Large Scale Machine Learning

Part 2: Distributed Logistic Regression

Synchronous Distributed SGD

- The goal is to reduce communication cost
  - Exploiting sparsity
  - Compression
  - Increasing computation

- Works reasonably well in practice
  - Can also give a good initial solution to be fine tuned with more complex methods

In Spark

```
// Load training data in LIBSVM format.
val data = MLUtils.loadLibSVMFile(sc, "data/mllib/sample_libsvm_data.txt")

// Split data into training (60%) and test (40%).
val splits = data.randomSplit(Array(0.6, 0.4), seed = 11L)
val training = splits(0).cache()
val test = splits(1)

// Run training algorithm to build the model
val model = new LogisticRegressionWithSGD()
                 .setNumClasses(10)
                 .run(training)

// Compute raw scores on the test set.
val predictionAndLabels = test.map { case LabeledPoint(label, features) =>
    val prediction = model.predict(features)
    (prediction, label)
}
```

Other ways to distribute

- Two major costs: Communication and synchronization

- We talked about reducing communication cost in different ways while staying in a synchronous centralized manner

- We will discuss two additional ways
    - Asynchronous updates: Parameter Server
    - Efficient Aggregation: All Reduce

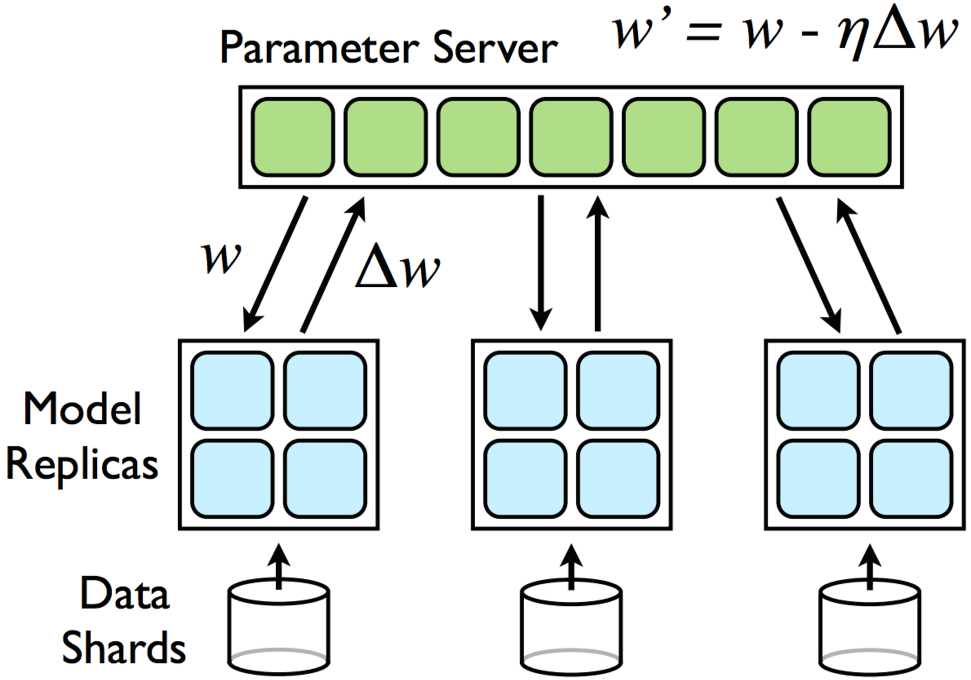
Parameter Server design rationale

- Model updates to be *generally* in the right direction
    - Not important to have strong consistency guarantees all the time
- Model updates are often sparse
    - No need to pull all model parameters and push all of them
    - Separate computation and storage

