

-Instructions of as use the runtoolsvectors.py program.

In the Windows operating system:

After the download the [repositorio](#) access the [runtoolsvectors.py](#) program given two clicks and follow the news instructions in display.

In the Linux operating system:

After the download the [repositorio](#) access the [runtoolsvectors.py](#) program. Exist two options to run the [runtoolsvectors.py](#) program.

1.) In any Terminal linux type the following command:

```
python3 runtoolsvectors.py
```

 after key ENTER

2.) Open the [runtoolsvectors.py](#) file using the IDLE3 and use the Run ==> Run Module options. After follow the news instructions.

[Warning]: When the [runtoolsvectors.py](#) program request type the [\[components\]](#) or [\[coordinates\]](#), always key ENTER after to continue, Ok!

Now will use of the [runtoolsvectors.py](#) program in the examples below.

Example1 – Given the vectors $a = \langle 7, -2, -11 \rangle$ and $b = \langle -18, 3, 17 \rangle$, find the vectors: addition $a + b$ and subtraction $a - b$.

Solution: Begin the [runtoolsvectors.py](#) program and select the [option\[13\]](#). After follow the instructions requested in accordance with showed in the display.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**
--[Version: 1.3 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)], [Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P,Q,and R key [11]

- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q]
key [12]

- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the [VectorA] and [length] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an
Vector A or [two points]: P and Q key [15]

- To find the [Addition] and [Subtraction] between [Two Vectors]: A and B
multiplied by scalars: [coeffic1 and coeffic2] key [16]

[\$] Select an previous [option] that will used--Ok!

(°>°) Provide the [new] value? 13

[The typed number]: 13 is a [valid integer number!]]

[CALCULATE THE [ADDITION] AND [SUBTRACTION] OF THE [VECTORS]: A AND B]

- Attribute the [Components] of the [1° vectorA]!

- Enter the 1° [Component(x)]!

(^<^) Enter the [new] value? 7

[[The typed number]: 7.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(^<^) Enter the [new] value? -2

[[The typed number]: -2.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(^<^) Enter the [new] value? -11

[[The typed number]: -11.0 is a [valid integer or float number!]]

- Provide the [Components] of the [2° vectorB]!

- Enter the 1° [Component(x)]!

(^<^) Enter the [new] value? -18

[[The typed number]: -18.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(^<^) Enter the [new] value? 3

[[The typed number]: 3.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(^<^) Enter the [new] value? 17

[[The typed number]: 17.0 is a [valid integer or float number!]]

-- The [vectorA]: vectorA [7.0, -2.0, -11.0]

-- The [vectorB]: vectorB [-18.0, 3.0, 17.0]

[ANSWER]

-- The [Addition]: vectorA+B [-11.0, 1.0, 6.0]

-- The [Subtraction]: vectorA-B [25.0, -5.0, -28.0]

. . . Key [ENTER] to exit -- Ok! . . .

Example2 – If $\mathbf{a} = 2\mathbf{i} - 3\mathbf{k}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, and $\mathbf{c} = 4\mathbf{j} - \mathbf{k}$ then find the Scalar Triple Product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$, the volume $V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$ of the parallelepiped as too the volume $V = (1/6) * |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$ of the Tetrahedron.

Solution: Before of begin the runtoolsvectors.py program isolate the components of the vectors so: $a_1 = 2$, $a_2 = 0$, $a_3 = -3$, $b_1 = 1$, $b_2 = 1$, $b_3 = 1$, $c_1 = 0$, $c_2 = 4$, and $c_3 = -1$ to type and select the option[4]. After follow the instructions requested in accordance with showed in the display.

```

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**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**
--[Version: 1.3 -- Stable]--

```

```

**[INSTRUCTIONS OF USE]**

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- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
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- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
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- To find the [Addition] and [Subtraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

((>>) Provide the [new] value? 4

```

**[The typed number]: 4 is a [valid integer number!] ]**

```

```

**[CALCULATE THE [SCALAR TRIPLE PRODUCT] BETWEEN [THREE VECTORS: A, B, AND C] IN TRI-DIMENSIONAL(XYZ) SPACE]**

```

****[COMPONENTS OF THE VECTORS: A, B and C]****

- Attribute the [Components] of the [1° vectorA]!

- Enter the 1° [Component(x)]!

(^a<a) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(^a<a) Enter the [new] value? 0

[[The typed number]: 0.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(^a<a) Enter the [new] value? -3

[[The typed number]: -3.0 is a [valid integer or float number!]]

- Provide the [Components] of the [2° vectorB]!

- Enter the 1° [Component(x)]!

(^a<a) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(^a<a) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(^a<a) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

- Provide the [Components] of the [3° vectorC]!

- Enter the 1° [Component(x)]!

(^a<a) Enter the [new] value? 0

[[The typed number]: 0.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(^a<a) Enter the [new] value? 4

[[The typed number]: 4.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(^a<a) Enter the [new] value? -1

[[The typed number]: -1.0 is a [valid integer or float number!]]

[ANSWER]

-- The [vectorA]: vectorA [2.0, 0.0, -3.0]

-- The [vectorB]: vectorB [1.0, 1.0, 1.0]

-- The [vectorC]: vectorC [0.0, 4.0, -1.0]

-- The [SCALAR TRIPLE PRODUCT]: a * (b x c) is: -22.0

-- The [VOLUME(V) PARALLELEPIPED]: 22.00

-- The [HEIGHT(H) PARALLELEPIPED]: 3.57

-- The [Tetrahedron volume]: 3.67

. . . Key [ENTER] to exit -- Ok! . . .

Example3 – A triangle in \mathbb{R}^3 has vertices P(0, 2, -1), Q(1, 1, 3), and R(1, 0, -4).

- Find the perimeter(P) of the triangle(PQR)
- Find the area(A) of the triangle(PQR)
- Find the three vertex angles of the triangle(PQR). (Round to the nearest degree)

Solution: – Run the runtoolsvectors.py program and key the option[8] and enter the coordinates of the vertices: P, Q, and R.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**  
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**  
--[Version: 1.3 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
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- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
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- To find the [Addition] and [Subraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

(°>°) Provide the [new] value? 8

[The typed number]: 8 is a [valid integer number!]]

