

# Sequential Data Structures

## (v5)

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# Bibliography

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  - Chapter 2
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  - Chapter 4 (nested classes)
- The Java Tutorials (nested classes)

# Data Structures

- A **data structure** is a way to **organize the data** involved in a program
  - It must provide the **operations** the program needs
  - With a reasonable **cost** (constant, logarithmic, linear, linearithmic)
- For us, the study of data structures will have different legs
  - The **data structure itself** (theory, independent of the language)
  - Its **implementation in Java** (so, we'll need some java specific elements)
  - And its **real implementation in the Java Collections Framework (JCF)**
    - So, we'll study real and very well engineered code
- And we will need to understand **all these three things !!**

# Why the implementations in the JCF?

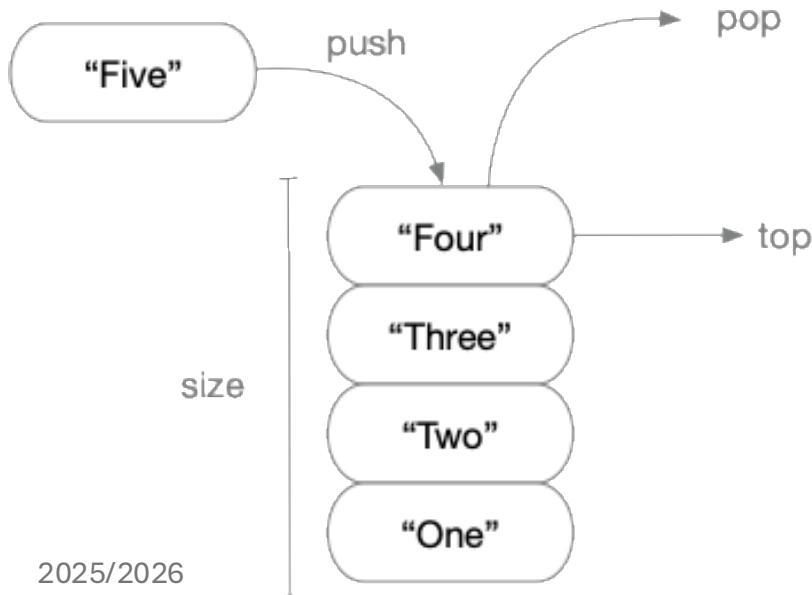
- They are the **most used** in Java, because it belongs to the Java standard library
  - So, in our daily Java programming, we'll probably use the JCF classes & interfaces
- So, knowing well the JCF will **improve** our Java **programming skills**
- If you want to consider other libraries for data structures in Java, you can read the TFG: [Comparativa entre diferentes librerías de Estructuras de Datos en Java](#), by José Ramón Ariza Pérez

# Sequential Data Structures

- A **sequential** data structure provides **insertion/deletion/access** to the elements by **position**
  - **First** (usually 0), second, ..., **last**
  - The difference among them are the **accessible positions** and the **available operations**
- **Stack:**
  - We can insert/delete/access only at **one end**
- **Queue:**
  - We can **insert** at **one end**, and **delete/access** at the **the other end**
- **Deque:**
  - We can **insert/delete/access** at **both ends**
- **List:**
  - We can **insert/delete/access** at **any position**

# Stack

```
public interface MyStack<E> {  
    void push(E element);  
    void pop();  
    E top();  
    int size();  
    default boolean isEmpty() {  
        return size() == 0;  
    }  
}
```



- I've named the type `MyStack<E>` to not confound it with the predefined `Stack<E>`
  - the class `Stack<E>` in JCF is **deprecated**
  - **one should use a `Deque<E>` instead**
- **Push:**
  - adds the given element at the top of the stack
- **Pop:**
  - removes the element at the top
- **Top:**
  - Returns the element at the top
- **Size:**
  - Returns the number of elements in the stack
- **NOTE:** We leave their implementations as exercises because they are a reduced set of operations on lists.

# Lists

- A **list** is a data structure in which **each element has a position in the list**, and insertions, deletions and accesses can be done at **any position**
- We will
  - define a very minimal interface on lists
  - present a simple implementation in Java using (extensible) arrays
  - describe the real implementations in the JCF, which use two different approaches (with different costs)
- And, in doing so, we also will
  - present iterators on lists
  - add nested (and inner) classes to our knowledge of Java

# Lists

```
public interface MyList<E>
    extends Iterable<E> {

    void add(int index, E element);
    E set(int index, E element);
    E get(int index);
    void remove(int index);

    int size();

    Iterator<E> iterator();

    default boolean isEmpty() {
        return size() == 0;
    }
}
```

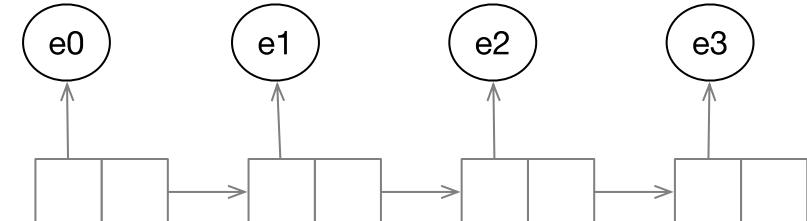
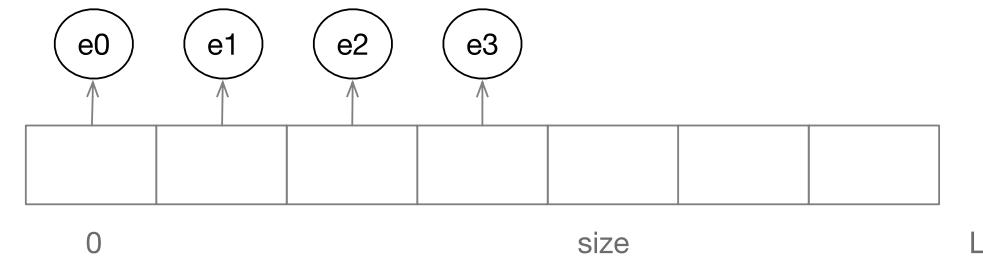
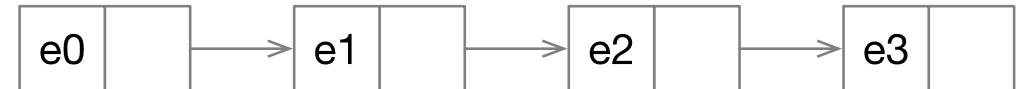
- We have defined `MyList<E>` as an `Iterable<E>`, to be able to use lists in `foreach` loops.
- Valid indexes:
  - `add`:  $[0, \text{size}()]$
  - `set/get/remove`:  $[0, \text{size}()]$
  - If the index is not valid, the method throws `IndexOutOfBoundsException`
- `set` returns the old value at this index
- `iterator` creates an iterator that will access the elements from index 0 to `size() - 1`

# List

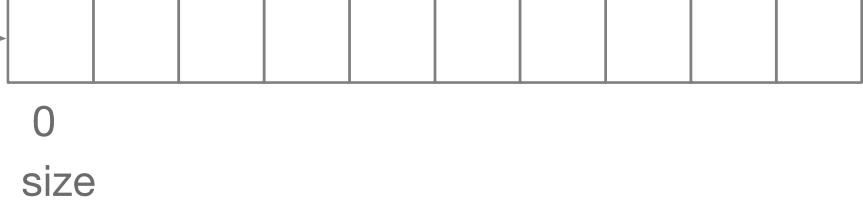
- Also, the lists we'll implement have an **unbounded size**
- There are **two different ways** to implement this type:
  - Using an (extensible) **array of elements**
  - Using a **linked structure of nodes**



0                    size                    L



# ArrayList

```
public class MyArrayList<E> implements myList<E> {  
  
    private static final int DEFAULT_SIZE = 10;  
  
    private Object[] elements;  
    private int size;  
  
    elements →   
    0  
    size  
  
    public MyArrayList() {  
        elements = new Object[DEFAULT_SIZE];  
        size = 0;  
    }  
}
```

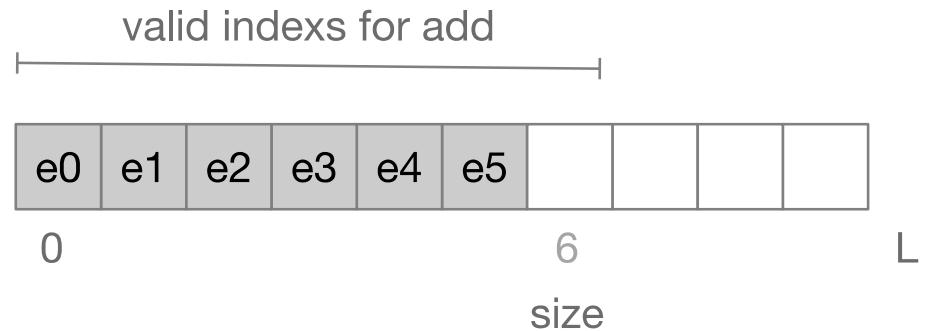
- Our first strategy for the implementation will be to use a resizable array to hold the elements of the list
- In this implementation the elements of the list are the elements of the prefix of the array  $[0, size)$
- It's an array of Object as we cannot create an array of E's.

# ArrayList

```
@Override  
public void add(int index, E e) {  
    checkIndexForAdd(index);  
    ensureCapacity();  
    moveSuffixOneRight(index);  
    elements[index] = e;  
    size += 1;  
}
```

```
private void checkIndexForAdd(int index) {  
    if (index < 0 || index > size)  
        throw new IndexOutOfBoundsException(  
            "adding at %d for a list of size %d".formatted(index, size));  
}
```

1. We check if the index is valid
2. We ensure that we have enough capacity in the array for the new element
3. We create a hole at index, moving the elements in the suffix [index, size) one position to the right
4. We add the element to the array at position index
5. We increment the size of the list



# ArrayList

```
private void ensureCapacity() {
    if (size < elements.length) return;
    Object[] newElements = new Object[elements.length * 2];
    for (int i = 0; i < size; i++) {
        newElements[i] = elements[i];
    }
    elements = newElements;
}

private void moveSuffixOneRight(int index) {
    // loop from right to left (the suffix moves one to the right)
    // index is the destination
    for (int i = size; i > index; i--) {
        elements[i] = elements[i - 1];
    }
}
elements[index] = e;
size += 1;
```

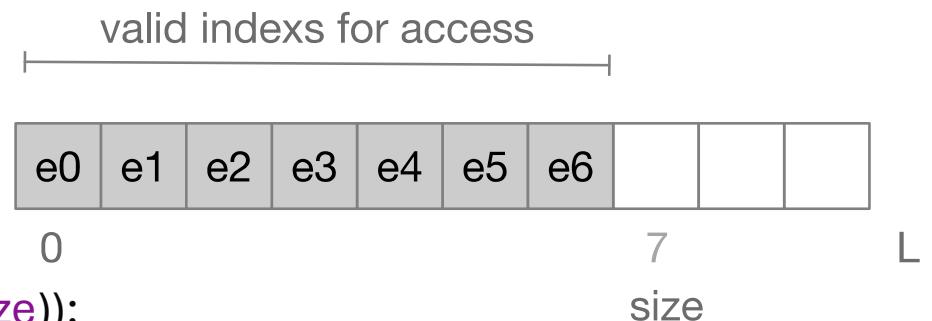


# ArrayList

```
@Override  
public E set(int index, E element) {  
    checkIndexForAccess(index);  
    @SuppressWarnings("unchecked")  
    E oldElement = (E) elements[index];  
    elements[index] = element;  
    return oldElement;  
}
```

```
private void checkIndexForAccess(int index) {  
    if (index < 0 || index >= size)  
        throw new IndexOutOfBoundsException(  
            "accessing at %d for a list of size %d".formatted(index, size));  
}
```

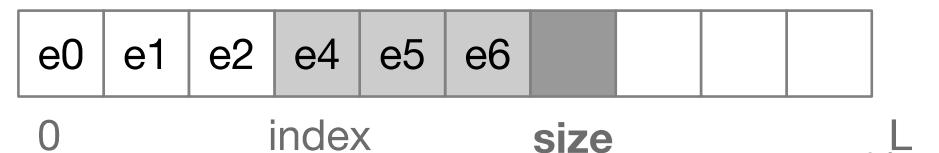
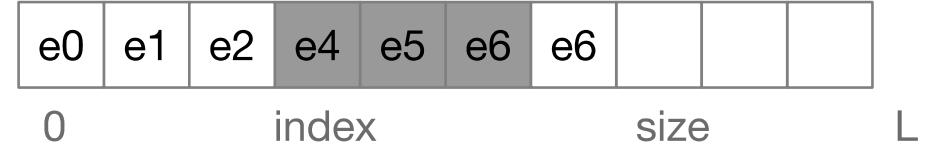
1. We check the index is valid
2. We get hold of the old value at the index position
  - The array is of Objects, but we need to return an E, so we cast
  - But the compiler cannot check this cast is OK (we can because the only things we have added are E's)
  - So, we tell the compiler to not issue a warning
3. We substitute with the new value
4. We return the old one



# ArrayList

```
@Override  
public void remove(int index) {  
    checkIndexForAccess(index);  
    moveSuffixOneLeft(index);  
    elements[size - 1] = null;  
    size -= 1;  
}  
  
private void moveSuffixOneLeft(int index) {  
    // loop from left to right (the suffix moves one to the left)  
    // index is the destination  
    for(int i = index; i < size - 1; i++) {  
        elements[i] = elements[i + 1];  
    }  
}  
elements[size - 1] = null;  
size -= 1;
```

1. We check if the index is valid
2. We move the suffix  $[index+1, size)$  one position to the left
3. We remove the stalling reference to the last element of the list (for Garbage Collection)
4. We decrement the size



# ArrayList

```
@Override  
@SuppressWarnings("unchecked")  
public E get(int index) {  
    checkIndexForAccess(index);  
    return (E) elements[index];  
}
```

