CSC236 Tutorial 4

1. Let F be a set defined as follows:

   - any tree consisting of a single node is an element of F;
   - if $t_1, t_2 \in F$, so is a binary tree consisting of a root with t_1 and t_2 as sub-trees;
   - nothing else belongs to F.

   Use structural induction to prove that every $t \in F$ has exactly one more leaf than interior nodes.

2. Let G be a set defined as follows:

   - if x is a propositional variable, then $x \in G$;
   - if $f_1, f_2 \in G$, then $\neg f_1 \in G$, $(f_1 \vee f_2) \in G$, and $(f_1 \wedge f_2) \in G$;
   - nothing else belongs to G.

   Use structural induction to prove that for every $f \in G$, there exists $f' \in G$ such that f and f' are logically equivalent, and f' does not contain $\wedge$ symbol.

   (Recall that propositional formulas f_1 and f_2 are *logically equivalent* if f_1 and f_2 evaluate to the same value, no matter how their variables are set.)