$$d_{K} = \frac{(2n-K)!}{2^{n-K}(K!)(n-K)!}$$

$$B_{n}(s) \text{ Satisfies the following recursive equation}$$

$$B_{n}(s) = (2n-1)B_{n-1}(s) + s^{2}B_{n-2}(s)$$
with
$$B_{0}(s) = 1, \quad B_{1}(s) = s+1$$

(4) cut-off knepuncy voices with the order of the filter (disadvantage) $\omega_c = do$ $v_c = do$

IIR Digital Filter Design Méthods Endirect methods

1- Impulse Invariant Method:

Impulse response of the digital filter is the sampled version of the impulse response of the analogy filter has filter. Assume that the analog filter has

$$H_{A}(s) = \sum_{i=1}^{n} \frac{Ai}{(s+\alpha i)}$$

- di: distinct polls of HLS).

$$Ai = (s + \alpha i) H(s) |_{s = -\alpha i}$$

If all the polis are real and distinct then

$$h(t) = \mathcal{L}[H(s)] = \sum_{i=1}^{n} A_i e$$

The impulse response of the digital filter is

$$h_A(t)$$
 $t = KT = h_D(kT) = \sum_{i=1}^{N} A_i e^{-d_i kT}$

The transfer function HD(2) is

$$H_{D}(z) = \sum_{k=0}^{\infty} h_{D}(kT) z^{-k}$$

$$= \sum_{k=0}^{\infty} \sum_{i=1}^{n} \frac{-d_i k T}{2} = \sum_{i=1}^{n} \frac{-d_i T}{2} \left(\frac{-1}{2} - \frac{d_i T}{2} \right)$$

$$= \sum_{i=1}^{n} \frac{Ai}{(1-e^{\alpha iT} 2^{-1})}$$

Comparison with

$$H_A(s) = \frac{s}{s} \frac{Ai}{(s+\alpha i)}$$

gives the mapping as

$$H_D(z) = H_A(s) | (s+\alpha i) \rightarrow (1-ze^{-\alpha iT})$$

Thus $H_D(Z)$ can be obtained from $H_A(S)$ without enclosing $h_A(t)$ or $h_D(KT)$.

For Stable analog filter diso for Vie[1,n]

The corresponding digital filter has poles at

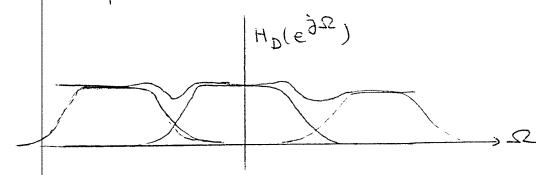
 $z = e^{-\alpha i T}$ with $1z | \langle 1 \rangle$ when $\alpha i > 0$ i.e. the digital filter is also fifter.

using the sampling, the frequency responses are related by

$$H_D(e^{\frac{1}{3}\Omega}) = \frac{1}{T} \sum_{n=-\infty}^{\infty} H_A(\Omega + 2\pi n) \Omega : \text{ Digital}$$

 $H_{D}(e^{\frac{1}{3}\Omega}) = \frac{1}{T} \sum_{n=-\infty}^{\infty} H_{A}(\Omega + 2\pi n) \Omega : \text{ Digital freq.}$ or Since $\omega T = \Omega$ $H_{D}(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} H_{A}(\omega + n\omega_{s})$ $If H_{A}(\omega) = 0 \quad \text{when} \quad |\omega| > \frac{\pi}{T} = \frac{\omega_{s}}{2} \quad \text{i.e. Boundlamited}$ then $H_{D}(e^{\frac{1}{3}\Omega}) = \frac{1}{T} H_{A}(\Omega) \quad \text{i.e. no aliensing}$

unfortunately $H_A(\omega)$ is not boundlimited in pronctical cases an consequentely aliasing problem is inevitable



As a result this method is appropriate only for Bandlimit LPF and HPF and BPF would require additional. bandlimiting to avoid severe aliasmy distortism.

$$\frac{2-e}{2-e} = \frac{-3\Omega}{-3\Omega}$$
Compar with $2=re$ = $r=e$ and

2- For Complex Conjugate poles one can alternatively use

$$\frac{S+\sigma}{S^2+2\sigma S+\sigma^2+\omega^2} \xrightarrow{\text{mapped}} \frac{1-\frac{1}{2}e^{-1}-\sigma T}{1-2\frac{1}{2}e^{-1}\cos\omega T+\frac{1}{2}e^{-2\sigma T}}$$

$$\frac{\omega}{S^2 + 2\sigma S + \sigma^2 + \omega^2} \rightarrow \frac{\overline{z}^2 e^{-\sigma T} S \dot{\omega} \omega T}{1 - 2\overline{z}^2 e^{-\sigma T} Cos\omega T + \overline{z}^2 e^{-2\sigma T}}$$

Example (LIPE)

Design a digital filter of Butterworth Type with

uning impulse invinant metural.