1. We check if the index is valid
2. We return the element at the index
  - We must do an unchecked cast
  - We suppress the warning

```
@Override  
public int size() {  
    return size;  
}
```

1. We return the size of the list

```
@Override  
public Iterator<E> iterator() {  
    return new Itr();  
}
```

1. An iterator is an object of type Iterator<E>
  - We create an instance of the class Itr
  - This class will implement the iterator interface for the ArrayList

# ArrayList

```
public class MyArrayList<E> implements MyList<E> {  
    // ...  
  
    @Override  
    public Iterator<E> iterator() {  
        return new Itr();  
    }  
  
    private class Itr implements Iterator<E> {  
        int cursor = 0;  
        int lastReturned = -1;  
  
        // ...  
    }  
}
```

- The Itr class is defined inside the ArrayList class
  - It is **not static**, so it's an **inner class**
  - As it is **private**, it can be accessed only from the ArrayList class
- Every instance of the inner class is **connected** to an instance of the outer class
  - **MyArrayList.this** is a reference to this instance (**qualified this**)
    - If the instance variable of the outer class is not hidden you can access it without it
  - Its constructor must be invoked
    - from an instance method of the outer class (uses this as outer instance)
    - or via an explicit reference to the outer instance

# ArrayList

```
public class MyArrayList<E> implements myList<E> {  
    // ...  
  
    @Override  
    public Iterator<E> iterator() {  
        return new Iterator<>(){  
            int cursor = 0;  
            int lastReturned = -1;  
  
            // ...  
        };  
    }  
}
```

- In this case, we could have used an **anonymous inner class**
- In the same expression
  - We create an instance
  - Of an anonymous class
  - That implements Iterator<E>
    - As E can be deduced from the context, we use the diamond operator <>

# ArrayList

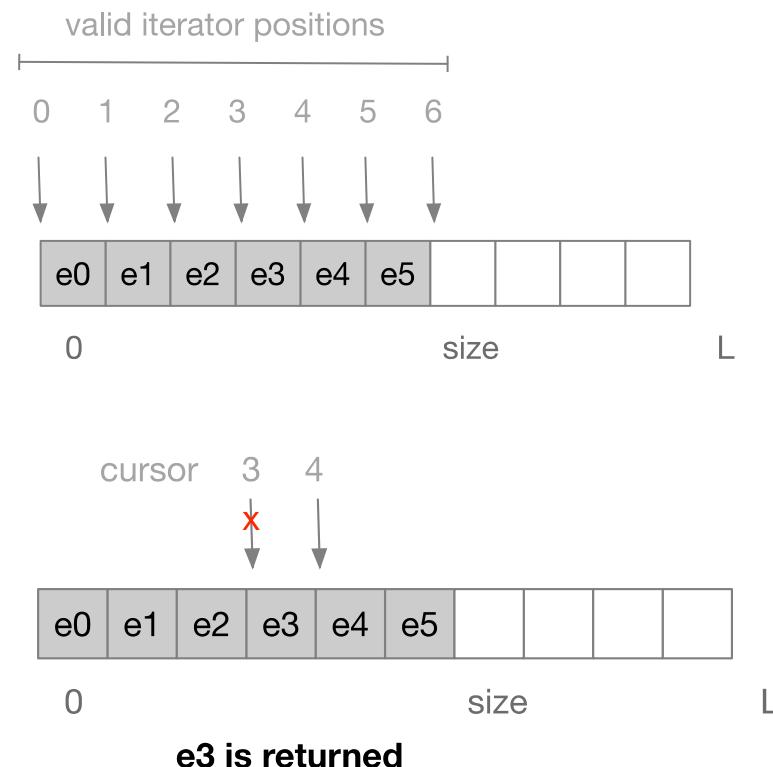
```
private class Itr implements Iterator<E> {
    int cursor = 0;
    int lastReturned = -1;

    @Override
    public boolean hasNext() {
        return cursor < MyArrayList.this.size;
    }

    @Override
    public E next() {
        if (!hasNext()) throw new NoSuchElementException(
            "iterator has no more elements");
        @SuppressWarnings("unchecked")
        E next = (E) MyArrayList.this.elements[cursor];
        lastReturned = cursor;
        cursor += 1;
        return next;
    }
}
```

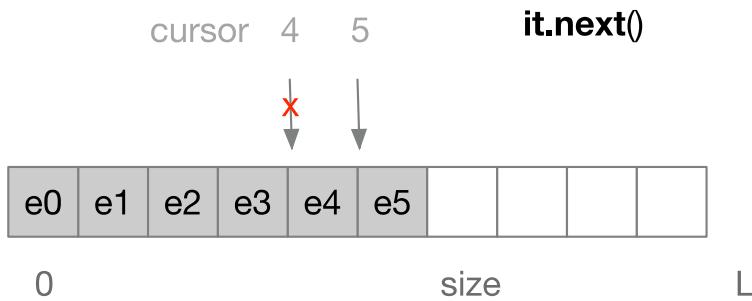
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- The iterator has a cursor that refers to a position in the array (or beyond the last one)
- We have also the index of the last returned next (for remove)
- We have a next element if we are not beyond the last element



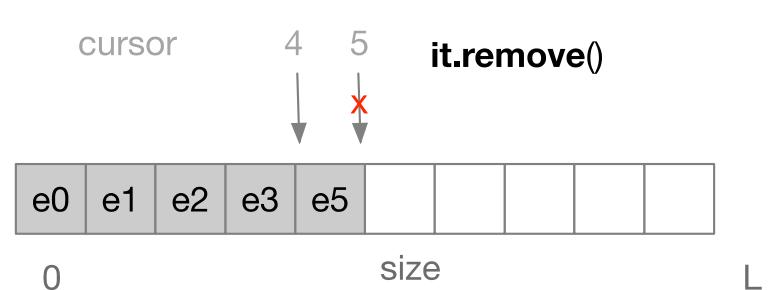
# ArrayList

```
@Override  
public void remove() {  
    checkNextWasCalled();  
    MyArrayList.this.remove(lastReturned);  
    cursor -= 1;  
    lastReturned = -1;  
}  
  
private void checkNextWasCalled() {  
    if (lastReturned == -1)  
        throw new IllegalStateException(  
            "no next before remove");  
}
```



e4 is returned

- The remove method deletes from the list the element returned by last
- It is an error to not have made a call to next before a call to remove
- NOTE: In this implementation we do not take into account co-modification, that is, that the list has been modified while using the iterator



# ArrayList

```
@Override  
public boolean equals(Object obj) {  
    if (obj == this) return true;  
    if (!(obj instanceof MyArrayList<?> other))  
        return false;  
    if (size != other.size) return false;  
    for (int i = 0; i < size; i++) {  
        if (!Objects.equals(elements[i], other.elements[i]))  
            return false;  
    }  
    return true;  
}
```

We use  
Objects.equals to  
avoid problems with  
null references

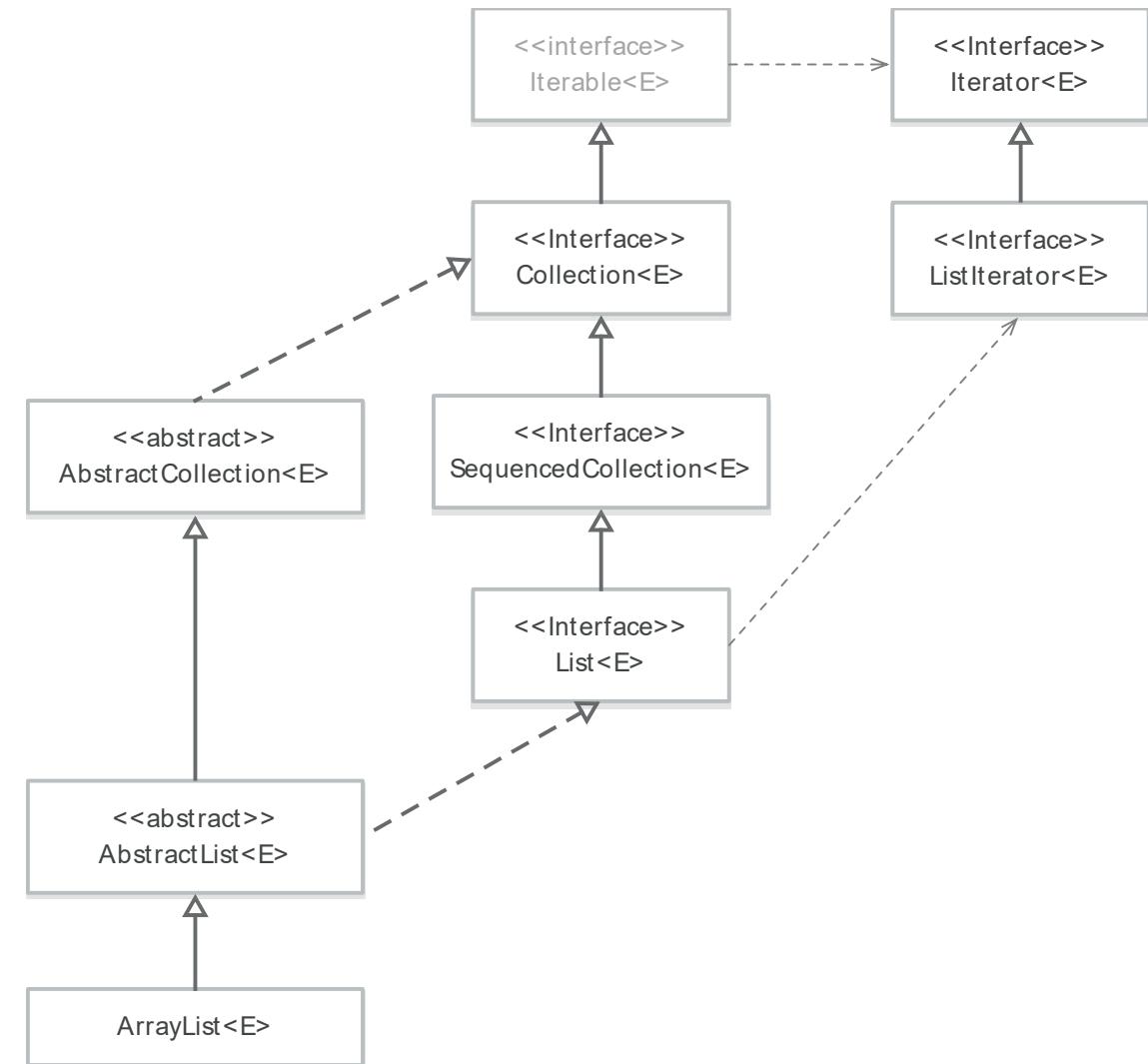
- Note the use of an unbounded wildcard in instanceof
  - Cannot use MyArrayList<E> due to erasure
- In this implementation we only allow to compare between array lists
  - In the JCF implementation equals works between different implementations of lists as well

# ArrayList in the JCF

- Now, we have understood the idea behind implementing lists using extensible arrays
- The real implementation in the class `ArrayList<E>` is a little bit more complicated
  - The interface `List<E>` has much more methods
  - Not all the methods are implemented in the `ArrayList<E>` class
    - We have abstract superclasses that implement some of them
    - Why? Because they are inherited by other implementations of `List<E>`
- Now, we will present the overall structure of the classes and interfaces of the JCF relevant to the implementation of `ArrayList<E>`
- NOTE: For backwards compatibility, most of the implementation does not use some of the new elements of the language (e.g. default methods)

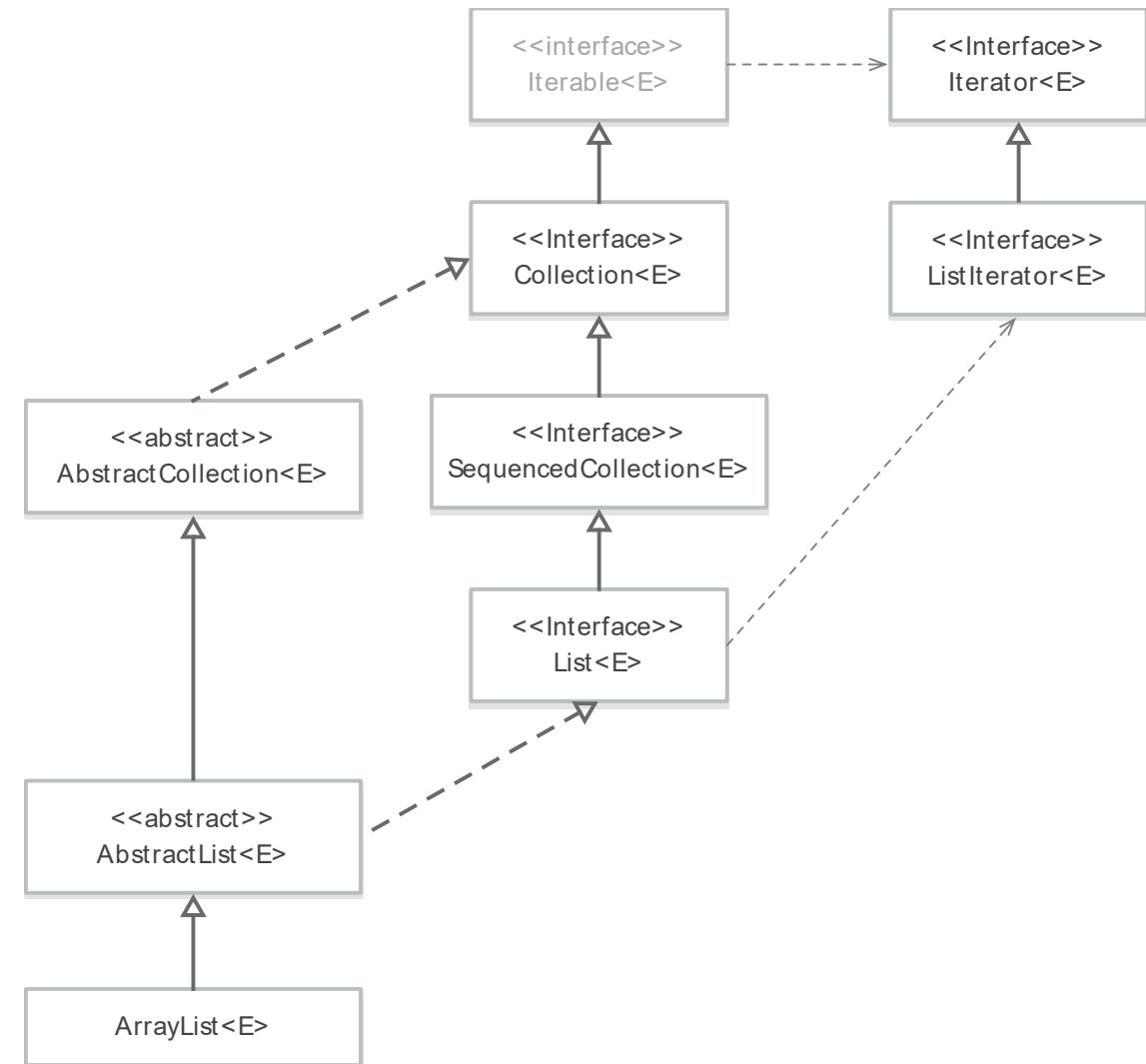
# ArrayList in the JCF

- As we saw before, the interface List<E> gathers the methods declared in
  - Iterable<E>
  - Collection<E>
  - SequenceCollection<E>
- ListIterator<E>
  - It is an extension of the Iterator<E> interface
  - It allows moving from right to left
  - It allows modifying (not just removing) the element pointed by the iterator and adding new elements



