

Instructions of as use the runtoolsvectors.py program.

In the **Windows operating system**:

After the download of the **repository** access the runtoolsvectors.py program given two clicks and follow the news instructions in display.

In the **Linux operating system**:

After the download of the **repository** access the runtoolsvectors.py program using two options of run.

1.) In any Terminal linux type the following command:

```
python3 runtoolsvectors.py after key ENTER
```

2.) Open the runtoolsvectors.py file using the IDLE3 and use the **Run ==> Run Module options**. After follow the news instructions.

[Warning]: When the **runtoolsvectors.py program** request type the **[components]** or **[coordinates]**, always key **ENTER** after to continue, Ok!

Now will use of the runtoolsvectors.py program in the examples below.

Example1 – Given the vectors $a = \langle 7, -2, -11 \rangle$ and $b = \langle -18, 3, 17 \rangle$, find the addition $a + b$ and too the subtraction $a - b$.

Solution: Begin the runtoolsvectors.py program and select the **option[13]**. After follow the instructions requested in accordance with showed in the display.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] in the [SPACE] ]**
--[Version: 1.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] givens the points: P, Q, R key [8]
- To find the [Area] of the [Parallelogram] given [two vectors A and B] key [9]
- To get the [Volume(V)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and the [Subtraction] between [Two Vectors] A and B key [13]
- To find the [VectorA] and [lenght] of givens Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] givens an Vector A or two points: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors] A and B multiplied by scalars[coeffic1 and coeffic2] key [16]

```
[§] Select an previous [option] that will used--Ok!
```

```
[0>0] What is the new value? 13
```

```
**[The typed number]: 13 is a [valid integer number!]
```

```
]
```

```
**[ CALCULATE THE [ADDITION] AND [SUBTRACTION] OF THE [VECTORS]: A AND B ]**
```

```
- Attribute the [Components] of the [1° vectorA]!
```

```
- Enter the 1° [Component(x)]!
```

```
-_- What is the new value? 7
```

```
**[The typed number]: 7.0 is a [valid float number!]
```

```
]
```

```
- Introduce the 2° [Component(y)]!
```

```
-_- What is the new value? -2
```

```
**[The typed number]: -2.0 is a [valid float number!]
```

```
]
```

```
- Give the 3° [Component(z)]!
```

```
-_- What is the new value? -11
```

```
**[The typed number]: -11.0 is a [valid float number!]
```

```
]
```

```
- Provide the [Components] of the [2° vectorB]!
```

```
- Enter the 1° [Component(x)]!
```

```
-_- What is the new value? -18
```

```
**[The typed number]: -18.0 is a [valid float number!]
```

```
]
```

```
- Introduce the 2° [Component(y)]!
```

```
-_- What is the new value? 3
```

```
**[The typed number]: 3.0 is a [valid float number!]
```

```
]
```

```
- Give the 3° [Component(z)]!
```

```
-_- What is the new value? 17
```

```
**[The typed number]: 17.0 is a [valid float number!]
```

```
]
```

```
-- The [vectorA]: vectorA [7.0, -2.0, -11.0]
```

```
-- The [vectorB]: vectorB [-18.0, 3.0, 17.0]
```

```
*[ANSWER]*
```

```
-- The [Addition]: vectorA+B [-11.0, 1.0, 6.0]
```

```
-- The [Subtraction]: vectorA-B [25.0, -5.0, -28.0]
```

```
. . . Key [ENTER] to exit -- Ok! . . .
```

Example2 – If $\mathbf{a} = 2\mathbf{i} - 3\mathbf{k}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, and $\mathbf{c} = 4\mathbf{j} - \mathbf{k}$ then find the Scalar Triple Product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$, the volume $V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$ of the parallelepiped as too the volume $V = (1/6) \cdot \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ of the Tetrahedron.

Solution: Before of begin the `runtoolsvectors.py` program do know the components of this vectors so: $a_1 = 2$, $a_2 = 0$, $a_3 = -3$, $b_1 = 1$, $b_2 = 1$, $b_3 = 1$, $c_1 = 0$, $c_2 = 4$, and $c_3 = -1$ and select the option[4]. After follow the instructions requested in accordance with showed in the display.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
```

```
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] in the [SPACE] ]**
```

```
--[Version: 1.0 -- Stable]--
```

****[INSTRUCTIONS OF USE]****

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] givens the points: P, Q, R key [8]
- To find the [Area] of the [Parallelogram] given [two vectors A and B] key [9]
- To get the [Volume(V)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
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- To determine the [Direction Cosines] and [Direction Angles] givens an Vector A or two points: P and

Q key [15]

- To find the [Addition] and [Subtraction] between [Two Vectors] A and B multiplyed by scalars[coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

[^{0>0}] What is the new value? 4

**[The typed number]: 4 is a [valid integer number!]

]

****[CALCULATE THE [SCALAR TRIPLE PRODUCT] BETWEEN [THREE VECTORS: A, B, AND C] IN TRI-DIMENSIONAL(XYZ) SPACE]****

- Attribute the [Components] of the [1^o vectorA]!

- Enter the 1^o [Component(x)]!

-- What is the new value? 2

**[The typed number]: 2.0 is a [valid float number!]

]

- Introduce the 2^o [Component(y)]!

-- What is the new value? 0

**[The typed number]: 0.0 is a [valid float number!]

]

- Give the 3^o [Component(z)]!

-- What is the new value? -3

**[The typed number]: -3.0 is a [valid float number!]

]

- Provide the [Components] of the [2^o vectorB]!

- Enter the 1^o [Component(x)]!

-- What is the new value? 1

**[The typed number]: 1.0 is a [valid float number!]

]

- Introduce the 2^o [Component(y)]!

-- What is the new value? 1

**[The typed number]: 1.0 is a [valid float number!]

]

- Give the 3^o [Component(z)]!

```

    - _ - What is the new value? 1
      **[The typed number]: 1.0 is a [valid float number!]
]

    - Provide the [Components] of the [3o vectorC]!

