Step (3) Realization

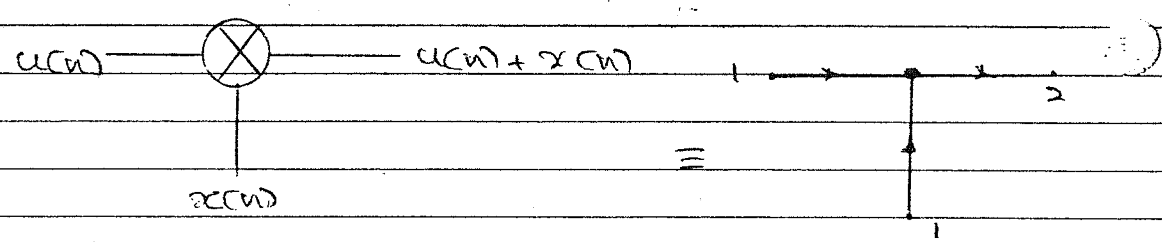
Process of converting a transfer function into filter networks

Different Realizations

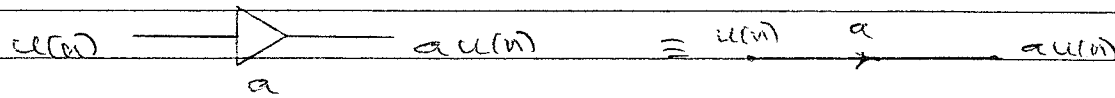
(1) Direct
(2) canonical form
(3) Cascade
(4) Parallel
(5) Ladder lattice
(6) Wave

Three building blocks that are used;

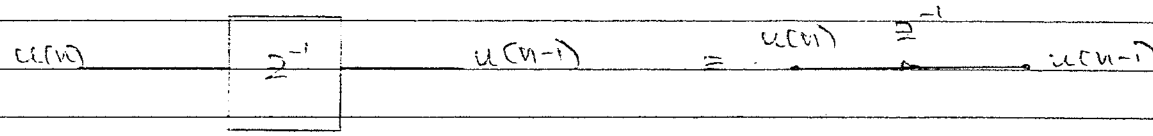
Adders:



Multipliers:



Delay:



Realization of FIR filters

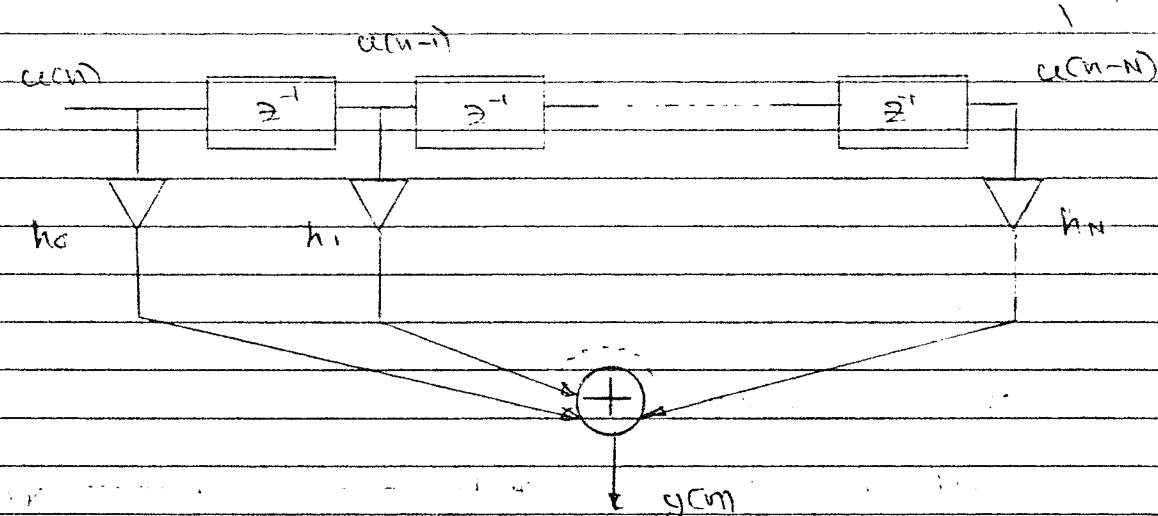
(a) Direct
(b) Cascade
(c) Frequency Sampling
(d) Lattice

