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IM^{2}C

Summary Sheet

For millennia, humans have coexisted and depended on domesticated animals for comfort and aid. By the 19th century, pet ownership had grown into a more intimate relationship within our lives. Pet ownership has since exploded, with a substantial portion of our population currently owning, or having priorly owned a pet. In the wake of the recent COVID-19 pandemic, pet ownership saw yet another surge in growth. A consequence of this growth, however, is a correlated surge in pet abandonment, leaving their "COVID pets" and placing a burden on pet shelters. The popularity of pet ownership has not properly translated to ownership readiness in recent years. Our task is to remedy this growing problem by implementing a model that pet shelters can use to evaluate "pet ownership readiness."

Our model confronts the high rates of pet abandonment by implementing an analytic network process (ANP). Taking into account two primary categories: ability and environment, subdivided into nine parameters: past ownership, income, family size, number of different pets, number of the same pet, time commitment, house size, youngest age of the household, and household stability, we created a model is capable of judging a household's readiness of owning a pet through comparison to metrics from the average successful pet-owning family. Usage of the analytic network process is key: with a lack of quantitative data on the factors that make a good pet owner, the ANP allows us to convert qualitative data into quantitative data that can then be used to determine the proper weightings for each of these parameters and provide an accurate mathematical relationship between them. The issue of "pet ownership readiness" then becomes a comparison between expected (aggregate) outputs with actual (specific) outputs. If our given family provides an output that matches or exceeds the expected output for each of our categories, then we can conclude that the family is ready to care for the pet under that category. On the flip side, if our given family provides an output that is below the expected output for our category model, then we can conclude that the family is **not** ready to care for the pet under that category. A combination of the outputs under both categories is guaranteed to return a final response of "yes", "no", or "maybe" when asked if a given family is ready to care for a new pet. A yes or a no are simple outputs to interpret, though a maybe would leave the final decision of readiness up to the pet shelter's discretion. Our model is simple to use and provides a direct answer for the user, making it an effective way for pet shelters to evaluate pet ownership readiness.

Beyond this, by inputting specific metrics for various regions, our model can predict pet ownership readiness in the future and across the globe. Extrapolating demographic data from our model can predict the behavior of pet ownership rates as well as pet retention rates several years into the future. This data is crucial in evaluating demographic trends pet shelters want to continue, or on the flipside, avoid. Our model expands on several concerns of pet abandonment, and attempts to serve uses outside of a pure scoring algorithm. Rather, it serves as a real-life tool to make educated decisions and better society.

Table of Contents

1 Introduction

To preface our approach in creating our model, it is important to highlight the specific goals that our model should reach. Within the problem of pet abandonment, we must assess and reason for the specific parameters that we've chosen while integrating them in a thorough and meaningful way.

1.1 Problem Restatement

Question 1. **Warm up (with cats).** Develop a mathematical model that can be utilized by an animal shelter, pet store, or similar entity to evaluate a household's readiness for cat ownership. In other words, your model needs to be able to receive information from a given household and determine if the household is prepared to own a cat.

a. What does a cat-ready household 'look like'? Validate your model's ability to capture the diversity of households that could have a cat as a pet. Provide at least three examples of households that qualify for cat ownership in a country/region of your choosing and at least three that do not qualify. Be sure to choose examples that highlight the factors, or combination of factors, that your model associates most readily with households that qualify for cat ownership.
b. Assess your model on a broader scale (and/or adjust it if necessary) by using it to determine the current number of households that are prepared to own a cat in three countries/regions of your choosing.

Question 2. **Generalize your model** from question #1 so that it still accepts ten (or fewer) inputs but now returns output that addresses a household's pet preparedness for cats as well as four additional pet species of your choice.

a. Demonstrate your model's utility by reviewing the pet preparedness of at least six households located in the same country / region you originally considered in question #1a. You may choose to analyze the same households you used earlier, but make sure to discuss the significance of the examples you've chosen to highlight.
b. Some households possess multiple pets. How does your model address this situation?

Question 3. **The future of pet ownership.** Using your previous model(s) as a tool for potential pet ownership, project future pet demographics. Specifically, develop a mathematical model that projects pet ownership and retention in five, ten and 15 years by pet species. Consider the same three countries/regions you identified in question #1b and the same five pet species (cats plus your four pet choices) you used in question #2.

1.2 Analysis

Question 1: We are asked to develop a model with no more than ten parameters to evaluate a household's readiness to own specifically a **cat**. To understand what a "ready" household looks like, we can first make our model, and then apply it to an average successful pet-owning household, in which the parameters will have inputs of one. Relative comparisons between a given family and the average family will then determine household readiness – in this case, for a cat and its behaviors/requirements. We will test this model with experimental and average data from three distinct regions, cumulatively providing variety in our assessment.

Question 2: From our model in question #1, we are asked to generalize its applications outside of cats. Of course, we will need to adjust the values of our parameters to better suit the behaviors and requirements of our **four** other pets, but our general weightings between our parameters – that is, how much more influential certain parameters are over others in determining readiness – should stay relatively the same. Our average household may vary slightly, but our general comparison model will not change. We will test this model with experimental data from six new households within our regions defined in question #1. To address the plurality of pets in certain households, we must set aside room in our parameters for the number of different pets and the number of the same pet within a household. These two parameters **must** be considered somewhere in our model.

