Monotonicity, Day 1

Luka Crnič, Yosef Grodzinsky, 7/8/23

https://lukacrnic.com/monotonicity

- logic in reasoning
- logic in grammar
- logic in language processing

lessons learned (and still learning)

- no autonomy of grammar from logic
- (partly) unfortunate split of the two endeavors

what we will (re)learn here

- intricate ways in which logic affects language
  - monotonicity-sensitive phenomena (esp. npis)
  - description requires environments (not operators)
    + hint at why this may be the case (explanation)
  - focus on modal and comparative sentences

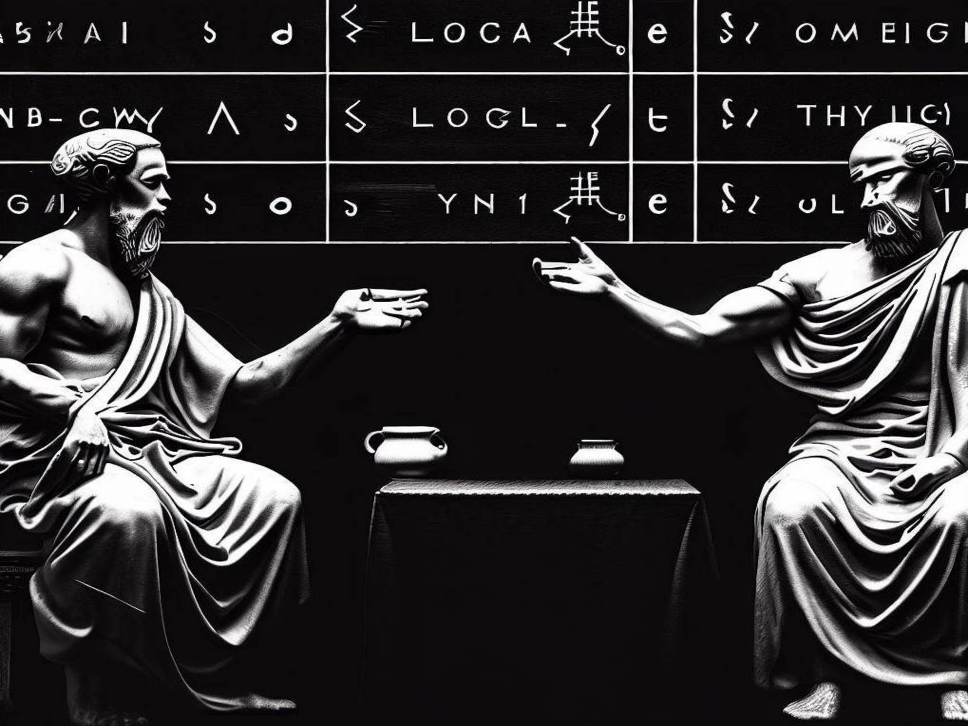
lessons learned (and still learning)

- grammatical processes significany affect language processing, and they have a pronounced reflection in the brain (and similarly for logical processes).

what we will (re)learn here

- logic and quantification in behavioral and fMRI experiments
  - monotonicity-related experiments
  - description requires environments (not operators)
  - (possible) neural locus of processing monotonicity

convergence of results in grammar/logic/processing!



the organon

- includes Aristotle's theory of inference ("the syllogistic")
- syllogisms involving quantificational operators: all, none, some (not)
- representation of their monotonicity properties (environment-based)

peripatetics

- (wholly) hypothetical syllogisms
- (pre) modus tollens (esp Theophrastus)
- representation of their monotonicity properties (environment-based)

syllogisms and monotonicity patterns in quantified sentences

Every A is B
Every B is C
∴ Every A is C

Table 1: Barbara

$A \subseteq B$		$A \subseteq B$	
$\Uparrow$	Every A is C	Every C is A	$\Downarrow$
	Every B is C	Every C is B	

Table 2: DM in the "subject" predicate, UM in the "predicate" predicate

- prelim terminology: if replacing a predicate (A) with a weaker predicate (B, where A⊆B) in a sentence S results in a stronger/weaker meaning of S, we say that we have 'Downward-Monotonicity'/'Upward-Monotonicity' in S with respect to A.

