1. Geometric Jacobian

The Jacobian matrix $\boldsymbol{J}$ relate the linear velocities $\dot{\mathbf{p}}$ and the angular velocities $\boldsymbol{\omega}$ with the joint velocities in the following way

$$\boldsymbol{v} = \begin{bmatrix} \dot{\mathbf{p}} \\ \boldsymbol{\omega} \end{bmatrix} = \boldsymbol{J}\dot{\boldsymbol{q}}$$

We have two Jacobian's matrix, *the Analytic Jacobian*, and the *Geometric Jacobian*. When we have a minimal orientation representation, we use the analytic Jacobian. When we use a non-minimal orientation representation, we can use the geometric Jacobian, which is defined as:

$$\boldsymbol{J_G} = \begin{bmatrix} \boldsymbol{J}_{P_i} \\ \boldsymbol{J}_{O_i} \end{bmatrix} = \begin{cases} \begin{bmatrix} z_{i-1} \\ \mathbf{0} \end{bmatrix}, & for\ prismatic\ joint \\ \begin{bmatrix} z_{i-1} \times (\boldsymbol{p}_e - \boldsymbol{p}_{i-1}) \\ z_{i-1} \end{bmatrix}, & for\ revolute\ joint \end{cases}$$

$$T_0^i = \begin{array}{c} \begin{array}{cccc} \phantom{r_{11}} & \phantom{r_{12}} & \mathrm{Z}_{i-1} & \mathrm{O}_i \end{array} \\ \begin{bmatrix} r_{11} & r_{12} & r_{13} & dx \\ r_{21} & r_{22} & r_{23} & dy \\ r_{31} & r_{32} & r_{33} & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

So, in our case we only use the term $\begin{bmatrix} z_{i-1} \times (\boldsymbol{p}_e - \boldsymbol{p}_{i-1}) \\ z_{i-1} \end{bmatrix}$ because the robot is composed only revolute joints. Now, we build the Geometric Jacobian as follows

$$\boldsymbol{J_G}(\boldsymbol{q}) = \begin{bmatrix} z_0 \times (\boldsymbol{p}_e - \boldsymbol{p}_0) & z_1 \times (\boldsymbol{p}_e - \boldsymbol{p}_1) & z_2 \times (\boldsymbol{p}_e - \boldsymbol{p}_2) & z_3 \times (\boldsymbol{p}_e - \boldsymbol{p}_3) & z_4 \times (\boldsymbol{p}_e - \boldsymbol{p}_4) & z_5 \times (\boldsymbol{p}_e - \boldsymbol{p}_5) \\ z_0 & z_1 & z_2 & z_3 & z_4 & z_5 \end{bmatrix}$$