CSC236 Tutorial 10

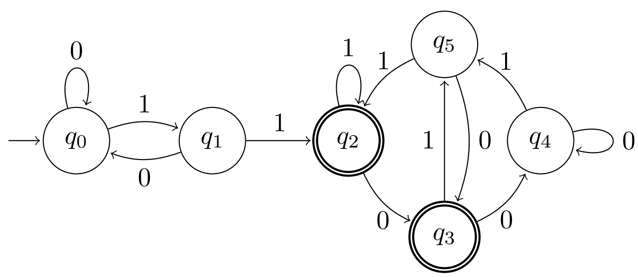
Sample Solutions

1. Let $\Sigma = \{0, 1\}$. Let $L = \{x : x \in \Sigma^*, 11$ is a substring of $x, |x| \geq 2$ and the second to last symbol of x is $1\}$.

   (a) Give a DFSA that accepts L.
   It is possible to give a correct DFSA with 6 states. One mark will be deducted for each extra state that your DFSA uses.

   (b) Provide an appropriate state invariant for your DFSA in part (a).
   Do not use regular expressions in your state invariant.

   Solution:



   (a)

   (b) Here are the state invariants

$$\delta^*(q_0, x) = \begin{cases} q_0 \text{ iff } x \text{ is empty or (11 is not a substring of } x \text{ and } x \text{ ends with 0);} \\ q_1 \text{ iff 11 is not a substring of } x \text{ and } x \text{ ends with 1;} \\ q_2 \text{ iff 11 is a substring of } x \text{ and } x \text{ ends with 11;} \\ q_3 \text{ iff 11 is a substring of } x \text{ and } x \text{ ends with 10;} \\ q_4 \text{ iff 11 is a substring of } x \text{ and } x \text{ ends with 00;} \\ q_5 \text{ iff 11 is a substring of } x \text{ and } x \text{ ends with 01.} \end{cases}$$

   The initial state is q_0. The only accepting states are q_2 and q_3.

2. Prove that $L_1 = \{w \in \{a, b\}^* : w$ has the same number of a's and b's$\}$ is not regular.

   Solution:
   Note that $L_1 \cap a^*b^* = \{a^ib^i | i \in \mathbb{N}\}$. Now assume towards contradiction that L_1 is regular. Since ab is regular, and regular languages are closed under intersection, then the intersection is also regular. But we know that $\{a^ib^i | i \in N\}$ is not regular. Contradiction. L_1 is therefore not regular.

   Note: L_1 could also be proved non-regular using the pumping lemma.

3. Prove that $L_2 = \{w \in \{a,b\}^* | w = a^i b^j, i \neq j\}$ is not regular.

   Solution:

   Note that $\overline{L_2} \cap a^*b^* = \{a^i b^i | i \geq 0\}$. Assume towards contradiction that L_2 is regular. Then $\overline{L_2}$ is also regular, because regular languages are closed under complement. But then, since regular languages are closed under intersection and a^*b^* is regular, we get that $\{a^i b^i | i \in N\}$ is also regular. Contradiction. L_2 is therefore not regular.

4. The set of non-palindromes, i.e. the language $\{a,b\}^* - PAL$, where $PAL = \{w \in \{a,b\}^* | w = w^R\}$. (You can use the fact that PAL is known to be non-regular.)

   Solution:

   Suppose to the contrary that $\{a,b\}^* - PAL$ is regular. Then $\overline{\{a,b\}^* - PAL}$ = PAL is regular, due to closure under complement. But this contradicts the non-regularity of P AL.