```
**[ GIVENS THE COORDINATES OF THE POINTS P, Q AND R FIND THE DIMENSIONS:  
SIDE(A), SIDE(B),SIDE(C), PERIMETER(P), HEIGHTS(H1,H2,H3), AND THE ]**  
**[ AREA(A) OF THE TRIANGLE(PQR) ]**
```

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

* Introduce the 1° [Coordinate(x)].

(^a<a) Enter the [new] value? 0

[[The typed number]: 0.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].

(^a<a) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].

(^a<a) Enter the [new] value? -1

[[The typed number]: -1.0 is a [valid integer or float number!]]

-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?

* Introduce the 1° [Coordinate(x)].

(^a<a) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].

(^a<a) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].

(^a<a) Enter the [new] value? 3

[[The typed number]: 3.0 is a [valid integer or float number!]]

-- Provide the (coordinates: xR, yR, zR) of the (Point R)?

* Introduce the 1° [Coordinate(x)].

(^a<a) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].

(^a<a) Enter the [new] value? 0

[[The typed number]: 0.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].

(^a<a) Enter the [new] value? -4

[[The typed number]: -4.0 is a [valid integer or float number!]]

- The (Point P): P (0.0, 2.0, -1.0)

- The (Point Q): Q (1.0, 1.0, 3.0)

- The (Point R): R (1.0, 0.0, -4.0)

[ANSWER]

- The [vectorA]=vectorPQ [1.0, -1.0, 4.0]

- The [vectorB]=vectorPR [1.0, -2.0, -3.0]

- The [vectorC]=vectorQR [0.0, -1.0, -7.0]

-- The [sideA] of the triangle(PQR)] is: 4.24

-- The [sideB] of the triangle(PQR)] is: 3.74

-- The [sideC] of the triangle(PQR)] is: 7.07

-- The [Perimeter] of the [triangle(PQR)] is 15.06

```
-- The [terms] of the [Scalar Product(PQ°QR)] is: [0.0, 1.0, -28.0]
-- The [terms] of the [Scalar Product(QR°PR)] is: [0.0, 2.0, 21.0]
-- The [terms] of the [Scalar Product(PR°PQ)] is: [1.0, 2.0, -12.0]
```

```
-- The [Scalar Product(PQ°QR)] is: -27.00
-- The [Scalar Product(QR°PR)] is: 23.00
-- The [Scalar Product(PR°PQ)] is: -9.00
```

```
-- The [Height(h1) relative as sideQR] is 1.85
-- The [Height(h2) relative as sidePR] is 3.49
-- The [Height(h3) relative as sidePQ] is 3.08
```

```
-- The [Cross Product]: vectorAxB [11.0, 7.0, -1.0]
-- The [Area(A) of a Triangle(PQR)] is: 6.54
```

```
... Key [ENTER] to exit -- Ok! ...
```

Now run the `runtoolsvectors.py` program again and key the `option[11]` and enter again the coordinates of the vertices: P, Q, and R.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**
--[Version: 1.3 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)], [Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
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- To find the [Addition] and [Subtraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

(°>°) Provide the [new] value? 11

**[The typed number]: 11 is a [valid integer number!] **

[GIVENS THE COORDINATES OF THE POINTS P, Q AND R]

[FIND THE [INNER ANGLES] OF THE TRIANGLE(PQR)]

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

* Introduce the 1° [Coordinate(x)].

(^<^) Enter the [new] value? 0

**[[The typed number]: 0.0 is a [valid integer or float number!] **

* Enter with the 2° [Coordinate(y)].

(^<^) Enter the [new] value? 2

**[[The typed number]: 2.0 is a [valid integer or float number!] **

* Give the 3° [Coordinate(z)].

(^<^) Enter the [new] value? -1

**[[The typed number]: -1.0 is a [valid integer or float number!] **

-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?

* Introduce the 1° [Coordinate(x)].

(^<^) Enter the [new] value? 1

**[[The typed number]: 1.0 is a [valid integer or float number!] **

* Enter with the 2° [Coordinate(y)].

(^<^) Enter the [new] value? 1

**[[The typed number]: 1.0 is a [valid integer or float number!] **

* Give the 3° [Coordinate(z)].

(^<^) Enter the [new] value? 3

**[[The typed number]: 3.0 is a [valid integer or float number!] **

-- Provide the (coordinates: xR, yR, zR) of the (Point R)?

* Introduce the 1° [Coordinate(x)].

(^<^) Enter the [new] value? 1

**[[The typed number]: 1.0 is a [valid integer or float number!] **

* Enter with the 2° [Coordinate(y)].

(^<^) Enter the [new] value? 0

**[[The typed number]: 0.0 is a [valid integer or float number!] **

* Give the 3° [Coordinate(z)].

(^<^) Enter the [new] value? -4

**[[The typed number]: -4.0 is a [valid integer or float number!] **

- The (Point P): P (0.0, 2.0, -1.0)

- The (Point Q): Q (1.0, 1.0, 3.0)

- The (Point R): R (1.0, 0.0, -4.0)

[ANSWER]

- The [vectorB]=vectorPR [1.0, -2.0, -3.0]

- The [vectorC]=vectorPQ [1.0, -1.0, 4.0]

- The Dot Product: [b * c] is: -9.00

- The value of the [ANGLE THETA] was calculate is: 124.54

- The [vectorA]=vectorQR [0.0, -1.0, -7.0]

- The [vectorC]=vectorQP [-1.0, 1.0, -4.0]

- The Dot Product: [a * c] is: 27.00

- The value of the [ANGLE BETA] was calculate is: 25.84

- The [vectorA]=vectorRQ [0.0, 1.0, 7.0]

- The [vectorB]=vectorRP [-1.0, 2.0, 3.0]

- The Dot Product: [a * b] is: 23.00

- The value of the [ANGLE GAMA] was calculate is: 29.62

-- The triangle is [Scalene]!

- THE [ADD] OF THE INNER ANGLE OF THE TRIANGLE] is: 180.00

--[END CALCULUS-OK!]

. . . Key [ENTER] to exit -- Ok! . . .

[Warning]:

When any user run the **runtoolsvectors.py program** to solve exercises that provide vertexes A, B, and C to triangles do the following:

$(x_A, y_A, z_A) = (x_P, y_P, z_P) \Rightarrow x_P = x_A, y_P = y_A, \text{ and } z_P = z_A$ replace the coordinates: x_A, y_A , and z_A into the coordinates: x_P, y_P , and z_P of the point: P when the **runtoolsvectors.py program** request to enter the coordinates: x_P, y_P and z_P . Of the same manner to the vertexes B and C so:

$(x_B, y_B, z_B) = (x_Q, y_Q, z_Q) \Rightarrow x_Q = x_B, y_Q = y_B, \text{ and } z_Q = z_B$ replace the coordinates: x_B, y_B , and z_B into the coordinates: x_Q, y_Q and z_Q of the point: Q when the **runtoolsvectors.py program** request to enter the coordinates: x_Q, y_Q and z_Q .

$(x_C, y_C, z_C) = (x_R, y_R, z_R) \Rightarrow x_R = x_C, y_R = y_C, \text{ and } z_R = z_C$ replace the coordinates: x_C, y_C , and z_C into the coordinates: x_R, y_R and z_R of the point: R when the **runtoolsvectors.py program** request to enter the coordinates: x_R, y_R and z_R .

Of the same manner do to the vertexes: A, B, C, and D of the **parallelogram** as too to the **parallelepiped**. Replace your coordinates when the **runtoolsvectors.py program** request enter the coordinates of the points: P, Q, R, and S.

Will be confused standardize the **letters: A, B, and C to the vectors** and too to the vertexes points: **A, B, and C of the triangle** into the code of functions developed.

Example4 – Find the dimensions of the triangle ABC with vertexes points: A(0, -1,2), B(1,2,-1), and C(3, -1,2).

Solution: Do: $A(0,-1,2) = P(x_P, y_P, z_P) \Rightarrow x_P = 0, y_P = -1, \text{ and } z_P = 2$
 $B(1,2,-1) = Q(x_Q, y_Q, z_Q) \Rightarrow x_Q = 1, y_Q = 2, \text{ and } z_Q = -1$
 $C(3, -1,2) = R(x_R, y_R, z_R) \Rightarrow x_R = 3, y_R = -1, \text{ and } z_R = 2$

Begin using the option[8] and provide the coordinates: $x_P = 0, y_P = -1, z_P = 2, x_Q = 1, y_Q = 2, z_Q = -1, x_R = 3, y_R = -1, \text{ and } z_R = 2$ when the **runtoolsvectors.py** program request.

