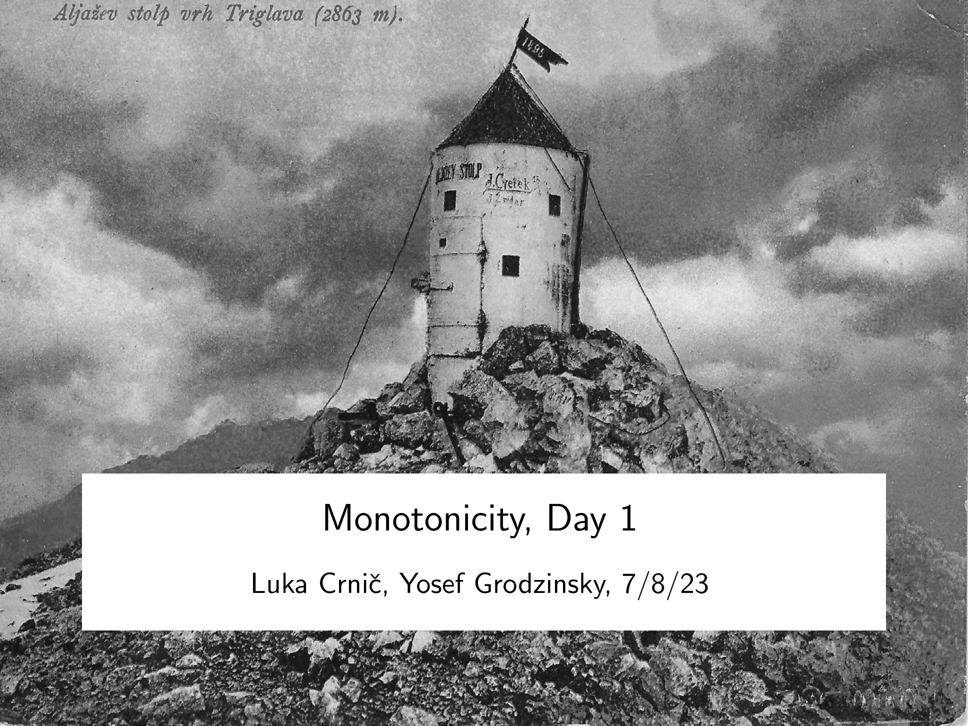

Monotonicity, Day 1

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https://lukacrnic.com/monotonicity

- logic in reasoning
- logic in grammar
- logic in language processing

lessons learned (and still learning)

- no autonomy of grammar from logic
- (partly) unfortunate split of the two endeavors

what we will (re)learn here

- intricate ways in which logic affects language
  - monotonicity-sensitive phenomena (esp. npis)
  - description requires environments (not operators)
    - + hint at why this may be the case (explanation)
  - focus on modal and comparative sentences

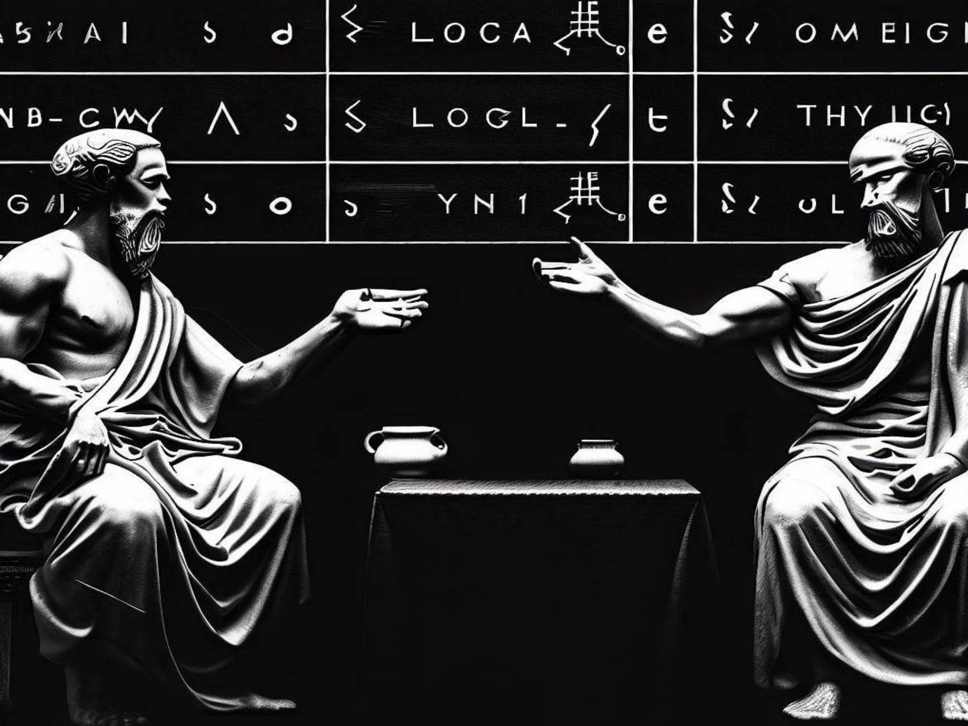
lessons learned (and still learning)