    - Enter the 1o [Component(x)]!
    - _ - What is the new value? 0
      **[The typed number]: 0.0 is a [valid float number!]
]

    - Introduce the 2o [Component(y)]!
    - _ - What is the new value? 4
      **[The typed number]: 4.0 is a [valid float number!]
]

    - Give the 3o [Component(z)]!
    - _ - What is the new value? -1
      **[The typed number]: -1.0 is a [valid float number!]
]

*[ANSWER]*

```

```

-- The [vectorA]: vectorA [2.0, 0.0, -3.0]
-- The [vectorB]: vectorB [1.0, 1.0, 1.0]
-- The [vectorC]: vectorC [0.0, 4.0, -1.0]

```

```

--The [SCALAR TRIPLE PRODUCT]: a * ( b x c ) is: -22.0
-- The [VOLUME(V) PARALLELEPIPED]: 22.00
-- The [HEIGHT(H) PARALLELEPIPED]: 3.57
-- The [Tetrahedron volume]: 3.67

```

. . . Key [ENTER] to exit -- Ok! . . .

Example3 – A triangle in \mathbb{R}^3 has vertices P(0, 2, -1), Q(1, 1, 3), and R(1, 0, -4).

- Find the perimeter(P) of the triangle(PQR)
- Find the area(A) of the triangle(PQR)
- Find the three vertex angles of the triangle(PQR). (Round to the nearest degree)

Solution: – Run the runtoolsvectors.py program and key the option[8] and enter the coordinates of the vertices: P, Q, and R.

```

**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] in the [SPACE] ]**
--[Version: 1.0 -- Stable]--

```

```

**[INSTRUCTIONS OF USE]**

```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] givens the points: P, Q, R key [8]
- To find the [Area] of the [Parallelogram] given [two vectors A and B] key [9]

- To get the [Volume(V)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and the [Subtraction] between [Two Vectors] A and B key [13]
- To find the [VectorA] and [lenght] of givens Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] givens an Vector A or two points: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors] A and B multiplyed by scalars[coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

[>] What is the new value? 8

**[The typed number]: 8 is a [valid integer number!]

]

[GIVENS THE COORDINATES OF THE POINTS P, Q AND R FIND THE DIMENSIONS SIDE(A), SIDE(B),]

[SIDE(C), PERIMETER(P), HEIGHTS(H1,H2,H3), AND AREA(A) OF THE TRIANGLE(PQR)]

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

* Introduce the 1º [Coordinate(x)].

-- What is the new value? 0

**[The typed number]: 0.0 is a [valid float number!]

]

* Enter with the 2º [Coordinate(y)].

-- What is the new value? 2

**[The typed number]: 2.0 is a [valid float number!]

]

* Give the 3º [Coordinate(z)].

-- What is the new value? -1

**[The typed number]: -1.0 is a [valid float number!]

]

-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?

* Introduce the 1º [Coordinate(x)].

-- What is the new value? 1

**[The typed number]: 1.0 is a [valid float number!]

]

* Enter with the 2º [Coordinate(y)].

-- What is the new value? 1

**[The typed number]: 1.0 is a [valid float number!]

]

* Give the 3º [Coordinate(z)].

-- What is the new value? 3

**[The typed number]: 3.0 is a [valid float number!]

]

-- Provide the (coordinates: xR, yR, zR) of the (Point R)?

* Introduce the 1º [Coordinate(x)].

-- What is the new value? 1

**[The typed number]: 1.0 is a [valid float number!]

]

* Enter with the 2º [Coordinate(y)].

-- What is the new value? 0

**[The typed number]: 0.0 is a [valid float number!]

]

```
* Give the 3o [Coordinate(z)].
-_- What is the new value? -4
  **[The typed number]: -4.0 is a [valid float number!]
```

]

```
- The (Point P): P (0.0, 2.0, -1.0)
- The (Point Q): Q (1.0, 1.0, 3.0)
- The (Point R): R (1.0, 0.0, -4.0)
```

```
*[ANSWER]*
```

```
- The [vectorA]=vectorPQ [1.0, -1.0, 4.0]
- The [vectorB]=vectorPR [1.0, -2.0, -3.0]
- The [vectorC]=vectorQR [0.0, -1.0, -7.0]
```

```
-- The [sideA] of the triangle(PQR)] is: 4.24
-- The [sideB] of the triangle(PQR)] is: 3.74
-- The [sideC] of the triangle(PQR)] is: 7.07
-- The [Perimeter] of the [triangle(PQR)] is 15.06
```

```
-- The [terms] of the [Scalar Product(PQoQR)] is: [0.0, 1.0, -28.0]
-- The [terms] of the [Scalar Product(QRoPR)] is: [0.0, 2.0, 21.0]
-- The [terms] of the [Scalar Product(PRoPQ)] is: [1.0, 2.0, -12.0]
```

```
-- The [Scalar Product(PQoQR)] is: -27.00
-- The [Scalar Product(QRoPR)] is: 23.00
-- The [Scalar Product(PRoPQ)] is: -9.00
```

```
-- The [Height(h1) relative as sideQR] is 1.85
-- The [Height(h2) relative as sidePR] is 3.49
-- The [Height(h3) relative as sidePQ] is 3.08
```

```
-- The [Cross Product]: vectorAxB [11.0, 7.0, -1.0]
-- The [Area(A) of a Triangle(PQR)] is: 6.54
```