Consider a FIR filter given by

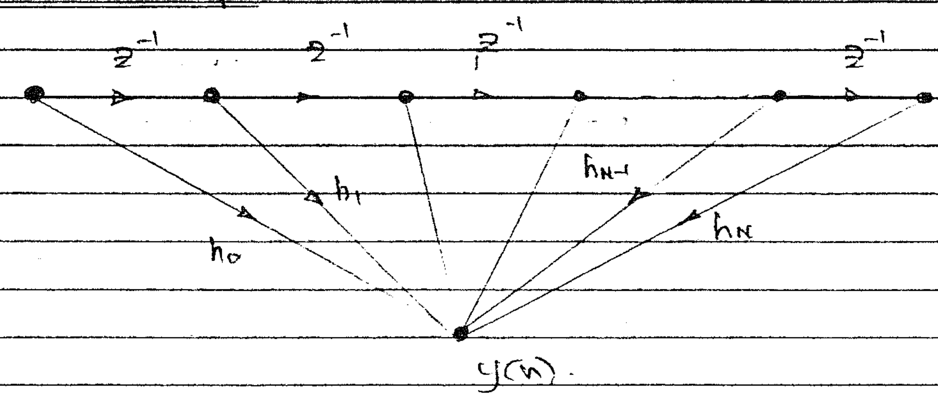
$$Y(z) = H(z)\, u(z)$$

$$H(z) = \sum_{n=0}^{N} h(n)\, z^{-n}$$

N - Delays
N+1 multipliers
1 Adder



Signal Flow Graph



cascade

$$H(z) = \prod_{k=1}^{K} H_k(z) \qquad K = \left[\frac{M+1}{2}\right]$$

where $H_K(z) = b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2}$

Realization of IIR Filters

(a) From the Transfer Function or Difference Equations

Consider an IIR filter with

$$H(z) = \frac{Y(z)}{U(z)} = \frac{\left(\sum_{i=0}^{M} b_i z^{-i}\right)}{\left(\sum_{i=1}^{N} a_i z^{-i}\right)}$$