Every A is B	$A \subseteq B$	
No B is C	No **A** is A	$\Uparrow$
∴ No A is C	No **B** is C	

Table 3: Celarent (modified order); DM in the "subject" predicate

Every A is B	$A \subseteq B$	
No C is B	No C **is A**	$\Uparrow$
∴ No C is A	No C **is B**	

Table 4: Camestres; DM in the "predicate" predicate

	Every A is B
	Some C is A
∴	Some C is B

$A \subseteq B$

$$\boxed{\begin{array}{l} \text{Some C is A} \\ \text{Some C is B} \end{array}} \Downarrow$$

Table 5: Darii; UM in the "predicate" predicate

	Every A is B
	Some C is not B
∴	Some C is not A

$A \subseteq B$

$$\boxed{\begin{array}{l} \text{Some C is not A} \\ \text{Some C is not B} \end{array}} \Uparrow$$

Table 6: Baroco; DM in the (negated) "predicate" predicate

syllogisms and monotonicity patterns in conditional sentences

	If x is A, then x is B
	x is not B
$\therefore$	x is not A

$A \subseteq B$

not	A	x
not	B	x

$\Uparrow$

Table 7: Syllogism ‘from a hypothesis’; DM in the “predicate” predicate

	If A, then B
	If B, then C
$\therefore$	If A, then C

$A \subseteq B$

$\Uparrow$

if	A	then C
if	B	then C

$A \subseteq B$

if C then	A
if C then	B

$\Downarrow$

Table 8: Wholly hypothetical syllogism; DM in antecedent, UM in consequent

(cf. Bobzien 2000, 2002, ia)

sourcing logic in language

- impression from the preceding: logic as something we do with language

- but: logic (also) as something we do in language (constantly, unawares*)
- demonstrable in many ways: scalar implicatures, weak islands and their obviation, aspectual modification, exceptive modification, scope economy, definiteness effect, moore sentences, embedding epistemic modals, etc.

- we will focus on a specific class of such phenomena, ie, on specific expressions whose acceptability depends on more than their syntactic properties:
  - so-called negative polarity items (npis; *any*, *ever*, etc)

suggestive parallels: npis - monotonicity patterns

*Every student is any good.

$A \subseteq B$

Every C	is A
Every C	is B

$\Downarrow$

*Some student is any good.

$A \subseteq B$

Some C	is A
Some C	is B

$\Downarrow$

*If Aristotle wrote Organon, he is any good.

$A \subseteq B$

if C then	A
if C then	B

$\Downarrow$

If Aristotle is any good, he wrote Organon.

$A \subseteq B$

$\Uparrow$

if $\boxed{A}$ then C
if $\boxed{B}$ then C

Some student is not any good.

$A \subseteq B$

Some C is not $\boxed{A}$
Some C is not $\boxed{B}$

$\Uparrow$

No student who smiled is any good.

$A \subseteq B$

No C is $\boxed{A}$
No C is $\boxed{B}$

$\Uparrow$

No student who is any good smiled.

$A \subseteq B$

$\Uparrow$

No $\boxed{A}$ is A
No $\boxed{B}$ is C

generalization from suggestive parallels

(1) An NPI is acceptable iff it is contained in a term of a quantificational or a conditional sentence that exhibits downward-monotonicity wrt the term.

(2) Conditional sentence:

If $[_A$ Aristotle is anyone of significance$]$, Boethius is happy

is DM wrt A; *anyone of significance* is contained in A

obvious undergeneration issues

(3) a. *Aristotle gave talks after he was as anyone of significance.
b. Aristotle gave talks before he was anyone of significance.

(4) a. Boethius was smarter than any other philosopher was.
b. Boethius was as smart as any other philosopher was.

classical entailment

(5) A sentence S entails another sentence S' iff

for every point of evaluation α, $[\![S]\!]^{\alpha} \rightarrow [\![S']\!]^{\alpha}$.

