CSC236 Thursday Lecture Feb 15, 2018

4 → Review of Recursive algo correctness – RecBin Search

- additional precondition: A is non-empty
- RecBinSearch starts by checking if one element (base case)
- then divides non-one problem into halves, focus on target half

- how do we prove correctness? iterate over length of A
- Complete induction because $\lfloor n/2 \rfloor \neq n \pm 1 \ \forall n \in \mathbb{N}$ (need all values below n covered

• Claim: $\forall n \in \mathbb{N}$, $P(n)$ holds.

- Base $n=1$; we know $n = e+1-s$ ∴ when $n=1$, $e=s$.
  ∴ if case at line 1 true ∴ two cases:
  - A] $A[s]==x$ ∴ line 3 executed, RecBinSearch terminates returning s, which is correct output
  - B] $A[s] != x$ ∴ line 5 executed, RecBinSearch terminates and returns -1, which is correct because A doesn't contain x.

  ∴ conclude $P(1)$ holds in all circumstances.

- Let k be an arbitrary natural number ≥ 1, (IH:) Assume $P(j)$ holds $\forall\ 1 \leq j < k$. We must show that $P(k)$ holds.
  ∴ lines 1, 6, 7 are executed ∴ $m = \lfloor \frac{s+e}{2} \rfloor$ ∴ $s \leq m < e$
  - A] $A[m] \geq x$ ∴ line 9 executed ∴ RBS called on input with $n' = m+1-s < e+1-s = n$ ∴ this recursive call covered under IH ∴ returns (terminates) with correct t (via IH)
  - B] $A[m] < x$ ∴ line 10, 11 executes ∴ RBS called on input where $n' = e+1-m-1 = e-m < e+1-m \leq e+1-s = n$ ∴ covered under IH ∴ this recursive call returns correct answer t (via IH) for $A[m+1, e]$ ∴ this higher call terminates with correct answer for $A[s, e]$

5 * TRY EXERCIZES ON THIS PAGE

7 • Loop invariant: assertion true before entering loop & at the end of every iteration

8 • loop measure must be strictly decreasing

9 $LI(k): k \le len(A) \to sum_k = \sum_{j=0}^{i_k-1} A[j] \wedge 0 \le i_k = k \le len(A)$

Claim $LI(k)$ holds $\forall k \in \mathbb{N}$. Prove via induction.

Base $n=0$: $LI(0)$: "If $0 \le len(A)$ then $\sum_{j=0}^{i_0-1} A[j] = sum_0 \wedge 0 \le i_0 \le len(A)$"

via precondition A nonempty $\therefore 0 \le len(A)$;
via algo itself $sum_0 = 0$; via algo $i_0 = 0$; $\Bigg\}$ $\therefore LI(0)$ holds
$\sum_{j=0}^{-1} = 0$ via conventions

Let k be arbitrary natural number & let us assume that $P(k)$ holds.

$sum_{k+1} = sum_k + A[i_k] = \sum_{j=0}^{i_k-1} A[j] + A[i_k] = \sum_{j=0}^{i_{k+1}-1} A[j]$ $\therefore sum_{k+1}$ holds

$i_{k+1} = i_k + 1$