# ArrayList in the JCF

- AbstractCollection<E>
  - Implements some of the methods of Collection
  - By calling other implemented methods of AbstractCollection<E>
  - By calling abstract methods of Collection<E> that will be implemented by its subclasses
- AbstractList<E>
  - Uses a similar strategy for List<E>



# Collection<E>

- **Represents a group of objects** known as its elements
  - Some collections allow duplicates, some don't
  - Some are ordered, some unordered
  - Some define encounter order, some don't
- Some **methods** are specified to be **optional**
  - They can throw **UnsupportedOperationException**
- **View** collections
  - Most collections manage storage of its elements, view collections rely on a backing collection
- **Unmodifiable collections**
  - A collection whose **mutator operations** throw **UnsupportedOperationException**
  - Any view collection backed by it must be unmodifiable as well
  - NOTE: They are not immutable because the elements can be mutable

# Collection<E>

- boolean add(E e)
- boolean addAll(Collection<? extends E> c)
- void clear()
- boolean contains(Object a)
- boolean contains(Collection<?> c)
- boolean equals(Object o)
- int hashCode()
- boolean isEmpty()
- Iterator<E> iterator()
- default Stream<E> parallelStream()
- boolean remove(Object o)
- boolean removeAll(Collection<?> c)
- default boolean removeIf(Predicate<? super E> filter)
- boolean retainAll(Collection<?> c)
- int size()
- default SplitIterator<E> splitIterator()
- default Stream<E> stream()
- Object[] toArray()
- default <T> T[] toArray(IntFunction<T[]> generator)
- <T> T[] toArray(T[] a)

# AbstractCollection<E>

- It provides a skeletal implementation of Collection<E> to minimize the effort to implement it
- **Unmodifiable** collection
  - **iterator** and **size**
- **Modifiable** collection needs also
  - **add**
  - **remove** operation **in** the Iterator<E> returned by **iterator**
- Its subclasses should generally provide also
  - No argument and Collection constructor
- The **specification** of each non-abstract method **describe in detail its implementation**
  - It may be overridden if the collection admits a more efficient implementation

# AbstractCollection<E>

- We will show the implementation of some of the methods
- But it is advisable to read the code for all of them
  - Those marked in grey in the previous list are skippable (they are about streams and/or use lambdas, which go beyond this course)
- **Reading code is one of the most important capabilities a programmer must acquire !!**
  - And the code of the JCF is (with exceptions) a good way to learn the style of programming required to implement data structures
- How to access the implementation:
  - From IntelliJ, Shift Shift and ask for the class (e.g. `java.util.ArrayList`)
    - Use a project configured with Java 21 (last LTS)
  - [Open JDK GitHub repository](#)

# AbstractCollection<E>

```
public abstract class AbstractCollection<E> implements Collection<E> {  
  
    protected AbstractCollection() {}  
  
    public boolean isEmpty() {  
        return size() == 0;  
    }  
  
    public boolean contains(Object o) {  
        Iterator<E> it = iterator();  
        if (o==null) {  
            while (it.hasNext())  
                if (it.next()==null)  
                    return true;  
        } else {  
            while (it.hasNext())  
                if (o.equals(it.next()))  
                    return true;  
        }  
        return false;  
    }  
}
```

```
    public boolean add(E e) {  
        throw new UnsupportedOperationException();  
    }  
  
    public boolean remove(Object o) {  
        Iterator<E> it = iterator();  
        if (o==null) {  
            while (it.hasNext()) {  
                if (it.next()==null) {  
                    it.remove();  
                    return true;  
                }  
            }  
        } else {  
            while (it.hasNext()) {  
                if (o.equals(it.next())) {  
                    it.remove();  
                    return true;  
                }  
            }  
        }  
        return false;  
    }  
}
```

# AbstractCollection<E>

```
public boolean containsAll(Collection<?> c) {
    for (Object e : c)
        if (!contains(e))
            return false;
    return true;
}

public boolean addAll(Collection<? extends E> c) {
    boolean modified = false;
    for (E e : c)
        if (add(e))
            modified = true;
    return modified;
}

public boolean removeAll(Collection<?> c) {
    Objects.requireNonNull(c);
    boolean modified = false;
    Iterator<?> it = iterator();
    while (it.hasNext()) {
        if (c.contains(it.next()))
            it.remove();
            modified = true;
    }
    return modified;
}

public void clear() {
    Iterator<E> it = iterator();
    while (it.hasNext())
        it.next();
        it.remove();
    }
}
```

# AbstractCollection<E>

```
public boolean retainAll(Collection<?> c) {
    Objects.requireNonNull(c);
    boolean modified = false;
    Iterator<E> it = iterator();
    while (it.hasNext()) {
        if (!c.contains(it.next())) {
            it.remove();
            modified = true;
        }
    }
    return modified;
}

public String toString() {
    Iterator<E> it = iterator();
    if (!it.hasNext())
        return "[]";

    StringBuilder sb = new StringBuilder();
    sb.append("[");
    for (;;) {
        E e = it.next();
        sb.append(e == this ? "(this Collection)" : e);
        if (!it.hasNext())
            return sb.append("]").toString();
        sb.append(",").append(" ");
    }
}

// toArray method omitted for mental sanity ☺
```

# SequencedCollection<E>

- A collection that
  - has a well-defined encounter order
    - Conceptually, they have a linear arrangement from first to last
  - supports operations at both ends
  - is reversible
- Mostly implemented by default methods
  - It was added in Java 21
  - Without breaking any existing code !!

# SequencedCollection<E>

```
public interface SequencedCollection<E> extends Collection<E> {  
  
    SequencedCollection<E> reversed();  
  
    default void addFirst(E e) {  
        throw new UnsupportedOperationException();  
    }  
  
    default void addLast(E e) {  
        throw new UnsupportedOperationException();  
    }  
  
    default E getFirst() {  
        return this.iterator().next();  
    }  
  
    default E getLast() {  
        return this.reversed().iterator().next();  
    }  
  
    default E removeFirst() {  
        var it = this.iterator();  
        E e = it.next();  
        it.remove();  
        return e;  
    }  
  
    default E removeLast() {  
        var it = this.reversed().iterator();  
        E e = it.next();  
        it.remove();  
        return e;  
    }  
}
```

# List<E>

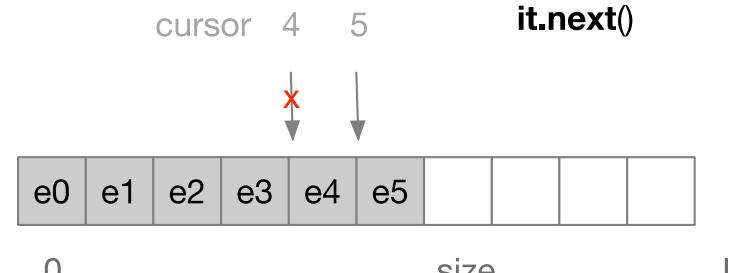
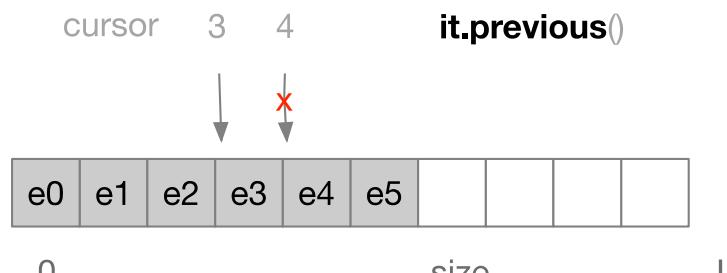
- An ordered (by position) collection
  - The user has precise control of where each element is added
  - The user can access elements by position
- Typically allows duplicates and null elements
- Refines the specification of some methods
  - iterator, add, remove, equals, hashCode
- Adds methods for indexed access (with int parameter for index)
  - add, addAll, get, remove, set
- Provides for a special iterator called ListIterator<E>
- Methods for search an element
  - indexOf, lastIndexOf
- Unmodifiable Lists
  - Static methods List.of, List.copyOf to create unmodifiable lists

## Iterator<E>

- In our toy implementation of lists, we have not considered what happens when one modifies the list while using an iterator
  - It will fail with a `ConcurrentModificationException` when detects this situation
  - To do so,
    - the list has a `modCount` attribute that is incremented when modifying the list
    - when creating an iterator, the current value is saved
    - the operations in the iterator check if the saved value is still the same
    - NOTE: It is defined as transient to not include it in the serialization of the list

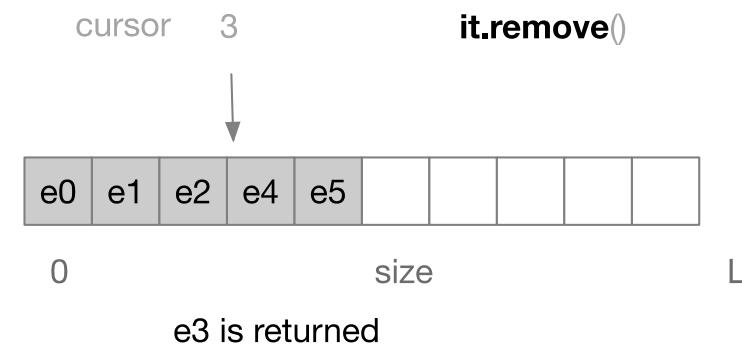
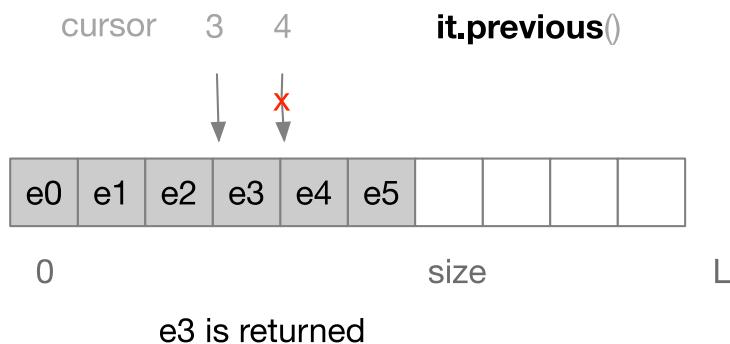
# ListIterator<E>

- An extension of `Iterator<E>` that allows
    - Traversing the list in each direction
    - Modifying the list while iterating
    - Obtaining the iterator’s “current” position in the list
  - A `ListIterator<E>` has no “current” position in the list
    - Conceptually, it is between the element that will be returned by previous and the element that will be returned by next



# ListIterator<E>

- You can also add at “the position” of the iterator
- And you can modify the value at the position returned by previous / next
- Obviously, the remove operator also works for the last returned previous element



# AbstractList<E>

- It provides a **skeletal implementation** of **List<E>** to minimize the effort to implement it
- Your implementation must be **backed by a “random access” data store** (such as an array)
  - For a “sequential access” implementation, use **AbstractSequentialList<E>**
- To implement an **unmodifiable** list
  - **get(int)** and **size**
- To implement a **modifiable** list
  - **set(int, E)**
  - If it's **variable size**: **add(int, E)** and **remove(int)**
- The **specification** of each non-abstract method **describes its implementation**
  - Meant to be extended
  - Override them if the collection allows for a more efficient implementations (e.g. **ArrayList<E>** overrides all of them)

# AbstractList<E>

```
public abstract class AbstractList<E>
    extends AbstractCollection<E> implements List<E> {

    protected transient int modCount = 0;

    protected AbstractList() {}

    public boolean add(E e) {
        add(size(), e);
        return true;
    }

    public abstract E get(int index);

    public E set(int index, E element) {
        throw new UnsupportedOperationException();
    }

    public void add(int index, E element) {
        throw new UnsupportedOperationException();
    }

    public E remove(int index) {
        throw new UnsupportedOperationException();
    }
```

