CSC236 - Week 3

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Building Recursively Defined Sets:

1. Define the smallest, simplest, elementary objects in the set.
2. Indicate how larger, more complex objects in the set can be constructed out of simpler ones.
3. Close the definition of the set.

Examples of Recursively Defined Sets:

- **The set of natural numbers,** $\mathbb{N}$.
  - $0 \in \mathbb{N}$
  - $k \in \mathbb{N} \rightarrow (k+1) \in \mathbb{N}$
  - nothing else belongs to $\mathbb{N}$
- **Non-empty binary trees**.
  - a single node is a binary tree
  - given disjoint non-empty binary trees T_1, T_2, and single node r,
    the tree with root r connected to the roots of one or both of $\{T_1, T_2\}$ is a non-empty binary tree
  - nothing else is a non-empty binary tree

Structural Induction: Prove that P holds for all elements of a recursively defined set.

1. Show that every elementary object in the set satisfies P.
2. Assume that P holds for smaller, simpler elements in the set. Show that every possible element constructed out of smaller elements for which P holds also satisfies P.

Examples of Structural Induction:

- Prove that every non-empty binary tree has one more node than edge.
- Consider the following recursively defined set $S \subseteq \mathbb{N}^2$:

  this is an item

  this is an item

  this is an item