$$y(n) = \sum_{i=0}^{M} b_i x(n-i) - \sum_{j=1}^{N} a_j y(n-j)$$

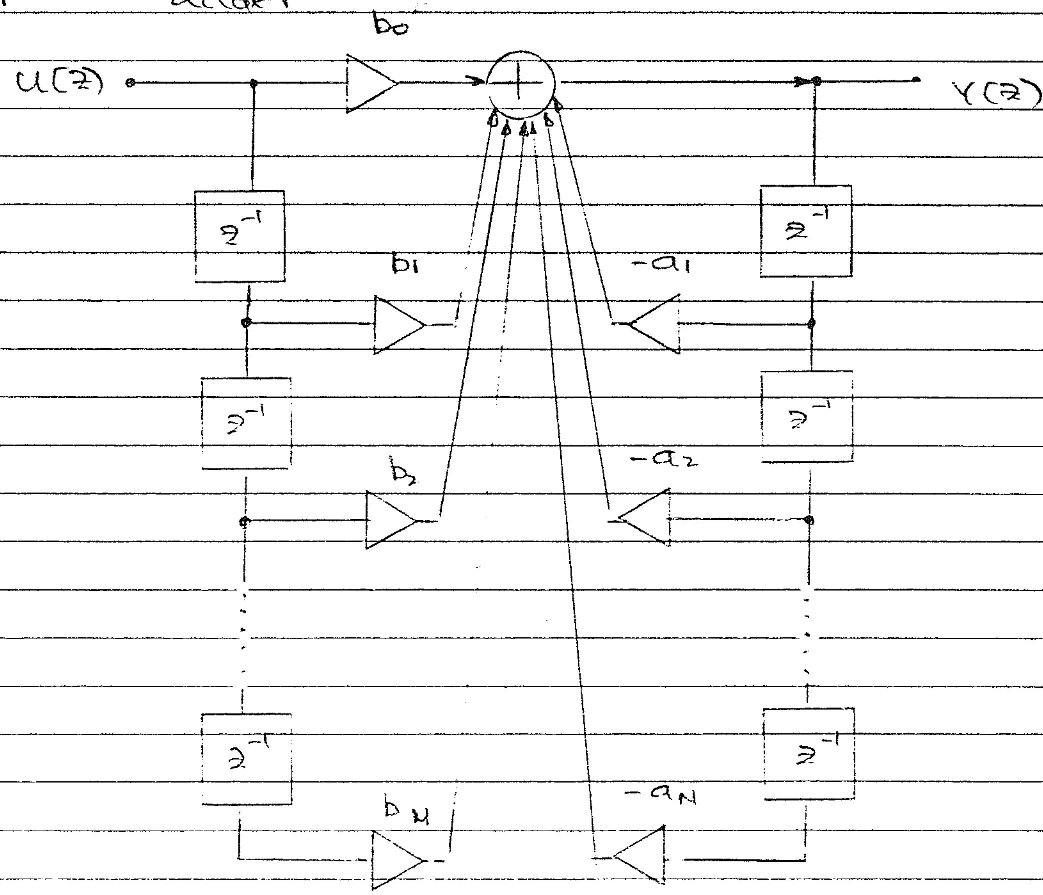
(i) Direct Realization:

$a_0 = 1$ (normalize)

$(M+N-1)$ multiplications

$(M+N)$ delay

1 adder



Direct Form

(2) Canonical Realization

$$H(z) = \frac{Y(z)}{U(z)} = \underbrace{\frac{1}{\left(\sum_{i=0}^{N} a_i z^{-i}\right)}}_{H_1(z)} \cdot \underbrace{\left(\sum_{i=0}^{M} b_i z^{-i}\right)}_{H_2(z)}$$