Question 3: To understand where pet ownership might go in the future, we can test several random points of experimental data within our model, and note how many of these experimental families are ready or not ready to own a pet. This will produce the trend for retention, and can be graphed with a separate, linearly-regressed model. A deviation away from the trend-line will create the predicted boundaries of pet ownership (for our five chosen pets in question #2) within the next five, ten, and 15 years within our three chose regions from question #1.

1.3 Assumptions

Assumption 1: Our given family will be able to provide information for all of our parameters.
Justification: If we lose information from one of our parameters, our model will have an inconclusive input, which will yield an inconclusive result. To guarantee that a family can provide information for all of our parameters, we will ensure that they are answerable with a discrete numerical value.

Assumption 2: Averages we find through research regarding traits of the population (i.e. income) are valid representations of the 50% margin of the population.
Justification: Availability of statistical data regarding this is not reasonable and/or sufficient to make justifications. For countries that are millions in population, the feasibility of this data becomes vague. Thus, simplification is necessary.

Assumption 3: The US’s average income standard is not reflective of that of other countries.
Justification: The living wages of different regions are different, so it would only make sense to compare a region’s average income to its own living wage.

Assumption 4: The form of our function and our model will not inherently produce a change in its output behavior.

Justification: An exponential function, polynomial function, rational function, or a mix of the three can always be regressed into each of the different forms. This means that our model does not have to be strictly bound to any of these forms.

Assumption 5: Some factors in our model remain constant for different countries (i.e. relative number of different pets, same pets, time commitment, youngest age, past ownership, household stability).
Justification: These factors are not dependent on location; they rely more on the individuals.

Assumption 6: All pet-ready households have access to get a pet, and calculations made to model pet-ready projection over the next several years are made assuming all pet-ready households will get a pet.
Justification: Many households could be considered as pet-ready, but there is little statistical value if none of those households, when modeling for projected pet-ready growth and projected pet-retention, have pets.

1.4 Methodology
To approach our model, as hinted in our analysis, we will utilize a comparison system of evaluation. By comparing the surveyed data from a given family to the numerical data of the average successful pet-owning family, we can create a recommendation to the pet shelter. This will follow a process indicated by *Figure 1*.

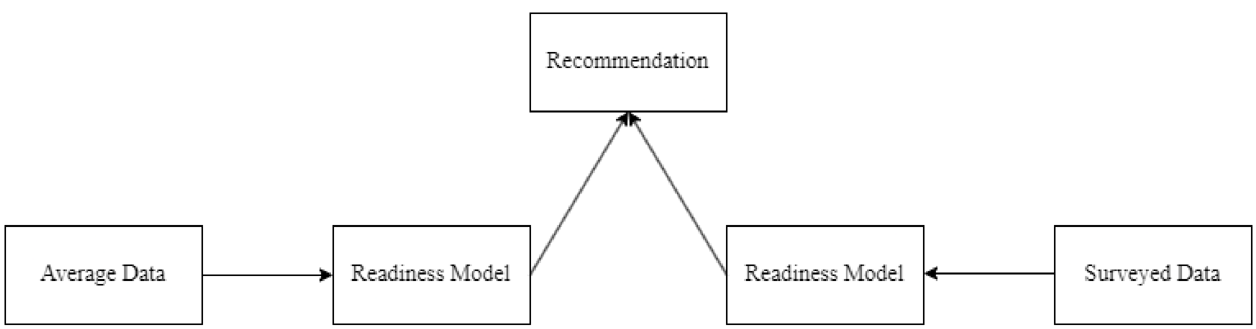

Figure 1: The general approach to utilizing our readiness model

So what will our readiness model eventually look like? We will split the model into two separate, but similar functions, that evaluate a score based on different parameters. These separate functions will be more abstractly labeled as “categories”. One category will be evaluated by a family’s **ability** to care for a pet, while the other will be evaluated by the **environment** of the household. The demands for the household ability generally vary by pet, and the demands for the household environment generally vary by family. This will make it easier to eventually generalize our model for different pets. The corresponding parameters for each category are given in *Figure 2*.

Ability	Environment
Past ownership (PO)	Number of different pets (DP)
Income (I)	Number of the same pet (SP)
House size (SF)	Youngest age of the household (Y)
Time commitment (T)	Household stability (S)
	Family size (F)

Figure 2: The nine chosen parameters indicated under each category for our model

The parameters DP and SP were understood to be required by our analysis of question #2, leaving us room with a few other parameters of our choosing. PO, I, SF, and T were the first sensible parameters indicative of direct ability and expertise to care for a new pet, and were, therefore, created for our ability category. The remaining parameters, Y, S and F, were chosen for their general application to pet ownership, seeing significance not only for cats, but also for several other species of pets. These parameters would make our model more generalizable when answering question #2. Outside of these nine parameters, we could not find a simple and generalizable parameter for pet ownership, so our model will just rely on the parameters provided above. Ultimately, we will evaluate each category individually, and then combine their results to provide a pure recommendation to the pet shelter. A diagram for this process is visualized in *figure 3*.

