CSC236 Week 1 - Introductory Lecture

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- predicate: statement about a set of variables
  - Ex] O(n): n is a natural number; $O : \mathbb{N} \to$ Boolean
  - Ex] D(a,b): a divides b; $D : \mathbb{N} \times \mathbb{N} \to$ Boolean
  - predicates may have infinitely many variables
  - in this course, we will mostly be focused on unary predicates
- Simple Induction:
  - base case - P(x); predicate is true for some natural number x
    - often, x is 0, however some problems have a base case other than 0
  - induction - $\forall k \in \mathbb{N},\ P(k) \to P(k+1)$
  - If P is true for the first element in an ordered set, and we know that P being true for any arbitrary element means that it is true for the next element, then we can conclude that P is true for all elements in the ordered set.
  - *Principle of Simple Induction (PSI):* $[P(0) \wedge [\forall k \in \mathbb{N},\ P(k) \to P(k+1)]] \to \forall n \in \mathbb{N},\ P(n)$
- Writing proofs with simple induction:
  1. Define predicate.
  2. Prove base case.
  3. Set up and prove induction step.
  4. Reference PSI, state conclusions.
- *Example of simple induction problem:*

  Let $\{a_0, a_1, ...\}$ be a sequence of natural numbers such that $a_0 = 1$, and $\forall n \geq 1,\ a_n = 2a_{n-1} + 1$. Prove that $\forall n \in \mathbb{N},\ a_n = 2^{n+1} - 1$.

  Define predicate: Let $P(n) : a_n = 2^{n+1} - 1$. Must show that $\forall n \in \mathbb{N},\ P(n)$.
  Base case: $n = 0, P(0) : 2^{0+1} - 1 = 2^1 - 1 = 2 - 1 = 1 = a_0$, therefore P(0) is true.
  Induction step: Assume P(k) is true for some arbitrary $k \in \mathbb{N}$, so we have $a_k = 2^{k+1} - 1$. From definition $a_n = 2a_{n-1} + 1$, we get $a_{k+1} = 2a_k + 1 = 2(2^{k+1} - 1) + 1 = 2^{k+2} - 2 + 1 = 2^{k+2} - 1$, and thus $P(k+1) : a_{k+1} = 2^{k+2} - 1$ is true. Thus assuming P(k) gives us P(k+1), so $P(k) \to P(k+1)$.
  Reference PSI, state conclusions: We have $P(0) \wedge [\forall k \in \mathbb{N},\ P(k) \to P(k+1)]$, so via PSI we can conclude that $\forall n \in \mathbb{N},\ P(n)$.