PENGGAMBARAN SISTEM KENDALI

- PENDAHULUAN
- FUNGSI ALIH
- DIAGRAM BLOK
- REDUKSI DIAGRAM BLOK
- SIGNAL FLOW GRAPH
- FORMULA MASON

PENDAHULUAN

Langkah-langkah dalam analisis dan desain sistem kendali:

- Penurunan model matematis sistem fisis (Persamaan Differensial)
- Peroleh model linear dari komponen-komponen sistem.
- Gunakan Transformasi Laplace untuk komponen-komponen sistem tsb.
- Turunkan hubungan antara output dengan input masing-masing komponen (Fungsi Alih).
- Diagram blok sistem diperoleh melalui interkoneksi komponen-komponen tsb.
- Gunakan reduksi diagram blok untuk memperoleh fungsi alih sistem.
- Gunakan Signal Flow Graph untuk menggambarkan sistem yang kompleks dan untuk memperoleh fungsi alih sistem melalui Formula Mason.
- Gunakan beberapa metoda analisis dan desain untuk mendapatkan rancangan yang diinginkan.

FUNGSI ALIH

- Digunakan untuk memudahkan melihat karakteristik suatu sistem.
- Karakterisitik suatu sistem tak dipengaruhi oleh jenis input.
- Hanya berlaku untuk sistem linear, invariant waktu.
- Definisi: Perbandingan fungsi Laplace output dengan fungsi Laplace input dengan semua kondisi mula dianggap nol.

Persamaan Differensial orde-n:

$$a_0 \overset{(n)}{y} + a_1 \overset{(n-1)}{y} + \cdots + a_{n-1}\dot{y} + a_n y = b_0 \overset{(m)}{x} + b_1 \overset{(m-1)}{x} + \cdots + b_{m-1}\dot{x} + b_m x \qquad (n \geq m)$$

Bentuk Laplace nya (untuk semua kondisi mula =0):

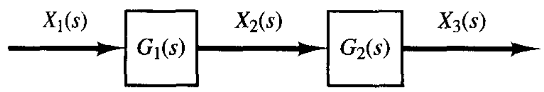
$$[a_0 s^n + a_1 s^{n-1} + ... + a_{n-1}s + a_n]Y(s) = [b_0 s^m + b_1 s^{m-1} + ... + b_{m-1}s \ + b_m]X(s)$$

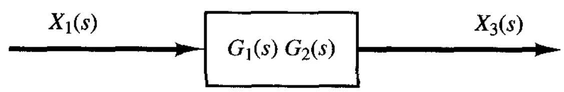
Fungsi Alih (untuk input = X(s), output = Y(s)):

$$\text{Transfer function} = G(s) = \left.\frac{\mathcal{L}[\text{output}]}{\mathcal{L}[\text{input}]}\right|_{\text{zero initial conditions}} = \frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$$

• Fungsi Alih Komponen-komponen Terhubung Secara Serial

1. Tanpa faktor Pembebanan:

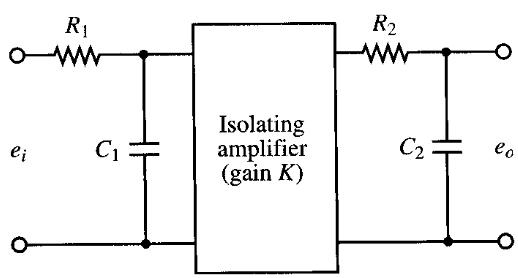




$$G_1(s) = \frac{X_2(s)}{X_1(s)} \qquad \text{and} \qquad G_2(s) = \frac{X_3(s)}{X_2(s)}$$

$$G(s) = \frac{X_3(s)}{X_1(s)} = \frac{X_2(s)X_3(s)}{X_1(s)X_2(s)} = G_1(s)G_2(s)$$

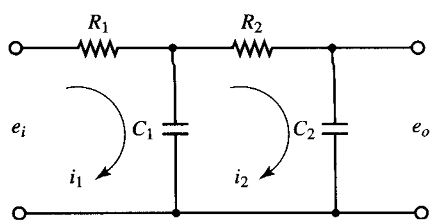
Contoh:



$$\frac{E_o(s)}{E_i(s)} = \left(\frac{1}{R_1C_1s + 1}\right)(K)\left(\frac{1}{R_2C_2s + 1}\right)$$
$$= \frac{K}{(R_1C_1s + 1)(R_2C_2s + 1)}$$

2. Ada Faktor Pembebanan

Banyak sistem kendali memiliki komponen yang membebani satu sama lain.
Misal : Tingkat kedua rangkaian (R_2C_2) membebani tingkat pertama (R_1C_1).



Persamaan Rangkaian:

$$\frac{1}{C_1}\int (i_1 - i_2)\,dt + R_1 i_1 = e_i$$

$$\frac{1}{C_1}\int (i_2 - i_1)\,dt + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt = 0$$

$$\frac{1}{C_2}\int i_2\,dt = e_o$$

Dalam Bentul Laplace

$$\frac{1}{C_1 s}[I_1(s) - I_2(s)] + R_1 I_1(s) = E_i(s)$$

$$\frac{1}{C_1 s}[I_2(s) - I_1(s)] + R_2 I_2(s) + \frac{1}{C_2 s} I_2(s) = 0$$

$$\frac{1}{C_2 s} I_2(s) = E_o(s)$$

Fungsi Alih:

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{(R_1C_1s + 1)(R_2C_2s + 1) + R_1C_2s}$$

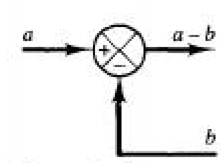
$$= \frac{1}{R_1C_1R_2C_2s^2 + (R_1C_1 + R_2C_2 + R_1C_2)s + 1}$$

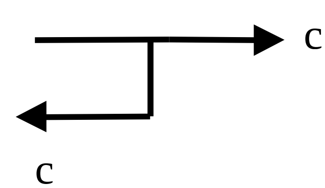
DIAGRAM BLOK

- Tidak praktis menggambarkan karakteristik setiap komponen dalam suatu sistem kendali.
- Karakteristik sekelompok komponen yang membentuk suatu fungsi tertentu (sub-sistem) diwakili oleh satu blok fungsi alih.
- Diagram blok: Interkoneksi antar beberapa blok fungsional sehingga membentuk suatu sistem kendali (loop terbuka / tertutup).
- Diagram blok dapat menggambarkan sifat-sifat dinamis suatu sistem dan aliran sinyal, tetapi tak menggambarkan konstruksi fisik sistem tsb.
- Suatu sistem fisis yang berbeda dapat saja memiliki diagram blok yang sama (misal: analogi sistem mekanis ←-> elektrik).
- Komponen-komponen dasar:

