Supporting Information for "Rates of decline of atmospheric radiocarbon and return to pre-modern values"

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Introduction

This supporting information contains additional details about the statistical models used for time series decomposition. More specific results are presented in this file, including a detailed presentation with the fitted model for each time series (Table S1) and figures representing the ETS decomposition for each series (Figure S1 to Sx).

As additional material (files included separately), I provide R code and data files to reproduce all results presented in this manuscript.

Text S1. Extended description of Exponential Smoothing State-Space Models

The ETS framework of *Hyndman et al.* [2008] decomposes a time series in Error, Trend and Seasonal components according to 30 different competing models that combine these terms either in additive or multiplicative form. For the radiocarbon series, the following type of models were obtained.

ETS(M,Ad,M): multiplicative error, additive trend, and multiplicative seasonality. This model is expressed mathematically as

$$\mu_t = (l_{t-1} + \phi b_{t-1}) s_{t-m}$$

$$l_t = (l_{t-1} + \phi b_{t-1})(1 + \alpha \varepsilon_t)$$

$$b_t = \phi b_{t-1} + \beta (l_{t-1} + \phi b_{t-1}) \varepsilon_t$$

$$s_t = s_{t-m}(1 + \gamma \varepsilon_t)$$

where μ_t is the predicted value at time t, l_t is the level term, b_t the growth term, s_t the seasonality, and ε_t the error. The parameters α, β, γ, and ϕ, are estimated by maximum likelihood.

ETS(M,M,M): multiplicative error, multiplicative trend, multiplicative seasonality. This model is represented as

$$
\begin{aligned}
\mu_t &= l_{t-1} b_{t-1} s_{t-m} \\
l_t &= l_{t-1} b_{t-1} (1 + \alpha \varepsilon_t) \\
b_t &= b_{t-1} (1 + \beta \varepsilon_t) \\
s_t &= s_{t-m} (1 + \gamma \varepsilon_t)
\end{aligned}
$$

ETS(M,Md,M): multiplicative error, multiplicative damped trend, multiplicative seasonality. The model is expressed as

$$
\begin{aligned}
\mu_t &= l_{t-1} b_{t-1}^{\phi} s_{t-m} \\
l_t &= l_{t-1} b_{t-1}^{\phi} (1 + \alpha \varepsilon_t) \\
b_t &= b_{t-1}^{\phi} (1 + \beta \varepsilon_t) \\
s_t &= s_{t-m} (1 + \gamma \varepsilon_t)
\end{aligned}
$$

ETS(M,Ad,N): multiplicative error, additive trend, no seasonality. The system of equations is given by

$$
\begin{aligned}
\mu_t &= l_{t-1} + \phi b_{t-1} \\
l_t &= (l_{t-1} + \phi b_{t-1})(1 + \alpha \varepsilon_t) \\
b_t &= \phi b_{t-1} + \beta (l_{t-1} + \phi b_{t-1}) \varepsilon_t
\end{aligned}
$$

To each series, the ETS framework fits different models for the error, trend, and seasonal components, and selects the best model according to the Akaike's information criterion AIC.

Data Set S1.

Dataset `S1_Cities.csv` contains radiocarbon data of dandelion plants (*Taraxacum spp.*) collected in different cities. Radiocarbon data is presented in pM (percent Modern) and D14 (Δ^{14}C). Coordinates of collection and sampling dates are also provided in this file.

Data Set S2.

Dataset `S2_Global.csv` contains global average values of radiocarbon compiled by *Hua et al.* [2013]. Values are presented in Δ^{14}C with mean and standard deviation.

Data Set S3.

Dataset `S3_Levin.txt` contains Monthly radiocarbon data from European stations reported in *Levin et al.* [2013]. Values are presented in Δ^{14}C.

R code.

File `forecast.R` contains code to process the data and reproduce all figures and results presented in the manuscript. The code runs under R version 3.0.3 or higher.

References

Hua, Q., M. Barbetti, and A. Rakowski (2013), Atmospheric radiocarbon for the period 1950–2010, *Radiocarbon*, *55*(4), 2059–2072.

Hyndman, A. R., A. Koehler, K. Ord, and R. Snyder (2008), *Forecasting with Exponential Smoothing*, Springer Series in Statistics, 3-7 pp., Springer Berlin Heidelberg.

Levin, I., B. Kromer, and S. Hammer (2013), Atmospheric $\Delta^{14}CO_2$ trend in Western European background air from 2000 to 2012, *Tellus B*, *65*(0).

