Large Scale Machine Learning

Part 2: Distributed Logistic Regression

Previously on Large Scale ML

- Definition of Large Scale ML
- Overview of Large Scale ML software and hardware paradigms
- Large Scale ML on your machine

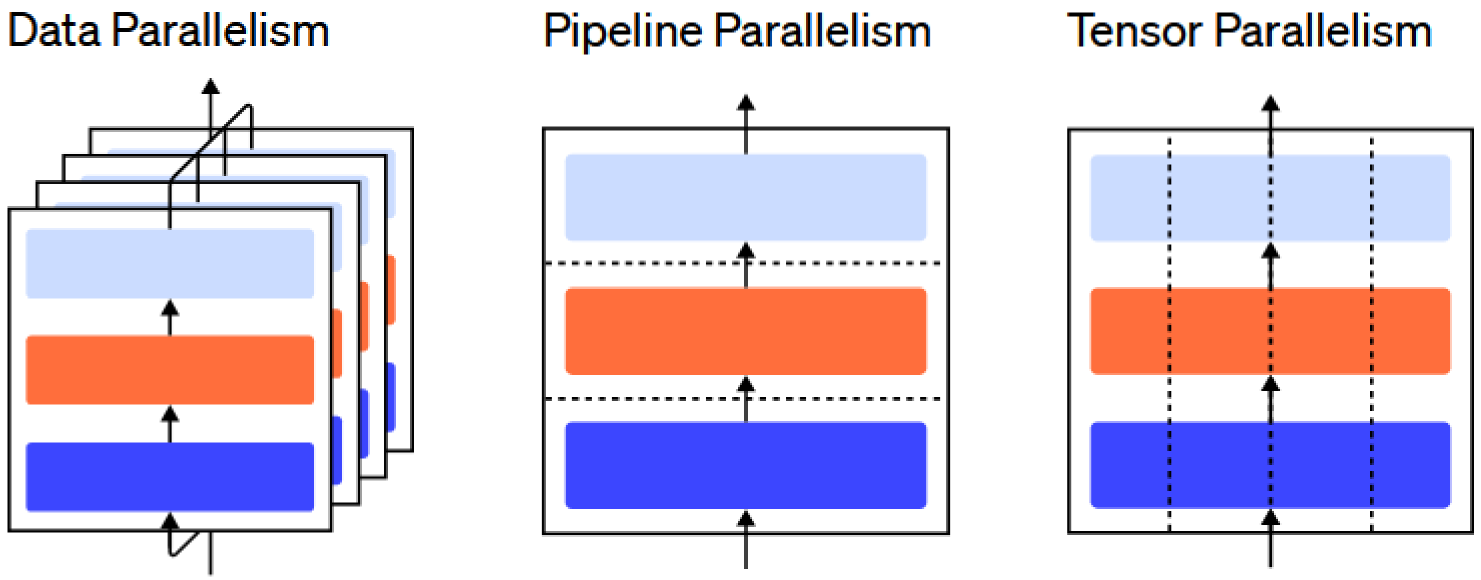
This episode

- Distributed Logistic Regression
  - Synchronous Distributed Gradient Descent
  - Second order methods
- Other distribution strategies
  - Parameter Server
  - AllReduce

Distributed Computing

- Using multiple networked computation nodes (computers)
- Nodes communicate and coordinate their actions by passing data on the network
- Distributed ML
    - Designing algorithms that work efficiently on distributed systems

How to distribute ?



Logistic Regression

- We are going to use logistic regression as a show case for some of the techniques shown
  - Most of what we are going to say applied to most parametric models that can be optimized with Gradient Descent

- Logistic Regression is used everywhere
  - Most common classification algorithm, can be quite very flexible
  - Criteo trains thousands of LR models per day and uses them to do billions of predictions

Logistic Regression

- Let’s start with a well-known algorithm to show case the design ideas
- M data points $(x, y)_i \; - \; i \; = 1..M$
- Parametrized model function $f_\theta(x) = \hat{y}$ Logistic model : $f_\theta(x)$:= Sigmoid$(x.\theta)$
- Loss function
  - For each data point

$$l(y, \hat{y}) = l\big(y, f_\theta(x)\big) = -\,\mathrm{y}\,.\,\mathrm{log}(\hat{y}) - (1\; - y).log(1\; - \;\hat{y})$$

Log Proba($Y_i = y_i$ | model $_\theta$, x_i)

  - For the whole dataset

$$L(\theta) = \frac{1}{M}\sum_{i=1}^{M} l(y_i, \widehat{y_i})$$

- Objective

$$argmin_\theta \; L(\theta)$$

Gradient and stochastic gradient

- Gradient descent
  - Repeat until convergence

$$\theta_{t+1} \longleftarrow \theta_t \; - \; \alpha_t \nabla L(\theta)$$

- Stochastic gradient descent
  - Repeat until convergence
    - Sample random i from $\{1..M\}$

$$\theta_{t+1} \longleftarrow \theta_t \; - \; \alpha_t \nabla_\theta l(y_i \, , \widehat{y_i})$$

Gradient and stochastic gradient

- Mini-batch stochastic gradient descent
  - Repeat until convergence
    - Sample a batch of data of size b

$$L_b(\theta) = \frac{1}{b}\sum_{i=1}^{b} l(y_i, \hat{y}_i)$$

$$\theta_{t+1} \longleftarrow \theta_t - \alpha_t \nabla_\theta L_b(\theta)$$

- Stochastic and mini-batch gradient descent rely on

$$E\left(\nabla_\theta l(y_i, \hat{y}_i)\right) = E\left(\nabla_\theta L_b(\theta)\right) = \nabla_\theta L(\theta)$$