```
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- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
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- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the [VectorA] and [lenght] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an Vector A or [two points]: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

```
[§] Select an previous [option] that will used--Ok!
```

```
(°>°) Provide the [new] value? 8
```

```
**[The typed number]: 8 is a [valid integer number!] ]**
```

**[GIVENS THE COORDINATES OF THE POINTS P, Q AND R FIND THE DIMENSIONS:
SIDE(A), SIDE(B),SIDE(C), PERIMETER(P), HEIGHTS(H1,H2,H3), AND THE AREA(A)]**
[OF THE TRIANGLE(PQR)]

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

* Introduce the 1° [Coordinate(x)].

(^a<^a) Enter the [new] value? 0

[[The typed number]: 0.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].

(^a<^a) Enter the [new] value? -1

[[The typed number]: -1.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].

(^a<^a) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?

* Introduce the 1° [Coordinate(x)].

(^a<^a) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].

(^a<^a) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].

(^a<^a) Enter the [new] value? -1

[[The typed number]: -1.0 is a [valid integer or float number!]]

-- Provide the (coordinates: xR, yR, zR) of the (Point R)?

* Introduce the 1° [Coordinate(x)].

(^a<^a) Enter the [new] value? 3

[[The typed number]: 3.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].

(^a<^a) Enter the [new] value? -1

[[The typed number]: -1.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].

(^a<^a) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

- The (Point P): P (0.0, -1.0, 2.0)

- The (Point Q): Q (1.0, 2.0, -1.0)

- The (Point R): R (3.0, -1.0, 2.0)

[ANSWER]

- The [vectorA]=vectorPQ [1.0, 3.0, -3.0]

- The [vectorB]=vectorPR [3.0, 0.0, 0.0]

- The [vectorC]=vectorQR [2.0, -3.0, 3.0]

```
-- The [sideA] of the triangle(PQR)] is: 4.36
-- The [sideB] of the triangle(PQR)] is: 3.00
-- The [sideC] of the triangle(PQR)] is: 4.69
-- The [Perimeter] of the [triangle(PQR)] is 12.05

-- The [terms] of the [Scalar Product(PQ°QR)] is: [2.0, -9.0, -9.0]
-- The [terms] of the [Scalar Product(QR°PR)] is: [6.0, -0.0, 0.0]
-- The [terms] of the [Scalar Product(PR°PQ)] is: [3.0, 0.0, -0.0]

-- The [Scalar Product(PQ°QR)] is: -16.00
-- The [Scalar Product(QR°PR)] is: 6.00
-- The [Scalar Product(PR°PQ)] is: 3.00

-- The [Height(h1) relative as sideQR] is 2.71
-- The [Height(h2) relative as sidePR] is 4.24
-- The [Height(h3) relative as sidePQ] is 2.92

-- The [Cross Product]: vectorAxB [0.0, -9.0, -9.0]
-- The [Area(A) of a Triangle(PQR)] is: 6.36
```

. . . Key [ENTER] to exit -- Ok! . . .

Example5 – Find $\sin\theta$ where θ is the angle between the vectors: $a = i + 2j + 3k$ and $b = 4i + 5j + 6k$.

Solution: Before of begin the `runtoolsvectors.py` program isolate the components of the vectors so: $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $b_1 = 4$, $b_2 = 5$, and $b_3 = 6$ to type. After begin the `runtoolsvectors.py` program using the option [3].

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**
--[Version: 1.3 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)], [Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]

- To calculate the three [Inner Angles] of the [Triangle] given three points: P,Q,and R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the[VectorA] and [length] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an Vector A or [two points]: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

(°>°) Provide the [new] value? 3

[The typed number]: 3 is a [valid integer number!]]

[FIND THE VALUE OF THE SINETHETA]

- Attribute the [Components] of the [1° vectorA]!

- Enter the 1° [Component(x)]!

(^<^) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(^<^) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(^<^) Enter the [new] value? 3

[[The typed number]: 3.0 is a [valid integer or float number!]]

- Provide the [Components] of the [2° vectorB]!

- Enter the 1° [Component(x)]!

(^<^) Enter the [new] value? 4

[[The typed number]: 4.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(^<^) Enter the [new] value? 5

[[The typed number]: 5.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(^<^) Enter the [new] value? 6

[[The typed number]: 6.0 is a [valid integer or float number!]]

-- The [vectorA]: vectorA [1.0, 2.0, 3.0]

-- The [vectorB]: vectorB [4.0, 5.0, 6.0]

[ANSWER]

-- The [length] of the [vectorA]=|vectorA|: 3.74

-- The [length] of the [vectorB]=|vectorB| 8.77

-- The [length] of the [vectorAxB]=|vectorAxB| 7.35

-- The value of the [SINETheta] is: 0.22

. . . Key [ENTER] to exit -- Ok! . . .

Example6 – Find the height(h), Area(A), and the angle[Theta in degree(°)] of the parallelogram(PQRS) determined by the adjacent vectors: $a = 4i - j + k$ and $b = 2i + 3j - k$.