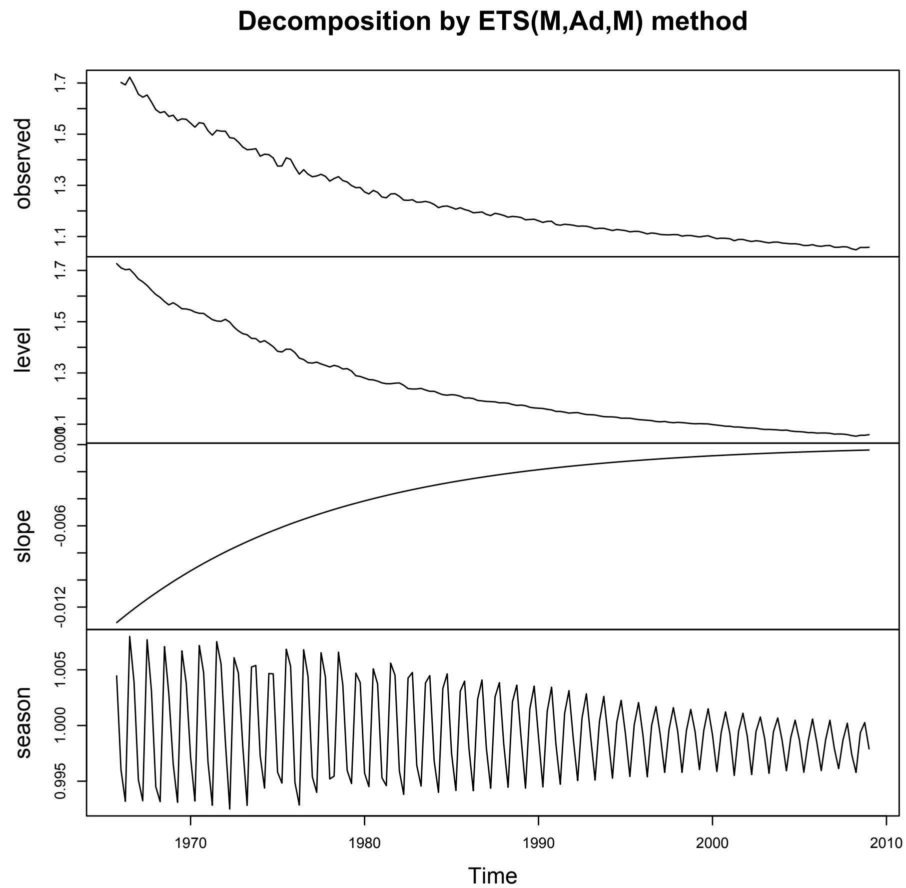

Figure S1. Time series decomposition of atmospheric radiocarbon for the northern hemisphere zone 1 series.