```
. . . Key [ENTER] to exit -- Ok! . . .
```

Now run the runtoolsvectors.py program again and key the option[11] and enter again the coordinates of the vertices: P, Q, and R.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] in the [SPACE] ]**
--[Version: 1.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] givens the points: P, Q, R key [8]
- To find the [Area] of the [Parallelogram] given [two vectors A and B] key [9]
- To get the [Volume(V)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
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- To find the [Addition] and [Subtraction] between [Two Vectors] A and B multiplyed by scalars[coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] What is the new value? 11

**[The typed number]: 11 is a [valid integer number!]

]

[GIVENS THE COORDINATES OF THE POINTS P, Q AND R]

[FIND THE [INNER ANGLES] OF THE TRIANGLE(PQR)]

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

* Introduce the 1° [Coordinate(x)].

-- What is the new value? 0

**[The typed number]: 0.0 is a [valid float number!]

]

* Enter with the 2° [Coordinate(y)].

-- What is the new value? 2

**[The typed number]: 2.0 is a [valid float number!]

]

* Give the 3° [Coordinate(z)].

-- What is the new value? -1

**[The typed number]: -1.0 is a [valid float number!]

]

-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?

* Introduce the 1° [Coordinate(x)].

-- What is the new value? 1

**[The typed number]: 1.0 is a [valid float number!]

]

* Enter with the 2° [Coordinate(y)].

-- What is the new value? 1

**[The typed number]: 1.0 is a [valid float number!]

]

* Give the 3° [Coordinate(z)].

-- What is the new value? 3

**[The typed number]: 3.0 is a [valid float number!]

]

-- Provide the (coordinates: xR, yR, zR) of the (Point R)?

* Introduce the 1° [Coordinate(x)].

-- What is the new value? 1

**[The typed number]: 1.0 is a [valid float number!]

]

* Enter with the 2° [Coordinate(y)].

-- What is the new value? 0

**[The typed number]: 0.0 is a [valid float number!]

]

* Give the 3° [Coordinate(z)].

- - What is the new value? -4
**[The typed number]: -4.0 is a [valid float number!]

]

- The (Point P): P (0.0, 2.0, -1.0)
- The (Point Q): Q (1.0, 1.0, 3.0)
- The (Point R): R (1.0, 0.0, -4.0)

[ANSWER]

- The [vectorB]=vectorPR [1.0, -2.0, -3.0]
- The [vectorC]=vectorPQ [1.0, -1.0, 4.0]

- The Dot Product: [b * c] is: -9.00
- The value of the [ANGLE THETA] was calculate is: 124.54

- The [vectorA]=vectorQR [0.0, -1.0, -7.0]
- The [vectorC]=vectorQP [-1.0, 1.0, -4.0]

- The Dot Product: [a * c] is: 27.00
- The value of the [ANGLE BETA] was calculate is: 25.84

- The [vectorA]=vectorRQ [0.0, 1.0, 7.0]
- The [vectorB]=vectorRP [-1.0, 2.0, 3.0]

- The Dot Product: [a * b] is: 23.00
- The value of the [ANGLE GAMA] was calculate is: 29.62

-- The triangle is [Scalene]!

- THE [ADD] OF THE INNER ANGLE OF THE TRIANGLE] is: 180.00

--[END CALCULUS-OK!!--

. . . Key [ENTER] to exit -- Ok! . . .

[Warning]:

When any user run the **runtoolsvectors.py program** to solve exercises that provide vertices A, B, and C to triangles do the following:

$(x_A, y_A, z_A) = (x_P, y_P, z_P) \implies x_P = x_A, y_P = y_A, \text{ and } z_P = z_A$ replace the coordinates: x_A, y_A , and z_A into the coordinates: x_P, y_P , and z_P of the point: P when the runtoolsvectors.py program request to enter the coordinates: x_P, y_P and z_P . Of the same manner to the vertices B and C so:

$(x_B, y_B, z_B) = (x_Q, y_Q, z_Q) \implies x_Q = x_B, y_Q = y_B, \text{ and } z_Q = z_B$ replace the coordinates: x_B, y_B , and z_B into the coordinates: x_Q, y_Q and z_Q of the point: Q when the runtoolsvectors.py program request to enter the coordinates: x_Q, y_Q and z_Q .

$(x_C, y_C, z_C) = (x_R, y_R, z_R) \implies x_R = x_C, y_R = y_C, \text{ and } z_R = z_C$ replace the coordinates: x_C, y_C , and z_C into the coordinates: x_R, y_R and z_R of the point: R when the runtoolsvectors.py program request to enter the coordinates: x_R, y_R and z_R .

Of the same manner do to the vertices: A, B, C, and D of the parallelogram as too to the parallelepiped. Replace your coordinates when the runtoolsvectors.py programs request enter the coordinates of the points: P, Q, R, and S.

Will be confused standardize the letters: A, B, and C to the vectors and too to the vertices points: A, B, and C of the triangle into the code of functions developed.

Example4 – Find the dimensions of the triangleABC with vertices points: A(0, -1,2), B(1,2,-1), and C(3, -1,2).

Solution: Do: $A(0, -1, 2) = P(x_P, y_P, z_P) \Rightarrow x_P = 0, y_P = -1, \text{ and } z_P = 2$
 $B(1, 2, -1) = Q(x_Q, y_Q, z_Q) \Rightarrow x_Q = 1, y_Q = 2, \text{ and } z_Q = -1$
 $C(3, -1, 2) = R(x_R, y_R, z_R) \Rightarrow x_R = 3, y_R = -1, \text{ and } z_R = 2$

Begin using the option[8] and provide the coordinates: $x_P = 0, y_P = -1, z_P = 2, x_Q = 1, y_Q = 2, z_Q = -1, x_R = 3, y_R = -1, \text{ and } z_R = 2$ when the **runtoolsvectors.py** program request.

```

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```

```

**[INSTRUCTIONS OF USE]**

```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
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```

[$] Select an previous [option] that will used--Ok!

```

```

[0>0] What is the new value? 8

```

```

**[The typed number]: 8 is a [valid integer number!]

```

```

]

```

```

**[ GIVENS THE COORDINATES OF THE POINTS P, Q AND R FIND THE DIMENSIONS SIDE(A),
SIDE(B), ]**

```

```

**[ SIDE(C), PERIMETER(P), HEIGHTS(H1,H2,H3), AND AREA(A) OF THE TRIANGLE(PQR) ]**

```

```

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

```

```

* Introduce the 1º [Coordinate(x)].

```

```

_ _ - What is the new value? 0

```

```

**[The typed number]: 0.0 is a [valid float number!]

```

```

]

```

```

    * Enter with the 2º [Coordinate(y)].
    _ _ What is the new value? -1
      **[The typed number]: -1.0 is a [valid float number!]
  ]

  * Give the 3º [Coordinate(z)].
  _ _ What is the new value? 2
    **[The typed number]: 2.0 is a [valid float number!]
]

-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?

* Introduce the 1º [Coordinate(x)].
_ _ What is the new value? 1
  **[The typed number]: 1.0 is a [valid float number!]
]

```

```

    * Enter with the 2º [Coordinate(y)].
    _ _ What is the new value? 2
      **[The typed number]: 2.0 is a [valid float number!]
  ]

  * Give the 3º [Coordinate(z)].
  _ _ What is the new value? -1
    **[The typed number]: -1.0 is a [valid float number!]
]

-- Provide the (coordinates: xR, yR, zR) of the (Point R)?