(sloppy terminology: entailment between syntactic, semantic objects)

generalizing entailment

(6) conjoinable/boolean types

a. t is a conjoinable type

b. if α is a type, and β is a conjoinable type, $(\alpha\beta)$ is a conjoinable type

(7) An object C entails another object C', C$\Rightarrow$C', iff

i) C and C' are of type t and C $\rightarrow$ C', or

ii) C and C' are of a conjoinable type $(\alpha\beta)$, and for all X of type α s.t. $[\![C]\!](X)$ and $[\![C']\!](X)$ are defined, C(X)$\Rightarrow$C'(X).

(Strawson entailment, see below; von Fintel 1999)

upward monotonicity

(8) A function F of type $(\alpha\beta)$ is upward-monotone (UM) iff α and β are conjoinable types, and for all A, A' of type α: A $\Rightarrow$ A', F(A) $\Rightarrow$ F(A').

downward monotonicity

(9) A function F of type $(\alpha\beta)$ is downward-monotone (DM) iff α and β are conjoinable types, and for all A, A' of type α: A $\Rightarrow$ A', F(A') $\Rightarrow$ F(A).

(10) $[\![\text{not}]\!] = [\lambda p.\ \neg p]$ is a DM function.

For any S,S': if S$\Rightarrow$S' and $[\![\text{not}]\!]$(S'), then $[\![\text{not}]\!]$(S) (modus tollens).

(11) $[\![\text{every}]\!] = [\lambda P.\lambda Q.\ \forall x{:}P(x)\rightarrow Q(x)]$ is a DM function.

Assume P$\Rightarrow$P', $[\![\text{every}]\!]$(P')(Q) and $\neg[\![\text{every}]\!]$(P)(Q) for some Q.
Hence: $\exists x$: $P(x) \wedge \neg Q(x)$. Hence: $\exists x$: $P'(x) \wedge \neg Q(x)$.
Hence: $\neg[\![\text{every}]\!]$(P')(Q). $\lightning$

(12) $[\![\text{every student}]\!] = [\lambda P.\ \forall x{:}\ \text{student}(x) \rightarrow P(x)]$ is a UM function.

Assume P$\Rightarrow$P', $[\![\text{every student}]\!]$(P) and $\neg[\![\text{every student}]\!]$(P').
Hence: $\exists x$: student x $\wedge \neg P'(x)$. Hence: $\exists x$: student x $\wedge \neg P(x)$.
Hence: $\neg[\![\text{every student}]\!]$(P). $\lightning$

(13) **Op-Condition:** An npi is acceptable iff it is c-commanded at LF by a constituent that denotes a downward-monotone function.

predictions 1: *any-DP* acceptable in the scope of *not, every, if*

[$\boxed{\text{not}}$ [Aristotle is anyone of significance]]

not c-commands *anyone of significance*, and ⟦not⟧ is a DM function

[[$\boxed{\text{Every}}$ [student who read any book]] smiled]

every c-commands *any book*, and ⟦every⟧ is a DM function

[$\boxed{\text{[no medieval philosopher]}}$ [was anyone of significance]]

no medieval philosopher c-commands *anyone of significance*, and ⟦no medieval philosopher⟧ is a DM function

(13) **Op-Condition:** An NPI is acceptable iff it is c-commanded at LF by a constituent that denotes a downward-monotone function.

predictions 2: *any-DP* unacceptable in the (immediate) scope of *every NP, if S*

*[$\boxed{\text{[Every student]}}$ [is anyone of significance]]

every student is the only pertinent expression that c-commands *anyone of significance*, and ⟦every student⟧ is a UM function

The meanings of *before, after, as, more*, etc., (or the meanings of their *compositiones*) must yet be provided in order to determine the predictions. See below.

upward monotonicity

(14) A constituent C of a conjoinable type β is upward-monotone with respect to the position of a constituent A of a conjoinable type α that C dominates iff $[\lambda X_\alpha . [\![C]\!]^{[A \to X]}]$ is a UM function. (cf. Gajewski 2005)

alternative statement (not equivalent!)

