exit -- Ok! ...

New Example2.2 - Now determine the measures of the sides: sideA, sideB, and sideC, perimeter(P), height(h1,h2,h3), and Area(A) of the TrianglePQR whose vertexes are P(-3,1), Q(-1,7), and R(-4, -2).

Solution: To find the sides: sideA, sideB, and sideC as too the perimeter(P), and Area(A) key in option: [2] and after ENTER. Wait the runVector2d.py program request enter the [coordinates] of the points: P, Q, and R according viewed below.

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
 To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
 To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 2

```
**[ TO CALCULATE THE SIDES: SIDE_A, SIDE_B, AND SIDE_C, THE PERIMETER(P),  
  THE HEIGHTS: H1,H2, AND H3, AND THE ]**  
**[ AREA(A) OF THE [TRIANGLE(PQR)] GIVEN THE [COORDINATES] OF THE  
  [ VERTEX POINTS: P, Q, AND R [ IN THE PLANE] ]**
```

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].

(^{a<a}) Enter the [new] value? -3

* Enter with the 2°[Coordinate(y)].

(^{a<a}) Enter the [new] value? 1

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1°[Coordinate(x)].

(^{a<a}) Enter the [new] value? -1

* Enter with the 2°[Coordinate(y)].

(^{a<a}) Enter the [new] value? 7

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1°[Coordinate(x)].

(^{a<a}) Enter the [new] value? -4

* Enter with the 2°[Coordinate(y)].

(^{a<a}) Enter the [new] value? -2

- The (Point P): P (-3.0, 1.0)

- The (Point Q): Q (-1.0, 7.0)

- The (Point R): R (-4.0, -2.0)

[Answer]

```
[ NO EXIST THE TRIANGLE(PQR) WITH THE [COORDINATES] OF THE GIVEN POINTS: P(xP,yP), Q(xQ,yQ), AND ]*  
*[ R(xR,yR) -- OK! BY [THEOREM OF THE PLANE GEOMETRY] THE BUILDING OF THE TRIANGLE(PQR) USING THE ]*  
*[ COMPASS AND RULER ONLY IS POSSIBLE WHEN: ]*
```

```
**[ sideA < sideB + sideC ]** and  
**[ sideB < sideA + sideC ]** and  
**[ sideC < sideA + sideB ]**
```

. . . Key [ENTER] to exit -- Ok! . . .

Example3 – Find the measures of the sides: sideA, sideB, and sideC, the perimeter(P), the height(h1,h2.h3), and Area(A), Scalar Products: $c \cdot a$, $b \cdot c$, $a \cdot b$, and the Inner Angles of the TrianglePQR whose vertexes are P(-6,0), Q(10,-5), and R(-2, 4).

Solution: To find the sides: sideA, sideB, and sideC as too the perimeter(P), and Area(A) key in option: [2] and after Key ENTER. Wait the runVector2d.py program request and enter the [coordinates] of the vertice points: P, Q, and R according viewed below.

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with he [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)] To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)] To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meetted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 2

```
**[ TO CALCULATE THE SIDES: SIDE_A, SIDE_B, AND SIDE_C, THE PERIMETER(P),
  THE HEIGHTS: H1,H2, AND H3, AND THE ]**
**[ AREA(A) OF THE [TRIANGLE(PQR)] GIVEN THE [COORDINATES] OF THE
  [ VERTICE POINTS: P, Q, AND R [ IN THE PLANE] ]**
```

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].

(^<^a) Enter the [new] value? -6

* Enter with the 2°[Coordinate(y)].

(^<^a) Enter the [new] value? 0

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? 10

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? -5

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? -2

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? 4

- The (Point P): P (-6.0, 0.0)

- The (Point Q): Q (10.0, -5.0)

- The (Point R): R (-2.0, 4.0)

[Answer]

```
=====
**[ EXIST THE TRIANGLE(PQR) WITH THE GIVEN POINTS: P, Q, AND R -- OK! ]**
=====
```

-- The [vectorA] = vectorQR [-12.0, 9.0]

-- The [vectorB] = vectorRP [-4.0, -4.0]

-- The [vectorC] = vectorPQ [16.0, -5.0]

-- The [sideA] = ||vectorQR|| of the triangle(PQR) is: 15.00

-- The [sideB] = ||vectorRP|| of the triangle(PQR) is: 5.66

-- The [sideC] = ||vectorPQ|| of the triangle(PQR) is: 16.76

-- The [terms] of the [Scalar Product(PQ°QR)] is: [-192.0, -45.0]

-- The [terms] of the [Scalar Product(QR°RP)] is: [48.0, -36.0]

-- The [terms] of the [Scalar Product(RP°PQ)] is: [-64.0, 20.0]

-- The [Scalar Product(PQ°QR)] is: -237.00

-- The [Scalar Product(QR°RP)] is: 12.00

-- The [Scalar Product(RP°PQ)] is: -44.00

-- The [Perimeter] of the [triangle(PQR)] is 37.42

-- The [Height(h1) relative as sideQR] is 5.60

-- The [Height(h2) relative as sideRP] is 14.85

-- The [Height(h3) relative as sidePQ] is 5.01

-- The [Area(A)] of the [triangle(PQR)] is 42.00

. . . Key [ENTER] to exit -- Ok! . . .

To calculate all the three Inner Angles of the TrianglePQR type the option: [6] when requested by runVector2d.py program.

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--
```

****[INSTRUCTIONS OF USE]****

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 6

```
**[ TO CALCULATE THE [PERIMETER(P), HEIGHTS(H1,H2,H3), AND AREA(A) OF THE ]**  
**[ [TRIANGLE-PQR] GIVENS THE [COORDINATES] OF THE [POINTS: P, Q, AND R] **  
    IN THE PLANE ]**
```

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].

(^{a<a}) Enter the [new] value? -6

* Enter with the 2°[Coordinate(y)].

(^{a<a}) Enter the [new] value? 0

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1°[Coordinate(x)].

(^{a<a}) Enter the [new] value? 10

* Enter with the 2°[Coordinate(y)].

(^{a<a}) Enter the [new] value? -5

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1°[Coordinate(x)].

(^{a<a}) Enter the [new] value? -2

* Enter with the 2^o[Coordinate(y)].
(^a<^a) Enter the [new] value? 4

- The (Point P): P (-6.0, 0.0)
- The (Point Q): Q (10.0, -5.0)
- The (Point R): R (-2.0, 4.0)

[Answer]

- The [vectorB]=vectorPR [4.0, 4.0]
- The [vectorC]=vectorPQ [16.0, -5.0]

- The [vectorA]=vectorQR [-12.0, 9.0]
- The [vectorC]=vectorQP [-16.0, 5.0]

- The [vectorA]=vectorRQ [12.0, -9.0]
- The [vectorB]=vectorRP [-4.0, -4.0]

=====
[EXIST THE TRIANGLE(PQR) WITH THE GIVEN POINTS: P, Q, AND R -- OK!]
=====

-- The [sideA]=|vectorA| of the triangle(PQR)] is: 15.00

-- The [terms] of the [Scalar Product(AC°AB)] is the vector[AC°AB]: [64.0, -20.0]
-- The [value] of the [Scalar Product(AC°AB)] is: 44.00
- The value of the [ALPHA ANGLE] was calculate is: 62.35

-- The [sideB]=|vectorB| of the triangle(PQR)] is: 5.66

-- The [terms] of the [Scalar Product(BC°BA)] is the vector[BC°BA]: [192.0, 45.0]
-- The [value] of the [Scalar Product(BC°BA)] is: 237.00
- The value of the [BETA ANGLE] was calculate is: 19.52

-- The [sideC]=|vectorC| of the triangle(PQR)] is: 16.76

-- The [terms] of the [Scalar Product(CB°CA)] is the vector[CB°CA]: [-48.0, 36.0]
-- The [value] of the [Scalar Product(CB°CA)] is: -12.00
- The value of the [GAMA ANGLE] was calculate is: 98.13

-- The triangle(PQR) is [Scalene]!

-- The [Add] of the [Inner Angles: ALPHA, BETA, GAMA] is 180.0

- THE [ADD] OF THE [INNER ANGLES] OF THE [TRIANGLE-PQR] is 180.00

--[END CALCULUS-OK!!--

. . . Key [ENTER] to exit -- Ok! . . .

Example4 – Find the cossine of the angle between the vectors $a = i - 3j$ and $b = -4i + j$.

Solution: Use the option: [5] and enter the [components]: $a_1 = 1$, $a_2 = -3$, $b_1 = -4$, and $b_2 = 1$ when requested by runVector2d.py program.

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM) and N(xN,yN) between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)] To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)] To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 5