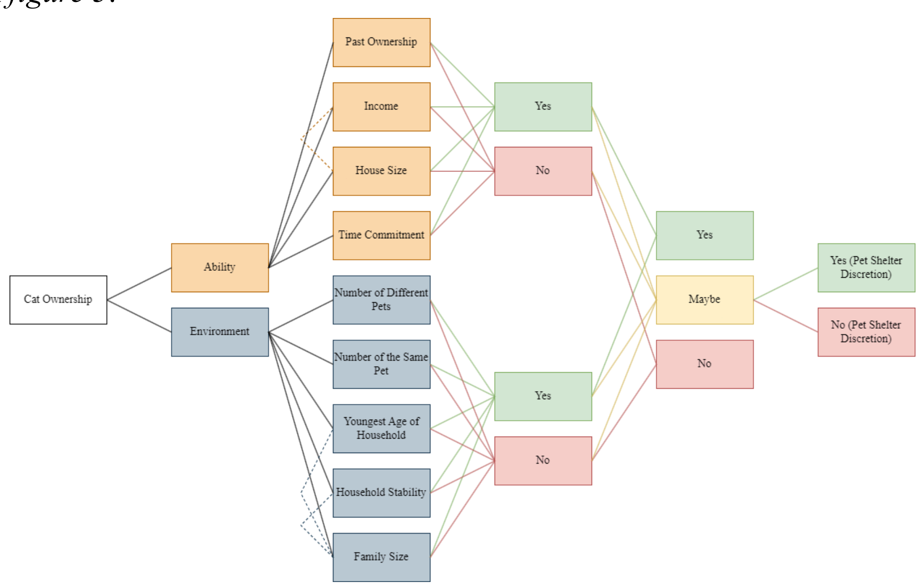

Figure 3: A diagram for our evaluation process using our models on ability and environment. Dotted lines indicate qualitative correspondence, and will provide an opening during our paired comparison when determining the weighting for each parameter.

2 Our Model
Our final model will be constructed as a polynomial function of our nine parameters, whose coefficients will vary depending on the importance of these parameters with respect to each other. To do this, we must create a relevant model of weighting that shows how much of a numerical effect each parameter will induce on each other, and then collapse this array of values into a singular list of consistent coefficients.

2.1 Weighting
When creating the model for ability and environment, the most important step is to identify the proper weightings between each parameter. To do this, we can utilize a tool called the analytical network process. We are creating a matrix (in our case, 9x9) that highlights the relative importance of each parameter on each other. This follows the Saaty Scale – a relative scale of importance – as illustrated in *figure 4*.

Numerical Value	Meaning
1	Equal Importance
3	Slight importance over the other
5	Moderate importance over the other
7	Strong importance over the other
9	Extreme importance over the other
2, 4, 6, 8	Intermediate values between two adjacent values

Figure 4: Saaty's Scale of relative importance

By using Saaty's Scale and comparing different parameters' importances over each other, we can fill our 9x9 matrix. The importance of a parameter A over the same parameter A will always be one by intuition, and the importance of parameter B over the parameter C is the reciprocal of the importance of parameter C over the parameter B. Visually, our matrix of paired comparisons will look similar to *figure 5*.

	A	B	C	D
A	1	ba	ca	da
B	ab = $\frac{1}{ba}$	1	cb	db

C	ac = $\frac{1}{ca}$	bc = $\frac{1}{cb}$	1	dc
D	ad = $\frac{1}{da}$	bd = $\frac{1}{db}$	cd = $\frac{1}{dc}$	1

Figure 5: A 4x4 matrix of pairwise comparisons following Saaty's Scale with four parameters

By performing this on our 9x9 matrix of parameters, we will get proper weightings for each parameter relative to one another. This is performed outside of our categories, as they will become distinct when defining the two separate functions. This will produce a 9x9 matrix illustrated by *figure 6*.

	PO	I	F	DP	SP	T	SF	Y	S
PO	1	1.4	0.6	1	1.1	1.8	0.8	0.3	1.8
I	0.714	1	0.4	0.5	0.6	1.2	0.7	0.3	1.1
F	1.667	2.5	1	0.9	1	2	1.4	0.9	2
DP	1	2	1.111	1	1.1	1.9	1.2	1	2
SP	0.90	1.667	1	0.909	1	1.8	0.9	1.1	1.9
T	0.556	0.833	0.5	0.526	0.556	1	0.4	0.4	1.1
SF	1.25	1.429	0.714	0.8333	1.111	2.5	1	0.7	1.8
Y	3.333	3.3333	1.111	1	0.909	2.5	1.429	1	1.8
S	0.556	0.909	0.5	0.5	0.5265	0.909	0.556	0.556	1

Figure 6: Our 9x9 matrix of relative weightings, rounded to three decimal places. Weightings were derived from research followed by qualitative weighting based on Saaty's scale.

This matrix alone, however, tells us little about how our functions should be defined. Our goal is to extrapolate nine coefficients from this matrix that correspond to each parameter in the form:

$$f = c_1 P_1 + c_2 P_2 + c_3 P_3 + ...$$

We can divide this into two categories:

$$A = c_1 PO + c_2 I + c_3 SF + c_4 T$$

$$E = c_5 DP + c_6 SP + c_7 Y + c_8 S + c_9 F$$

Note: *A* describes the family readiness score for ability, and *E* describes the family readiness score for environment.