Distributed Gradient Descent 101

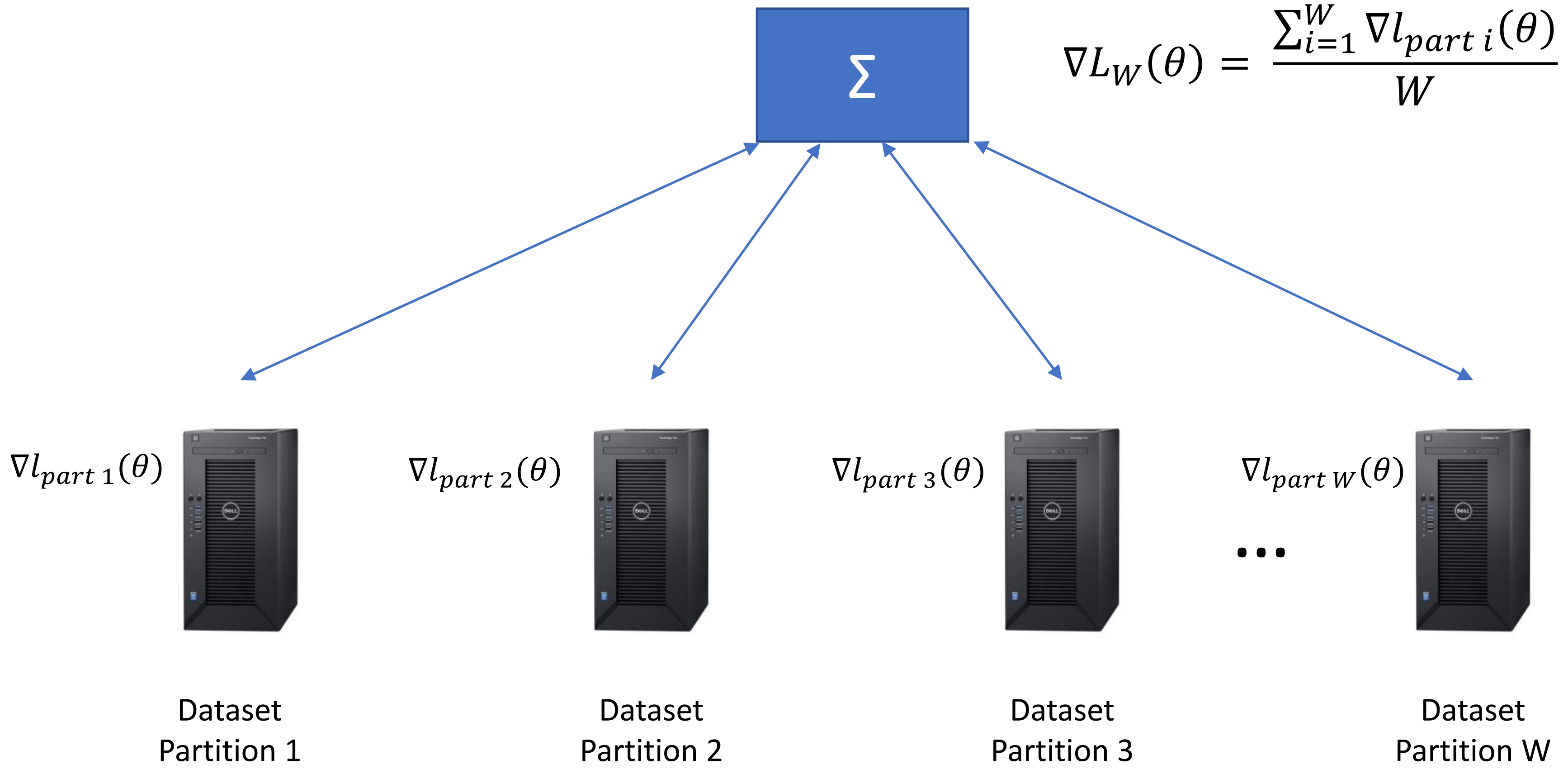
- W workers
- Each worker
  - Reads a partition of data
  - Computes a local gradient
  - Sends the gradient over to be aggregated
  - Model is updated and re-pushed to the workers
  - Rince and repeat

Distributed Gradient Descent 101

- In mini-batch SGD gradients will only be computed on a batch of data
  - Each worker will process a small batch of data

- In Full Gradient Descent it will computed on the whole dataset
  - Assuming n workers, each worker will process 1/n of the dataset

