CSC236 Tutorial 8

1. Give a proof of correctness for the program below with respect to the given specification.
   Hint: For any integers a, b such that $a + 1 < b$,

   $$a < \lfloor \frac{a+b}{2} \rfloor < b.$$

   Precondition: A is a list of integers, $0 \leq p < q \leq len(A)$.
   Postcondition: Return the index of a minimum element in $A[p:q]$. That is, return a number i such that $p \leq i < q$ and $A[i]$ is a smallest integer of $A[p:q]$ (i.e., no cell of $A[p:q]$ contains a smaller integer).

```
def IndexMin(A, p, q):
    if p + 1 == q:
        return p
    else:
        m = ⌊(p+q)/2⌋
        j = IndexMin(A, p, m)
        k = IndexMin(A, m, q)
        if A[j] ≤ A[k]:
            return j
        else:
            return k
```

2. Consider the following program.
   Precondition: A is a nonempty list of integers.
   Postcondition: The elements of A are rearranged in sorted (nondecreasing) order.

```
def Sort(A):
    k = 0
    while k < len(A) − 1:
            j = IndexMin(A, k, len(A)) # see Question 1 for the specification of IndexMin
            A[k], A[j] = A[j], A[k] # swap A[k] and A[j]
            k = k + 1
```

   (a) Give an appropriate loop invariant for the purpose of proving both partial correctness and termination for the above program with respect to its given specification. For this part a proof is not required.

   (b) Define an appropriate loop measure for the purpose of proving termination. For this part a proof is not required.

   (c) Assume your loop invariant from part (a) is correct and use it to prove partial correctness.