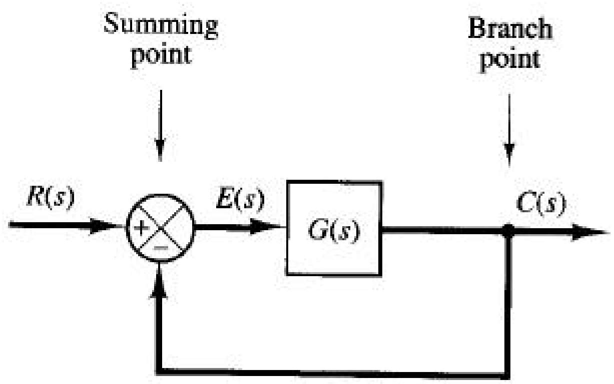
  - Blok fungsional

  - Titik penjumlah (summing point)
  - Percabangan

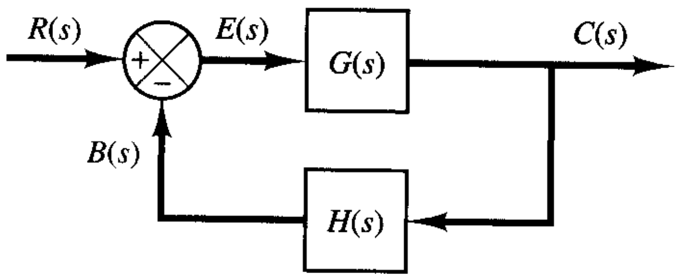




Contoh:



FUNGSI ALIH SISTEM LOOP TERBUKA, FUNGSI ALIH LINTASAN MAJU DAN FUNGSI ALIH SISTEM LOOP TERTUTUP



Fungsi Alih Loop terbuka:

$$\frac{B(s)}{E(s)} = G(s)H(s)$$

Fungsi Alih Lintasan Maju:

$$\frac{C(s)}{E(s)} = G(s)$$

Fungsi Alih Loop tertutup:

$$C(s) = G(s)E(s)$$
$$E(s) = R(s) - B(s)$$
$$= R(s) - H(s)C(s)$$

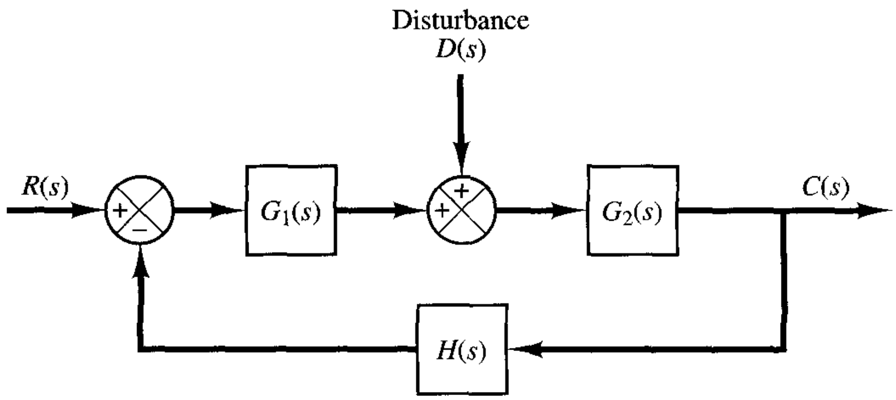
Atau:

$$C(s) = G(s)[R(s) - H(s)C(s)],$$

Sehingga:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

MODEL SISTEM LOOP TERTUTUP DENGAN GANGGUAN



- Anggap sistem mula-mula tanpa errror, sehingga respons sistem terhadap gangguan saja:

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$

- Bila gangguan dianggap tak ada, maka respons sistem terhadap input referensi:

$$\frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$

- Respons total terhadap keduanya:

$$\begin{aligned} C(s) &= C_R(s) + C_D(s) \\ &= \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}[G_1(s)R(s) + D(s)] \end{aligned}$$

Bila:

$$|G_1(s)H(s)| \gg 1 \text{ dan } |G_1(s)G_2(s)H(s)| \gg 1$$

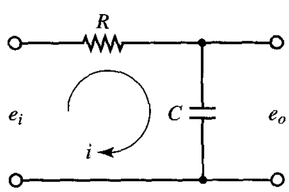
Maka: $\frac{C_D(s)}{D(s)} \approx 0$, sehingga pengaruh gangguan dapat ditekan (baca: keuntungan sistem loop tertutup).

MENGGAMBAR DIAGRAM BLOK

Prosedur:

1. Tulis persamaan dinamis setiap komponen sistem.
2. Nyatakan dalam bentuk Laplace nya dengan asumsi kondisi mula = 0.
3. Gambarkan masing-masing komponen dalam bentuk blok-blok fungsional.
4. Gabungkan blok-blok tsb sehingga membentuk diagram blok lengkap sistem (loop tertutup).

Contoh:



$$i = \frac{e_i - e_o}{R}$$

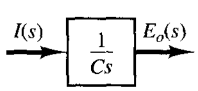
$$e_o = \frac{\int i\,dt}{C}$$

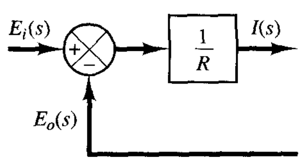
Bentuk laplace nya:

$$I(s) = \frac{E_i(s) - E_o(s)}{R}$$

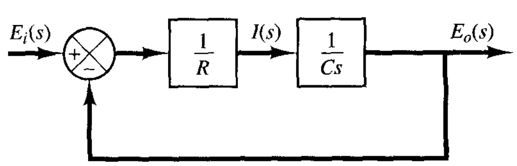
$$E_o(s) = \frac{I(s)}{Cs}$$

Blok-blok pembentuk sistem:





Penggabungan:



REDUKSI DIAGRAM BLOK

- Blok-blok hanya dapat dihubungkan secara seri bila tak ada pengaruh pembebanan.
- Blok-blok yang terhubung seri tanpa faktor pembebanan dapat diganti dengan blok tunggal dengan fungsi alihnya adalah perkalian masing-masing fungsi alih blok-blok tsb.
- Diagram blok kompleks dapat disederhanakan melalui reduksi bertahap dengan aturan-aturan tertentu.
- Perkalian fungsi alih beberapa blok dalam arah lintasan maju harus tetap.
- Perkalian fungsi alih beberapa blok dalam loop harus tetap.