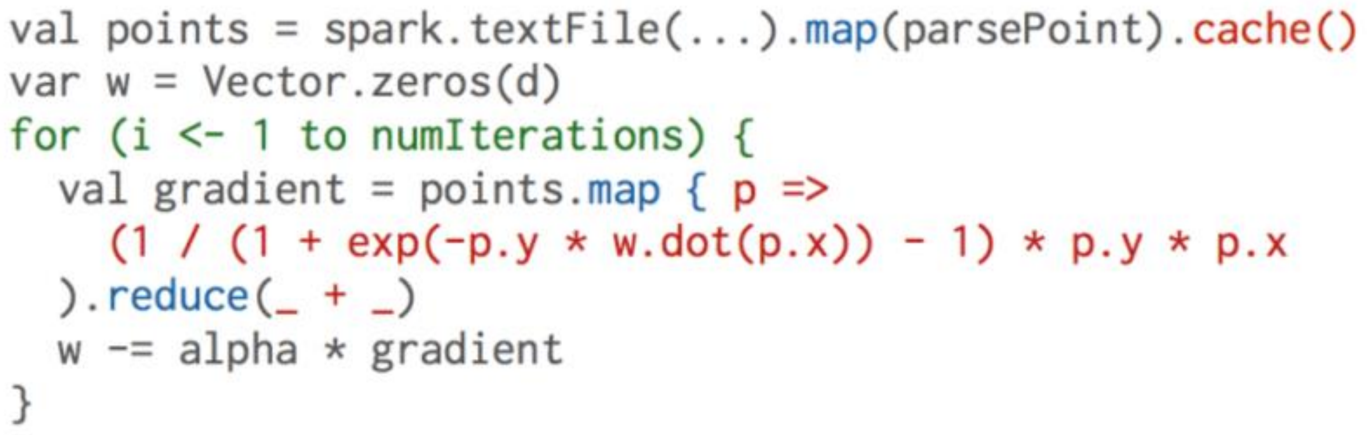
Distributed Gradient Descent 101


Σ
$\nabla l_{part\ 1}(\theta)$
$\nabla l_{part\ 2}(\theta)$
$\nabla l_{part\ 3}(\theta)$
$\nabla l_{part\ W}(\theta)$
...
Dataset Partition 1
Dataset Partition 2
Dataset Partition 3
Dataset Partition W