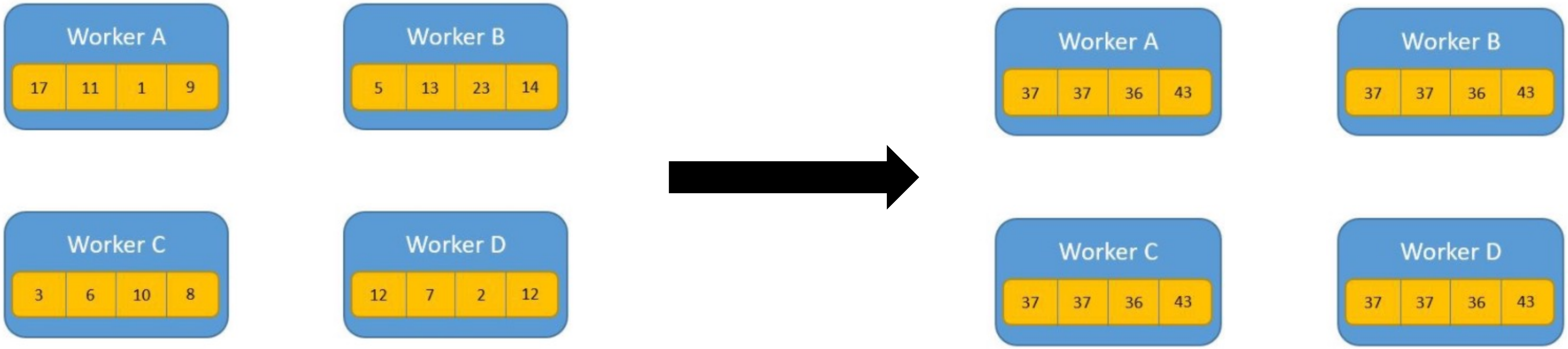
Parameter Server design rationale

- Lock free asynchronous updates
  - HOGWILD!

- Introducing asynchrony makes the system fault tolerant
  - If a worker fails while computing gradient, no need to wait, just restart the computation