Tabel: Aturan-Aturan Penyederhanaan Diagram Blok

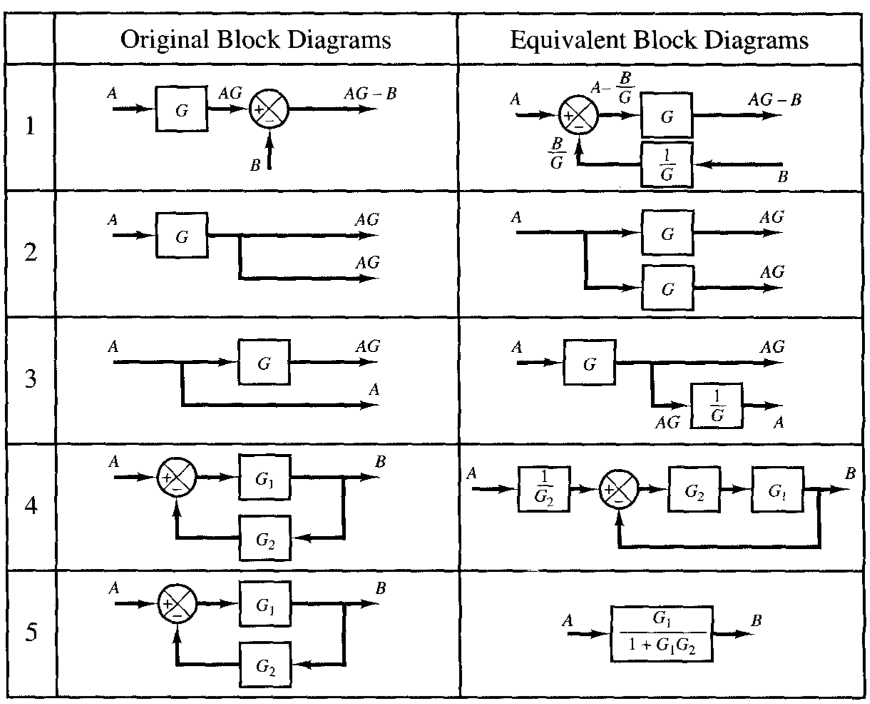
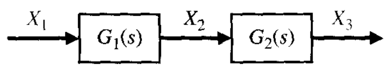
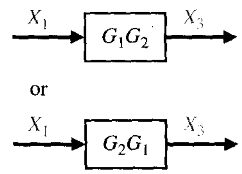
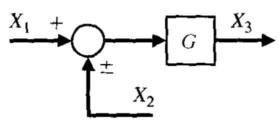
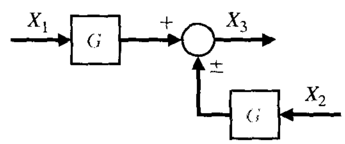
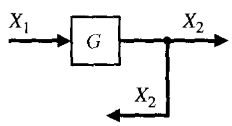
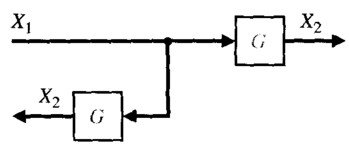
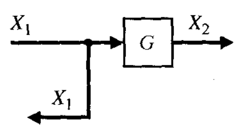
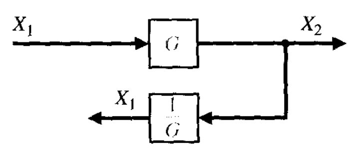
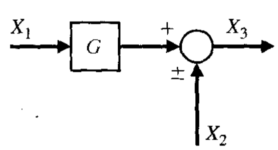
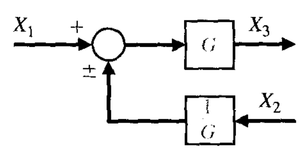
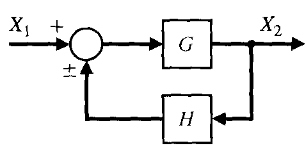
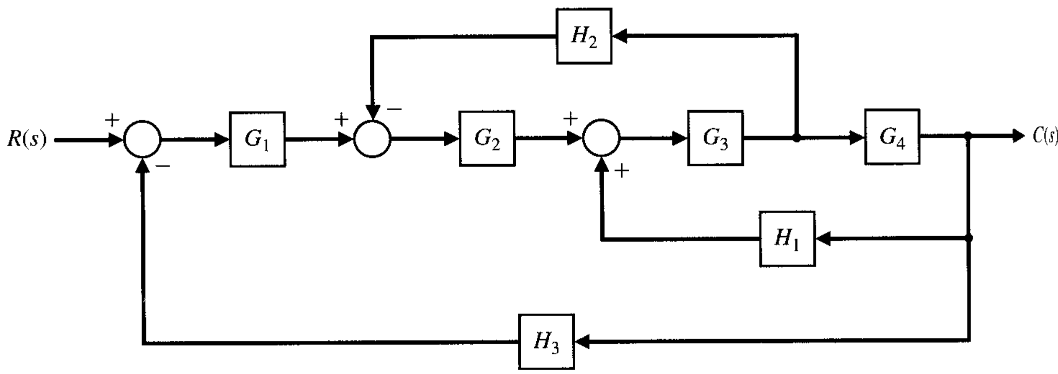
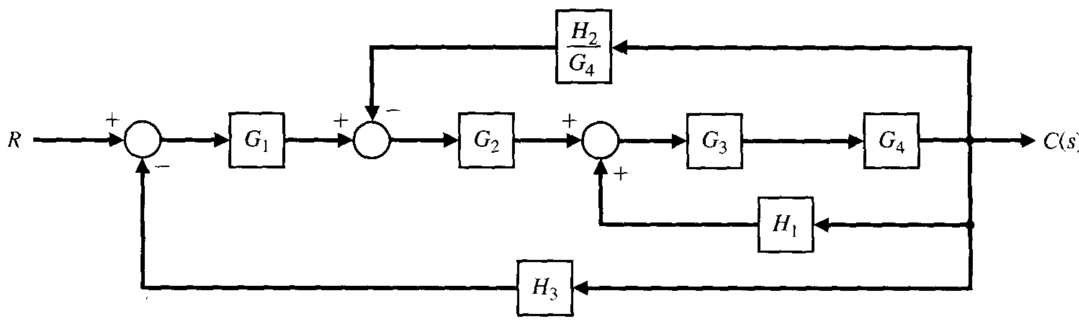


TABLE Block Diagram Transformations

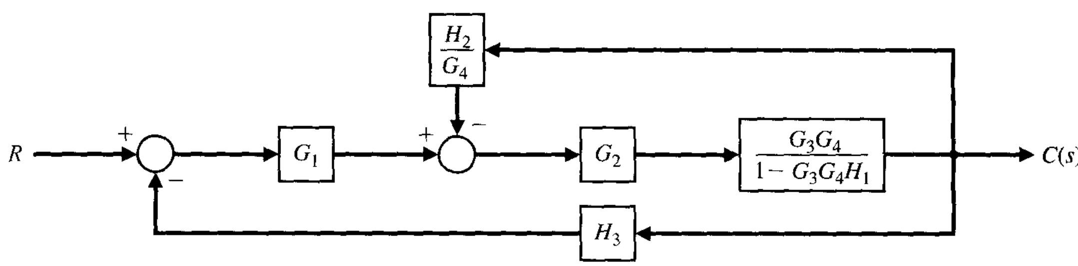
Transformation	Original Diagram	Equivalent Diagram
1. Combining blocks in cascade		
2. Moving a summing point behind a block		
3. Moving a pickoff point ahead of a block		
4. Moving a pickoff point behind a block		
5. Moving a summing point ahead of a block		
6. Eliminating a feedback loop		

Contoh:

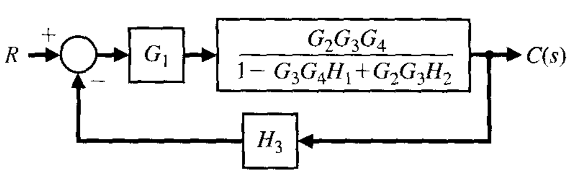

H_2
$R(s)$
G_1
G_2
G_3
G_4
$C(s)$
H_1
H_3

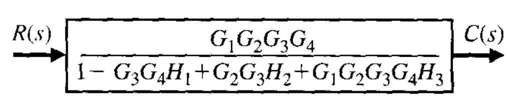

$\frac{H_2}{G_4}$
R
G_1
G_2
G_3
G_4
$C(s)$
H_1
H_3

(a)