```
**[ TO CALCULATE THE [VALUE] OF THE [COSSINE THETA] BETWEEN
THE [TWO VECTORS]: A AND B]**
```

- Attribute the [Components] of the [1° vectorA]!

- Enter the [coefficient] of the [Component: (i) or (a1)]?
($a < a$) Enter the [new] value? 1

- Introduce the [coefficient] of the [Component: (j) or (a2)]?
($a < a$) Enter the [new] value? -3

- Provide the [Components] of the [2° vectorB]?

- Enter the [coefficient] of the [Component: (i) or (b1)]?
($a < a$) Enter the [new] value? -4

- Introduce the [coefficient] of the [Component: (j) or (b2)]?
(^a<^a) Enter the [new] value? 1

[Answer]

-- The [vectorA]: vectorA [1.0, -3.0]

-- The [vectorB]: vectorB [-4.0, 1.0]

-- The [VectorA*B]: [-4.0, -3.0]

-- The [value] finded of |A*B| is: 13.04

-- The [length] of a ||vectorA||: 3.16

-- The [length] of a ||vectorB||: 4.12

-- The [Scalar Product] of the [VectorA*B] is: -7.00

[Answer]

-- The value of the [COSSINE THETA IN RADIANS] calculated is: -0.54

. . . Key [ENTER] to exit -- Ok! . . .

New Example5 – Find the follow [distances] between the given points: P(3,-5), Q(2,8), and R(-3, 4).

Solution: Use the option: [10] and after provide the [coordinates] of the given previous points when requested by runVector2d.py program.

Note: The old option: [6] in runvectors2dim.py file was excluded in new runVector2d.py program file. The option: [10] will calculate the following new [distances]: PQ, PR, and QR in accordance with the given points. Enter the [coordinates] of the previous points after type the option: [10] and follow the new instructions of the runVector2d.py program in display. See the processing below, OK!

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**  
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**  
--[Version: 2.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with he [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]

- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are metted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 10

[TO FIND THE [VECTOR(A)] THAT REPRESENT THE TWO GIVENS POINTS: P, AND Q]
 [AND TOO THE [LENGTH] OF THE VECTOR(A) = |VECTOR(PQ)|]

[TO GET THE [VECTOR(B)] THAT REPRESENT THE TWO GIVENS POINTS: P, AND R]
 [AND TOO THE [LENGTH] OF THE VECTOR(B) = |VECTOR(PR)|]

[TO FIND THE [VECTOR(C)] THAT REPRESENT THE TWO GIVENS POINTS: Q, AND R]
 [AND TOO THE [LENGTH] OF THE VECTOR(C) = |VECTOR(QR)|]

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].

(^<^) Enter the [new] value? 3

* Enter with the 2°[Coordinate(y)].

(^<^) Enter the [new] value? -5

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1°[Coordinate(x)].

(^<^) Enter the [new] value? 2

* Enter with the 2°[Coordinate(y)].

(^<^) Enter the [new] value? 8

- The (Point P): P (3.0, -5.0)

- The (Point Q): Q (2.0, 8.0)

[Answer]

-- The [vectorA]: vectorA=vectorPQ [-1.0, 13.0]

-- The [lenghtPQ = distPQ] is the ||vectorPQ||: 13.04

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].

(^<^) Enter the [new] value? 3

* Enter with the 2°[Coordinate(y)].

(^<^) Enter the [new] value? -5

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? -3

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? 4

- The (Point P): P (3.0, -5.0)

- The (Point R): R (-3.0, 4.0)

[Answer]

-- The [vectorB]: vectorB=vectorPR [-6.0, 9.0]

-- The [lengthPR = distPR] is the ||vectorPR||: 10.82

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? 2

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? 8

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? -3

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? 4

- The (Point Q): Q (2.0, 8.0)

- The (Point R): R (-3.0, 4.0)

[Answer]

-- The [vectorC]: vectorC=vectorQR [-5.0, -4.0]

-- The [lengthQR = distQR] is the ||vectorQR||: 6.40

. . . Key [ENTER] to exit -- Ok! . . .

Warning: The **Example4** from the instructions-v1.3_User.pdf file finded only the distance(D) between two points: P and Q.

New information to all the users:

In this repository was added a new ParallelogramPQRS.pdf file with the following drawings: drawing1 and drawing2 to all user view and follow the previous **Examples: 6**(drawing1) and **New Example7**(drawing2).

Example6 – Find the sides: A and B, Heights: (h1 and h2), the Perimeter(P), and Area(S) of the parallelogramPQRS given the [coordinates] of the vertice points: P(-1,2), Q(2,0), R(7,1), and S(4,3).

Solution: Use the **option: [3]** and after Key in **option: [2]** and provide the [coordinates] of the previous points when requested by runVector2d.py program.

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**  
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**  
--[Version: 2.0 -- Stable]--
```

****[INSTRUCTIONS OF USE]****

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 3

****[TO GET THE SIDES: SIDE_A AND SIDE_B, PERIMETER(P), [HEIGHTS: H1 AND H2], AND [AREA(S)] OF THE]****

****[[PARALLELOGRAM(PQRS)]GIVENS THE [COORDINATES] OF THE [VERTICES POINTS: P, Q, R AND S] IN THE PLANE]****

- Key [1] if in problem is given the [coordinates] of the three vertices: P, Q, and R. After follow the new [instructions] in [Display] to find the [coordinates: xS, and yS] relative as point: S below - Ok!

- Key [2] if in problem is given the [coordinates] of the four vertice points: P, Q, R, and S.

[°>°] Provide the [new] value? 2

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].

(^a<^a) Enter the [new] value? -1

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? 2

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1°[Coordinate(x)].

(^a<^a) Enter the [new] value? 2

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? 0

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? 7

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? 1

-- Give the [coordinates]: (xS, yS) of the (point S)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? 4

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? 3

- The (Point P): P (-1.0, 2.0)

- The (Point Q): Q (2.0, 0.0)

- The (Point R): R (7.0, 1.0)

- The (Point S): S (4.0, 3.0)

[Answer]

-- The [vectorPQ]: vectorPQ [3.0, -2.0]

-- The [vectorPS]: vectorPS [5.0, 1.0]

-- The [sideA] relative as |vectorPQ|: 3.61

-- The [sideB] relative as |vectorPS|: 5.10

-- The [Perimeter(P)] is: 17.41

-- The [terms] of the Scalar Product(vectorPS^oPQ) is: [15.0, -2.0]

-- The [Scalar Product(PQ^oPS)] is: 13.0

-- The Cossine of theta: 0.71

-- The [Theta angle] between vectors: vectorPQ and vectorPS is: 45.00

-- The Sine of alpha: 0.71

-- The [Alpha angle] between vectors: vectorPQ and vectorQR is: 135.00

-_- The [Height(h1)] relative as [sideB] is: 2.55

----- or -----

°<° The [Height(h2)] relative as [sideA] is: 3.61

-- The [Area(S)] of the [Parallelogram(PQRS)]: 13.00

. . . Key [ENTER] to exit -- OK! . . .

New Example7 – Given the [coordinates] of the three vertex points: P(-6,0), Q(3,1), and R(1,-4) of a parallelogramPQRS. Find the following dimensions: the [coordinates: xS and yS], the sides: A and B, the inner angles: PSQ = θ and SPR = α , the Heights: (h1 and h2), the Perimeter(P), and Area(S).

Solution: Type the option: [3] and in option: [1]. After key in option: [3] and provide the [coordinates] of the previous points when requested by runVector2d.py program.

```

**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--

```

****[INSTRUCTIONS OF USE]****

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)] To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)] To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[>] Provide the [new] value? 3

```

**[ TO GET THE SIDES: SIDE_A AND SIDE_B, PERIMETER(P), [HEIGHTS: H1 AND H2], AND
  [AREA(S)] OF THE ]**
**[ [PARALLELOGRAM(PQRS)] GIVEN THE [COORDINATES] OF THE [ VERTICES POINTS: P,
  Q, R AND S] IN THE PLANE ]**

```

- Key [1] if if problem is given the [coordinates] of the three vertices: P, Q, and R. After follow the new [instructions] in [Display] to find the [coordinates: xS, and yS] relative as point: S below - Ok!

- Key [2] if in problem is given the [coordinates] of the four vertices: P, Q, R, and S.

[>] Provide the [new] value? 1

-- Type [1] to find the [coordinates: xM and yM] of the middle point: M in [diagonal: SP] Use the [coordinates: xQ, yQ, xR, and yR] of the points: Q and R.

-- Type [2] to find the [coordinates: xL and yL] of the middle point: L in [diagonal: SQ]
Use the [coordinates: xP, yP, xR, and yR] of the points: P and R.

-- Type [3] to find the [coordinates: xK and yK] of the middle point: K in [diagonal: SR]
Use the [coordinates: xP, yP, xQ, and yQ] of the points: P and Q.

[0>0] Provide the [new] value? 3

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1^o[Coordinate(x)].

(^{a<a}) Enter the [new] value? -6

* Enter with the 2^o[Coordinate(y)].

(^{a<a}) Enter the [new] value? 0

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1^o[Coordinate(x)].

(^{a<a}) Enter the [new] value? 3

* Enter with the 2^o[Coordinate(y)].

(^{a<a}) Enter the [new] value? 1

- The (Point P): P (-6.0, 0.0)

- The (Point Q): Q (3.0, 1.0)

[Answer]

-- The MidPoint_K(xK,yK): (-1.5, 0.5)

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1^o[Coordinate(x)].

(^{a<a}) Enter the [new] value? 1

* Enter with the 2^o[Coordinate(y)].

(^{a<a}) Enter the [new] value? -4

- The (Point R): R (1.0, -4.0)

[Answer]

-- The [Coordinates] of the point S:

-- The [Coordinate]: xS is -4.00

-- The [Coordinate]: yS is 5.00

-- The [vectorRP]: vectorRP [-7.0, 4.0]

-- The [vectorRQ]: vectorRQ [2.0, 5.0]

-- The [sideA] relative as |vectorRP|: 8.06

-- The [sideB] relative as |vectorRQ|: 5.39

-- The [Perimeter(P)] is: 26.89

-- The [terms] of the Scalar Product(vectorRP^oRQ) is: [-14.0, 20.0]

-- The [Scalar Product(RP^oRQ)] is: 6.0

-- The Cossine of theta: 0.14

-- The [Theta angle] between vectors: vectorRP and vectorRQ is: 82.06

-- The Sine of alpha: 0.99

-- The [Alpha angle] between vectors: vectorRP and vectorPS is: 97.94


```

-- The [Height(h1)] relative as [sideB] is: 7.98
or
<° The [Height(h2)] relative as [sideA] is: 5.33

```

```
-- The [Area(S)] of the [Parallelogram(PQRS)]: 43.00
```

```
... Key [ENTER] to exit -- Ok! ...
```

Warning: When the problem provide only the [coordinates] of the three vertex points: P, Q, and R, draw this points in \mathbb{R}^2 cartesian plane to view if the [diagonal] will be a of the segments: SP or SQ or SR.