* Introduce the 1º [Coordinate(x)].
_ _ What is the new value? 3
  **[The typed number]: 3.0 is a [valid float number!]
]

```

```

    * Enter with the 2º [Coordinate(y)].
    _ _ What is the new value? -1
      **[The typed number]: -1.0 is a [valid float number!]
  ]

  * Give the 3º [Coordinate(z)].
  _ _ What is the new value? 2
    **[The typed number]: 2.0 is a [valid float number!]
]

```

```

- The (Point P): P (0.0, -1.0, 2.0)
- The (Point Q): Q (1.0, 2.0, -1.0)
- The (Point R): R (3.0, -1.0, 2.0)

```

```

*[ANSWER]*

```

```

- The [vectorA]=vectorPQ [1.0, 3.0, -3.0]
- The [vectorB]=vectorPR [-3.0, 0.0, 0.0]
- The [vectorC]=vectorQR [2.0, -3.0, 3.0]

```

```

-- The [sideA] of the triangle(PQR)] is: 4.36
-- The [sideB] of the triangle(PQR)] is: 3.00
-- The [sideC] of the triangle(PQR)] is: 4.69
-- The [Perimeter] of the [triangle(PQR)] is 12.05

```

```

-- The [terms] of the [Scalar Product(PQºQR)] is: [2.0, -9.0, -9.0]
-- The [terms] of the [Scalar Product(QRºRP)] is: [-6.0, -0.0, 0.0]
-- The [terms] of the [Scalar Product(RPºPQ)] is: [-3.0, 0.0, -0.0]

```

```

-- The [Scalar Product(PQºQR)] is: -16.00
-- The [Scalar Product(QRºRP)] is: -6.00

```

```
-- The [Scalar Product(RP°PQ)] is: -3.00
```

```
-- The [Height(h1) relative as sideQR] is 2.71  
-- The [Height(h2) relative as sideRP] is 4.24  
-- The [Height(h3) relative as sidePQ] is 2.92
```

```
-- The [Cross Product]: vectorAxB [0.0, 9.0, 9.0]  
-- The [Area(A) of a Triangle(PQR)] is: 6.36
```

```
. . . Key [ENTER] to exit -- Ok! . . .
```

Example5 – Find $\sin\theta$ where θ is the angle between the vectors: $a = i + 2j + 3k$ and $b = 4i + 5j + 6k$.

Solution: Before of begin the runtoolsvectors.py program do know the components of this vectors so: $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $b_1 = 4$, $b_2 = 5$, and $b_3 = 6$. After begin the runtoolsvectors.py program using the option [3].

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**  
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] in the [SPACE] ]**  
--[Version: 1.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] givens the points: P, Q, R key [8]
- To find the [Area] of the [Parallelogram] given [two vectors A and B] key [9]
- To get the [Volume(V)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and the [Subtraction] between [Two Vectors] A and B key [13]
- To find the [VectorA] and [lenght] of givens Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] givens an Vector A or two points: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors] A and B multiplyed by scalars[coeffic1 and coeffic2] key [16]

```
[$] Select an previous [option] that will used--Ok!
```

```
[°>°] What is the new value? 3
```

```
**[The typed number]: 3 is a [valid integer number!]
```

```
]
```

```
**[FIND THE VALUE OF THE SINETHETA]**
```

```
- Attribute the [Components] of the [1° vectorA]!
```

```
- Enter the 1° [Component(x)]!
```

```
-_- What is the new value? 1
```

```
**[The typed number]: 1.0 is a [valid float number!]
```

```
]
```

```
- Introduce the 2° [Component(y)]!
```

```
-_- What is the new value? 2
```

```
**[The typed number]: 2.0 is a [valid float number!]
```

```

]

- Give the 3o [Component(z)]!
-_- What is the new value? 3
  **[The typed number]: 3.0 is a [valid float number!]

]

- Provide the [Components] of the [2o vectorB]!

- Enter the 1o [Component(x)]!
-_- What is the new value? 4
  **[The typed number]: 4.0 is a [valid float number!]

]

- Introduce the 2o [Component(y)]!
-_- What is the new value? 5
  **[The typed number]: 5.0 is a [valid float number!]

]

- Give the 3o [Component(z)]!
-_- What is the new value? 6
  **[The typed number]: 6.0 is a [valid float number!]

]

-- The [vectorA]: vectorA [1.0, 2.0, 3.0]
-- The [vectorB]: vectorB [4.0, 5.0, 6.0]

```

[ANSWER]

```

-- The [length] of the [vectorA]=|vectorA|: 3.74
-- The [length] of the [vectorB]=|vectorB| 8.77
-- The [length] of the [vectorAxB]=|vectorAxB| 7.35
-- The value of the [SINETheta] is: 0.22

```

. . . Key [ENTER] to exit -- Ok! . . .

Example6 – Find the area(A) and height(h) of the parallelogram determined by the vectors: $a = 4i - j + k$ and $b = 2i + 3j - k$.

Solution – Begin the runtoolsvectors.py program using the option[9] and follow the instructions in to display.

```

**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] in the [SPACE] ]**
--[Version: 1.0 -- Stable]--

```

[INSTRUCTIONS OF USE]

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] givens the points: P, Q, R key [8]
- To find the [Area] of the [Parallelogram] given [two vectors A and B] key [9]
- To get the [Volume(V)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]

- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and the [Subtraction] between [Two Vectors] A and B key [13]
- To find the [VectorA] and [lenght] of givens Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] givens an Vector A or two points: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors] A and B multiplyed by scalars[coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] What is the new value? 9

**[The typed number]: 9 is a [valid integer number!]

]

[FIND THE AREA(A) AND HEIGHT(H) PARALLELOGRAM GIVEN THE VECTORS: A AND B]

- Attribute the [Components] of the [1° vectorA]!

- Enter the 1° [Component(x)]!

-_- What is the new value? 4

**[The typed number]: 4.0 is a [valid float number!]

]

- Introduce the 2° [Component(y)]!

-_- What is the new value? -1

**[The typed number]: -1.0 is a [valid float number!]

]

- Give the 3° [Component(z)]!

-_- What is the new value? 1

**[The typed number]: 1.0 is a [valid float number!]

]

- Provide the [Components] of the [2° vectorB]!

- Enter the 1° [Component(x)]!

-_- What is the new value? 2

**[The typed number]: 2.0 is a [valid float number!]

]

- Introduce the 2° [Component(y)]!

-_- What is the new value? 3

**[The typed number]: 3.0 is a [valid float number!]

]

- Give the 3° [Component(z)]!

-_- What is the new value? -1

**[The typed number]: -1.0 is a [valid float number!]

]

-- The [vectorA]: vectorA [4.0, -1.0, 1.0]

-- The [vectorB]: vectorB [2.0, 3.0, -1.0]

[ANSWER]

-- The [Height(h) Parallelogram] is: 3.62

-- The [Area(A) Parallelogram] is: 15.36

. . . Key [ENTER] to exit -- Ok! . . .

Example7 – Find the cosine of the angle between the vectors $a = i - 3j + 2k$ and $b = 3i + 3j + 2k$.

Solution – Begin the runtoolsvectors.py program using the option[6] and follow the instructions in to display.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] in the [SPACE] ]**
--[Version: 1.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] givens the points: P, Q, R key [8]
- To find the [Area] of the [Parallelogram] given [two vectors A and B] key [9]
- To get the [Volume(V)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and the [Subtraction] between [Two Vectors] A and B key [13]
- To find the [VectorA] and [lenght] of givens Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] givens an Vector A or two points: P and

Q key [15]

- To find the [Addition] and [Subtraction] between [Two Vectors] A and B multiplyed by scalars[coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] What is the new value? 6