To modify the information in our matrix, we can condense it into a 1x9 preference vector **P**. This, however, to adjust for scaling, must be done to a normalized matrix. That way, weighting over each parameter is evenly distributed, and a more accurate relative weighting can be established. *Figure 7* shows the normalized matrix for our Saaty Scale weightings.

	PO	I	F	DP	SP	T	SF	Y	S
PO	0.0910	0.093	0.086	0.139	0.139	0.115	0.095	0.048	0.124
I	0.065	0.066	0.058	0.070	0.076	0.077	0.083	0.048	0.076
F	0.152	0.166	0.144	0.126	0.127	0.128	0.167	0.144	0.138
DP	0.091	0.133	0.160	0.139	0.139	0.122	0.143	0.160	0.138
SP	0.083	0.111	0.144	0.127	0.127	0.115	0.107	0.176	0.131
T	0.051	0.055	0.072	0.073	0.070	0.064	0.048	0.064	0.076
SF	0.114	0.095	0.103	0.116	0.14	0.160	0.119	0.112	0.124
Y	0.303	0.221	0.160	0.139	0.115	0.160	0.170	0.160	0.124
S	0.051	0.060	0.072	0.070	0.067	0.058	0.066	0.089	0.069

Figure 7: Normalized matrix for each assigned weighting (rounded to three decimals)

The average value for each row will then be assigned an index in our 1x9 vector **P**. These relative weightings tell us the fraction of a parameter's input necessary to sum with all other weighted parameters to achieve a sum of one (on average) – a byproduct of normalization. In other words, it provides a list of the inverses of the coefficients for each parameter.

We can find the reciprocal of each vector index to create a corresponding list of coefficients, as indicated in *figure 8*.

$$\mathbf{P} = \begin{bmatrix} 0.104 \\ 0.069 \\ 0.143 \\ 0.136 \\ 0.124 \\ 0.064 \\ 0.120 \\ 0.173 \\ 0.069 \end{bmatrix}$$

Parameter	Coefficient
PO	9.657
I	14.542

F	6.972
DP	7.345
SP	8.033
T	15.700
SF	8.303
Y	5.792
S	14.960

Figure 8: The corresponding coefficients for each of our parameters to define our functions with (rounded to three decimal places)

This provides:

$$A = (9.657)PO + (14.542)I + (8.303)SF + (15.700)T$$
$$E = (7.345)DP + (8.033)SP + (5.792)Y + (14.960)S + (6.972)F$$

2.2 Consistency

Before implementing and testing our functions thoroughly, we must check the accuracy of our coefficients and ensure that they were picked/weighted on a similar scale. This can be done using the consistency ratio, *CR*.

$$CR = \frac{CI}{RI}$$

In this equation, *CR* is the consistency ratio, *CI* is the consistency index, and *RI* is the random consistency index. A *CR* less than 0.1 would generally prove our weightings in our pairwise comparison matrix (*figure 6*) are logically consistent with each other. *CI* is defined as

$$CI = \frac{\lambda_{max} - n}{n-1}$$

where n is the number of criteria (or parameters), and λ_{max} is the eigenvalue that is achieved satisfying the equation:

$$\mathbf{A} \bullet \mathbf{v} = \lambda_{max} \bullet \mathbf{v}$$

A in this case is the normalized matrix (*figure 7*) which resembles a grouping of operators, and **v** is our preference vector **P**. λ_{max} is found as the average of every index in **P** dotted with every corresponding index in **A**. Through this dot operation, we see that λ_{max}, rounded to three decimal places, is 9.169, and therefore,

$$CI = \frac{\lambda_{max} - n}{n-1} = \frac{9.169-9}{9-1} = 0.021$$

RI is an assigned numerical value that increases based on the number of criteria (or parameters). For $n = 9$, $RI = 1.45$, and so,

$$CR = \frac{CI}{RI} = \frac{0.021}{1.45} = 0.015 << 0.1$$

This allows us to conclude that our model is consistent across all nine parameters, and will provide a rational recommendation when inputting properly scaled values for our parameters.

3 Tests and Results

3.1 Usage

From the previous section, we've established that our models for ability and environment will be written as

$$A = (9.657)PO + (14.542)I + (8.303)SF + (15.700)T$$
$$E = (7.345)DP + (8.033)SP + (5.792)Y + (14.960)S + (6.972)F$$

We will now integrate our numerical parameter values into our model. The key is that our model is entirely relativistic: we will compare our inputs for a given family directly to the corresponding value from an average successful pet-owning family. For example, if a provided family has owned eight pets, and the average successful pet-owning family has owned three pets, our input for the parameter *PO* would not be eight pets, but instead, $\frac{8\,pets}{3\,pets}$, or $\frac{8}{3}$. In this way, we can guarantee that our score is unitless, and additionally, that our scaling per input will be proper. With this in mind, our testing results should be consistent across all regions and all species of pets.

