EE 512

Digital Signal Processing

Session 3

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Read chapter 4
problems
Computer Assignment
2.63, 2.64

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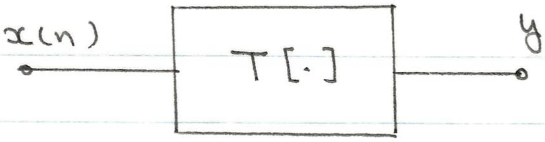
Discrete - Time Systems

A discrete-time system is formally a mapping or transformation which maps the input signal into the output signal



$$y(n) = T[x(n)]$$

Linear Discrete-Time Systems

The operator T is linear iff

$$T[a x_1(n) + b x_2(n)] = a\, T[x_1(n)] + b\, T[x_2(n)]$$
$$= a\, y_1(n) + b\, y_2(n)$$

a, b : arbitrary constants

Problem 12.(b), (e)

(b) Determine whether the system described by

$$y(n) = \sum_{k=n_0}^{n} x(k)$$

is linear or not?

$$y(n) = T[x(n)] = \sum_{k=n_0}^{n} x(k)$$

Then

$$T[a x_1(n) + b x_2(n)] = a \sum_{k=n_0}^{n} x_1(n) + b \sum_{k=n_0}^{n} x_2(n)$$

$$= a\, y_1(n) + b\, y_2(n)$$ i.e. linear

(e) $y(n) = e^{x(n)} = T[x(n)]$

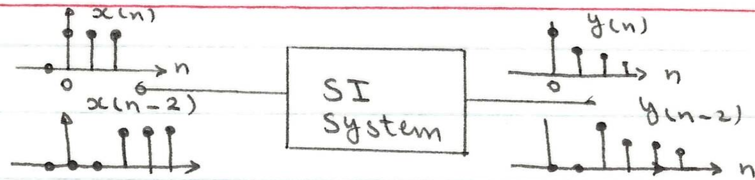
Let $x(n) = a.x_1(n) + b x_2(n)$ then

$$T[a x_1(n) + b x_2(n)] = e^{a x_1(n) + b x_2(n)} \neq a e^{x_1(n)} + b e^{x_2(n)}$$

i.e. Non linear

Time - Invariant or Shift - Invariant Discrete - Time Systems

Let $\quad y(n) = T[x(n)]$



and $\quad \hat{x}(n) = x(n-n_0) \qquad n_0$: An integer

i.e. $\hat{x}(n)$ is the shifted version of $x(n)$ by n_0 samples. Then if

$$\hat{y}(n) = T[\hat{x}(n)]$$
$$= y(n-n_0)$$

the system is said to be time - invariant or shift - invariant.

Example

Consider the previous problems and determine the shift - invariancy

(b) $\quad y(n) = \sum_{k=n_0}^{n} x(k)$

Let $\hat{x}(n) = x(n-N)$

then $\hat{y}(n) = T[x(n-N)] = \sum_{k=n_0}^{n} \hat{x}(k) = \sum_{k=n_0}^{n} x(k-N)$

Define $k - N = \ell$, then

$$\hat{y}(n) = \sum_{\ell=n_0-N}^{n-N} x(\ell)$$

On the other hand $\qquad y(n-N) = \sum_{k=n_0}^{n-N} x(k)$

Thus $\quad \hat{y}(n) \neq y(n-N)$

i.e. the system is shift varying.

(e) $y(n) = e^{x(n)}$

Let $\hat{x}(n) = x(n-N)$, then

$$\hat{y}(n) = T[\hat{x}(n)] = e^{x(n-N)}$$

On the other hand

$$y(n-N) = e^{x(n-N)}$$

i.e. $\hat{y}(n) = y(n-N)$ and system is shift invariant.

Causal Systems

A system is causal if the output for any $n = n_0$ depends only on the inputs for $n \leq n_0$ only i.e.

$$x_1(n) = x_2(n), \quad n < n_0 \overset{\text{implies that}}{\Longrightarrow} y_1(n) = y_2(n), \quad n < n_0$$

$$y(n) = F[x(n), x(n-1) \ldots]$$

Not an issue for prerecorded data (not real-time)

Examples

(b) $y(n) = \sum_{k=n_0}^{n} x(k)$ $\quad n > n_0$

Since $y(n)$ depends on future values of $x(n)$ when $n < n_0$ the system is not causal.

(e) $y(n) = e^{x(n)}$

Causal since it does not depend on future values of $x(n)$.

Definition

The response of a linear system to a unit pulse $\delta(n-m)$ is called the unit pulse response i.e.

$h(n,m) = T[\delta(n-m)]$ Response at nth instant to a unit pulse applied at the mth instant.

T: linear operator

If the system is linear shift invariant (LSI)

$$h(n,m) = h(n-m)$$

For causal LSI systems

$$h(n) = 0 \qquad n < 0$$

Representation of LSI Systems

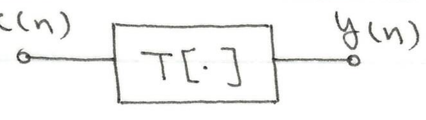
An LSI system can be described by one of the following methods of representation

1) Convolution Sum 2) Transfer Function

3) Difference Equations 4) State-Space Equations.

Convolution Sum

Let consider a linear discrete-time system given by



$$y(n) = T[x(n)]$$

Recall that $$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

Using this property and invoking linearity we have

$$y(n) = T\left[\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, T\left[\delta(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n,k)$$

If the system is LSI then

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \qquad \text{Convolution Sum}$$

$$= x(n) * h(n) \qquad \text{Convolution}$$

For Causal LSI Systems

$$y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k) = \sum_{k=-\infty}^{n} x(k)\, h(n-k)$$

As a result, in an LSI System the output is obtained by convolving the input with the unit pulse response if all the initial Conditions of the system are zero. Thus, $h(n)$ completely characterizes the I/O properties of the system.

Note that the Convolution Sum should not be thought of as an approximation to the Convolution integral. In contrast to the Convolution integral which plays a theoretical role in analog linear systems, we see that in addition to its theoretical importance, Convolution sum may serve for realization of discrete-time Systems.