- grammatical processes significanly affect language processing, and they have a pronounced reflection in the brain (and similarly for logical processes).

what we will (re)learn here

- logic and quantification in behavioral and fMRI experiments
  - monotonicity-related experiments
  - description requires environments (not operators)
  - (possible) neural locus of processing monotonicity

convergence of results in grammar/logic/processing!



the organon

- includes Aristotle's theory of inference ("the syllogistic")
- syllogisms involving quantificational operators: all, none, some (not)
- representation of their monotonicity properties (environment-based)

peripatetics

- (wholly) hypothetical syllogisms
- (pre) modus tollens (esp Theophrastus)
- representation of their monotonicity properties (environment-based)

syllogisms and monotonicity patterns in quantified sentences

Every A is B
Every B is C

∴ Every A is C

Table 1: Barbara

A ⊆ B

Every	A	is C
Every	B	is C

A ⊆ B

Every	C	is A
Every	C	is B

Table 2: DM in the "subject" predicate, UM in the "predicate" predicate

- prelim terminology: if replacing a predicate (A) with a weaker predicate (B, where A⊆B) in a sentence S results in a stronger/weaker meaning of S, we say that we have 'Downward-Monotonicity'/'Upward-Monotonicity' in S with respect to A.

syllogisms and monotonicity patterns in quantified sentences

Every A is B	$A \subseteq B$
No B is C	No **A** is C
$\therefore$ No A is C	No **B** is C $\Uparrow$

Table 3: Celarent (modified order); DM in the “subject” predicate

Every A is B	$A \subseteq B$
No C is B	No C **is A**
$\therefore$ No C is A	No C **is B** $\Uparrow$

Table 4: Camestres; DM in the “predicate” predicate

syllogisms and monotonicity patterns in quantified sentences (w negation)

Every A is B

Some C is A

$\therefore$ Some C is B

$A \subseteq B$

Some C	is A	$\Downarrow$
Some C	is B	

Table 5: Darii; UM in the “predicate” predicate

Every A is B

Some C is not B

$\therefore$ Some C is not A

$A \subseteq B$

Some C is not	A	$\Uparrow$
Some C is not	B	

Table 6: Baroco; DM in the (negated) “predicate” predicate

syllogisms and monotonicity patterns in conditional sentences

$$\begin{array}{l} \text{If x is A, then x is B} \\ \text{x is not B} \\ \hline \therefore \text{ x is not A} \end{array}$$

$A \subseteq B$

not	A	x
not	B	x

$\Uparrow$

Table 7: Syllogism 'from a hypothesis'; DM in the "predicate" predicate

$$\begin{array}{l} \text{If A, then B} \\ \text{If B, then C} \\ \hline \therefore \text{ If A, then C} \end{array}$$

$A \subseteq B$

$\Uparrow$

if	A	then C
if	B	then C

$A \subseteq B$

if C then	A
if C then	B

$\Downarrow$

Table 8: Wholly hypothetical syllogism; DM in antecedent, UM in consequent

(cf. Bobzien 2000, 2002, ia)

- impression from the preceding: logic as something we do with language

- but: logic (also) as something we do in language (constantly, unawares*)
- demonstrable in many ways: scalar implicatures, weak islands and their obviation, aspectual modification, exceptive modification, scope economy, definiteness effect, moore sentences, embedding epistemic modals, etc.