$$\nabla L_W(\theta) = \frac{\sum_{i=1}^{W} \nabla l_{part\ i}(\theta)}{W}$$

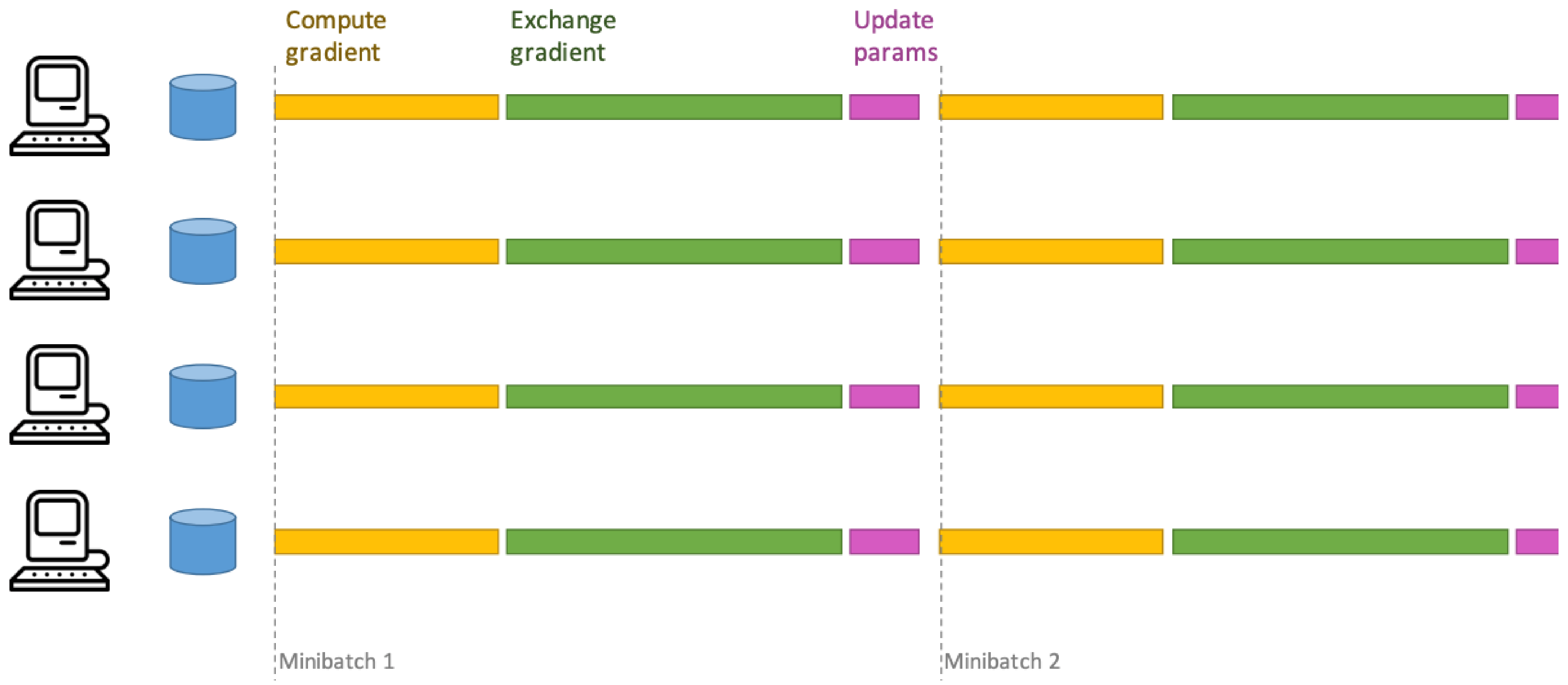
Distributed Gradient Descent 101

- Easy to implement a simple version in Spark

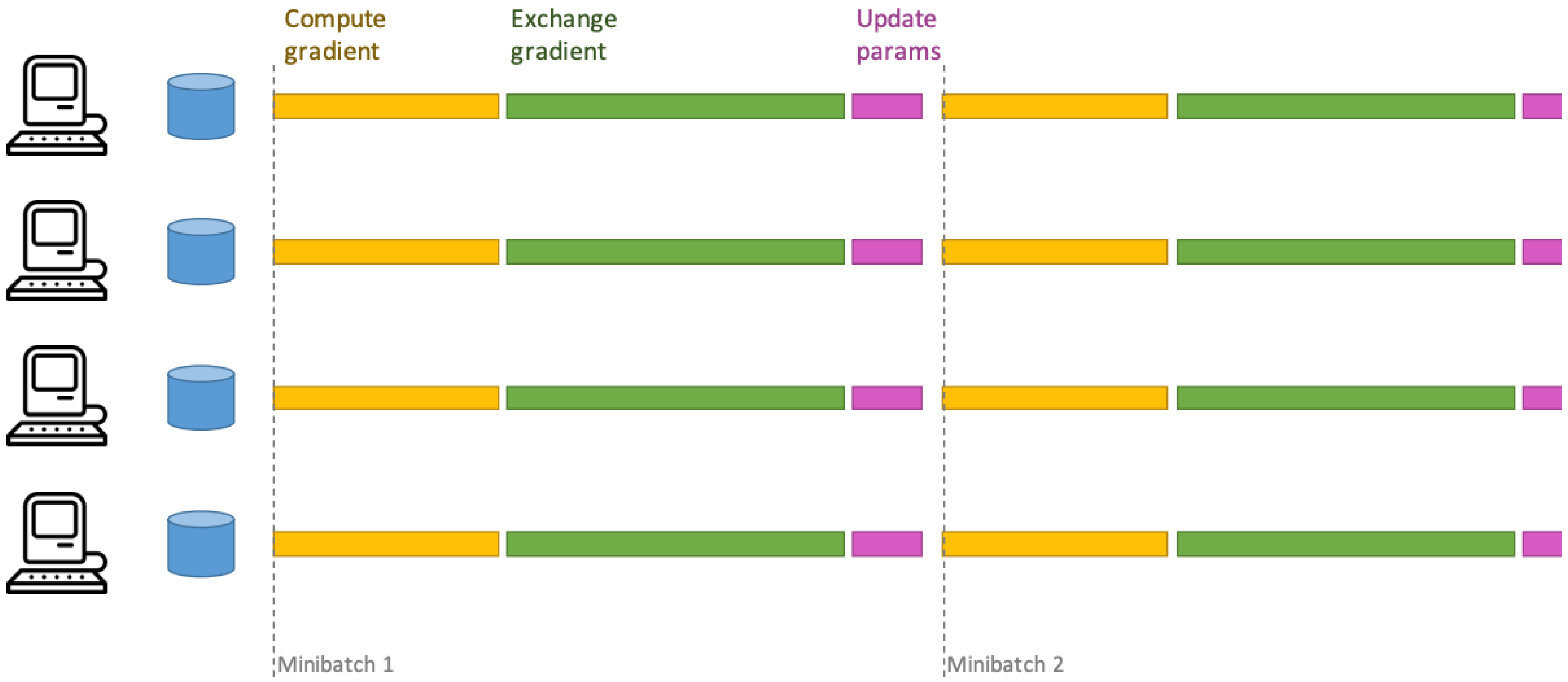


```
val points = spark.textFile(...).map(parsePoint).cache()
var w = Vector.zeros(d)
for (i <- 1 to numIterations) {
  val gradient = points.map { p =>
    (1 / (1 + exp(-p.y * w.dot(p.x)) - 1) * p.y * p.x
  ).reduce(_ + _)
  w -= alpha * gradient
}
```