Solution – Begin the runtoolsvectors.py program using the option[9] and follow the instructions in to display.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**  
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**  
--[Version: 1.3 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)], [Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P,Q,and R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the [VectorA] and [lenght] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an Vector A or [two points]: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

(°>°) Provide the [new] value? 9

```
**[The typed number]: 9 is a [valid integer number!] ]**
```

[FIND THE HEIGHT(H), AREA(A), AND THE ANGLE(THETA) OF THE PARALLELOGRAM(PQRS) DETERMINED BY TWO ADJACENT VECTORS: A AND B OR TO VERTICES: P, Q,R, AND S]

[Instructions to enter the [Coordinates] or [Components] of the ParallelogramPQRS]

- To enter the [Coordinates] of the points: P, Q, R, and S key [1].
- For the [Components] of the adjacent Vectors: [vectorA]=vectorPQ e
- [vectorB]=vectorPS key [2].

(°>°) Provide the [new] value? 2

[The typed number]: 2 is a [valid integer number!]]

[COMPONENTS OF THE ADJACENT VECTORS: A and B]

- Attribute the [Components] of the [1° vectorA]!

- Enter the 1° [Component(x)]!

(^<^) Enter the [new] value? 4

[[The typed number]: 4.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(^<^) Enter the [new] value? -1

[[The typed number]: -1.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(^<^) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

- Provide the [Components] of the [2° vectorB]!

- Enter the 1° [Component(x)]!

(^<^) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(^<^) Enter the [new] value? 3

[[The typed number]: 3.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(^<^) Enter the [new] value? -1

[[The typed number]: -1.0 is a [valid integer or float number!]]

-- The [vectorA]: vectorA [4.0, -1.0, 1.0]

-- The [vectorB]: vectorB [2.0, 3.0, -1.0]

[ANSWER]

-- The [height]: 3.62

-- The [Area]: 15.36

-- The angle [Theta in degree(°)] between the [adjacent vectors]: [vectorA] and

-- [vectorB] is: 75.41

. . . Key [ENTER] to exit -- Ok! . . .

Note: In the (new)Example15 of this tutorial all users will see the runtoolsvectors.py program calculate the height, Area, and The angle[Theta in degree(°)] of parallelogram PQRS when are given the [Coordinates] of the points: P, Q, R, and S. The [Version: 1.2—Stable] have not this operation.

Example7 – Find the cosine of the angle between the vectors $a = i - 3j + 2k$ and $b = 3i + 3j + 2k$.

Solution – Begin the runtoolsvectors.py program using the option[6] and follow the instructions in to display.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**  
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**  
--[Version: 1.3 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)], [Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P,Q,and R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the [VectorA] and [lenght] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an Vector A or [two points]: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

```
[§] Select an previous [option] that will used--Ok!
```

```
(°>°) Provide the [new] value? 6
```

```
**[The typed number]: 6 is a [valid integer number!] ]**
```

```
**[FIND THE [VALUE] OF THE [COSSINETHETA] BETWEEN VECTORS: A AND B]**
```


- Attribute the [Components] of the [1° vectorA]!

- Enter the 1° [Component(x)]!

(a<a) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(a<a) Enter the [new] value? -3

[[The typed number]: -3.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(a<a) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

- Provide the [Components] of the [2° vectorB]!

- Enter the 1° [Component(x)]!

(a<a) Enter the [new] value? 3

[[The typed number]: 3.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(a<a) Enter the [new] value? 3

[[The typed number]: 3.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(a<a) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

-- The [vectorA]: vectorA [1.0, -3.0, 2.0]

-- The [vectorB]: vectorB [3.0, 3.0, 2.0]

[ANSWER]

-- The length [|vectorA|] is: 3.74

-- The length [|vectorB|] is: 4.69

-- The value [ScalarVectorA*B] is: -2.00

-- The value of the [CossineTheta] is: -0.11

. . . Key [ENTER] to exit -- Ok! . . .

Example8 – Find the direction cosines and direction angles of the vector represented by vectorPQ gives two points: P(2,-3, 5) and Q(1,0,-1).

Solution – Begin the runtoolsvectors.py program using the option[15] and after key the option[2] to enter the coordinates: xP = 2, yP = -3, zP = 5, xQ = 1, yQ = 0, and zQ = -1.

[WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM]

[TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE]]

--[Version: 1.3 -- Stable]--

[INSTRUCTIONS OF USE]

- To find the [value] of the [Dot Product] of two vectors key [1]

- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]

- To calculate the [value] of the [SineTheta] between [two vectors] A and B

key [3]

- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)],[Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P,Q,and R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the[VectorA] and [lenght] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an Vector A or [two points]: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

(°>°) Provide the [new] value? 15

[The typed number]: 15 is a [valid integer number!]]

[GIVENS AN VECTOR(A) OR TWO POINTS: P AND Q DETERMINE THE [DIRECTION ANGLES] AND [DIRECTION COSINES]]

- Type [1] to enter the [components] of vectorA.

- Type [2] to enter the [coordinates] of two given points: P and Q.

(°>°) Provide the [new] value? 2

[The typed number]: 2 is a [valid integer number!]]

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

* Introduce the 1° [Coordinate(x)].

(^a<^a) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].

(^a<^a) Enter the [new] value? -3

[[The typed number]: -3.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].

(^a<^a) Enter the [new] value? 5

[[The typed number]: 5.0 is a [valid integer or float number!]]

-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?

* Introduce the 1° [Coordinate(x)].

(a<a) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].

(a<a) Enter the [new] value? 0

[[The typed number]: 0.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].

(a<a) Enter the [new] value? -1

[[The typed number]: -1.0 is a [valid integer or float number!]]

- The (Point P): P (2.0, -3.0, 5.0)

- The (Point Q): Q (1.0, 0.0, -1.0)

[ANSWER]

-- The [vectorA]: vectorA=vectorPQ [-1.0, 3.0, -6.0]

-- The [length] of the vectorA=|vectorA|: 6.78

[(COSINEALPHA)² + (COSINEBETA)² + (COSINEGAMA)² = 1]

-- The [value] of the [CosineAlpha] is: -0.15

-- The [value] of the [CosineBeta] is: 0.44

-- The [value] of the [CosineGama] is: -0.88

-- The [value] of the [AlphaAngle] is: 98.48

-- The [value] of the [BetaAngle] is: 63.75

-- The [value] of the [GamaAngle] is: 152.21

. . . Key [ENTER] to exit -- Ok! . . .

Example9 – Calculate the direction cosines and direction angles of the given vector $a = i - 2j - 3k$.

Solution – After was begin the runtoolsvectors.py program key the option[15] and wait the display present instructions to select the options: [1] or [2]. Follow type the option[1] and provide the [components]; a1 = 1, a2 = -2, and a3 = -3.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**
--[Version: 1.3 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]

- To calculate the [angle] between [two vectors] in space key [5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)], [Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, and R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the [VectorA] and [lenght] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an Vector A or [two points]: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

(°>°) Provide the [new] value? 15

[The typed number]: 15 is a [valid integer number!]]