(15) A constituent C of a conjoinable type β is upward-monotone with respect to a constituent A of a conjoinable type α that C dominates iff $\forall X$: $[\![A]\!] \Rightarrow [\![X]\!] \to [\![C]\!] \Rightarrow [\![C[A/X]]\!]$ (or $\forall X$: $[\![X]\!] \Rightarrow [\![A]\!] \to [\![C[A/X]]\!] \Rightarrow [\![C]\!]$)

terminological convention: upward-monotonicity wrt ~~the position of~~ a phrase

downward monotonicity

(16) A constituent C of a conjoinable type β is downward-monotone with respect to the position of a constituent A of a conjoinable type α that C dominates iff $[\lambda X_\alpha.[\![C]\!]^{[A \to X]}]$ is a DM function. (cf. Gajewski 2005)

alternative statement (not equivalent!)

(17) A constituent C of a conjoinable type β is downward-monotone with respect to a constituent A of a conjoinable type α that C dominates iff $\forall X$: $[\![A]\!] \Rightarrow [\![X]\!] \to [\![C[A/X]]\!] \Rightarrow [\![C]\!]$ $\big($or $\forall X$: $[\![X]\!] \Rightarrow [\![A]\!] \to [\![C]\!] \Rightarrow [\![C[A/X]]\!]\big)$

terminological convention: downward-monotonicity wrt ~~the position of~~ a phrase

(18) [*not S*] is DM wrt *S*.

$\lambda X.[\![\text{not S}]\!]^{[S \to X]} = [\![\text{neg}]\!]$. $[\![\text{neg}]\!]$ is a DM function (see above).

(19) [*every NP*] is DM wrt *NP*, for any NP.

$\lambda X.[\![\text{every NP}]\!]^{[NP \to X]} = [\![\text{every}]\!]$. $[\![\text{every}]\!]$ is a DM function (see above).

(20) [*every student who read a book*] is DM wrt *a book*.

$\lambda X.\ [\![\text{every student who read a book}]\!]^{[\text{a book} \to X]} =$

$[\lambda X.\lambda P.\ \forall x\text{: } X(\lambda z.\ \text{student x read z}) \to P(x)]$ is a DM function.

Assume: $Z \Rightarrow Z'$, $[\forall x\text{: } Z'(\lambda z.\ \text{student x read z}) \to P(x)]$ for some P, and $[\neg\forall x\text{: } Z(\lambda z.\ \text{student x read z}) \to P(x)]$.

Hence: $\exists x\text{: } Z(\lambda z.\ \text{student x read z}) \wedge \neg P(x)$.

Hence: $\exists x\text{: } Z'(\lambda z.\ \text{student x read z}) \wedge \neg P(x)$.

Hence: $\neg\forall x\text{: } Z'(\lambda z.\ \text{student x read z}) \to P(x)$. $\lightning$

(21) **Env-Condition:** An npi is acceptable iff it occurs at LF in a constituent that is downward-monotone with respect to its position.

predictions 1: *any-DP* acceptable in the scope of *every, not, if* (in our above examples, not in every other configuration)

$\boxed{[_S \text{ not [Aristotle is anyone of significance]]}}$

S is DM wrt *anyone of significance*.

$\boxed{[_S\ \boxed{[_{DP} \text{ every student who read any book]}}\ \text{] smiled]}}$

Both *S* and *DP* are DM wrt *any book*.

$\boxed{[_S \text{ no medieval philosopher was anyone of significance]}}$

S is DM wrt *anyone of significance*.

(21) **Env-Condition:** An NPI is acceptable iff it occurs at LF in a constituent that is downward-monotone with respect to its position.

predictions 2: *any-DP* unacceptable in the scope of *every NP, if S* (in our above examples, not in every other configuration)

$\boxed{[_S \text{ every student } \boxed{[_{VP} \text{ is anyone of significance}]}\ ]}$

Neither *S* nor *VP* are DM wrt *anyone of significance*.

