

# Failure of exhaustification and obviation

Luka Crnić

Last updated on November 19, 2019

Dense scales of alternatives give rise to questions for the prevailing theories of exhaustification, in particular, questions pertaining to what alternatives may be pruned when. Specifically, exhaustification inferences observed in sentences in which a dense scale is induced in the scope of a universal quantifier are unexpected – unless a subset of alternatives is systematically pruned at all levels of representation (a subset that has been assumed to be unprunable elsewhere, see, esp., Chierchia 2013 on intervention).

## 1 Background

We begin by briefly reviewing two assumptions about alternatives and exhaustification, both of which have been independently supported (Section 1). We, then, show that their combination prevents a derivation of certain observed exhaustification inferences (Section 2) unless systematic pruning of certain alternatives is assumed (Section 3).

### 1.1 Pruning of alternatives

Every theory of exhaustification must deal with the so-called symmetry problem (see, e.g., Katzir 2007; Fox 2007; Singh 2008; Fox & Katzir 2011; Katzir 2014; Trinh & Haida 2015; Breheny et al. 2018 for a discussion). A well-worn illustration of it is provided in (1): the sentence conveys (implicates) that Mary did not read all of the books, that is, the negation of sentence ‘Mary read all of the books’; it does not convey that Mary read all of the books, that is, the negation of sentence ‘Mary read some but not all of the books’ coupled with the meaning that Mary read some of the books.

- (1) Mary read some of the books.  
 $\rightsquigarrow \neg$  Mary read all of the books.  
 $\nrightarrow \neg$  Mary read some but not all of the books.

If both abovementioned sentences would count as alternatives in the application of exhaustification, and would get negated, we would obtain a contradiction; alternatives whose negations are incompatible with each other in this way are called ‘symmetric.’ That only specific ones of them (if any) get negated is the ‘symmetry problem’ (see Breheny et al. 2018 for a review).

- (2) S1 and S2 are symmetric alternatives to sentence S iff  
a.  $\llbracket S1 \rrbracket \vee \llbracket S2 \rrbracket = \llbracket S \rrbracket$  and  
b.  $\llbracket S1 \rrbracket \wedge \llbracket S2 \rrbracket = \emptyset$ .

The observed resolution in (1) is captured by assuming that ‘Mary read some but not all of the books’ is not an alternative to the sentence that enters into exhaustification – in contrast to ‘Mary read all of the books’. Katzir (2007) achieves this by restricting the alternatives of a

sentence that may enter into exhaustification to those that are at most as complex as the sentence (see Katzir 2007 for details, and Trinh & Haida 2015; Trinh 2019 for qualifications).

- (3) a.  $ALT(S) = \{S' \mid S' \text{ is derived from } S \text{ by substitution of lexical items in } S \text{ with other lexical items and by substitution of constituents in } S \text{ by its subconstituents}\}$   
b.  $\text{Domain of EXH} \subseteq ALT(S)$
- (4) a. Mary read all of the books  $\in ALT(\text{Mary read some of the books})$   
b. Mary read some but not all of the books  $\notin ALT(\text{Mary read some of the books})$

Now, in some cases, the complexity constraint encoded in (3) cannot privilege any one alternative, and so none of them gets negated (rather than all of them, which would yield a contradiction). This has been argued to underly the contrast between the sentences in (5), which is a pattern first observed by Hurford (1974).

- (5) a. John read some or all of the books.  
b. #John is in France or Paris.

The sentence in (5-a) is acceptable, while the sentence in (5-b) is not. The simple meanings of the two disjuncts stand in an entailment relation in both sentences (John reading all of the books entails John reading some of the books, and John being in Paris entails John being in France), which Hurford diagnosed as problematic. Now, ‘John read some of the books’ may be strengthened to John reading some but not all books (which is obviously incompatible with the meaning of the second disjunct in (5-a), that John read all of the books), while ‘John is in France’ cannot be strengthened to him being in France but not Paris, as given in (6). On the assumption that exhaustification may occur in the scope of disjunction, this accounts for the asymmetry in (5) (e.g., Singh 2008; Chierchia et al. 2011 for details).