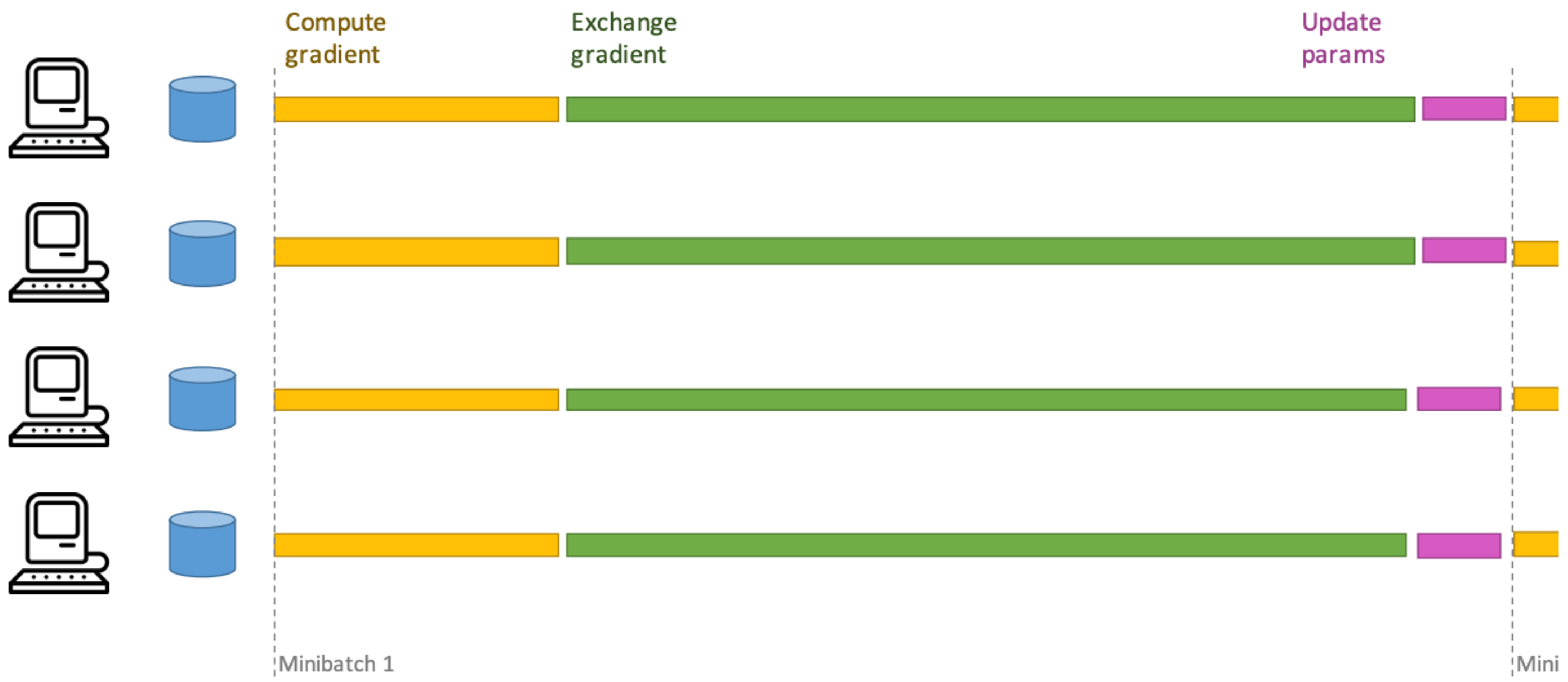
Synchronous Data Parallel SGD



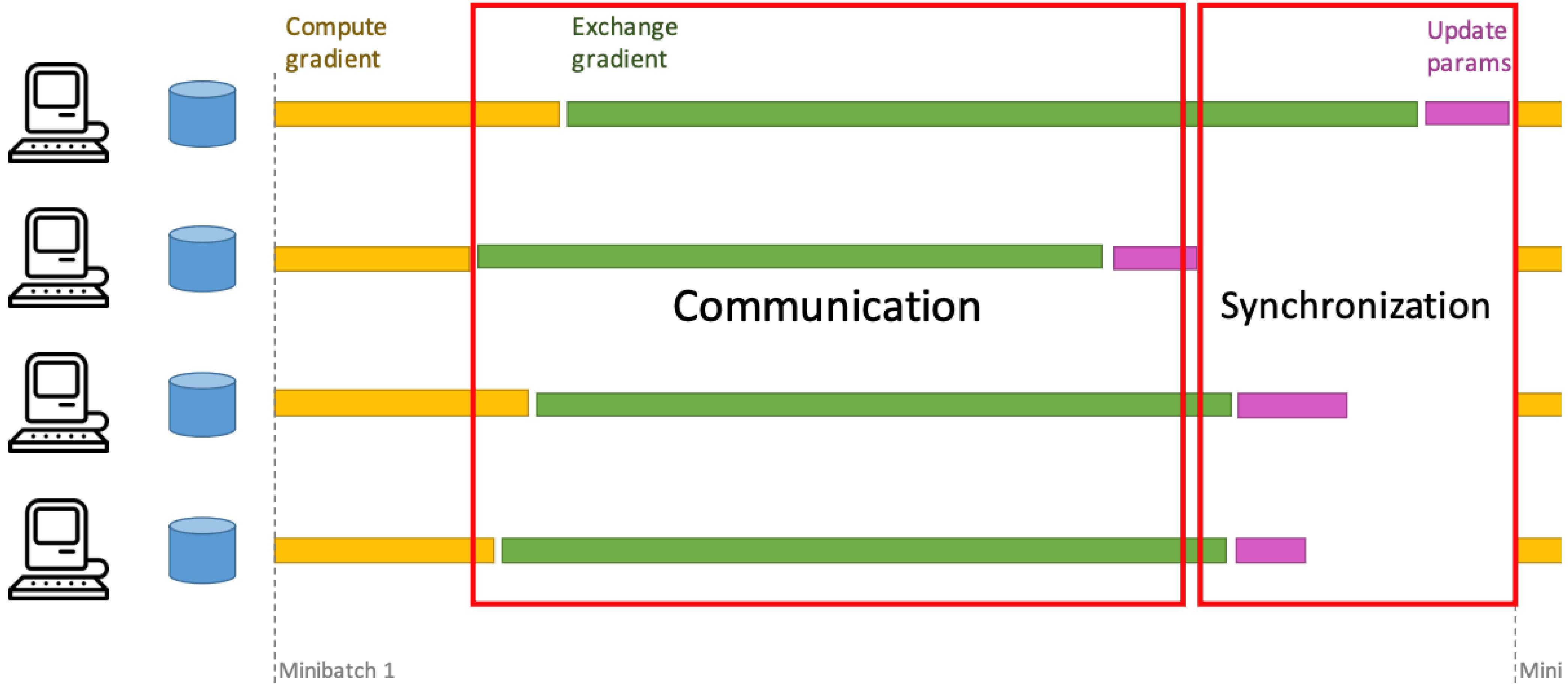
Synchronous Data Parallel SGD



Synchronous Data Parallel SGD



Synchronous Data Parallel SGD



Main Overheads

- Objective: compute the gradient in parallel

- Two main overheads
    - Communication: sending gradient updates to the driver / other workers
    - Synchronization: waiting for all the workers to execute the current iteration

- This is what is preventing from getting the theoretical speed up
    - With n workers
    - Gradient computation: T / n
    - Comm and sync cost: does not decrease with n but increase !