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```
public int indexOf(Object o) {
    ListIterator<E> it = listIterator();
    if (o==null) {
        while (it.hasNext())
            if (it.next()==null)
                return it.previousIndex();
    } else {
        while (it.hasNext())
            if (o.equals(it.next()))
                return it.previousIndex();
    }
    return -1;
}

public int lastIndexOf(Object o) {
    ListIterator<E> it = listIterator(size());
    if (o==null) {
        while (it.hasPrevious())
            if (it.previous()==null)
                return it.nextIndex();
    } else {
        while (it.hasPrevious())
            if (o.equals(it.previous()))
                return it.nextIndex();
    }
    return -1;
}
```

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# AbstractList<E> (Iterator<E>)

```
public Iterator<E> iterator() { return new Itr(); }

private class Itr implements Iterator<E> {
    int cursor = 0;
    int lastRet = -1;
    int expectedModCount = modCount;

    public boolean hasNext() {
        return cursor != size();
    }

    public E next() {
        checkForComodification();
        try {
            int i = cursor;
            E next = get(i);
            lastRet = i;
            cursor = i + 1;
            return next;
        } catch (IndexOutOfBoundsException e) {
            checkForComodification();
            throw new NoSuchElementException(e);
        }
    }

    public void remove() {
        if (lastRet < 0)
            throw new IllegalStateException();
        checkForComodification();

        try {
            AbstractList.this.remove(lastRet);
            if (lastRet < cursor)
                cursor--;
            lastRet = -1;
            expectedModCount = modCount;
        } catch (IndexOutOfBoundsException e) {
            throw new ConcurrentModificationException();
        }
    }

    final void checkForComodification() {
        if (modCount != expectedModCount)
            throw new ConcurrentModificationException();
    }
}
```

# AbstractList<E> (ListIterator<E>)

```
public ListIterator<E> listIterator() {
    return listIterator(0);
}

public ListIterator<E> listIterator(final int index) {
    rangeCheckForAdd(index);
    return new ListItr(index);
}

private class ListItr extends Itr implements ListIterator<E> {
    ListItr(int index) {
        cursor = index;
    }

    public boolean hasPrevious() {
        return cursor != 0;
    }

    public int nextIndex() {
        return cursor;
    }

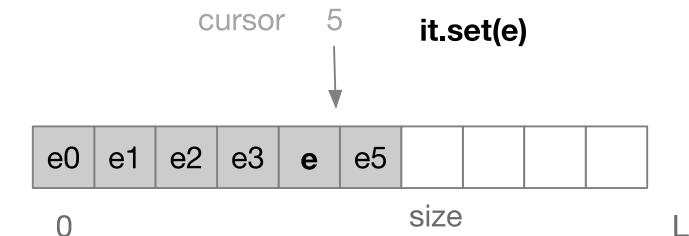
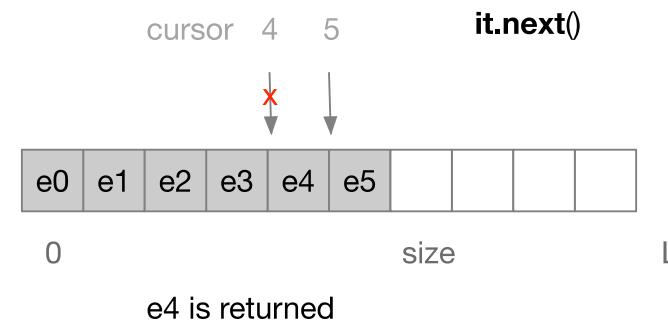
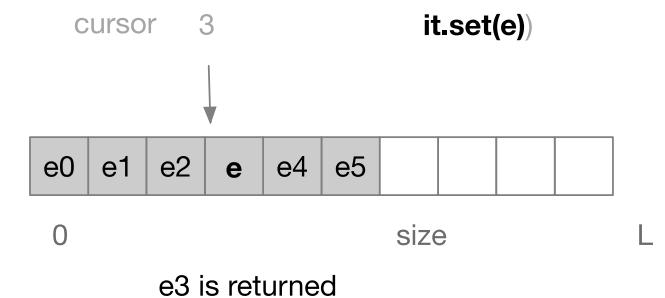
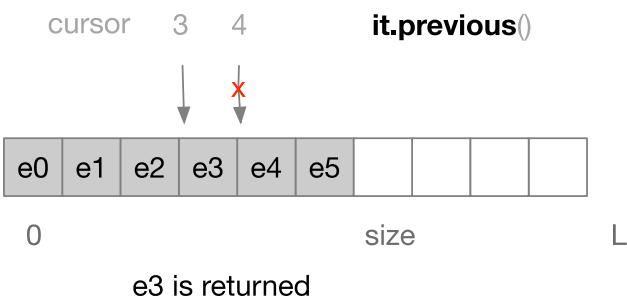
    public int previousIndex() {
        return cursor-1;
    }

    public E previous() {
        checkForComodification();
        try {
            int i = cursor - 1;
            E previous = get(i);
            lastRet = cursor = i;
            return previous;
        } catch (IndexOutOfBoundsException e) {
            checkForComodification();
            throw new NoSuchElementException(e);
        }
    }
}
```

# AbstractList<E> (ListIterator<E>)

```
public void set(E e){  
    if (lastRet < 0)  
        throw new IllegalStateException();  
    checkForComodification();  
  
    try {  
        AbstractList.this.set(lastRet, e);  
        expectedModCount = modCount;  
    } catch (IndexOutOfBoundsException ex) {  
        throw new ConcurrentModificationException();  
    }  
}
```

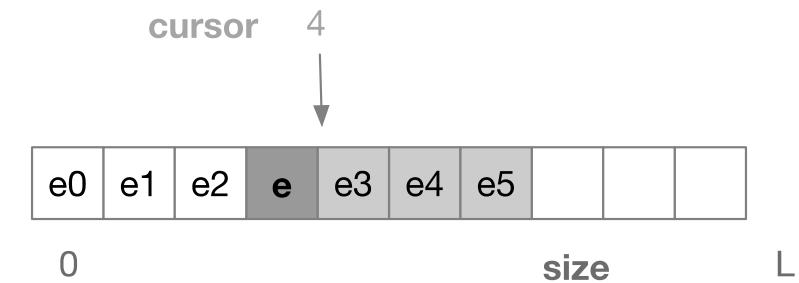
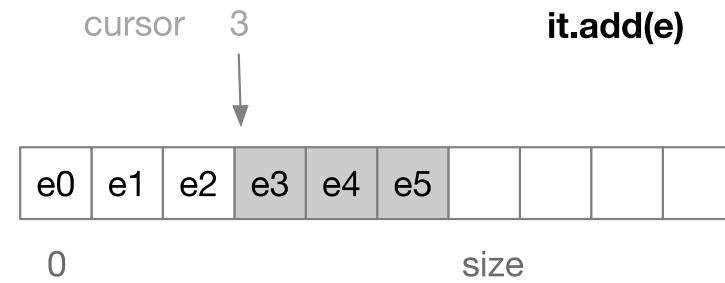
- Replaces the last element returned by previous / next
- If can be made when neither remove nor add has been called after the last call of previous / next



# AbstractList<E> (ListIterator<E>)

```
public void add(E e){  
    checkForComodification();  
  
    try {  
        int i = cursor;  
        AbstractList.this.add(i, e);  
        lastRet = -1;  
        cursor = i + 1;  
        expectedModCount = modCount;  
    } catch (IndexOutOfBoundsException ex){  
        throw new ConcurrentModificationException();  
    }  
}
```

- The element is inserted
  - before the element that would be returned by next (if any)
  - after the element that would be returned by previous (if any)
- A subsequent call to next wouldn't be affected
- A subsequent call to previous returns the new element



# ArrayList<E>

- Resizable-array implementation of List<E>
- Implements all optional operations and permits null elements
- Besides implementing List<E>, it provides some methods to manipulate the underlying array
- It has a capacity, which is the size of the underlying array that is always at least equal to the size of the list
  - The growth policy only requires amortized constant time for add
- The implementation is not synchronized
- Iterators are fail-fast:
  - Fail if a structural modification of the list is detected after creating the iterator
  - Do not guarantee correctness, only detects some bugs

# ArrayList<E>

```
public class ArrayList<E> extends AbstractList<E>
    implements List<E>, RandomAccess, Cloneable, java.io.Serializable {

    @java.io.Serial
    private static final long serialVersionUID = 8683452581122892189L;

    private static final int DEFAULT_CAPACITY = 10;

    private static final Object[] EMPTY_ELEMENTDATA = {};

    private static final Object[] DEFAULTCAPACITY_EMPTY_ELEMENTDATA = {};

    transient Object[] elementData;

    private int size;

    // ...
}
```

- **RandomAccess**: marker that indicates constant time random access
  - **Cloneable**: cloning machinery
  - **Serializable**: serializable machinery
- 
- **DEFAULT\_CAPACITY**: default capacity of the list
  - **EMPTY\_ELEMENTDATA**: shared array instance used for empty instances
  - **DEFAULTCAPACITY\_EMPTY\_ELEMENTDATA**: shared empty array used for default sized empty instances (distinguished from the previous to know how much inflate on first insertion)

# ArrayList<E>

```
public ArrayList(int initialCapacity) {  
    if (initialCapacity > 0) {  
        this.elementData = new Object[initialCapacity];  
    } else if (initialCapacity == 0) {  
        this.elementData = EMPTY_ELEMENTDATA;  
    } else {  
        throw new IllegalArgumentException("Illegal Capacity: "+  
            initialCapacity);  
    }  
  
    public ArrayList() {  
        this.elementData = DEFAULTCAPACITY_EMPTY_ELEMENTDATA;  
    }  
  
    public ArrayList(Collection<? extends E> c) {  
        Object[] a = c.toArray();  
        if ((size = a.length) != 0) {  
            if (c.getClass() == ArrayList.class) {  
                elementData = a;  
            } else {  
                elementData = Arrays.copyOf(a, size, Object[].class);  
            }  
        } else {  
            // replace with empty array.  
            elementData = EMPTY_ELEMENTDATA;  
        }  
    }
```

- ArrayLists without capacity are **initialized lazily** (on first insertion)
- As we do not have the guarantee that the array returned by toArray can be aliased we do a **defensive copy**
- As we know its implementation in ArrayList, and we know it's a fresh copy, we do not do a defensive copy in that case

# ArrayList<E>

```
public int size() {
    return size;
}

public boolean isEmpty() {
    return size == 0;
}

public void trimToSize() {
    modCount++;
    if (size < elementData.length) {
        elementData = (size == 0)
            ? EMPTY_ELEMENTDATA
            : Arrays.copyOf(elementData, size);
    }
}
```

```
private Object[] grow(int minCapacity) {
    int oldCapacity = elementData.length;
    if (oldCapacity > 0 || elementData != DEFAULTCAPACITY_EMPTY_ELEMENTDATA) {
        int newCapacity = ArraysSupport.newLength(oldCapacity,
            minCapacity - oldCapacity, /* minimum growth */
            oldCapacity >> 1           /* preferred growth */);
        return elementData = Arrays.copyOf(elementData, newCapacity);
    } else {
        return elementData = new Object[Math.max(DEFAULT_CAPACITY, minCapacity)];
    }
}

private Object[] grow() {
    return grow(size + 1);
}

public void ensureCapacity(int minCapacity) {
    if (minCapacity > elementData.length
        && !(elementData == DEFAULTCAPACITY_EMPTY_ELEMENTDATA
            && minCapacity <= DEFAULT_CAPACITY)) {
        modCount++;
        grow(minCapacity);
    }
}
```

# ArraySupport

- in package jdk.internal.util
- not accessible out of JDK

```
public class ArraysSupport {  
  
    // ...  
  
    public static final int SOFT_MAX_ARRAY_LENGTH = Integer.MAX_VALUE - 8;  
  
    public static int newLength(int oldLength, int minGrowth, int prefGrowth) {  
        int prefLength = oldLength + Math.max(minGrowth, prefGrowth); // might overflow  
        if (0 < prefLength && prefLength <= SOFT_MAX_ARRAY_LENGTH) {  
            return prefLength;  
        } else {  
            return hugeLength(oldLength, minGrowth);  
        }  
    }  
  
    private static int hugeLength(int oldLength, int minGrowth) {  
        int minLength = oldLength + minGrowth;  
        if (minLength < 0) { // overflow  
            throw new OutOfMemoryError(  
                "Required array length " + oldLength + " + " + minGrowth + " is too large");  
        } else if (minLength <= SOFT_MAX_ARRAY_LENGTH) {  
            return SOFT_MAX_ARRAY_LENGTH;  
        } else {  
            return minLength;  
        }  
    }  
}
```

# ArrayList<E>

```
private void add(E e, Object[] elementData, int s) {  
    if (s == elementData.length)  
        elementData = grow();  
    elementData[s] = e;  
    size = s + 1;  
}
```

```
public boolean add(E e) {  
    modCount++;  
    add(e, elementData, size);  
    return true;  
}
```

```
public void addFirst(E element) {  
    add(0, element);  
}
```

```
public void add(int index, E element) {  
    rangeCheckForAdd(index);  
    modCount++;  
    final int s;  
    Object[] elementData;  
    if ((s = size) == (elementData = this.elementData).length)  
        elementData = grow();  
    System.arraycopy(elementData, index,  
                     elementData, index + 1,  
                     s - index);  
    elementData[index] = element;  
    size = s + 1;  
}
```

```
public void addLast(E element) {  
    add(element);  
}
```

# ArrayList<E>

```
@SuppressWarnings("unchecked")
E elementData(int index) {
    return (E) elementData[index];
}

@SuppressWarnings("unchecked")
static <E> E elementAt(Object[] es, int index) {
    return (E) es[index];
}

public E get(int index) {
    Objects.checkIndex(index, size);
    return elementData(index);
}

public E getFirst() {
    if (size == 0) {
        throw new NoSuchElementException();
    } else {
        return elementData(0);
    }
}

public E getLast() {
    int last = size - 1;
    if (last < 0) {
        throw new NoSuchElementException();
    } else {
        return elementData(last);
    }
}

public E set(int index, E element) {
    Objects.checkIndex(index, size);
    E oldValue = elementData(index);
    elementData[index] = element;
    return oldValue;
}
```

# ArrayList<E>