In any Parallelogram PQRS have by definition:

$SQ \parallel PR$ and $SP \parallel QR$

The [Height(h1)] is the projection of PP1 in sideSQ

The [Height(h2)] is the projection of QQ1 in sideSP

By drawing2 have the right triangles: PP1S and QQ1S.

The height(h1) = projection: $PP1 < sideSP$ (hypotenuse) and the height(h2) = projection: $QQ1 < sideSQ$ (hypotenuse).

By theorem:

\equiv : identical or Congruent

$SQ \equiv PR$ and $SP \equiv QR$

The diagonals: $PQ \equiv SR$

The Inner Angles: $PSQ = \theta = PRQ$ and $SPR = \alpha = RQS$

The add of the inner angles: $\theta + \alpha = 180^\circ$

The diagonals: PQ and SR meet in the middle-point: $K(xK, yK)$.

Warning: The old VectorModule2Dim.py module and runvectors2dim.py program files version: 1.3 will not find the height: h2 as too no the Inner Angle: α .

Example8 – Find the addition and subtraction of the vectors $a = -11i + 4j$ and $b = 3i - j$.

Solution: Use the option: [8] and enter the [components]: $a1 = -11$, $a2 = 4$, $b1 = 3$, and $b2 = -1$ when requested by runVector2d.py program.

```

**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--

```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM) and N(xN,yN) between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]

- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
- To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
- To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are metted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 8

****[TO FIND THE [ADDITION AND SUBTRACTION] OF [TWO VECTORS: A and B] IN THE PLANE]****

- Attribute the [Components] of the [1° vectorA]!

- Enter the [coefficient] of the [Component: (i) or (a1)]?

(^a<^a) Enter the [new] value? -11

- Introduce the [coefficient] of the [Component: (j) or (a2)]?

(^a<^a) Enter the [new] value? 4

- Provide the [Components] of the [2° vectorB]?

- Enter the [coefficient] of the [Component: (i) or (b1)]?

(^a<^a) Enter the [new] value? 3

- Introduce the [coefficient] of the [Component: (j) or (b2)]?

(^a<^a) Enter the [new] value? -1

-- The [vectorA]: vectorA [-11.0, 4.0]

-- The [vectorB]: vectorB [3.0, -1.0]

[Answer]

- The [Addition]: vector[a+b]: [-8.0, 3.0]

- The [Subtraction]: vector[a-b]: [-14.0, 5.0]

... Key [ENTER] to exit -- Ok! ...

Example9 – Find the Theta angle between the **vectors** $a = -3i + 5$ and $b = 7i - 2j$.

Solution: Use the **option: [4]** and enter the [components]: $a_1 = -3$, $a_2 = 5$, $b_1 = 7$, and $b_2 = -2$ when requested by **runVector2d.py** program.

****[WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM]****
****[TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE]]****
 --[Version: 2.0 -- Stable]--

****[INSTRUCTIONS OF USE]****

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 4

****[WILL FIND THE [VALUE] OF THE [THETA ANGLE] BETWEEN [TWO VECTORS] GIVEN: A AND B]****

- Attribute the [Components] of the [1° vectorA]!

- Enter the [coefficient] of the [Component: (i) or (a1)]?
(^a<^a) Enter the [new] value? -3

- Introduce the [coefficient] of the [Component: (j) or (a2)]?
(^a<^a) Enter the [new] value? 5

- Provide the [Components] of the [2° vectorB]?

- Enter the [coefficient] of the [Component: (i) or (b1)]?
(^a<^a) Enter the [new] value? 7

- Introduce the [coefficient] of the [Component: (j) or (b2)]?
(^a<^a) Enter the [new] value? -2

[Answer]

-- The [vectorA]: vectorA [-3.0, 5.0]
-- The [vectorB]: vectorB [7.0, -2.0]

```
-- The VectorA*B: [-21.0, -10.0]
-- The [length] of a ||vectorA||: 5.83
-- The [length] of a ||vectorB||: 7.28
-- The [value] found of |A*B| is: 42.45
-- The [Scalar Product] of the [VectorA*B] is: -31.00

-- The value of the [THETA ANGLE IN DEGREES] calculated is: 136.91
```

... Key [ENTER] to exit -- Ok! ...

Example10 – The three displacement vectors A, B, and C are specified by their magnitudes $A = 10$, $B = 7$, and $C = 8$, respectively, and by their respective direction angles with the horizontal direction $\theta = 35^\circ$, $\beta = 110^\circ$, and $\phi = 30^\circ$. The physical units of the magnitudes are centimeters. Find the Resultant(|R) Vector.

Solution: Use the option: [9] and enter the data problem when requested by runVector2d.py program.

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--

**[INSTRUCTIONS OF USE]**

- To find the [Scalar Product: a * b] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [ points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS) ] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)] To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)] To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [ Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR) ] of the triangle(PQR) with the [Coordinates] given of the [ Middle points: K(xK,yK), L(xL,yL), and M(xM,yM) ] key [14]
- To get the [Coordinates] of any a of the [ Points: D(xD,yD) or E(xE,yE), or F(xF,yF) ] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[$] Select an previous [option] that will used--Ok!