Communication cost

- Sparse model example
  - Total model weights M
  - But only f << M non zero features per row
  - For the log-loss defined earlier, gradient will be sparse and needs to be represented by a sparse vector

- This is the key to efficiency
  - Gradient update is sparse + sparse operation

Why is the gradient sparse

$$\nabla_\theta l(y, \hat{y}) = -\nabla_\theta [y \,.\, log\, f_\theta(x) + (1-y).\, log(1 - f_\theta(x))]$$

$$\nabla_\theta l(y, \hat{y}) = \left(y - \frac{1}{1 + e^{-x^T\theta}}\right) x$$

$$\nabla_\theta l(y, \hat{y}) = \kappa\, x$$

Sparse feature vector = sparse gradient

Some bad news

- What about regularization ?
  - L2 Regularization term $\frac{1}{2}\lambda\ \theta^T\theta$

$$\nabla_\theta \frac{1}{2}\lambda\ \theta^T\theta = \lambda\theta$$

  - The update rule becomes
    - $\theta \longleftarrow \theta\ - \alpha\ (\kappa\ x + \lambda\theta)$
    - It’s dense now ☹ no matter how sparse the feature vector is

Regularized SGD with Sparse Updates

- Let's concentrate on the first dimension of the model θ_0

- Suppose we have an update with the first dimension being zero

$$\theta_0 \longleftarrow \theta_0 - \alpha\,(\kappa\, x_0 + \lambda\theta_0)$$
$$\theta_0 \longleftarrow \theta_0 \,- \alpha\lambda\theta_0$$
$$\theta_0 \longleftarrow (1\,-\alpha\lambda)\theta_0$$

- After two updates

$$\theta_0 \longleftarrow (1\,-\alpha\lambda)^2\theta_0$$

Regularized SGD with Sparse Updates

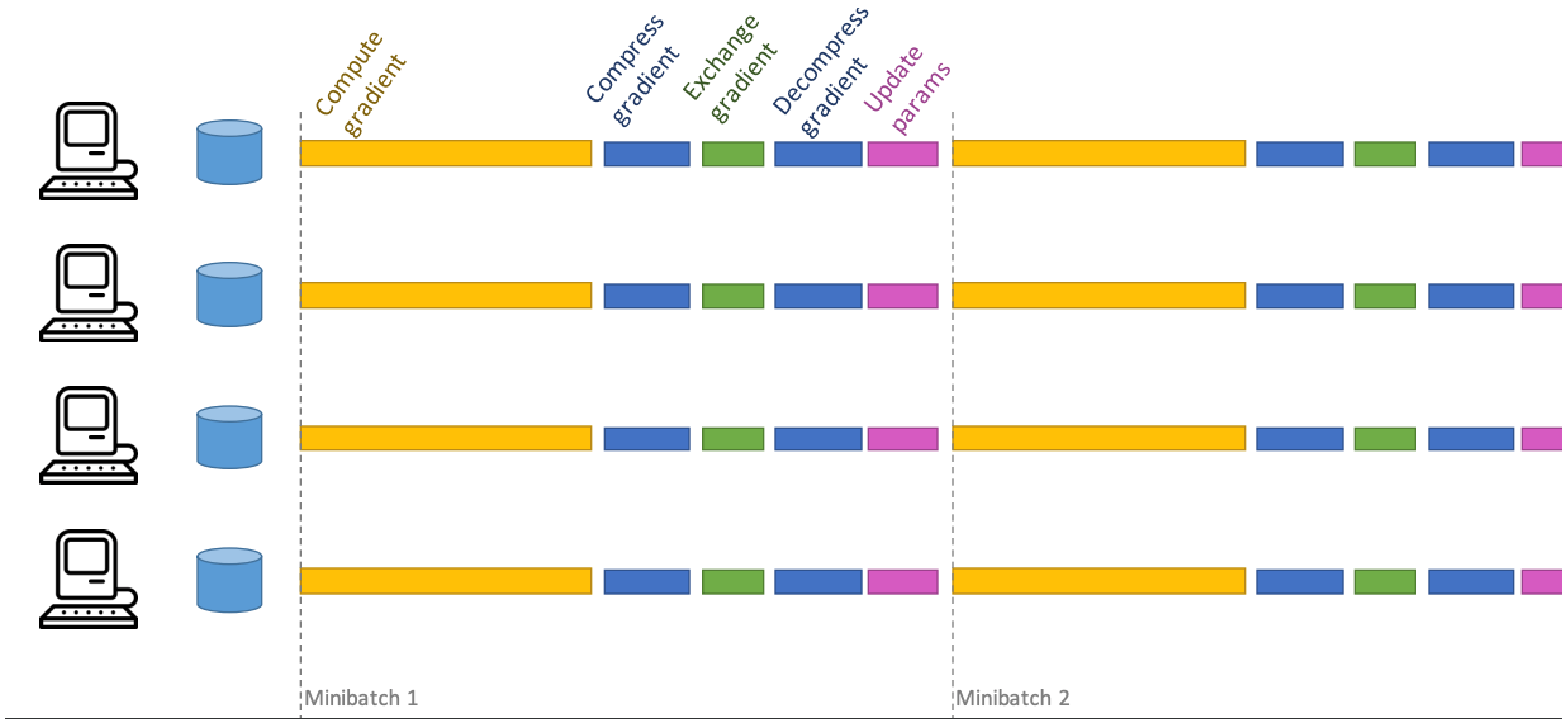
- We can then compress K computations

$$\theta_0 \longleftarrow (1 - \alpha\lambda)^K \theta_0$$

- We remember how many steps we didn’t update a particular dimension

- Catch up when needed

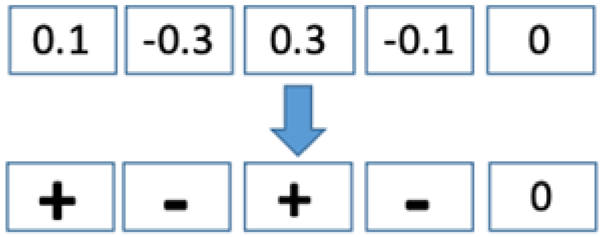
More general pattern for reducing communication cost



How can we compress a vector ?