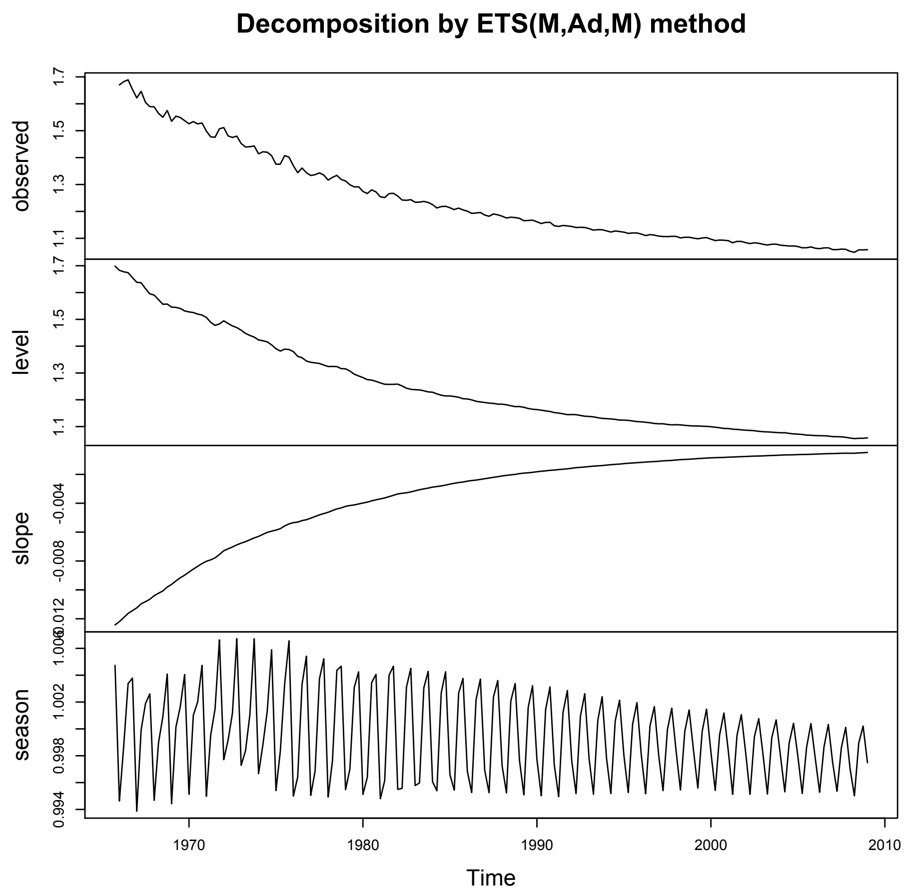

Figure S2. Time series decomposition of atmospheric radiocarbon for the northern hemisphere zone 2 series.

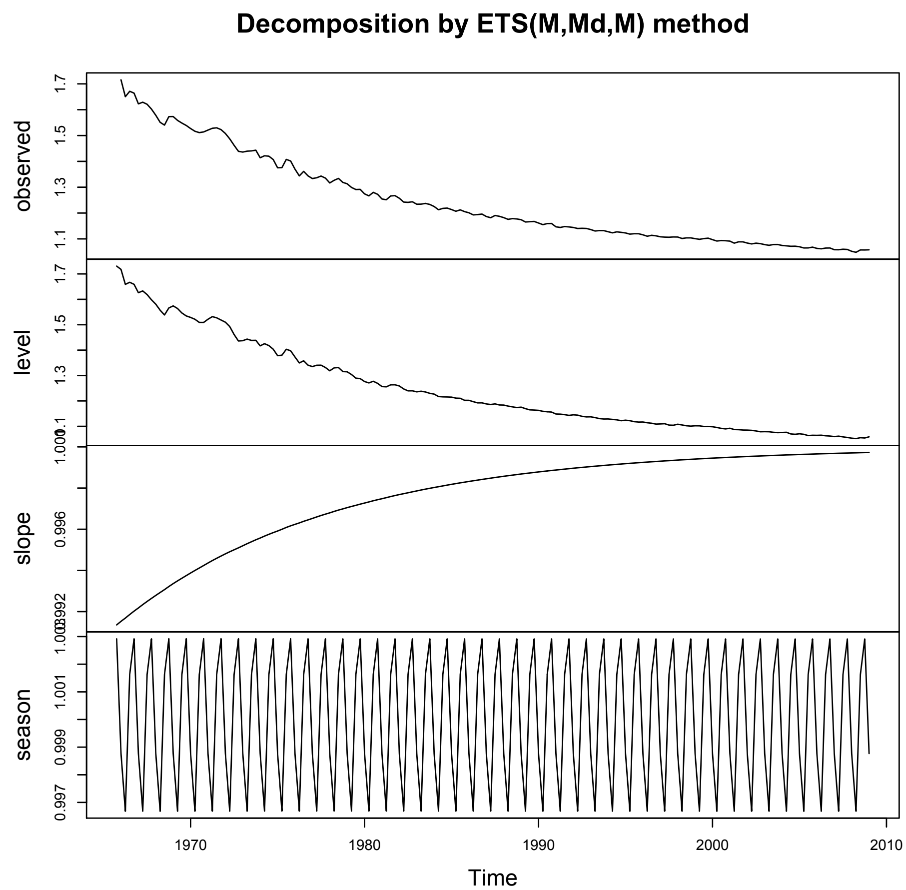

Figure S3. Time series decomposition of atmospheric radiocarbon for the northern hemisphere zone 3 series.