The only inconsistency that we may need to keep in mind is for the parameter *Y*. The youngest age in a household is only significant for values at or around 1 (that is, the given family's youngest age is around 4 years). As *Y* increases above a limit, 2.5 (that is, the given family's youngest age is 10 years), then *Y* suddenly becomes a much less significant piece of information in our model – a **fundamental weakness** of our previous model. For family's whose youngest age is above 10, we will redefine our model as follows:

$$A = (9.657)PO + (14.542)I + (8.303)SF + (15.700)T$$
$$E = (7.345)DP + (8.033)SP + (5.792)(2.5) + (14.960)S + (6.972)F$$

3.2 Experimental Households

To test the effectiveness of our model, we came up with 6 various living situations, listed in *figure 9*. Our process relied on diversity throughout our parameters, which meant creating a mix of what we believed would be low-scoring ability and environment metrics with what we believed would be high-scoring ability and environment metric.

Family	PO	I	F	DP	SP	T	SF	Y	S	Bias
1	2	80000	4	1	2	4	1800	10	9	Yes
2	0	120000	3	0	0	4	2200	8	8	Maybe

3	2	40000	2	4	1	1	1000	16	4	Maybe
4	0	30000	2	1	0	1	800	40	3	No
5	0	30000	6	2	0	2	1200	2	3	No
6	4	90000	6	7	4	22	2500	5	7	Yes

Figure 9: Our experimental households

These experimental families were created with a bias to assign a pre-evaluated readiness score to each household. Households 4 and 5 are indicative of what we believe as unready households because of their relatively low income and lack of previous experience with pets. Households 2 and 3 are indicative of what we believe as partially ready households (for which our model will return a "maybe" response) because each have strengths and weaknesses in their respective sectors (ability/environment), and households 1 and 6 are indicative of what we believe as ready households because of their relative experience, economic status, and environment.

To test the accuracy and consistency of our model, we have to compare our experimental bias with our model's recommendation. Our model will rely on assigning a readiness score for the two categories – ability and environment – and comparing those scores to the average score of a successful pet-owning family in the United States. *Figure 10* lists the parameter values of the average successful pet-owning family in the United States, which will define the average readiness score for our comparison.

Parameter	Value
PO	3.24 (pets)[1]
I	74755 ($/year)[2]
F	3.13 (people)[3]
DP	1.46 (pets)[1]
SP	1.78 (pets)[1]
T_{cat}	2 (hours/day)[4]
SF	2299 (ft^2)[5]
Y	4 (years)[6]
S	5

Figure 10: The parameter values for an average successful pet-owning family in the United States, with units. Note that DP and SP are condensed averages from ***all species*** *of pets, cumulatively totalling to the parameter PO. T is the only parameter that varies by species.*

The average readiness score for an American household successfully caring for a cat is

$$A = (9.657)(1) + (14.542)(1) + (8.303)(1) + (15.700)(1) = 48.202$$
$$E = (7.345)(1) + (8.033)(1) + (5.792)(1) + (14.960)(1) + (6.972)(1) = 43.102$$

The results of our model on the experimental families in *figure 9* are shown in *figure 11*.

Family	Ability Score (A)	Environment Score (E)	Model Recommendation	Bias
1	59.424	64.374	Yes	Yes
2	62.688	42.203	Maybe	Maybe
3	25.204	55.539	Maybe	Maybe
4	16.575	32.942	No	No
5	25.870	35.300	No	No
6	211.157	94.818	Yes	Yes

Figure 11: Model recommendation vs. pre-evaluated prediction for cats

3.3 Extrapolating to Other Pets

As stated under *figure 10*, the time commitment parameter is the only parameter that varies significantly between different species of pets. *Figure 12* provides a chart of the varying nature of T for four other different species of pets.

Species	Time commitment
Dog	2 (hours/day)[7]
Bird	0.5 (hours/day)[8]
Fish	0.75 (hours/day)[9]
Hamster	0.333 (hours/day)[10]

Figure 12: Time commitment averages for four other chosen species

With different averages for T, we can perform the same process of creating a readiness score by comparing our given family's available time commitment to the average time commitment of a successful pet-owning family. Since T is a parameter of ability **only**, our families' environment should not vary by different pets. In that case, we can redefine the divisor for our given families time commitment relative to the pet, and create a new ability score. The average ability will still be held constant at a score of 48.202 since the average successful pet-owning family will still contain a T value of 1 (our average successful pet-owning family will be committing the exact time requirements listed in *figure 12*). *Figures 13-16* show the new recommendations and biases based on the varying species of pet.