[GIVENS AN VECTOR(A) OR TWO POINTS: P AND Q DETERMINE THE [DIRECTION ANGLES] AND [DIRECTION COSINES]]

- Type [1] to enter the [components] of vectorA.

- Type [2] to enter the [coordinates] of two given points: P and Q.

(°>°) Provide the [new] value? 1

[The typed number]: 1 is a [valid integer number!]]

- Attribute the [Components] of the [1° vectorA]!

- Enter the 1° [Component(x)]!

(^<^a) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(^<^a) Enter the [new] value? -2

[[The typed number]: -2.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(^<^a) Enter the [new] value? -3

[[The typed number]: -3.0 is a [valid integer or float number!]]

[ANSWER]

-- The [vectorA]: vectorA [1.0, -2.0, -3.0]

-- The [lenght] of the vectorA=|vectorA|: 3.74

```
**[ (COSINEALPHA)2 + (COSINEBETA)2 + (COSINEGAMA)2 = 1 ]**
```

```
-- The [value] of the [CosineAlpha] is: 0.27
-- The [value] of the [CosineBeta] is: -0.53
-- The [value] of the [CosineGama] is: -0.80
```

```
-- The [value] of the [AlphaAngle] is: 74.50
-- The [value] of the [BetaAngle] is: 122.31
-- The [value] of the [GamaAngle] is: 143.30
```

. . . Key [ENTER] to exit -- Ok! . . .

Example10 – Given the vectors: $a = i + 2j - 3k$ and $b = -2i - j + 5k$ find the [vectors and modules]: $a + b$, $a - b$, $|a + b|$, $|a - b|$, and $2a - 3b$.

Solution – Begin the `runtoolsvectors.py` program and key the `option[16]` and wait the display present instructions to select the options: [1] or [2]. Follow type the `option[1]` and provide the [components]; $a_1 = 1$, $a_2 = 2$, $a_3 = -3$, $b_1 = -2$, $b_2 = -1$, and $b_3 = 5$. Following the program will wait the user enter the [coefficients]. Do (1°)[coefficient] = 1 and (2°)[coefficient] = 1 and key ENTER to process.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**
--[Version: 1.3 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)], [Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P,Q,and R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the [VectorA] and [lenght] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an Vector A or [two points]: P and Q key [15]

- To find the [Addition] and [Subtraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

(°>°) Provide the [new] value? 15

[The typed number]: 15 is a [valid integer number!]]

[GIVENS AN VECTOR(A) OR TWO POINTS: P AND Q DETERMINE THE [DIRECTION ANGLES] AND [DIRECTION COSINES]]

- Type [1] to enter the [components] of vectorA.

- Type [2] to enter the [coordinates] of two given points: P and Q.

(°>°) Provide the [new] value? 1

[The typed number]: 1 is a [valid integer number!]]

- Attribute the [Components] of the [1° vectorA]!

- Enter the 1° [Component(x)]!

(^<^a) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(^<^a) Enter the [new] value? -2

[[The typed number]: -2.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(^<^a) Enter the [new] value? -3

[[The typed number]: -3.0 is a [valid integer or float number!]]

[ANSWER]

-- The [vectorA]: vectorA [1.0, -2.0, -3.0]

-- The [lenght] of the vectorA=|vectorA|: 3.74

[(COSINEALPHA)² + (COSINEBETA)² + (COSINEGAMA)² = 1]

-- The [value] of the [CosineAlpha] is: 0.27

-- The [value] of the [CosineBeta] is: -0.53

-- The [value] of the [CosineGama] is: -0.80

-- The [value] of the [AlphaAngle] is: 74.50

-- The [value] of the [BetaAngle] is: 122.31

-- The [value] of the [GamaAngle] is: 143.30

. . . Key [ENTER] to exit -- Ok! . . .

Example11 – Givens the vectors: $a = i + 2j - 3k$ and $b = -2i - j + 5k$ find the [vectors and modules]: $a + b$, $a - b$, $|a + b|$, $|a - b|$, and $2a - 3b$.

Solution – Begin the `runtoolsvectors.py` program and key the `option[16]` and wait the display present instructions to select the options: [1] or [2]. Follow type the `option[1]` and provide the [components]; $a_1 = 1$, $a_2 = 2$, $a_3 = -3$, $b_1 = -2$, $b_2 = -1$, and $b_3 = 5$. Following the program will wait the user enter the [coefficients]. Do (1°)[coefficient] = 1 and (2°)[coefficient] = 1 and key ENTER to process.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**
--[Version: 1.3 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)], [Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P,Q,and R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subraction] between [Two Vectors] A and B key [13]
- To calculate the [VectorA] and [lenght] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an Vector A or [two points]: P and Q key [15]
- To find the [Addition] and [Subraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

```
[§] Select an previous [option] that will used--Ok!
```

```
(°>°) Provide the [new] value? 16
```

```
**[The typed number]: 16 is a [valid integer number!] ]**
```

```
**[ GIVENS [TWO VECTORS: A AND B] MULTIPLYED BY THE [SCALARS]: [COEFFIC1] AND [COEFFIC2] OR ]**
```

```
**[ GIVENS [THE POINTS: P, Q, AND R] FIND [TWO VECTORS]: VECTOR(A)=VECTOR(PQ) AND VECTOR(B)=VECTOR(PR) ]**
```

```
**[ AND MULTIPLY BY THE [SCALARS]: [COEFFIC1] AND [COEFFIC2] AND TOO GET THE [ADDITION] AND [SUBTRACTION] ]**
```

- To enter the [Components] of the [vectors: A and B] type [1].
- To introduce the [Coordinates] of the given points: P, Q, and R type [2].
(°>°) Provide the [new] value? 1
[The typed number]: 1 is a [valid integer number!]]

- Provide the [Components] of the [vectors]: vectorA and vectorB.

- Attribute the [Components] of the [1° vectorA]!

- Enter the 1° [Component(x)]!
(^<^) Enter the [new] value? 1
[[The typed number]: 1.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!
(^<^) Enter the [new] value? 2
[[The typed number]: 2.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!
(^<^) Enter the [new] value? -3
[[The typed number]: -3.0 is a [valid integer or float number!]]

- Provide the [Components] of the [2° vectorB]!

- Enter the 1° [Component(x)]!
(^<^) Enter the [new] value? -2
[[The typed number]: -2.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!
(^<^) Enter the [new] value? -1
[[The typed number]: -1.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!
(^<^) Enter the [new] value? 5
[[The typed number]: 5.0 is a [valid integer or float number!]]

-- The [vectorA]: vectorA [1.0, 2.0, -3.0]
-- The [vectorB]: vectorB [-2.0, -1.0, 5.0]

- Enter with new [value] to the (1°)[coefficient]?
(^<^) Enter the [new] value? 1
[[The typed number]: 1.0 is a [valid integer or float number!]]