(>>) Provide the [new] value? 9
```

```
**[TO GET THE VALUE OF THE RESULTANT(|R) VECTOR]**
```

+ How much [Vectors] will necessary to get the [Resultant(R) Vector]?
[°>°] Provide the [new] value? 3

[Warning]

- All the [Vectorials Components] will can be:
[positive] or [negative] or Zero!

+ What are the arguments: [angle] and [radius] of the (1°) [vector]?

- Enter with the [new value] of the [angle] in degree?

(^a<^a) Enter the [new] value? 35

- Provide the [new value] of the [radius] of vector?

[°<°] Type a [new] value? 10

_ _ . . . [Running] . . . _ _

[Answer]

- The component of the (1°) [vector]: $V_x(1) = 8.19$

- The component of the (1°) [vector]: $V_y(1) = 5.74$

[Warning]:

[THE [PREVIOUS COMPONENTS]: $V_x(1)$ AND $V_y(1)$ BEEN CALCULED; BUT]

[[NO EXIST RESULTANT(R) VECTOR] TO A ONLY VECTOR!]

+ What are the arguments: [angle] and [radius] of the (2°) [vector]?

- Enter with the [new value] of the [angle] in degree?

(^a<^a) Enter the [new] value? -110

- Provide the [new value] of the [radius] of vector?

[°<°] Type a [new] value? 7

_ _ . . . [Running] . . . _ _

[Answer]

- The component of the (2°) [vector]: $V_x(2) = -2.39$

- The component of the (2°) [vector]: $V_y(2) = -6.58$

+ What are the arguments: [angle] and [radius] of the (3°) [vector]?

- Enter with the [new value] of the [angle] in degree?

(^a<^a) Enter the [new] value? 30

- Provide the [new value] of the [radius] of vector?

[°<°] Type a [new] value? 8

_ _ . . . [Running] . . . _ _

[Answer]

- The component of the (3°) [vector]: $V_x(3) = 6.93$

- The component of the (3°) [vector]: $V_y(3) = 4.00$

+ The sum of all the components of x_axis: $R_x = 12.73$
 + The sum of all the components of y_axis: $R_y = 3.16$

+ The resultant vector: [Resultant(R)_Vector] = 13.11
 + The [theta angle] in [degrees] = 13.94

[End Processing of the [RESULTANTVECTOR CLASS]--Ok!]

. . . Key [ENTER] to exit -- Ok! . . .

(New) Example10 – Find the components: V_x and V_y of a vector in polar form given the angle $\Theta = 57^\circ$ and radius $r = 23$.

Solution: Use the option: [9] and enter the datas problem when requested by runVector2d.py program. After follow keying [1] in keyboard to a only vector.

[WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM]
 [TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE]]
 --[Version: 2.0 -- Stable]--

[INSTRUCTIONS OF USE]

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
 To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
 To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meetted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 9

[TO GET THE VALUE OF THE RESULTANT(|R) VECTOR]

+ How much [Vectors] will necessary to get the [Resultant(R) Vector]?
[°>°] Provide the [new] value? 1

[Warning]

- All the [Vectorials Components] will can be:
[positive] or [negative] or Zero!

+ What are the arguments: [angle] and [radius] of the (1°) [vector]?

- Enter with the [new value] of the [angle] in degree?

(^<^a) Enter the [new] value? 57

- Provide the [new value] of the [radius] of vector?

[°<°] Type a [new] value? 23

_ _ . . . [Running] . . . _ _

* [answer] *

- The component of the 1 vector: $V_x(1) = 12.53$

- The component of the 1 vector: $V_y(1) = 19.29$

--[Warning]:

[THE [PREVIOUS COMPONENTS]: $V_x(1)$ AND $V_y(1)$ BEEN CALCULED; BUT]

[[NO EXIST RESULTANT(R) VECTOR] TO A ONLY VECTOR!]

[End Processing of the [RESULTANTVECTOR.PY FUNCTION]--Ok!]

. . . Key [ENTER] to exit -- Ok! . . .

Now view the [new] exceptions when the user attribute values [Negative] or [Zero] to the radius of any vector.
Below will resolve the same previous Example 10.

[WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM]

[TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE]]

--[Version: 2.0 -- Stable]--

[INSTRUCTIONS OF USE]

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]

- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with he [coordinates] of the given [points: $P(x_P, y_P)$, $Q(x_Q, y_Q)$, and $R(x_R, y_R)$] key [2]

- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h_1 and h_2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: $P(x_P, y_P)$, $Q(x_Q, y_Q)$, $R(x_R, y_R)$, and $S(x_S, y_S)$] key [3]

- To find the [angle] between [two vectors] in plane key [4]

- To determine the [value] of the [CossineTheta] between the [two vectors]:
A and B key [5]

- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: $P(x_P, y_P)$, $Q(x_Q, y_Q)$, and $R(x_R, y_R)$] key [6]

- To Determine the following [Middle Points]: $K(x_K, y_K)$, $L(x_L, y_L)$, $M(x_M, y_M)$] and $N(x_N, y_N)$ between [points: $P(x_P, y_P)$, $Q(x_Q, y_Q)$, $R(x_R, y_R)$, and $S(x_S, y_S)$] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]

- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]

- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B]
Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are metted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 9

****[TO GET THE VALUE OF THE RESULTANT(|R) VECTOR]****

+ How much [Vectors] will necessary to get the [Resultant(R) Vector]?

[°>°] Provide the [new] value? 1

****[Warning]****

- All the [Vectorials Components] will can be:
[positive] or [negative] or Zero!

+ What are the arguments: [angle] and [radius] of the (1°) [vector]?

- Enter with the [new value] of the [angle] in degree?

(^<^a) Enter the [new] value? 57

- Provide the [new value] of the [radius] of vector?

[°<°] Type a [new] value? -23

===

°>°

[Warning!]:

[~] ****[No exist none [vector] when the [component]: radius is:]**

****[A Negative float number: radius < 0] or [Zero: radius = 0] -- Ok!]****

[°<°] Type a [new] value? 0

===

°>°

[Warning!]:

[~] ****[No exist none [vector] when the [component]: radius is:]**

****[A Negative float number: radius < 0] or [Zero: radius = 0] -- Ok!]****

[°<°] Type a [new] value? ENTER

===

@<@

[Warning!]: could not convert string to float: "

[-] **[NO KEY ANY OTHER CHARACTER OR [ENTER] IN THE KEYBOARD]******

[TYPE ALWAYS A NEW [POSITIVE FLOAT NUMBER] TO THE [RADIUS]]
[IN NEXT INSTRUCTION -- OK!]

[°<°] Type a [new] value? 23

- _ . . . [Running] . . . _ -

[Answer]

- The component of the (1)^o [vector]: Vx(1)= 12.53
- The component of the (1)^o [vector]: Vy(1)= 19.29

[Warning]:

[THE [PREVIOUS COMPONENTS]: Vx(1) AND Vy(1) BEEN CALCULATED; BUT]
[[NO EXIST RESULTANT(R) VECTOR] TO A ONLY VECTOR!]

[End Processing of the [RESULTANTVECTOR CLASS]--Ok!]

. . . Key [ENTER] to exit -- Ok! . . .

Example11 – Given the vectors: $a = 2i + 3j$ and $b = 5i - j$ find the [vectors and modules]: $a + b$, $a - b$, $|a + b|$, $|a - b|$, and $2a - 3b$.

Solution – Begin run the runVector2d.py program and key the option[11] and wait the display present instructions to select the options: [1] or [2]. Follow type the option[1] and provide the [components]; $a_1 = 2$, $a_2 = 3$, $b_1 = 5$, and $b_2 = -1$. Following the program will wait the user enter the [coefficients]. Do (1^o) [coefficient] = 1 and (2^o) [coefficient] = 1 and after key ENTER to process.

[WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM]
[TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE]]
--[Version: 2.0 -- Stable]--

[INSTRUCTIONS OF USE]

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM) and N(xN,yN) between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]

- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 11

**[GIVENS [TWO VECTORS: A AND B] MULTIPLYED BY THE [SCALARS]:
[COEFFIC1] AND [COEFFIC2] OR]**

**[GIVENS [THE POINTS: P, Q, AND R] FIND [TWO VECTORS]:
VECTOR(A)=VECTOR(PQ) AND VECTOR(B)=VECTOR(PR)]**

**[AND MULTIPLY BY THE [SCALARS]: [COEFFIC1] AND [COEFFIC2] AND
TOO GET THE [ADDITION] AND [SUBTRACTION]]**

- To enter the [Components] of the [vectors] type [1].

- To introduce the [Coordinates] of the [points] type [2].

(°>°) Provide the [new] value? 1

- Provide the [Components] of the [vectors]: vectorA and vectorB.

- Attribute the [Components] of the [1° vectorA]!

- Enter the [coefficient] of the [Component: (i) or (a1)]?

(°<°) Enter the [new] value? 2

- Introduce the [coefficient] of the [Component: (j) or (a2)]?

(°<°) Enter the [new] value? 3

- Provide the [Components] of the [2° vectorB]?

- Enter the [coefficient] of the [Component: (i) or (b1)]?

(°<°) Enter the [new] value? 5

- Introduce the [coefficient] of the [Component: (j) or (b2)]?

(°<°) Enter the [new] value? -1

-- The [vectorA]: vectorA [2.0, 3.0]

-- The [vectorB]: vectorB [5.0, -1.0]

- Enter with new [value] to the (1°)[coefficient]?

(°<°) Enter the [new] value? 1

- Give the new [value] to the (2°)[coefficient]?

(°<°) Enter the [new] value? 1

[Answer]

+ The [product]: (coeffic1)*[vectorA] = coeffic1*vectorA [2.0, 3.0]

+ The [product]: (coeffic2)*[vectorB] = coeffic2*vectorB [5.0, -1.0]

-- The [Addition] = (coeffic1)vectorA+(coeffic2)vectorB [7.0, 2.0]

-- The [Subtraction] = (coeffic1)vectorA-(coeffic2)vectorB [-3.0, 4.0]

-- The [length] of the |vectorAddition| is: 7.28

-- The [length] of the |vectorSubtraction| is: 5.00

. . . Key [ENTER] to exit -- Ok! . . .

After the **previous processing** the [vectors and modules]: $a + b$, $a - b$, $|a + b|$, $|a - b|$ was found.

Now to get the vector: $2a - 3b$ and module: $|2a - 3b|$, run the runVector2d.py program again and key the option[11] and wait the display present instructions to select the options: [1] or [2]. Follow type the option[1] and provide again the [components]: $a_1 = 2$, $a_2 = 3$, $b_1 = 5$, and $b_2 = -1$. The program will wait the user enter the [coefficients]. Do (1°)[coefficient] = 2 and (2°)[coefficient] = -3 and after key ENTER to process.

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)] To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)] To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 11