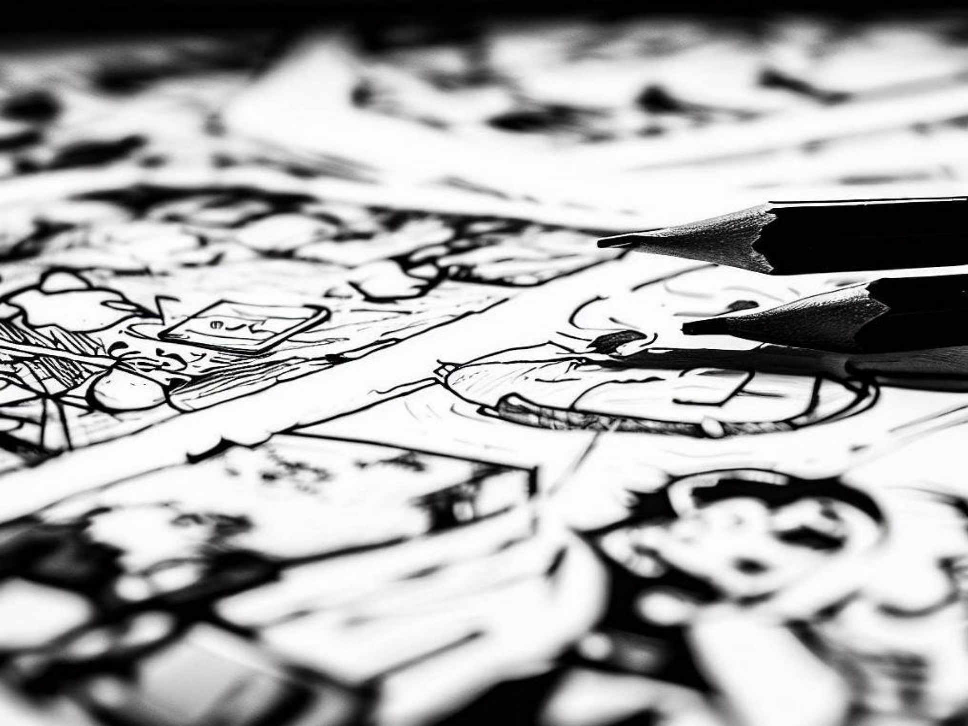
The meanings of sentences with *before, after, more, as*, etc, (or the meanings of their subconstituents) must yet be provided to determine the predictions.

intermediate summary: the players

Op-Condition: An npi is acceptable iff it is c-commanded at LF by a constituent that denotes a downward-monotone function.

Env-Condition: An npi is acceptable iff it occurs at LF in a constituent that is downward-monotone with respect to its position.

so far neither condition has an upper hand, they may appear indistinguishable



reminder: strength of entailment (in the conditions)

classical entailment

(22) An object C (classically) entails another object C', C⇒C', iff

i) C and C' are of type t and C $\rightarrow$ C', or

ii) C and C' are of a conjoinable type ($\alpha\beta$), and for all X of type α, C(X) $\Rightarrow$ C'(X)

Strawson entailment (what we adopted)

(23) An object C (Strawson) entails another object C', C⇒C', iff

i) C and C' are of type t and C $\rightarrow$ C', or

ii) C and C' are of a conjoinable type ($\alpha\beta$), and for all X of type α s.t. $[\![C]\!]$(X) and $[\![C']\!]$(X) are defined, C(X)⇒C'(X).

classical entailment $\subseteq$ Strawson entailment $\Big(\subseteq$ contextual (Strawson) entailment$\Big)$

one puzzle about npis in before-clauses

(24) Aristotle gave talks before he was anyone of significance.

(25) $\exists$t: Aristotle gave talks at t $\wedge$

$\exists$t': t<t' Aristotle was of significance at t' $\wedge$

$\forall$t'': Aristotle was of significance at t'' $\rightarrow$ t<t'

Strawson entailment + conditions: weak enough

veridical presupposition (cf Landman, Condoravdi, Ogihara)

(26) $[\![\text{before}]\!] = [\lambda p\text{: } \exists t(p(t)).\ \lambda t.\ \forall t'\text{: } p(t') \rightarrow t < t']$

is a DM function (hence, Op-Condition predicts acceptability)

(27) [λX: $\exists$t(Aristotle was X at t). $\exists$t: Aristotle gave talks at t $\wedge$

$\forall$t': Aristotle was X at t' $\rightarrow$ t<t']

is a DM function (hence, Env-Condition predicts acceptability)

(28) *The student who attended any class smiled.

Strawson entailment + conditions: too weak

(29) $[\![\text{the}]\!] = [\lambda P\text{: } \exists!x(P(x)).\ \lambda Q.\ \exists x\text{: } P(x) \wedge Q(x)]$ is a DM function.