```
**[The typed number]: 6 is a [valid integer number!]
```

]

```
**[FIND THE [VALUE] OF THE [COSSINETHETA] BETWEEN VECTORS: A AND B]**
```

- Attribute the [Components] of the [1° vectorA]!

- Enter the 1° [Component(x)]!

_- What is the new value? 1

```
**[The typed number]: 1.0 is a [valid float number!]
```

]

- Introduce the 2° [Component(y)]!

_- What is the new value? -3

```
**[The typed number]: -3.0 is a [valid float number!]
```

]

- Give the 3° [Component(z)]!

_- What is the new value? 2

```
**[The typed number]: 2.0 is a [valid float number!]
```

]

- Provide the [Components] of the [2° vectorB]!

- Enter the 1° [Component(x)]!

_- What is the new value? 3

```
**[The typed number]: 3.0 is a [valid float number!]
```

]

```

- Introduce the 2o [Component(y)]!
-_- What is the new value? 3
  **[The typed number]: 3.0 is a [valid float number!]
]

- Give the 3o [Component(z)]!
-_- What is the new value? 2
  **[The typed number]: 2.0 is a [valid float number!]
]

-- The [vectorA]: vectorA [1.0, -3.0, 2.0]
-- The [vectorB]: vectorB [3.0, 3.0, 2.0]

```

[ANSWER]

```

-- The length [|vectorA|] is: 3.74
-- The length [|vectorB|] is: 4.69
-- The value [ScalarVectorA*B] is: -2.00
-- The value of the [CossineTheta] is: -0.11

```

. . . Key [ENTER] to exit -- Ok! . . .

Example8 – Find the direction cosines and direction angles of the vector represented by vectorPQ gives two points: P(2,-3, 5) and Q(1,0,-1).

Solution – Begin the runtoolsvectors.py program using the option[15] and after key the option[2] to enter the coordinates: xP = 2, yP = -3, zP = 5, xQ = 1, yQ = 0, and zQ = -1.

```

**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] in the [SPACE] ]**
--[Version: 1.0 -- Stable]--

```

[INSTRUCTIONS OF USE]

```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] gives the points: P, Q, R key [8]
- To find the [Area] of the [Parallelogram] given [two vectors A and B] key [9]
- To get the [Volume(V)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and the [Subtraction] between [Two Vectors] A and B key [13]
- To find the[VectorA] and [lenght] of givens Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] gives an Vector A or two points: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors] A and B multiplyed by scalars[coeffic1 and coeffic2] key [16]

```

[§] Select an previous [option] that will used—Ok!

[^o>^o] What is the new value? 15

```

    **[The typed number]: 15 is a [valid integer number!]
]

**[ GIVENS AN VECTOR(A) OR TWO POINTS: P AND Q DETERMINE THE [DIRECTION ANGLES]
AND [DIRECTION COSINES] ]**

- Type [1] to enter the [components] of vectorA.
- Type [2] to enter the [coordinates] of two givens points: P and Q.
[0>0] What is the new value? 2
    **[The typed number]: 2 is a [valid integer number!]
]

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

* Introduce the 1º [Coordinate(x)].
-_- What is the new value? 2
    **[The typed number]: 2.0 is a [valid float number!]
]

* Enter with the 2º [Coordinate(y)].
-_- What is the new value? -3
    **[The typed number]: -3.0 is a [valid float number!]
]

* Give the 3º [Coordinate(z)].
-_- What is the new value? 5
    **[The typed number]: 5.0 is a [valid float number!]
]

-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?

* Introduce the 1º [Coordinate(x)].
-_- What is the new value? 1
    **[The typed number]: 1.0 is a [valid float number!]
]

* Enter with the 2º [Coordinate(y)].
-_- What is the new value? 0
    **[The typed number]: 0.0 is a [valid float number!]
]

* Give the 3º [Coordinate(z)].
-_- What is the new value? -1
    **[The typed number]: -1.0 is a [valid float number!]
]

- The (Point P): P (2.0, -3.0, 5.0)
- The (Point Q): Q (1.0, 0.0, -1.0)

*[ANSWER]*

-- The [vectorA]: vectorA [-1.0, 3.0, -6.0]
-- The [lenght] of the vectorA=|vectorA|: 6.78

**[ (COSINEALPHA)² + (COSINEBETA)² + (COSINEGAMA)² = 1 ]**

-- The [value] of the [CosineAlpha] is: -0.15
-- The [value] of the [CosineBeta] is: 0.44
-- The [value] of the [CosineGama] is: -0.88

-- The [value] of the [AlphaAngle] is: 98.48
-- The [value] of the [BetaAngle] is: 63.75
-- The [value] of the [GamaAngle] is: 152.21

```


. . . Key [ENTER] to exit -- Ok! . . .

Example9 – Calculate the direction cosines and direction angles of the given vector $a = i - 2j - 3k$.

Solution – After was begin the runtoolsvectors.py program key the option[15] and wait the display present instructions to select the options: [1] or [2]. Follow type the option[1] and provide the [components]; $a_1 = 1$, $a_2 = -2$, and $a_3 = -3$.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] in the [SPACE] ]**
--[Version: 1.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] givens the points: P, Q, R key [8]
- To find the [Area] of the [Parallelogram] given [two vectors A and B] key [9]
- To get the [Volume(V)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the[VectorA] and [lenght] of givens Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] givens an Vector A or two points: P and Q key [15]

- To find the [Addition] and [Subtraction] between [Two Vectors] A and B multiplied by scalars[coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

[0>0] What is the new value? 15