- (6) a. John read some of the books.  $\rightsquigarrow \neg \text{John read all of the books.}$   
b. John is in France.  $\nrightarrow \neg \text{John is in Paris.}$

What remains to be accounted for is the difference between the disjuncts in the two sentences, specifically, why ‘John is in France’ does not allow for the strengthening that he is in France but not in Paris. The culprit has been taken to be the different types of alternatives induced by the sentence and how these feature in exhaustification. Different proposals about capturing this have been put forward. We describe the proposal by Katzir (2014) below.

**Katzir’s proposal.** Katzir (2014) proposes that the innocently excludable alternatives are determined on the basis of all the formal alternatives.

- (7)  $\text{Excl}(S) = \bigcap \{M \mid M \text{ is a maximal subset of } ALT(S) \text{ such that } \{\neg \llbracket S' \rrbracket \mid S' \in M\} \cup \{\llbracket S \rrbracket\} \text{ is consistent}\}$

Only subsequently may exhaustification be restricted. In the case at hand, again, maximal sets of excludable alternatives differ in what alternatives of the form ‘John is in x’ where x is a locale in France are left out (see (6) above). Their intersection thus lacks any alternatives of the form ‘John is in x’ where x is a locale in France.

- (8)  $\text{Excl}(\text{John is in France}) = \{\text{John is in Italy, John is in Asia, ...}\}$

Relevant innocently excludable alternatives get negated in exhaustification, as encoded in (9)

(Katzir, 2014). Given that none of these alternatives are of the form ‘John is in x’ where x is a locale in France, exhaustification cannot suspend entailment between ‘John is in France’ and ‘John is in Paris.’ Accordingly, the disjunction ‘John is in France or Paris’ cannot be rescued from violating Hurford’s constraint.

$$(9) \quad \llbracket \text{exh}_C S \rrbracket = \llbracket S \rrbracket \wedge \forall S' \in \text{Excl}(S) \cap C: \neg \llbracket S' \rrbracket$$

## 1.2 Density

The sentence in (10-a) lacks the scalar implicature described in (10-b) (cf. Krifka 1999). Fox & Hackl (2006) provide a far-reaching account of this state of affairs.

- (10) John weighs more than 80kg.  
 $\not\sim \neg$  John weighs more than 81kg.

Fox & Hackl account relies on the assumption that the alternatives to the sentence in (10) are those provided in (11) (the sentence has other alternatives, but it is safe to ignore them), and that these form a ‘dense’ set, as characterized in (12) (which is parasitic on the density of the weight measurement scale).

- (11)  $\text{ALT}(\text{John weighs more than } 80 \text{ kg}) = \{\text{John weighs more than } d \mid d \geq 80\text{kg}\}$   
(12)  $\forall p, r \in \text{ALT}(\text{John weighs more than } 80\text{kg}): \text{if } p \not\Rightarrow r,$   
then there exists an  $q \in \text{ALT}(\text{John weighs more than } 80\text{kg})$  such that  $p \Rightarrow q \not\Rightarrow r$ .

Given the assumption of density, the exhaustification of the sentence in (10) yields a contradiction – there are no maximal subsets of (12) whose members may all be jointly negated consistently with the sentence being true (see Fox & Hackl 2006; Gajewski 2008 for further details). Let us sketch the underlying reasoning. First: M in (13) cannot be the maximal set of excludable alternatives. Namely, on the assumption of density, the sentence entails that there is a degree greater than 80 kg such that John weighs more than it, as given in (14), and so negating all the alternatives in (13) would necessarily result in a contradiction.

- (13)  $M = \{\text{John weighs more than } d \mid d > 80\text{kg}\}$   
(14)  $\text{John weighs more than } 80 \text{ kg} \Rightarrow \text{There exists } d > 80\text{kg} \text{ s.t. John weighs more than } d$

Second: No alternative may be precluded from a purported maximal subset M of excludable alternatives in (11). Assume that an alternative is precluded, say, ‘John weighs more than d’ for some  $d > 80\text{kg}$ . However, it is consistent, due to density, that John weighs more than 80kg, while ‘John weighs more than d’ is false, as stated in (15). This contradicts the initial assumption that ‘John weighs more than d’ is not in M, that is, that M is maximal.

- (15) For any alternative in  $\text{ALT}(\text{John weighs more than } 80 \text{ kg})$  that is not in M, say, ‘John weighs more than d’ for some  $d > 80 \text{ kg}$ ,  $[\text{John weighs more than } 80 \text{ kg} \wedge \neg \text{John weighs more than } d]$  is consistent.

