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well ordering
$\updownarrow$
simple induction
$\updownarrow$
complete induction

- predicate: statement about set of variables
  - Ex] $O(n)$: n is an odd natural number $O: \mathbb{N} \to$ Boolean
    $D(a,b)$: a divides b $D: \mathbb{N} \times \mathbb{N} \to \{\text{true}, \text{false}\}$
    $F(\text{Tom}, \text{Bob})$: Tom fathered Bob F: People × People → Boolean
  - may have infinite variables, may need inputs strictly specified
  - this course: unary predicates abstractly defined

- Simple Induction:
  - Base Case: $P(x)$
  - Induction: $\forall k \in \mathbb{N}, P(k) \to P(k+1)$ // k is arbitrary
  - combined these show that P is true for all $\mathbb{N}$

* PSI: $[P(0) \wedge [\forall k \in \mathbb{N}, P(k) \to P(k+1)]] \to \forall n \in \mathbb{N}, P(n)$

- Writing Proofs with Simple Induction:
  1) Define predicate
  2) Prove Base case
  3) Set up & prove inductive step
  4) STATE PSI; conclude proved

- countable: pairing between objects & natural numbers

Ex
- Let $\{a_0, a_1, \ldots\}$ be sequence of natural numbers s.t. $a_0 = 1$; $\forall n \geq 1$ $a_n = 2a_{n-1} + 1$. Prove $\forall n \in \mathbb{N}$ $a_n = 2^{n+1} - 1$.
  - Let $P(n)$: $a_n = 2^{n+1} - 1$. Must show $\forall n \in \mathbb{N}, P(n)$.
  - $P(0) = 2^{0+1} - 1 = 2 - 1 = 1 = a_0$ $\therefore$ $P(0)$ is true!
  - Assume $P(k)$ is true $\therefore P(k) = 2^{k+1} - 1$ for arbitrary $k \in \mathbb{N}$
    We know $P(k+1) = 2a_k + 1 = 2(2^{k+1} - 1) + 1 = 2^{k+2} - 2 + 1 = 2^{k+2} - 1$
    $\therefore [P(k+1) = 2^{k+2} - 1] \cong [P(n) = 2^{n+1} - 1$ where $n = k+1)$
    $\therefore P(k) \to P(k+1)$
  - We have $[P(0) \wedge [\forall k \in \mathbb{N}\ P(k) \to P(k+1)]]$ $\therefore$ via PSI $P(n) \forall n \in \mathbb{N}$