$\frac{H_2}{G_4}$
R
G_1
G_2
$\frac{G_3G_4}{1-G_3G_4H_1}$
$C(s)$
H_3

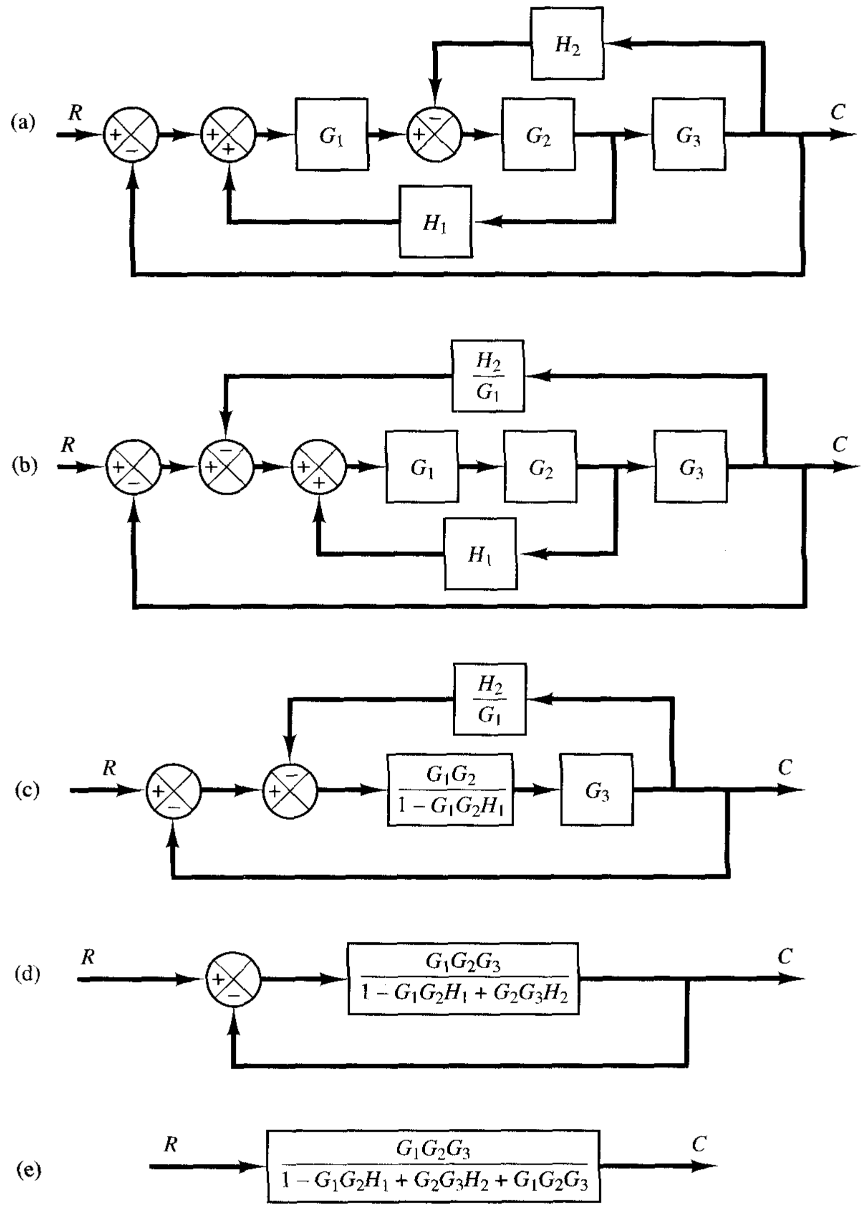
(b)


R
G_1
$\frac{G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2}$
$C(s)$
H_3

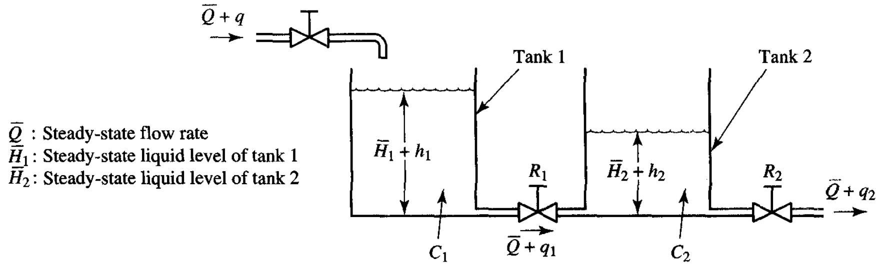

$R(s)$
$\frac{G_1G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4H_3}$
$C(s)$

(c) (d)

Contoh:


(a)
R
H_2
G_1
G_2
G_3
H_1
C
(b)
R
$\frac{H_2}{G_1}$
G_1
G_2
G_3
H_1
C
(c)
R
$\frac{H_2}{G_1}$
$\frac{G_1G_2}{1 - G_1G_2H_1}$
G_3
C
(d)
R
$\frac{G_1G_2G_3}{1 - G_1G_2H_1 + G_2G_3H_2}$
C
(e)
R
$\frac{G_1G_2G_3}{1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3}$
C

Contoh:

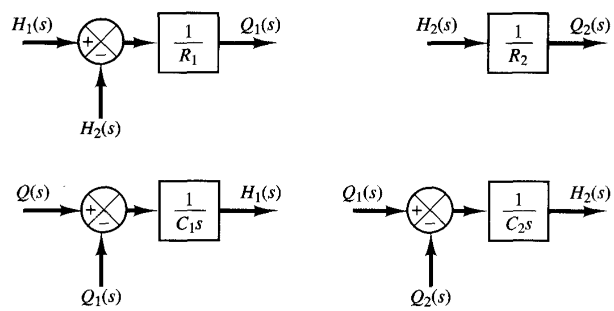


$$\frac{h_1 - h_2}{R_1} = q_1$$

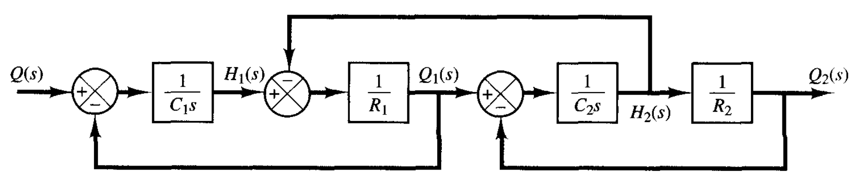
$$C_1 \frac{dh_1}{dt} = q - q_1$$

$$\frac{h_2}{R_2} = q_2$$

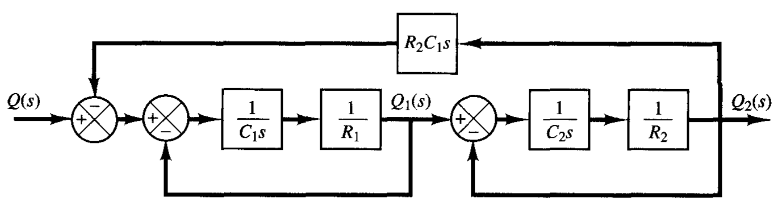
$$C_2 \frac{dh_2}{dt} = q_1 - q_2$$



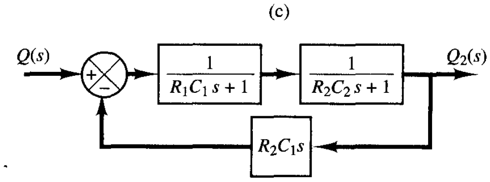
(a)



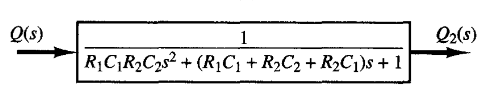
(b)



(c)



(d)



(e)