Family	Ability Score (A)	Environment Score (E)	Model Recommendation	Bias
1	59.424	64.374	Yes	Yes
2	62.688	42.203	Maybe	Maybe
3	25.204	55.539	Maybe	Maybe
4	16.575	32.942	No	No
5	25.870	35.300	No	No
6	211.157	94.818	Yes	Yes

Figure 13: Model recommendation vs. pre-evaluated prediction for dogs

Family	Ability Score (A)	Environment Score (E)	Model Recommendation	Bias
1	153.624	64.374	Yes	Yes
2	156.888	42.203	Maybe	Maybe
3	48.745	55.539	Yes	Yes
4	40.125	32.942	No	No
5	72.970	35.300	No	No
6	729.257	94.818	Yes	Yes

Figure 14: Model recommendation vs. pre-evaluated prediction for birds

Family	Ability Score (A)	Environment Score (E)	Model Recommendation	Bias
1	111.757	64.374	Yes	Yes
2	114.721	42.203	Maybe	Maybe
3	38.287	55.539	Maybe	Maybe
4	29.658	32.942	No	No
5	52.037	35.300	Maybe	Maybe
6	498.990	94.818	Yes	Yes

Figure 15: Model recommendation vs. pre-evaluated prediction for fish

Family	Ability Score (A)	Environment Score (E)	Model Recommendation	Bias
1	216.613	64.374	Yes	Yes
2	219.877	42.203	Maybe	Maybe
3	64.501	55.539	Yes	Yes
4	55.872	32.942	Maybe	Maybe
5	52.724	35.300	Yes	Yes
6	1075.694	94.818	Yes	Yes

Figure 16: Model recommendation vs. pre-evaluated prediction for hamsters

3.4 Extrapolating to Other Countries

While selecting other countries to extend our model to, it is important to note some factors are unmodified from our U.S. values due to inaccessible data and/or because some data is not specific to location. The average data that **do differ** for three additional countries – Brazil, Canada, and India – are provided in *figure 17*.

Country	Number of households (in millions) [11,12,13]	Living wage salary ($) [14,15,16]	Average salary ($) [17,18,19]	Family size (people) [20,21,22]	House size (ft^2) [23,24,25]
Brazil	74.1	8100	8140	2.77	1288
Canada	16.2	50400	59300	2.9	1700
India	302.4	2484	4590.6	4.44	1300

Figure 17 - Average successful pet-owning family within our three other selected countries

Based on our data, we modified our model by changing the calculation method for the weight of income. Instead of taking a family's income and directly dividing by the average income for the region (i.e. $\frac{\$80,000}{\$74,755}$ for family 1 in the United States), we will take $\frac{Income}{Living\ Wage}$ to see how fit a family is (in terms of income). This is because the living standards in other countries differ from living standards in the United States. We can then determine the ability score and environmental score based on the specified data from other regions.

To determine the number of households that qualify for owning a pet, we considered how to utilize the values our model outputted. Since we used average values for most of our calculations, we assumed the average values correlate to the 50% margin of households.
How much the readiness score deviates from the qualifying score can be determined by using a percent difference:

$$\left(1 + \frac{(ReadinessScore - QualifyingScore)}{\left(\frac{ReadinessScore + QualifyingScore}{2}\right)}\right)$$

We then take this value and multiply it by the values we are referencing: the 50% of households. Thus, the number of households that qualify for owning a pet can be modeled by:

$$\left(1 + \frac{(ReadinessScore - QualifyingScore)}{\left(\frac{ReadinessScore + QualifyingScore}{2}\right)}\right) \bullet NumberOfHouseholds\left(\frac{1}{2}\right)$$

After performing our calculations,

Country	Number of households (in millions)	Ability Score	Environmental Score	Total Readiness Score	Percent of Households that qualify (%)	Households that qualify (in millions)
Brazil	74.1	44.622	42.300	86.922	47.541	35.2
Canada	16.2	48.606	42.589	91.196	49.940	8.1
India	302.4	56.926	44.576	101.502	55.289	167.2

Figure 18: Results of our model on demographics within our three other selected countries

4 Pet Ownership Demographics

4.1 Income Against Time

One of the most intuitive changes in our parameters over time is income—historically, average income increases over time. We chose to model change in income over time via regression. This will provide us with the graphical representation in *Figure 19*.

Note: The following calculations are made to reflect USA projections.

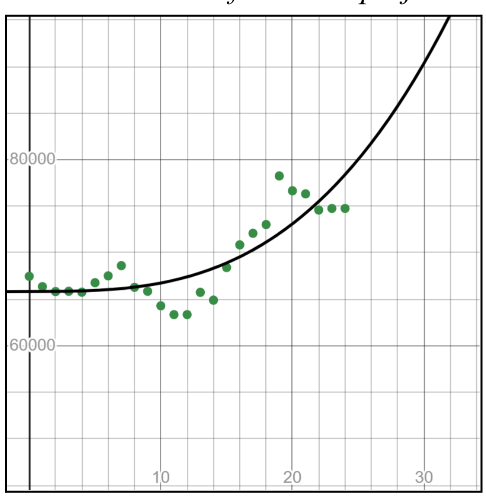

Figure 19: The regression model of USA income over time, where x represents years after 2000

This model can be represented by the following:

$$I(x) = 1.15769x^3 + 65071.3$$

With all the other values in our readiness-score model being constant, we can project the growth of our ability model by using the following:

$$A(x) = (9.657)PO + (14.542)(I(x + 2000)/74755) + (8.303)SF + (15.700)T$$

Note: we are taking $I(x + 2000)$ because it is currently 2024, and x represents years after 2000

Thus, the percent of households over time that are cat-ready can be represented by the following:

$$p(x) = \frac{1}{2}(1 + \frac{(A(x)+E - QualifyingScore)}{(\frac{A(x)+E + QualifyingScore}{2})})$$

Note: E and QualifyingScore are constant; only A(x) is projected to grow

Using this equation, the change in cat-ready households (in percent) over the next 5 years is:

$$p(5) - p(0) \simeq 1.269118\%$$

The overall change in cat-ready households (in percent) over the next 10 years is:

$$p(10) - p(0) \simeq 3.006550\%$$

The overall change in cat-ready households (in percent) over the next 15 years is:

$$p(15) - p(0) \simeq 5.243578\%$$

Note: These are the projections for the USA.