```
public E remove(int index) {
    Objects.checkIndex(index, size);
    final Object[] es = elementData;
    @SuppressWarnings("unchecked") E oldValue = (E) es[index];
    fastRemove(es, index);
    return oldValue;
}

public E removeFirst() {
    if (size == 0) {
        throw new NoSuchElementException();
    } else {
        Object[] es = elementData;
        @SuppressWarnings("unchecked") E oldValue = (E) es[0];
        fastRemove(es, 0);
        return oldValue;
    }
}

public E removeLast() {
    int last = size - 1;
    if (last < 0) {
        throw new NoSuchElementException();
    } else {
        Object[] es = elementData;
        @SuppressWarnings("unchecked") E oldValue = (E) es[last];
        fastRemove(es, last);
        return oldValue;
    }
}

private void fastRemove(Object[] es, int i) {
    modCount++;
    final int newSize;
    if ((newSize = size - 1) > i)
        System.arraycopy(es, i + 1, es, i, newSize - i);
    es[size = newSize] = null;
}
```

# ArrayList<E> (Iterator<E>)

```
/*
 * An optimized version of AbstractList.Itr
 */
private class Itr implements Iterator<E>{
    int cursor;    // index of next element to return
    int lastRet = -1; // index of last element returned; -1 if no such
    int expectedModCount = modCount;

    // prevent creating a synthetic constructor
    Itr() {}

    public boolean hasNext() {
        return cursor != size;
    }

    @SuppressWarnings("unchecked")
    public E next() {
        checkForComodification();
        int i = cursor;
        if (i >= size)
            throw new NoSuchElementException();
        Object[] elementData = ArrayList.this.elementData;
        if (i >= elementData.length)
            throw new ConcurrentModificationException();
        cursor = i + 1;
        return (E) elementData[lastRet = i];
    }

    public void remove() {
        if (lastRet < 0)
            throw new IllegalStateException();
        checkForComodification();
        try {
            ArrayList.this.remove(lastRet);
            cursor = lastRet;
            lastRet = -1;
            expectedModCount = modCount;
        } catch (IndexOutOfBoundsException ex) {
            throw new ConcurrentModificationException();
        }
    }
}
```

# ArrayList<E> (ListIterator<E>)

```
/*
 * An optimized version of AbstractList.ListItr
 */
private class ListItr extends Itr implements ListIterator<E> {
    ListItr(int index) {
        super();
        cursor = index;
    }

    public boolean hasPrevious() {
        return cursor != 0;
    }

    public int nextIndex() {
        return cursor;
    }

    public int previousIndex() {
        return cursor - 1;
    }

    @SuppressWarnings("unchecked")
    public E previous() {
        checkForComodification();
        int i = cursor - 1;
        if (i < 0)
            throw new NoSuchElementException();
        Object[] elementData = ArrayList.this.elementData;
        if (i >= elementData.length)
            throw new ConcurrentModificationException();
        cursor = i;
        return (E) elementData[lastRet = i];
    }
}
```

# ArrayList<E> (ListIterator<E>)

```
public void set(E e) {  
    if (lastRet < 0)  
        throw new IllegalStateException();  
    checkForComodification();  
  
    try {  
        ArrayList.this.set(lastRet, e);  
    } catch (IndexOutOfBoundsException ex) {  
        throw new ConcurrentModificationException();  
    }  
}
```

```
public void add(E e) {  
    checkForComodification();  
  
    try {  
        int i = cursor;  
        ArrayList.this.add(i, e);  
        cursor = i + 1;  
        lastRet = -1;  
        expectedModCount = modCount;  
    } catch (IndexOutOfBoundsException ex) {  
        throw new ConcurrentModificationException();  
    }  
}
```

# ArrayList<E> and AbstractList<E>

- Both implement, the former totally and the later partially, List<E> with **data baked by an efficient random-access store**:
  - ArrayList<E> uses an array
  - AbstractList<E> uses efficient get / set / add / remove by index
- But, what if the implementation of List<E> is backed by a **sequence of linked nodes**?
  - So that **we do not have efficient access by index**
- The classes AbstractSequentialList<E> and LinkedList<E> provides us with a partial and a total implementation of this design

# AbstractSequentialList<E>

- It's an extension of [AbstractList<E>](#) that provides a skeletal implementation of the [List<E>](#) interface by a “sequential access” data store
- This class is the opposite of [AbstractList<E>](#) in the sense that the random-access operations get / set / add / remove are implemented on top of the [ListIterator<E>](#) returned by listIterator(int)
  - in [AbstractList<E>](#) it is the other way around
- To implement a [List<E>](#) the programmer must implement
  - **size** and **listIterator(int)** with **next**, **previous**, **hasXXX**, and **XXXIndex**
  - For **modifiable** list, iterator's **set(E)**
  - For **variable size**, iterator's **add(E)** and **remove**

# AbstractSequentialList<E>

```
public abstract class AbstractSequentialList<E>
    extends AbstractList<E>{

    protected AbstractSequentialList() {}

    public E get(int index) {
        try {
            return listIterator(index).next();
        } catch (NoSuchElementException exc) {
            throw new IndexOutOfBoundsException("Index: "+index);
        }
    }

    public E set(int index, E element) {
        try {
            ListIterator<E> e = listIterator(index);
            E oldVal = e.next();
            e.set(element);
            return oldVal;
        } catch (NoSuchElementException exc) {
            throw new IndexOutOfBoundsException("Index: "+index);
        }
    }
}
```

```
public void add(int index, E element) {
    try {
        listIterator(index).add(element);
    } catch (NoSuchElementException exc) {
        throw new IndexOutOfBoundsException("Index: "+index);
    }
}

public E remove(int index) {
    try {
        ListIterator<E> e = listIterator(index);
        E outCast = e.next();
        e.remove();
        return outCast;
    } catch (NoSuchElementException exc) {
        throw new IndexOutOfBoundsException("Index: "+index);
    }
}
```

- **as we'll see, the creation of a listIterator at a given index is linear, so all these indexed operations will be linear as well**

# AbstractSequentialList<E>

```
public boolean addAll(int index, Collection<? extends E> c){  
    try {  
        boolean modified = false;  
        ListIterator<E> e1 = listIterator(index);  
        for (E e : c) {  
            e1.add(e);  
            modified = true;  
        }  
        return modified;  
    } catch (NoSuchElementException exc) {  
        throw new IndexOutOfBoundsException("Index: "+index);  
    }  
  
    public Iterator<E> iterator(){  
        return listIterator();  
    }  
  
    public abstract ListIterator<E> listIterator(int index);  
}
```

- All these random-access operations by index do a sequential search
- *listIterator(int)* is a **concrete** operation in *AbstractList<E>*, but here we mark it as **abstract**
  - we **do not inherit** its base class implementation
  - and our implementors **must also implement it !!**

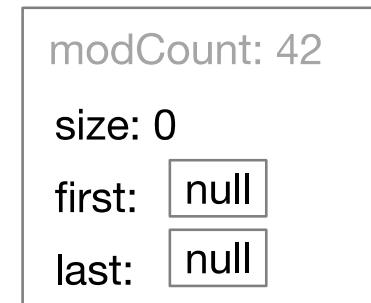
## LinkedList<E>

- Doubly-linked implementation of the List<E> and Deque<E> interfaces
- Implements all optional operations and permits null elements
- All the operations perform as one expects them for a doubly-linked list
  - Index operations perform a traverse from the closest end
- The implementation is not synchronized
- Iterators are fail-fast:
  - They fail if a structural modification of the list is detected after creating the iterator
  - And they do not guarantee correctness, only detects some bugs

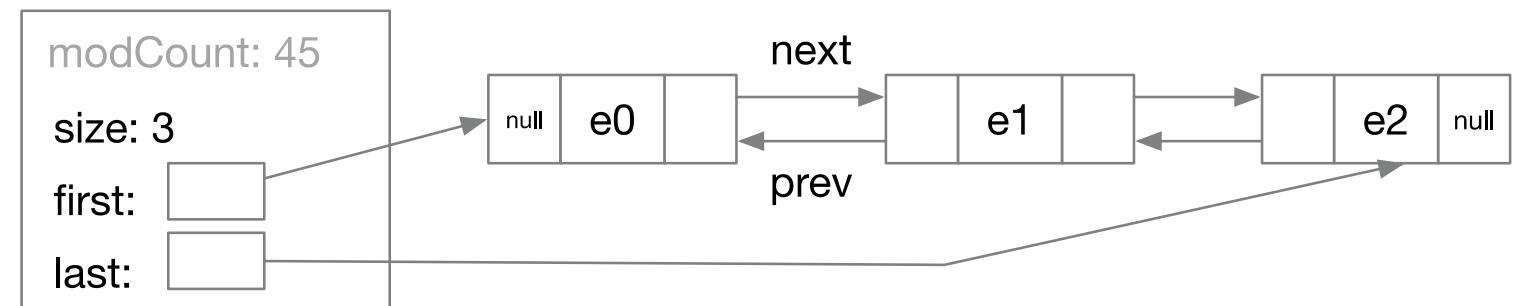
# LinkedList<E>

```
public class LinkedList<E> extends AbstractSequentialList<E> implements List<E>, Deque<E>, Cloneable, java.io.Serializable {  
  
    transient int size = 0;  
  
    /**  
     * Pointer to first node.  
     */  
    transient Node<E> first;  
  
    /**  
     * Pointer to last node.  
     */  
    transient Node<E> last;  
  
    /*  
     void dataStructureInvariants() {  
         assert(size == 0)  
             ? (first == null && last == null)  
             : (first.prev == null && last.next == null);  
    }  
*/
```

**Empty list:**



**A list with three elements:**



# LinkedList<E> (Node<E>)

```
private static class Node<E> {
    E item;
    Node<E> next;
    Node<E> prev;

    Node(Node<E> prev, E element, Node<E> next) {
        this.item = element;
        this.next = next;
        this.prev = prev;
    }
}
```

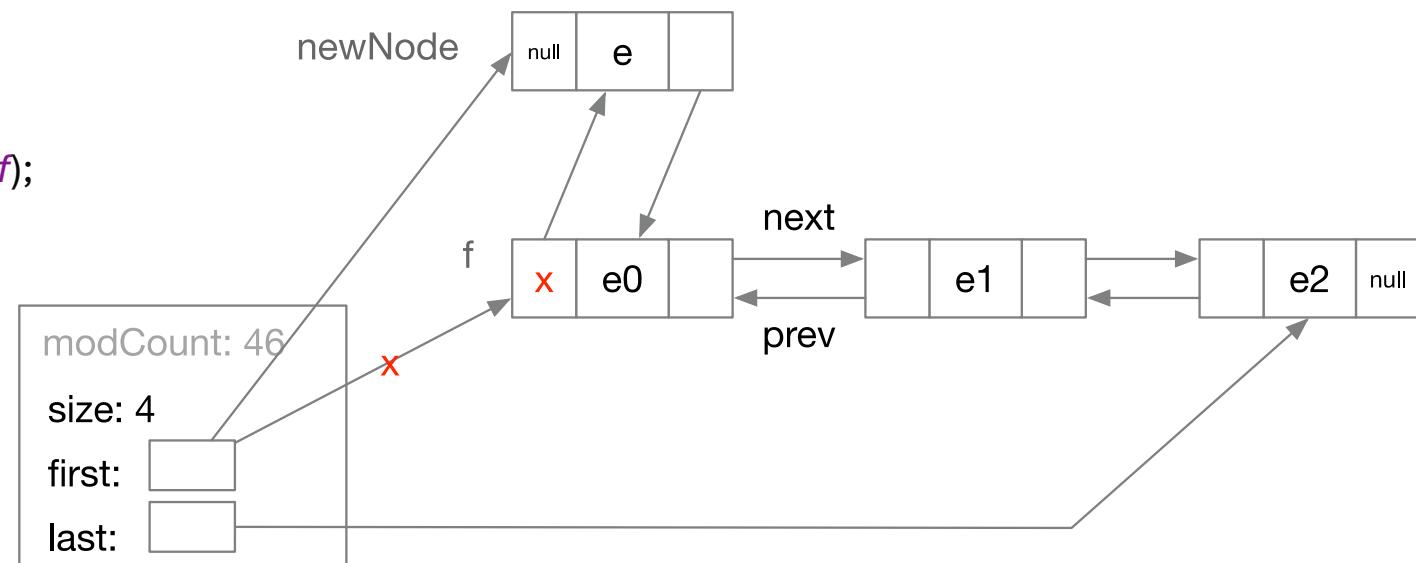
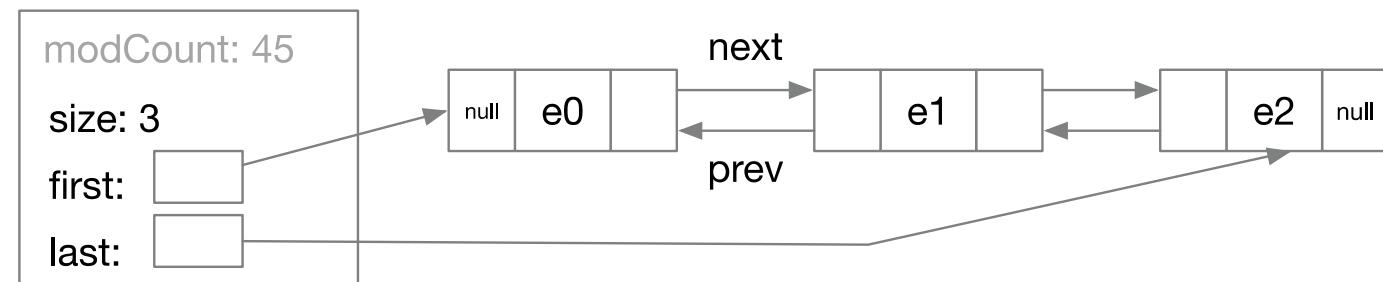
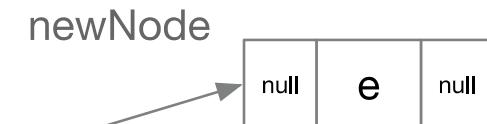
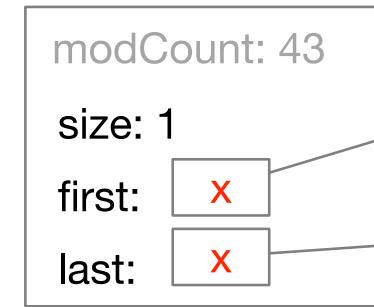
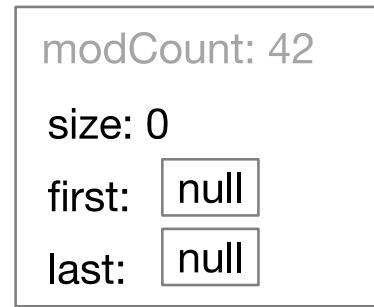
- A Node<E> is implemented by a **static nested class**
  - It has no access to the attributes of the LinkedList (no qualified this)
  - The parameter <E> is different from that of the LinkedList<E>
- It's a **doubly-linked list** because each Node<E> has a references to
  - Its **previous** node in the list
  - Its **next** node in the list

# LinkedList<E>