- Lossy compression

- Quantization function
  - v vector of real values



$avg_+ = +0.2$

$avg_- = -0.2$

$$Q_i(v) = \begin{cases} avg_+ & \text{if } v_i \geq 0, \\ avg_- & \text{otherwise} \end{cases}$$

where $avg_+ = \text{mean}([v_i \text{ for } i: v_i \geq 0])$, $avg_- = \text{mean}([v_i \text{ for } i: v_i < 0])$

Why this shouldn’t work

- Remember: Mini-batch gradient descent rely on

$$E\left(\nabla_\theta L_b(\theta)\right) = \nabla_\theta L(\theta)$$

- But

$$E(Q(\nabla_\theta L_b(\theta))) \neq E\left(\nabla_\theta L_b(\theta)\right)$$

- We don’t have an unbiased estimate anymore

Nice trick: Stochastic Quantization

- Quantization function

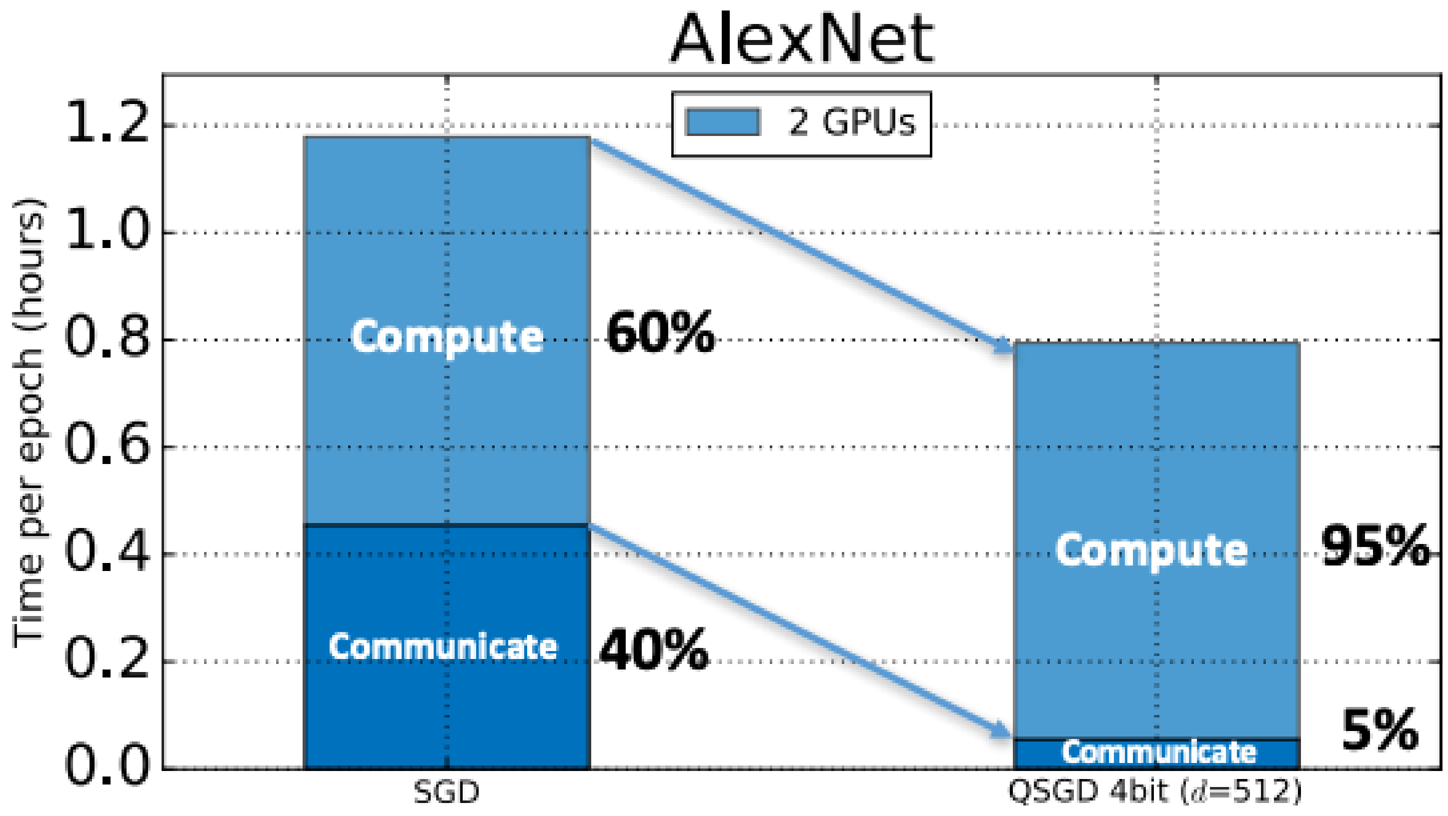
$$Q(vi) = \|v\|_2 \cdot \mathrm{sgn}(v_i) \cdot \xi_i(v_i)$$

where $\xi_i(v_i) = 1$ with probability $|v_i|/\|v\|_2$ and 0 otherwise.