AllReduce



Worker A

17	11	1	9

Worker B

5	13	23	14

Worker C

3	6	10	8

Worker D

12	7	2	12

Worker A

37	37	36	43

Worker B

37	37	36	43

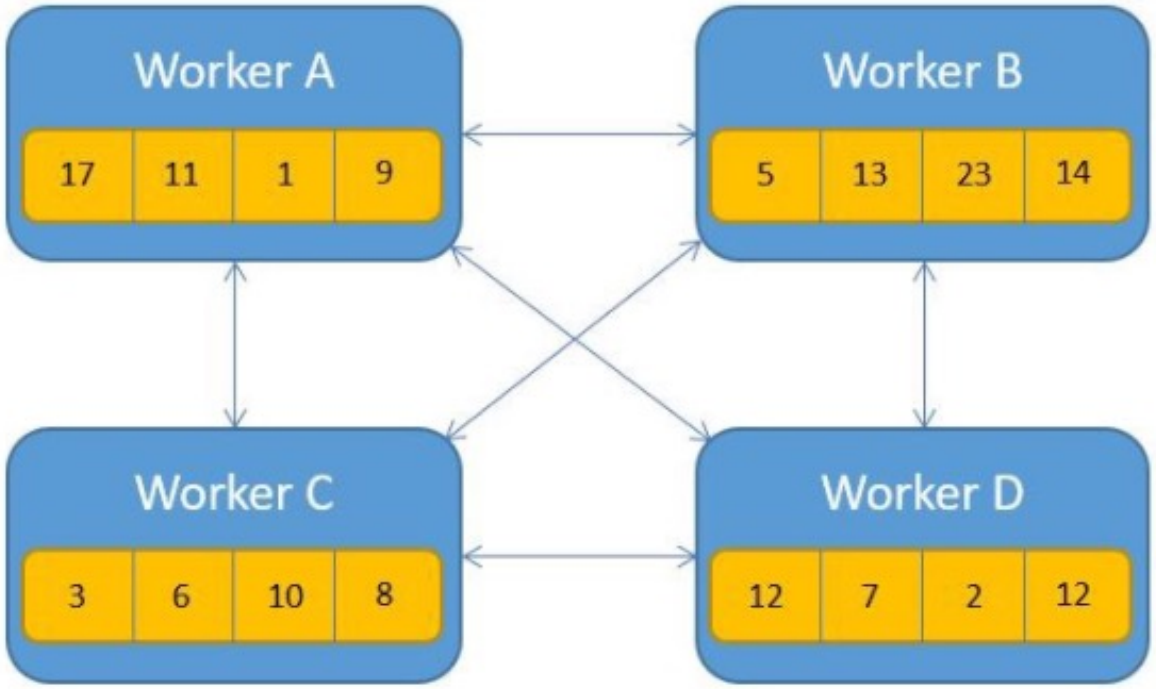
Worker C

37	37	36	43

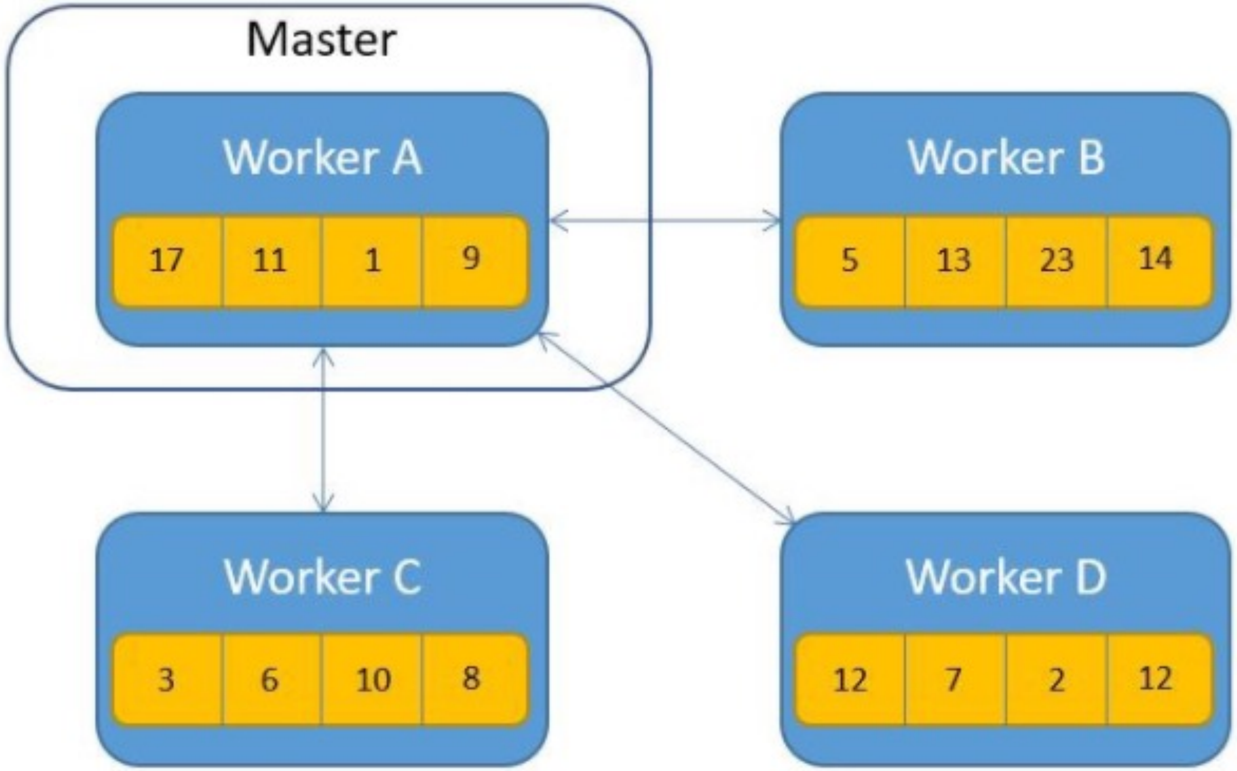
Worker D

37	37	36	43

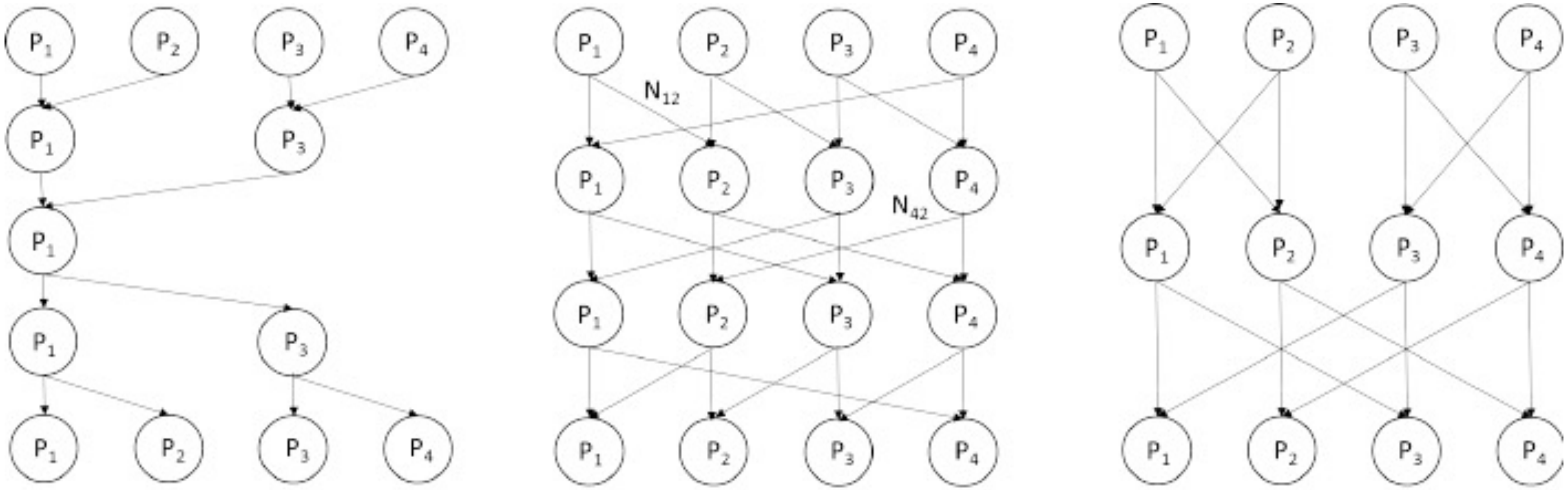
AllReduce: all to all communication



AllReduce: Reduce then Broadcast