```
public LinkedList() { }
```

```
public LinkedList(Collection<? extends E> c) {
    this();
    addAll(c);
}

/**
 * Links e as first element.
 */
private void linkFirst(E e) {
    final Node<E> f = first;
    final Node<E> newNode = new Node<>(null, e, f);
    first = newNode;
    if (f == null)
        last = newNode;
    else
        f.prev = newNode;
    size++;
    modCount++;
}
```



# LinkedList<E>

```
/**  
 * Links e as last element.  
 */  
  
void linkLast(E e) {  
    final Node<E> l = last;  
    final Node<E> newNode = new Node<>(l, e, null);  
    last = newNode;  
    if (l == null)  
        first = newNode;  
    else  
        l.next = newNode;  
    size++;  
    modCount++;  
}
```

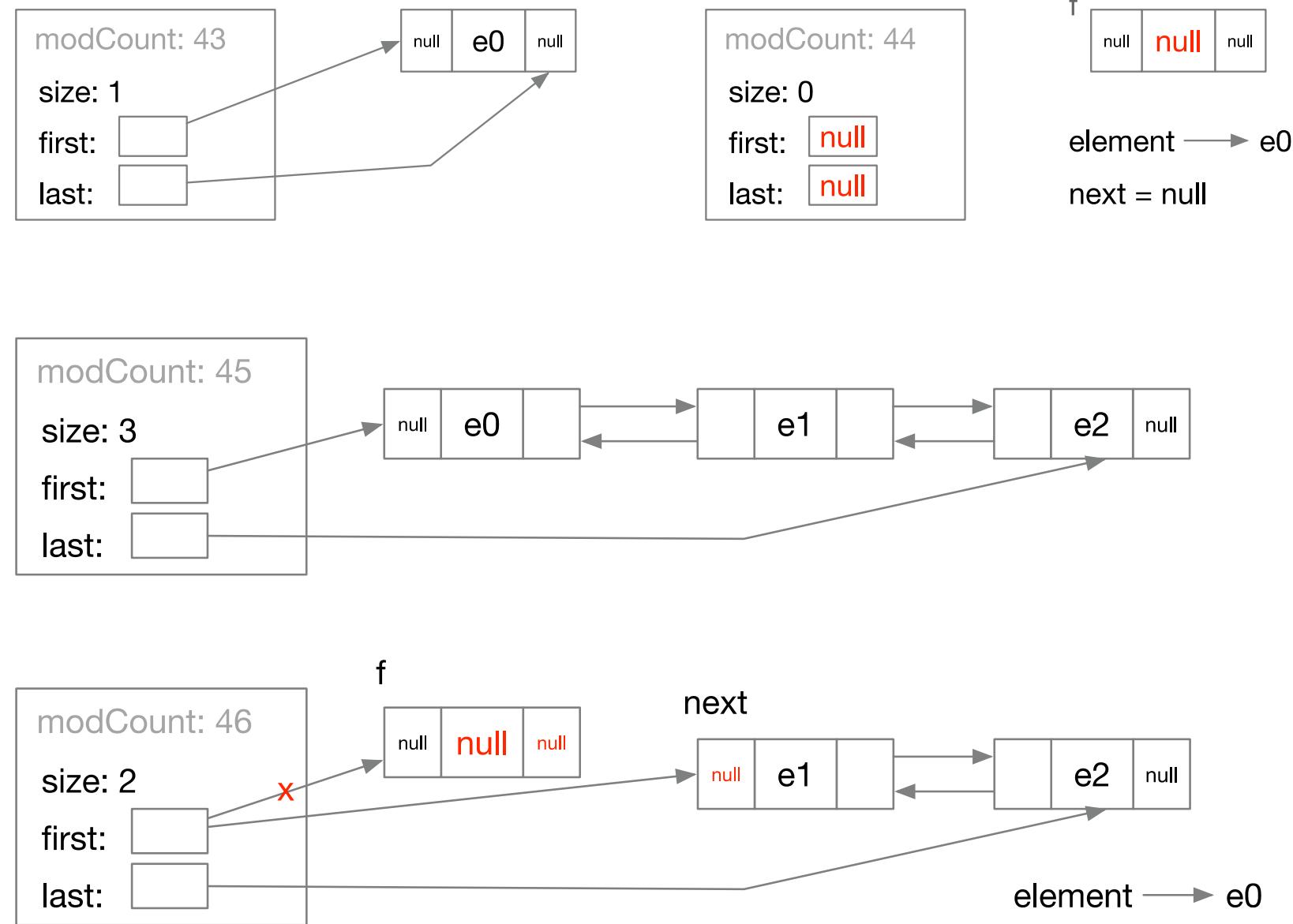
```
/**  
 * Inserts element e before non-null Node succ.  
 */  
  
void linkBefore(E e, Node<E> succ) {  
    // assert succ != null;  
    final Node<E> pred = succ.prev;  
    final Node<E> newNode = new Node<>(pred, e, succ);  
    succ.prev = newNode;  
    if (pred == null)  
        first = newNode;  
    else  
        pred.next = newNode;  
    size++;  
    modCount++;  
}
```

# LinkedList<E>

```

/**
 * Unlinks non-null first node f.
 */
private E unlinkFirst(Node<E> f){
    // assert f == first && f != null;
    final E element = f.item;
    final Node<E> next = f.next;
    f.item = null;
    f.next = null; // help GC
    first = next;
    if (next == null)
        last = null;
    else
        next.prev = null;
    size--;
    modCount++;
    return element;
}

```



# LinkedList<E>

```
/*
 * Unlinks non-null last node l.
 */
private E unlinkLast(Node<E> l) {
    // assert l == last && l != null;
    final E element = l.item;
    final Node<E> prev = l.prev;
    l.item = null;
    l.prev = null; // help GC
    last = prev;
    if (prev == null)
        first = null;
    else
        prev.next = null;
    size--;
    modCount++;
    return element;
}
```

```
/*
 * Unlinks non-null node x.
 */
E unlink(Node<E> x) {
    // assert x != null;
    final E element = x.item;
    final Node<E> next = x.next;
    final Node<E> prev = x.prev;
    if (prev == null) {
        first = next;
    } else {
        prev.next = next;
        x.prev = null;
    }
    if (next == null) {
        last = prev;
    } else {
        next.prev = prev;
        x.next = null;
    }
    x.item = null;
    size--;
    modCount++;
    return element;
}
```

# LinkedList<E>

```
public E getFirst() {
    final Node<E> f = first;
    if (f == null)
        throw new NoSuchElementException();
    return f.item;
}

public E getLast() {
    final Node<E> l = last;
    if (l == null)
        throw new NoSuchElementException();
    return l.item;
}

public E removeFirst() {
    final Node<E> f = first;
    if (f == null)
        throw new NoSuchElementException();
    return unlinkFirst(f);
}

public E removeLast() {
    final Node<E> l = last;
    if (l == null)
        throw new NoSuchElementException();
    return unlinkLast(l);
}

public void addFirst(E e) {
    linkFirst(e);
}

public void addLast(E e) {
    linkLast(e);
}

public boolean add(E e) {
    linkLast(e);
    return true;
}

public boolean contains(Object o) {
    return indexOf(o) >= 0;
}

public int size() {
    return size;
}
```

# LinkedList<E>

```
public E get(int index) {
    checkElementIndex(index);
    return node(index).item;
}

public E set(int index, E element) {
    checkElementIndex(index);
    Node<E> x = node(index);
    E oldVal = x.item;
    x.item = element;
    return oldVal;
}

public void add(int index, E element) {
    checkPositionIndex(index);

    if (index == size)
        linkLast(element);
    else
        linkBefore(element, node(index));
}
```

```
public E remove(int index) {
    checkElementIndex(index);
    return unlink(node(index));
}

private boolean isElementIndex(int index) {
    return index >= 0 && index < size;
}

private void checkElementIndex(int index) {
    if (!isElementIndex(index))
        throw new IndexOutOfBoundsException(outOfBoundsMsg(index));
}

private boolean isPositionIndex(int index) {
    return index >= 0 && index <= size;
}

private void checkPositionIndex(int index) {
    if (!isPositionIndex(index))
        throw new IndexOutOfBoundsException(outOfBoundsMsg(index));
}
```

# LinkedList<E>

```
Node<E> node(int index) {
    // assert isElementIndex(index);

    if (index < (size >> 1)) {
        Node<E> x = first;
        for (int i = 0; i < index; i++)
            x = x.next;
        return x;
    } else {
        Node<E> x = last;
        for (int i = size - 1; i > index; i--)
            x = x.prev;
        return x;
    }
}
```

- This is the method that **makes all indexed operations linear** in the number of elements in the list
- Depending whether the index is in the first or the last half of the list, it counts nodes from the start or the end.

# LinkedList<E> (ListIterator<E>)

```
public ListIterator<E> listIterator(int index) {  
    checkPositionIndex(index);  
    return new ListItr(index);  
}  
  
private class ListItr implements ListIterator<E>  
{  
    private Node<E> lastReturned;  
    private Node<E> next;  
    private int nextIndex;  
    private int expectedModCount = modCount;  
  
    ListItr(int index){  
        // assert isPositionIndex(index);  
        next = (index == size) ? null : node(index);  
        nextIndex = index;  
    }  
}
```

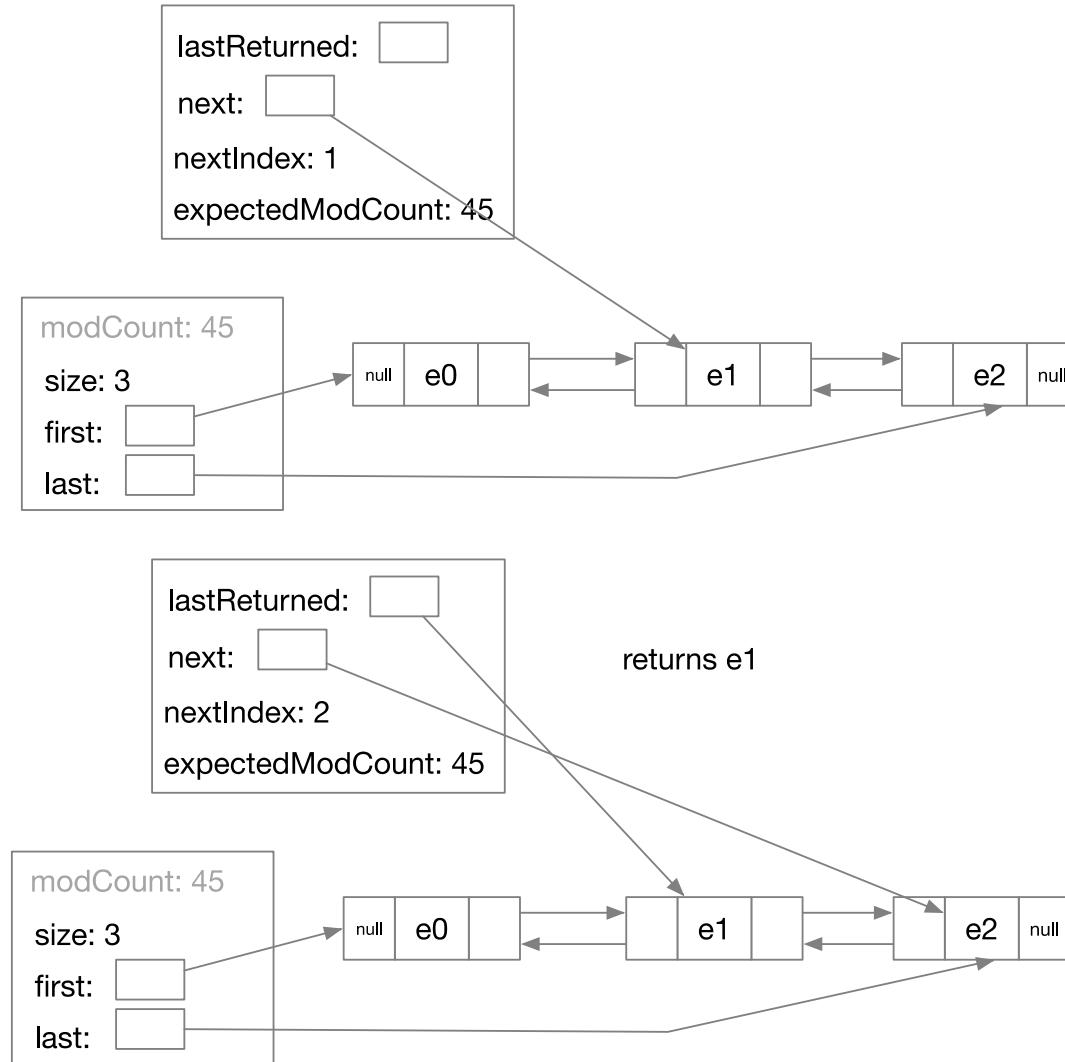
- ListItr is an inner class that implements iterators over LinkedList<E>
  - It's non-static so it has access to the instance of list it is iterating
- It is represented by:
  - **lastReturned**: reference to the node of the last returned element by next / previous (for add / set / remove) or null
  - **next**: node of the element to be returned by next
  - **nextIndex**: index of the element to be returned by next
  - **expectedModCount**: to detect co-modification

# LinkedList<E> (ListIterator<E>)