- we will focus on a specific class of such phenomena, ie, on specific expressions whose acceptability depends on more than their syntactic properties:
  - so-called negative polarity items (npis; *any*, *ever*, etc)

suggestive parallels: npis - monotonicity patterns

*Every student is any good.

$A \subseteq B$

Every C	is A
Every C	is B

$\Downarrow$

*Some student is any good.

$A \subseteq B$

Some C	is A
Some C	is B

$\Downarrow$

*If Aristotle wrote Organon, he is any good.

$A \subseteq B$

if C then	A
if C then	B

$\Downarrow$

If Aristotle is any good, he wrote Organon. $\Uparrow$

$A \subseteq B$

if	A	then	C
if	B	then	C

Some student is not any good.

$A \subseteq B$

Some C is not	A
Some C is not	B

$\Uparrow$

No student who smiled is any good.

$A \subseteq B$

No C is	A
No C is	B

$\Uparrow$

No student who is any good smiled. $\Uparrow$

$A \subseteq B$

No	A	is A
No	B	is C

generalization from suggestive parallels

(1) An NPI is acceptable iff it is contained in a term of a quantificational or a conditional sentence that exhibits downward-monotonicity wrt the term.

(2) Conditional sentence:

If [$_A$ Aristotle is anyone of significance], Boethius is happy

is DM wrt A; *anyone of significance* is contained in A

obvious undergeneration issues

(3) a. *Aristotle gave talks after he was as anyone of significance.
b. Aristotle gave talks before he was anyone of significance.

(4) a. Boethius was smarter than any other philosopher was.
b. Boethius was as smart as any other philosopher was.

classical entailment

(5) A sentence S entails another sentence S' iff

for every point of evaluation α, $[\![\text{S}]\!]^\alpha \rightarrow [\![\text{S'}]\!]^\alpha$.

(sloppy terminology: entailment between syntactic, semantic objects)

generalizing entailment

(6) conjoinable/boolean types

a. t is a conjoinable type

b. if α is a type, and β is a conjoinable type, $(\alpha\beta)$ is a conjoinable type

(7) An object C entails another object C', C$\Rightarrow$C', iff

i) C and C' are of type t and C $\rightarrow$ C', or

ii) C and C' are of a conjoinable type $(\alpha\beta)$, and for all X of type α s.t. $[\![\text{C}]\!](\text{X})$ and $[\![\text{C'}]\!](\text{X})$ are defined, C(X)$\Rightarrow$C'(X).

(Strawson entailment, see below; von Fintel 1999)

upward monotonicity

(8) A function F of type $(\alpha\beta)$ is upward-monotone (UM) iff α and β are conjoinable types, and for all A, A' of type α: A $\Rightarrow$ A', F(A) $\Rightarrow$ F(A').

downward monotonicity

(9) A function F of type $(\alpha\beta)$ is downward-monotone (DM) iff α and β are conjoinable types, and for all A, A' of type α: A $\Rightarrow$ A', F(A') $\Rightarrow$ F(A).

(10) $[\![\text{not}]\!] = [\lambda p.\ \neg p]$ is a DM function.

For any S,S': if S$\Rightarrow$S' and $[\![\text{not}]\!]$(S'), then $[\![\text{not}]\!]$(S) (modus tollens).

(11) $[\![\text{every}]\!] = [\lambda P.\lambda Q.\ \forall x{:}P(x)\rightarrow Q(x)]$ is a DM function.

Assume P$\Rightarrow$P', $[\![\text{every}]\!]$(P')(Q) and $\neg[\![\text{every}]\!]$(P)(Q) for some Q.
Hence: $\exists x$: P(x) $\wedge$ $\neg$Q(x). Hence: $\exists x$: P'(x) $\wedge$ $\neg$Q(x).
Hence: $\neg[\![\text{every}]\!]$(P')(Q). $\lightning$

(12) $[\![\text{every student}]\!] = [\lambda P.\ \forall x{:}\ \text{student}(x) \rightarrow P(x)]$ is a UM function.