- Give the new [value] to the (2°)[coefficient]?
(^<^) Enter the [new] value? 1
[[The typed number]: 1.0 is a [valid integer or float number!]]

[ANSWER]

+ The vectorAcoeffic1 [1.0, 2.0, -3.0]
+ The vectorBcoeffic2 [-2.0, -1.0, 5.0]

-- The vectorAddition [-1.0, 1.0, 2.0]
-- The vectorSubtraction [3.0, 3.0, -8.0]

-- The [length] of the [vectorAddition] is: 2.45
-- The [length] of the [vectorSubtraction] is: 9.06

. . . Key [ENTER] to exit -- Ok! . . .

Now to get the vector: $2a - 3b$, run the `runtoolsvectors.py` program again and key the option[16] and wait the display present instructions to select the options: [1] or [2]. Follow type the option[1] and provide again the [components]; $a_1 = 1$, $a_2 = 2$, $a_3 = -3$, $b_1 = -2$, $b_2 = -1$, and $b_3 = 5$. Following the program will wait the user enter the [coefficients]. Do $(1^\circ)[\text{coefficient}] = 2$ and $(2^\circ)[\text{coefficient}] = -3$ and key ENTER to process.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**
--[Version: 1.3 -- Stable]---
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)],[Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P,Q,and R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subraction] between [Two Vectors] A and B key [13]
- To calculate the[VectorA] and [lenght] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an Vector A or [two points]: P and Q key [15]
- To find the [Addition] and [Subraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used—Ok!

(°>°) Provide the [new] value? 16

```
**[The typed number]: 16 is a [valid integer number!] ]**
```

[GIVENS [TWO VECTORS: A AND B] MULTIPLIED BY THE [SCALARS]: [COEFFIC1] AND [COEFFIC2] OR]

[GIVENS [THE POINTS: P, Q, AND R] FIND [TWO VECTORS]: VECTOR(A)=VECTOR(PQ) AND VECTOR(B)=VECTOR(PR)]

[AND MULTIPLY BY THE [SCALARS]: [COEFFIC1] AND [COEFFIC2] AND TOO GET THE [ADDITION] AND [SUBTRACTION]]

- To enter the [Components] of the [vectors: A and B] type [1].
- To introduce the [Coordinates] of the given points: P, Q, and R type [2].

(°>°) Provide the [new] value? 1

[The typed number]: 1 is a [valid integer number!]]

- Provide the [Components] of the [vectors]: vectorA and vectorB.

- Attribute the [Components] of the [1° vectorA]!

- Enter the 1° [Component(x)]!

(°<°) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(°<°) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(°<°) Enter the [new] value? -3

[[The typed number]: -3.0 is a [valid integer or float number!]]

- Provide the [Components] of the [2° vectorB]!

- Enter the 1° [Component(x)]!

(°<°) Enter the [new] value? -2

[[The typed number]: -2.0 is a [valid integer or float number!]]

- Introduce the 2° [Component(y)]!

(°<°) Enter the [new] value? -1

[[The typed number]: -1.0 is a [valid integer or float number!]]

- Give the 3° [Component(z)]!

(°<°) Enter the [new] value? 5

[[The typed number]: 5.0 is a [valid integer or float number!]]

-- The [vectorA]: vectorA [1.0, 2.0, -3.0]

-- The [vectorB]: vectorB [-2.0, -1.0, 5.0]

- Enter with new [value] to the (1°)[coefficient]?

(°<°) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

- Give the new [value] to the (2°)[coefficient]?

(°<°) Enter the [new] value? -3

[[The typed number]: -3.0 is a [valid integer or float number!]]

[ANSWER]

+ The vectorAcoeffic1 [2.0, 4.0, -6.0]

+ The vectorBcoeffic2 [6.0, 3.0, -15.0]

-- The vectorAddition [8.0, 7.0, -21.0]

-- The vectorSubtraction [-4.0, 1.0, 9.0]

-- The [length] of the [vectorAddition] is: 23.54

-- The [length] of the [vectorSubtraction] is: 9.90

. . . Key [ENTER] to exit -- Ok! . . .

Example12 – Given the points: P(2, 2, 3) and Q(4, -5, 6), find the [distance] between the points.

Solution – Begin the runtoolsvectors.py program and key the option[7] and provide the [coordinates]: xP = 2, yP = 2, zP = 3, xQ = 4, yQ = -5, and zQ = 6 when request by program.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**  
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**  
--[Version: 1.3 -- Stable]--
```

****[INSTRUCTIONS OF USE]****

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)], [Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P,Q,and R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the [VectorA] and [lenght] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an Vector A or [two points]: P and Q key [15]

- To find the [Addition] and [Subtraction] between [Two Vectors]: A and B
multiplied by scalars: [coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

(⁰>⁰) Provide the [new] value? 7

[The typed number]: 7 is a [valid integer number!]]

[DETERMINE THE DISTANCE(d) BETWEEN TWO GIVENS POINTS P and Q]

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

* Introduce the 1^o [Coordinate(x)].

(^a<^a) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

* Enter with the 2^o [Coordinate(y)].

(^a<^a) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

* Give the 3^o [Coordinate(z)].

(^a<^a) Enter the [new] value? 3

[[The typed number]: 3.0 is a [valid integer or float number!]]

-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?

* Introduce the 1^o [Coordinate(x)].

(^a<^a) Enter the [new] value? 4

[[The typed number]: 4.0 is a [valid integer or float number!]]

* Enter with the 2^o [Coordinate(y)].

(^a<^a) Enter the [new] value? -5

[[The typed number]: -5.0 is a [valid integer or float number!]]

* Give the 3^o [Coordinate(z)].

(^a<^a) Enter the [new] value? 6

[[The typed number]: 6.0 is a [valid integer or float number!]]

- The (Point P): P (2.0, 2.0, 3.0)

- The (Point Q): Q (4.0, -5.0, 6.0)

[ANSWER]

-- The [vectorPQ]: vectorPQ [2.0, -7.0, 3.0]

-- The [Quadratic Components] of the [vectorCQD]:vectorCQD [4.0, 49.0, 9.0]

-- The [Distance(d)] geted between (Two Points) P and Q is: 7.87

. . . Key [ENTER] to exit -- OK! . . .

Example13 – Givens the points: P(5, -9, 7) and Q(-2, 3, 3), find the [MidPoint] between the points.

Solution – Begin the `runtoolsvectors.py` program and key the option[12] and provide the [coordinates]: xP = 5, yP = -9, zP = 7, xQ = -2, yQ = 3, and zQ = 3 when the program request.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**
--[Version: 1.3 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)], [Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P,Q,and R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subraction] between [Two Vectors] A and B key [13]
- To calculate the [VectorA] and [lenght] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an Vector A or [two points]: P and Q key [15]
- To find the [Addition] and [Subraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

(°>°) Provide the [new] value? 12