```
**[ GIVENS [TWO VECTORS: A AND B] MULTIPLIED BY THE [SCALARS]:
[COEFFIC1] AND [COEFFIC2] OR ]**
```

```
**[ GIVENS [THE POINTS: P, Q, AND R] FIND [TWO VECTORS]:
VECTOR(A)=VECTOR(PQ) AND VECTOR(B)=VECTOR(PR) ]**
```

```
**[ AND MULTIPLY BY THE [SCALARS]: [COEFFIC1] AND [COEFFIC2] AND
TOO GET THE [ADDITION] AND [SUBTRACTION] ]**
```

- To enter the [Components] of the [vectors] type [1].
- To introduce the [Coordinates] of the [points] type [2].

(⁰>⁰) Provide the [new] value? 1

- Provide the [Components] of the [vectors]: vectorA and vectorB.

- Attribute the [Components] of the [1^o vectorA]!

- Enter the [coefficient] of the [Component: (i) or (a1)]?

(^a<^a) Enter the [new] value? 2

- Introduce the [coefficient] of the [Component: (j) or (a2)]?

(^a<^a) Enter the [new] value? 3

- Provide the [Components] of the [2^o vectorB]?

- Enter the [coefficient] of the [Component: (i) or (b1)]?

(^a<^a) Enter the [new] value? 5

- Introduce the [coefficient] of the [Component: (j) or (b2)]?

(^a<^a) Enter the [new] value? -1

-- The [vectorA]: vectorA [2.0, 3.0]

-- The [vectorB]: vectorB [5.0, -1.0]

- Enter with new [value] to the (1^o)[coefficient]?

(^a<^a) Enter the [new] value? 2

- Give the new [value] to the (2^o)[coefficient]?

(^a<^a) Enter the [new] value? -3

[Answer]

+ The [product]: (coeffic1)*[vectorA] = coeffic1*vectorA [4.0, 6.0]

+ The [product]: (coeffic2)*[vectorB] = coeffic2*vectorB [-15.0, 3.0]

-- The [Addition] = (coeffic1)vectorA+(coeffic2)vectorB [-11.0, 9.0]

-- The [Subtraction] = (coeffic1)vectorA-(coeffic2)vectorB [19.0, 3.0]

-- The [length] of the |vectorAddition| is: 14.21

-- The [length] of the |vectorSubtraction| is: 19.24

. . . Key [ENTER] to exit -- Ok! . . .

Example12 – Given the points: P(2, 0), Q(0, 3), and R(3, 4) find the [vectors and modules]: $a + b$, $a - b$, $|a + b|$, and $|a - b|$.

Solution – Again run the runVector2d.py program and key the option[11] and wait the display present instructions to select the options: [1] or [2]. Follow type the option[2] and provide the [coordinates]; $x_P = 2$, $y_P = 0$, $x_Q = 0$, $y_Q = 3$, $x_R = 3$ and $y_R = 4$. Following the program will wait the user enter the [coefficients]. Do (1^o)[coefficient] = 1 and (2^o)[coefficient] = 1 and after key ENTER to process.

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]

- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with he [coordinates] of the given [points: P(x_P, y_P), Q(x_Q, y_Q), and R(x_R, y_R)] key [2]

- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 11

[GIVENS [TWO VECTORS: A AND B] MULTIPLYED BY THE [SCALARS]: [COEFFIC1] AND [COEFFIC2] OR]

[GIVENS [THE POINTS: P, Q, AND R] FIND [TWO VECTORS]: VECTOR(A)=VECTOR(PQ) AND VECTOR(B)=VECTOR(PR)]

[AND MULTIPLY BY THE [SCALARS]: [COEFFIC1] AND [COEFFIC2] AND TOO GET THE [ADDITION] AND [SUBTRACTION]]

- To enter the [Components] of the [vectors] type [1].
- To introduce the [Coordinates] of the [points] type [2].

(°>°) Provide the [new] value? 2

- Give the [Coordinates] of the [givens points: P and Q].

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].

(°<°) Enter the [new] value? 2

* Enter with the 2°[Coordinate(y)].

(°<°) Enter the [new] value? 0

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1°[Coordinate(x)].

(°<°) Enter the [new] value? 0

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? 3

- The (Point P): P (2.0, 0.0)

- The (Point Q): Q (0.0, 3.0)

[Answer]

-- The [vectorA]: vectorA=vectorPQ [-2.0, 3.0]

-- The [lengthPQ = distPQ] is the ||vectorPQ||: 3.61

- Enter the [Coordinates] of the [gives points: P and R].

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].

(^a<^a) Enter the [new] value? 2

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? 0

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1°[Coordinate(x)].

(^a<^a) Enter the [new] value? 3

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? 4

- The (Point P): P (2.0, 0.0)

- The (Point R): R (3.0, 4.0)

[Answer]

-- The [vectorB]: vectorB=vectorPR [1.0, 4.0]

-- The [lengthPR = distPR] is the ||vectorPR||: 4.12

-- The [vectorA]: vectorA [-2.0, 3.0]

-- The [vectorB]: vectorB [1.0, 4.0]

- Enter with new [value] to the (1°)[coefficient]?

(^a<^a) Enter the [new] value? 1

- Give the new [value] to the (2°)[coefficient]?

(^a<^a) Enter the [new] value? 1

[Answer]

+ The [product]: (coeffic1)*[vectorA] = coeffic1*vectorA [-2.0, 3.0]

+ The [product]: (coeffic2)*[vectorB] = coeffic2*vectorB [1.0, 4.0]

-- The [Addition] = (coeffic1)vectorA+(coeffic2)vectorB [-1.0, 7.0]

-- The [Subtraction] = (coeffic1)vectorA-(coeffic2)vectorB [-3.0, -1.0]

-- The [length] of the |vectorAddition| is: 7.07

-- The [length] of the |vectorSubtraction| is: 3.16

. . . Key [ENTER] to exit -- Ok! . . .

[Warning]: After process all the previous Examples using the runVector2d.py program get the same answer in accordance with the old runvectors2dim.py program file version: 1.3 . Now will introduce a New Example-13 that created to all users view the new performance of the runVector2d.py program in solve new problems.

New information to all the users:

In this repository too was added a new TrianglePQR.pdf file.

New Example-13: Given the vertice points: P(2,3), Q(-3,-2), and R(5,-4) of a TrianglePQR in \mathbb{R}^2 Plane. Determine the following dimensions below. **[Warning]:** below of each item will process the solution.

1. Find the sides: sideA, sideB, and sideC, the Perimeter, Heights: h1, h2, and h3, the Area(S).

Solution – Begin the runVector2d.py program and key the option[2] and wait the display present instructions.

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--

**[INSTRUCTIONS OF USE]**

- To find the [Scalar Product: a * b] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [ points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS) ] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)] To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)] To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [ Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR) ] of the triangle(PQR) with the [Coordinates] given of the [ Middle points: K(xK,yK), L(xL,yL), and M(xM,yM) ] key [14]
- To get the [Coordinates] of any a of the [ Points: D(xD,yD) or E(xE,yE), or F(xF,yF) ] that are meetted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[$] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 2

**[ TO CALCULATE THE SIDES: SIDE_A, SIDE_B, AND SIDE_C, THE PERIMETER(P), THE HEIGHTS: H1,H2, AND H3, AND THE ]**
**[ AREA(A) OF THE [TRIANGLE(PQR)] GIVEN THE [COORDINATES] OF THE [ VERTICE POINTS: P, Q, AND R [ IN THE PLANE] ]**
```

--Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1º[Coordinate(x)].

(^a<^a) Enter the [new] value? 2

* Enter with the 2º[Coordinate(y)].

(^a<^a) Enter the [new] value? 3

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1º[Coordinate(x)].

(^a<^a) Enter the [new] value? -3

* Enter with the 2º[Coordinate(y)].

(^a<^a) Enter the [new] value? -2

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1º[Coordinate(x)].

(^a<^a) Enter the [new] value? 5

* Enter with the 2º[Coordinate(y)].

(^a<^a) Enter the [new] value? -4

- The (Point P): P (2.0, 3.0)

- The (Point Q): Q (-3.0, -2.0)

- The (Point R): R (5.0, -4.0)

[Answer]

==
[EXIST THE TRIANGLE(PQR) WITH THE GIVEN POINTS: P, Q, AND R -- OK!]
==

-- The [vectorA] = vectorQR [8.0, -2.0]

-- The [vectorB] = vectorRP [-3.0, 7.0]

-- The [vectorC] = vectorPQ [-5.0, -5.0]

-- The [sideA] = ||vectorQR|| of the triangle(PQR)] is: 8.25

-- The [sideB] = ||vectorRP|| of the triangle(PQR)] is: 7.62

-- The [sideC] = ||vectorPQ|| of the triangle(PQR)] is: 7.07

-- The [terms] of the [Scalar Product(PQºQR)] is: [-40.0, 10.0]

-- The [terms] of the [Scalar Product(QRºRP)] is: [-24.0, -14.0]

-- The [terms] of the [Scalar Product(RPºPQ)] is: [15.0, -35.0]

-- The [Scalar Product(PQºQR)] is: -30.00

-- The [Scalar Product(QRºRP)] is: -38.00

-- The [Scalar Product(RPºPQ)] is: -20.00

-- The [Perimeter] of the [triangle(PQR)] is 22.93

-- The [Height(h1) relative as sideQR] is 6.06

-- The [Height(h2) relative as sideRP] is 6.57

-- The [Height(h3) relative as sidePQ] is 7.07

-- The [Area(A)] of the [triangle(PQR)] is 25.00

... Key [ENTER] to exit -- Ok! ...

2. Will Get the three Inner Angles: Alpha(α), Beta(β), and Gama(γ) of the TrianglePQR.

Solution – Again begin the runVector2d.py program and key the option[6] and wait the display present instructions.

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM) and N(xN,yN) between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)] To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)] To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[>] Provide the [new] value? 6

```
**[ TO FIND THE THREES SIDES: SIDEA, SIDEB, AND SIDEC, THE [INNER ANGLES:
ALPHA, BETA, AND GAMA]**
**[ OF THE [TRIANGLE(PQR)] GIVENS THE [COORDINATES] OF THE POINTS: P, Q,
AND R IN THE PLANE ]**
```

--Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1^o[Coordinate(x)].

(^{a<a}) Enter the [new] value? 2

* Enter with the 2^o[Coordinate(y)].

(^{a<a}) Enter the [new] value? 3

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? -3

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? -2

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? 5

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? -4

- The (Point P): P (2.0, 3.0)

- The (Point Q): Q (-3.0, -2.0)

- The (Point R): R (5.0, -4.0)

[Answer]

- The [vectorB]=vectorPR [3.0, -7.0]

- The [vectorC]=vectorPQ [-5.0, -5.0]

- The [vectorA]=vectorQR [8.0, -2.0]

- The [vectorC]=vectorQP [5.0, 5.0]

- The [vectorA]=vectorRQ [-8.0, 2.0]

- The [vectorB]=vectorRP [-3.0, 7.0]

==
[EXIST THE TRIANGLE(PQR) WITH THE GIVEN POINTS: P, Q, AND R -- OK!]
==

-- The [sideA]=|vectorA| of the triangle(PQR)] is: 8.25

-- The [terms] of the [Scalar Product(AC°AB)] is the vector[AC°AB]: [-15.0, 35.0]

-- The [value] of the [Scalar Product(AC°AB)] is: 20.00

- The value of the [ALPHA ANGLE] was calculate is: 68.20

-- The [sideB]=|vectorB| of the triangle(PQR)] is: 7.62

-- The [terms] of the [Scalar Product(BC°BA)] is the vector[BC°BA]: [40.0, -10.0]

-- The [value] of the [Scalar Product(BC°BA)] is: 30.00

- The value of the [BETA ANGLE] was calculate is: 59.04

-- The [sideC]=|vectorC| of the triangle(PQR)] is: 7.07

-- The [terms] of the [Scalar Product(CB°CA)] is the vector[CB°CA]: [24.0, 14.0]

-- The [value] of the [Scalar Product(CB°CA)] is: 38.00

- The value of the [GAMA ANGLE] was calculate is: 52.77

-- The triangle(PQR) is [Scalene]!

-- The [Add] of the [Inner Angles: ALPHA, BETA, GAMA] is 180.0

- THE [ADD] OF THE [INNER ANGLES] OF THE [TRIANGLE-PQR] is 180.00

--[END CALCULUS-OK!!--

. . . Key [ENTER] to exit -- Ok! . . .

[Warning]: The next items: 3. until 6. and too the next **New Example 14**, only will can be solved using the runVector2d.py program file.

3. Determine the [coordinates] of the three Middle points: K(xK,yK), L(xL, yL), and M(xM,yM).

Solution – Now begin he runVector2d.py program and type the option[7] and wait the display present instructions. After key the option[1].