```
**[The typed number]: 15 is a [valid integer number!]
```

]

```
**[ GIVENS AN VECTOR(A) OR TWO POINTS: P AND Q DETERMINE THE [DIRECTION ANGLES]
AND [DIRECTION COSINES] ]**
```

- Type [1] to enter the [components] of vectorA.
- Type [2] to enter the [coordinates] of two givens points: P and Q.

[0>0] What is the new value? 1

```
**[The typed number]: 1 is a [valid integer number!]
```

]

- Attribute the [Components] of the [1° vectorA]!

- Enter the 1° [Component(x)]!

_- What is the new value? 1

```
**[The typed number]: 1.0 is a [valid float number!]
```

]

- Introduce the 2° [Component(y)]!

_- What is the new value? -2

```

    **[The typed number]: -2.0 is a [valid float number!]
]

- Give the 3o [Component(z)]!
- _ What is the new value? -3
    **[The typed number]: -3.0 is a [valid float number!]
]

-- The [vectorA]: vectorA [1.0, -2.0, -3.0]
-- The [length] of the vectorA=|vectorA|: 3.74

**[ (COSINEALPHA)2 + (COSINEBETA)2 + (COSINEGAMA)2 = 1 ]**

-- The [value] of the [CosineAlpha] is: 0.27
-- The [value] of the [CosineBeta] is: -0.53
-- The [value] of the [CosineGama] is: -0.80

-- The [value] of the [AlphaAngle] is: 74.50
-- The [value] of the [BetaAngle] is: 122.31
-- The [value] of the [GamaAngle] is: 143.30

... Key [ENTER] to exit -- Ok! ...

```

Example10 – Givens the vectors: $a = i + 2j - 3k$ and $b = -2i - j + 5k$ find the [vectors and modules]: $a + b$, $a - b$, $|a + b|$, $|a - b|$, and $2a - 3b$.

Solution – Begin the runtoolsvectors.py program and key the option[16] and wait the display present instructions to select the options: [1] or [2]. Follow type the option[1] and provide the [components]; $a_1 = 1$, $a_2 = 2$, $a_3 = -3$, $b_1 = -2$, $b_2 = -1$, and $b_3 = 5$. Following the program will wait the user enter the [coefficients]. Do (1^o)[coefficient] = 1 and (2^o)[coefficient] = 1 and key ENTER to process.

```

**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] in the [SPACE] ]**
--[Version: 1.0 -- Stable]--

```

[INSTRUCTIONS OF USE]

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] givens the points: P, Q, R key [8]
- To find the [Area] of the [Parallelogram] given [two vectors A and B] key [9]
- To get the [Volume(V)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the[VectorA] and [length] of givens Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] givens an Vector A or two points: P and Q key [15]
 - To find the [Addition] and [Subtraction] between [Two Vectors] A and B multiplied by scalars[coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

```

[o>o] What is the new value? 16
**[The typed number]: 16 is a [valid integer number!]

```

]

```
**[ GIVENS [TWO VECTORS: A AND B] MULTIPLIED BY THE [SCALARS]: [COEFFIC1] AND [COEFFIC2] OR ]**  
**[ GIVENS [THE POINTS: P, Q, AND R] FIND [TWO VECTORS]: VECTOR(A)=VECTOR(PQ) AND VECTOR(B)=VECTOR(PR) ]**  
**[ AND MULTIPLY BY THE [SCALARS]: [COEFFIC1] AND [COEFFIC2] AND TOO GET THE [ADDITION] AND [SUBTRACTION] ]**
```

- To enter the [Components] of the [vectors: A and B] type [1].
- To introduce the [Coordinates] of the [points: P, Q, and R] type [2].

[0>0] What is the new value? 1

**[The typed number]: 1 is a [valid integer number!]

]

- Provide the [Components] of the [vectors]: vectorA and vectorB.

- Attribute the [Components] of the [1° vectorA]!

- Enter the 1° [Component(x)]!

-_- What is the new value? 1

**[The typed number]: 1.0 is a [valid float number!]

]

- Introduce the 2° [Component(y)]!

-_- What is the new value? 2

**[The typed number]: 2.0 is a [valid float number!]

]

- Give the 3° [Component(z)]!

-_- What is the new value? -3

**[The typed number]: -3.0 is a [valid float number!]

]

- Provide the [Components] of the [2° vectorB]!

- Enter the 1° [Component(x)]!

-_- What is the new value? -2

**[The typed number]: -2.0 is a [valid float number!]

]

- Introduce the 2° [Component(y)]!

-_- What is the new value? -1

**[The typed number]: -1.0 is a [valid float number!]

]

- Give the 3° [Component(z)]!

-_- What is the new value? 5

**[The typed number]: 5.0 is a [valid float number!]

]

-- The [vectorA]: vectorA [1.0, 2.0, -3.0]

-- The [vectorB]: vectorB [-2.0, -1.0, 5.0]

- Enter with new [value] to the (1°)[coefficient]? 1

- Give the new [value] to the (2°)[coefficient]? 1

[ANSWER]

+ The [vectorProduct]: [vectorA]*[scalar(coeffic1)]=vectorAcoeffic1 [1.0, 2.0, -3.0]

+ The [vectorproduct]: [vectorB]*[scalar(coeffic2)]=vectorBcoeffic2 [-2.0, -1.0, 5.0]

```
-- The [vectorAddition]=[vectorA(coeffic1)+vectorB(coeffic2)] [-1.0, 1.0, 2.0]
-- The [vectorSubtraction]=[vectorA(coeffic1)-vectorB(coeffic2)] [3.0, 3.0, -8.0]
```

```
-- The [length] of the [vectorAddition] is: 2.45
-- The [length] of the [vectorSubtraction] is: 9.06
```