Assume P$\Rightarrow$P', $[\![\text{every student}]\!]$(P) and $\neg[\![\text{every student}]\!]$(P').
Hence: $\exists x$: student x $\wedge$ $\neg$P'(x). Hence: $\exists x$: student x $\wedge$ $\neg$P(x).
Hence: $\neg[\![\text{every student}]\!]$(P). $\lightning$

(13) **Op-Condition:** An npi is acceptable iff it is c-commanded at LF by a constituent that denotes a downward-monotone function.

predictions 1: *any-DP* acceptable in the scope of *not, every, if*

[$\boxed{\text{not}}$ [Aristotle is anyone of significance]]

not c-commands *anyone of significance*, and ⟦not⟧ is a DM function

[[$\boxed{\text{Every}}$ [student who read any book]] smiled]

every c-commands *any book*, and ⟦every⟧ is a DM function

[$\boxed{\text{[no medieval philosopher]}}$ [was anyone of significance]]

no medieval philosopher c-commands *anyone of significance*, and ⟦no medieval philosopher⟧ is a DM function

(13) **Op-Condition:** An NPI is acceptable iff it is c-commanded at LF by a constituent that denotes a downward-monotone function.

predictions 2: *any-DP* unacceptable in the (immediate) scope of *every NP, if S*

*[$\boxed{\text{[Every student]}}$ [is anyone of significance]]

every student is the only pertinent expression that c-commands *anyone of significance*, and ⟦every student⟧ is a UM function

The meanings of *before, after, as, more*, etc., (or the meanings of their *compositiones*) must yet be provided in order to determine the predictions. See below.

upward monotonicity

(14) A constituent C of a conjoinable type β is upward-monotone with respect to the position of a constituent A of a conjoinable type α that C dominates iff $[\lambda X_\alpha . [\![C]\!]^{[A \rightarrow X]}]$ is a UM function. (cf. Gajewski 2005)

alternative statement (not equivalent!)

(15) A constituent C of a conjoinable type β is upward-monotone with respect to a constituent A of a conjoinable type α that C dominates iff $\forall X\text{: } [\![A]\!] \Rightarrow [\![X]\!] \rightarrow [\![C]\!] \Rightarrow [\![C[A/X]]\!]$ (or $\forall X\text{: } [\![X]\!] \Rightarrow [\![A]\!] \rightarrow [\![C[A/X]]\!] \Rightarrow [\![C]\!]$)

terminological convention: upward-monotonicity wrt ~~the position of~~ a phrase

downward monotonicity

(16) A constituent C of a conjoinable type β is downward-monotone with respect to the position of a constituent A of a conjoinable type α that C dominates iff $[\lambda X_\alpha . [\![C]\!]^{[A \to X]}]$ is a DM function. (cf. Gajewski 2005)

alternative statement (not equivalent!)

(17) A constituent C of a conjoinable type β is downward-monotone with respect to a constituent A of a conjoinable type α that C dominates iff $\forall X\!: [\![A]\!] \Rightarrow [\![X]\!] \to [\![C[A/X]]\!] \Rightarrow [\![C]\!]$ $\big($or $\forall X\!: [\![X]\!] \Rightarrow [\![A]\!] \to [\![C]\!] \Rightarrow [\![C[A/X]]\!]\big)$

terminological convention: downward-monotonicity wrt ~~the position of~~ a phrase

(18) [*not S*] is DM wrt *S*.

$\lambda X.[\![\text{not S}]\!]^{[S \to X]} = [\![\text{neg}]\!]$. $[\![\text{neg}]\!]$ is a DM function (see above).

(19) [*every NP*] is DM wrt *NP*, for any NP.

$\lambda X.[\![\text{every NP}]\!]^{[NP \to X]} = [\![\text{every}]\!]$. $[\![\text{every}]\!]$ is a DM function (see above).

(20) [*every student who read a book*] is DM wrt *a book*.

$\lambda X.\ [\![\text{every student who read a book}]\!]^{[\text{a book} \to X]} =$

$[\lambda X.\lambda P.\ \forall x\text{: } X(\lambda z.\ \text{student x read z}) \to P(x)]$ is a DM function.

Assume: $Z \Rightarrow Z'$, $[\forall x\text{: } Z'(\lambda z.\ \text{student x read z}) \to P(x)]$ for some P, and $[\neg\forall x\text{: } Z(\lambda z.\ \text{student x read z}) \to P(x)]$.

Hence: $\exists x\text{: } Z(\lambda z.\ \text{student x read z}) \wedge \neg P(x)$.

Hence: $\exists x\text{: } Z'(\lambda z.\ \text{student x read z}) \wedge \neg P(x)$.