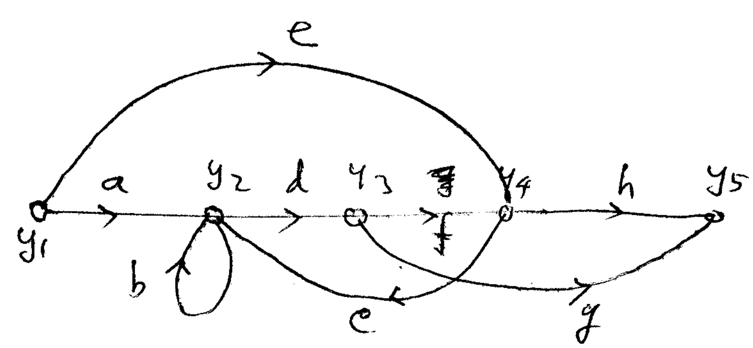
SIGNAL FLOW GRAPH

- Diagram blok menggambarkan sistem kendali secara grafik.
- Untuk sistem kompleks, SFG lebih praktis digunakan.
- SFG menggambarkan hubungan variabel-variabel sistem secara sederhana.
- Secara matematis: SFG adalah suatu diagram yang menggambarkan sekumpulan persamaan aljabar linear sbb:

$$y_i = \sum_{j=1}^{n} a_{ij} y_j ; \quad i = 1,2,\ldots n$$

melalui percabangan dan simpul(node).

Contoh:



Persamaan aljabar linear:

$y_2 = ay_1 + by_2 + cy_4$

$y_3 = dy_2$

$y_4 = ey_1 + fy_3$

$y_5 = gy_3 + hy_4$

BEBERAPA DEFINISI

- Source (input node): simpul yang hanya memiliki percabangan keluar saja (y_i)

- Sink (output node) : simpul yang hanya memiliki percabangan masuk saja (y_5)

- Path (lintasan) : sekelompok cabang yang berhubungan dan memiliki arah yang sama: eh; adfh dan b.

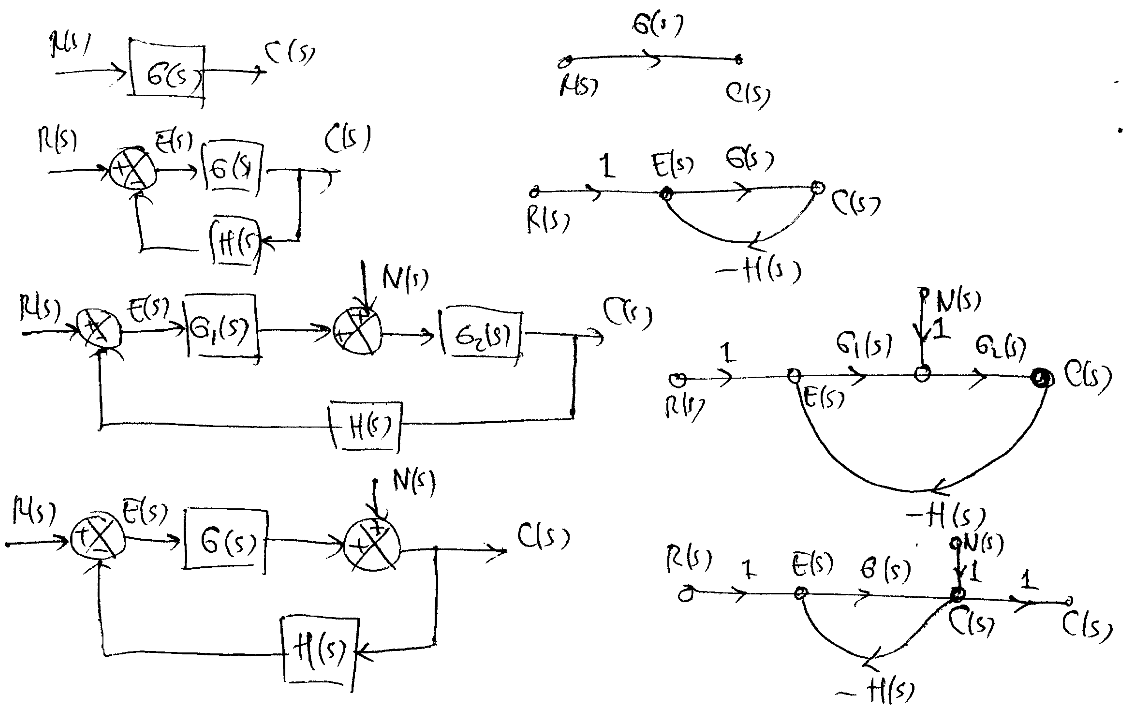
- Lintasan maju : lintasan yang dimulai dari source dan berakhir di sink, tetapi tak ada node yang dilalui lebih dari satu kali: eh, ecdg, adg dan adfh

- Penguatan Lintasan: perkalian penguatan (koefisien) pada cabang-cabang sepanjang lintasan.

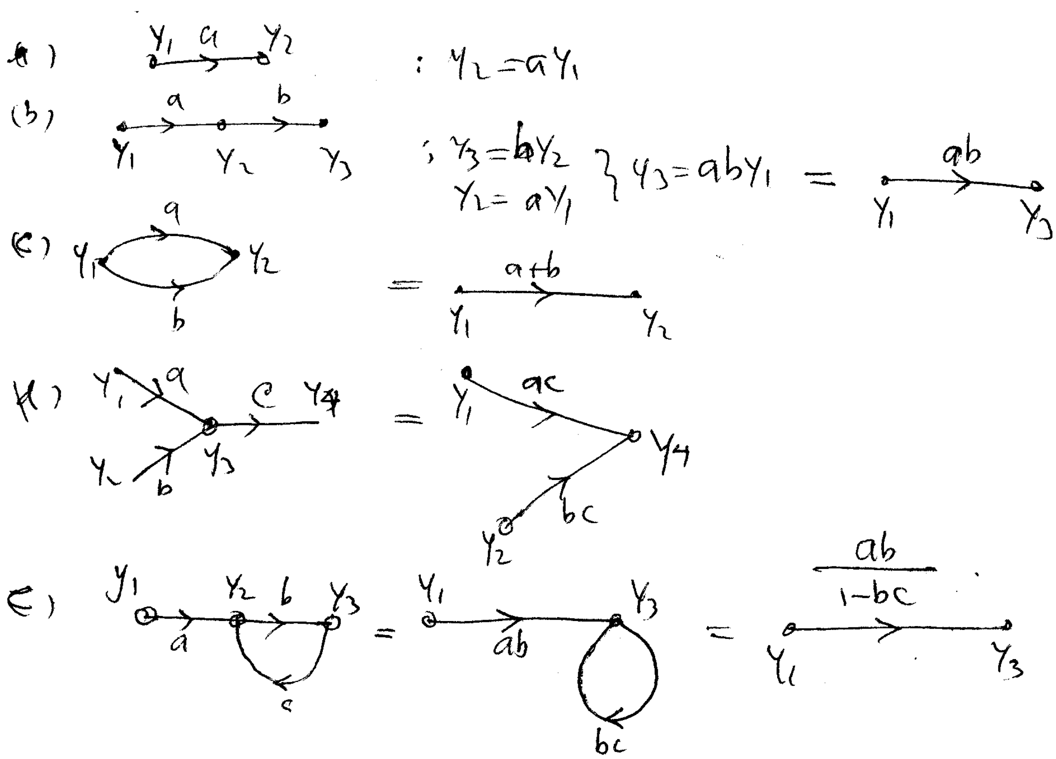
- Loop Umpanbalik : lintasan yang berawal dan berakhir pada node yang sama, tetapi node tsb tak boleh dilalui lebih dari satu kali: b, dfc.