```
public boolean hasNext() {
    return nextIndex < size;
}

public E next() {
    checkForComodification();
    if (!hasNext())
        throw new NoSuchElementException();

    lastReturned = next;
    next = next.next;
    nextIndex++;
    return lastReturned.item;
}
```

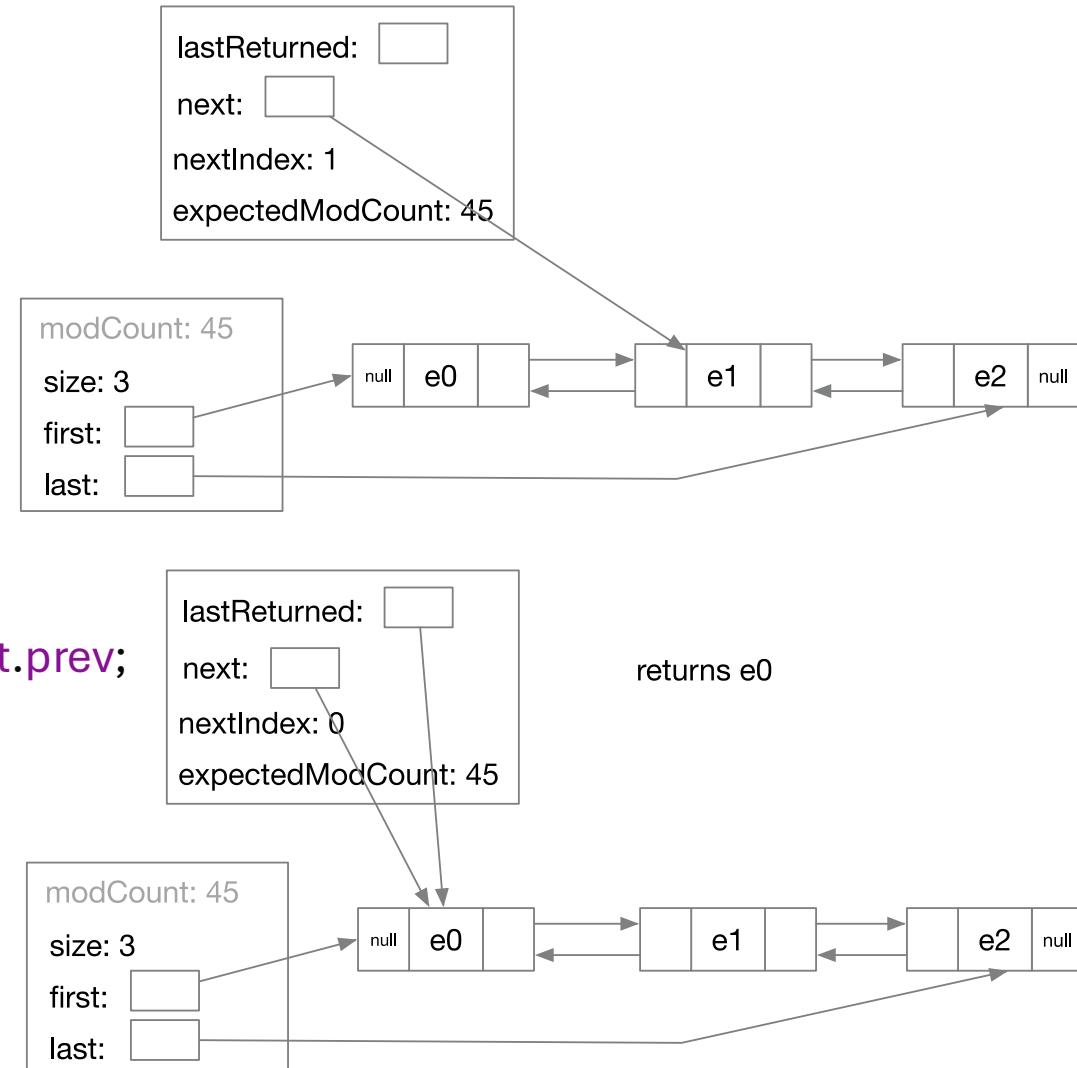


# LinkedList<E> (ListIterator<E>)

```
public boolean hasPrevious() {
    return nextIndex > 0;
}

public E previous() {
    checkForComodification();
    if (!hasPrevious())
        throw new NoSuchElementException();

    lastReturned = next = (next == null) ? last : next.prev;
    nextIndex--;
    return lastReturned.item;
}
```

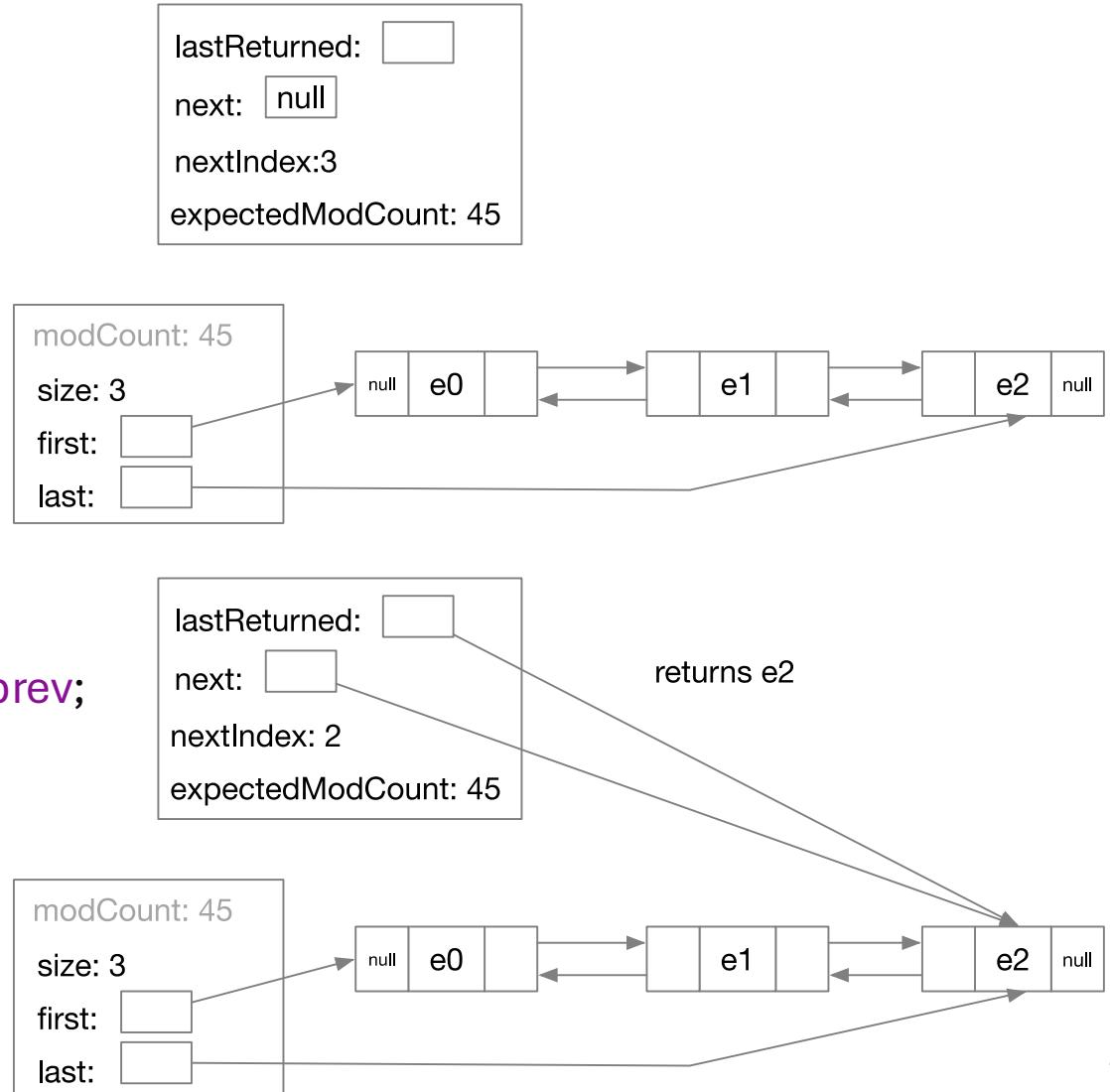


# LinkedList<E> (ListIterator<E>)

```
public boolean hasPrevious() {
    return nextIndex > 0;
}

public E previous() {
    checkForComodification();
    if (!hasPrevious())
        throw new NoSuchElementException();

    lastReturned = next = (next == null) ? last : next.prev;
    nextIndex--;
    return lastReturned.item;
}
```



# LinkedList<E> (ListIterator<E>)

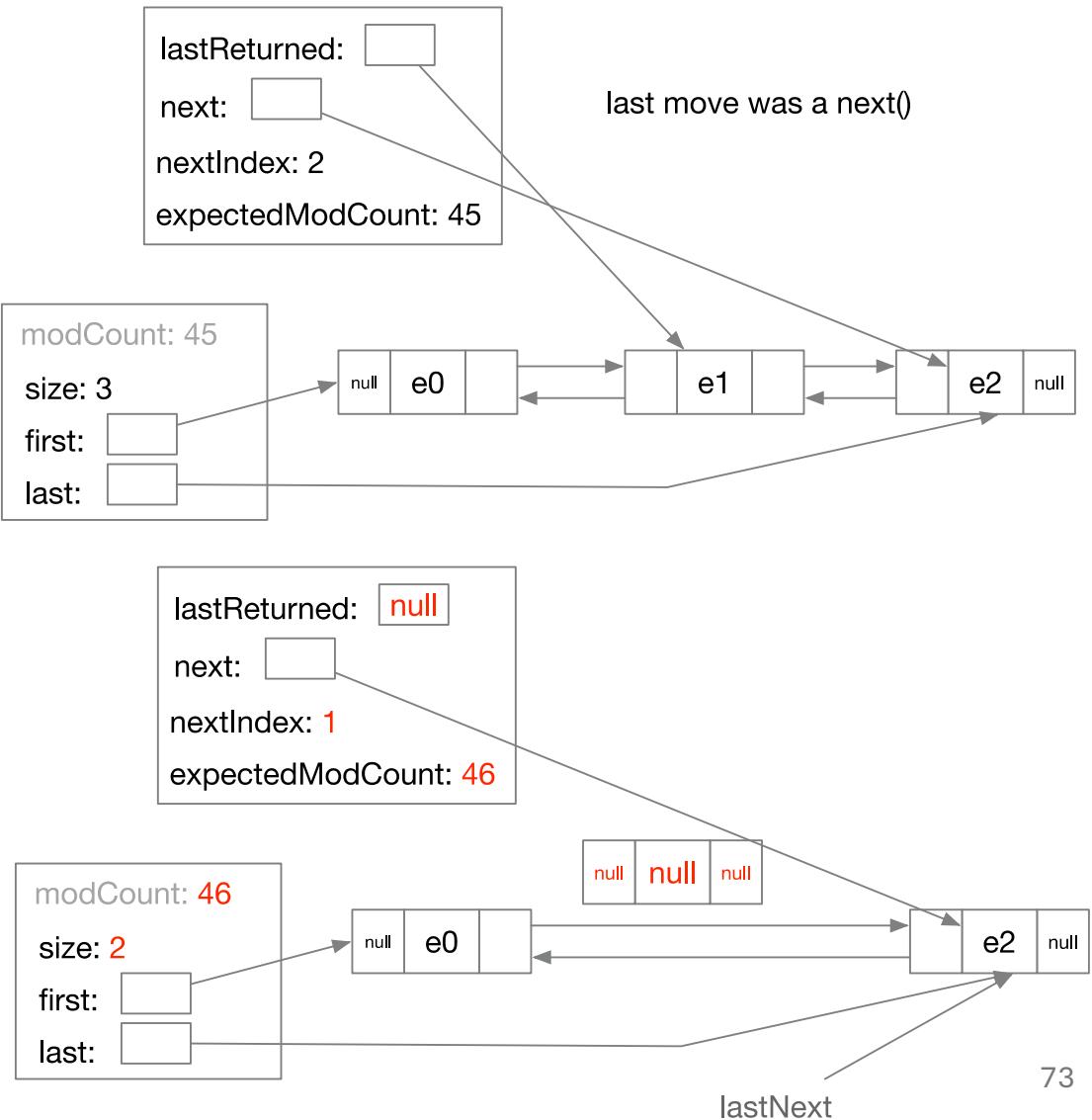
```
public int nextIndex() {  
    return nextIndex;  
}  
  
public int previousIndex() {  
    return nextIndex - 1;  
}  
  
public void set(E e) {  
    if (lastReturned == null)  
        throw new IllegalStateException();  
    checkForComodification();  
    lastReturned.item = e;  
}
```

```
public void add(E e) {  
    checkForComodification();  
    lastReturned = null;  
    if (next == null)  
        linkLast(e);  
    else  
        linkBefore(e, next);  
    nextIndex++;  
    expectedModCount++;  
}  
  
final void checkForComodification() {  
    if (modCount != expectedModCount)  
        throw new ConcurrentModificationException();  
}
```

# LinkedList<E> (ListIterator<E>)

