CSC236H Tutorial 5

1. Let $T(n)$ denote the worst-case running time of the algorithm below on inputs of size n.

   ```
   # A is a non-empty list of integers, i is a natural number.
   def recSS(A, i):
       if i < len(A) - 1:
           small = i
           for j in range(i + 1, len(A)):
               if A[j] < A[small]:
                   small = j
           temp = A[i]
           A[i] = A[small]
           A[small] = temp
           recSS(A, i + 1)
   ```

   Note that the above has an implicit base case $i =$ `len(A) - 1`, for which it does nothing.

   (a) Write a recurrence relation satisfied by T. Make sure to define n precisely (as a function of the algorithm's parameters) and justify that your recurrence is correct (by referring to the algorithm to describe how you obtained each term in your answer).

   (b) Give an asymptotic upper-bound for the worst-case running time of the algorithm.

2. When an annual interest rate of i is compounded m times per year, the interest rate paid per period is $\frac{i}{m}$. For instance, if $3\% = 0.03$ annual interest is compounded quarterly, then the interest rate paid per quarter is $\frac{0.03}{4} = 0.0075$.
   For each integer $k \geq 0$, let $Q(k)$ denote the amount on deposit at the end of the $k-$th period, assuming no additional deposits or withdrawals.

   (a) Let d denote the amount of an initial deposit into a bank account earning interest at a rate of i which is compounded m times per year.
   Find a recurrence relation relating $Q(k)$ to $Q(k-1)$.

   (b) Find a closed-form formula for $Q(k)$.
   Note that as discussed in class, you are required to do repeated substitutions, guess a pattern, use the pattern to find a closed-form expression for Q, and finally proved the correctness of the closed-form expression using induction.

3. Give an asymptotic upper bound for each of the following functions.

(a)

$$T_1(n) = \begin{cases} a, & n = 1 \text{ or } n = 2 \\ 9T_1(\frac{n}{3}) + \frac{n^3}{\log n}, & n \geq 3 \end{cases}$$

(b)

$$T_2(n) = \begin{cases} a, & n = 1 \\ 2T_2(n/2) + 4n, & n \geq 2 \end{cases}$$

a

(c)

$$T_3(n) = \begin{cases} a, & n = 1 \\ 2T_3(n/7) + \log n + \sqrt{n}, & n \geq 2 \end{cases}$$

$f(n) = \frac{n^3}{\log n}$; $b = 3$; $a = 9$