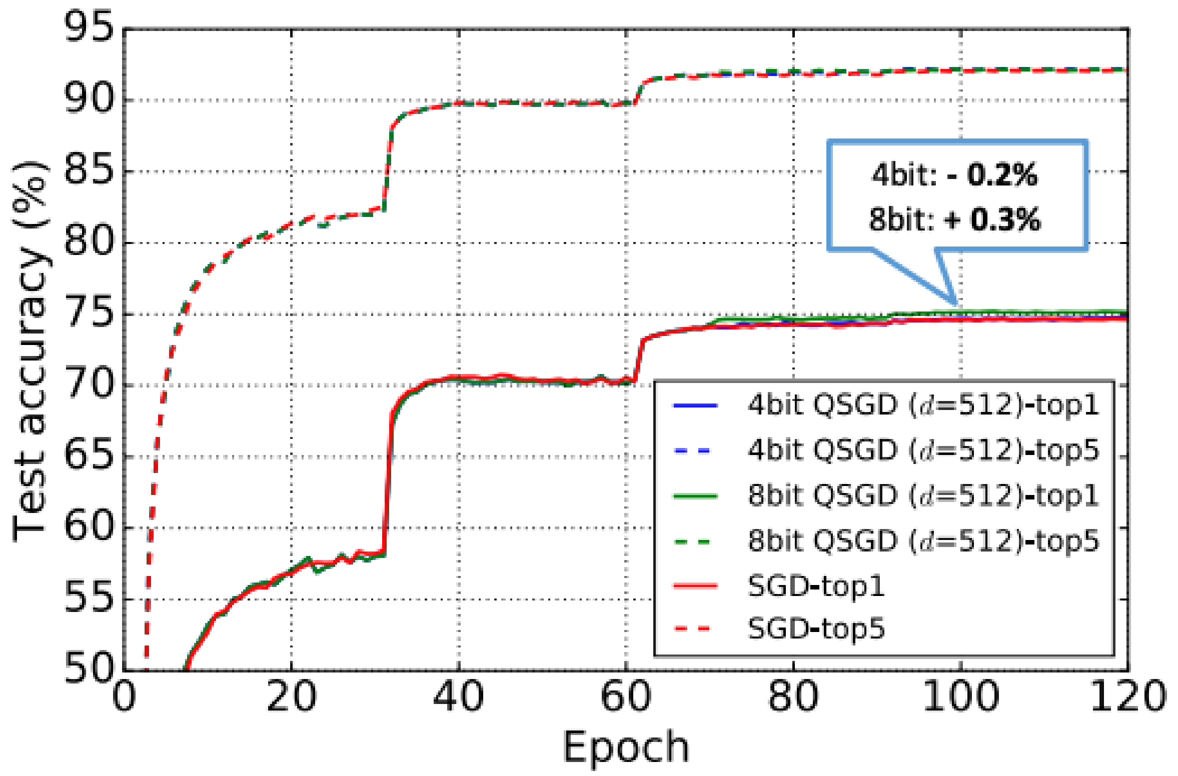
- Now it’s unbiased

$$\boldsymbol{E}\left[\boldsymbol{Q}[\boldsymbol{v}_i]\right] = \|v\|_2 \cdot \mathrm{sgn}(v_i) \cdot |v_i|/\|v\|_2 = \mathrm{sgn}(v_i) \cdot |v_i|$$

Nice trick: Stochastic Quantization



Nice trick: Stochastic Quantization

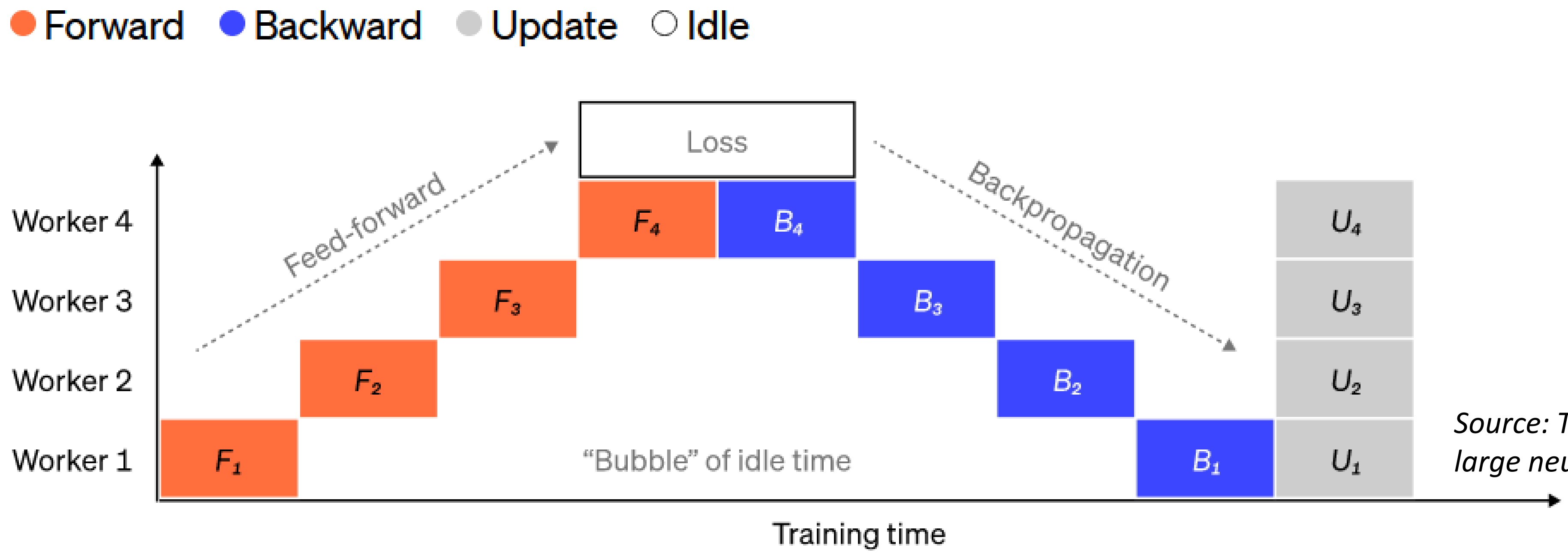


ResNet50 on ImageNet
8 GPU nodes

How about synchronization ?

- We will talk about it a bit later but here is simple solution
  - Drop the gradients of slow workers by imposing a timeout to the computation

- Other ‘Synchronization’ costs may arise when working with (deep) nets:



Source: Techniques for training large neural networks (openai)