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--
```

[INSTRUCTIONS OF USE]

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with he [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meetted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 7

```
=====
**[ WILL DETERMINE THE [MIDPOINTS]: ]**
=====
```

- Type [1] to find the [Middle points] of the sides: PQ, PR, and QR of a Triangle(PQR).
- Type [2] to determine the [Middle Point: K(xK,yK)] relative as points: P(xP,yP) and Q(xQ,yQ).
- Type [3] to get the [Middle Point: L(xL,yL)] relative as points: Q(xQ,yQ) and R(xR,yR).

- Type [4] to find the [Middle Point: $M(x_M, y_M)$] relative as points: $R(x_R, y_R)$ and $S(x_S, y_S)$.
- Type [5] to calculate the [Middle Point: $N(x_N, y_N)$] relative as points: $S(x_S, y_S)$ and $P(x_P, y_P)$.

[>] Provide the [new] value? 1

- [Middle point: $K(x_K, y_K)$] relative as side PQ.
- [Middle point: $L(x_L, y_L)$] relative as side PR.
- [Middle point: $M(x_M, y_M)$] relative as side QR.

-- Enter the [coordinates]: (x_P, y_P) of the (point P)?

* Introduce the 1^o[Coordinate(x)].

(^a<a) Enter the [new] value? 2

* Enter with the 2^o[Coordinate(y)].

(^a<a) Enter the [new] value? 3

-- Introduce the [coordinates]: (x_Q, y_Q) of the (point Q)?

* Introduce the 1^o[Coordinate(x)].

(^a<a) Enter the [new] value? -3

* Enter with the 2^o[Coordinate(y)].

(^a<a) Enter the [new] value? -2

- The (Point P): $P(2.0, 3.0)$

- The (Point Q): $Q(-3.0, -2.0)$

[Answer]

-- The MidPoint_K(x_K, y_K): $(-0.5, 0.5)$

-- Enter the [coordinates]: (x_P, y_P) of the (point P)?

* Introduce the 1^o[Coordinate(x)].

(^a<a) Enter the [new] value? 2

* Enter with the 2^o[Coordinate(y)].

(^a<a) Enter the [new] value? 3

-- Provide the [coordinates]: (x_R, y_R) of the (point R)?

* Introduce the 1^o[Coordinate(x)].

(^a<a) Enter the [new] value? 5

* Enter with the 2^o[Coordinate(y)].

(^a<a) Enter the [new] value? -4

- The (Point P): $P(2.0, 3.0)$

- The (Point R): $R(5.0, -4.0)$

[Answer]

-- The MidPoint_L(x_L, y_L): $(3.5, -0.5)$

-- Introduce the [coordinates]: (x_Q, y_Q) of the (point Q)?

* Introduce the 1^o[Coordinate(x)].

(^a<a) Enter the [new] value? -3

* Enter with the 2^o[Coordinate(y)].

(^a<a) Enter the [new] value? -2

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? 5

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? -4

- The (Point Q): Q (-3.0, -2.0)

- The (Point R): R (5.0, -4.0)

[Answer]

-- The MidPoint_M(xM,yM): (1.0, -3.0)

. . . Key [ENTER] to exit -- Ok! . . .

[Warning] : The previous options: [2] until [5] will find only the [coordinates] of each Middle-points: K(xK,yK) or L(xL,yL) or M(xM,yM) or N(xN,yN) between the points: P, Q, R, and S.

4. Calculate the three medians: PM, QL, and RK.

Solution – Run the runVector2d.py program and type the option[12] and wait the display present instructions. After key the option[1]

[Warning]: The previous option: [7] and [1] calculated the three Middle points: K(xK, yK), L(xL,yL), and M(xM,yM) of the sides: sideA, sideB, and sideC of the TrianglePQR. Use this **previous option only** when the **problem request** to find only the three Middle points: K, L, and M of the TrianglePQR. Here will not necessary use this previous option because of new option: [12] and [1] created in **Medians** class of the Vector2DClassModule.py module will find the Middle point M(xM,yM) before of determine the [Median. PM].

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**  
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**  
--[Version: 2.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]

- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are metted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 12

```
=====
**[ TO DETERMINE THE MEDIANS OF THE TRIANGLE(PQR) ]**
=====
```

- Key [1] to find the new medianPM or
- Key [2] to find the new medianQL or
- Key [3] to find the new medianRK.

[°>°] Provide the [new] value? 1

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1°[Coordinate(x)].

(^<^) Enter the [new] value? -3

* Enter with the 2°[Coordinate(y)].

(^<^) Enter the [new] value? -2

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1°[Coordinate(x)].

(^<^) Enter the [new] value? 5

* Enter with the 2°[Coordinate(y)].

(^<^) Enter the [new] value? -4

- The (Point Q): Q (-3.0, -2.0)

- The (Point R): R (5.0, -4.0)

[Answer]

-- The MidPoint_M(xM,yM): (1.0, -3.0)

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].

(^<^) Enter the [new] value? 2

* Enter with the 2°[Coordinate(y)].

(^<^) Enter the [new] value? 3

[Answer]

-- The terms of the [vectorPM]: [-1.0, -6.0]

-- The [median: PM] finded is 6.08

. . . Key [ENTER] to exit -- Ok! . . .

Propose Exercise:

Now all users using the [Coordinates: $x_P=2$, $y_P=3$, $x_Q=-3$, $y_Q=-2$, $x_R=5$, and $y_R=-4$] of the previous vertex points: P, Q, and R of the TrianglePQR. Calculate the medians: QL and RK. Again begin run the runVector2d.py program and follow key the options: [12] and [2] to find the median: QL and after follow type in options: [12] and [3] to determine the median: RK.

[Answer]

-- The terms of the [vectorQL]: [6.5, 1.5]

-- The [median: QL] finded is 6.67

-- The terms of the [vectorRK]: [-5.5, 4.5]

-- The [median: RK] finded is 7.11

5. Find the [coordinates: x_D , y_D , x_E , y_E , x_F , and y_F] of the three points: D, E, and F that the bisectrices: PD, QE, and RF meet in sides: QR, PR, and PQ of the TrianglePQR.

Solution – Again run the runVector2d.py program and type the option[15] and wait the display present instructions. After key the option[1]

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--
```

[INSTRUCTIONS OF USE]

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(x_P, y_P), Q(x_Q, y_Q), and R(x_R, y_R)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h_1 and h_2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(x_P, y_P), Q(x_Q, y_Q), R(x_R, y_R), and S(x_S, y_S)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(x_P, y_P), Q(x_Q, y_Q), and R(x_R, y_R)] key [6]
- To Determine the following [Middle Points]: K(x_K, y_K), L(x_L, y_L), M(x_M, y_M) and N(x_N, y_N) between [points: P(x_P, y_P), Q(x_Q, y_Q), R(x_R, y_R), and S(x_S, y_S)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(x_P, y_P), and Q(x_Q, y_Q)] To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(x_P, y_P), and R(x_R, y_R)] To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(x_Q, y_Q), and R(x_R, y_R)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(x_G, y_G)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(x_P, y_P), Q(x_Q, y_Q), and R(x_R, y_R)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(x_K, y_K), L(x_L, y_L), and M(x_M, y_M)] key [14]
- To get the [Coordinates] of any a of the [Points: D(x_D, y_D) or E(x_E, y_E), or F(x_F, y_F)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]

- To determine any a of the [bisectrix: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 15

[TO GET ANY A OF THE [POINTS: D(xD,yD) or E(xE,yE), or F(xF,yF)] IN THE SIDES:]
[QR, PR, AND PQ OF A TRIANGLE(PQR)]

- What's the point: D, or E, or F will get?

- Type [1] to find the [coordinate] of the point: D(xD,yD).
- Type [2] to find the [coordinate] of the point: E(xE,yE).
- Type [3] to find the [coordinate] of the point: F(xF,yF).

[°>°] Provide the [new] value? 1

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].
[°<°] Enter the [new] value? 2

* Enter with the 2°[Coordinate(y)].
[°<°] Enter the [new] value? 3

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1°[Coordinate(x)].
[°<°] Enter the [new] value? -3

* Enter with the 2°[Coordinate(y)].
[°<°] Enter the [new] value? -2

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1°[Coordinate(x)].
[°<°] Enter the [new] value? 5

* Enter with the 2°[Coordinate(y)].
[°<°] Enter the [new] value? -4

[Answer]

-- The [coordinate: xD] is 0.85
-- The [coordinate: yD] is -2.96

. . . Key [ENTER] to exit -- Ok! . . .

Continue run the runVector2d.py program and type the option[15] and wait the display present instructions.
After key the option[2]

[WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM]
[TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE]]
--[Version: 2.0 -- Stable]--

[INSTRUCTIONS OF USE]

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM) and N(xN,yN) between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 15

[TO GET ANY A OF THE [POINTS: D(xD,yD) or E(xE,yE), or F(xF,yF)] IN THE SIDES:]
 [QR, PR, AND PQ OF A TRIANGLE(PQR)]

- What's the point: D, or E, or F will get?

- Type [1] to find the [coordinate] of the point: D(xD,yD).
- Type [2] to find the [coordinate] of the point: E(xE,yE).
- Type [3] to find the [coordinate] of the point: F(xF,yF).

[°>°] Provide the [new] value? 2

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].

(^a<^a) Enter the [new] value? 2

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? 3

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1°[Coordinate(x)].