Consequently, since there are no maximal sets of excludable alternatives, every alternative is in the intersection of such (non-existent) sets (Gajewski, 2008). If one further assumes that no specific contextual pressures influence pruning of the alternatives (that is, all the alternatives count as relevant), all the alternatives must be negated, and their joint negation contradicts the

sentence (see Fox & Hackl 2006 for the discussion of the necessary architectural considerations, and Fox 2000; Gajewski 2002; Chierchia 2013 for related discussion). Accordingly, since sentence (10) cannot be exhaustified consistently (unless special contextual pressures arise, cf. Fox & Hackl 2006, Section 5), no scalar implicatures are generated.

## 2 The puzzle

How do the two types of assumptions from the preceding section gel together? We study this by looking at scalar implicatures of measure phrase comparatives in the scope of universal quantifiers.

### 2.1 The observation

Fox & Hackl (2006) observe that the exhaustification inferences with measure phrase comparatives become available in the scope of universal modal and nominal quantifiers. This is illustrated in (16), with a universal modal, and (17), with a universal nominal quantifier (see Alexandropoulou et al. 2016 for related experimental evidence).

(16) You are required to weigh more than 80kg (to box as a heavy weight).

$\rightsquigarrow \neg$ You are required to weigh more than 81kg

(17) Every student weighed more than 80kg.

$\rightsquigarrow \neg$ Every student weighed more than 81kg

The obviation effect should be a consequence of consistent exhaustification. The exhaustified meaning given in (18) is indeed consistent (on the assumption of an infinitely large domain of boys, see Fox & Hackl for discussion). However, it turns out that (18) does not correspond to the exhaustified meaning that we obtain by applying *exh* to (17), at least not without an appropriate pruning of alternatives.

(18) Every student weighed more than 80kg  $\wedge$

$\forall d: d > 80\text{kg} \rightarrow \neg$ Every student weighed more than  $d$

### 2.2 The problem

The obviation effects do not follow immediately from the assumptions described in the preceding section. Since the case of modal obviation can be resolved by assuming that some modal alternatives are missing (see Bar-Lev & Fox 2019 for discussion, building on some observations by Chierchia 2013), we focus on the case of nominal quantification in the following, for which such an assumption cannot be maintained. The formal alternatives to sentence (17) are provided in (19). Can we identify maximal subsets of excludable alternatives in (19) given sentence (17)? The answer is ‘no’: admitting existential alternatives sneaks in the same pathology described above. (Note that Fox & Hackl 2006, in line with Fox 2007, do not consider existential alternatives in their derivations.)

(19) ALT(Every boy weighs more than 80kg) =

{Every boy weighs more than  $d$ , Some boy weighs more than  $d \mid d \geq 80\text{kg}$ }

Let us sketch the underlying reasoning behind there being no maximal sets of excludable alternatives; it mirrors the one sketched above. First:  $M$  in (20) cannot be the maximal set of excludable alternatives. Namely, on the assumption of density, the sentence entails that there is a degree greater than 80kg such that some boy weighs more than it, and so negating all the existential alternatives in (19) would necessarily result in a contradiction (all universal modal alternatives may be negated, as mentioned above, and discussed by Fox & Hackl 2006).

(20)  $M = \{\text{Every boy weighs more than } d, \text{Some boy weighs more than } d \mid d > 80\text{kg}\}$

(21) Every boy weighs more than 80kg  $\Rightarrow$   
 There exists  $d > 80\text{kg}$  s.t. Some boy weighs more than  $d$

Second: No alternative can be missing from a maximal subset  $M$  of excludable alternatives in (20). If an alternative is missing, say, ‘Some boy weighs more than  $d$ ’ for some  $d > 80\text{kg}$ , then all the alternatives ‘Some boy weighs more than  $d$ ’ for  $d > d' > 80\text{kg}$  are missing from  $M$  (that there are such alternatives follows from density). However, it is consistent, again due to density, that one of them is true and ‘Some boy weighs more than  $d$ ’ is false, as stated in (22). This clashes with the assumption that ‘Some boy weighs more than  $d$ ’ is not in  $M$ , that is, that  $M$  is maximal.