Assume $P \Rightarrow P'$, $[\![\text{the}]\!](P')(Q)$ and $\neg[\![\text{the}]\!](P)(Q)$ for some Q (hence all defined). Hence: $\neg\exists x{:}P(x) \wedge Q(x)$ and $\exists!x{:}P'(x)$.
Hence: $\neg\exists x{:}P'(x) \wedge Q(x)$. Hence: $\neg[\![\text{the}]\!](P')(Q)$. $\lightning$

(30) $\lambda X.[\![\text{the student who attended any class smiled}]\!]^{[\text{any class} \rightarrow X]}$
$= [\lambda X\text{: } \exists!x\text{: } X(\lambda z.\ \text{student x attended z}).$
$\exists x\text{: } X(\lambda z.\ \text{student x attended z}) \wedge \text{student x smiled}]$
is a DM function.

Assume $Z \Rightarrow Z'$, $[\exists x{:}Z'(\lambda z.\text{student x attended z}) \wedge \text{student x smiled}]$, $[\neg(\exists x{:}Z(\lambda z.\text{student x attended z}) \wedge \text{student x smiled})]$, and $\exists!x{:}Z/Z'(\lambda z.\ \text{student x attended z})$.
Hence: $[\neg\exists x{:}Z'(\lambda z.\ \text{student x attended z}) \wedge \text{student x smiled})]$. $\lightning$

Strawson equivalence (unlike in all preceding examples)

(31) $[\![\text{the}]\!] = [\lambda P\text{: } \exists!x(P(x)).\ \lambda Q.\ \exists x\text{: } P(x) \wedge Q(x)]$ is a UM function.

Assume $P \Rightarrow P'$, $[\![\text{the}]\!](P)(Q)$ and $\neg[\![\text{the}]\!](P')(Q)$ for some Q (hence all defined). Hence: $\neg\exists x\text{: } P'(x) \wedge Q(x)$. Hence: $\neg\exists x\text{: } P(x) \wedge Q(x)$.
Hence: $\neg[\![\text{the}]\!](P)(Q)$. $\lightning$

counteracting excessive weakness *(but why should this hold?!)*

(32) **Op-Condition:** An NPI is acceptable iff it is c-commanded at LF by a constituent that denotes a DM (and not UM) function.

(33) **Env-Condition:** An NPI is acceptable iff it occurs at LF in a constituent that is DM (and not UM) with respect to its position.

illustration 3: embedding

(34) Never have fewer than 2 students attended any of my classes.

operators

(35) [never [$\boxed{\text{fewer than 2 students}}$ [attended any of my classes]]]

(36) $[\![\text{fewer than 2 students}]\!](P) = |\{x \mid \text{student}(x) \wedge P(x)\}| < 2$

is a DM function

environments

(37) $\boxed{\text{[never [} \boxed{\text{fewer than 2 students [attended any of my classes]]}} \text{]}}$

(38) $\lambda Q.\ [\![\text{fewer than 2 st. attended any of my classes}]\!]^{[\text{any of my classes} \to Q]} =$

$\lambda Q.\ |\{x \mid Q(\lambda z.\ \text{student } x \text{ attended } z)\}| < 2$

is a DM function

(39) a. If the students$_i$ liked any of their$_i$ classes, we are happy

b. *If exactly 22 students$_i$ liked any of their$_i$ classes, we are happy

operators

(40) Op-Condition is satisfied in both (a) and (b)!

(41) $[\![\text{if}]\!] = [\lambda p.\lambda q.\ p \rightarrow q]$ is a DM function.

an additional constraint is needed: immediate scope constraint (Linebarger 1980)

(42) a. If the students$_i$ liked any of their$_i$ classes, we are happy

b. *If exactly 22 students$_i$ liked any of their$_i$ classes, we are happy

environments

(43) Env-Condition is satisfied in (a), <u>but not in (b)!</u>

(44) λX. $[\![$if the st's liked any of their$_i$ classes, we are happy$]\!]^{[\text{any..classes}\rightarrow X]}$ =

[λX. $\neg$(X(λz. S liked z)) $\vee$ (we are happy)] is a DM function.