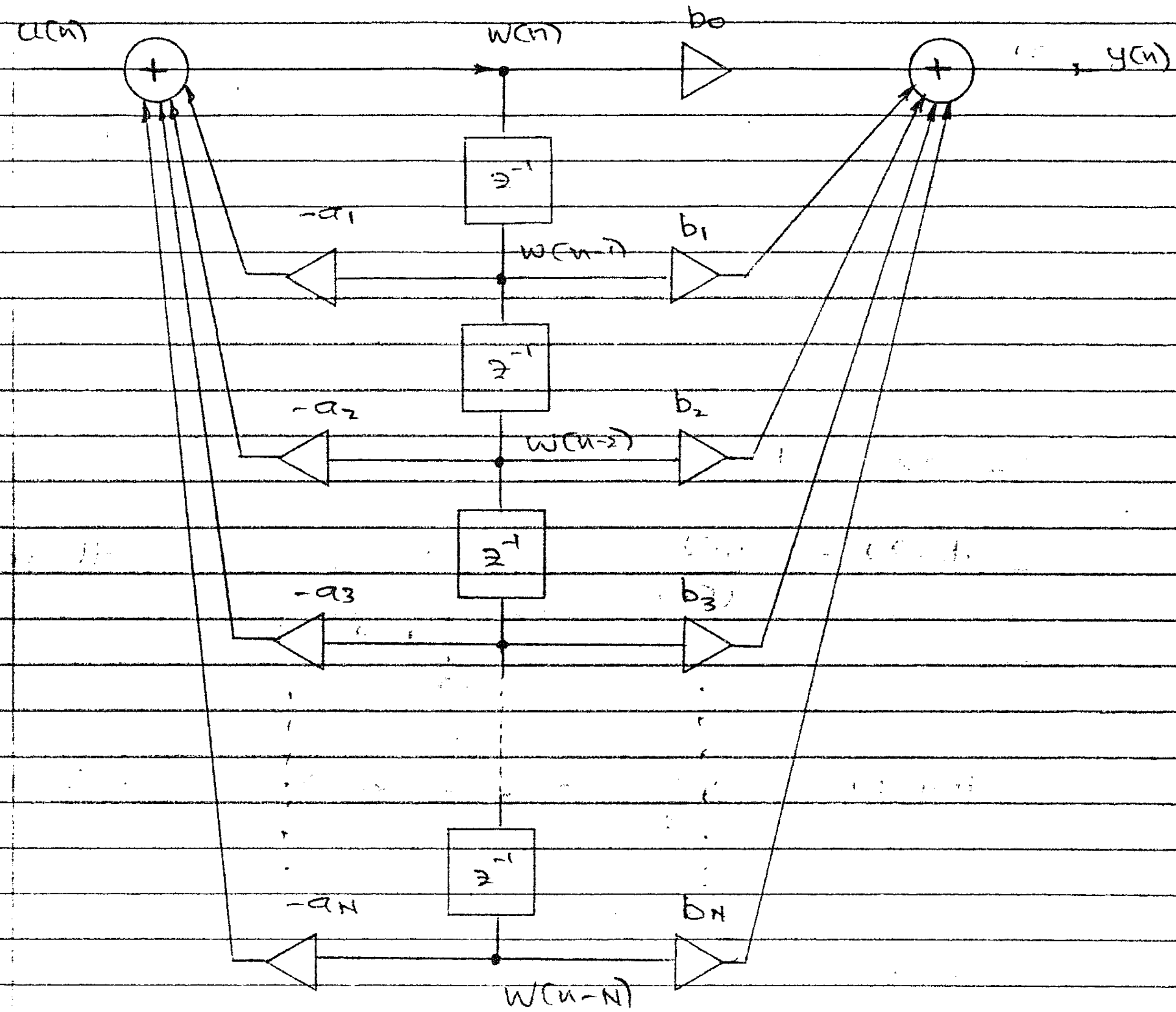
Assume $M = N$

$$H_1(z) = \frac{W(z)}{U(z)} = \frac{1}{\left(\sum_{i=0}^{N} a_i z^{-i}\right)}$$ all pole system

$$H_2(z) = \frac{Y(z)}{W(z)} = \sum_{i=0}^{M} b_0 z^{-i}$$ FIR system

$$W(n) = u(n) - \sum_{i=1}^{N} a_i W(n-i)$$

$$y(n) = \sum_{i=0}^{M} b_i W(n-i)$$



canonical form.

(3) <u>Cascade Realization:</u>

Applied to higher order recursive filters

$$H(z) = b_0 \prod_{i=1}^{L} H_i(z)$$

$$H_i(z) = \frac{1 + b_{1i} z^{-1}}{1 + a_{1i} z^{-1}} \qquad 1^{st} \text{ order block}$$

$$H_i(z) = \frac{1 + b_{1i} z^{-1} + b_{2i} z^{-2}}{1 + a_{1i} z^{-1} + a_{2i} z^{-2}} \qquad 2^{nd} \text{ order block}$$

$$L = \left[\frac{N+1}{2}\right]$$

$[\cdot]$: integer part

(4) <u>Parallel Realization</u>

Expand $H(z)$ using PFE

$$H(z) = A + \sum_{i=1}^{L} H_i(z)$$

$$H_i(z) = \frac{\gamma_i}{z - p_i}$$

If p_i is complex pair it up with p_i^*

$$H_i(z) = \frac{\gamma_i}{z - p_i} + \frac{\gamma_i^*}{z - p_i^*}$$

$$= \frac{z(\gamma_i + \gamma_i^*) + (\gamma_i p_i^* + \gamma_i^* p_i)}{z^2 - z(p_i + p_i^*) + p_i p_i^*}$$

or

$$H_i(z) = \frac{\beta_{1i} z^{-1} + \beta_{2i} z^{-2}}{1 + \alpha_{1i} z^{-1} + \alpha_{2i} z^{-2}}$$