```
public void remove() {
    checkForComodification();
    if (lastReturned == null)
        throw new IllegalStateException();

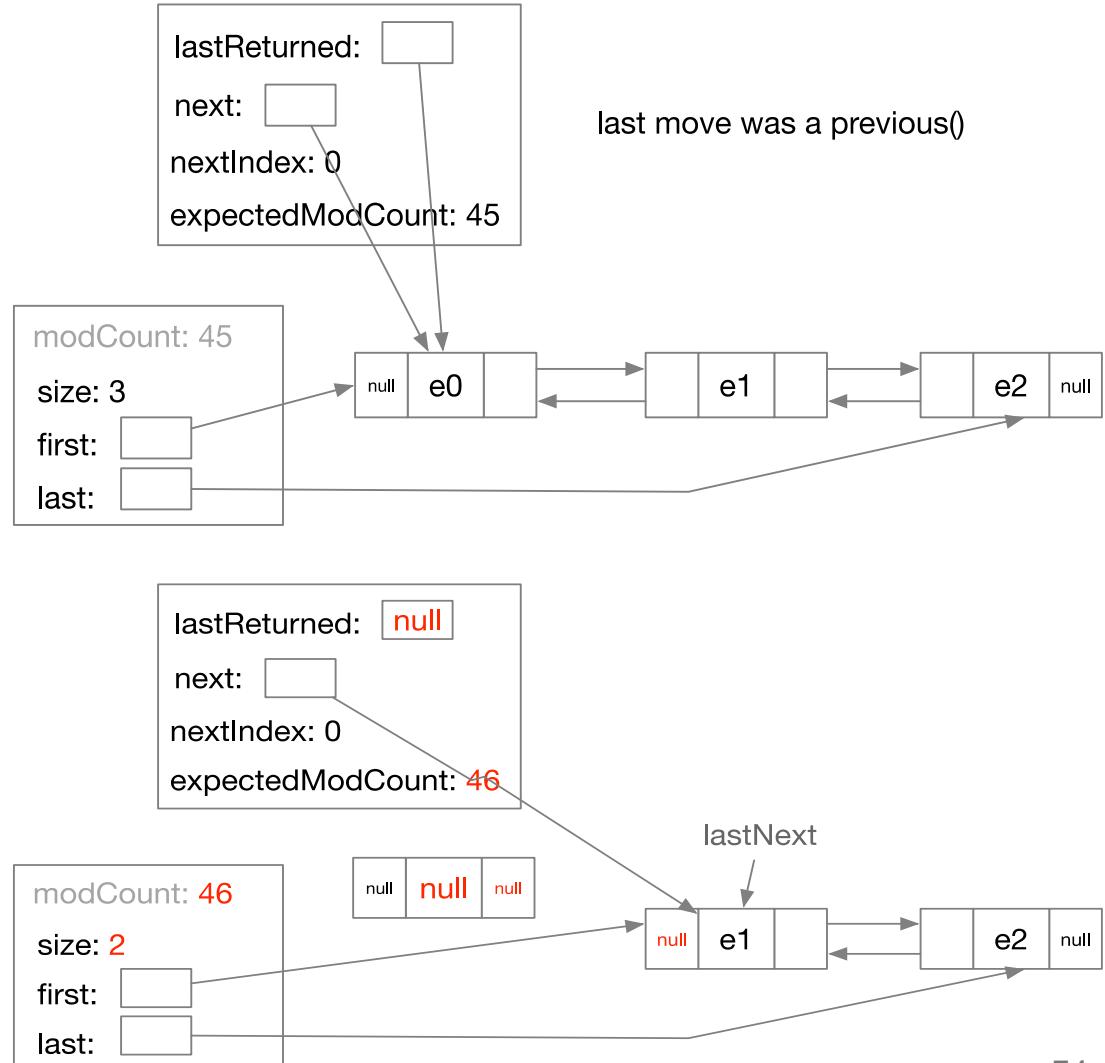
    Node<E> lastNext = lastReturned.next;
    unlink(lastReturned);
    if (next == lastReturned)
        next = lastNext;
    else
        nextIndex--;
    lastReturned = null;
    expectedModCount++;
}
```



# LinkedList<E> (ListIterator<E>)

```
public void remove() {
    checkForComodification();
    if (lastReturned == null)
        throw new IllegalStateException();

    Node<E> lastNext = lastReturned.next;
    unlink(lastReturned);
    if (next == lastReturned)
        next = lastNext;
    else
        nextIndex--;
    lastReturned = null;
    expectedModCount++;
}
```



# Costs of the operations

- The **most important factors** to consider are:
  - **ArrayList<E>** must **move elements** to make room for the new element, or to remove the "hole" created by a removal
    - And that takes **linear time**  $\mathcal{O}(n)$
  - **LinkedList<E>** must do a **sequential search** in all the operations that use the **index**
    - And that takes **linear time**  $\mathcal{O}(n)$
  - Besides, in the **ArrayList<E>** implementation, even when we are adding the last element, and we do not need to displace elements, if the array is **full**, we must do a **resize**
    - Another **linear time**  $\mathcal{O}(n)$

# Amortized Cost

- Let's begin with the **cost of resizing** the array when adding the last element
- This operation is **not always done**, only when the **array is full**
  - **Most of the time it is not needed**, so most of the time, add is  $\mathcal{O}(1)$
  - But, sometimes, due to resizing, the operation is  $\mathcal{O}(n)$
- **Amortized cost allows us to share the cost of a sequence of operations** between them
  - The idea is well known in accountability
  - E.g. we compute the cost of a new computer not to the first project we do with it but among all the projects we forecast to do with it during all its live

# Amortized Cost

- To simplify the math, we'll consider
  - The initial size of the array is 1
  - We double its size when full
- Suppose we are about to add the element  $n + 1$ , and the array is full
  - What is the cost of all the copies done until now?
  - $1 + 2 + 4 + 8 + \dots + n = 2n - 1$
- So, this is the cost to amortize among the  $n + 1$  insertions
  - $\frac{2n-1}{n+1} = \mathcal{O}(1)$
- This approach to assign amortized costs is called **aggregate analysis** (a most complete strategy is the **potential method**).

# Amortized Cost

- Let's generalize this result
  - Initial size is  $I$
  - We'll use  $k > 1$  as the increment factor
- As before, we're about to insert element  $n + 1$  and the array is full
  - What is the cost of all the copies done until now?
  - $I \cdot k^0 + I \cdot k^1 + I \cdot k^2 + \dots + I \cdot n = I \cdot \sum_{i=0}^{\log_k n} k^i$
  - $I \cdot \sum_{i=0}^{\log_k n} k^i = I \cdot \frac{k \cdot k^{\log_k n} - k^0}{k-1} = I \cdot \frac{k \cdot n - 1}{k-1} \approx \mathcal{O}(n)$
- So, as before, the amortized cost is  $\frac{\mathcal{O}(n)}{n+1} = \mathcal{O}(1)$ 
  - NOTE: The amortized cost would be linear if, instead of multiplying by  $k$ , we'd add a fixed amount of elements at each resize, cause the total cost of the copies would be quadratic.

# Costs of the operations on List<E>

Operation	ArrayList<E>	LinkedList<E>
Adding first element	$\mathcal{O}(n)$	$\mathcal{O}(1)$
Adding with index	$\mathcal{O}(n)$	$\mathcal{O}(n)$
Adding last element	$\mathcal{O}(n)$ , $\mathcal{O}(1)$ amortized	$\mathcal{O}(1)$
Removing first element	$\mathcal{O}(n)$	$\mathcal{O}(1)$
Removing with index	$\mathcal{O}(n)$	$\mathcal{O}(n)$
Removing last element	$\mathcal{O}(1)$	$\mathcal{O}(1)$
Getting first element	$\mathcal{O}(1)$	$\mathcal{O}(1)$
Getting with index	$\mathcal{O}(1)$	$\mathcal{O}(n)$
Getting last element	$\mathcal{O}(1)$	$\mathcal{O}(1)$
Setting first element	$\mathcal{O}(1)$	$\mathcal{O}(1)$
Setting with index	$\mathcal{O}(1)$	$\mathcal{O}(n)$
Setting last element	$\mathcal{O}(1)$	$\mathcal{O}(1)$

# Costs of the operations on List<E>

Operation	ArrayList<E>	LinkedList<E>
Create iterator at first position	$\mathcal{O}(1)$	$\mathcal{O}(1)$
Create iterator at end position	$\mathcal{O}(1)$	$\mathcal{O}(1)$
Create iterator at index	$\mathcal{O}(1)$	$\mathcal{O}(n)$
Adding with iterator	$\mathcal{O}(n)$ , $\mathcal{O}(1)$ amortized if at end	$\mathcal{O}(1)$
Removing with iterator	$\mathcal{O}(n)$ , $\mathcal{O}(1)$ if at end	$\mathcal{O}(1)$
Getting with iterator	$\mathcal{O}(1)$	$\mathcal{O}(1)$
Setting with iterator	$\mathcal{O}(1)$	$\mathcal{O}(1)$

# Costs of the operations on List<E>

- In general, if we do not know the implementation, and we must do sequential operations, use iterators and not indices
  - If you must modify the underlying list, do the operation at the end (no elements are moved even in the ArrayList<E> case)
- Marker interface RandomAccess signals if the implementation provides efficient index access
- If the implementation of the function must use index operation
  - Consider asking for an ArrayList<E> instead of a List<E>

# Queue<E> (Deque<E>)

- A Queue<E> is a structure that follows a FIFO policy (First In, First Out)
  - So, you can remove/get the first element, and add the last one
  - A Deque<E> is a structure that combines both allowing adding, removal and access at both ends
- To compare, a Stack<E> follows a LIFO policy (Last In, First Out)
  - We can implement it using both the ideas of ArrayList<E> and LinkedList<E> (using single-linked nodes)
- The LinkedList<E> class implements both Queue<E> and Deque<E> interfaces
- So, can we implement Queue<E> (or Deque<E>) backed by an array data store?

# Queue<E>

- It is an extension of Collection<E> which adds operations that follow the FIFO policy
- The two sets of operations differ in the way to signal errors:
  - **add** throws **IllegalStateException** if the queue has capacity restrictions and is full; **offer** returns **false** in this case
  - **remove / element** throw **NoSuchElementException** if the queue is empty; **poll / peek** return **null** (which can cause ambiguity)

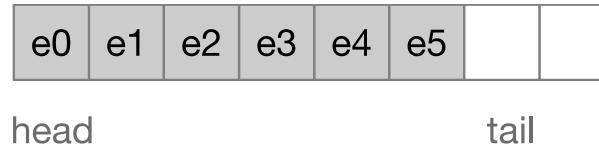
Operation	Throws Exception	Returns special value
Insert (new last element)	boolean add(E e)	boolean offer(E e)
Remove (the first element)	E remove()	E poll()
Examine (the first element)	E element()	E peek()

# Queue<E>

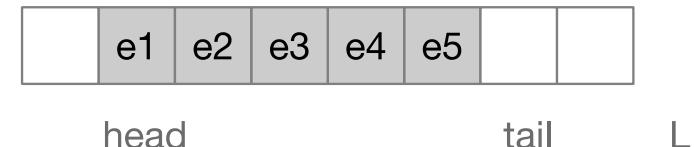
- Using directly an ArrayList<E> won't work.
- If we store the elements in the normal order:
  - add/offer: constant amortized time
  - remove/poll: linear time
  - element/peek: constant time
- If we store the elements in the reverse order:
  - add/offer: linear time
  - remove/poll: constant time
  - element/peek: constant time
- In both cases one of the operations is linear !!

# Queue<E>

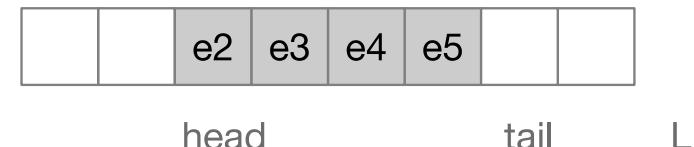
- We can use a strategy known as **circular array**, which consists of
  - An array of elements
  - Two indexes:
    - head: index of the first element of the queue
    - tail: index for the next element to be added to the queue
- And these indexes, when arriving at the end of the array, come around to the beginning !!
- When head == tail we need size to know if the queue is empty or full !!



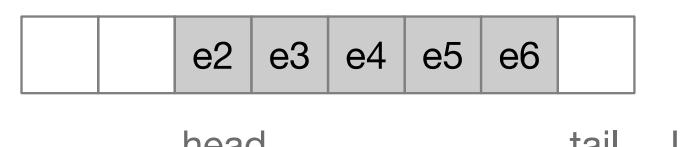
remove() returns e0



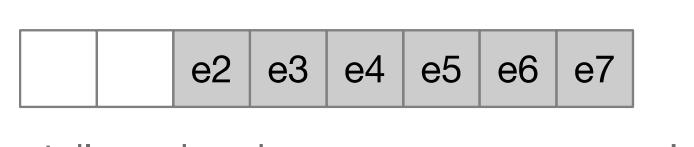
remove() returns e1



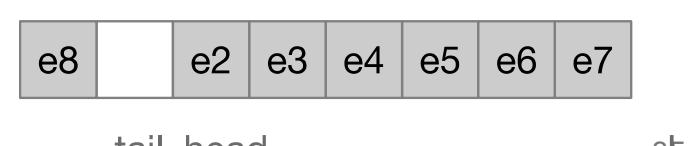
add(e6) returns true



add(e7) returns true



add(e8) returns true



# ArrayDeque<E>

- This idea is what is used by the class ArrayDeque<E> which implements both Queue<E> and Deque<E>
- The implementation is a little bit much complicated because the array is resizable.
- If you want to understand array manipulation code is a very interesting class to study !!