- Penguatan Loop : perkalian penguatan (koefisien) pada cabang-cabang yang membentuk loop umpanbalik.

HUBUNGAN ANTARA SFG DAN DIAGRAM BLOK



BEBERAPA PENYEDERHANAAN SFG



FORMULA MASON

- SFG mengandung informasi yang sama dengan diagram blok.
- SFG memudahkan penentuan fungsi alih melalui formula penguatan Mason, tanpa perlu melakukan reduksi diagram blok secara bertahap.
- Formula Penguatan Mason:

$$P = \frac{1}{\Delta} \sum_{k=1}^{m} P_k \, \Delta_k$$

Dengan:

P_k : penguatan lintasan maju ke k

: determinan grafik

= 1- $\sum L_1$ + $\sum L_2$ - $\sum L_3$ + ….+$(-1)^m \sum L_m$

$\sum L_1$: Jumlah penguatan setiap loop (tertutup)

$\sum L_2$: Jumlah perkalian dari semua kombinasi penguatan 2 loop yang tak bersentuhan satu sama lain (tak memiliki node bersama).

$\sum L_3$: Jumlah perkalian dari semua kombinasi penguatan 3 loop yang tak bersentuhan satu sama lain.

k : Nilai bila bagian grafik tidak menyentuh lintasan maju ke k, atau nilai sisa jika lintasan yang menghasilkan P_k dihilangkan.

Contoh

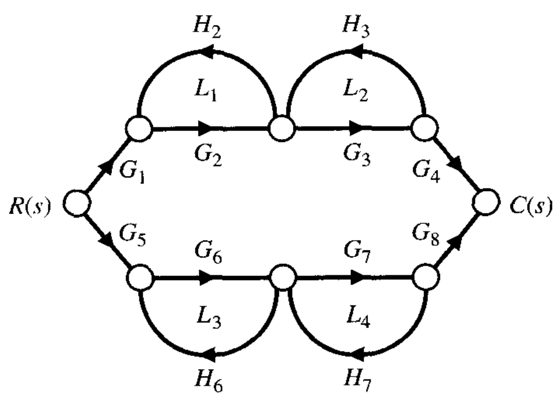
Transfer function of interacting system

A two-path signal-flow graph is shown . An example of a control system with multiple signal paths is a multilegged robot. The paths connecting the input $R(s)$ and output $C(s)$ are

$$\text{path 1: } P_1 = G_1G_2G_3G_4 \quad \text{and} \quad \text{path 2: } P_2 = G_5G_6G_7G_8.$$

There are four self-loops:

$$L_1 = G_2H_2, \quad L_2 = H_3G_3, \quad L_3 = G_6H_6, \quad L_4 = G_7H_7.$$



Loops L_1 and L_2 do not touch L_3 and L_4. Therefore the determinant is

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4).$$

The cofactor of the determinant along path 1 is evaluated by removing the loops that touch path 1 from Δ. Therefore we have

$$L_1 = L_2 = 0 \quad \text{and} \quad \Delta_1 = 1 - (L_3 + L_4).$$

Similarly, the cofactor for path 2 is

$$\Delta_2 = 1 - (L_1 + L_2).$$

Therefore the transfer function of the system is

$$\frac{C(s)}{R(s)} = T(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3G_4(1 - L_3 - L_4) + G_5G_6G_7G_8(1 - L_1 - L_2)}{1 - L_1 - L_2 - L_3 - L_4 + L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4}. \blacksquare$$

Contoh 2:

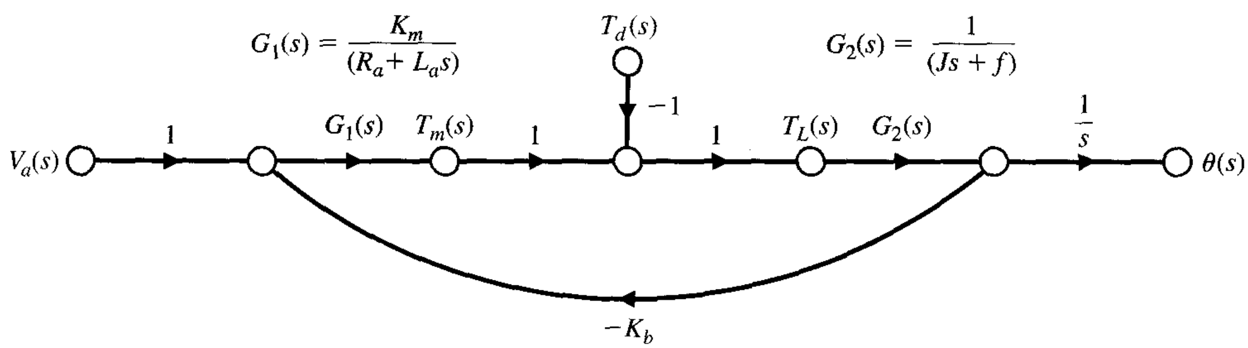
Armature-controlled motor

The block diagram of the armature-controlled dc motor is shown Using Mason's rule, let us obtain the transfer function for $\theta(s)/V_a(s)$ with $T_d(s) = 0$. The forward path is $P_1(s)$, which touches the one loop, $L_1(s)$, where

$$P_1(s) = \frac{1}{s} G_1(s) G_2(s) \quad \text{and} \quad L_1(s) = -K_b G_1(s) G_2(s).$$

Therefore the transfer function is

$$T(s) = \frac{P_1(s)}{1 - L_1(s)} = \frac{(1/s) G_1(s) G_2(s)}{1 + K_b G_1(s) G_2(s)} = \frac{K_m}{s[(R_a + L_a s)(Js + f) + K_a K_m]},$$



Contoh 3:

Transfer function of multiple-loop system

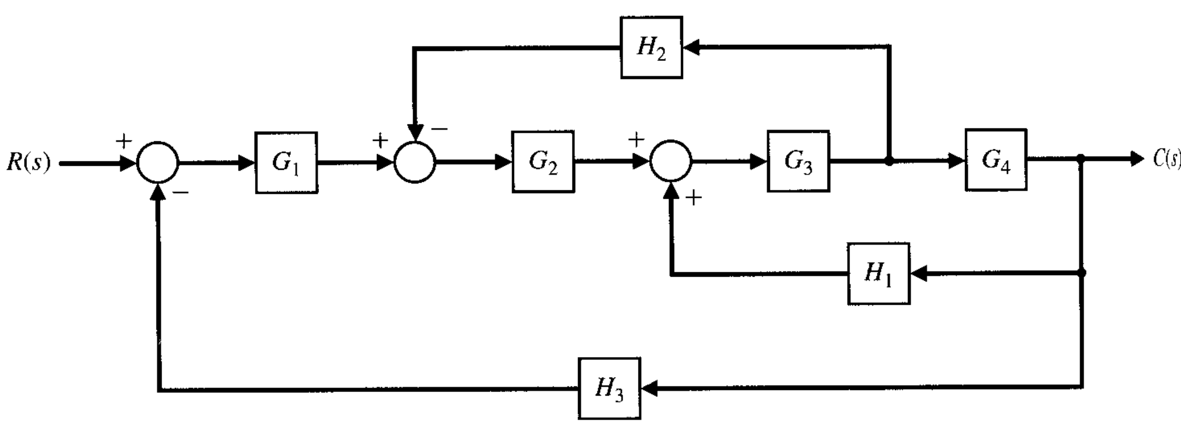
A multiple-loop feedback system is shown in block diagram form. There is no reason to redraw the diagram in signal-flow graph form, and so we shall proceed as usual by using the signal-flow gain formula, There is one forward path $P_1 = G_1G_2G_3G_4$. The feedback loops are