(^a<^a) Enter the [new] value? -3

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? -2

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? 5

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? -4

[Answer]

-- The [coordinate: xE] is 3.38

-- The [coordinate: yE] is -0.23

Continue the run the runVector2d.py program and type the option[15] and wait the display present instructions. After key the option[3]

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--
```

[INSTRUCTIONS OF USE]

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

(^o>^o) Provide the [new] value? 15

```
**[ TO GET ANY A OF THE [POINTS: D(xD,yD) or E(xE,yE), or F(xF,yF)] IN THE SIDES: ]**
**[ QR, PR, AND PQ OF A TRIANGLE(PQR) ]**
```

- What's the point: D, or E, or F will get?

-- Type [1] to find the [coordinate] of the point: D(xD,yD).
-- Type [2] to find the [coordinate] of the point: E(xE,yE).
-- Type [3] to find the [coordinate] of the point: F(xF,yF).

[0>0] Provide the [new] value? 3

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1^o[Coordinate(x)].

(a<a) Enter the [new] value? 2

* Enter with the 2^o[Coordinate(y)].

(a<a) Enter the [new] value? 3

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1^o[Coordinate(x)].

(a<a) Enter the [new] value? -3

* Enter with the 2^o[Coordinate(y)].

(a<a) Enter the [new] value? -2

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1^o[Coordinate(x)].

(a<a) Enter the [new] value? 5

* Enter with the 2^o[Coordinate(y)].

(a<a) Enter the [new] value? -4

[Answer]

-- The [coordinate: xF] is -0.40

-- The [coordinate: yF] is 0.60

. . . Key [ENTER] to exit -- Ok! . . .

6. Calculate the **bisectrices: PD, QE, and RF** of the TrianglePQR.

Solution – Again run the **runVector2d.py** program and type the **option[16]** and wait the display present instructions. After key the **option[1]**

***-_[Warning]*:** The previous **option: [15]** with the **options: [1] or [2] or [3]** calculated the [coordinates] of the three points: D(xD, yD), E(xE,yE), and F(xF,yF) in sides: sideA, sideB, and sideC of the TrianglePQR. Used this **previous option only** when the **problem request** to find **only [coordinates]** of the three points: D, E, and F in TrianglePQR. Here will not necessary use this **previous option** because of new **option: [16]** with the **options: [1], [2], and [3]** was developed in **BisectrixTriangle** class of the **Vector2DClassModule.py** module will find the point: D(xD,yD) before of determine the [Bisectrix: PD]. Of the same manner to the points: E(xE,yE), and F(xF,yF) that will find the Bisectrix: QE and RF of the TrianglePQR.

****[WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM]****
****[TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE]]****
--[Version: 2.0 -- Stable]--

****[INSTRUCTIONS OF USE]****

- To find the [Scalar Product: a * b] between [two vectors] key **[1]**

- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM) and N(xN,yN) between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are metted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 16

[TO FIND ANY A OF THE [BISECTRIXS: PD, QE, and RF] OF INNER ANGLES OF A TRIANGLE(PQR)]

- What's the bisectrix: PD, or QE, or RF of a Triangle(PQR) will determine?

- Key [1] to calculate the [bisectrixPD]
- Key [2] to calculate the [bisectrixQE]
- Key [3] to calculate the [bisectrixRF]

[°>°] Provide the [new] value? 1

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].

(^a<^a) Enter the [new] value? 2

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? 3

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1°[Coordinate(x)].

(^a<^a) Enter the [new] value? -3

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? -2

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? 5

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? -4

[Answer]

-- The [coordinate: xD] is 0.85

-- The [coordinate: yD] is -2.96

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? 2

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? 3

[Answer]

-- The value finded of the [bisectrixPD] of the Triangle(PQR) is 6.07

. . . Key [ENTER] to exit -- Ok! . . .

Now run the runVector2d.py program and type the option[16] and wait the display present instructions. After key the option[2]

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**  
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**  
--[Version: 2.0 -- Stable]--
```

[INSTRUCTIONS OF USE]

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with he [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]

- To find the [Centroid: $G(x_G, y_G)$] of the triangle(PQR) key [13]
- To calculate the [Vertices: $P(x_P, y_P)$, $Q(x_Q, y_Q)$, and $R(x_R, y_R)$] of the triangle(PQR) with the [Coordinates] given of the [Middle points: $K(x_K, y_K)$, $L(x_L, y_L)$, and $M(x_M, y_M)$] key [14]
- To get the [Coordinates] of any a of the [Points: $D(x_D, y_D)$ or $E(x_E, y_E)$, or $F(x_F, y_F)$] that are metted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 16

[TO FIND ANY A OF THE [BISECTRIXS: PD, QE, and RF] OF INNER ANGLES OF A TRIANGLE(PQR)]

- What's the bisectrix: PD, or QE, or RF of a Triangle(PQR) will determine?

- Key [1] to calculate the [bisectrixPD]
- Key [2] to calculate the [bisectrixQE]
- Key [3] to calculate the [bisectrixRF]

[°>°] Provide the [new] value? 2

-- Enter the [coordinates]: (x_P , y_P) of the (point P)?

* Introduce the 1°[Coordinate(x)].

(^{a<a}) Enter the [new] value? 2

* Enter with the 2°[Coordinate(y)].

(^{a<a}) Enter the [new] value? 3

-- Introduce the [coordinates]: (x_Q , y_Q) of the (point Q)?

* Introduce the 1°[Coordinate(x)].

(^{a<a}) Enter the [new] value? -3

* Enter with the 2°[Coordinate(y)].

(^{a<a}) Enter the [new] value? -2

-- Provide the [coordinates]: (x_R , y_R) of the (point R)?

* Introduce the 1°[Coordinate(x)].

(^{a<a}) Enter the [new] value? 5

* Enter with the 2°[Coordinate(y)].

(^{a<a}) Enter the [new] value? -4

[Answer]

-- The [coordinate: x_E] is 3.38

-- The [coordinate: y_E] is -0.23

-- Introduce the [coordinates]: (x_Q , y_Q) of the (point Q)?

* Introduce the 1°[Coordinate(x)].

(^{a<a}) Enter the [new] value? -3

* Enter with the 2°[Coordinate(y)].

(^{a<a}) Enter the [new] value? -2

[Answer]

-- The value finded of the [bisectrixQE] of the Triangle(PQR) is 6.63

. . . Key [ENTER] to exit -- Ok! . . .

Again run the runVector2d.py program and type the option[16] and wait the display present instructions. After key the option[3]

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**  
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**  
--[Version: 2.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)] To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)] To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are metted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 16

```
**[ TO FIND ANY A OF THE [BISECTRIXS: PD, QE, and RF] OF INNER ANGLES OF A TRIANGLE(PQR)]**
```

- What's the bisectrix: PD, or QE, or RF of a Triangle(PQR) will determine?

```
-- Key [1] to calculate the [bisectrixPD]  
-- Key [2] to calculate the [bisectrixQE]  
-- Key [3] to calculate the [bisectrixRF]
```

[°>°] Provide the [new] value? 3

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].

[°<°] Enter the [new] value? 2

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? 3

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1°[Coordinate(x)].

(^a<^a) Enter the [new] value? -3

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? -2

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1°[Coordinate(x)].

(^a<^a) Enter the [new] value? 5

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? -4

[Answer]

-- The [coordinate: xF] is -0.40

-- The [coordinate: yF] is 0.60

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1°[Coordinate(x)].

(^a<^a) Enter the [new] value? 5

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? -4

[Answer]

-- The value finded of the [bisectrixRF] of the Triangle(PQR) is 7.09

. . . Key [ENTER] to exit -- Ok! . . .

New Example 14 – Given the [coordinates] of the Middle-points: K(-2,0), L(1,4), and M(4,1) of the sides: PQ, PR, and QR of a trianglePQR find the [coordinates: xP, yP, xQ, yQ, xR, and yR] of the three vertex points: P, Q, and R. After determine the [coordinates: xG, and yG] of the [Center of Gravity: G].

Solution: - Run the runVector2d.py program and type the option[14] and wait the display present instructions. After key the option[1].

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**  
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**  
--[Version: 2.0 -- Stable]--
```

[INSTRUCTIONS OF USE]

- To find the [Scalar Product: a * b] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with he [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]:

A and B key [5]

- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)] To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)] To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 14

[TO FIND THE VERTICES: P(xP,yP), Q(xQ,yQ), AND R(xR,yR) OF A TRIANGLE(PQR)]

- Type [1] to find the vertice: P(xP,yP) triangle(PQR).
- Type [2] to get the vertice: Q(xQ,yQ) triangle(PQR).
- Type [3] to find the vertice: R(xR,yR) triangle(PQR).

[°>°] Provide the [new] value? 1

-- Give the [coordinates]: (xK, yK) of the (Mid_point K)?

* Introduce the 1°[Coordinate(x)].

(^<^a) Enter the [new] value? -2

* Enter with the 2°[Coordinate(y)].

(^<^a) Enter the [new] value? 0

-- Give the [coordinates]: (xL, yL) of the (Mid_point L)?

* Introduce the 1°[Coordinate(x)].

(^<^a) Enter the [new] value? 1

* Enter with the 2°[Coordinate(y)].

(^<^a) Enter the [new] value? 4

-- Give the [coordinates]: (xM, yM) of the (Mid_point M)?

* Introduce the 1°[Coordinate(x)].

(^<^a) Enter the [new] value? 4

* Enter with the 2°[Coordinate(y)].

(^<^a) Enter the [new] value? 1

[Answer]

- The (Point P): P (-5.0, 3.0)

. . . Key [ENTER] to exit -- Ok! . . .

Solution: - Continue the run the runVector2d.py program and type the option[14] and wait the display present instructions. After key the option[2].

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**  
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**  
--[Version: 2.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with he [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] Provide the [new] value? 14

```
**[ TO FIND THE VERTICES: P(xP,yP), Q(xQ,yQ), AND R(xR,yR) OF A TRIANGLE(PQR) ]**
```

- Type [1] to find the vertice: P(xP,yP) triangle(PQR).
- Type [2] to get the vertice: Q(xQ,yQ) triangle(PQR).
- Type [3] to find the vertice: R(xR,yR) triangle(PQR).

[°>°] Provide the [new] value? 2

-- Give the [coordinates]: (xK, yK) of the (Mid_point K)?

* Introduce the 1°[Coordinate(x)].

[°<°] Enter the [new] value? -2

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? 0

-- Give the [coordinates]: (xL, yL) of the (Mid_point L)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? 1

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? 4

-- Give the [coordinates]: (xM, yM) of the (Mid_point M)?

* Introduce the 1^o[Coordinate(x)].

(^a<^a) Enter the [new] value? 4

* Enter with the 2^o[Coordinate(y)].

(^a<^a) Enter the [new] value? 1

[Answer]

- The (Point Q): Q (1.0, -3.0)

. . . Key [ENTER] to exit -- Ok! . . .

Solution: - Again run the runVector2d.py program and type the option[14] and wait the display present instructions. After key the option[3].

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**  
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**  
--[Version: 2.0 -- Stable]--
```

[INSTRUCTIONS OF USE]

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)] To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)] To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]

- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

(°>°) Provide the [new] value? 14

[TO FIND THE VERTICES: P(xP,yP), Q(xQ,yQ), AND R(xR,yR) OF A TRIANGLE(PQR)]

- Type [1] to find the vertice: P(xP,yP) triangle(PQR).
- Type [2] to get the vertice: Q(xQ,yQ) triangle(PQR).
- Type [3] to find the vertice: R(xR,yR) triangle(PQR).