```
**[The typed number]: 12 is a [valid integer number!] ]**
```

```
**[ WILL FIND THE [MIDPOINT(M)] OF THE [LINE SEGMENT] BETWEEN THE GIVEN POINTS : P AND Q ]**
```

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

* Introduce the 1° [Coordinate(x)].

(^<^a) Enter the [new] value? 5

```
**[ [The typed number]: 5.0 is a [valid integer or float number!] ]**
```

```
* Enter with the 2° [Coordinate(y)].
(a<a) Enter the [new] value? -9
**[ [The typed number]: -9.0 is a [valid integer or float number!] ]**

* Give the 3° [Coordinate(z)].
(a<a) Enter the [new] value? 7
**[ [The typed number]: 7.0 is a [valid integer or float number!] ]**
```

```
-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?
```

```
* Introduce the 1° [Coordinate(x)].
(a<a) Enter the [new] value? -2
**[ [The typed number]: -2.0 is a [valid integer or float number!] ]**

* Enter with the 2° [Coordinate(y)].
(a<a) Enter the [new] value? 3
**[ [The typed number]: 3.0 is a [valid integer or float number!] ]**

* Give the 3° [Coordinate(z)].
(a<a) Enter the [new] value? 3
**[ [The typed number]: 3.0 is a [valid integer or float number!] ]**
```

```
- The (Point P): P (5.0, -9.0, 7.0)
- The (Point Q): Q (-2.0, 3.0, 3.0)
```

```
*[ANSWER]*
```

```
-- The (MidPoint M): M (1.5, -3.0, 5.0)
```

```
. . . Key [ENTER] to exit -- Ok! . . .
```

Example14 – Given the points: P(-1, 2, 0), Q(2, 1, -3), R(1, 0, 1), and S(3, -2, 3), find the Volume(V) and Height(h) of the parallelepiped as too the volume of the Tetrahedron.

Solution – Run the runtoolsvectors.py program and key the option[10] and provide the [coordinates]: xP = -1, yP = 2, zP = 0, xQ = -2, yQ = 1, zQ = -3, xR = 1, yR = 0, zR = 1, xS = 3, yS = -2, and zS = 3 when the program request.

```
**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**
--[Version: 1.3 -- Stable]--
```

```
**[INSTRUCTIONS OF USE]**
```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]

- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)], [Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P, Q, and R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the [VectorA] and [lenght] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an Vector A or [two points]: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

(°>°) Provide the [new] value? 10

[The typed number]: 10 is a [valid integer number!]]

[GIVENS THE COORDINATES OF THE POINTS P, Q, R AND S FIND THE]

[VOLUME(V) AND HEIGHT(H) OF THE PARALLELEPIPED AND TETRAHEDRON]

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

* Introduce the 1° [Coordinate(x)].

(^<^) Enter the [new] value? -1

[[The typed number]: -1.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].

(^<^) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].

(^<^) Enter the [new] value? 0

[[The typed number]: 0.0 is a [valid integer or float number!]]

-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?

* Introduce the 1° [Coordinate(x)].

(^<^) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].

(^<^) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].
 (a<a) Enter the [new] value? -3
 [[The typed number]: -3.0 is a [valid integer or float number!]]

-- Provide the (coordinates: xR, yR, zR) of the (Point R)?

* Introduce the 1° [Coordinate(x)].
 (a<a) Enter the [new] value? 1
 [[The typed number]: 1.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].
 (a<a) Enter the [new] value? 0
 [[The typed number]: 0.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].
 (a<a) Enter the [new] value? 1
 [[The typed number]: 1.0 is a [valid integer or float number!]]

-- Give the [Coordinates: xS, yS, zS] of the [point S]?

* Introduce the 1° [Coordinate(x)].
 (a<a) Enter the [new] value? 3
 [[The typed number]: 3.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].
 (a<a) Enter the [new] value? -2
 [[The typed number]: -2.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].
 (a<a) Enter the [new] value? 3
 [[The typed number]: 3.0 is a [valid integer or float number!]]

- The (Point P): P (-1.0, 2.0, 0.0)
 - The (Point Q): Q (2.0, 1.0, -3.0)
 - The (Point R): R (1.0, 0.0, 1.0)
 - The (Point S): S (3.0, -2.0, 3.0)

[ANSWER]

- The [vectorA]=vectorPQ [3.0, -1.0, -3.0]
 - The [vectorB]=vectorPR [2.0, -2.0, 1.0]
 - The [vectorC]=vectorPS [4.0, -4.0, 3.0]

-- [THE SCALAR TRIPLE PRODUCT]: a * (b x c) is: -4.0
 -- The [VOLUME(V) PARALLELEPIPED]: 4.00
 -- The [HEIGHT(H) PARALLELEPIPED]: 0.33
 -- The [tetrahedron volume]: 0.67

. . . Key [ENTER] to exit -- Ok! . . .

(new)Example15 – Calculate the dimensions: height, angle [Theta in degree(°)], and Area of the parallelogram PQRS given the [Coordinates] of the points: P(2, -3, 1), Q(6, 5, -1), R(7, 2, 2), and S(3, -6, 4).

Solution:


```

**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**
--[Version: 1.3 -- Stable]--

```

```

**[INSTRUCTIONS OF USE]**

```

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)], [Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P,Q,and R key [11]
- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subraction] between [Two Vectors] A and B key [13]
- To calculate the[VectorA] and [lenght] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an Vector A or [two points]: P and Q key [15]
- To find the [Addition] and [Subraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

(°>°) Provide the [new] value? 9

[The typed number]: 9 is a [valid integer number!]]

```

**[ FIND THE HEIGHT(H), AREA(A), AND THE ANGLE(THETA) OF THE PARALLELOGRAM(PQRS)
  DETERMINED BY TWO ADJACENT VECTORS: A AND B OR TO VERTICES: P, Q,R, AND S ]**

```