AllReduce

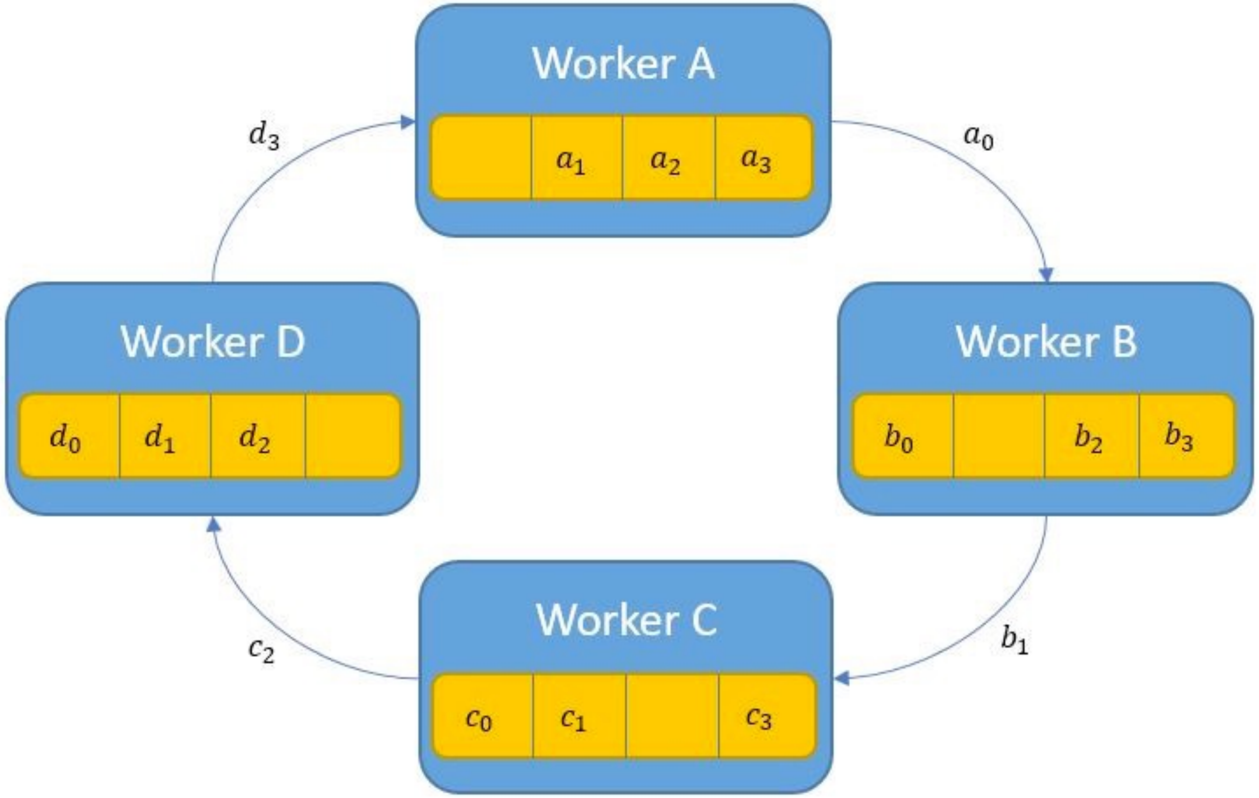


(a) Tree AllReduce

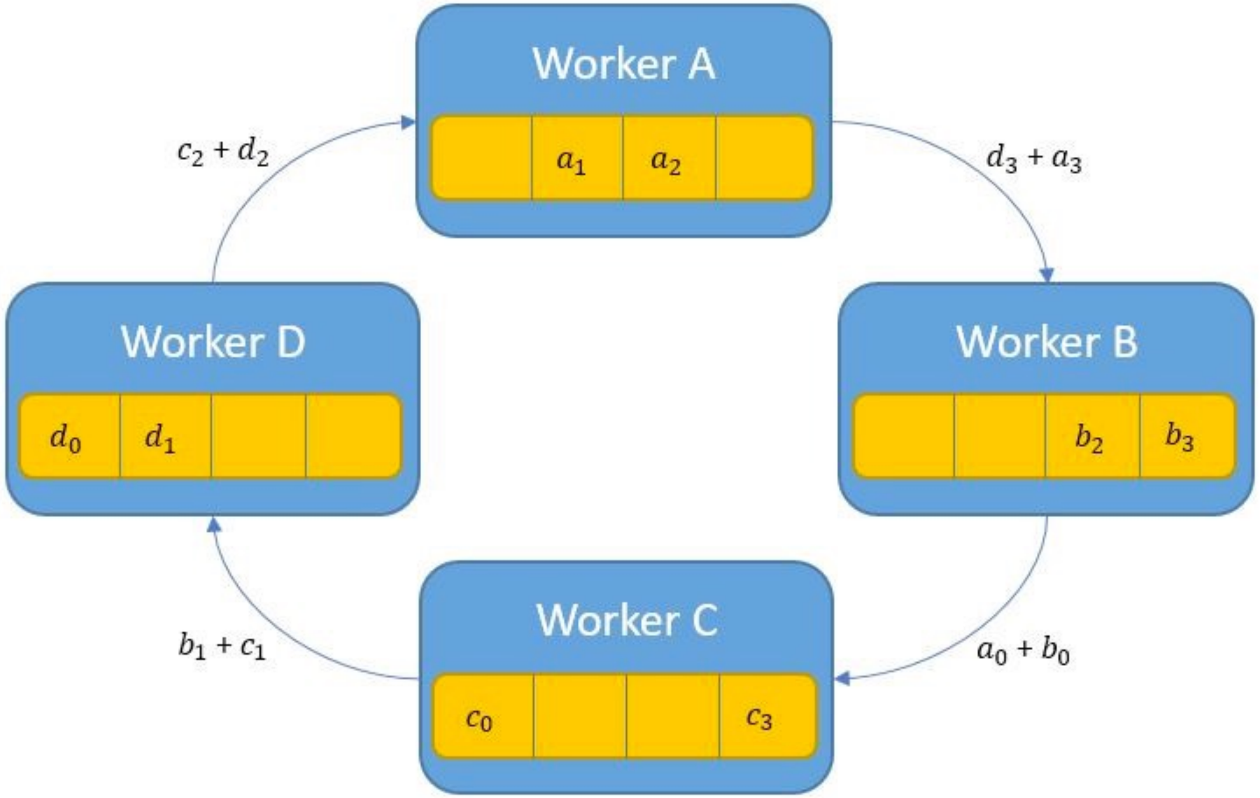
(b) Round-robin AllReduce

(c) Butterfly AllReduce

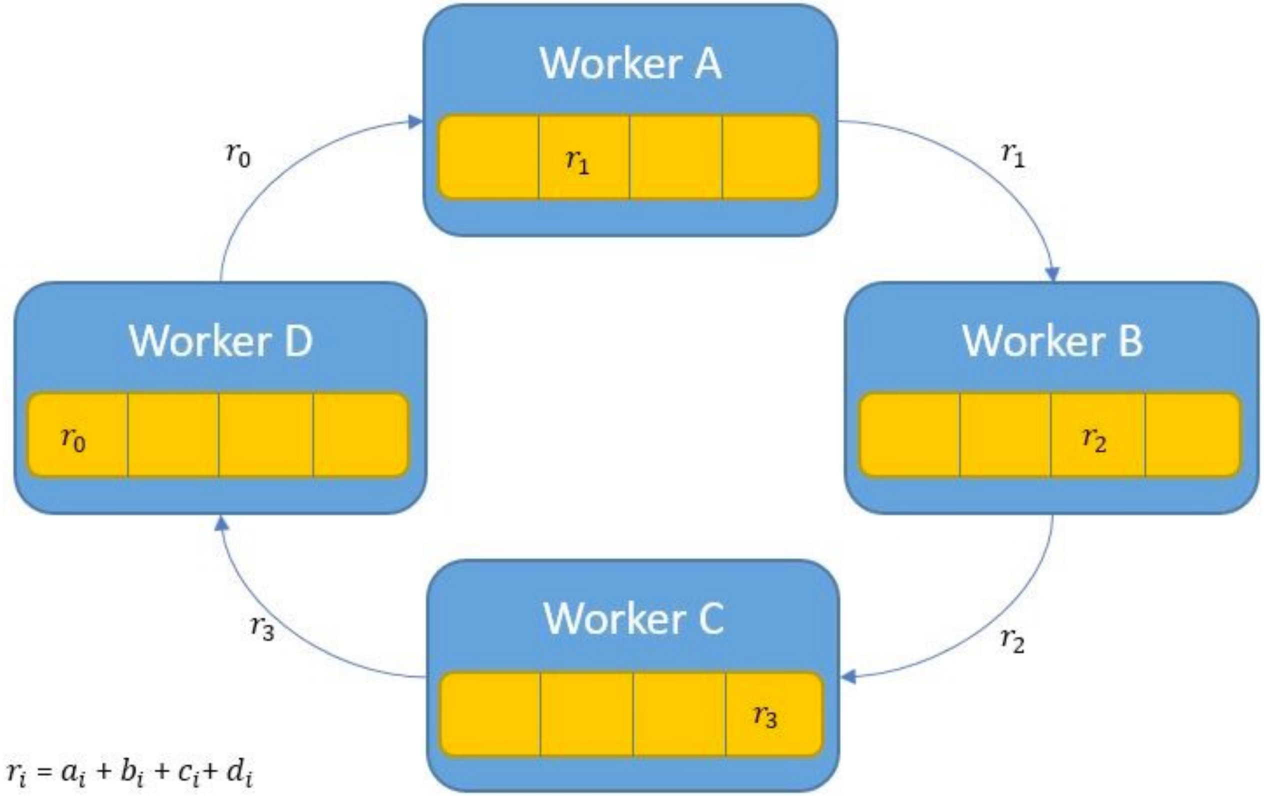
Ring AllReduce



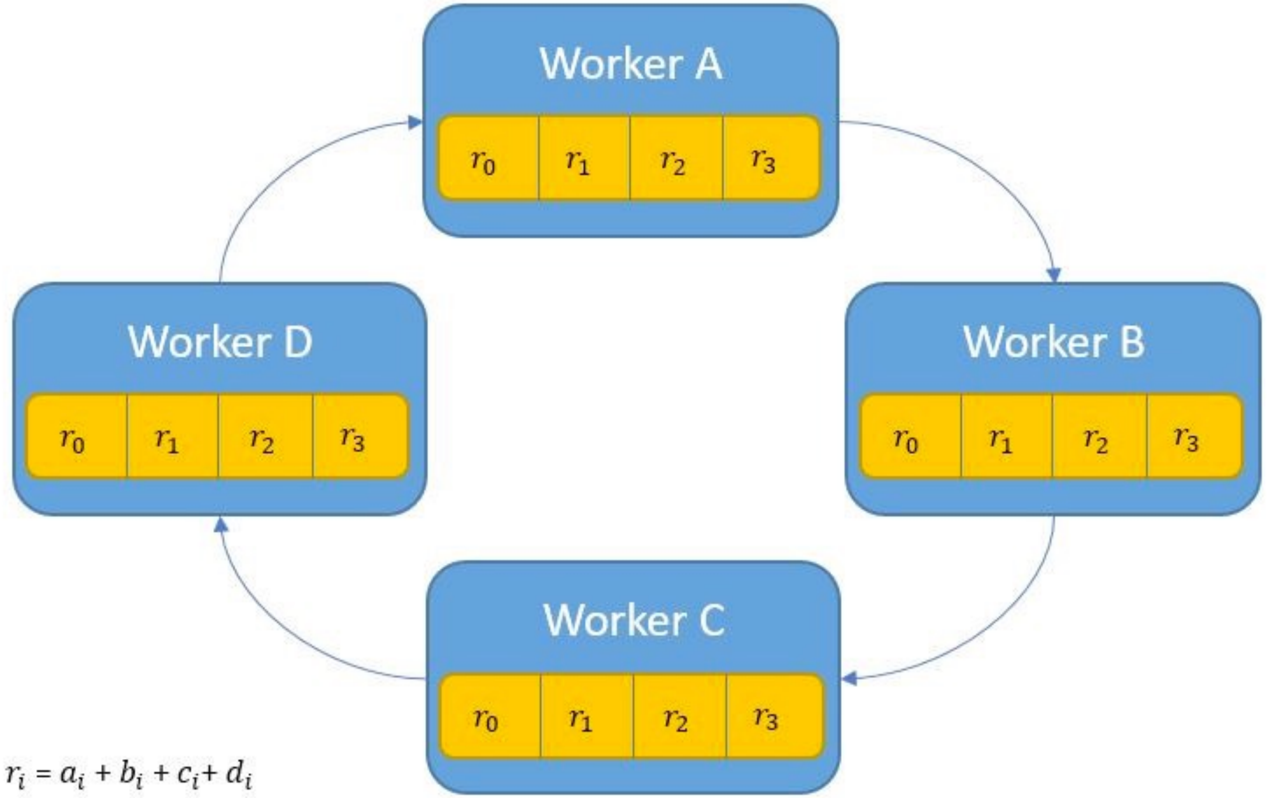
Ring AllReduce



Ring AllReduce



Ring AllReduce



$r_i = a_i + b_i + c_i + d_i$

How to speedup gradient descent?

