1. **A Simple Real Business Cycle Model** The planner's problem in the simple RBC is given by,

$$\max_{c_t,\, n_t,\, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \psi \frac{n_t^{1+\phi}}{1+\phi} \right)$$

$$s.t. \quad k_{t+1} = a_t k_t^{\alpha} n_t^{1-\alpha} - c_t + (1+\delta)\, k_t$$

$$k_0 \, given; \; \psi > 0; \; \phi > 0$$

a_t follows a stationary, mean zero AR(1) in log:

$$\ln a_t = \rho \ln a_{t-1} + \epsilon_t, \quad 0 < \rho < 1, \; \epsilon_t \backsim N\left(0, \, \sigma^2\right)$$

(a) The Lagrangian and the first order conditions are:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \psi \frac{n_t^{1+\phi}}{1+\phi} \right) - \lambda_t \left(k_{t+1} - a_t k_t^{\alpha} n_t^{1-\alpha} + c_t - (1+\delta)\, k_t \right)$$

F.O.C.

$$\frac{1}{c_t} = E_t \left[\frac{\beta}{c_{t+1}} \left(\alpha k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + (1-\delta) \right) \right]$$

$$\psi n_t^{\phi} = \frac{1}{c_t} (1-\alpha)\, a_t k_t^{\alpha} n_t^{-\alpha}$$

(b) Non-stochastic steady states,

$$\frac{k}{n} = \left[\frac{\alpha}{\frac{1}{\beta} - (1-\delta)} \right]^{\frac{1}{1-\alpha}}$$

$$\frac{y}{n} = \left(\frac{k}{n} \right)^{\alpha} = \left[\frac{\alpha}{\frac{1}{\beta} - (1-\delta)} \right]^{\frac{\alpha}{1-\alpha}}$$

$$\frac{c}{n} = \left(\frac{k}{n} \right)^{\alpha} - \delta \left(\frac{k}{n} \right)$$

$$n = \left[\frac{1}{\psi} \left[\frac{(1-\alpha) \left(\frac{k}{n} \right)^{\alpha}}{\left(\frac{k}{n} \right)^{\alpha} - \delta \left(\frac{k}{n} \right)} \right] \right]^{\frac{1}{1+\phi}}$$

(c) To solve the policy function I follow Blanchard-Kahn's solution procedure. The log-linearized characteristic equations are:

$$\tilde{n}_t = \frac{1}{\phi + \alpha} \left[\tilde{a}_t + \alpha \tilde{k}_t - \tilde{c}_t \right]$$

$$-\tilde{c}_t = -\left(1 + \frac{\alpha\beta(1-\alpha)}{\phi+\alpha} \left(\frac{k}{n} \right)^{\alpha-1} \right) \tilde{c}_{t+1} + \left(\alpha\beta \left(\frac{k}{n} \right)^{\alpha-1} \left(\frac{1+\phi}{\phi+\alpha} \right) \right) \tilde{a}_{t+1}$$
$$- \left(\alpha\beta(1-\alpha) * \left(\frac{k}{n} \right)^{\alpha-1} \left(\frac{\phi}{\phi+\alpha} \right) \right) \tilde{k}_{t+1}$$

$$\tilde{k}_{t+1} = \left(\frac{1}{\beta} + \alpha \left(\frac{1-\alpha}{\phi+\alpha} \right) \left(\left(\frac{k}{n} \right)^{\alpha-1} \right) \right) \tilde{k}_t + \left(\left(\frac{k}{n} \right)^{\alpha-1} \left(\frac{1+\phi}{\phi+\alpha} \right) \right) \tilde{a}_t - \left(\left(\frac{c}{k} \right) + \left(\frac{1-\alpha}{\phi+\alpha} \right) \left(\left(\frac{k}{n} \right)^{\alpha-1} \right) \right) \tilde{c}_t$$

$$\tilde{a}_{t+1} = \rho \tilde{a}_t + \epsilon_t$$

From here, I form the respective matrices and get a policy function Φ such that,

$$\tilde{c}_t = \Phi \begin{bmatrix} \tilde{k}_t \\ \tilde{a}_t \end{bmatrix}$$

(d) Figure (1) shows different IRFs of the model when varying the values of ϕ. Since, ϕ implicitly determines the value of the Frisch elasticity, low values of ϕ imply large values of such elasticity and therefore, a more elastic labor supply. In consequence, we observe given a productivity shock, when ϕ is closer to zero, consumption reacts way more than in the cases where ϕ is higher. As ϕ decreases, the amplification in the model increases. In other words, smaller values of ϕ yield to larger fluctuations in output given an exogenous shock in productivity.