Data on other countries and pets can be found in Figure 20.

4.2 Feasibility

It is important to check the feasibility of our model to represent the relative change in pet ownership over the next several years before we make our calculations on the rest of the countries and pet species. To do this, we looked at the pet-ownership growth in the past ~30 years and compared the % growth to the % growth we calculated for the next 30 years. In 1988, 56% of households owned a pet. In 2024, 66% of households owned a pet. Thus, we can calculate the change in pet ownership in 36 years:

$$(\frac{0.66-0.56}{0.56})100\% \simeq 17.857143\%$$

Our next 30 years is projected to grow:

$$p(30) - p(0) \simeq 14.862818\%$$

This is a reasonably feasible growth model.

4.3 Other Pets and Countries

	USA	Brazil	Canada	India
Cat-ready households in 5 years (% change)	1.269	1.339	0.846	2.144
Cat-ready households in 10 years (% change)	3.006	4.643	1.679	4.202
Cat-ready households in 15 years (% change)	5.243	10.348	2.500	6.180

Dog-ready households in 5 years (% change)	1.269	1.339	0.846	2.144
Dog-ready households in 10 years (% change)	3.006	4.643	1.679	4.202
Dog-ready households in 15 years (% change)	5.243	10.348	2.500	6.180
Bird-ready households in 5 years (% change)	0.806	0.836	0.522	1.336
Bird-ready households in 10 years (% change)	1.917	2.920	1.039	2.630
Bird-ready households in 15 years (% change)	3.359	6.584	1.549	3.882
Fish-ready households in 5 years (% change)	0.974	1.017	0.638	1.626
Fish-ready households in 10 years (% change)	2.312	3.541	1.268	3.196
Fish-ready households in 15 years (% change)	4.044	7.950	1.889	4.711
Hamster-ready households in 5 years (% change)	0.625	0.643	0.400	1.026
Hamster-ready households in 10 years (% change)	1.488	2.253	0.795	2.024
Hamster-ready households in 15 years (% change)	2.614	5.109	1.187	2.995

Figure 20: Percent change in households that are pet-ready by country and species with respect to population of households.
Note: Smaller values do not indicate smaller % of total households that are pet-ready.
These values represent the change in % of households that are pet-ready from now.
Cat and dog values are equal due to no differences in inputs (they have similar standards).

For each of the countries, formulas for $A(x)$ and $E(x)$ were modified (similarly to what was done in 3.4). For example, in Brazil, our equation for A(x) was represented by:

$$A = (9.657)PO + (14.542)I + (8.303)SF + (15.700)T$$
$$= (9.657)(1) + (14.542)(I(x)/8100) + (8.303)(1288/2299) + (15.700)(2/t)$$

Where,

$PO = 1$

$I = (I(x)/8100)$, where $I(x)$ represents income v.s. time for Brazil (x years after 2024)

Note: computation differs slightly from what is demonstrated in 4.1

$SF = \frac{1288}{2299}$, where $1288ft^2$ is Brazil's average household size

$T = \frac{2}{t}$, where t represents the time commitment standard for a specific species

And,

$$E = (7.345)DP + (8.033)SP + (5.792)Y + (14.960)S + (6.972)F$$
$$= (7.345)(1) + (8.033)(1) + (5.792)(1) + (14.960)(1) + (6.972)(\frac{h}{3.24})$$

Where,

$DP,\ SP,\ Y,\ S = 1$

$F = \frac{h}{3.24}$, where h represents the average household size for a specific country

Recall,

$$p(x) = \frac{1}{2}(1 + \frac{(A(x)+E - QualifyingScore)}{(\frac{A(x)+E + QualifyingScore}{2})})$$

We used $p(x)$ and the processes above for each country and species of pet to generate the values in *Figure 20*.

4.4 Retention

With the assumption that all households that are pet-ready will consider getting a pet, utilizing our model from 4.1.3 gives us the households projected to get a pet:

$$p(x) \bullet h$$

Where $x =$ years after 2024 (5 years, 10 years, 15 years)

$h =$ number of households in selected country

To measure the amount of households that would retain these pets, we determined retention ratios by using research done by American Humane[30]:

Cat retention ratio: $\frac{127\ retained\ cats}{142\ total\ cats} = 0.894$

Dog retention ratio: $\frac{148\ retained\ dogs}{165\ total\ dogs} = 0.897$

Other pet retention ratio: $\frac{22\ retained\ other\ pets}{24\ total\ other\ pets} = 0.917$

Thus, households retaining pets over the course of 5 years, 10 years, and 15 years can be represented by

$$r \bullet p(x) \bullet h$$

where r is the retention ratio. This equation generated the values that are illustrated in depth in *figure 21*.

