CSC236H
Introduction to the Theory of Computation

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- A predicate is a statement about a set of variables.
- Examples:
  - $O(n)$: n is an odd natural number.
  - $D(a, b)$: a divides b.
  - $F(Tom, Bob)$: Tom is the father of Bob.

A simple induction proof consists of two parts:

- **Base Case**: $P(0)$.
    - Note: the base case may be different from 0!
- **Induction Step**: $\forall k \in \mathbb{N}, P(k) \rightarrow P(k+1)$.

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- **Induction Step**: $\forall k \in \mathbb{N}, P(k) \to P(k+1)$.

- **Conclusion**: by the **Principle of Simple Induction** it follows that $\forall n \in \mathbb{N}, P(n)$.

PSI: $[P(0) \wedge \forall k \in \mathbb{N}, P(k) \to P(k+1)] \to \forall n \in \mathbb{N}, P(n)$.

Informal Justification:

Suppose $P(0)$ is true.

Suppose for all $k \in \mathbb{N}$, $P(k)$ implies $P(k+1)$.

- Step 1: Define the predicate.
- Step 2: Prove the predicate holds for the **Base Case**.
- Step 3: Set up the **Induction Step (IS)**, indicate **Induction Hypothesis (IH)**, indicate **What to Prove (WTP)**.
- Step 4: Prove that the predicate holds for all natural numbers using **IH** (make sure to explicitly indicate where you use IH).

Let $a_0, a_1, ...$ be a sequence of natural numbers such that:

$a_0 = 1$, and for all $n \geq 1$, $a_n = 2a_{n-1} + 1$.

Prove that for all $n \in \mathbb{N}$, $a_n = 2^{n+1} - 1$.

Prove that for all natural numbers $n > 4$, $2^n > n^2$.

A complete induction proof consists of two parts:

- **Base Case**: $P(0)$.
- **Induction Step**: $\forall k \in \mathbb{N}, [P(0) \wedge P(1) \wedge ... \wedge P(k-1) \rightarrow P(k)]$.

- Step 1: Define the predicate.

- Step 2: Prove the predicate holds for the **Base Case**.

- Step 3: Set up the **Induction Step (IS)**, indicate **Induction Hypothesis (IH)**, indicate **What to Prove (WTP)**.

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