```
. . . Key [ENTER] to exit -- Ok! . . .
```

Now to get the vector: $2a - 3b$, run the runtoolsvectors.py program again and key the option[16] and wait the display present instructions to select the options: [1] or [2]. Follow type the option[1] and provide again the [components]; $a_1 = 1$, $a_2 = 2$, $a_3 = -3$, $b_1 = -2$, $b_2 = -1$, and $b_3 = 5$. Following the program will wait the user enter the [coefficients]. Do (1^o)[coefficient] = 2 and (2^o)[coefficient] = -3 and key ENTER to process.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] in the [SPACE] ]**
--[Version: 1.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] givens the points: P, Q, R key [8]
- To find the [Area] of the [Parallelogram] given [two vectors A and B] key [9]
- To get the [Volume(V)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the[VectorA] and [lenght] of givens Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] givens an Vector A or two points: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors] A and B multiplied by scalars[coeffic1 and coeffic2] key [16]

```
[§] Select an previous [option] that will used--Ok!
```

```
[0>0] What is the new value? 16
```

```
**[The typed number]: 16 is a [valid integer number!]
```

```
]
```

```
**[ GIVENS [TWO VECTORS: A AND B] MULTIPLYED BY THE [SCALARS]: [COEFFIC1] AND [COEFFIC2] OR ]**
```

```
**[ GIVENS [THE POINTS: P, Q, AND R] FIND [TWO VECTORS]: VECTOR(A)=VECTOR(PQ) AND VECTOR(B)=VECTOR(PR) ]**
```

```
**[ AND MULTIPLY BY THE [SCALARS]: [COEFFIC1] AND [COEFFIC2] AND TOO GET THE [ADDITION] AND [SUBTRACTION] ]**
```

- To enter the [Components] of the [vectors: A and B] type [1].
- To introduce the [Coordinates] of the [points: P, Q, and R] type [2].

```
[0>0] What is the new value? 1
```

```
**[The typed number]: 1 is a [valid integer number!]
```

```
]
```

- Provide the [Components] of the [vectors]: vectorA and vectorB.

- Attribute the [Components] of the [1^o vectorA]!

- Enter the 1^o [Component(x)]!

-- What is the new value? 1

**[The typed number]: 1.0 is a [valid float number!]

]

- Introduce the 2^o [Component(y)]!

-- What is the new value? 2

**[The typed number]: 2.0 is a [valid float number!]

]

- Give the 3^o [Component(z)]!

-- What is the new value? -3

**[The typed number]: -3.0 is a [valid float number!]

]

- Provide the [Components] of the [2^o vectorB]!

- Enter the 1^o [Component(x)]!

-- What is the new value? -2

**[The typed number]: -2.0 is a [valid float number!]

]

- Introduce the 2^o [Component(y)]!

-- What is the new value? -1

**[The typed number]: -1.0 is a [valid float number!]

]

- Give the 3^o [Component(z)]!

-- What is the new value? 5

**[The typed number]: 5.0 is a [valid float number!]

]

-- The [vectorA]: vectorA [1.0, 2.0, -3.0]

-- The [vectorB]: vectorB [-2.0, -1.0, 5.0]

- Enter with new [value] to the (1^o)[coefficient]? 2

- Give the new [value] to the (2^o)[coefficient]? -3

[ANSWER]

+ The [vectorProduct]: [vectorA]*[scalar(coeffic1)]=vectorAcoeffic1 [2.0, 4.0, -6.0]

+ The [vectorproduct]: [vectorB]*[scalar(coeffic2)]=vectorBcoeffic2 [6.0, 3.0, -15.0]

-- The [vectorAddition]=[vectorA(coeffic1)+vectorB(coeffic2)] [8.0, 7.0, -21.0]

-- The [vectorSubtraction]=[vectorA(coeffic1)-vectorB(coeffic2)] [-4.0, 1.0, 9.0]

-- The [length] of the [vectorAddition] is: 23.54

-- The [length] of the [vectorSubtraction] is: 9.90

. . . Key [ENTER] to exit -- Ok! . . .

Example11 – Givens the points: P(2, 2, 3) and Q(4, -5, 6), find the [distance] between the points.

Solution – Begin the runtoolsvectors.py program and key the option[7] and provide the [coordinates]: xP = 2, yP = 2, zP = 3, xQ = 4, yQ = -5, and zQ = 6 when request by program.

```

**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] in the [SPACE] ]**
--[Version: 1.0 -- Stable]--

```

```

**[INSTRUCTIONS OF USE]**

```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] gives the points: P, Q, R key [8]
- To find the [Area] of the [Parallelogram] given [two vectors A and B] key [9]
- To get the [Volume(V)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the[VectorA] and [lenght] of givens Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] givens an Vector A or two points: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors] A and B multiplied by scalars[coeffic1 and coeffic2] key [16]

```

[§] Select an previous [option] that will used--Ok!

```

```

[0>0] What is the new value? 7

```

```

**[The typed number]: 7 is a [valid integer number!]

```

```

]

```

```

**[DETERMINE THE DISTANCE(d) BETWEEN TWO GIVENS POINTS P and Q]**

```

```

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

```

```

* Introduce the 1º [Coordinate(x)].

```

```

_ _ - What is the new value? 2

```

```

**[The typed number]: 2.0 is a [valid float number!]

```

```

]

```

```

* Enter with the 2º [Coordinate(y)].

```

```

_ _ - What is the new value? 2

```

```

**[The typed number]: 2.0 is a [valid float number!]

```

```

]

```

```

* Give the 3º [Coordinate(z)].

```

```

_ _ - What is the new value? 3

```

```

**[The typed number]: 3.0 is a [valid float number!]

```

```

]

```

```

-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?

```

```

* Introduce the 1º [Coordinate(x)].

```

```

    - _ - What is the new value? 4
      **[The typed number]: 4.0 is a [valid float number!]
]

* Enter with the 2º [Coordinate(y)].
- _ - What is the new value? -5
      **[The typed number]: -5.0 is a [valid float number!]
]

* Give the 3º [Coordinate(z)].
- _ - What is the new value? 6
      **[The typed number]: 6.0 is a [valid float number!]
]

- The (Point P): P (2.0, 2.0, 3.0)
- The (Point Q): Q (4.0, -5.0, 6.0)

```

```
*[ANSWER]*
```

```

-- The [vectorPQ]: vectorPQ [2.0, -7.0, 3.0]
-- The [Quadratic Components] of the [vectorCQD]:vectorCQD [4.0, 49.0, 9.0]
-- The [Distance(d)] geted between (Two Points) P and Q is: 7.87

```

```
. . . Key [ENTER] to exit -- Ok! . . .
```

Example12 – Givens the points: P(5, -9, 7) and Q(-2, 3, 3), find the [MidPoint] between the points.

Solution – Begin the runtoolsvectors.py program and key the option[7] and provide the [coordinates]: xP = 5, yP = -9, zP = 7, xQ = -2, yQ = 3, and zQ = 3 when the program request.