(22) For any alternative in (20) that is not in  $M$ , say, ‘Some boy weighs more than  $d$ ’ for some  $d > 80\text{ kg}$ , [Every boy weighs more than 80 kg  $\wedge \neg$ Some boy weighs more than  $d$ ] is consistent.

Consequently, since there are no maximal subsets of excludable alternatives, exhaustification would negate all the alternatives, contradicting the sentence – unless some alternatives would be pruned.

### 3 Mandatory pruning

The conclusion is that on the assumption of Katzir’s algorithm for determining Innocently Excludable alternatives, (7), and on the assumption of density for the scales discussed, (12), obviation effects can be accounted for only on the assumption that some of the alternatives are systematically pruned; one such pruning is provided in (23).

(23) a.  $[\text{exh}_R [\text{every student weighs more than } 80 \text{ kg}]]$   
 b.  $R = \text{ALT}(\text{every student weighs more than } 80\text{kg}) \setminus$   
 $\{\text{some student weighs more than } d \mid d > 80\text{kg}\}$

While this is unproblematic given what we said so far – we did not impose any additional constraints on pruning besides it happening after Innocently Excludable alternatives are determined –, it does raise at least two questions: (1) Why is pruning of existential alternatives possible (and easy) but not that of, say, universal alternatives not based on whole numbers? (2) Why is pruning of existential alternatives possible in the cases at hand but perhaps not in the domain of NPI licensing (see Chierchia 2013 on intervention)?

## References

- Alexandropoulou, Stavroula, Jakub Dotlacil, Yaron McNabb & Rick Nouwen. 2016. Pragmatic inferences with numeral modifiers: Novel experimental data. In *Semantics and linguistic theory*, vol. 25, 533–549.
- Bar-Lev, Moshe E & Danny Fox. 2019. Free choice, simplification, and innocent inclusion. MS, Hebrew University of Jerusalem & MIT.
- Breheny, Richard, Nathan Klinedinst, Jacopo Romoli & Yasutada Sudo. 2018. The symmetry problem: current theories and prospects. *Natural Language Semantics* 26(2). 85–110.
- Chierchia, Gennaro. 2013. *Logic in grammar*. Oxford: Oxford University Press.
- Chierchia, Gennaro, Danny Fox & Benjamin Spector. 2011. The grammatical view of scalar implicatures and the relationship between semantics and pragmatics. In Paul Portner, Claudia Maienborn & Klaus von Heusinger (eds.), *Handbook of semantics*, Mouton de Gruyter.
- Fox, Danny. 2000. *Economy and Semantic Interpretation*. MIT Press.
- Fox, Danny. 2007. Free choice and the theory of scalar implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and Implicature in Compositional Semantics*, 71–120. Palgrave Macmillan.
- Fox, Danny & Martin Hackl. 2006. The universal density of measurement. *Linguistics and Philosophy* 29. 537–586.
- Fox, Danny & Roni Katzir. 2011. On the characterization of alternatives. *Natural Language Semantics* 19(1). 87–107.
- Gajewski, Jon. 2002. L-analyticity and natural language. Manuscript, MIT.
- Gajewski, Jon. 2008. Npi any and connected exceptive phrases. *Natural Language Semantics* 16(1). 69–110.
- Hurford, James R. 1974. Exclusive or inclusive disjunction. *Foundations of language* 11(3). 409–411.
- Katzir, Roni. 2007. Structurally defined alternatives. *Linguistics and Philosophy* 30. 669–690.
- Katzir, Roni. 2014. On the roles of markedness and contradiction in the use of alternatives. In Salvatore Pistoia-Reda (ed.), *Pragmatics, semantics and the case of scalar implicatures*, 40–71. Springer.
- Krifka, Manfred. 1999. At least some determiners aren't determiners. In Ken Turner (ed.), *The semantics/pragmatics interface from different points of view*, vol. 1, 257–291. Elsevier.
- Singh, Raj. 2008. On the interpretation of disjunction: Asymmetric, incremental, and eager for inconsistency. *Linguistics and Philosophy* 31(2). 245–260.
- Trinh, Tue. 2019. Exhaustification and contextual restriction. *Frontiers in Communication* 4. 47.
- Trinh, Tue & Andreas Haida. 2015. Constraining the derivation of alternatives. *Natural Language Semantics* 23(4). 249–270.