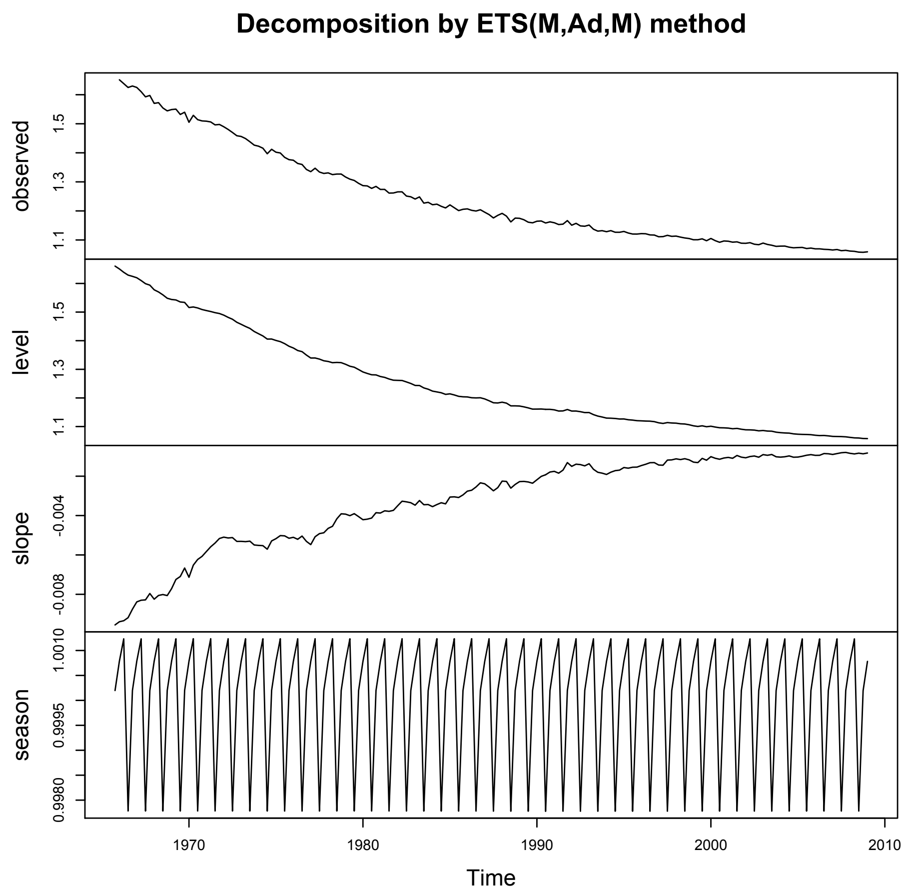

Figure S4. Time series decomposition of atmospheric radiocarbon for the southern hemisphere zones 1-2 series.

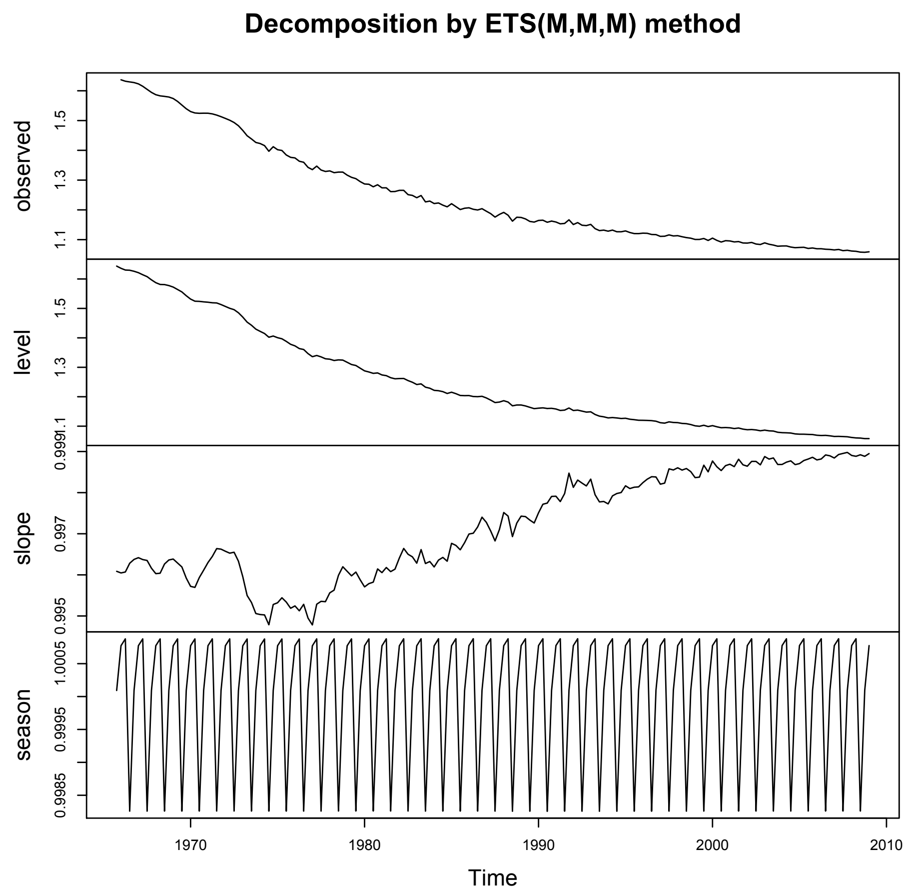

Figure S5. Time series decomposition of atmospheric radiocarbon for the southern hemisphere zone 3 series.