```

**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] in the [SPACE] ]**
--[Version: 1.0 -- Stable]--

```

```
**[INSTRUCTIONS OF USE]**
```

```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the
Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] givens the points: P, Q, R key [8]
- To find the [Area] of the [Parallelogram] given [two vectors A and B] key [9]
- To get the [Volume(V)] of the [Parallelepiped and Tetrahedron] given four points: P, Q,
R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, R key
[11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q]
key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]

```

- To calculate the [VectorA] and [lenght] of givens Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] givens an Vector A or two points: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors] A and B multiplied by scalars[coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

[°>°] What is the new value? 12

**[The typed number]: 12 is a [valid integer number!]

]

[WILL FIND THE [MIDPOINT(M)] OF THE [LINE SEGMENT] BETWEEN THE POINTS GIVEN: P AND Q]

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

* Introduce the 1° [Coordinate(x)].

-- - What is the new value? 5

**[The typed number]: 5.0 is a [valid float number!]

]

* Enter with the 2° [Coordinate(y)].

-- - What is the new value? -9

**[The typed number]: -9.0 is a [valid float number!]

]

* Give the 3° [Coordinate(z)].

-- - What is the new value? 7

**[The typed number]: 7.0 is a [valid float number!]

]

-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?

* Introduce the 1° [Coordinate(x)].

-- - What is the new value? -2

**[The typed number]: -2.0 is a [valid float number!]

]

* Enter with the 2° [Coordinate(y)].

-- - What is the new value? 3

**[The typed number]: 3.0 is a [valid float number!]

]

* Give the 3° [Coordinate(z)].

-- - What is the new value? 3

**[The typed number]: 3.0 is a [valid float number!]

]

- The (Point P): P (5.0, -9.0, 7.0)

- The (Point Q): Q (-2.0, 3.0, 3.0)


```
*[ANSWER]*
```

```
-- The (MidPoint M): M (1.5, -3.0, 5.0)
```

```
. . . Key [ENTER] to exit -- Ok! . . .
```

Example13 – Given the points: P(-1, 2, 0), Q(2, 1, -3), R(1, 0, 1), and S(3, -2, 3), find the Volume(V) and Height(h) of the parallelepiped as too the volume of the Tetrahedron.

Solution – Run the runtoolsvectors.py program and key the option[10] and provide the [coordinates]: xP = -1, yP = 2, zP = 0, xQ = -2, yQ = 1, zQ = -3, xR = 1, yR = 0, zR = 1, xS = 3, yS = -2, and zS = 3 when the program request.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**  
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] in the [SPACE] ]**  
--[Version: 1.0 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between two vectors A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] gives the points: P, Q, R key [8]
- To find the [Area] of the [Parallelogram] given [two vectors A and B] key [9]
- To get the [Volume(V)], [Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the[VectorA] and [lenght] of givens Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] givens an Vector A or two points: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors] A and B multiplied by scalars[coeffic1 and coeffic2] key [16]

```
[§] Select an previous [option] that will used--Ok!
```

```
[^>^] What is the new value? 10
```

```
**[The typed number]: 10 is a [valid integer number!]
```

```
]
```

```
**[ GIVENS THE COORDINATES OF THE POINTS P, Q, R AND S FIND THE]**  
**[ VOLUME(V) AND HEIGHT(H) OF THE PARALLELEPIPED AND TETRAHEDRON ]**
```

```

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

* Introduce the 1º [Coordinate(x)].
- _ - What is the new value? -1
    **[The typed number]: -1.0 is a [valid float number!]
]

* Enter with the 2º [Coordinate(y)].
- _ - What is the new value? 2
    **[The typed number]: 2.0 is a [valid float number!]
]

* Give the 3º [Coordinate(z)].
- _ - What is the new value? 0
    **[The typed number]: 0.0 is a [valid float number!]
]

-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?

* Introduce the 1º [Coordinate(x)].
- _ - What is the new value? 2
    **[The typed number]: 2.0 is a [valid float number!]
]

* Enter with the 2º [Coordinate(y)].
- _ - What is the new value? 1
    **[The typed number]: 1.0 is a [valid float number!]
]

* Give the 3º [Coordinate(z)].
- _ - What is the new value? -3
    **[The typed number]: -3.0 is a [valid float number!]
]

-- Provide the (coordinates: xR, yR, zR) of the (Point R)?

* Introduce the 1º [Coordinate(x)].
- _ - What is the new value? 1
    **[The typed number]: 1.0 is a [valid float number!]
]

* Enter with the 2º [Coordinate(y)].
- _ - What is the new value? 0
    **[The typed number]: 0.0 is a [valid float number!]
]

* Give the 3º [Coordinate(z)].
- _ - What is the new value? 1
    **[The typed number]: 1.0 is a [valid float number!]
]

-- Give the [Coordinates: xS, yS, zS] of the [point S]?

* Introduce the 1º [Coordinate(x)].
- _ - What is the new value? 3
    **[The typed number]: 3.0 is a [valid float number!]
]

* Enter with the 2º [Coordinate(y)].
- _ - What is the new value? -2
    **[The typed number]: -2.0 is a [valid float number!]
]

```

]

* Give the 3º [Coordinate(z)].

- _ - What is the new value? 3

**[The typed number]: 3.0 is a [valid float number!]

]

- The (Point P): P (-1.0, 2.0, 0.0)
- The (Point Q): Q (2.0, 1.0, -3.0)
- The (Point R): R (1.0, 0.0, 1.0)
- The (Point S): S (3.0, -2.0, 3.0)

[ANSWER]

- The [vectorA]=vectorPQ [3.0, -1.0, -3.0]
- The [vectorB]=vectorPR [2.0, -2.0, 1.0]
- The [vectorC]=vectorPS [4.0, -4.0, 3.0]

- [THE SCALAR TRIPLE PRODUCT]: $a \cdot (b \times c)$ is: -4.0
- The [VOLUME(V) PARALLELEPIPED]: 4.00
- The [HEIGHT(H) PARALLELEPIPED]: 0.33
- The [tetrahedron volume]: 0.67

. . . Key [ENTER] to exit -- Ok! . . .

Developer: **Cristovom A. Girodo**