$$L_1 = -G_2G_3H_2, \qquad L_2 = G_3G_4H_1, \qquad L_3 = -G_1G_2G_3G_4H_3.$$

All the loops have common nodes and therefore are all touching. Furthermore, the path P_1 touches all the loops, so $\Delta_1 = 1$. Thus the closed-loop transfer function is

$$T(s) = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1}{1 - L_1 - L_2 - L_3}$$

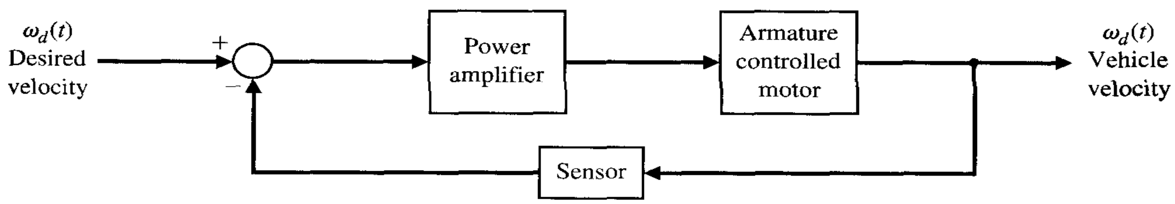
$$= \frac{G_1G_2G_3G_4}{1 + G_2G_3H_2 - G_3G_4H_1 + G_1G_2G_3G_4H_3}. \blacksquare$$



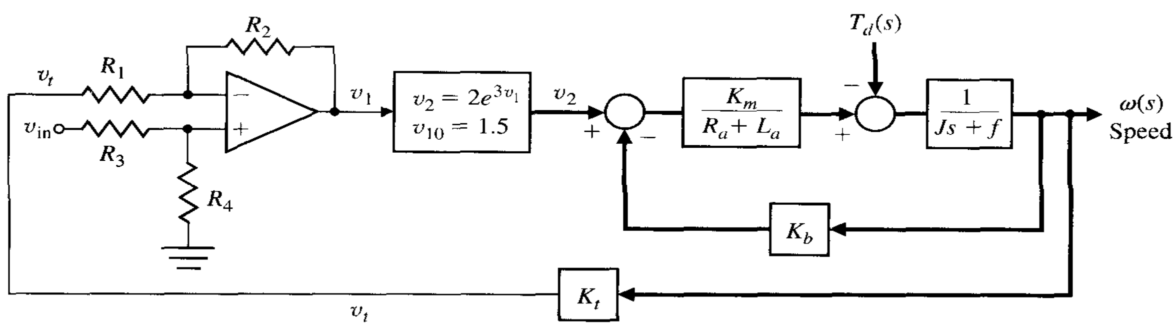
Contoh:

Electric traction motor control

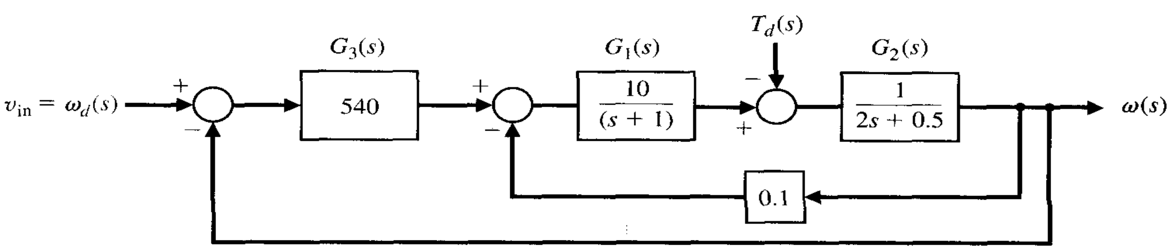
A majority of modern trains and local transit vehicles utilize electric traction motors. The electric motor drive for a railway vehicle is shown in block diagram form in Fig. 2.33(a) incorporating the necessary control of the velocity of the vehicle. The goal of the design is to obtain a system model and the closed-loop transfer function of the system, $\omega(s)/\omega_d(s)$, select appropriate resistors R_1, R_2, R_3, and R_4, and then predict the system response.



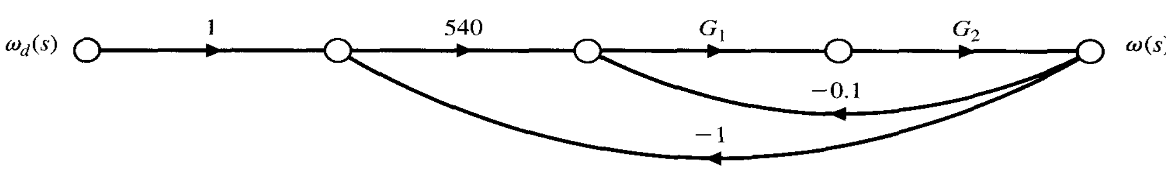
(a)



(b)



(c)



(d)

The first step is to describe the transfer function of each block. We propose the use of a tachometer to generate a voltage proportional to velocity and to connect that voltage, v_t, to one input of a difference amplifier, as shown in Fig. 2.33(b). The power amplifier is nonlinear and can be approximately represented by $v_2 = 2e^{3v_1} = 2\exp(3v_1) = g(v_1)$, an exponential function with a normal operating point, $v_{10} = 1.5V$. Using the technique in Section 2.3, we then obtain a linear model

$$v_2 = \left[\left. \frac{dg(v_1)}{dv_1} \right|_{v_{10}} \right] \Delta v_1 = 2[3\exp(3v_{10})]\,\Delta v_1 = 2[270]\,\Delta v_1 = 540\,\Delta v_1. \qquad (2.108)$$

Then, discarding the delta notation and writing the Laplace transform, we have

$$V_2(s) = 540V_1(s).$$

The transfer function of the differential amplifier is

$$v_1 = \frac{1 + R_2/R_1}{1 + R_3/R_4} v_{in} - \frac{R_2}{R_1} v_t. \qquad (2.109)$$

We wish to obtain an input control that sets $\omega_d(t) = v_{in}$ where the units of ω_d are rad/s and the units of v_{in} are volts. Then, when $v_{in} = 10\ V$, the steady-state speed is $\omega = 10$ rad/s. We note that $v_t = K_t\omega_d$ in steady state and we expect, in balance, the steady-state output, v_1, to be

$$v_1 = \frac{1 + R_2/R_1}{1 + R_3/R_4} v_{in} - \left(\frac{R_2}{R_1}\right) K_t(v_{in}). \qquad (2.110)$$