From the tables we can get the normalized transfer function

$$H_{n}(s) = \frac{1}{1+2s+2s^{2}+5^{3}} = \frac{1}{(s+1)(s^{2}+s+1)}$$

$$= \frac{A_1}{(S+1)} + \frac{A_2}{(S+\alpha)} + \frac{A_3}{(S+\alpha^*)}$$

$$\alpha = \frac{1}{2} \left(1 - \frac{1}{2} \sqrt{3} \right)$$

Donest Motheds (after Bolinia 2-) Least Square Innerse Design This method leads to a set of linear equations {ho(n)}: n∈[0, L-1] Desired Impulse Response (first L Samples) Filter transfer function H(5) = 1- Sak 3-K must find an's Generalization is proposed by Shanks and Burrus and Pontes The output of the innerse of HIZ) must approximate a unit Sample (S(n)) when the input is h(n). If v(n) denotes Let v(n): the output of innerse system with the ramifer function YH(Z) thin h_D(n) /H(z) V(n) $\Lambda(5) = \frac{H(5)}{H^{3}(5)}$ Thus we can write $b_0 N(n) = h_0(n) - \sum_{r=1}^{\infty} a_r h_0(n-r)$ Recall that we require N(n) = s(n) thus bo = ho(0) and that vin) be as small as possible for n)o. Choose the remaining coeff to minimize $E = \sum_{n=1}^{\infty} (N(n))^2$ $= \frac{1}{2} \sum_{n=1}^{\infty} (h_D(n))^2 - 2 \sum_{n=1}^{\infty} h_D(n) \sum_{n=1}^{\infty} \sum_{r=1}^{\infty} (h_r - r) + \sum_{n=1}^{\infty} \sum_{r=1}^{\infty} h_r + h_r - r) \prod_{n=1}^{\infty} \frac{1}{r}$

$$\frac{\partial E}{\partial a_i} = 0 \qquad i \in [1, N]$$

$$\sum_{r=1}^{N} \sum_{n=1}^{M} h_n(n-r) dh_n(n-r) = \sum_{n=1}^{M} h_n(n) h_n(n-r)$$

$$\text{Define the autocorrelation function}$$

$$\text{Qii,r} = \sum_{n=1}^{\infty} h_n(n-r) h_n(n-r)$$

$$\text{Then the above equation becomes a normal equation}$$

$$\sum_{r=1}^{N} a_r \text{Qii,r} = \text{Qii,0} \qquad i \in [1, N]$$

$$\text{Then equation of the equation}$$

These equations can be solved using GE or Lu factorization.

Toeplitz form The correlation matrix is Toeplitz and efficient procedure known as Levinson algorithm exist for the solution

$$a_{1}q(i,1) + - - + a_{N}q(i,N) = q(i,0)$$

$$\begin{array}{c|c}
 & (2,1) & (2,2) \\
\hline
 & (2,1) & (2,2) \\
\hline
 & (2,N) \\
\hline
 & (2,N)
\end{array}$$

$$\begin{array}{c|c}
 & (1,0) \\
\hline
 & (N,0)
\end{array}$$

the snow introduced E = Z (Rem), HZE we have to selects ai's such that E is winimum. [n, 1] = 5 V $E = \frac{1}{\sqrt{2}} \sum_{n=1}^{\infty} (\mu^{2}(n)) - 5 \sum_{n=1}^{\infty} \mu^{2}(n) \sum_{n=1}^{\infty} \alpha^{2} \mu^{2}(n-1)$ + Z Z ar procurs = -> \frac{2}{2} how how how = > > \frac{2}{2} how how how = > > \frac{2}{2} how how = > > \frac{2}{2} how = > \frac{2}{2} h then I der prizz = prize __ (*) ielinj We can comprise ai's. using

Direct Method This is direct design welhods for IER fillers. (1) Least Square Inverse Method Described in pulse regime

Let 2 horris: 1st L-point of the impulse vesponse ne [o, L-1] $H(2) = \frac{b_0}{1 - \sum_{i=1}^{N} a_i a_i^2} = \frac{\sum_{i=1}^{N} h(n) \frac{1}{2}^n}{k = 0}$ objective: find ab's ACM & ECM) = HD(2). [1 - 5 ak 2-12)] date IRT porces = poces = \frac{1}{2} = 1 \text{ at p p (n-p)} require that usin & son > bo = ho co) all oran , n>0 should be very small.

eg r=1

P(0,1) = \$\frac{1}{27}\$ ho (n-1) ho (n)

N=1

= ho (0). ho (1) + ho (1) ho (2) + ...

.... + hock-i). hock

system of himser en' can be solved simultaneously to set the values of ai's.

use Levinson Algorithm to solve the above.

*	The second secon		
	(2) L	inaar Programming Approach:	
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