

Quasi-geostrophic flow with tracers

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1 Preliminaries

More or less according to Vallis (2006), two-layer quasi-geostrophic flow is governed by the equations

$$\tilde{q}_{1t} + J(\tilde{\psi}_1, \tilde{q}_1) = F(\Delta_h \tilde{\psi}_1) \quad (1)$$

$$\tilde{q}_{2t} + J(\tilde{\psi}_2, \tilde{q}_2) = -r \Delta_h \tilde{\psi}_2 + F(\Delta_h \tilde{\psi}_2), \quad (2)$$

where the potential vorticities Q_i are defined by

$$\tilde{q}_1 = \Delta_h \tilde{\psi}_1 + F_1(\tilde{\psi}_1 - \tilde{\psi}_2) + \beta y, \quad (3)$$

$$\tilde{q}_2 = \Delta_h \tilde{\psi}_2 - F_2(\tilde{\psi}_1 - \tilde{\psi}_2) + \beta y, \quad (4)$$

and the Burger numbers F_1 and F_2 are defined by $F_i = f_0^2/g'H_i$, where H_i is the height of layer i and g' is the reduced gravity. The number of parameters in the problem are reduced if the deformation ‘radius’ R and layer depth ratio δ are defined through

$$R^2 \stackrel{\text{def}}{=} \frac{g'H_1H_2}{f_0^2(H_1 + H_2)}, \quad \text{and} \quad \delta \stackrel{\text{def}}{=} \frac{H_1}{H_2}, \quad (5)$$

so that

$$F_1 = \frac{R^{-2}}{1 + \delta}, \quad \text{and} \quad F_2 = \frac{\delta R^{-2}}{1 + \delta}. \quad (6)$$

We decompose $\tilde{\psi}_i$ into $\tilde{\psi}_i = -U_i y + \psi_i$, so that

$$\tilde{q}_1 = \underbrace{\beta y - F_1(U_1 - U_2)}_{\stackrel{\text{def}}{=} Q_1} + \underbrace{\Delta_h \psi_1 + F_1(\psi_1 - \psi_2)}_{\stackrel{\text{def}}{=} q_1}, \quad (7)$$

$$\tilde{q}_2 = \underbrace{\beta y + F_2(U_1 - U_2)}_{\stackrel{\text{def}}{=} Q_2} + \underbrace{\Delta_h \psi_2 - F_2(\psi_1 - \psi_2)}_{\stackrel{\text{def}}{=} q_2}, \quad (8)$$

The potential vorticity conservation equations become

$$q_{1t} + J(\psi_1, q_1) + U_1 q_{1x} + \psi_{1x} Q_{1y} = F(\Delta_h \psi_1), \quad (9)$$

$$q_{2t} + J(\psi_2, q_2) + U_2 q_{2x} + \psi_{2x} Q_{2y} = -r \Delta_h \psi_2 + F(\Delta_h \psi_2). \quad (10)$$

2 Linear stability analysis

3 Numerics

When we compute the right hand side, we either dealias or use an exponential filter of the form

$$\mathcal{F} = \exp \left\{ -d \left[(k/k_c)^2 + (\ell/\ell_c)^2 - 1 \right]^{n/2} \right\}. \quad (11)$$