(45) λX. $[\![$if ex22st liked any of their$_i$ classes, we are happy$]\!]^{[\text{any..classes}\rightarrow X]}$ =

[λX. $\neg$($[\![$ex22st$]\!]$(λy. X(λz. y liked z))) $\vee$ (we are happy)]

is not a DM function.

no additional constraint is needed here

(46) a. Every student who attended any ESSLLI courses had a blast.
b. The students who attended any ESSLLI courses had a blast.

(47) $\forall$x: ($\exists$y: student x attended ESSLLI course y) $\rightarrow$ student x had a blast

operators

(48) Op-Condition is satisfied in (a), but not (obviously) in (b)!

(49) ⟦every⟧ is a DM function.

(50) ⟦the⟧ is not of a conjoinable type (Frege, Strawson).
*(cf. not every student vs. *not the students)*
possible path: dist operator c-commanding the definite description?

illustration 5: plural definites and commitments

(51) a. Every student who attended any ESSLLI courses had a blast.
b. The students who attended any ESSLLI courses had a blast.

(52) $\forall$x: ($\exists$y: student x attended ESSLLI course y) $\rightarrow$ student x had a blast

environments

(53) [λX. $\forall$x: (X(λz. student x attended z)) $\rightarrow$ student x had a blast]
is a DM function.

(cf Gajewski & Hsieh 2014 for some puzzles)

main goals in the following

the candidate descriptions and their parameters

(54) **Op-Condition:** $\boxed{\text{An NPI}}$ is acceptable iff it is c-commanded at LF by $\boxed{\text{a constituent}}$ that denotes $\boxed{\text{a DM (and not UM) function.}}$

(55) **Env-Condition:** $\boxed{\text{An NPI}}$ is acceptable iff it occurs at LF in $\boxed{\text{a constituent}}$ that is $\boxed{\text{DM (and not UM)}}$ with respect to $\boxed{\text{its position.}}$

are these conditions empirically adequate? distinguishable? necessary?

- we provide support for environments over operators on the basis of
  - npis in modal sentences
  - npis in comparative sentences
- we improve on Env-Condition (and hint at an explanation for it)
- we connect our conclusions to those about continuous data

the acceptability and variation challenge

(56) Tina is allowed to attend any class.

(57) *Tina is allowed to ever attend a class.

operators and environments

(58) $[\![\text{allowed}]\!]$ is not a DM function.

(59) $\lambda X.\ [\![\text{T is allowed to attend any class}]\!]^{[\text{any class} \rightarrow X]}$ is not a DM function.

illustration of non-DMness

(60) Tina is allowed to attend a(ny) class
$\not\Rightarrow$ Tina is allowed to attend two classes/every class/most classes

the strength challenge

(61) Tina is allowed to attend any class. *(also: imperatives, generics)*

(62) *Tina is required to attend any class.

the plural/mass challenge

(63) Tina is allowed to attend any class.

(64) *Tina is allowed to attend any classes.

(65) *Tina is allowed to donate any blood.

approaching the acceptability, variation, and strength challenge

(66) Gali is allowed to attend any class

$\not\Rightarrow$ Gali is allowed to attend two classes/every class/most classes

(67) Gali is allowed to attend any class

$\boxed{\Rightarrow}$ Gali is allowed to attend any difficult class/any logic class/etc

(68) Gali is required to attend a class

$\not\Rightarrow$ Gali is required to attend a difficult class/any logic class/etc.

potential revisions (cf Kadmon & Landman on *any*)

(69) **Env-Condition (old):** $\boxed{\text{An NPI}}$ is acceptable iff it occurs at LF in a constituent that is DM with respect to $\boxed{\text{its position.}}$

(70) **Env-Condition-any:** An *any*-DP is acceptable iff it occurs at LF in a constituent that is DM with respect to the position of its complement.

(71) **Env-Condition-ever:** An *ever*-AdvP is acceptable iff it occurs at LF in a constituent that is DM with respect to its position.

(all but) impossible revision

(72) **Op-Condition:** An NPI is acceptable iff it is c-commanded at LF by a constituent that denotes a DM function.