```

**[ Instructions to enter the [Coordinates] or [Components] of the Parallelogram PQRS ]**

```

- To enter the [Coordinates] of the points: P, Q, R, and S key [1].
- For the **[Components]** of the adjacent Vectors: [vectorA]=vectorPQ e
- [vectorB]=vectorPS key [2].

(°>°) Provide the [new] value? 1

[The typed number]: 1 is a [valid integer number!]]

****[COORDINATES OF THE POINTS: P, Q, R, and S]****

-- Enter the (coordinates: xP, yP, zP) of the (Point P)?

* Introduce the 1° [Coordinate(x)].

(^a<^a) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].

(^a<^a) Enter the [new] value? -3

[[The typed number]: -3.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].

(^a<^a) Enter the [new] value? 1

[[The typed number]: 1.0 is a [valid integer or float number!]]

-- Introduce the (coordinates: xQ, yQ, zQ) of the (Point Q)?

* Introduce the 1° [Coordinate(x)].

(^a<^a) Enter the [new] value? 6

[[The typed number]: 6.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].

(^a<^a) Enter the [new] value? 5

[[The typed number]: 5.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].

(^a<^a) Enter the [new] value? -1

[[The typed number]: -1.0 is a [valid integer or float number!]]

-- Provide the (coordinates: xR, yR, zR) of the (Point R)?

* Introduce the 1° [Coordinate(x)].

(^a<^a) Enter the [new] value? 7

[[The typed number]: 7.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].

(^a<^a) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].

(^a<^a) Enter the [new] value? 2

[[The typed number]: 2.0 is a [valid integer or float number!]]

-- Give the [Coordinates: xS, yS, zS] of the [point S]?

* Introduce the 1° [Coordinate(x)].

(^a<^a) Enter the [new] value? 3

[[The typed number]: 3.0 is a [valid integer or float number!]]

* Enter with the 2° [Coordinate(y)].

(^a<^a) Enter the [new] value? -6

[[The typed number]: -6.0 is a [valid integer or float number!]]

* Give the 3° [Coordinate(z)].

(^a<^a) Enter the [new] value? 4

[[The typed number]: 4.0 is a [valid integer or float number!]]

- The (Point P): P (2.0, -3.0, 1.0)
- The (Point Q): Q (6.0, 5.0, -1.0)
- The (Point R): R (7.0, 2.0, 2.0)
- The (Point S): S (3.0, -6.0, 4.0)

[ANSWER]

- The [vectorA]=vectorPQ [4.0, 8.0, -2.0]
- The [vectorB]=vectorPS [1.0, -3.0, 3.0]

- The [height]: 3.31
- The [Area]: 30.33
- The angle [Theta in degree($^{\circ}$)] between the [adjacent vectors]: [vectorA] and [vectorB] is: 49.40

. . . Key [ENTER] to exit -- Ok! . . .

[Warning]: The **new version: 1.3** of the `runtoolsvectors.py` program and `VectorModDev.py` module keep all the previous operation, but present more performance than the **last version: 1.2**.

[Warning]: In the **new version: 1.2** of the `runtoolsvectors.py` program was improve the security of datas enter to the user type only *positive integer number* in the menu options. View below using the `runtoolsvectors.py` program when any user type: -2 or p or @ or J or 0(zero) or key ENTER.

```

**[ WELCOME IN USING THE [RUNTOOLSVECTORS.PY] PROGRAM ]**
**[ TO [SOLVE] VARIOUS PROBLEMS OF [VECTORS] IN THE [SPACE] ]**
--[Version: 1.3 -- Stable]--

```

[INSTRUCTIONS OF USE]

- To find the [value] of the [Dot Product] of two vectors key [1]
- To get the [Cross Product] of two [vectors: A and B] and the [Area(A)] of the Triangle(PQR) key [2]
- To calculate the [value] of the [SineTheta] between [two vectors] A and B key [3]
- To find the [value] of the [Scalar Triple Product] of three vectors A, B, and C key [4]
- To calculate the [angle] between [two vectors] in space key[5]
- To get the [value] of the [CossineTheta] between two vectors A and B key [6]
- To find the [Distance] between two points P and Q key [7]
- To calculate the [Dimensions] of the [Triangle(PQR)] given the points: P, Q, and R key [8]
- To find the [Height(h)], [Area(A)] and angle[Theta] of the [Parallelogram(PQRS)] determined to [two adjacent vectors]: A and B or to [Coordinates] of the vertices: P, Q, R, and S key [9]
- To get the [Volume(V)], [Height(h)] of the [Parallelepiped and Tetrahedron] given four points: P, Q, R, S key [10]
- To calculate the three [Inner Angles] of the [Triangle] given three points: P,Q,and R key [11]

- To find the [MID-POINT M] OF THE LINE SEGMENT BETWEEN [THE POINTS: P AND Q] key [12]
- To get the [Addition] and [Subtraction] between [Two Vectors] A and B key [13]
- To calculate the [VectorA] and [lenght] between the given Points: P, AND Q key [14]
- To determine the [Direction Cosines] and [Direction Angles] given an Vector A or [two points]: P and Q key [15]
- To find the [Addition] and [Subtraction] between [Two Vectors]: A and B multiplied by scalars: [coeffic1 and coeffic2] key [16]

[§] Select an previous [option] that will used--Ok!

(°>°) Provide the [new] value? -2

[NO TYPE AN [NEGATIVE INTEGER NUMBER] or equal [ZERO]--Ok!]

(°>°) Provide the [new] value? p

```
_/§\
@>@ [Warning!]: invalid literal for int() with base 10: 'p'
\~/ [ TYPE AN [NEW POSITIVE INTEGER NUMBER ]
[ IN NEXT INSTRUCTION -- OK! ]
```

(°>°) Provide the [new] value? @

```
_/§\
@>@ [Warning!]: invalid literal for int() with base 10: '@'
\~/ [ TYPE AN [NEW POSITIVE INTEGER NUMBER ]
[ IN NEXT INSTRUCTION -- OK! ]
```

(°>°) Provide the [new] value? J

```
_/§\
@>@ [Warning!]: invalid literal for int() with base 10: 'J'
\~/ [ TYPE AN [NEW POSITIVE INTEGER NUMBER ]
[ IN NEXT INSTRUCTION -- OK! ]
```

(°>°) Provide the [new] value? 0

[NO TYPE AN [NEGATIVE INTEGER NUMBER] or equal [ZERO]--Ok!]

(°>°) Provide the [new] value? Typed ENTER

```
_/§\
@>@ [Warning!]: invalid literal for int() with base 10: ""
\~/ [ TYPE AN [NEW POSITIVE INTEGER NUMBER ]
[ IN NEXT INSTRUCTION -- OK! ]
```

(°>°) Provide the [new] value? 23

[The typed number]: 23 is a [valid integer number!]]

[NEITHER OF THE PREVIOUS OPTIONS WAS SELECTED]
[RUN THE RUNTOOLSVECTORS.PY PROGRAM AGAIN -- OK!]

. . . Key [ENTER] to exit -- Ok! . . .

[Warning]: The `runtoolsvectors.py` program will follow only when the user types any *positive integer number* between 1 to 16 – Ok! If type any positive integer number bigger than 16 the user will have that run the `runtoolsvectors.py` program again. View the last previous test.

Note: The `Components of Vectors` and `Coordinates of Points` will can be: Negative or Zero or Positive; but the `options of menu` of the `runtoolsvectors.py` program only can be typed *positive integer number*.

Developer: ***Cristovom A. Girodo***