Note: various different pet species resulted in a different scaling of growth.

	USA	Brazil	Canada	India
Households retaining cat in 5 years (in millions)	61.026	31.671	6.487	128.751
Households retaining cat in 10 years (in millions)	63.067	33.860	6.608	134.316
Households retaining cat in 15 years (in millions)	65.696	37.639	6.727	139.661
Households retaining dog in 5 years (in millions)	61.230	31.778	6.509	129.183
Households retaining dog in 10 years (in millions)	63.279	33.974	6.630	134.766
Households retaining dog in 15 years (in millions)	65.916	37.766	6.749	140.130
Households retaining bird in 5 years (in millions)	86.454	46.970	9.99	191.366
Households retaining bird in 10 years (in millions)	87.792	48.386	10.067	194.953
Households retaining bird in 15 years (in millions)	89.530	50.876	10.142	198.427
Households retaining fish in 5 years (in millions)	77.157	41.355	8.702	168.381
Households retaining fish in 10 years (in millions)	78.770	43.071	8.795	172.733
Households retaining fish in 15 years (in millions)	80.858	46.066	8.887	176.935
Households retaining hamster in 5 years (in millions)	97.672	53.697	11.525	218.891
Households retaining hamster in 10 years (in millions)	98.713	54.791	11.584	221.658
Households retaining hamster in 15 years (in millions)	100.07	56.732	11.642	224.349

Figure 21: Households retaining pets in 5, 10, 15 years

5 Strengths and Weaknesses

5.1 Strengths

- A great strength of our model is our exceptional consistency, as tested with the consistency ratio. Our scaling, coefficients, and weighting were produced on a proper scale, which guarantees a heightened level of accuracy when implementing our model to various experimental households.
- The regression model of USA income over time is realistic to the actual growth of income in future years.
- Another strength of our model is its ease of use: we developed a model that works primarily off of a “plug and compare” basis, allowing our readiness score to be easy to find, and easy to interpret.
- Finally, our model can be mutated and skewed in several ways, making it easy for us to morph our outputs into demographic data or percentage data – information that is relevant in drawing several conclusions in the latter sections of our paper.

5.2 Weaknesses

- An additional form of our model appears when a simple parameter, the youngest age of the household, exceeds 10. This means that we have to create a unique case to consider separately when a certain parameter strikes varying values, in a way making our model piecewise. This is of course not ideal, but is a weakness that we were willing to accept to preserve accuracy.
- The regression model of USA income over time assumes that income will grow similarly to how it has been growing in recent years, specifically within a 10 year margin.
- The feasibility of our income against time is based on historical data, which is not entirely accurate when predicting future data. Additionally, our historical data did not entirely match our projected income behavior in the next 30 years, though their general trends do correlate strongly.
- Our pet retention ratios are limited by the data provided by American Humane, so the projections of pet retention in 5 years, 10 years, and 15 years may not be entirely accurate.
- The publication of our data for our models are inconsistent, meaning many sources are not published in the same year.
- The average incomes and salaries of various regions don’t necessarily represent the median, which means that our use of dividing total households by two (see *figure 18*) to represent the upper 50% of readiness scores may be inaccurate depending on how far the mean salary deviates from the median.

6 Letter to IMMC-A Directors

Dear IMMC-A directors,

In recent years, and especially in the wake of the COVID-19 pandemic, it is irrefutable to conclude that pet ownership – and consequentially, pet abandonment – has increased by a massive factor. This has become a question of humanity and responsibility in our society today, and requires attention and resolve.

We have devised a model that can be used by pet providers to evaluate a household's readiness in owning a new pet. Depending on the species of pet, slightly different metrics will be used to determine a minimum score that our contending household must achieve in two categories: ability (to care for the new pet) and environment (of the household that the new pet will possibly go to). The precondition of our model relies on the fact that the contending household can and will provide inputs for our nine parameters as listed: past ownership, income, house size, time commitment, number of different pets, number of the same pet, youngest age of the household, household stability (on a 1-10 scale), and family size. Afterwards, by simply plugging in our parameters (which are relative values to the corresponding average successful pet-owning parameter), our model will output a score. Comparing our score to the minimal readiness score for each of the two categories will land the pet provider at different conclusions, which they can consider at their own will. Ultimately, our model aims to provide a precise and concise suggestion which may be meaningful for a pet provider to consider.

Our model is generally consistent across all parameters, except for one distinct case: when the youngest age of the household is greater than ten years our model splits into a slightly different form. This distinction is made because of the scaled relevance of the youngest age parameter. As soon as the youngest age exceeds ten, no matter its magnitude, the accepted result is the same. A 50 year old and a 10 year old are generally accepted to be mature enough to properly handle all pets, and so, they procure the same influence in our model. The same can not be said for toddlers and young children between infancy and 10 years of age.

The generalizability of our model is another key advantage that allows us to extrapolate our model to various averages in different regions and different times. From these values, we can predict demographic data on pet ownership (and successful pet ownership) outside of the United States and into the future. This is a crucial step forward to attacking and predicting the effects of pet abandonment both in the present and into the future.

Sincerely,
Team # US-14844

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