[°>°] Provide the [new] value? 3

-- Give the [coordinates]: (xK, yK) of the (Mid_point K)?

* Introduce the 1°[Coordinate(x)].

(^<^a) Enter the [new] value? -2

* Enter with the 2°[Coordinate(y)].

(^<^a) Enter the [new] value? 0

-- Give the [coordinates]: (xL, yL) of the (Mid_point L)?

* Introduce the 1°[Coordinate(x)].

(^<^a) Enter the [new] value? 1

* Enter with the 2°[Coordinate(y)].

(^<^a) Enter the [new] value? 4

-- Give the [coordinates]: (xM, yM) of the (Mid_point M)?

* Introduce the 1°[Coordinate(x)].

(^<^a) Enter the [new] value? 4

* Enter with the 2°[Coordinate(y)].

(^<^a) Enter the [new] value? 1

[Answer]

- The (Point R): R (7.0, 5.0)

. . . Key [ENTER] to exit -- Ok! . . .

Geted the following vertice points:

- The (Point P): P (-5.0, 3.0)
- The (Point Q): Q (1.0, -3.0)
- The (Point R): R (7.0, 5.0)

Now follow type the option[13] to determine the [Center of Gravity].

Solution: - Run the runVector2d.py program and type the option[13] and wait the display present instructions.

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--
```

****[INSTRUCTIONS OF USE]****

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h)], and [Area(A)] of the TrianglePQR with the [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meeted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

(°>°) Provide the [new] value? 13

****[TO DETERMINE THE [CENTROID: G(xG,yG)] OF A TRIANGLE(PQR) GIVEN:]****
****[THE [COORDINATES] OF THE [POINTS]: P(xP,yP), Q(xQ,yQ), AND R(xR,yR)]****

-- Enter the [coordinates]: (xP, yP) of the (point P)?

* Introduce the 1°[Coordinate(x)].
(°<°) Enter the [new] value? -5

* Enter with the 2°[Coordinate(y)].
(°<°) Enter the [new] value? 3

-- Introduce the [coordinates]: (xQ, yQ) of the (point Q)?

* Introduce the 1°[Coordinate(x)].
(°<°) Enter the [new] value? 1

* Enter with the 2°[Coordinate(y)].
(°<°) Enter the [new] value? -3

-- Provide the [coordinates]: (xR, yR) of the (point R)?

* Introduce the 1°[Coordinate(x)].
(°<°) Enter the [new] value? 7

* Enter with the 2°[Coordinate(y)].

(^a<^a) Enter the [new] value? 5

- The (Point P): P (-5.0, 3.0)

- The (Point Q): Q (1.0, -3.0)

- The (Point R): R (7.0, 5.0)

[Answer]

-- The [Coordinates] of the [Centroid_ G(xG,yG)] in the Triangle(PQR):

-- The [Coordinate]: xG is 1.00

-- The [Coordinate]: yG is 1.67

. . . Key [ENTER] to exit -- Ok! . . .

[Warning]: In new version: 2.0 of the runVector2d.py program was improve the security of datas enter to the user type only *positive integer number* in the menu options. View below using the runVector2d.py program when any user type: -2 or p or @ or J or 0(zero) or key ENTER.

```
**[ WELCOME IN USING THE [RUNVECTOR2D.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [PLANE] ]**
--[Version: 2.0 -- Stable]--
```

[INSTRUCTIONS OF USE]

- To find the [Scalar Product: $a \cdot b$] between [two vectors] key [1]
- To calculate the sides: sideA, sideB and sideC, [Perimeter(P)], [Heights(h), and [Area(A)] of the TrianglePQR with he [coordinates] of the given [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [2]
- To get the sides: sideA, and sideB, the Perimeter(P), the [Heights: h1 and h2], and [Area(S)] of the [Parallelogram(PQRS)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] key [3]
- To find the [angle] between [two vectors] in plane key [4]
- To determine the [value] of the [CossineTheta] between the [two vectors]: A and B key [5]
- To find the three sides and [Inner Angles: ALPHA, BETA, and GAMA] of the [Triangle(PQR)] with the [coordinates] of the [points: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] key [6]
- To Determine the following [Middle Points]: K(xK,yK), L(xL,yL), M(xM,yM)] and N(xN,yN)] between [points: P(xP,yP), Q(xQ,yQ), R(xR,yR), and S(xS,yS)] or in the sides: PQ, PR, and QR of a Triangle(PQR).key [7]
- To get the [Addition and Subtraction] of [Two Vectors: A and B] key [8]
- To calculate the [Resultant(|R) Vector] key [9]
- To find the [VectorA and lenght|A|] with the [coordinates] of the [points: P(xP,yP), and Q(xQ,yQ)]
To get the [VectorB and lenght|B|] with the [coordinates] of the [points: P(xP,yP), and R(xR,yR)]
To find the [VectorC and lenght|C|] with the [coordinates] of the [points: Q(xQ,yQ), and R(xR,yR)] key [10]
- To get the [Addition] and [Subtraction] between [Two Vectors: A and B] Multiplied by scalars: [coeffic1 and coeffic2] key [11]
- To determine the [Medians: PM, QL, and RK] of the triangle(PQR) key [12]
- To find the [Centroid: G(xG, yG)] of the triangle(PQR) key [13]
- To calculate the [Vertices: P(xP,yP), Q(xQ,yQ), and R(xR,yR)] of the triangle(PQR) with the [Coordinates] given of the [Middle points: K(xK,yK), L(xL,yL), and M(xM,yM)] key [14]
- To get the [Coordinates] of any a of the [Points: D(xD,yD) or E(xE,yE), or F(xF,yF)] that are meetted with the [bisectrixs: PD or QE or RF] in the sides: QR or PR or PQ of a Triangle(PQR) key [15]
- To determine any a of the [bisectrixs: PD or QE or RF] of a Triangle(PQR) key [16]

[§] Select an previous [option] that will used--Ok!

[>] Provide the [new] value? -2

[NO TYPE AN [NEGATIVE INTEGER NUMBER] or equal [ZERO]--Ok!]

[>] Provide the [new] value? p

```
////
< [Warning!]: invalid literal for int() with base 10: 'p'
[~] [ TYPE AN [NEW POSITIVE INTEGER NUMBER ]
      [ IN NEXT INSTRUCTION -- OK! ]
```

[>] Provide the [new] value? @

```
////
< [Warning!]: invalid literal for int() with base 10: '@'
[~] [ TYPE AN [NEW POSITIVE INTEGER NUMBER ]
      [ IN NEXT INSTRUCTION -- OK! ]
```

[>] Provide the [new] value? J

```
////
< [Warning!]: invalid literal for int() with base 10: 'J'
[~] [ TYPE AN [NEW POSITIVE INTEGER NUMBER ]
      [ IN NEXT INSTRUCTION -- OK! ]
```

[>] Provide the [new] value? 0

[NO TYPE AN [NEGATIVE INTEGER NUMBER] or equal [ZERO]--Ok!]

[>] Provide the [new] value? ENTER

```
////
< [Warning!]: invalid literal for int() with base 10: "
[~] [ TYPE AN [NEW POSITIVE INTEGER NUMBER ]
      [ IN NEXT INSTRUCTION -- OK! ]
```

[>] Provide the [new] value? 23

```
**[NEITHER OF THE PREVIOUS OPTIONS WAS SELECTED]**
[ RUN THE RUNVECTOR2D.PY PROGRAM AGAIN -- OK! ]
```

. . . Key [ENTER] to exit -- Ok! . . .

[Warning]: The runVector2d.py program will follow only when the user types any *positive integer number* between 1 to 16 – Ok! If type any positive integer number bigger than 16 the user will have that run the runVector2d.py program again. View the last previous test.

Note: The Components of Vectors and Coordinates of Points will can be: Negative or Zero or Positive; but the options of menu of the runVector2d.py program only can be typed *positive integer number*.

With the use of the [version: 2.0] runVector2d.py program and Vector2DClassModule.py module in previous examples all users viewed the new performance that insert into as *new code*.

[Warning]:

Any user that try solve all the Examples of the instructions1-v2.0_User.pdf without use the runVector2d.py program and Vector2DClassModule.py module files using pencil and paper will duty use a Scientific Calculator as help to determine the values: roots of the Vectors: |A|, |B|, and |C| modulus.

Developer: **Cristovom A. Girodo**