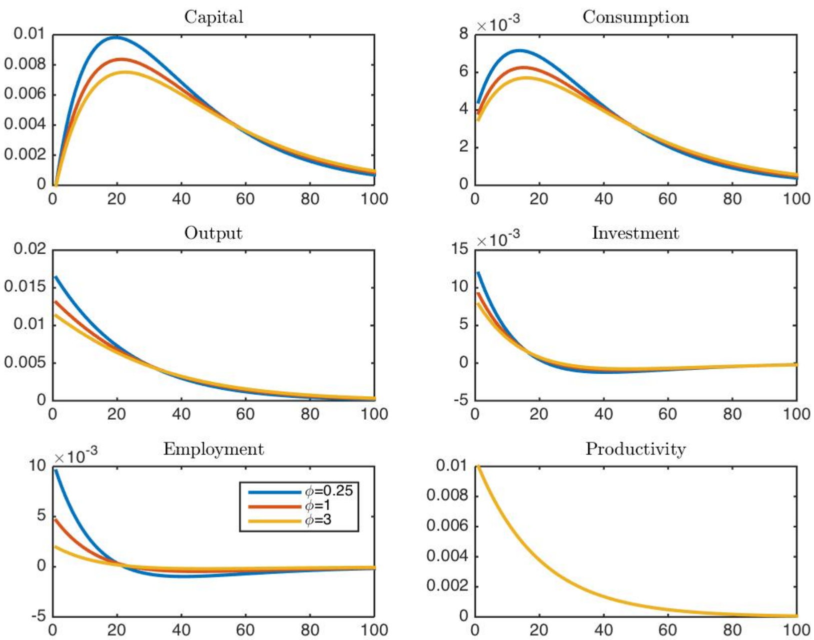

Figure 1: IRFs of basic RBC with different values of Frisch Elasticity of Labor Supply.

(e) Figure(2) shows different IRFs when changing the persistence of the productivity shock. As ρ increases, the more consumption reacts to the TFP shock. We see that when the persistence of the shock is really low, there is almost no reaction in consumption. Employment and investment react less when the shock is more persistent, but more capital is accumulated in the long run.

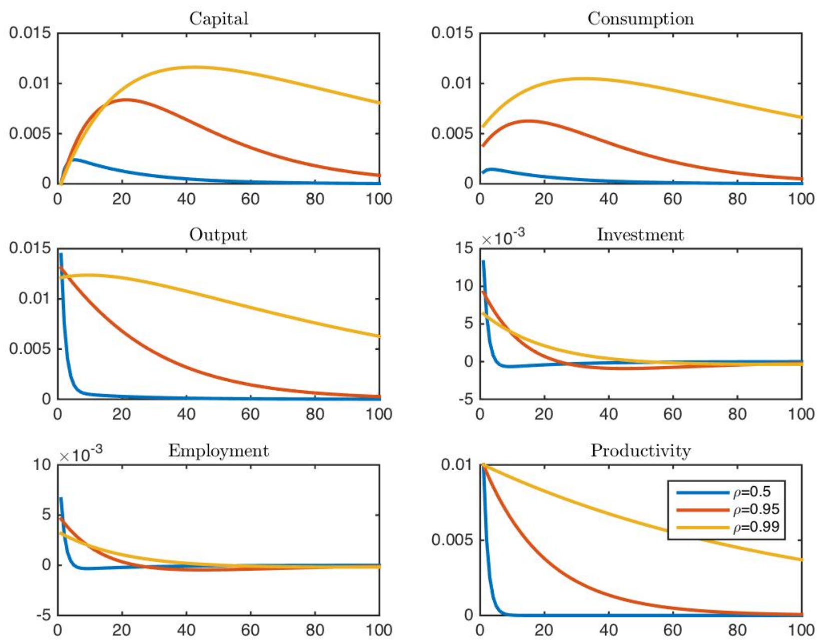

Figure 2: Impulse Responses to shocks with different persistence.