- Up to now: faster gradient computation
    - Distributed
    - Optimize communication cost
    - Reduce synchronization overhead

- Everything we said is widely applicable
    - You just need a differentiable model
    - Model parameters fit on one machine
    - Loss is a sum of pointwise losses over training examples

- Another way: smarter optimization algorithms
    - Line search
    - 2nd order methods

Line search

- Heavily used with Full Batch Gradient Descent
  - Repeat until convergence
    - Compute descent direction $\nabla L(\theta)$
    - Choose α_t to « loosely » minimize $L(\theta - \alpha_t \nabla L(\theta))$

$$\theta_{t+1} \longleftarrow \theta_t - \alpha_t \nabla L(\theta)$$

Theoretical guarantees

- Using second order Taylor expansion

$$L(\theta + \Delta\theta) = L(\theta) + \Delta\theta^T \nabla L(\theta) + \frac{1}{2}\Delta\theta^T \left(\nabla^2 L(\theta)\right)\Delta\theta$$

- If L is l gradient Lipschitz, $\nabla^2 L(\theta) \leq l$

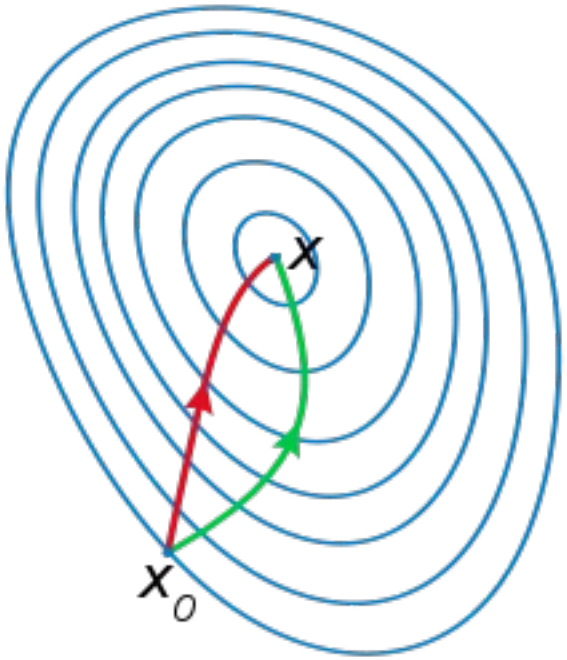
$$L(\theta + \Delta\theta) \leq L(\theta) + \Delta\theta^T \nabla L(\theta) + \frac{l}{2}\Delta\theta^T \Delta\theta$$