Hence: $\neg\forall x\text{: } Z'(\lambda z.\ \text{student x read z}) \to P(x)$. $\lightning$

(21) **Env-Condition:** An npi is acceptable iff it occurs at LF in a constituent that is downward-monotone with respect to its position.

predictions 1: *any-DP* acceptable in the scope of *every, not, if* (in our above examples, not in every other configuration)

$\boxed{[_S \text{ not [Aristotle is anyone of significance]]}}$

S is DM wrt *anyone of significance*.

$\boxed{[_S\ \boxed{[_{DP} \text{ every student who read any book]}}\ \text{] smiled]}}$

Both *S* and *DP* are DM wrt *any book*.

$\boxed{[_S \text{ no medieval philosopher was anyone of significance]}}$

S is DM wrt *anyone of significance*.

(21) **Env-Condition:** An NPI is acceptable iff it occurs at LF in a constituent that is downward-monotone with respect to its position.

predictions 2: *any-DP* unacceptable in the scope of *every NP, if S* (in our above examples, not in every other configuration)

$\boxed{[_S \text{ every student } \boxed{[_{VP} \text{ is anyone of significance}]}\ ]}$

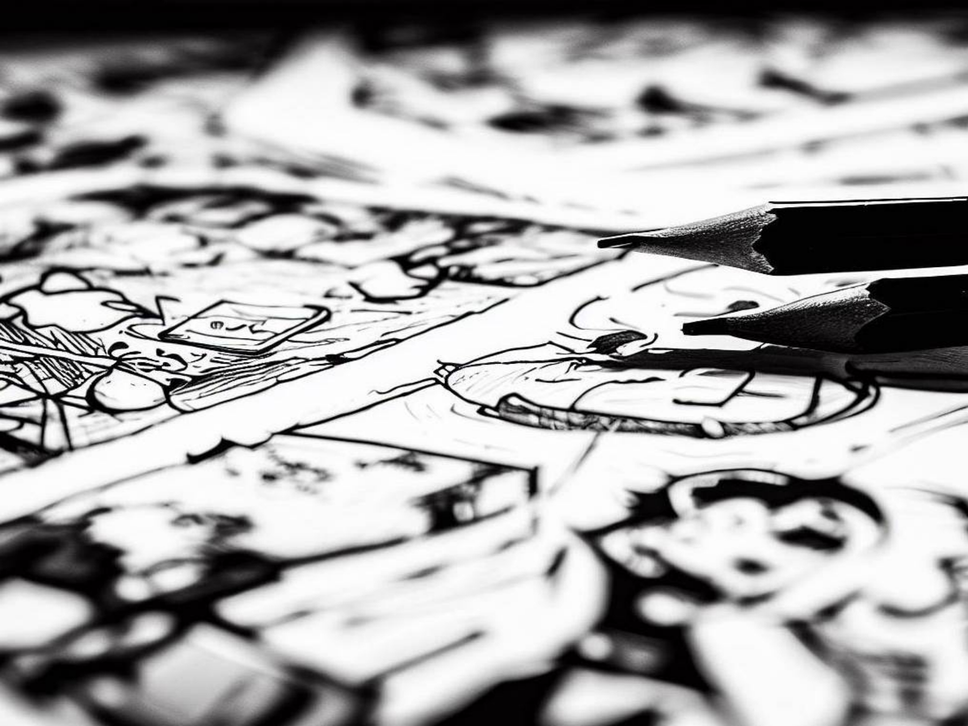
Neither *S* nor *VP* are DM wrt *anyone of significance*.

The meanings of sentences with *before, after, more, as*, etc, (or the meanings of their subconstituents) must yet be provided to determine the predictions.

Op-Condition: An npi is acceptable iff it is c-commanded at LF by a constituent that denotes a downward-monotone function.