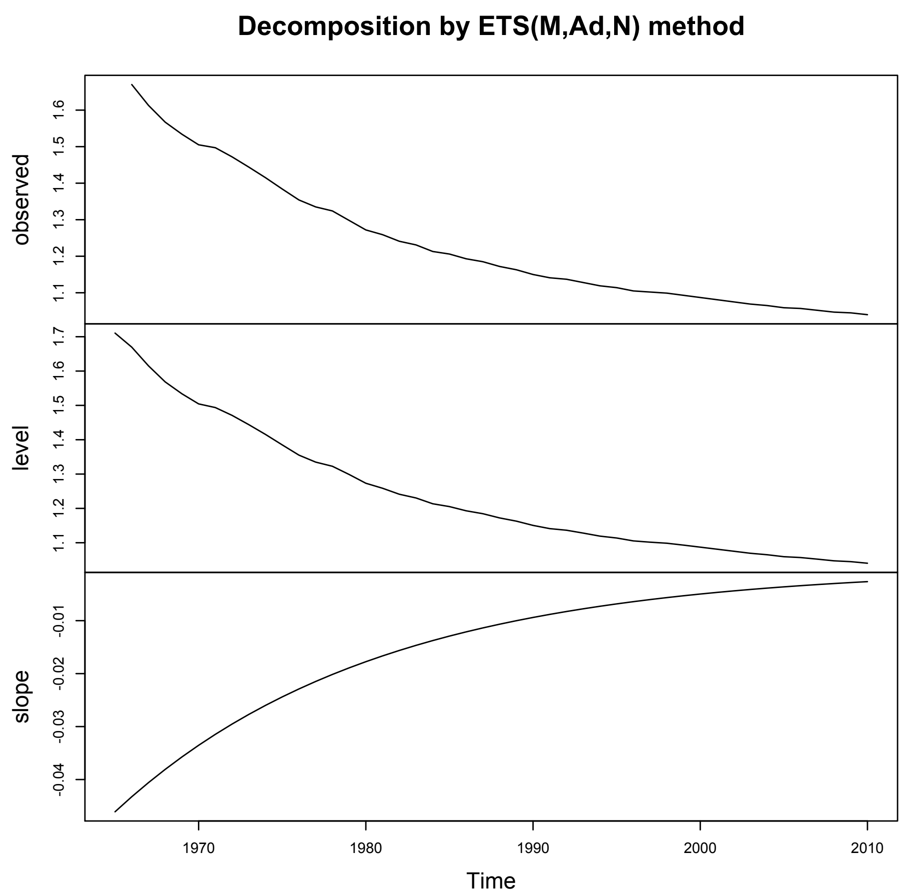

Figure S6. Time series decomposition of atmospheric radiocarbon for the global average series.

Table S1. Model structure and parameter values obtained by the ETS decomposition for the atmospheric radiocarbon zones of *Hua et al.* [2013].

Series	ETS model	Parameter values	AIC
NH zone 1	M,Ad,M	$\alpha = 0.681$, $\beta = 1 \times 10^{-4}$, $\gamma = 0.1187$, $\phi = 0.98$	-850.54
NH zone 2	M,Ad,M	$\alpha = 0.433$, $\beta = 0.0025$, $\gamma = 0.0925$, $\phi = 0.98$	-775.12
NH zone 3	M,Md,M	$\alpha = 0.936$, $\beta = 4 \times 10^{-4}$, $\gamma = 2 \times 10^{-4}$, $\phi = 0.98$	-775.55
SH zones 1 and 2	M,Ad,M	$\alpha = 0.487$, $\beta = 0.0259$, $\gamma = 1 \times 10^{-4}$, $\phi = 0.98$	-886.39
SH zone 3	M,M,M	$\alpha = 0.661$, $\beta = 0.04$, $\gamma = 1 \times 10^{-4}$	-894.74
Global	M,Ad,N	$\alpha = 0.862$, $\beta = 1e - 04$, $\phi = 0.94$	-294.77