When the system is in balance, $v_1 = 0$, and when $K_t = 0.1$, we have

$$\frac{1 + R_2/R_1}{1 + R_3/R_4} = \left(\frac{R_2}{R_1}\right) K_t = 1.$$

This relation can be achieved when

$$R_2/R_1 = 10 \qquad \text{and} \qquad R_3/R_4 = 10.$$

The parameters of the motor and load are given in Table 2.9. The overall system is shown in Fig. 2.33(b). Using Mason's signal-flow rule with the signal-flow diagram of Fig. 2.33(d), we have

$$\begin{aligned} \frac{\omega(s)}{\omega_d(s)} &= \frac{540G_1(s)G_2(s)}{1 + 0.1G_1G_2 + 540G_1G_2} = \frac{540G_1G_2}{1 + 540.1G_1G_2} \\ &= \frac{5400}{(s+1)(2s+0.5) + 5401} = \frac{5400}{2s^2 + 2.5s + 5401.5} \qquad (2.111) \\ &= \frac{2700}{s^2 + 1.25s + 2700.75}. \end{aligned}$$

Since the characteristic equation is second order, we note that $\omega_n = 52$ and $\zeta = 0.012$, and we expect the response of the system to be highly oscillatory (underdamped). ■

Contoh:

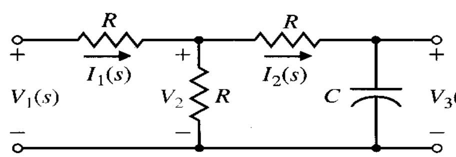
Design of a low-pass filter

Our goal is to design a first-order low-pass filter that passes signals at a frequency below 106.1 Hz and attenuates signals with a frequency above 106 Hz. In addition, the dc gain should be ½.

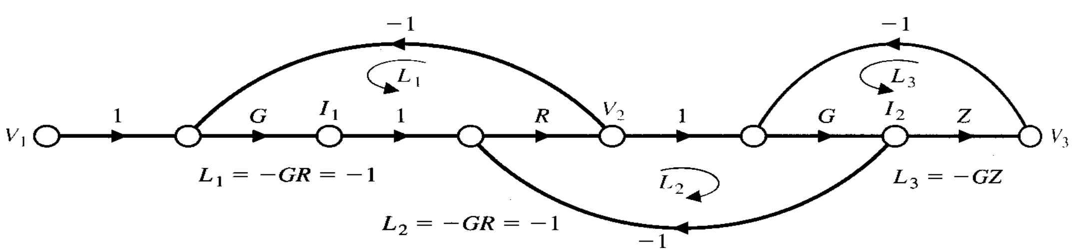
A ladder network with one energy storage element, as shown in Fig. 2.37(a), will a as a first-order low-pass network. Note that the dc gain will be equal to ½ (open-circuit t capacitor). The current and voltage equations are

$$\begin{aligned} I_1 &= (V_1 - V_2)G \\ I_2 &= (V_2 - V_3)G \\ V_2 &= (I_1 - I_2)R \\ V_3 &= I_2 Z, \end{aligned}$$

where $G = 1/R$, $Z(s) = 1/Cs$, and $I_1(s) = I_1$ (we omit the (s)). The signal-flow gra constructed for the four equations is shown in Fig. 2.37(b). The three loops are $L_1 = -($ $= -1$, $L_2 = -GR = -1$, and $L_3 = -GZ$. All loops touch the forward path. Loops



(a)



(b)

and L_3 are nontouching. Therefore the transfer function is

$$T(s) = \frac{V_3}{V_1} = \frac{P_1}{1 - (L_1 + L_2 + L_3) + L_1 L_3} = \frac{GZ}{3 + 2GZ}$$

$$= \frac{1}{3RCs + 2} = \frac{(1/3RC)}{(s + 2/3RC)}.$$

Note that the dc gain is ½, as expected. The pole is desired at $p = 2\pi(106.1) = 666.7 = 2000/3$. Therefore we require $RC = 0.001$. Select $R = 1\ \mathrm{k}\Omega$ and $C = 1\ \mu\mathrm{F}$. Hence we achieve the filter

$$T(s) = \frac{333.35}{(s + 666.7)}. \blacksquare$$

Contoh:

Transfer function of complex system

Finally, we shall consider a reasonably complex system that would be difficult to reduce by block diagram techniques. A system with several feedback loops and feedforward paths is shown in Fig. 2.30. The forward paths are

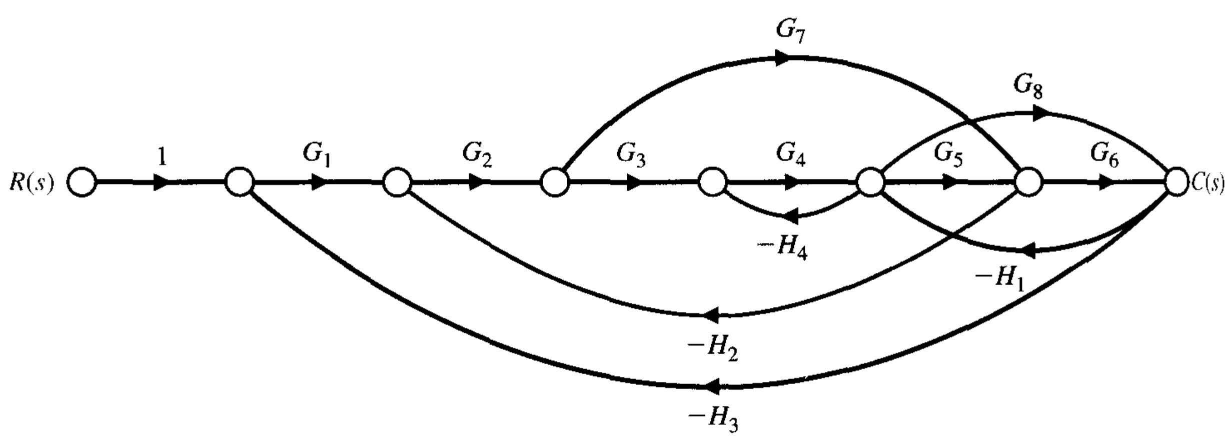
$$P_1 = G_1G_2G_3G_4G_5G_6, \qquad P_2 = G_1G_2G_7G_6, \qquad P_3 = G_1G_2G_3G_4G_8.$$

The feedback loops are

$$L_1 = -G_2G_3G_4G_5H_2, \qquad L_2 = -G_5G_6H_1, \qquad L_3 = -G_8H_1, \qquad L_4 = -G_7H_2G_2,$$
$$L_5 = -G_4H_4, \qquad L_6 = -G_1G_2G_3G_4G_5G_6H_3, \qquad L_7 = -G_1G_2G_7G_6H_3,$$
$$L_8 = -G_1G_2G_3G_4G_8H_3.$$

Loop L_5 does not touch loop L_4 or loop L_7; loop L_3 does not touch loop L_4; and all other loops touch. Therefore the determinant is

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8) + (L_5L_7 + L_5L_4 + L_3L_4). \tag{2.103}$$



The cofactors are

$$\Delta_1 = \Delta_3 = 1 \quad \text{and} \quad \Delta_2 = 1 - L_5 = 1 + G_4H_4.$$

Finally, the transfer function is

$$T(s) = \frac{C(s)}{R(s)} = \frac{P_1 + P_2\Delta_2 + P_3}{\Delta}. \blacksquare$$