- That is minimized with

$$\Delta\theta = -\frac{1}{l}\nabla L(\theta)$$

Second order optimization methods

- We want to speed up our convergence in another way
- Using the “curvature” information can help a lot



Newton’s method

$$L(\theta + \Delta\theta) = L(\theta) + \Delta\theta^T \nabla L(\theta) + \frac{1}{2}\Delta\theta^T\left(\nabla^2 L(\theta)\right)\Delta\theta$$

- Minimizing Taylor expansion wrt. $\Delta\theta$ gives

$$\nabla L(\theta_{n+1}) = g_n + H_n \Delta\theta$$

with $\theta_{n+1} = \theta + \Delta\theta, \nabla^2 L(\theta_n) = H_n$ and $\nabla L(\theta_n) = g_n$,

- We want to locally minimize L, so by taking the above gradient to 0, we obtain

$$\Delta\theta = -H_n^{-1} g_n$$

Or cheaper we can solve the linear system $H_n \Delta\theta = -g_n$

Quasi-newton methods

- Two main steps to newton iteration
  - Compute $\nabla^2 L(\theta_n) = H_n$
  - Solve linear system $H_n \Delta\theta = -g_n$

- Both of these steps can be very expensive

- Quasi-newton methods approximate H_n^{-1} by some matrix B_n and updates it appropriately at each step

Barzilai Borwein step size

- We want to approximate H_n^{-1}, but how ?
- Secant condition, H_n^{-1} such that

$$\Delta\theta = H_n^{-1}\Delta g_n$$

- Most simple approximation : $H_n = \frac{1}{\alpha_n}\boldsymbol{I}$
- Not enough degrees of freedom to achieve secant condition but we can minimize the residuals

$$\alpha_n = argmin_\alpha \left\| \frac{1}{\alpha_n}\Delta\theta - \Delta g_n \right\|_2^2 \quad \Rightarrow \quad \alpha_n = \frac{\|\Delta\theta\|_2^2}{\Delta\theta^T \Delta g_n}$$

Quasi-newton methods : generic

- Until convergence
  - Compute update direction ($B_n \approx H_n^{-1}$)
    $$\Delta\theta = -B_n g_n$$
  - Line search for learning rate
    $$\alpha \leftarrow \min_{\alpha \geq 0} L(\theta_n - \alpha\Delta\theta)$$
  - Update parameters
    $$\theta_{n+1} \leftarrow \theta_n - \alpha\Delta\theta$$
  - Store the parameters and gradient deltas
    $$g_{n+1} \leftarrow \nabla L(\theta_{n+1})$$
    $$\Delta\theta_{n+1} \leftarrow \theta_{n+1} - \theta_n$$
    $$\Delta g_{n+1} \leftarrow g_{n+1} - g_n$$
  - Update inverse hessian approximation
    $$B_{n+1} \leftarrow QuasiUpdate(B_n, \Delta\theta_{n+1}, \Delta g_{n+1})$$

BFGS

- Update B with a rank two matrix, of the form

$$B_{n+1} \leftarrow B_n + auu^T + bvv^T$$

- Limited memory BFGS or L-BFGS for short
  - Perform the update without actually materializing B matrix and performing an explicit matrix vector multiplication
  - Can be achieved by storing the latest few values and gradients

Why are we talking about this ?

- We can now perform second order updates just with the (full) gradient

- Gradient computation is embarassingly parallel

- L-BFGS works very well in practice and converges in very few epochs (so we increase the computation to reduce the communication overhead)

Conclusion

- Distributed Machine Learning is about trade-offs
  - Communication VS computation cost

- For simple models (like Logistic Regression), a synchronous approach works well
  - Exploit sparsity
  - Use more complex optimization schemes

- There are several ways to distribute and aggregate computation
  - Centralized synchronous or asynchronous model, AllReduce …

AllReduce

- AllReduce
  - Average all the values in all the computation nodes and send the averages value to all the nodes
  - Example: averaging gradients !

- Implementation
  - Naive: Reduce and then Broadcast like the first Spark snippet
  - Optimized: do both at the same time