Env-Condition: An npi is acceptable iff it occurs at LF in a constituent that is downward-monotone with respect to its position.

so far neither condition has an upper hand, they may appear indistinguishable



classical entailment

(22) An object C (classically) entails another object C', C$\Rightarrow$C', iff

i) C and C' are of type t and C $\rightarrow$ C', or

ii) C and C' are of a conjoinable type ($\alpha\beta$), and for all X of type α, C(X) $\Rightarrow$ C'(X)

Strawson entailment (what we adopted)

(23) An object C (Strawson) entails another object C', C$\Rightarrow$C', iff

i) C and C' are of type t and C $\rightarrow$ C', or

ii) C and C' are of a conjoinable type ($\alpha\beta$), and for all X of type α s.t. $[\![C]\!]$(X) and $[\![C']\!]$(X) are defined, C(X)$\Rightarrow$C'(X).

classical entailment $\subseteq$ Strawson entailment $\left(\subseteq\right.$ contextual (Strawson) entailment$\left.\right)$

one puzzle about npis in before-clauses

(24) Aristotle gave talks before he was anyone of significance.

(25) $\exists$t: Aristotle gave talks at t $\wedge$

$\exists$t': t<t' Aristotle was of significance at t' $\wedge$

$\forall$t'': Aristotle was of significance at t'' $\rightarrow$ t<t'

Strawson entailment + conditions: weak enough

veridical presupposition (cf Landman, Condoravdi, Ogihara)

(26) $[\![\text{before}]\!] = [\lambda p\text{: } \exists t(p(t)).\ \lambda t.\ \forall t'\text{: } p(t') \rightarrow t<t']$

is a DM function (hence, Op-Condition predicts acceptability)

(27) [λX: $\exists$t(Aristotle was X at t). $\exists$t: Aristotle gave talks at t $\wedge$

$\forall$t': Aristotle was X at t' $\rightarrow$ t<t'']

is a DM function (hence, Env-Condition predicts acceptability)

(28) *The student who attended any class smiled.

Strawson entailment + conditions: too weak

(29) $[\![\text{the}]\!] = [\lambda P\text{: } \exists!x(P(x)).\ \lambda Q.\ \exists x\text{: } P(x) \wedge Q(x)]$ is a DM function.

Assume $P \Rightarrow P'$, $[\![\text{the}]\!](P')(Q)$ and $\neg[\![\text{the}]\!](P)(Q)$ for some Q (hence all defined). Hence: $\neg\exists x\text{:}P(x) \wedge Q(x)$ and $\exists!x\text{:}P'(x)$.
Hence: $\neg\exists x\text{:}P'(x) \wedge Q(x)$. Hence: $\neg[\![\text{the}]\!](P')(Q)$. ↯

(30) $\lambda X.[\![\text{the student who attended any class smiled}]\!]^{[\text{any class} \rightarrow X]}$
$= [\lambda X\text{: } \exists!x\text{: } X(\lambda z.\ \text{student x attended z}).$
$\exists x\text{: } X(\lambda z.\ \text{student x attended z}) \wedge \text{student x smiled})]$
is a DM function.

Assume $Z \Rightarrow Z'$, $[\exists x\text{:}Z'(\lambda z.\text{student x attended z}) \wedge \text{student x smiled}]$, $[\neg(\exists x\text{:}Z(\lambda z.\text{student x attended z}) \wedge \text{student x smiled})]$, and $\exists!x\text{:}Z/Z'(\lambda z.\ \text{student x attended z})$.
Hence: $[\neg\exists x\text{:}Z'(\lambda z.\ \text{student x attended z}) \wedge \text{student x smiled})]$. ↯

Strawson equivalence (unlike in all preceding examples)

(31) $[\![\text{the}]\!] = [\lambda P{:}\ \exists! x(P(x)).\ \lambda Q.\ \exists x{:}\ P(x) \wedge Q(x)]$ is a UM function.

Assume $P \Rightarrow P'$, $[\![\text{the}]\!](P)(Q)$ and $\neg[\![\text{the}]\!](P')(Q)$ for some Q (hence all defined). Hence: $\neg\exists x{:}\ P'(x) \wedge Q(x)$. Hence: $\neg\exists x{:}\ P(x) \wedge Q(x)$. Hence: $\neg[\![\text{the}]\!](P)(Q)$. $\lightning$

counteracting excessive weakness *(but why should this hold?!)*

(32) **Op-Condition:** An NPI is acceptable iff it is c-commanded at LF by a constituent that denotes a DM (and not UM) function.

(33) **Env-Condition:** An NPI is acceptable iff it occurs at LF in a constituent that is DM (and not UM) with respect to its position.

(cf Lahiri 1998, Cable 2002, Guerzoni & Sharvit 2007)